

Nonlinear $\mathcal{N} = 2$ global supersymmetry

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ABSTRACT: We study the partial breaking of $\mathcal{N} = 2$ global supersymmetry, using a novel formalism that allows for the off-shell nonlinear realization of the broken supersymmetry, extending previous results scattered in the literature. We focus on the Goldstone degrees of freedom of a massive $\mathcal{N} = 1$ gravitino multiplet which are described by deformed $\mathcal{N} = 2$ vector and single-tensor superfields satisfying nilpotent constraints. We derive the corresponding actions and study the interactions of the superfields involved, as well as constraints describing incomplete $\mathcal{N} = 2$ matter multiplets of non-linear supersymmetry (vectors and single-tensors).

KEYWORDS: Extended Supersymmetry, Supersymmetry Breaking

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Contents

1	Introduction	2
2	Partial supersymmetry breaking with one hypermultiplet	4
2.1	Single-tensor multiplet formulation	5
2.1.1	Alternative proof	6
2.1.2	The vacuum	7
2.2	Dual hypermultiplet formulation	9
2.3	Several single-tensor multiplets	11
3	Nonlinear deformations	11
3.1	Deformations of the Maxwell superfield	11
3.2	Deformations of the single-tensor superfield	13
4	Constrained multiplets	14
4.1	The infinite-mass limit	15
4.2	Solutions of the constraints	16
5	The “long” super-Maxwell superfield	18
5.1	The chiral-antichiral $\mathcal{N} = 2$ superfield	19
5.2	The long and short super-Maxwell superfields	20
5.3	Long superfield and nonlinear deformations	21
6	Interactions	21
6.1	The Chern-Simons interaction	21
6.1.1	The Chern-Simons interaction with deformed Maxwell multiplet	22
6.1.2	The Chern-Simons interaction with deformed single-tensor multiplet	23
6.2	Constrained matter multiplets	25
6.2.1	The goldstino in the Maxwell multiplet	26
6.2.2	The goldstino in the single-tensor multiplet	28
7	Conclusions	30
A	Conventions and some useful identities	30
B	More on the Maxwell supermultiplet	31
C	More on $\text{Im } \mathbb{V}_\mu$	34

1 Introduction

The spontaneous breaking of global symmetries is described at low energies by a nonlinear σ -model of the corresponding Goldstone modes which have nonlinear transformations. These can often be obtained by applying an appropriate constraint on a linear σ -model. In the case of supersymmetry, the Goldstone modes are fermions, the goldstini, and the nonlinear σ -model for $\mathcal{N} = 1$ is the Volkov-Akulov action [1]. In analogy with ordinary symmetries, it can be obtained (up to field redefinitions) by a chiral superfield X satisfying a nilpotent constraint $X^2 = 0$ which eliminates its scalar component (sgoldstino) in terms of the goldstino bilinear [2–5]:

$$X = -\frac{\kappa\kappa}{2F} + \sqrt{2}\theta\kappa - \theta^2 F, \tag{1.1}$$

where κ is the two-component Goldstone fermion, θ the usual fermionic coordinates and F the (nonzero) auxiliary field. The most general Kähler potential is then quadratic $K = X\bar{X}$ and the superpotential linear in X , $P = \zeta X$, with a proportionality constant ζ fixing the scale of the supersymmetry breaking. Indeed, solving for F , one finds $F = \zeta + \text{fermions}$ and one obtains (on-shell) the Volkov-Akulov action [2, 6].

Besides the use of nonlinear supersymmetry as an effective low-energy theory at energies below the sgoldstino mass, it can also be realized exactly in particular vacua of type I string theory, when D-branes are combined with anti-orientifold planes that break the linear supersymmetries preserved by the D-branes, while they preserve the other half that are realized nonlinearly. In such vacua of “brane supersymmetry breaking”, superpartners of brane excitations do not exist, and supersymmetry is nonlinearly realized with the presence of a massless goldstino in the open string spectrum [7–11].

The generalization of these results to extended supersymmetry, in particular to $\mathcal{N} = 2$, broken at two different scales, is a challenging and not straightforward problem. An interesting case is $\mathcal{N} = 2$ with one linear and one nonlinear supersymmetry, which is the standard situation of D-branes in a $\mathcal{N} = 2$ supersymmetric bulk and describes the low-energy limit of partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking. The goldstino of the nonlinear supersymmetry should then belong to a multiplet of the $\mathcal{N} = 1$ linear supersymmetry, which can be either a vector or a chiral multiplet. In fact, both cases have to be studied, since they constitute the Goldstone degrees of freedom of a massive spin-3/2 multiplet. Indeed, a massless spin-3/2 multiplet contains a gravitino and a graviphoton, while a massive one contains, in addition, a spin-1 and a (Majorana) spinor, so that the Goldstone modes are a vector, two 2-component spinors and two scalars [12].

When the second and nonlinear supersymmetry is taken into account, the above two $\mathcal{N} = 1$ multiplets should be described by constrained $\mathcal{N} = 2$ superfields associated with a Maxwell multiplet and a hypermultiplet. The latter comes with an extra complication since it has no off-shell formulation in the standard $\mathcal{N} = 2$ superspace. Fortunately, the presence of bosonic shift symmetries associated with the would-be Goldstone bosons providing the longitudinal components of the spin-1 fields, implies that the chiral multiplet can be dualized to a linear multiplet having an off-shell description when promoted to a (constrained) $\mathcal{N} = 2$ single-tensor superfield.

In this work we analyze the partial breaking of global $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry [13], extending known results in the literature on Maxwell multiplets [13–17] and single-tensor multiplets [18, 19], we derive the corresponding $\mathcal{N} = 2$ constrained superfields and study their possible interactions. The easiest way to introduce a breaking of $\mathcal{N} = 2$ supersymmetry is by a (constant) deformation of the supersymmetry transformations of the fermions that cannot be absorbed in expectation values of the auxiliary fields, unlike the $\mathcal{N} = 1$ case [16]. Partial breaking arises when the deformation parameters satisfy particular relations, guaranteeing the existence of one goldstino associated with a linear combination of the two supersymmetries. The goldstino superfield of one nonlinear supersymmetry can then be obtained by imposing a nilpotent (double chiral) constraint, in analogy with $X^2 = 0$ of $\mathcal{N} = 1$.

The outline of this paper is the following. In section 2, we present a model of spontaneous partial breaking of $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry using one single-tensor multiplet, which contains a $\mathcal{N} = 1$ linear multiplet L and one chiral multiplet. The theory admits a special superpotential that allows for partial supersymmetry breaking, in analogy with the magnetic Fayet-Iliopoulos (FI) term in the Maxwell multiplet model of [13]. This correspondence exchanges the $\mathcal{N} = 1$ chiral field-strength superfield of the $\mathcal{N} = 2$ Maxwell multiplet with the antichiral superfield $D_\alpha L$. Thus, the $\mathcal{N} = 2$ Maxwell superfield is chiral under both supersymmetries (CC), while the single-tensor superfield is chiral under the first and antichiral under the second (CA). In section 3, we discuss nonlinear deformations of the $\mathcal{N} = 2$ Maxwell and single-tensor superfields, write the most general actions and compute the scalar potentials that have $\mathcal{N} = 1$ supersymmetric minima. In section 4, we consider the infinite-mass limit that freezes half of the degrees of freedom, and derive the constrained multiplets and the corresponding nilpotent constraints. We then give the solutions of the constraints (off-shell) and derive the generalizations of the goldstino Volkov-Akulov action in the presence of a linear supersymmetry, in addition to the nonlinear one. These are the supersymmetric Dirac-Born-Infeld (DBI) action and a similar action for the linear multiplet, in agreement with previous results. We then turn to the study of interactions. To this end, we introduce in section 5 “long” $\mathcal{N} = 2$ superfields for the Maxwell and single-tensor multiplets with opposite relative chiralities compared to the “short” ones, namely CA for the Maxwell and CC for the single-tensor, so that one can write a Chern-Simons type of interaction that we discuss in section 6. This interaction leads to a super-Brout-Englert-Higgs mechanism without gravity, in which the linear multiplet is absorbed by the vector which becomes massive [19]. In section 6, we also study more general constraints that describe incomplete $\mathcal{N} = 2$ matter multiplets of non-linear supersymmetry (vectors or single-tensors), half of the components of which are projected out. Finally, section 7 contains concluding remarks and open problems, while there are three appendices with our conventions (appendix A) and the technical details of the Maxwell multiplet (appendices B and C).

In the following, $\mathcal{W}, \mathcal{Z}, \mathcal{Y} \dots$ denote $\mathcal{N} = 2$ superfields with $8_B + 8_F$ components, while hatted superfields $\widehat{\mathcal{W}}, \widehat{\mathcal{Z}} \dots$ have $16_B + 16_F$ fields. They are chiral with respect to the first supersymmetry (which shifts Grassmann coordinates θ_α) and either chiral or antichiral under the second supersymmetry (shifting $\tilde{\theta}_\alpha$). All other superfields are $\mathcal{N} = 1$ superfields.

2 Partial supersymmetry breaking with one hypermultiplet

In this section we show the existence of partial supersymmetry breaking in a large class of $\mathcal{N} = 2$ theories with a single hypermultiplet. The hypermultiplet couplings have a (translational) isometry allowing for a description in terms of a dual single-tensor multiplet which admits, like the Maxwell multiplet, a fully off-shell formulation. We use this formulation to obtain these theories, dualize back to the hypermultiplet formulation and then display the strong similarity between partial breaking with a Maxwell (namely the APT model [13]) and partial breaking with a single-tensor multiplet.

The single-tensor $\mathcal{N} = 2$ multiplet [20–23] describes an antisymmetric tensor with gauge symmetry

$$\delta B_{\mu\nu} = 2 \partial_{[\mu} \Lambda_{\nu]}, \tag{2.1}$$

three real scalar fields and two Weyl (or massless Majorana) spinors. In the same manner that an antisymmetric tensor is dual to a pseudoscalar with axionic shift symmetry, a single-tensor multiplet is equivalent to a hypermultiplet with shift symmetry. In both cases, the symmetry implies masslessness. In analogy with the Yang-Mills or Maxwell multiplet but in contrast with the hypermultiplet, the single-tensor multiplet admits an off-shell formulation.

In terms of $\mathcal{N} = 1$ superfields, the single-tensor multiplet has two descriptions which can be viewed as the supersymmetrization either of the gauge invariant three-form field strength

$$H_{\mu\nu\rho} = 3 \partial_{[\mu} B_{\nu\rho]} \tag{2.2}$$

or of a two-form potential $B_{\mu\nu}$ and of its gauge transformation. The first description [21] associates a real linear superfield L , $\overline{DD}L = 0$, which includes $H_{\mu\nu\rho}$, with a chiral superfield Φ , $\overline{D}_{\dot{\alpha}}\Phi = 0$, for a total of $8_B + 8_F$ off-shell fields. The second supersymmetry variations δ^* can be written as

$$\delta^* L = -\frac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta} \overline{D}\overline{\Phi}), \quad \delta^* \Phi = \sqrt{2}i \eta \overline{D}L, \quad \delta^* \overline{\Phi} = \sqrt{2}i \eta DL, \tag{2.3}$$

where η_α is the spinor parameter of the second supersymmetry. Since the linearity condition $\overline{DD}L = 0$ is solved by

$$L = D^\alpha \chi_\alpha - \overline{D}_{\dot{\alpha}} \overline{\chi}^{\dot{\alpha}} \equiv D\chi - \overline{D}\overline{\chi}, \tag{2.4}$$

where the chiral spinor superfield χ_α includes $B_{\mu\nu}$, there is a second description with two chiral superfields Φ and Y associated with χ_α , for a total of $16_B + 16_F$ fields.¹ The variations are [19]

$$\begin{aligned} \delta^* Y &= \sqrt{2} \eta \chi, \\ \delta^* \chi_\alpha &= -\frac{i}{\sqrt{2}} \Phi \eta_\alpha - \frac{\sqrt{2}}{4} \eta_\alpha \overline{DD}\overline{Y} - \sqrt{2}i(\sigma^\mu \overline{\eta})_\alpha \partial_\mu Y, \\ \delta^* \Phi &= 2\sqrt{2}i \left[\frac{1}{4} \overline{DD}\eta\chi + i\partial_\mu \chi \sigma^\mu \overline{\eta} \right]. \end{aligned} \tag{2.5}$$

¹The superfield Φ appears in both descriptions.

They close the $\mathcal{N} = 2$ superalgebra off-shell. The supersymmetric extension of the gauge symmetry (2.1) is then

$$\delta_{\text{gauge}}\chi_\alpha = -\frac{i}{4}\overline{D}D_\alpha\widehat{V}_2, \quad \delta_{\text{gauge}}Y = \frac{1}{2}\overline{D}D\widehat{V}_1, \quad \delta_{\text{gauge}}\Phi = 0, \quad (2.6)$$

with \widehat{V}_1 and \widehat{V}_2 real: the gauge transformation of the single-tensor multiplet in the description (χ_α, Φ, Y) is generated by a $\mathcal{N} = 2$ Maxwell multiplet, which removes $8_B + 8_F$ fields. There is a gauge with $Y = 0$, residual $\mathcal{N} = 1$ supersymmetry and gauge invariance generated by \widehat{V}_2 .

The kinetic $\mathcal{N} = 2$ lagrangian in the description (L, Φ) takes the simple form [21]

$$\mathcal{L}_{\text{kin.}} = \int d^2\theta d^2\bar{\theta} \mathcal{H}(L, \Phi, \bar{\Phi}), \quad (2.7)$$

where \mathcal{H} is any real function solving the three-dimensional Laplace equation

$$\frac{\partial^2 \mathcal{H}}{\partial L^2} + 2 \frac{\partial^2 \mathcal{H}}{\partial \Phi \partial \bar{\Phi}} = 0. \quad (2.8)$$

A unique superpotential $\tilde{m}^2\Phi$ is allowed, since, under the second supersymmetry,

$$\delta^* \int d^2\theta (\tilde{m}^2\Phi) = \sqrt{2}i \tilde{m}^2 \int d^2\theta \overline{\eta}D L \quad (2.9)$$

which is a derivative. For the real linear superfield L , $\overline{D}_\alpha L$ is a chiral superfield with expansion

$$\overline{D}_\alpha L = i\overline{\varphi}_\alpha - (\theta\sigma^\mu)_{\dot{\alpha}}(v_\mu + i\partial_\mu C) - \theta\theta(\partial_\mu\varphi\sigma^\mu)_{\dot{\alpha}}, \quad v_\mu = \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma} \quad (2.10)$$

(in chiral coordinates), where the real scalar C is the lowest component of L . Note also that the superpartner of L (under the second supersymmetry) is

$$\Phi = z + \sqrt{2}\theta\psi - \theta\theta f. \quad (2.11)$$

2.1 Single-tensor multiplet formulation

To derive a theory with partial supersymmetry breaking, we first consider a generic $\mathcal{N} = 1$ chiral function $W(\Phi)$, with second supersymmetry variation

$$\delta^* \int d^2\theta W(\Phi) = \sqrt{2}i \int d^2\theta W_\Phi \overline{\eta}D L, \quad W_\Phi = \frac{dW}{d\Phi}. \quad (2.12)$$

It is not a derivative unless $W(\Phi) \sim \Phi$. Since²

$$\overline{D}D(\overline{\theta}\eta L) = -2\overline{\eta}D L = \overline{D}D(\overline{\theta}\eta L + \theta\eta L), \quad (2.13)$$

the variation can also be written as³

$$\delta^* \int d^2\theta W(\Phi) + \text{h.c.} = 2\sqrt{2}i \int d^2\theta d^2\bar{\theta} [W_\Phi - \overline{W}_{\overline{\Phi}}](\eta\theta + \overline{\eta}\bar{\theta})L. \quad (2.14)$$

²These equalities respect the first supersymmetry (which shifts θ and $\bar{\theta}$).

³We usually omit derivatives when comparing lagrangian terms.

Consider now the function

$$\mathcal{H}(L, \Phi, \bar{\Phi}) = i \left[-L^2 [W_\Phi - \bar{W}_{\bar{\Phi}}] + \bar{\Phi} W - \Phi \bar{W} \right], \quad (2.15)$$

which is obviously a solution of the Laplace equation, while the action corresponding to

$$\begin{aligned} \mathcal{L} &= i \int d^2\theta d^2\bar{\theta} \left[-L^2 [W_\Phi - \bar{W}_{\bar{\Phi}}] + \bar{\Phi} W - \Phi \bar{W} \right] + \int d^2\theta (\tilde{m}^2 \Phi) + \text{h.c.} \\ &= \int d^2\theta \left[\frac{i}{2} W_\Phi (\bar{D}L)(\bar{D}L) - \frac{i}{4} W \bar{D}\bar{D}\bar{\Phi} + \tilde{m}^2 \Phi \right] + \text{h.c.} \end{aligned} \quad (2.16)$$

is invariant under linear (off-shell) $\mathcal{N} = 2$ supersymmetry.

To break spontaneously the second supersymmetry, we first add the generic superpotential $\widetilde{M}^2 W(\Phi)$ to (2.16):

$$\begin{aligned} \mathcal{L}_{\text{nl}} &= \int d^2\theta \left[\frac{i}{2} W_\Phi (\bar{D}L)(\bar{D}L) - \frac{i}{4} W \bar{D}\bar{D}\bar{\Phi} + \tilde{m}^2 \Phi + \widetilde{M}^2 W \right] + \text{h.c.} \\ &= i \int d^2\theta d^2\bar{\theta} \left[-L^2 (W_\Phi - \bar{W}_{\bar{\Phi}}) + \bar{\Phi} W - \Phi \bar{W} \right] + \int d^2\theta \left[\tilde{m}^2 \Phi + \widetilde{M}^2 W \right] + \text{h.c.} \end{aligned} \quad (2.17)$$

The action corresponding to (2.17) is then invariant under linear $\mathcal{N} = 1$ supersymmetry as well as under the nonlinearly deformed second supersymmetry transformations

$$\begin{aligned} \delta^* L &= \delta_{\text{nl}}^* L - \frac{i}{\sqrt{2}} (\eta D\Phi + \bar{\eta} \bar{D}\bar{\Phi}), & \delta_{\text{nl}}^* L &= \sqrt{2} \widetilde{M}^2 (\bar{\theta}\eta + \theta\bar{\eta}), \\ \delta_{\text{nl}}^* \bar{D}_\alpha L &= -\sqrt{2} \widetilde{M}^2 \bar{\eta}_{\dot{\alpha}}, \end{aligned} \quad (2.18)$$

with $\delta^* \Phi$ unchanged, since

$$\delta_{\text{nl}}^* \mathcal{L}_{\text{kin.}} = -i\sqrt{2} \widetilde{M}^2 \int d^2\theta W_\Phi \bar{\eta} \bar{D}L + \text{h.c.} = -\widetilde{M}^2 \delta^* \int d^2\theta W(\Phi) + \text{h.c.} \quad (2.19)$$

\mathcal{L}_{nl} depends on two complex numbers, the deformation parameter \widetilde{M}^2 and the quantity \tilde{m}^2 in the linear $\mathcal{N} = 2$ superpotential. Note also that the deformation in (2.18) implies that the spinor $\bar{\varphi}_{\dot{\alpha}}$ in the expansion (2.10) of $\bar{D}_\alpha L$ transforms like a goldstino. In fact, the transformations (2.18) for the $\mathcal{N} = 1$ linear multiplet were first found in [18] by performing a chirality switch on the transformations of the $\mathcal{N} = 1$ Maxwell multiplet, first given in [14].

2.1.1 Alternative proof

Let us consider the $\mathcal{N} = 2$ supersymmetric lagrangian (2.7). Suppose that, to induce the partial breaking, we deform the second supersymmetry transformations of the single-tensor multiplet, in such a way that the spinor $\bar{\varphi}_{\dot{\alpha}}$ in the expansion of $\bar{D}_\alpha L$ transforms like a goldstino; the transformations take then the form (2.18). The deformation induces a new term in the variation of the lagrangian under the second supersymmetry:

$$\delta_{\text{def}}^* \mathcal{L}_{\text{kin.}} = \sqrt{2} \widetilde{M}^2 \int d^2\theta d^2\bar{\theta} \mathcal{H}_L(\theta\eta + \bar{\theta}\bar{\eta}), \quad (2.20)$$

where $\mathcal{H}_L = \frac{\partial \mathcal{H}}{\partial L}$ and \mathcal{H} satisfies the Laplace equation in the limit $\widetilde{M}^2 \rightarrow 0$. The expression (2.20) selects the $\theta\theta\bar{\theta}$ and $\bar{\theta}\bar{\theta}\theta$ components of \mathcal{H}_L . To obtain partial breaking, these components must transform as derivatives under the first, unbroken supersymmetry. This is the case if the highest component of \mathcal{H}_L is zero or a derivative,

$$\int d^2\theta d^2\bar{\theta} \mathcal{H}_L = \text{derivative}, \quad (2.21)$$

whose solution is

$$\mathcal{H}_L = \widetilde{\mathcal{G}}(\Phi) + \overline{\widetilde{\mathcal{G}}}(\bar{\Phi}) - 2L \left(\mathcal{G}_{\Phi\Phi}(\Phi) + \overline{\mathcal{G}}_{\bar{\Phi}\bar{\Phi}}(\bar{\Phi}) \right) \quad (2.22)$$

where $\mathcal{G}, \widetilde{\mathcal{G}}$ are holomorphic functions of Φ and $\mathcal{G}_{\Phi} = \frac{d}{d\Phi} \mathcal{G}(\Phi)$ (we use the derivatives merely for convenience). The prefactor -2 of L terms is conventional. Consequently,

$$\mathcal{H} = K(\Phi, \bar{\Phi}) + L \left(\widetilde{\mathcal{G}}(\Phi) + \overline{\widetilde{\mathcal{G}}}(\bar{\Phi}) \right) - L^2 \left(\mathcal{G}_{\Phi\Phi}(\Phi) + \overline{\mathcal{G}}_{\bar{\Phi}\bar{\Phi}}(\bar{\Phi}) \right), \quad (2.23)$$

where $K(\Phi, \bar{\Phi})$ is a function of $\Phi, \bar{\Phi}$ and, using the Laplace equation, we obtain

$$\mathcal{H} = \left(\bar{\Phi} \mathcal{G}_{\Phi}(\Phi) + \Phi \overline{\mathcal{G}}_{\bar{\Phi}}(\bar{\Phi}) \right) - L^2 \left(\mathcal{G}_{\Phi\Phi}(\Phi) + \overline{\mathcal{G}}_{\bar{\Phi}\bar{\Phi}}(\bar{\Phi}) \right), \quad (2.24)$$

since terms linear in L do not contribute to the integral $\int d^2\theta d^2\bar{\theta}$.

Now let us consider again the deformation (2.20) of the lagrangian. With the use of (2.24), it becomes (since terms proportional to L^0 do not contribute):

$$\begin{aligned} \delta_{\text{def}}^* \mathcal{L}_{\text{kin.}} &= -2\sqrt{2}\widetilde{M}^2 \int d^2\theta d^2\bar{\theta} L \left(\mathcal{G}_{\Phi\Phi}(\Phi) + \overline{\mathcal{G}}_{\bar{\Phi}\bar{\Phi}}(\bar{\Phi}) \right) (\theta\eta + \bar{\theta}\bar{\eta}) \\ &= \frac{\widetilde{M}^2}{\sqrt{2}} \int d^2\theta \overline{DD} \left[L \mathcal{G}_{\Phi\Phi}(\Phi) \bar{\theta}\bar{\eta} \right] + \text{h.c.} \\ &= -\widetilde{M}^2 \sqrt{2} \int d^2\theta (\bar{\eta} \overline{DL}) \mathcal{G}_{\Phi\Phi}(\Phi) + \text{h.c.} = i\widetilde{M}^2 \delta^* \int d^2\theta \mathcal{G}_{\Phi}(\Phi) + \text{h.c.} \end{aligned} \quad (2.25)$$

Consequently, the deformed lagrangian

$$\mathcal{L}_{\text{def,kin.}} = \int d^2\theta d^2\bar{\theta} \mathcal{H}(L, \Phi, \bar{\Phi}) - i\widetilde{M}^2 \int d^2\theta \mathcal{G}_{\Phi}(\Phi) + \text{h.c.} \quad (2.26)$$

is invariant under the first (linearly-realized) supersymmetry as well as under the second nonlinearly-realized one. It is also obvious that the lagrangians corresponding to (2.15) and (2.24) are equivalent upon identifying $\mathcal{G}_{\Phi}(\Phi) = iW(\Phi)$.

2.1.2 The vacuum

Theory (2.17) with $\widetilde{m}^2 = 0$ can be derived from a deformed chiral-antichiral $\mathcal{N} = 2$ superfield with the use of a prepotential function $\mathcal{G}(\mathcal{Z})$. Let us define⁴

$$\mathcal{Z} = \Phi + \sqrt{2}i \overline{\theta} \overline{DL} - \frac{1}{4} \overline{\theta}\bar{\theta} \left[4i\widetilde{M}^2 + \overline{DD}\bar{\Phi} \right]. \quad (2.27)$$

⁴We introduce a second set of Grassmann coordinates $\widetilde{\theta}_{\alpha}, \overline{\widetilde{\theta}}_{\dot{\alpha}}$ and use chiral-antichiral coordinates \widetilde{y}^{μ} such that $\overline{D}_{\dot{\alpha}} \widetilde{y}^{\mu} = \widetilde{D}_{\alpha} \widetilde{y}^{\mu} = 0$. Then, \mathcal{Z} is a function of $\widetilde{y}^{\mu}, \theta, \bar{\theta}$.

We then obtain

$$\int d^2\theta \int d^2\bar{\theta} \mathcal{G}(\mathcal{Z}) + \text{h.c.} = \int d^2\theta \left[\frac{1}{2} \mathcal{G}_{\Phi\Phi} (\bar{D}L)(\bar{D}L) - \frac{1}{4} \mathcal{G}_{\Phi} \bar{D}\bar{D}\bar{\Phi} - i \widetilde{M}^2 \mathcal{G}_{\Phi} \right] + \text{h.c.} \quad (2.28)$$

Clearly, $\mathcal{G}_{\Phi}(\Phi) = iW(\Phi)$. Notice that the deformation cannot be understood as the expectation value of a scalar of the $\mathcal{N} = 1$ superfields.

Partial supersymmetry breaking is achieved if theory (2.17) has a vacuum state invariant under the first (linear) supersymmetry. We then analyze the scalar potential, which, since L does not have auxiliary fields, follows from the auxiliary f (in Φ) only. The auxiliary field lagrangian is⁵

$$\begin{aligned} \mathcal{L}_{\text{aux.}} = & i(W_{\Phi} - \bar{W}_{\bar{\Phi}})f\bar{f} - \widetilde{m}^2 f - \widetilde{M}^2 W_{\Phi} f - \widetilde{m}^2 \bar{f} - \widetilde{M}^2 \bar{W}_{\bar{\Phi}} \bar{f} \\ & - \frac{i}{2} W_{\Phi\Phi} [\bar{f} \psi\psi - f \bar{\varphi}\bar{\varphi}] + \frac{i}{2} \bar{W}_{\bar{\Phi}\bar{\Phi}} [f \bar{\psi}\bar{\psi} - \bar{f} \varphi\varphi] = -V + \mathcal{L}_{\text{ferm.}}. \end{aligned} \quad (2.29)$$

It generates the scalar potential

$$V = \frac{1}{i(W_{\Phi} - \bar{W}_{\bar{\Phi}})} \left| \widetilde{m}^2 + \widetilde{M}^2 W_{\Phi} \right|^2. \quad (2.30)$$

The term depending on L in theory (2.17) does not contribute to the potential. Fermion mass terms read

$$\begin{aligned} \mathcal{L}_{\text{ferm.}} = & -\frac{1}{2} \widetilde{M}^2 W_{\Phi\Phi} \psi\psi - \frac{1}{2} \left[\widetilde{m}^2 + \widetilde{M}^2 W_{\Phi} \right] \frac{W_{\Phi\Phi}}{W_{\Phi} - \bar{W}_{\bar{\Phi}}} \psi\psi + \text{h.c.} \\ & - \frac{1}{2} \left[\widetilde{m}^2 + \widetilde{M}^2 W_{\Phi} \right] \frac{\bar{W}_{\bar{\Phi}\bar{\Phi}}}{W_{\Phi} - \bar{W}_{\bar{\Phi}}} \varphi\varphi + \text{h.c.} \end{aligned} \quad (2.31)$$

Three situations can occur.

Firstly, if $\widetilde{M}^2 = \widetilde{m}^2 = 0$, the theory has unbroken (linear) $\mathcal{N} = 2$ supersymmetry and all fields are massless. This is also the case if $\widetilde{M}^2 = 0$, $\widetilde{m}^2 \neq 0$ and if the theory is canonical (i.e. free), $W_{\Phi\Phi} = 0$, in which case the potential is an irrelevant constant $V \sim |\widetilde{m}|^4$.

Secondly, if the second supersymmetry is not deformed ($\widetilde{M}^2 = 0$), the theory is not free ($W_{\Phi\Phi} \neq 0$) and $\widetilde{m}^2 \neq 0$, $\mathcal{N} = 2$ breaks to $\mathcal{N} = 0$ with

$$\langle f \rangle = -\frac{\widetilde{m}^2}{2 \text{Im}\langle W_{\Phi} \rangle}. \quad (2.32)$$

The theory has a vacuum state if $\langle W_{\Phi\Phi} \rangle = 0$ has a solution, fermions remain then massless and the splitting of scalar masses is controlled by $\langle W_{\Phi\Phi\Phi} \rangle$. This is also the case if $\widetilde{m}^2 = 0$ and $\widetilde{M}^2 \neq 0$ with

$$\langle f \rangle = -\frac{\widetilde{M}^2 \langle W_{\Phi} \rangle}{2 \text{Im}\langle W_{\Phi} \rangle}. \quad (2.33)$$

⁵In this section, we use the same notation Φ for the superfield and its lowest component. The other components are ψ and f , as in the other sections. The kinetic metric of the multiplet is $i(W_{\Phi} - \bar{W}_{\bar{\Phi}})$.

Thirdly, partial breaking to $\mathcal{N} = 1$ occurs if $\widetilde{M}^2 \neq 0 \neq \widetilde{m}^2$ and if the theory is not canonical ($W_{\Phi\Phi} \neq 0$). At the vacuum state,

$$\langle W_{\Phi} \rangle = -\frac{\widetilde{m}^2}{\widetilde{M}^2} \quad \langle f \rangle = 0. \quad (2.34)$$

Positivity of kinetic terms requires $\text{Im}\langle W_{\Phi} \rangle < 0$. The linear superfield L remains of course massless, while the mass⁶ of Φ is controlled by $\langle W_{\Phi\Phi} \rangle$:

$$\mathcal{M}_{\Phi}^2 = \widetilde{M}^2 \widetilde{M}^2 \left| \frac{\langle W_{\Phi\Phi} \rangle}{2 \text{Im}\langle W_{\Phi} \rangle} \right|^2. \quad (2.35)$$

In principle, Φ can acquire a very large mass and decouple from the massless L .

The analogy with partial supersymmetry breaking in a $\mathcal{N} = 2$ Maxwell multiplet theory [13] is striking. Describing this multiplet with $\mathcal{N} = 1$ superfields $W_{\alpha} = -\frac{1}{4} \overline{D} \overline{D} D_{\alpha} V$ and X , with deformed supersymmetry variations

$$\delta^* W_{\alpha} = -\sqrt{2} M^2 \eta_{\alpha} + \sqrt{2} i \left[\frac{1}{4} \eta_{\alpha} \overline{D} \overline{D} \overline{X} + i(\sigma^{\mu} \overline{\eta})_{\alpha} \partial_{\mu} X \right], \quad \delta^* X = \sqrt{2} i \eta^{\alpha} W_{\alpha}, \quad (2.36)$$

the invariant lagrangian is written as

$$\mathcal{L}_{\text{Max.}} = \frac{1}{2} \int d^2\theta \left[\frac{1}{2} \mathcal{F}_{XX} W W - \frac{1}{4} \mathcal{F}_X \overline{D} \overline{D} \overline{X} + m^2 X - i M^2 \mathcal{F}_X \right] + \text{h.c.} + \mathcal{L}_{\text{F.I.}}, \quad (2.37)$$

where $\mathcal{F}(X)$ is the holomorphic prepotential and $\mathcal{L}_{\text{F.I.}} = \xi \int d^4\theta V$ is the Fayet-Iliopoulos (FI) term. Partial breaking arises if the theory is interacting, $\mathcal{F}_{XXX} \neq 0$, if $M^2 \neq 0 \neq m^2$ and $\xi = 0$. If we now compare with the lagrangian (2.17) and the deformed variation

$$\delta^* \overline{D}_{\dot{\alpha}} L = -\sqrt{2} \widetilde{M}^2 \overline{\eta}_{\dot{\alpha}} + \sqrt{2} i \left[\frac{1}{4} \overline{\eta}_{\dot{\alpha}} \overline{D} \overline{D} \overline{\Phi} - i(\eta \sigma^{\mu})_{\dot{\alpha}} \partial_{\mu} \Phi \right], \quad (2.38)$$

we observe that there is clearly a correspondence between Φ and X , $\mathcal{F}_X(X)$ and $W(\Phi)$ with a Lorentz chirality inversion from W_{α} to $\overline{D}_{\dot{\alpha}} L$. However, there are significant differences, namely the absence of auxiliary fields in L as well as the consequent inexistence of a corresponding “electric” FI term analogous to the ξD term for the Maxwell multiplet.

2.2 Dual hypermultiplet formulation

The duality transformation from the single-tensor to the hypermultiplet formulation is a Legendre transformation in $\mathcal{N} = 1$ superspace. Instead of expression (2.7), let us use

$$\mathcal{L}_{\text{kin.}} = \int d^2\theta d^2\overline{\theta} \left[\mathcal{H}(V, \Phi, \overline{\Phi}) - (S + \overline{S}) V \right]. \quad (2.39)$$

The field equation for S implies $V = L$ and the field equation for V yields

$$\mathcal{H}_V = S + \overline{S} \quad (\text{Legendre transformation}) \quad (2.40)$$

⁶Normalized with the metric $-2 \text{Im}\langle W_{\Phi} \rangle$.

which allows one to express V as a function of $S + \bar{S}$, Φ and $\bar{\Phi}$. The Kähler potential for the hypermultiplet with superfields S and Φ is then

$$\mathcal{K}(S + \bar{S}, \Phi, \bar{\Phi}) = \left[\mathcal{H}(V, \Phi, \bar{\Phi}) - (S + \bar{S})V \right]_{V(S+\bar{S}, \Phi, \bar{\Phi})}. \quad (2.41)$$

In our case, the Legendre transformation is simply

$$\mathcal{K}_V = 0 \quad \implies \quad S + \bar{S} = \mathcal{H}_V = -2iV(W_\Phi - \bar{W}_\Phi) \quad (2.42)$$

with also

$$\mathcal{H}_S = 0 \quad \implies \quad \mathcal{K}_S = -V. \quad (2.43)$$

The dual hypermultiplet theory reads

$$\begin{aligned} \mathcal{L}_{\text{dual}} &= i \int d^2\theta d^2\bar{\theta} \left[-\frac{1}{4} \frac{(S + \bar{S})^2}{W_\Phi - \bar{W}_\Phi} + W\bar{\Phi} - \bar{W}\Phi \right] + \int d^2\theta \left[\tilde{m}^2\Phi + \tilde{M}^2W \right] + \text{h.c.} \\ &= \int d^2\theta \left[-\frac{i}{2} W_\Phi (\bar{D}\mathcal{K}_S) (\bar{D}\mathcal{K}_S) - \frac{i}{4} W \bar{D}\bar{D}\Phi + \tilde{m}^2\Phi + \tilde{M}^2W \right] + \text{h.c.} \end{aligned} \quad (2.44)$$

The D -term in the first expression is the Kähler potential of a hyper-Kähler space, $\det \mathcal{K}_{m\bar{n}} = 1/2$. Since the superpotential depends on Φ only, the auxiliary component f_S of S does not contribute to the potential. Its field equation

$$(W_\Phi - \bar{W}_\Phi) f_S - (S + \bar{S}) W_{\Phi\Phi} f_\Phi = 0 \quad (2.45)$$

is actually the $\theta\theta$ component of the duality relation (2.42). The ground state in the partially broken phase is again characterized by relations (2.34) with, in addition, $\langle f_S \rangle = 0$. On-shell, relations (2.42) and (2.43) with L replacing V ,

$$\mathcal{H}_L = S + \bar{S}, \quad \mathcal{K}_S = -L, \quad (2.46)$$

are consistent using the field equations for L and S ,

$$\bar{D}\bar{D}D_\alpha \mathcal{H}_L = 0, \quad \bar{D}\bar{D}\mathcal{K}_S = 0, \quad (2.47)$$

as integrability conditions.

That the $\mathcal{N} = 1$ theory (2.44) has a second supersymmetry is not obvious. Since the Kähler potential \mathcal{K} generates a hyper-Kähler metric, the first term certainly has (on-shell) $\mathcal{N} = 2$ [24]. Following [21], one easily verifies that \mathcal{K} is invariant (up to a superspace derivative) under the variations

$$\delta^* \mathcal{K}_S = \frac{i}{\sqrt{2}} (\eta D\Phi + \bar{\eta} \bar{D}\bar{\Phi}), \quad \delta^* \Phi = -\sqrt{2}i \bar{\eta} \bar{D}\mathcal{K}_S, \quad \delta^* \bar{\Phi} = -\sqrt{2}i \eta D\mathcal{K}_S, \quad (2.48)$$

where $\mathcal{K}_S = \frac{\partial}{\partial S} \mathcal{K} = -\frac{i}{2} \frac{S + \bar{S}}{W_\Phi - \bar{W}_\Phi}$. These variations are simply obtained by inserting the second duality relation (2.46) in the single-tensor off-shell variations (2.3). The field equation $\bar{D}\bar{D}\mathcal{K}_S = 0$ provides the linearity and chirality of $\delta^* \mathcal{K}_S$ and $\delta^* \Phi$ respectively. The superpotential term $\tilde{m}^2\Phi$ is also invariant. The nonlinear deformation which allows for the presence of the superpotential \tilde{M}^2W is then

$$\delta_{\text{nl}}^* \bar{D}_\alpha \mathcal{K}_S = \sqrt{2} \tilde{M}^2 \bar{\eta}_\alpha, \quad (2.49)$$

in agreement with eqs. (2.18) and (2.46).

2.3 Several single-tensor multiplets

The extension to a theory with several single-tensor multiplets is straightforward. Consider the deformed $\mathcal{N} = 2$ chiral superfields

$$\mathcal{Z}^a = \Phi^a + \sqrt{2}i \widetilde{\theta} \overline{D}L^a - \frac{1}{4} \widetilde{\theta} \overline{\theta} \left[4i(\widetilde{M}^a)^2 + \overline{D}D \overline{\Phi}^a \right]. \quad (2.50)$$

The lagrangian

$$\begin{aligned} \mathcal{L} &= \int d^2\theta \int d^2\widetilde{\theta} \mathcal{G}(\mathcal{Z}^a) + \text{h.c.} \\ &= \int d^2\theta \left[\frac{1}{2} \mathcal{G}_{ab} (\overline{D}L^a) (\overline{D}L^b) - \frac{1}{4} \mathcal{G}_a \overline{D}D \overline{\Phi}^a - i(\widetilde{M}^a)^2 \mathcal{G}_a + \widetilde{m}_a^2 \Phi^a \right] + \text{h.c.}, \end{aligned} \quad (2.51)$$

where

$$\mathcal{G}_a = \frac{\partial}{\partial \Phi^a} \mathcal{G}(\Phi^c), \quad \mathcal{G}_{ab} = \frac{\partial^2}{\partial \Phi^a \partial \Phi^b} \mathcal{G}(\Phi^c),$$

is invariant under the nonlinear second supersymmetry variations

$$\delta^* L^a = \sqrt{2}(\widetilde{M}^a)^2(\theta\eta + \overline{\theta}\overline{\eta}) - \frac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta} \overline{D}\overline{\Phi}), \quad \delta^* \Phi^a = \sqrt{2}i \overline{\eta} \overline{D}L^a. \quad (2.52)$$

For $\widetilde{m}_a^2 \neq 0 \neq (\widetilde{M}^b)^2$, the condition for unbroken $\mathcal{N} = 1$ is the cancellation of all auxiliary fields f^a :

$$-i \langle \mathcal{G}_{ab} \rangle (\widetilde{M}^b)^2 + \widetilde{m}_a^2 = 0. \quad (2.53)$$

In this vacuum, the kinetic metric $2\langle \text{Re} \mathcal{G}_{ab} \rangle$ must be invertible and the mass matrix of the chiral multiplets Φ^a is then

$$\mathcal{M}_{ab} = -\frac{i}{2} \langle \text{Re} \mathcal{G}_{ac}^{-1} \rangle \langle \mathcal{G}_{bcd} \rangle (\widetilde{M}^d)^2, \quad (2.54)$$

controlled by the third derivatives of \mathcal{G} .

3 Nonlinear deformations

In the previous section, we made use of particular nonlinear deformations of the $\mathcal{N} = 2$ single-tensor and Maxwell multiplets to engineer theories with partial supersymmetry breaking. As illustrated by eq. (2.27), a nonlinear deformation of the single-tensor multiplet can be introduced as a spurious constant component inserted in a $\mathcal{N} = 2$ superfield. In this section, we study general nonlinear deformations of these multiplets, using their representation as chiral superfields in $\mathcal{N} = 2$ superspace.

3.1 Deformations of the Maxwell superfield

A chiral-chiral (CC) $\mathcal{N} = 2$ superfield describes the Maxwell multiplet:

$$\mathcal{W}(y, \theta, \widetilde{\theta}) = X + \sqrt{2}i \widetilde{\theta} W - \frac{1}{4} \widetilde{\theta} \overline{\theta} \overline{D}D \overline{X}, \quad \overline{D}_{\dot{\alpha}} \mathcal{W} = \overline{D}_{\dot{\alpha}} \overline{\mathcal{W}} = 0, \quad (3.1)$$

using chiral coordinates $\overline{D}_{\dot{\alpha}} y^{\mu} = \widetilde{D}_{\dot{\alpha}} y^{\mu} = 0$, with also

$$\begin{aligned} W_{\alpha} &= -i\lambda_{\alpha} + \theta_{\alpha} D - \frac{i}{2}(\sigma^{\mu}\overline{\sigma}^{\nu}\theta)_{\alpha} F_{\mu\nu} - \theta\theta(\sigma^{\mu}\partial_{\mu}\overline{\lambda})_{\alpha}, \\ X &= x + \sqrt{2}\theta\kappa - \theta\theta F, \end{aligned} \tag{3.2}$$

$$\frac{1}{4}\overline{D}\overline{D}\overline{X} = \overline{F} + \sqrt{2}i\theta\sigma^{\mu}\partial_{\mu}\overline{\kappa} + \theta\theta\Box\overline{x}.$$

The $SU(2)_R$ symmetry of the $\mathcal{N} = 2$ algebra acts linearly on the components of the Maxwell superfield \mathcal{W} . Defining fermion doublets

$$\theta^1 = \theta, \quad \theta^2 = \widetilde{\theta}, \quad \lambda_1 = \kappa, \quad \lambda_2 = \lambda, \tag{3.3}$$

leads to

$$\mathcal{W}(y, \theta, \widetilde{\theta}) = x + \sqrt{2}\theta^i\lambda_i - \theta^i\theta^j Y_{ij} + \dots \tag{3.4}$$

omitting terms which depend on derivatives of the fields. Since $\theta^i\theta^j = \theta^j\theta^i$,

$$Y_{ij} = Y_{ji} = [\vec{Y} \cdot \vec{\sigma}]_{ij} \tag{3.5}$$

and the vector \vec{Y} is in general a complex $SU(2)_R$ triplet. But in \mathcal{W} , the auxiliary fields correspond to

$$Y_{11} = F, \quad Y_{22} = \overline{F}, \quad Y_{12} = -\frac{i}{\sqrt{2}}D, \quad \vec{Y} = \left(\text{Im } F, \text{Re } F, \frac{D}{\sqrt{2}} \right) \tag{3.6}$$

and the $SU(2)_R$ -invariant “reality” condition

$$Y^{ij} \equiv Y_{ij}^* = \epsilon^{ik}\epsilon^{jl}Y_{kl} \tag{3.7}$$

is verified: a complex value of \vec{Y} violating this condition cannot be seen as a background value of $\mathcal{N} = 1$ superfields X or W_{α} .

Since gauginos are in the θ^i components, nonlinear deformations of their variations, as expected for goldstino fermions, should be introduced with

$$\begin{aligned} \mathcal{W}_{\text{nl}} &= A^2\theta\theta + B^2\widetilde{\theta}\widetilde{\theta} + 2\Gamma\theta\widetilde{\theta}, \quad (A, B, \Gamma \text{ complex}) \\ &= (Y_2 + iY_1)\theta\theta + (Y_2 - iY_1)\widetilde{\theta}\widetilde{\theta} - 2iY_3\theta\widetilde{\theta} \end{aligned} \tag{3.8}$$

added to \mathcal{W} . Then, $\vec{Y} = (-\frac{i}{2}[A^2 - B^2], \frac{1}{2}[A^2 + B^2], i\Gamma)$ and

$$\delta\kappa_{\alpha} = \sqrt{2}(A^2\epsilon_{\alpha} + \Gamma\eta_{\alpha}) + \dots \quad \delta\lambda_{\alpha} = \sqrt{2}(B^2\eta_{\alpha} + \Gamma\epsilon_{\alpha}) + \dots \tag{3.9}$$

If $\Gamma = \pm AB$, $\mathcal{W}_{\text{nl}} = (A\theta \pm B\widetilde{\theta})^2$, $\delta(B\kappa_{\alpha} \mp A\lambda_{\alpha}) = 0$ and the deformation partially breaks $\mathcal{N} = 2$ to $\mathcal{N} = 1$. We earlier used the particular case $A = \Gamma = 0$. The condition for partial breaking is in any case incompatible with the reality condition (3.7): the auxiliary fields F and D are not able to induce partial breaking with their background values; in other words, the deformation parameters cannot be absorbed in the background values of the auxiliary fields, in contrast with the case of the spontaneous breaking of $\mathcal{N} = 1$. An $SU(2)$ rotation can be used to cancel $Y_3 = i\Gamma$. With this choice, partial breaking occurs either if $A = 0$, and the goldstino is λ_{α} , or if $B = 0$ and the goldstino is κ_{α} .

3.2 Deformations of the single-tensor superfield

While a chiral-chiral (CC) superfield is relevant to study deformations of the Maxwell multiplet, the single-tensor multiplet is conveniently described using a chiral-antichiral (CA) $\mathcal{N} = 2$ superfield \mathcal{Z} ,

$$\bar{D}_{\dot{\alpha}} \mathcal{Z} = \tilde{D}_{\alpha} \mathcal{Z} = 0, \tag{3.10}$$

with the expansion

$$\mathcal{Z} = \Phi + \sqrt{2i} \bar{\theta} \overline{DL} - \frac{1}{4} \overline{\theta\theta} \overline{DD} \Phi \tag{3.11}$$

in the appropriate coordinates $(\tilde{y}, \theta, \bar{\theta})$, $\bar{D}_{\dot{\alpha}} \tilde{y}^{\mu} = \tilde{D}_{\alpha} \tilde{y}^{\mu} = 0$. A particular deformation with partial supersymmetry breaking has been earlier described [eq. (2.28)] and we wish to generalize it. Since fermion fields are in the components⁷

$$\sqrt{2} \theta \psi - \sqrt{2} \bar{\theta} \overline{\varphi} \tag{3.12}$$

of \mathcal{Z} , the deformation parameters will add

$$\mathcal{Z}_{\text{nl}} = \tilde{A}^2 \theta \theta + \tilde{B}^2 \overline{\theta\theta} \tag{3.13}$$

to \mathcal{Z} . In contrast with the Maxwell case, the mixed contribution $\theta_{\alpha} \bar{\theta}_{\dot{\alpha}}$ is a space-time vector and the deformations are encoded in two complex numbers \tilde{A}^2 and \tilde{B}^2 only. The nonlinear variations of the spinors are

$$\delta\psi_{\alpha} = \sqrt{2} (\tilde{A}^2 - f) \epsilon_{\alpha} + \dots \quad \delta\overline{\varphi}_{\dot{\alpha}} = -\sqrt{2} (\tilde{B}^2 + f) \eta_{\dot{\alpha}} + \dots \tag{3.14}$$

and generic values of \tilde{A}^2 and \tilde{B}^2 break both supersymmetries. Partial breaking occurs if either $\tilde{B}^2 = 0$ and the goldstino is ψ in Φ , or if $\tilde{A}^2 = 0$ with φ in L as the goldstino. An expectation value $\langle f \rangle$ of the auxiliary f in Φ corresponds to $\tilde{A}^2 = -\tilde{B}^2$ and cannot generate partial breaking on its own.

In the linear $\mathcal{N} = 2$ theory, all fields are massless since the single-tensor multiplet includes a tensor with gauge symmetry. A generic lagrangian generated by the CA superfield \mathcal{Z} is

$$\mathcal{L} = \int d^2\theta \left[\int d^2\bar{\theta} \overline{\mathcal{G}}(\mathcal{Z}) + \tilde{m}^2 \Phi \right] + \text{h.c.} = \mathcal{L}_{\text{lin.}} + \mathcal{L}_{\text{nl.}} \tag{3.15}$$

where \mathcal{L}_{nl} includes all terms generated by the deformations with parameters \tilde{A}^2 and \tilde{B}^2 . In the function $\mathcal{G}(\mathcal{Z})$, a term linear in \mathcal{Z} is irrelevant (it contributes with a derivative) and the component expansion of the lagrangian depends on the second and higher derivatives of \mathcal{G} . The only auxiliary field is f in Φ and $\mathcal{L}_{\text{lin.}}$ includes the terms

$$[\mathcal{G}''(z) + \overline{\mathcal{G}}''(\bar{z})] \bar{f} f + \left[\frac{1}{2} \mathcal{G}'''(z) [\bar{f} \psi \psi + f \overline{\varphi} \overline{\varphi}] - \tilde{m}^2 f \right] + \text{h.c.} \tag{3.16}$$

⁷The field components of Φ are z , ψ and f and $\bar{D}_{\dot{\alpha}} L$ is expanded in eq. (2.10).

The parameter \tilde{m}^2 induces $\langle f \rangle = \tilde{m}^2/2\langle \text{Re } \mathcal{G}'' \rangle$ which breaks both supersymmetries if the theory is not canonical, $\mathcal{G}''' \neq 0$. The nonlinear deformation produces the following terms:

$$\begin{aligned} \mathcal{L}_{\text{nl}} = & -\mathcal{G}''(z) \left[\overline{\tilde{B}}^2 f + \tilde{A}^2 \bar{f} + \tilde{A}^2 \overline{\tilde{B}}^2 \right] + \text{h.c.} \\ & -\frac{1}{2} \mathcal{G}'''(z) \left[\overline{\tilde{B}}^2 \psi\psi + \tilde{A}^2 \overline{\varphi\varphi} \right] + \text{h.c.} \end{aligned} \quad (3.17)$$

Hence,

$$2[\text{Re } \mathcal{G}''(z)] \bar{f} = \mathcal{G}''(z) \overline{\tilde{B}}^2 + \overline{\mathcal{G}''}(\bar{z}) \tilde{A}^2 + \tilde{m}^2 - \frac{1}{2} \mathcal{G}'''(z) \overline{\varphi\varphi} - \frac{1}{2} \overline{\mathcal{G}'''}(\bar{z}) \psi\psi$$

and the scalar potential and the fermion bilinear terms read respectively

$$\begin{aligned} V(z, \bar{z}) = & \frac{1}{2 \text{Re } \mathcal{G}''} \left| \overline{\tilde{B}}^2 \overline{\mathcal{G}''} + \tilde{A}^2 \mathcal{G}'' + \tilde{m}^2 \right|^2 + 2 \text{Re} [\tilde{A}^2 \overline{\tilde{B}}^2 \mathcal{G}''], \\ \mathcal{L}_{\text{ferm.}} = & \frac{1}{2} \psi\psi \left[\frac{\mathcal{G}'''}{2 \text{Re } \mathcal{G}''} (\overline{\tilde{B}}^2 \mathcal{G}'' + \tilde{A}^2 \overline{\mathcal{G}''} + \tilde{m}^2) - \overline{\tilde{B}}^2 \mathcal{G}''' \right] + \text{h.c.} \\ & + \frac{1}{2} \overline{\varphi\varphi} \left[\frac{\overline{\mathcal{G}''}}{2 \text{Re } \mathcal{G}''} (\overline{\tilde{B}}^2 \mathcal{G}'' + \tilde{A}^2 \overline{\mathcal{G}''} + \tilde{m}^2) - \tilde{A}^2 \overline{\mathcal{G}'''} \right] + \text{h.c.} \end{aligned} \quad (3.18)$$

The kinetic metric of the multiplet is $2 \text{Re } \mathcal{G}''(z)$. Notice that these formulas do not depend on the real scalar C in L , which always leads to a flat direction.

If $\tilde{A}\tilde{B} = 0$ with $\mathcal{L}_{\text{nl}} \neq 0$ and the ground state equation $\langle \overline{\tilde{B}}^2 \mathcal{G}'' + \tilde{A}^2 \overline{\mathcal{G}''} + \tilde{m}^2 \rangle = 0$ has a solution, one supersymmetry remains unbroken: $\langle f \rangle = 0$. This requires $\tilde{m}^2 \neq 0$, since positivity of the kinetic metric forbids $\langle \mathcal{G}'' \rangle = 0$. If $\tilde{B} \neq 0$, the mass terms are

$$2\langle \text{Re } \mathcal{G}'' \rangle \left[\mathcal{M}_\Phi \overline{\mathcal{M}_\Phi} z\bar{z} - \frac{1}{2} \mathcal{M}_\Phi \psi\psi - \frac{1}{2} \overline{\mathcal{M}_\Phi} \overline{\psi\psi} \right], \quad \mathcal{M}_\Phi = \frac{\overline{\tilde{B}}^2 \langle \mathcal{G}''' \rangle}{2\langle \text{Re } \mathcal{G}'' \rangle}.$$

This is the case already obtained in eqs. (2.34) and (2.35): the chiral $\mathcal{N} = 1$ superfield Φ has mass \mathcal{M}_Φ , and L is massless. If $\tilde{A} \neq 0$, the mass terms are

$$2\langle \text{Re } \mathcal{G}'' \rangle \left[\mathcal{M}_\Phi \overline{\mathcal{M}_\Phi} z\bar{z} - \frac{1}{2} \mathcal{M}_\Phi \varphi\varphi - \frac{1}{2} \overline{\mathcal{M}_\Phi} \overline{\varphi\varphi} \right], \quad \mathcal{M}_\Phi = \frac{\tilde{A}^2 \langle \mathcal{G}''' \rangle}{2\langle \text{Re } \mathcal{G}'' \rangle}.$$

The roles of ψ and φ are exchanged, the $\mathcal{N} = 1$ multiplet with mass \mathcal{M}_Φ has fields z and φ , while ψ is the $\mathcal{N} = 1$ partner of $H_{\mu\nu\rho}$ and C in the massless linear superfield.

If $\tilde{A}\tilde{B} \neq 0$, the non-zero second term in the scalar potential (which can have both signs) breaks both supersymmetries, assuming that V has a ground state $\langle z \rangle$.

4 Constrained multiplets

When supersymmetry is partially broken in the Maxwell or single-tensor (hypermultiplet) theory, a chiral multiplet (X or Φ) acquires an arbitrary mass. In the infinite-mass limit, the field equation of this superfield is a constraint which allows for the elimination of the massive chiral superfield. One is then left with a nonlinear realization of $\mathcal{N} = 2$ supersymmetry in terms of the $4_B + 4_F$ fields of the $\mathcal{N} = 1$ Maxwell or linear superfield.

4.1 The infinite-mass limit

We begin with partial breaking in the Maxwell theory. Since the two options $A^2 = 0$ and $B^2 = 0$ are equivalent, we only consider the first case and use the deformed chiral-chiral deformed superfield

$$\mathcal{W} = X + \sqrt{2}i\tilde{\theta}W + \tilde{\theta}\tilde{\theta} \left[B^2 - \frac{1}{4}\overline{DD}\overline{X} \right], \quad (4.1)$$

in terms of which the lagrangian is

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \int d^2\theta \left[\int d^2\tilde{\theta} \mathcal{F}(\mathcal{W}) + m^2 X \right] + \text{h.c.} + \mathcal{L}_{\text{F.I.}} \\ &= \frac{1}{4} \int d^2\theta \left[\mathcal{F}_{XX} WW - \frac{1}{2} \mathcal{F}_X \overline{DD}\overline{X} + 2m^2 X + 2B^2 \mathcal{F}_X \right] + \text{h.c.} + \mathcal{L}_{\text{F.I.}} \end{aligned} \quad (4.2)$$

Since the auxiliary fields f and D vanish in the ground state, the mass terms of the fermion χ in X are

$$-\frac{B^2}{4} \langle \mathcal{F}_{XXX} \rangle \chi\chi - \frac{\overline{B}^2}{4} \langle \overline{\mathcal{F}}_{\overline{X}\overline{X}\overline{X}} \rangle \overline{\chi}\overline{\chi}$$

and, since the kinetic metric is $\text{Re}\langle \mathcal{F}_{XX} \rangle$, the mass of X is

$$\mathcal{M}_X = \frac{B^2 \langle \mathcal{F}_{XXX} \rangle}{2 \text{Re}\langle \mathcal{F}_{XX} \rangle}. \quad (4.3)$$

The infinite-mass limit is $\langle \mathcal{F}_{XXX} \rangle \rightarrow \infty$ with fixed $\text{Re}\langle \mathcal{F}_{XX} \rangle$ (as the latter corresponds to the metric of the scalar manifold), thus disproving the claim made in [25]. Expanding the field equation of X and retaining only the term in $\langle \mathcal{F}_{XXX} \rangle$ leads to the constraint

$$WW - \frac{1}{2} X \overline{DD}\overline{X} + 2B^2 X = 0, \quad (4.4)$$

which was first given in [14]. Multiplying (4.4) by W_α or X leads also to $XW_\alpha = X^2 = 0$ and the constraint (4.4) is then equivalent to [16]

$$\mathcal{W}^2 = 0. \quad (4.5)$$

We now turn to the partial breaking in a single-tensor theory. Again, the two options $\tilde{A}^2 = 0$ and $\tilde{B}^2 = 0$ are equivalent, so we only consider the first case and use the deformed chiral-antichiral superfield

$$\mathcal{Z} = \Phi + \sqrt{2}i\tilde{\theta}\overline{DL} + \tilde{\theta}\tilde{\theta} \left[\tilde{B}^2 - \frac{1}{4}\overline{DD}\overline{\Phi} \right], \quad (4.6)$$

which induces the nonlinear deformation

$$\delta_{\text{nl}}^* \overline{D}_\alpha L = -i\sqrt{2} \tilde{B}^2 \tilde{\eta}_\alpha. \quad (4.7)$$

The theory (3.15) and the field equation for Φ respectively read

$$\begin{aligned} \mathcal{L} &= \int d^2\theta \left[\mathcal{G}_\Phi(\Phi) \left(-\frac{1}{4}\overline{DD}\overline{\Phi} + \tilde{B}^2 \right) + \frac{1}{2} \mathcal{G}_{\Phi\Phi}(\Phi) (\overline{DL})(\overline{DL}) + \tilde{m}^2 \Phi \right] + \text{h.c.}, \\ 0 &= \mathcal{G}_{\Phi\Phi}(\Phi) \left(-\frac{1}{4}\overline{DD}\overline{\Phi} + \tilde{B}^2 \right) + \frac{1}{2} \mathcal{G}_{\Phi\Phi\Phi}(\Phi) (\overline{DL})(\overline{DL}) + \tilde{m}^2. \end{aligned} \quad (4.8)$$

The lowest component is the field equation for the auxiliary field f ,

$$\mathcal{G}_{zz}(z) (\bar{f} - \overline{\widetilde{B}}^2) = \widetilde{m}^2$$

omitting fermions, and $\langle f \rangle = 0$ defines the ground state $\mathcal{G}_{zz}(\langle z \rangle) = -\widetilde{m}^2 / \overline{\widetilde{B}}^2$ and the kinetic metric normalization $2 \operatorname{Re} \mathcal{G}_{zz}(\langle z \rangle)$.

As explained earlier, the mass of Φ is controlled by $\mathcal{G}_{zzz}(\langle z \rangle)$ and this free parameter can be sent to infinity keeping $\mathcal{G}_{zz}(\langle z \rangle)$ finite as in the Maxwell case. In this limit,

$$\mathcal{G}_{zz}(\Phi) \sim \mathcal{G}_{zzz}(\langle z \rangle)[\Phi - \langle z \rangle], \quad \mathcal{G}_{zzz}(\Phi) \sim \mathcal{G}_{zzz}(\langle z \rangle)$$

and the field equation becomes⁸

$$\frac{1}{2} \Phi \overline{DD\Phi} - (\overline{DL})(\overline{DL}) = 2\overline{\widetilde{B}}^2 \Phi, \quad (4.9)$$

which does not depend on the function \mathcal{G} and which was first given in [18]. This equation allows to eliminate Φ . The solution expresses Φ as a function of $(\overline{DL})(\overline{DL})$, with

$$\Phi = -\frac{2(\overline{DL})(\overline{DL})}{\overline{\widetilde{B}}^2 - \overline{DD\Phi}} \implies \Phi \overline{D}_\alpha L = \Phi^n = 0 \quad (n \geq 2). \quad (4.10)$$

The second supersymmetry variation of the constraint (4.9) is

$$\delta^* \left[\frac{1}{2} \Phi \overline{DD\Phi} - (\overline{DL})(\overline{DL}) - 2\overline{\widetilde{B}}^2 \Phi \right] = -2\sqrt{2} \partial_\mu (\eta \sigma^\mu \overline{DL} \Phi). \quad (4.11)$$

The invariance of the constraint then follows from the results (4.10). Moreover, since

$$\mathcal{Z}^2 = \Phi^2 + 2\sqrt{2}i \Phi \overline{\theta DL} - \overline{\theta\theta} \left[\frac{1}{2} \Phi \overline{DD\Phi} - (\overline{DL})(\overline{DL}) - 2\overline{\widetilde{B}}^2 \Phi \right], \quad (4.12)$$

eq. (4.9) is equivalent to the $\mathcal{N} = 2$ condition

$$\mathcal{Z}^2 = 0. \quad (4.13)$$

4.2 Solutions of the constraints

The solution of (4.4), and thus of (4.5), was first given in [14]. In our conventions, it is

$$X = -\frac{W^2}{2B^2} \left[1 - \overline{D}^2 \left(\frac{\overline{W}^2}{4B^4 + a + 4B^4 \sqrt{1 + \frac{a}{2B^4} + \frac{b^2}{16B^8}}} \right) \right], \quad (4.14)$$

where

$$a = \frac{1}{2}(D^2 W^2 + \overline{D}^2 \overline{W}^2), \quad b = \frac{1}{2}(D^2 W^2 - \overline{D}^2 \overline{W}^2). \quad (4.15)$$

The bosonic part of lagrangian (4.2) then takes the form

$$\mathcal{L}|_{\text{bos}} = 8m^2 B^2 \left(1 - \sqrt{1 - \frac{1}{B^4} (-F_{\mu\nu} F^{\mu\nu} + 2D^2) - \frac{1}{4B^8} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2} \right). \quad (4.16)$$

⁸One can redefine $\Phi - \langle z \rangle \rightarrow \Phi$.

The equation of motion for D is then

$$D = 0, \tag{4.17}$$

and, substituting back into (4.16), one arrives at [14, 16]

$$\begin{aligned} \mathcal{L}|_{\text{bos}} &= 8m^2 B^2 \left(1 - \sqrt{1 + \frac{1}{B^4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4B^8} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2} \right) \\ &= 8m^2 B^2 \left(1 - \sqrt{-\det(\eta_{\mu\nu} - \frac{\sqrt{2}}{B^2} F_{\mu\nu})} \right). \end{aligned} \tag{4.18}$$

It is also possible to add the FI term

$$\xi \int d^2\theta d^2\bar{\theta} V = \frac{1}{2} \xi D \tag{4.19}$$

to the lagrangian (4.16). Solving the equation of motion for D then gives

$$-\frac{2}{B^4} D^2 = -\frac{\xi^2}{\xi^2 + 2 \cdot 16^2 m^4} \left(1 + \frac{1}{B^4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4B^8} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right), \tag{4.20}$$

and substituting back to (4.16), we find that the latter takes the form

$$\begin{aligned} \mathcal{L}|_{\text{bos}} &= 8m^2 B^2 \left(1 - \left(1 + \frac{\xi^2}{4 \cdot 8 \cdot 16m^4} \right) \sqrt{1 - \frac{1}{B^4} (-F_{\mu\nu} F^{\mu\nu} + 2D^2) - \frac{1}{4B^8} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2} \right) \\ &= 8m^2 B^2 \left(1 - \sqrt{1 + \frac{\xi^2}{8^3 m^4}} \sqrt{-\det(\eta_{\mu\nu} - \frac{\sqrt{2}}{B^2} F_{\mu\nu})} \right), \end{aligned} \tag{4.21}$$

which means that the addition of the FI term only changes the prefactor of the Born-Infeld lagrangian included in \mathcal{L} .

Following [14, 18] and [19], we now give the solution $\Phi = \Phi(\bar{D}L)$ of the constraint (4.10) or equivalently of (4.13). In our conventions, it is

$$\Phi = -\frac{1}{2\tilde{B}^2} \left[(\bar{D}L)^2 - \bar{D}^2 \left(\frac{(DL)^2 (\bar{D}L)^2}{4\tilde{B}^4 + \tilde{a} + 4\tilde{B}^4 \sqrt{1 + \frac{\tilde{a}}{2\tilde{B}^4} + \frac{\tilde{b}^2}{16\tilde{B}^8}}} \right) \right], \tag{4.22}$$

where we have assumed that \tilde{B} is real for simplicity and

$$\tilde{a} = \frac{1}{2} \left(\bar{D}^2 [(DL)^2] + D^2 [(\bar{D}L)^2] \right) = \tilde{a}, \quad \tilde{b} = \frac{1}{2} \left(\bar{D}^2 [(DL)^2] - D^2 [(\bar{D}L)^2] \right) = -\tilde{b}. \tag{4.23}$$

Due to the constraint (4.13), only if \mathcal{G} has linear dependence on \mathcal{Z} will it contribute to (3.15). However,

$$\int d^2\theta d^2\bar{\theta} \mathcal{Z} + \text{h.c.} \sim \int d^2\theta \left(\tilde{B}^2 - \frac{1}{4} \bar{D}\bar{D}\bar{\Phi} \right) + \text{h.c.} = \text{derivative}. \tag{4.24}$$

Consequently, (3.15) takes the form

$$\begin{aligned} \mathcal{L} &= \tilde{m}^2 \int d^2\theta \Phi + \text{h.c.} \\ &= -\frac{\tilde{m}^2}{2\tilde{B}^2} \int d^2\theta (\bar{D}L)^2 \left[1 - \bar{D}^2 \left(\frac{(DL)^2}{4\tilde{B}^4 + \tilde{a} + 4\tilde{B}^4 \sqrt{1 + \frac{\tilde{a}}{2\tilde{B}^4} + \frac{\tilde{b}^2}{16\tilde{B}^8}}} \right) \right] + \text{h.c.} \end{aligned} \tag{4.25}$$

Moreover, using (2.10), we find

$$\begin{aligned} (\overline{DL})^2|_{\text{bos}} &= \theta^2 (v_\mu v^\mu + 2iv_\mu \partial^\mu C - \partial_\mu C \partial^\mu C), \\ \tilde{a}|_{\text{bos}} &= 4(v^2 - (\partial C)^2), \quad \tilde{b}|_{\text{bos}} = -8i v \cdot \partial C. \end{aligned} \quad (4.26)$$

Then

$$\begin{aligned} \mathcal{L}|_{\text{bos}} &= \tilde{m}^2 \tilde{B}^2 \left(1 - \sqrt{1 + \frac{2}{\tilde{B}^4} (v^2 - (\partial C)^2) - \frac{4}{\tilde{B}^8} (v \cdot \partial C)^2} \right) \\ &= \tilde{m}^2 \tilde{B}^2 \left(1 - \sqrt{1 - \frac{2}{\tilde{B}^4} \left(\frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} + \partial_\mu C \partial^\mu C \right) - \frac{1}{9\tilde{B}^8} (\epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} \partial^\mu C)^2} \right). \end{aligned} \quad (4.27)$$

5 The “long” super-Maxwell superfield

In section 6 we will construct supersymmetric interactions of deformed or constrained single-tensor and Maxwell supermultiplets. We will find it useful to describe the Maxwell multiplet in terms of a chiral-antichiral superfield, with $16_B + 16_F$ components, as an alternative to the $8_B + 8_F$ chiral-chiral superfield (3.1). In the present and technical section, we thus proceed to construct this “long” $\mathcal{N} = 2$ superfield for the super-Maxwell theory.

To begin with, both types of superfields exist for the single-tensor multiplet. In particular, the latter can be described either by the “short” ($8_B + 8_F$) chiral-antichiral (CA) superfield (3.11),

$$\mathcal{Z} = \Phi + \sqrt{2}i \overline{\theta} \overline{D} L - \overline{\theta} \overline{\theta} \frac{1}{4} \overline{D} \overline{D} \overline{\Phi}, \quad (5.1)$$

(and its AC conjugate), or by a “long” chiral-chiral (CC) superfield [19]

$$\widehat{\mathcal{Z}} = Y + \sqrt{2} \tilde{\theta} \chi - \tilde{\theta} \tilde{\theta} \left[\frac{i}{2} \Phi + \frac{1}{4} \overline{D} \overline{D} \overline{Y} \right], \quad (5.2)$$

where Y , Φ and χ_α are chiral $\mathcal{N} = 1$ superfields with $16_B + 16_F$ field components. They are related by⁹

$$\mathcal{Z} = -\frac{i}{2} \tilde{D} \tilde{D} \widehat{\mathcal{Z}} + \frac{i}{2} \overline{D} \overline{D} \overline{\widehat{\mathcal{Z}}} \quad (5.3)$$

and the real linear superfield L is

$$L = D\chi - \overline{D}\overline{\chi}. \quad (5.4)$$

Chirality of χ_α implies linearity of L .

There is a gauge invariance acting on the long CC superfield. According to eqs. (5.1) and (5.4), $\mathcal{Z} = 0$ if $\Phi = 0$ and $D\chi = \overline{D}\overline{\chi}$. The second condition is a Bianchi identity verified by

$$\chi_\alpha = -\frac{i}{4} \overline{D} \overline{D} D_\alpha \Pi, \quad \overline{\chi}_{\dot{\alpha}} = -\frac{i}{4} D D \overline{D}_{\dot{\alpha}} \Pi, \quad (\Pi \text{ real}). \quad (5.5)$$

Hence, \mathcal{Z} is invariant under

$$\widehat{\mathcal{Z}} \longrightarrow \widehat{\mathcal{Z}} + \mathcal{W} \quad (5.6)$$

⁹Identities in Appendix A may help.

where \mathcal{W} is a Maxwell (chiral-chiral) superfield (3.1). This gauge invariance eliminates $8_B + 8_F$ components in $\widehat{\mathcal{Z}}$. We now proceed to construct a “long” chiral-antichiral $\mathcal{N} = 2$ superfield for the super-Maxwell theory.

5.1 The chiral-antichiral $\mathcal{N} = 2$ superfield

A generic chiral-antichiral superfield, $\overline{D}_{\dot{\alpha}}\widehat{\mathcal{W}} = \widetilde{D}_{\alpha}\widehat{\mathcal{W}} = 0$, has the expansion

$$\widehat{\mathcal{W}} = U + \sqrt{2}\widetilde{\theta}\overline{\Omega} - \widetilde{\theta}\overline{\theta}\left[\frac{i}{2}X + \frac{1}{4}\overline{DD}U\right], \quad (5.7)$$

where the $\mathcal{N} = 1$ superfields U, X and $\overline{\Omega}_{\dot{\alpha}}$ which include $16_B + 16_F$ fields, are chiral: they vanish under $\overline{D}_{\dot{\alpha}}$. In components, $\overline{\Omega}_{\dot{\alpha}}$ includes a complex vector \mathbb{V}_{μ} (8_B) and two Majorana fermions:

$$\overline{\Omega}_{\dot{\alpha}} = \overline{\omega}_{\dot{\alpha}} + (\theta\sigma^{\mu})_{\dot{\alpha}}\mathbb{V}_{\mu} - \theta\overline{\theta}\overline{\lambda}_{\dot{\alpha}}. \quad (5.8)$$

Such a chiral right-handed (the index $\dot{\alpha}$) spinor superfield can always be written as

$$\overline{\Omega}_{\dot{\alpha}} = \overline{D}_{\dot{\alpha}}\mathbb{L}, \quad \Omega_{\alpha} = -D_{\alpha}\overline{\mathbb{L}}, \quad (5.9)$$

where \mathbb{L} is complex linear, $\overline{DD}\mathbb{L} = 0$. In components, a complex linear superfield can be written

$$\mathbb{L}(x, \theta, \overline{\theta}) = \Phi(x, \theta, \overline{\theta}) - \overline{\theta}\omega - \theta\sigma^{\mu}\overline{\theta}\mathbb{V}_{\mu} + \theta\overline{\theta}\overline{\lambda} - \frac{i}{2}\overline{\theta}\overline{\theta}\theta\sigma^{\mu}\partial_{\mu}\overline{\omega} + \frac{i}{2}\theta\overline{\theta}\overline{\theta}\overline{\theta}\partial^{\mu}\mathbb{V}_{\mu} \quad (5.10)$$

with Φ chiral, $\overline{D}_{\dot{\alpha}}\Phi = 0$, an expansion which leads directly to $\overline{D}_{\dot{\alpha}}\mathbb{L} = \overline{\Omega}_{\dot{\alpha}}$ in eq. (5.8). In other words,

$$\widehat{\mathcal{W}} = U + \sqrt{2}\widetilde{\theta}\overline{D}\mathbb{L} - \widetilde{\theta}\overline{\theta}\left[\frac{i}{2}X + \frac{1}{4}\overline{DD}U\right] \quad (5.11)$$

in general.

Upon defining the chiral-chiral superfield

$$\mathcal{W} = -\frac{i}{2}\overline{DD}\widetilde{\mathcal{W}} + \frac{i}{2}\overline{DD}\overline{\widetilde{\mathcal{W}}}, \quad (5.12)$$

one finds

$$\begin{aligned} \mathcal{W} &= X + \sqrt{2}i\widetilde{\theta}^{\alpha}\overline{D}_{\dot{\alpha}}\left[D_{\alpha}\overline{\Omega}^{\dot{\alpha}} + \frac{1}{2}\overline{D}^{\dot{\alpha}}\Omega_{\alpha}\right] - \widetilde{\theta}\overline{\theta}\frac{1}{4}\overline{DD}X \\ &= X + \sqrt{2}i\widetilde{\theta}^{\alpha}W_{\alpha} - \widetilde{\theta}\overline{\theta}\frac{1}{4}\overline{DD}X, \end{aligned} \quad (5.13)$$

where W_{α} is the usual Maxwell chiral superfield

$$W_{\alpha} = -\frac{1}{4}\overline{DD}D_{\alpha}V$$

with, however,

$$V = 2(\mathbb{L} + \overline{\mathbb{L}}) \quad (5.14)$$

instead of V being simply a real superfield. This new condition follows from

$$\overline{D}_{\dot{\alpha}}\left[D_{\alpha}\overline{\Omega}^{\dot{\alpha}} + \frac{1}{2}\overline{D}^{\dot{\alpha}}\Omega_{\alpha}\right] = -\frac{1}{2}\overline{DD}D_{\alpha}(\mathbb{L} + \overline{\mathbb{L}}), \quad (5.15)$$

which is a consequence of (5.12). The $\mathcal{N} = 2$ gauge transformation of $\widehat{\mathcal{W}}$ leaving \mathcal{W} invariant can be read from expressions (5.13) and (5.14): $\mathcal{W} = 0$ if $X = 0$ and $\mathbb{L} = iL$, with a real linear L . In other words, \mathcal{W} is invariant under

$$\widehat{\mathcal{W}} \quad \longrightarrow \quad \widehat{\mathcal{W}} + \mathcal{Y}, \quad \mathcal{Y} = U + \sqrt{2i} \overline{\theta} \overline{D}L - \overline{\theta\theta} \frac{1}{4} \overline{D}D\overline{U}. \quad (5.16)$$

Eq. (5.1) indicates that this gauge variation is induced by a single-tensor supermultiplet in a “short” chiral-antichiral superfield.

5.2 The long and short super-Maxwell superfields

To summarize, to describe the single-tensor and the Maxwell multiplet, we have obtained two pairs of $\mathcal{N} = 2$ superfields respectively, with each pair containing one long ($16_B + 16_F$) and one short ($8_B + 8_F$) superfield:

	Long, $16_B + 16_F$	Short, $8_B + 8_F$	Gauge variation, $8_B + 8_F$	
Maxwell:	$\widehat{\mathcal{W}}$	\mathcal{W}	$\delta \widehat{\mathcal{W}} = \mathcal{Z}_{\text{gauge}}$	$\delta \mathcal{W} = 0$
Single-tensor:	$\widehat{\mathcal{Z}}$	\mathcal{Z}	$\delta \widehat{\mathcal{Z}} = \mathcal{W}_{\text{gauge}}$	$\delta \mathcal{Z} = 0$

Counting off-shell degrees of freedom in the “long” Maxwell multiplet is interesting. Firstly, X and U include $8_B + 8_F$ fields while the complex linear \mathbb{L} has $12_B + 12_F$ components.¹⁰ The superfield $\widehat{\mathcal{W}}$ depends however on $\overline{D}_{\dot{\alpha}}\mathbb{L}$ and one can write $\mathbb{L} = \Phi + \Delta\mathbb{L}$ (Φ chiral), with $8_B + 8_F$ fields in $\Delta\mathbb{L}$: the superfield $\widehat{\mathcal{W}}$ sees then only $16_B + 16_F$ fields. One actually expects that a larger supermultiplet with $24_B + 24_F$ fields exists, with all $\mathcal{N} = 2$ partners of \mathbb{L} . This is discussed in appendix B.

The variation (5.16) is not the gauge transformation of the super-Maxwell theory: it does not act on $V = 2(\mathbb{L} + \overline{\mathbb{L}})$. It only allows to eliminate U and $4_B + 4_F$ components of \mathbb{L} , leaving X, V, W_{α} and then also the $\mathcal{N} = 2$ superfield \mathcal{W} unchanged. The standard Maxwell gauge transformation $V \rightarrow V + \Lambda + \overline{\Lambda}$ is actually

$$\mathbb{L} \quad \longrightarrow \quad \mathbb{L} + \frac{1}{2} \Lambda, \quad \overline{D}_{\dot{\alpha}} \Lambda = 0, \quad (5.17)$$

which is a symmetry of $\widehat{\mathcal{W}}$.¹¹ A comparison of $2(\mathbb{L} + \overline{\mathbb{L}})$ with the standard expansion of the Maxwell real superfield indicates that the gauge field and the auxiliary $\theta\theta\overline{\theta\theta}$ component are respectively

$$\begin{aligned} A_{\mu} &= -4 \operatorname{Re} \mathbb{V}_{\mu}, \\ D &= -4 \partial^{\mu} \operatorname{Im} \mathbb{V}_{\mu}. \end{aligned} \quad (5.18)$$

Replacing the scalar D by the divergence of a vector field has nontrivial consequences which are precisely discussed in appendix C. In short, the role of the FI coefficient ξ is taken by an integration constant appearing when solving the field equation of $\operatorname{Im} \mathbb{V}_{\mu}$ and a well-defined procedure for the elimination of $\operatorname{Im} \mathbb{V}_{\mu}$ shows that the theories formulated with either D or $\operatorname{Im} \mathbb{V}_{\mu}$ are physically equivalent.

¹⁰It is a complex superfield ($16_B + 16_F$) with the chiral constraint $\overline{D}D\mathbb{L} = 0$, removing $4_B + 4_F$ fields.

¹¹See appendix B.

5.3 Long superfield and nonlinear deformations

According to relation (5.12), the nonlinear deformation \mathcal{W}_{nl} can be transferred to a deformation $\widehat{\mathcal{W}}_{\text{nl}}$ only if $A^2 = \overline{B}^2$, $\Gamma = 0$ since the only available chiral-antichiral deformation term would be

$$\widehat{\mathcal{W}}_{\text{nl}} = -\frac{i}{2} A^2 \theta\theta \overline{\theta\theta}. \quad (5.19)$$

This is the case if the deformation can be viewed as a background value of the auxiliary F in X , which never leads to partial breaking. A similar argument holds for the single-tensor superfield with relation (5.3). Then, to consider a general deformation and in particular if the interest is in partial supersymmetry breaking, the deformed short version of the superfields must be used. Since these short superfields have different chiralities, writing an interaction of two deformed supermultiplets is problematic.

6 Interactions

6.1 The Chern-Simons interaction

The interaction of a $\mathcal{N} = 2$ Maxwell multiplet with a single-tensor multiplet can be introduced either by a supersymmetrization of the Chern-Simons coupling $B \wedge F$ or by a supersymmetrization of $F_{\mu\nu} - B_{\mu\nu}$. These options are related via electric-magnetic duality. The supersymmetric interaction exists for off-shell fields and can be written in $\mathcal{N} = 2$ or $\mathcal{N} = 1$ superspace. The goal of this subsection is to discuss the Chern-Simons coupling of a nonlinear or constrained Maxwell or single-tensor multiplet with unbroken linear $\mathcal{N} = 1$, to its counterpart with linear $\mathcal{N} = 2$.

In terms of $\mathcal{N} = 1$ superfields, the $\mathcal{N} = 2$ Chern-Simons interaction can be written in two simple ways. Firstly, using (L, Φ) and (V_1, V_2) to describe the single-tensor and Maxwell multiplets respectively, the Chern-Simons interaction with (real) coupling g can be written as a $\mathcal{N} = 1$ D -term [16, 19]:

$$\mathcal{L}_{\text{CS}} = -g \int d^2\theta d^2\bar{\theta} \left[V_1(\Phi + \overline{\Phi}) + V_2 L \right]. \quad (6.1)$$

It is invariant under the second supersymmetry variations (2.3) and (B.1) and it is also gauge invariant. A second expression using an F -term exists in terms of χ_α , Φ for the single-tensor and X , W_α for the Maxwell multiplet, using the relations

$$L = D\chi - \overline{D}\chi, \quad W_\alpha = -\frac{1}{4} \overline{D}D D_\alpha V_2, \quad X = \frac{1}{2} \overline{D}D V_1$$

and some partial integrations:

$$\mathcal{L}_{\text{CS}} = g \int d^2\theta \left[\frac{1}{2} \Phi X + \chi^\alpha W_\alpha \right] + g \int d^2\bar{\theta} \left[\frac{1}{2} \overline{\Phi} \overline{X} - \overline{\chi}_{\dot{\alpha}} \overline{W}^{\dot{\alpha}} \right]. \quad (6.2)$$

The expressions (6.1) and (6.2) differ by a derivative term. The chiral form can be extended to a chiral integral over $\mathcal{N} = 2$ superspace, using the chiral-chiral superfields \mathcal{W} and $\widehat{\mathcal{Z}}$ for

the Maxwell and single-tensor multiplets respectively [19]:¹²

$$\mathcal{L}_{\text{CS}} = ig \int d^2\theta \int d^2\tilde{\theta} \mathcal{W} \widehat{\mathcal{Z}} + \text{h.c.} \quad (6.3)$$

All dependence on Y disappears in the imaginary part of $[\mathcal{W} \widehat{\mathcal{Z}}]_{\theta\bar{\theta}}$ (under a spacetime integral). This expression is also invariant under the gauge transformation (5.6) of $\widehat{\mathcal{Z}}$, since, for any pair of (short) Maxwell multiplets \mathcal{W}_1 and \mathcal{W}_2 ,

$$\text{Im} \int d^2\theta \int d^2\tilde{\theta} \mathcal{W}_1 \mathcal{W}_2 \quad \text{and} \quad \text{Im} \int d^2\theta W_1^\alpha W_{2\alpha}$$

are derivative terms.

Finally, one can also write the Chern-Simons lagrangian using the chiral-antichiral superfields \mathcal{Z} (short) and $\widehat{\mathcal{W}}$ (long) for the single-tensor and the Maxwell multiplet respectively¹³

$$\mathcal{L}_{\text{CS}} = ig \int d^2\theta \int d^2\tilde{\theta} \widehat{\mathcal{W}} \mathcal{Z} + \text{h.c.} \quad (6.4)$$

This can be verified either by direct calculation or by using relation (5.12) and partial integrations in expression (6.3) and of course $V_2 = 2(\mathbb{L} + \bar{\mathbb{L}})$. Equation (6.4) is invariant up to a derivative term under the gauge transformation (B.13) of $\widehat{\mathcal{W}}$, since, for any pair of (short) single-tensor multiplets $\mathcal{Z}_1, \mathcal{Z}_2$,

$$\text{Im} \int d^2\theta \int d^2\tilde{\theta} \mathcal{Z}_1 \mathcal{Z}_2 \quad \text{and} \quad \text{Im} \int d^2\theta (\bar{D}_{\dot{\alpha}} L_1) (\bar{D}^{\dot{\alpha}} L_2)$$

are derivative terms.

In terms of the $\mathcal{N} = 1$ component superfields,

$$\mathcal{L}_{\text{CS}} = g \int d^2\theta \left[\frac{1}{2} \Phi X + (\bar{D}L)(\bar{D}\bar{L}) \right] + g \int d^2\bar{\theta} \left[\frac{1}{2} \bar{\Phi} \bar{X} + (DL)(D\bar{L}) \right]. \quad (6.5)$$

In components, using expansions (2.10) and (5.10), we find that (under a spacetime integral)

$$\begin{aligned} \mathcal{L}_{\text{CS}} = & -\frac{1}{2} g (x f + \bar{x} \bar{f} + z F + \bar{z} \bar{F} + \kappa \psi + \bar{\kappa} \bar{\psi}) + ig \lambda \varphi - ig \bar{\lambda} \bar{\varphi} \\ & - \frac{1}{8} g \epsilon_{\mu\nu\rho\sigma} B^{\mu\nu} F^{\rho\sigma} + 2g C \partial_\mu \text{Im} \mathbb{V}^\mu - g \partial_\mu \varphi \sigma^\mu \bar{\omega} - g \omega \sigma^\mu \partial_\mu \bar{\varphi}, \end{aligned} \quad (6.6)$$

where $F^{\rho\sigma} \equiv \partial^\rho A^\sigma - \partial^\sigma A^\rho$.

6.1.1 The Chern-Simons interaction with deformed Maxwell multiplet

The nonlinearly-deformed Maxwell multiplet is described by the CC superfield \mathcal{W} , including the deformation terms (3.8). This leads to the Chern-Simons interaction

$$\begin{aligned} \mathcal{L}_{\text{nl}} = & ig \int d^2\theta \int d^2\tilde{\theta} \widehat{\mathcal{Z}} \mathcal{W} + \text{h.c.} \\ = & \mathcal{L}_{\text{CS}} + ig \int d^2\theta \left[B^2 Y - \sqrt{2} \Gamma \theta \chi - A^2 \theta \theta \left(\frac{i}{2} \Phi + \frac{1}{4} \overline{DDY} \right) \right] + \text{h.c.}, \end{aligned} \quad (6.7)$$

¹²See eqs. (3.1) and (5.2).

¹³Eqs. (3.11) and (5.11).

where \mathcal{L}_{CS} is given by (6.2). For the partial breaking, using $A = \Gamma = 0$, we obtain

$$\mathcal{L}_{\text{nl}} = g \int d^2\theta \left[\frac{1}{2} \Phi X + \chi^\alpha W_\alpha + iB^2 Y \right] + \text{h.c.} \quad (6.8)$$

The second supersymmetry variation $\sqrt{2}iB^2\eta\chi$ of iB^2Y is cancelled by the nonlinear variation of W_α , $\delta^*W_\alpha = -\sqrt{2}iB^2\eta_\alpha + \text{linear}$. However, the equation of motion of Y is inconsistent. One can get around this problem by using $l > 1$ deformed Maxwell multiplets (namely one “long” single-tensor and at least two “short” and deformed Maxwell multiplets), as then the relevant equation of motion would take the form of a tadpole-like condition

$$g_a B_a^2 = 0, \quad a = 1, \dots, l, \quad (6.9)$$

where g_a would be the coupling of each Chern-Simons interaction. This is in agreement with the claim made in [26] and [27], namely that one cannot couple hypermultiplets to a single Maxwell multiplet in a theory with partial breaking induced by the latter.

The Chern-Simons interaction (6.8) can be combined with the kinetic lagrangian

$$\mathcal{L}_{\text{kin.}} = \int d^2\theta d^2\bar{\theta} \mathcal{H}(L, \Phi, \bar{\Phi}) + \frac{1}{2} \int d^2\theta \int d^2\bar{\theta} \mathcal{F}(\mathcal{W}) + \text{h.c.} \quad (6.10)$$

for the two multiplets, as well as with an FI contribution

$$\mathcal{L}_{\text{FI}} = \xi \int d^2\theta d^2\bar{\theta} V_2 + \frac{1}{2} m^2 \int d^2\theta X + \text{h.c.} \quad (6.11)$$

The theory depends then on a function \mathcal{H} solving the Laplace equation and on an arbitrary holomorphic function \mathcal{F} . Imposing the constraint $\mathcal{W}^2 = 0$ (where \mathcal{W} is deformed) eliminates X , which becomes a function $X(WW)$ of WW and its derivatives. Moreover, due to the constraint, the lagrangian no longer depends on \mathcal{F} and it reduces to

$$\int d^2\theta d^2\bar{\theta} \mathcal{H}(L, \Phi, \bar{\Phi}) + \xi \int d^2\theta d^2\bar{\theta} V_2 + \frac{1}{2} m^2 \int d^2\theta X + \text{h.c.} \quad (6.12)$$

The resulting theory has a linear $\mathcal{N} = 1$ as well as a second nonlinear supersymmetry and has been analyzed in [19].

6.1.2 The Chern-Simons interaction with deformed single-tensor multiplet

In the analogous procedure for the nonlinear single-tensor multiplet, the CA superfield (3.11) with deformation (3.13) is coupled to the long Maxwell CA superfield (5.11):

$$\begin{aligned} \mathcal{L}_{\text{nl}} &= ig \int d^2\theta \int d^2\bar{\theta} \widetilde{\mathcal{W}} \mathcal{Z} + \text{h.c.} \\ &= \mathcal{L}_{\text{CS}} + ig \int d^2\theta \left[\widetilde{B}^2 U - \widetilde{A}^2 \theta\theta \left(\frac{i}{2} X + \frac{1}{4} \overline{DDU} \right) \right] + \text{h.c.}, \end{aligned} \quad (6.13)$$

where \mathcal{L}_{CS} is given by (6.5). Requiring now partial breaking with $\widetilde{A} = 0$ yields

$$\mathcal{L}_{\text{nl}} = g \int d^2\theta \left[\frac{1}{2} \Phi X + (\overline{DL})(\overline{DL}) + i\widetilde{B}^2 U \right] + \text{h.c.} \quad (6.14)$$

Since¹⁴

$$\delta^* i\tilde{B}^2 U = \sqrt{2}i\tilde{B}^2 \overline{\eta D} \mathbb{L}, \quad \delta^* \overline{D}_\alpha L = -\sqrt{2}i\tilde{B}^2 \overline{\eta}_\alpha,$$

\mathcal{L}_{nl} is invariant under a linear $\mathcal{N} = 1$ and under a second nonlinear supersymmetry. However, the equation of motion of U is inconsistent as that of Y of the previous subsection — this problem can be solved by coupling the “long” Maxwell multiplet(s) to at least two “short” and deformed single-tensor multiplets.¹⁵

The complete theory has then lagrangian

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{nl}} + & \left[\frac{1}{2} \int d^2\theta \int d^2\tilde{\theta} \mathcal{F}(\mathcal{W}) + \int d^2\theta \int d^2\tilde{\theta} \mathcal{G}(\mathcal{Z}) + \int d^2\theta \tilde{m}^2 \Phi \right] + \text{h.c.} \\ & + \xi \int d^2\theta d^2\tilde{\theta} V_2, \end{aligned} \quad (6.15)$$

where \mathcal{Z} is deformed and we have added an FI term for V_2 . Upon imposing the constraint (4.13), \mathcal{G} does not contribute to (6.15), since

$$\int d^2\theta \int d^2\tilde{\theta} \mathcal{Z} + \text{h.c.} \sim \int d^2\theta \overline{D}^2 \overline{\Phi} + \text{h.c.} = \text{deriv. term} \quad (6.16)$$

and the bosonic part of (6.15) becomes

$$\begin{aligned} \mathcal{L}_{\text{bos}} = & \frac{1}{2} \int d^2\theta \int d^2\tilde{\theta} \mathcal{F}(\mathcal{W})|_{\text{bos}} + \text{h.c.} - 2\xi \partial^\mu \text{Im } \mathbb{V}_\mu \\ & + 2g \left(-\frac{1}{26} \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} A^\mu + C \partial^\mu \text{Im } \mathbb{V}_\mu - \tilde{B}^2 \text{Im } F_U \right) \\ & + (g \text{Re } x + 2\tilde{m}^2) \tilde{B}^2 \\ & \cdot \left(1 - \sqrt{1 - \frac{2}{\tilde{B}^4} \left(\frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} + \partial_\mu C \partial^\mu C \right)} - \frac{1}{9\tilde{B}^8} (\epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} \partial^\mu C)^2 \right), \end{aligned} \quad (6.17)$$

where \tilde{B} has been assumed to be real and F_U is the auxiliary field of U . Notice that the lagrangian (4.27) has acquired a field-dependent coefficient $(g \text{Re } x + 2\tilde{m}^2) \tilde{B}^2$ as its analogue, the Born-Infeld lagrangian, does in ref. [19].

The solution of the equation of motion for the auxiliary field F of X is $F = 0$. Moreover, the equation of motion for the auxiliary field $\text{Im } \mathbb{V}_\mu$ is

$$\partial_\mu (16 \text{Re } \mathcal{F}_{xx} \partial^\nu \text{Im } \mathbb{V}_\nu + 2g C) = 0, \quad (6.18)$$

whose solution is

$$16 \text{Re } \mathcal{F}_{xx} \partial^\nu \text{Im } \mathbb{V}_\nu + 2g C = -\lambda, \quad (6.19)$$

¹⁴See appendix B.

¹⁵Note that there is no reason to identify the imaginary part of the auxiliary field of U with a four-form field as was done for Y in [19]. In particular, the variation of Y under the gauge transformation of $\widehat{\mathcal{Z}}$ is $\delta_{\text{gauge}} Y = -\frac{1}{2} \overline{DD} \Delta'$ [19], where Δ' is a real superfield, while the variation of U under the gauge transformation of $\widehat{\mathcal{W}}$ is $\delta_{\text{gauge}} U = \Sigma_c$ (see (B.12) of appendix B) and the chiral superfield Σ_c is not necessarily identified with $\overline{DD} \Delta''$, where Δ'' is a real superfield.

where λ is an arbitrary integration constant. For reasons explained in appendix C, we make the identification

$$\lambda = 2\xi. \tag{6.20}$$

The scalar potential of the theory is then

$$V = \frac{1}{32 \operatorname{Re} \mathcal{F}_{xx}} (2gC - 2\xi)^2, \tag{6.21}$$

whose supersymmetric vacuum is at

$$\langle C \rangle = \frac{\xi}{g}. \tag{6.22}$$

In this vacuum, x corresponds to a flat direction of the potential and is massless. The canonically normalized mass $\mathcal{M}_{C,\text{can}}^2$ that C acquires is then

$$\mathcal{M}_{C,\text{can}}^2 = \frac{1}{4} \frac{1}{\operatorname{Re} \mathcal{F}_{xx}} \frac{g^2 \tilde{B}^2}{\operatorname{Re} x + 4\tilde{m}^2}. \tag{6.23}$$

Moreover, the interaction term $-\frac{1}{12}g \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} A^\mu$ generates a mass term for A_μ and we find that the canonically normalized mass $\mathcal{M}_{A_\mu,\text{can}}^2$ is

$$\mathcal{M}_{A_\mu,\text{can}}^2 = \mathcal{M}_{C,\text{can}}^2. \tag{6.24}$$

The spectrum consists then of a massive $\mathcal{N} = 1$ vector multiplet and a massless $\mathcal{N} = 1$ chiral multiplet X ; the Chern-Simons coupling results in the vector multiplet W absorbing the goldstino multiplet, while X remains massless. Consequently, we observe a mechanism analogous to the super-Brout-Englert-Higgs effect without gravity [19], which is induced by the Chern-Simons coupling of the previous subsection (6.1.1).

6.2 Constrained matter multiplets

In subsection 6.1, we described the couplings of the deformed $\mathcal{N} = 2$ goldstino multiplet to unconstrained matter $\mathcal{N} = 2$ multiplets. They are based on a Chern-Simons interaction that couples a Maxwell to a single-tensor multiplet, where one of the two contains the goldstino. In both cases, upon imposing a nilpotent constraint on the goldstino multiplet, the Chern-Simons interaction generates a super-Brout-Englert-Higgs phenomenon without gravity, where the goldstino is absorbed in a massive $\mathcal{N} = 1$ vector multiplet, while a massless chiral multiplet remains in the spectrum.

Here, we discuss generalisations of the nilpotent constraint in order to describe, besides the goldstino, incomplete matter multiplets of non-linear supersymmetry in which half of the degrees of freedom are integrated out of the spectrum, giving rise to constraints. Examples of such constraints in $\mathcal{N} = 1$ non-linear supersymmetry, which is described by the nilpotent goldstino superfield X with $X^2 = 0$, are given by

$$X\Phi = 0, \tag{6.25}$$

which eliminates the scalar component of the matter chiral superfield Φ , or

$$X\bar{\Phi} = \text{chiral}, \quad (6.26)$$

that eliminates the fermion component of Φ [5]. In $\mathcal{N} = 2$, we examine below both cases, with the goldstino being part of either a nilpotent (deformed) Maxwell multiplet \mathcal{W} with $\mathcal{W}^2 = 0$, or of a nilpotent (deformed) single-tensor multiplet \mathcal{Z} with $\mathcal{Z}^2 = 0$.

6.2.1 The goldstino in the Maxwell multiplet

Consider the case in which the goldstino is in a deformed Maxwell multiplet \mathcal{W}_0 , given by (4.1)

$$\mathcal{W}_0 = X_0 + \sqrt{2}i\tilde{\theta}W_0 + \tilde{\theta}\tilde{\theta} \left[B^2 - \frac{1}{4}\overline{DD}X_0 \right], \quad (6.27)$$

which satisfies the constraint $\mathcal{W}_0^2 = 0$, or, equivalently, eq. (4.4) [14]:

$$X_0 = -2 \frac{W_0W_0}{4B^2 - \overline{DD}X_0}. \quad (6.28)$$

To describe an incomplete $\mathcal{N} = 2$ vector multiplet with non-linear supersymmetry containing an $\mathcal{N} = 1$ vector W_1 , we consider the $\mathcal{N} = 2$ constraint

$$\mathcal{W}_0\mathcal{W}_1 = 0, \quad (6.29)$$

where \mathcal{W}_1 is an undeformed (and short) Maxwell multiplet given by (3.1):

$$\mathcal{W}_1 = X_1 + \sqrt{2}i\tilde{\theta}W_1 - \frac{1}{4}\tilde{\theta}\tilde{\theta}\overline{DD}X_1. \quad (6.30)$$

The constraint (6.29) then yields the following set of equations

$$\begin{aligned} X_0X_1 &= 0, \\ X_0W_{1\alpha} + X_1W_{0\alpha} &= 0, \\ X_1B^2 - \frac{1}{4}\overline{DD}(X_0\bar{X}_1 + X_1\bar{X}_0) + W_0W_1 &= 0. \end{aligned} \quad (6.31)$$

We now use (6.28) and the identity

$$(W_0W_1)W_{0\alpha} = -\frac{1}{2}(W_0W_0)W_{1\alpha} \quad (6.32)$$

to solve the second of equations (6.31), which yields

$$X_1 = -4 \frac{W_0W_1}{4B^2 - \overline{DD}X_0} + hW_0W_0, \quad (6.33)$$

where h is a chiral superfield. This expression verifies the first eq. (6.31) for all h and the third eq. (6.31) if

$$h = -2 \frac{\overline{DD}X_1}{(4B^2 - \overline{DD}X_0)^2} \quad (6.34)$$

and thus

$$X_1 = -4 \frac{W_0 W_1}{4B^2 - \overline{DDX}_0} - 2 \frac{\overline{DDX}_1}{(4B^2 - \overline{DDX}_0)^2} W_0 W_0. \quad (6.35)$$

One may further use the solution (4.14) for X_0 and solve (6.35) to obtain X_1 as a function of W_0 , W_1 and their derivatives; the constraint (6.29) eliminates X_1 .

Note that the constraint $\mathcal{W}_0^2 = \mathcal{W}_0 \mathcal{W}_1 = 0$ is a particular case of the system of equations

$$d_{abc} \mathcal{W}_b \mathcal{W}_c = 0; \quad a, b, c = 1, \dots, l \quad (6.36)$$

introduced in [28, 29] to obtain coupled DBI (Dirac-Born-Infeld) actions. In eqs. (6.36), all \mathcal{W}_a are in general deformed with different deformation parameters B_a and the constants d_{abc} are totally symmetric. Our constraints correspond to the case of two $\mathcal{N} = 2$ vector multiplets with $d_{000} = d_{001} = 1$ and all other d 's vanishing.

We can also describe incomplete $\mathcal{N} = 2$ single-tensor multiplets containing a single $\mathcal{N} = 1$ chiral multiplet. For that, let us consider the constraint

$$\mathcal{W}_0 \widehat{\mathcal{Z}} = 0, \quad (6.37)$$

where $\widehat{\mathcal{Z}}$ is a “long”¹⁶ single-tensor multiplet given by (5.2). Equation (6.37) then leads to

$$\begin{aligned} X_0 Y &= 0, \\ X_0 \chi_\alpha + i Y W_{0\alpha} &= 0, \\ Y B^2 - \frac{i}{2} \Phi X_0 - \frac{1}{4} \overline{DD}(X_0 \overline{Y} + Y \overline{X}_0) - i W_0 \chi &= 0, \end{aligned} \quad (6.38)$$

which, following the same steps as before, yield

$$Y = 4i \frac{W_0 \chi}{4B^2 - \overline{DDX}_0} - 2 \frac{2i\Phi + \overline{DDY}}{(4B^2 - \overline{DDX}_0)^2} W_0 W_0, \quad (6.39)$$

which again one may solve to eliminate $Y = Y(W_0, \chi, \Phi)$.

One can also check if the expression (6.39) is covariant under the gauge variation (5.6)

$$\widehat{\mathcal{Z}} \quad \longrightarrow \quad \widehat{\mathcal{Z}} + \mathcal{W}_g, \quad (6.40)$$

where \mathcal{W}_g is a “short” (undeformed) Maxwell multiplet with components $(X_g, W_{g\alpha})$, or, equivalently,

$$\delta Y = X_g, \quad \delta \chi_\alpha = i W_{g\alpha}, \quad \delta \Phi = 0. \quad (6.41)$$

Under (6.41), the expression (6.39) becomes

$$X_g = -4 \frac{W_0 W_g}{4B^2 - \overline{DDX}_0} - 2 \frac{\overline{DDX}_g}{(4B^2 - \overline{DDX}_0)^2} W_0 W_0, \quad (6.42)$$

which, as was previously shown, is actually the consequence of

$$\mathcal{W}_0 \mathcal{W}_g = 0, \quad (6.43)$$

¹⁶Note that it is easy to check that the constraint $\mathcal{W}_0 \mathcal{Z} = 0$, where \mathcal{Z} is a “short” single-tensor multiplet, leads to an overconstrained system of equations.

that is the variation of (6.37) under (6.40). The expression (6.39) is thus invariant only under the reduced gauge transformations (6.40) subject to the constraint (6.43). These are not sufficient to eliminate all unphysical components of $\widehat{\mathcal{Z}}$.

Alternatively, we can consider that we actually solve the constraints $\mathcal{W}_0(\widehat{\mathcal{Z}} - \mathcal{W}_g) = \mathcal{W}_0\mathcal{W}_g = 0$, where $\widehat{\mathcal{Z}} - \mathcal{W}_g$ is gauge invariant and \mathcal{W}_g can be eliminated by a gauge transformation (6.40). One can then choose $Y - X_g = 0$ and use eq. (6.39) to eliminate $\chi - iW_g$ in terms of the $\mathcal{N} = 1$ chiral superfield Φ :

$$\chi_\alpha - iW_{g\alpha} = \frac{\Phi}{(4B^2 - \overline{DD}X_0)} W_{0\alpha}. \quad (6.44)$$

In the physically-relevant linear superfield L however, W_g disappears:

$$L = D\chi - \overline{D}\chi = D(\chi - iW_g) - \overline{D}(\overline{\chi} - i\overline{W}_g),$$

since W_g verifies the Bianchi identity.

6.2.2 The goldstino in the single-tensor multiplet

Now let us consider the case in which the goldstino is in a deformed single-tensor multiplet \mathcal{Z}_0 , given by

$$\mathcal{Z}_0 = \Phi_0 + \sqrt{2}i\overline{\theta}DL_0 + \overline{\theta}\overline{\theta} \left[\overline{B}^2 - \frac{1}{4}\overline{DD}\Phi_0 \right], \quad (6.45)$$

which satisfies (4.13)

$$\mathcal{Z}_0^2 = 0, \quad (6.46)$$

or equivalently eq. (4.10) [18]:

$$\Phi_0 = -2 \frac{(\overline{D}L_0)(\overline{D}L_0)}{4\overline{B}^2 - \overline{DD}\Phi_0}. \quad (6.47)$$

To describe another incomplete $\mathcal{N} = 2$ single-tensor multiplet with non-linear supersymmetry containing an $\mathcal{N} = 1$ linear multiplet, we consider the $\mathcal{N} = 2$ constraint

$$\mathcal{Z}_0\mathcal{Z}_1 = 0, \quad (6.48)$$

where \mathcal{Z}_1 is an undeformed (and short) single-tensor multiplet given by (3.11)

$$\mathcal{Z}_1 = \Phi_1 + \sqrt{2}i\overline{\theta}DL_1 - \frac{1}{4}\overline{\theta}\overline{\theta}\overline{DD}\Phi_1. \quad (6.49)$$

Following the same steps as before, as well as the identity

$$(\overline{D}L_0\overline{D}L_1)\overline{D}_\alpha L_0 = -\frac{1}{2}(\overline{D}L_0\overline{D}L_0)\overline{D}_\alpha L_1, \quad (6.50)$$

we find

$$\Phi_1 = -4 \frac{\overline{D}L_0\overline{D}L_1}{4\overline{B}^2 - \overline{DD}\Phi_0} - 2 \frac{\overline{DD}\Phi_1}{(4\overline{B}^2 - \overline{DD}\Phi_0)^2} \overline{D}L_0\overline{D}L_0, \quad (6.51)$$

which one may solve to eliminate the chiral component Φ_1 in terms of L_1 and the goldstino multiplet L_0 . Note that the constraints $\mathcal{Z}_0^2 = \mathcal{Z}_0 \mathcal{Z}_1 = 0$ can be generalised to a system of equations

$$\tilde{d}_{abc} \mathcal{Z}_b \mathcal{Z}_c = 0; \quad a, b, c = 1, \dots, l, \quad (6.52)$$

in analogy with the system (6.36), where \tilde{d}_{abc} are totally symmetric constants, in order to obtain a coupled action of non-linear (deformed) single-tensor multiplets.

Finally, we consider the constraint

$$\mathcal{Z}_0 \widehat{\mathcal{W}} = 0, \quad (6.53)$$

where $\widehat{\mathcal{W}} = 0$ is a “long” Maxwell multiplet given by (5.7), and, using the same procedure as before, we obtain

$$U = 4i \frac{\overline{D}L_0 \overline{D}\mathbb{L}}{4\tilde{B}^2 - \overline{D}D\Phi_0} - 2 \frac{2iX + \overline{D}D\overline{U}}{(4\tilde{B}^2 - \overline{D}D\Phi_0)^2} \overline{D}L_0 \overline{D}L_0, \quad (6.54)$$

which eliminates U . Using the same reasoning as before, one can show that the solution (6.54) is invariant under the reduced gauge variation (5.16)

$$\widehat{\mathcal{W}} \quad \longrightarrow \quad \widehat{\mathcal{W}} + \mathcal{Z}_g, \quad (6.55)$$

where \mathcal{Z}_g is a “short” (undeformed) single-tensor multiplet, namely $\delta U = \Phi_g$, $\delta \mathbb{L} = iL_g$, $\delta X = 0$, satisfying the constraint

$$\mathcal{Z}_0 \mathcal{Z}_g = 0. \quad (6.56)$$

Following the same procedure as for the solution of the constraint (6.37), one can use the full gauge invariance to set $U = 0$. Eq. (6.54) can then be used to eliminate $\overline{\Omega}_{\dot{\alpha}} = \overline{D}_{\dot{\alpha}} \mathbb{L}$ in terms of the $\mathcal{N} = 1$ chiral superfield X :¹⁷

$$\overline{D}_{\dot{\alpha}} \mathbb{L} = \frac{X}{4\tilde{B}^2 - \overline{D}D\Phi_0} \overline{D}_{\dot{\alpha}} L_0. \quad (6.57)$$

This result defines \mathbb{L} up to the addition of an arbitrary chiral field: as expected, the constraint equation (6.57) is invariant under the Maxwell gauge transformation

$$\mathbb{L} \quad \longrightarrow \quad \mathbb{L} + \Lambda_c, \quad \overline{D}_{\dot{\alpha}} \Lambda_c = 0$$

(see appendix B). In addition, the physically-relevant $V = 2(\mathbb{L} + \overline{\mathbb{L}})$ is invariant under the gauge ambiguity (6.55).

¹⁷Since L is real linear, SL is complex linear for any chiral S .

7 Conclusions

In this work, we studied the off-shell partial breaking of global $\mathcal{N} = 2$ supersymmetry using constrained $\mathcal{N} = 2$ superfields. The corresponding Goldstone fermion belongs to a vector or a linear multiplet of the unbroken $\mathcal{N} = 1$ supersymmetry and is described by a deformed $\mathcal{N} = 2$ Maxwell or single-tensor superfield, respectively, satisfying a nilpotent constraint. Unlike $\mathcal{N} = 1$ non-linear supersymmetry, where the nilpotent constraint assumes a non-vanishing expectation value for the F-component of the goldstino superfield arising a priori from the underlying dynamics, in $\mathcal{N} = 2$, non-linear supersymmetry is imposed by hand through a non-trivial deformation that cannot be obtained by an expectation value of the auxiliary fields.

We then studied interactions between the goldstino and matter multiplets of $\mathcal{N} = 2$ supersymmetry (vectors and single-tensors that have off-shell descriptions), as well as generalisations of the nilpotent constraints describing incomplete matter multiplets. The interactions are of the Chern-Simons type and describe a super-Brout-Englert-Higgs phenomenon without gravity where the goldstino is absorbed into a massive $\mathcal{N} = 1$ vector multiplet. The constraints describe, in the case of a goldstino in a Maxwell multiplet, either incomplete $\mathcal{N} = 2$ vector multiplets containing only a $\mathcal{N} = 1$ vector, or incomplete (“long”) $\mathcal{N} = 2$ single-tensors containing a $\mathcal{N} = 1$ chiral multiplet. Similarly, in the case of a goldstino in a linear multiplet, the constraints describe either incomplete single-tensors containing a $\mathcal{N} = 1$ linear multiplet, or (“long”) Maxwell containing a $\mathcal{N} = 1$ chiral multiplet. We were not able to find constraints on incomplete $\mathcal{N} = 2$ matter multiplets that do the opposite, keeping the $\mathcal{N} = 1$ linear component of a single-tensor in the first case, or the $\mathcal{N} = 1$ vector component of the Maxwell multiplet in the latter case.

It would be interesting to study the interactions of the Goldstone degrees of freedom of a massive spin-3/2 multiplet consisting of an $\mathcal{N} = 1$ vector and an $\mathcal{N} = 1$ linear multiplet. It is not clear whether our results are sufficient to provide a description of such a system. Another open but related question is the coupling to supergravity realising partial breaking of $\mathcal{N} = 2$ supersymmetry and its rigid limit.

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A Conventions and some useful identities

The notation $[\dots]$ in (2.1) is used for antisymmetrization with weight one. Specifically,

$$\partial_{[\mu} B_{\nu\rho]} = \frac{1}{6} \partial_{\mu} B_{\nu\rho} \pm 5 \text{ permutations}.$$

The supersymmetric derivatives D_α and $\bar{D}_{\dot{\alpha}}$ are the usual $N = 1$ expressions verifying $\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$:

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \quad \bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu. \quad (\text{A.1})$$

As a consequence,

$$[D_\alpha, \bar{D}\bar{D}] = -4i(\sigma^\mu\bar{D})_\alpha\partial_\mu, \quad [\bar{D}_{\dot{\alpha}}, DD] = 4i(D\sigma^\mu)_{\dot{\alpha}}\partial_\mu. \quad (\text{A.2})$$

In \tilde{D}_α and $\tilde{\bar{D}}_{\dot{\alpha}}$, $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ are replaced by $\tilde{\theta}_\alpha, \tilde{\bar{\theta}}_{\dot{\alpha}}$. Note also that $(\bar{D}_{\dot{\alpha}})^\dagger = -D_\alpha$.

The Maxwell field-strength chiral superfields are defined as

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V, \quad \bar{W}_{\dot{\alpha}} = -(W_\alpha)^* \quad (\text{A.3})$$

where V is a real superfield. In addition,

$$\begin{aligned} 2i\partial_\mu\chi\sigma^\mu\tilde{\bar{\theta}} &= \tilde{\bar{\theta}}\bar{D}D\chi, & \bar{D}\bar{D}\tilde{\bar{\theta}}\chi &= -2\tilde{\bar{\theta}}\bar{D}\bar{D}\chi, \\ 2\tilde{\theta}\sigma^\mu\partial_\mu\bar{\omega} &= i\tilde{\theta}^\alpha\bar{D}_{\dot{\alpha}}D_\alpha\bar{\omega}^{\dot{\alpha}}, & \bar{D}\bar{D}\tilde{\theta}\omega &= \tilde{\theta}^\alpha\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\omega_\alpha, \end{aligned} \quad (\text{A.4})$$

where χ_α (left-handed) and $\bar{\omega}_{\dot{\alpha}}$ (right-handed) are $\mathcal{N} = 1$ chiral spinor superfields, $\bar{D}_{\dot{\alpha}}\chi_\beta = \bar{D}_{\dot{\alpha}}\bar{\omega}_{\dot{\beta}} = 0$, and

$$\frac{1}{16}\bar{D}\bar{D}DDY = -\square Y. \quad (\text{A.5})$$

where Y is a chiral superfield, $\bar{D}_{\dot{\alpha}}Y = 0$. Other useful identities are

$$\bar{\eta}_{\dot{\alpha}}\bar{D}\bar{D} = -2\bar{D}_{\dot{\alpha}}\bar{\eta}\bar{D}, \quad \bar{D}\bar{D}D_\alpha\mathbb{L} = -2\bar{D}_{\dot{\alpha}}D_\alpha\bar{D}^{\dot{\alpha}}\mathbb{L} = 4i(\sigma^\mu\bar{D})_\alpha\partial_\mu\mathbb{L},$$

where \mathbb{L} is a complex linear superfield.

B More on the Maxwell supermultiplet

The usual construction of the $\mathcal{N} = 2$ Maxwell multiplet starts with two real $\mathcal{N} = 1$ superfields V_1 and V_2 with second supersymmetry variations

$$\delta^*V_1 = -\frac{i}{\sqrt{2}}(\eta D + \bar{\eta}\bar{D})V_2, \quad \delta^*V_2 = \sqrt{2}i(\eta D + \bar{\eta}\bar{D})V_1. \quad (\text{B.1})$$

The parameters of the $U(1)$ gauge variations are in a single-tensor $\mathcal{N} = 2$ multiplet:

$$\delta_{\text{gauge}}V_1 = \Lambda_\ell, \quad \delta_{\text{gauge}}V_2 = \Lambda_c + \bar{\Lambda}_c, \quad (\text{B.2})$$

with Λ_ℓ real linear and Λ_c chiral: $\Lambda_\ell = \bar{\Lambda}_\ell$, $\bar{D}\bar{D}\Lambda_\ell = 0$, $\bar{D}_{\dot{\alpha}}\Lambda_c = 0$. Under the second supersymmetry,

$$\delta^*\Lambda_\ell = -\frac{i}{\sqrt{2}}(\eta D\Lambda_c + \bar{\eta}\bar{D}\bar{\Lambda}_c), \quad \delta^*\Lambda_c = \sqrt{2}i\bar{\eta}\bar{D}\Lambda_\ell, \quad \delta^*\bar{\Lambda}_c = \sqrt{2}i\eta D\bar{\Lambda}_\ell. \quad (\text{B.3})$$

The gauge field is the $\theta\sigma^\mu\bar{\theta}$ component of V_2 . The $\mathcal{N} = 2$ multiplet containing the field strength $F_{\mu\nu}$ uses the chiral superfields

$$X = \frac{1}{2}\overline{DD}V_1, \quad W_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V_2 \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V_2. \quad (\text{B.4})$$

Variations (B.1) imply:

$$\begin{aligned} \delta^* X &= \sqrt{2}i\eta^\alpha W_\alpha, & \delta^* \bar{X} &= \sqrt{2}i\bar{\eta}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}, \\ \delta^* W_\alpha &= \sqrt{2}i\left[\frac{1}{4}\eta_\alpha\overline{DD}\bar{X} + i(\sigma^\mu\bar{\eta})_\alpha\partial_\mu X\right], \\ \delta^* \bar{W}_{\dot{\alpha}} &= \sqrt{2}i\left[\frac{1}{4}\bar{\eta}_{\dot{\alpha}}DDX - i(\eta\sigma^\mu)_{\dot{\alpha}}\partial_\mu\bar{X}\right]. \end{aligned} \quad (\text{B.5})$$

These are the second supersymmetry variations of the components of the “short” chiral-chiral superfield (3.1):

$$\mathcal{W}(y, \theta, \bar{\theta}) = X + \sqrt{2}i\bar{\theta}W - \frac{1}{4}\bar{\theta}\bar{\theta}\overline{DD}\bar{X}.$$

To go to the “long” Maxwell multiplet, one introduces the complex linear \mathbb{L} with eq. (5.14),

$$V_2 = 2(\mathbb{L} + \bar{\mathbb{L}}), \quad \delta_{\text{gauge}}\mathbb{L} = \frac{1}{2}\Lambda_c, \quad (\text{B.6})$$

and variations (B.1) suggest to write

$$\delta^*\mathbb{L} = \frac{i}{\sqrt{2}}\bar{\eta}\bar{D}V_1, \quad \delta^*\bar{\mathbb{L}} = \frac{i}{\sqrt{2}}\eta DV_1, \quad (\text{B.7})$$

which verifies the linearity conditions $\overline{DD}\mathbb{L} = DD\bar{\mathbb{L}} = 0$. However, \mathbb{L} ($12_B + 12_F$) and V_1 ($8_B + 8_F$) do not form an off-shell representation of $\mathcal{N} = 2$: the algebra does not properly close¹⁸ and the number of off-shell fields is not a multiple of $8_B + 8_F$.

To find the complete multiplet, we rely upon the chiral-antichiral superfield written in its two forms (5.7) and (5.11):

$$\begin{aligned} \widehat{\mathcal{W}} &= U + \sqrt{2}\bar{\theta}\bar{\Omega} - \bar{\theta}\bar{\theta}\left[\frac{i}{2}X + \frac{1}{4}\overline{DD}\bar{U}\right], \\ \widehat{\mathcal{W}} &= U + \sqrt{2}\bar{\theta}\bar{D}\mathbb{L} - \bar{\theta}\bar{\theta}\left[\frac{i}{2}X + \frac{1}{4}\overline{DD}\bar{U}\right]. \end{aligned} \quad (\text{B.8})$$

Since the first expression is a chiral-antichiral superfield with $16_B + 16_F$ components,¹⁹ the second supersymmetry variations

$$\begin{aligned} \delta^* U &= \sqrt{2}\bar{\eta}\bar{\Omega}, \\ \delta^* \bar{\Omega}_{\dot{\alpha}} &= -\frac{i}{\sqrt{2}}\left[X\bar{\eta}_{\dot{\alpha}} + i\bar{D}_{\dot{\alpha}}(\eta DU + \bar{\eta}\bar{D}\bar{U})\right], \\ \delta^* X &= 2\sqrt{2}i\left[\frac{1}{4}\overline{DD}\bar{\eta}\bar{\Omega} - i\eta\sigma^\mu\partial_\mu\bar{\Omega}\right] \end{aligned} \quad (\text{B.9})$$

give an off-shell representation of $\mathcal{N} = 2$ supersymmetry.

¹⁸See below.

¹⁹ U and X have $4_B + 4_F$ components each, Ω_α includes $8_B + 8_F$ fields.

In the second expression (B.8), $\bar{\Omega}_{\dot{\alpha}}$ has been replaced by \mathbb{L} , introducing $4_B + 4_F$ supplementary components which are actually invisible in $\widehat{\mathcal{W}}$: the gauge variation (B.6) leaves $\widehat{\mathcal{W}}$ invariant. In addition, the variation $\delta^* \Omega_{\dot{\alpha}}$ cannot be written as $\bar{D}_{\dot{\alpha}} \delta^* \mathbb{L}$ without a supplementary condition on the chiral X . This is where

$$X = \frac{1}{2} \bar{D} \bar{D} V_1$$

helps by firstly adding $4_B + 4_F$ fields to reach $24_B + 24_F$ with U and \mathbb{L} and secondly by turning the second supersymmetry variations (B.9) into

$$\begin{aligned} \delta^* U &= \sqrt{2} \bar{\eta} \bar{D} \mathbb{L}, & \delta^* \bar{U} &= -\sqrt{2} \eta D \bar{\mathbb{L}}, \\ \delta^* \mathbb{L} &= \frac{i}{\sqrt{2}} \bar{\eta} \bar{D} V_1 + \frac{1}{\sqrt{2}} (\eta D U + \bar{\eta} \bar{D} \bar{U}), \\ \delta^* \bar{\mathbb{L}} &= \frac{i}{\sqrt{2}} \eta D V_1 - \frac{1}{\sqrt{2}} (\eta D U + \bar{\eta} \bar{D} \bar{U}), \\ \delta^* V_1 &= -\frac{i}{\sqrt{2}} (\eta D + \bar{\eta} \bar{D}) 2(\mathbb{L} + \bar{\mathbb{L}}) = -\frac{i}{\sqrt{2}} (\eta D + \bar{\eta} \bar{D}) V_2 \end{aligned} \tag{B.10}$$

which represents $\mathcal{N} = 2$ supersymmetry off-shell.²⁰ This is the long representation of the Maxwell $\mathcal{N} = 2$ supermultiplet with $\mathcal{N} = 1$ superfield content U , \mathbb{L} and V_1 for a total of $24_B + 24_F$ fields. Since

$$\delta^* V_2 = \sqrt{2} i (\eta D + \bar{\eta} \bar{D}) V_1, \tag{B.11}$$

the $16_B + 16_F$ multiplet with superfields V_1 and V_2 is included in the long representation.

The long multiplet has two gauge variations generated by two independent single-tensor multiplets with superfields $(\Lambda_{\ell}, \Lambda_c)$ and $(\Sigma_{\ell}, \Sigma_c)$ respectively. The gauge variations are

$$\begin{aligned} \delta_{\text{gauge}} U &= 0 & + \Sigma_c, \\ \delta_{\text{gauge}} \mathbb{L} &= \frac{1}{2} \Lambda_c + i \Sigma_{\ell}, & \delta_{\text{gauge}} \bar{\mathbb{L}} &= \frac{1}{2} \bar{\Lambda}_c - i \Sigma_{\ell}, \\ \delta_{\text{gauge}} V_1 &= \Lambda_{\ell} + 0, & \delta_{\text{gauge}} V_2 &= \Lambda_c + \bar{\Lambda}_c + 0. \end{aligned} \tag{B.12}$$

Standard Maxwell gauge transformations (B.2) are generated by $(\Lambda_{\ell}, \Lambda_c)$. They leave invariant U , $\bar{D}_{\dot{\alpha}} \mathbb{L}$, W_{α} and X and then also the $\mathcal{N} = 2$ superfields \mathcal{W} and $\widehat{\mathcal{W}}$.

The gauge transformation generated by $(\Sigma_{\ell}, \Sigma_c)$ acts on $\widehat{\mathcal{W}}$ according to

$$\delta_{\text{gauge}} \widehat{\mathcal{W}} = \Sigma_c + \sqrt{2} i \bar{\theta} \bar{D} \Sigma_{\ell} - \bar{\theta} \bar{\theta} \frac{1}{4} \bar{D} \bar{D} \bar{\Sigma}_c \equiv \mathcal{S}. \tag{B.13}$$

which is a short chiral-antichiral multiplet similar to eq. (5.1). This is the gauge transformation already discussed in paragraph 5.1, which leaves V_1 , V_2 , W_{α} , X and then \mathcal{W} invariant. There is a ($\mathcal{N} = 1$) gauge in which $U = 0$. In this gauge, however,

$$\begin{aligned} [\delta_1^*, \delta_2^*] \mathbb{L} &= 2i (\eta_1 \sigma^{\mu} \bar{\eta}_2 - \eta_2 \sigma^{\mu} \bar{\eta}_1) \partial_{\mu} \mathbb{L} - i \Lambda_{\ell} \\ \Lambda_{\ell} &= i (\bar{\eta}_2 \bar{D} \eta_1 D - \bar{\eta}_1 \bar{D} \eta_2 D) \bar{\mathbb{L}} - i (\eta_2 D \bar{\eta}_1 \bar{D} - \eta_1 D \bar{\eta}_2 \bar{D}) \mathbb{L}. \end{aligned} \tag{B.14}$$

²⁰Verifying explicitly the closure of the algebra is relatively easy.

Since Λ_ℓ is a real linear superfield, the algebra closes up to a gauge transformation of \mathbb{L} and the multiplet is not complete without U .²¹

The two sets of gauge variations (B.12) remove $16_B + 16_F$ components in the long supermultiplet, to obtain the $8_B + 8_F$ physically relevant components of the super-Maxwell theory: the gauge field $-4 \operatorname{Re} \mathbb{V}_\mu$ (3_B), the (auxiliary) longitudinal vector $D = -4 \partial^\mu \operatorname{Im} \mathbb{V}_\mu$ (1_B), the two complex scalars in X (4_B) and two Majorana gauginos (8_F).

C More on $\operatorname{Im} \mathbb{V}_\mu$

In the construction of the long Maxwell $\mathcal{N} = 2$ superfield, the abelian gauge field is not, as is usually the case, a component of a real superfield V , but it appears in the expansion of a complex linear superfield \mathbb{L} , with the relation $V = 2(\mathbb{L} + \bar{\mathbb{L}})$. As a consequence, the auxiliary scalar field D in the expansion of V is replaced by the divergence of a vector field. Comparing expansion (5.10) of \mathbb{L} with

$$V = \theta \sigma^\mu \bar{\theta} A_\mu + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D + \dots$$

one finds $A_\mu = -4 \operatorname{Re} \mathbb{V}_\mu$ and $D = -4 \partial^\mu \operatorname{Im} \mathbb{V}_\mu$. In the version of super-Maxwell theory²² with the auxiliary scalar D , its lagrangian is quadratic in D :

$$\mathcal{L}_D = \frac{1}{2} A D^2 + \frac{1}{2} (B + \xi) D, \quad A > 0, \quad (\text{C.1})$$

where A and B are functions of other scalar fields²³ and the constant ξ is the FI term. In particular, A would be the gauge kinetic metric in super-Maxwell theory (hence the positivity condition). To integrate over D , it is legitimate to solve the field equation $2AD + B + \xi = 0$ and substitute the result into \mathcal{L}_D to obtain the scalar potential

$$\mathcal{L}_D = -\frac{(B + \xi)^2}{8A} = -\mathcal{V}. \quad (\text{C.2})$$

This theory does not have any symmetry and the (supersymmetric) ground state is at $\langle B \rangle = -\xi$. The contribution of \mathcal{L}_D to the field equations of the scalars appearing as variables of A and B is of course given by

$$\partial_z \mathcal{L}_D = -\partial_z \frac{(B + \xi)^2}{8A} = -\partial_z \mathcal{V}. \quad (\text{C.3})$$

The replacement $V = 2(\mathbb{L} + \bar{\mathbb{L}})$ leads to $D = \partial^\mu V_\mu$ with $V_\mu = -4 \operatorname{Im} \mathbb{V}_\mu$ and then to a quadratic lagrangian for the divergence of a vector field,

$$\mathcal{L} = \frac{1}{2} A (\partial^\mu V_\mu)^2 + \frac{1}{2} (B + \xi) \partial^\mu V_\mu, \quad A > 0, \quad (\text{C.4})$$

²¹In this gauge, variations (B.7) hold.

²²This appendix applies to $\mathcal{N} = 1$ and $\mathcal{N} = 2$ super-Maxwell theories.

²³They do not depend on derivatives of fields. These scalar fields are collectively denoted by z .

instead of expression (C.1). Now, the FI term is a derivative which does not contribute to the dynamical equations and the field equation for V_μ is

$$\partial_\mu[2A \partial^\nu V_\nu + B] = 0. \tag{C.5}$$

Its solution

$$\partial^\nu V_\nu = -\frac{B+c}{2A} \tag{C.6}$$

involves an integration constant c which replaces the FI coefficient ξ . The more subtle point is the procedure to obtain the lagrangian after the integration of $\partial^\mu V_\mu$, since the right-hand side of the solution is not a derivative of off-shell fields.

This situation is not new in the literature. Redefine

$$V_\mu = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} A^{\nu\rho\sigma}, \quad F_{\mu\nu\rho\sigma} = 4 \partial_{[\mu} A_{\nu\rho\sigma]}. \tag{C.7}$$

Since

$$\partial^\mu V_\mu = \frac{1}{24} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}, \quad (\partial^\mu V_\mu)^2 = -\frac{1}{24} F^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}, \tag{C.8}$$

the lagrangian (C.4) becomes

$$\mathcal{L}_F = -\frac{1}{48} A F^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} + \frac{1}{48} (B + \xi) \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}. \tag{C.9}$$

It is part of $\mathcal{N} = 8$ supergravity, with $A = e$, and the introduction of the ξ term has been studied as a potential source for a cosmological constant [30]. Another example is the massive Schwinger model [31]²⁴ where the Maxwell lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \theta \epsilon^{\mu\nu} \partial_\mu A_\nu + A^\mu j_\mu \tag{C.10}$$

(j_μ is a conserved fermion current) does not propagate any field. In the gauge $A_0 = 0$,

$$\mathcal{L} = \frac{1}{2} (\partial_0 A_1)^2 + \theta \partial_0 A_1, \tag{C.11}$$

and the field equation $\partial_0^2 A_1 = j_1$ implies the presence of a physically-relevant arbitrary integration constant in $F_{01} = \partial_0 A_1$, to be identified with the parameter θ .

Returning to our lagrangian (C.4) and solution (C.6), if we substitute the solution into the lagrangian, $\partial^\mu V_\mu$ becomes a function of the scalar fields z , it is not any longer a derivative and the ξ -term would then become physically relevant and contribute to the field equation of z . We obtain

$$\mathcal{L} = -\frac{(B + \xi)^2}{8A} + \frac{(\xi - c)^2}{8A} = -\mathcal{V} \tag{C.12}$$

and the contribution of \mathcal{L} to the field equations of the scalar fields z is of course $\partial_z \mathcal{L} = -\partial_z \mathcal{V}$. Comparing with expression (C.3), equivalence is obtained if we identify the integration constant with the FI coefficient ξ ,

$$c = \xi. \tag{C.13}$$

²⁴As also explained in ref. [30].

except if A is constant (the super-Maxwell theory has then canonical kinetic terms), in which case the second constant term in the potential is irrelevant. With this procedure, both versions of the theory depend on a single arbitrary constant $c = \xi$, the FI coefficient of the super-Maxwell theory.

Notice that a derivative term may in general contribute to currents. The canonical energy-momentum tensor for a “lagrangian” $\mathcal{L}_\xi = \xi \partial^\mu V_\mu$ is

$$T_{\mu\nu} = \xi [\partial_\nu V_\mu - \eta_{\mu\nu} \partial^\rho V_\rho] \tag{C.14}$$

which is not zero, conserved ($\partial^\mu T_{\mu\nu} = 0$) and an improvement term (so that the total energy-momentum is zero, assuming the absence of boundary contributions):

$$T_{00} = \xi \vec{\nabla} \cdot \vec{V}, \quad T_{0i} = \xi \partial_i V_0. \tag{C.15}$$

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References

- [1] D.V. Volkov and V.P. Akulov, *Is the Neutrino a Goldstone Particle?*, *Phys. Lett. B* **46** (1973) 109 [[INSPIRE](#)].
- [2] M. Roček, *Linearizing the Volkov-Akulov model*, *Phys. Rev. Lett.* **41** (1978) 451 [[INSPIRE](#)].
- [3] U. Lindström and M. Roček, *Constrained local superfields*, *Phys. Rev. D* **19** (1979) 2300 [[INSPIRE](#)].
- [4] R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio and R. Gatto, *Nonlinear realization of supersymmetry algebra from supersymmetric constraint*, *Phys. Lett. B* **220** (1989) 569 [[INSPIRE](#)].
- [5] Z. Komargodski and N. Seiberg, *From linear SUSY to constrained superfields*, *JHEP* **09** (2009) 066 [[arXiv:0907.2441](#)] [[INSPIRE](#)].
- [6] S.M. Kuzenko and S.J. Tyler, *On the goldstino actions and their symmetries*, *JHEP* **05** (2011) 055 [[arXiv:1102.3043](#)] [[INSPIRE](#)].
- [7] S. Sugimoto, *Anomaly cancellations in type I D-9 – anti-D-9 system and the USp(32) string theory*, *Prog. Theor. Phys.* **102** (1999) 685 [[hep-th/9905159](#)] [[INSPIRE](#)].
- [8] I. Antoniadis, E. Dudas and A. Sagnotti, *Brane supersymmetry breaking*, *Phys. Lett. B* **464** (1999) 38 [[hep-th/9908023](#)] [[INSPIRE](#)].
- [9] E. Dudas and J. Mourad, *Consistent gravitino couplings in nonsupersymmetric strings*, *Phys. Lett. B* **514** (2001) 173 [[hep-th/0012071](#)] [[INSPIRE](#)].
- [10] G. Pradisi and F. Riccioni, *Geometric couplings and brane supersymmetry breaking*, *Nucl. Phys. B* **615** (2001) 33 [[hep-th/0107090](#)] [[INSPIRE](#)].
- [11] R. Kallosh, F. Quevedo and A.M. Uranga, *String theory realizations of the nilpotent goldstino*, *JHEP* **12** (2015) 039 [[arXiv:1507.07556](#)] [[INSPIRE](#)].
- [12] S. Ferrara and P. van Nieuwenhuizen, *Noether coupling of massive gravitinos to $N = 1$ supergravity*, *Phys. Lett. B* **127** (1983) 70.

- [13] I. Antoniadis, H. Partouche and T.R. Taylor, *Spontaneous breaking of $N = 2$ global supersymmetry*, *Phys. Lett. B* **372** (1996) 83 [[hep-th/9512006](#)] [[INSPIRE](#)].
- [14] J. Bagger and A. Galperin, *A new Goldstone multiplet for partially broken supersymmetry*, *Phys. Rev. D* **55** (1997) 1091 [[hep-th/9608177](#)] [[INSPIRE](#)].
- [15] E.A. Ivanov and B.M. Zupnik, *Modified $N = 2$ supersymmetry and Fayet-Iliopoulos terms*, *Phys. Atom. Nucl.* **62** (1999) 1043 [*Yad. Fiz.* **62** (1999) 1110] [[hep-th/9710236](#)] [[INSPIRE](#)].
- [16] I. Antoniadis, J.P. Derendinger and T. Maillard, *Nonlinear $N = 2$ supersymmetry, effective actions and moduli stabilization*, *Nucl. Phys. B* **808** (2009) 53 [[arXiv:0804.1738](#)] [[INSPIRE](#)].
- [17] S.M. Kuzenko, *The Fayet-Iliopoulos term and nonlinear self-duality*, *Phys. Rev. D* **81** (2010) 085036 [[arXiv:0911.5190](#)] [[INSPIRE](#)].
- [18] J. Bagger and A. Galperin, *The tensor Goldstone multiplet for partially broken supersymmetry*, *Phys. Lett. B* **412** (1997) 296 [[hep-th/9707061](#)] [[INSPIRE](#)].
- [19] N. Ambrosetti, I. Antoniadis, J.P. Derendinger and P. Tziveloglou, *Nonlinear supersymmetry, brane-bulk interactions and super-Higgs without gravity*, *Nucl. Phys. B* **835** (2010) 75 [[arXiv:0911.5212](#)] [[INSPIRE](#)].
- [20] B. de Wit and J.W. van Holten, *Multiplets of linearized $SO(2)$ supergravity*, *Nucl. Phys. B* **155** (1979) 530 [[INSPIRE](#)].
- [21] U. Lindström and M. Roček, *Scalar tensor duality and $N = 1$, $N = 2$ nonlinear σ -models*, *Nucl. Phys. B* **222** (1983) 285 [[INSPIRE](#)].
- [22] A. Karlhede, U. Lindström and M. Roček, *Selfinteracting tensor multiplets in $N = 2$ superspace*, *Phys. Lett. B* **147** (1984) 297 [[INSPIRE](#)].
- [23] N.J. Hitchin, A. Karlhede, U. Lindström and M. Roček, *HyperKähler metrics and supersymmetry*, *Commun. Math. Phys.* **108** (1987) 535 [[INSPIRE](#)].
- [24] L. Álvarez-Gaumé and D.Z. Freedman, *Geometrical structure and ultraviolet finiteness in the supersymmetric σ -model*, *Commun. Math. Phys.* **80** (1981) 443 [[INSPIRE](#)].
- [25] M. Roček and A.A. Tseytlin, *Partial breaking of global $D = 4$ supersymmetry, constrained superfields and three-brane actions*, *Phys. Rev. D* **59** (1999) 106001 [[hep-th/9811232](#)] [[INSPIRE](#)].
- [26] H. Partouche and B. Pioline, *Partial spontaneous breaking of global supersymmetry*, *Nucl. Phys. Proc. Suppl.* **56** (1997) 322 [[hep-th/9702115](#)] [[INSPIRE](#)].
- [27] K. Fujiwara, H. Itoyama and M. Sakaguchi, *Partial breaking of $N = 2$ supersymmetry and of gauge symmetry in the $U(N)$ gauge model*, *Nucl. Phys. B* **723** (2005) 33 [[hep-th/0503113](#)] [[INSPIRE](#)].
- [28] S. Ferrara, M. Porrati and A. Sagnotti, *$N = 2$ Born-Infeld attractors*, *JHEP* **12** (2014) 065 [[arXiv:1411.4954](#)] [[INSPIRE](#)].
- [29] S. Ferrara, M. Porrati, A. Sagnotti, R. Stora and A. Yeranyan, *Generalized Born-Infeld actions and projective cubic curves*, *Fortsch. Phys.* **63** (2015) 189 [[arXiv:1412.3337](#)] [[INSPIRE](#)].
- [30] A. Aurilia, H. Nicolai and P.K. Townsend, *Hidden constants: the theta parameter of QCD and the cosmological constant of $N = 8$ supergravity*, *Nucl. Phys. B* **176** (1980) 509 [[INSPIRE](#)].
- [31] S.R. Coleman, *More about the massive Schwinger model*, *Annals Phys.* **101** (1976) 239 [[INSPIRE](#)].