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Double and cyclic λ-deformations and their canonical equivalents

George Georgiou [a], Konstantinos Sfetsos [b], Konstantinos Siampos [c,*]

[a] Institute of Nuclear and Particle Physics, National Center for Scientific Research Demokritos, Ag. Paraskevi, GR-15310 Athens, Greece
[b] Department of Nuclear and Particle Physics, Faculty of Physics, National and Kapodistrian University of Athens, Athens 15784, Greece
[c] Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, Laboratory for High-Energy Physics, University of Bern, Sidlerstrasse 5, CH3012 Bern, Switzerland

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ABSTRACT

We prove that the doubly λ-deformed σ-models, which include integrable cases, are canonically equivalent to the sum of two single λ-deformed models. This explains the equality of the exact β-functions and current anomalous dimensions of the doubly λ-deformed σ-models to those of two single λ-deformed models. Our proof is based upon agreement of their Hamiltonian densities and of their canonical structure. Subsequently, we show that it is possible to take a well defined non-Abelian type limit of the doubly-deformed action. Last, but not least, by extending the above, we construct multi-matrix integrable deformations of an arbitrary number of WZW models.

0. Introduction and results

A new class of integrable theories based on current algebras for a semi-simple group was recently constructed [1]. The starting point was to consider two independent WZW models at the same positive integer level k and two distinct PCM models which were then left-right asymmetrically gauged with respect to a common global symmetry. The models are labeled by the level k and two general invertible matrices $\lambda_{1,2}$. For certain choices of $\lambda_{1,2}$ integrability is retained [1]. This idea can be generalized to include integrable deformations of exact CFTs on symmetric spaces. This construction is reminiscent to the one for single λ-deformations [2–4].

Subsequently, the quantum properties of the aforementioned multi-parameter integrable deformations were studied in [5], by employing a variety of techniques. One of the main results of that work was that the running of the couplings λ_1 and λ_2, as well as the anomalous dimensions of current operators depend only on one of the couplings, either λ_1 or λ_2 and are identical to those found for single λ-deformations [6–12]. These rather unexpected results seek for a simple explanation. The purpose of this work is to demonstrate that they are due to the fact that the doubly deformed models are canonically equivalent to the sum of two single λ-deformations, one with deformation matrix being λ_1 and the other with deformation matrix λ_2. Recall that all known forms of T-duality, i.e., Abelian, non-Abelian and Poisson–Lie T-duality can be formulated as canonical transformations in the phase space of the corresponding two-dimensional σ-models [13–17]. Moreover, it has been shown in various works that the running of couplings is preserved under these canonical transformations even though the corresponding σ-models fields are totally different [18–22]. All of the above strongly hint towards the validity of our assertion, which of course we will prove.

The plan of the paper is as follows: In section 1, after a brief review of the single and doubly λ-deformed models and of their *non-perturbative* symmetries, we will show that the doubly deformed models are canonically equivalent to the sum of two single λ-deformations. In section 2, we will present the type of non-Abelian T-duality that is based on the doubly deformed σ-models of [1]. Finally, in section 3, we will construct multi-matrix *integrable* deformations of an arbitrary number of independent WZW models by performing a left-right asymmetric gauging for each one of them but in such a way that the total classical gauge anomaly vanishes. This happens if these models are forced to obey the cyclic symmetry property or if they are infinitely many, resembling in structure either a closed or an infinitely open spin chain. Their action can be thought of as the all-loop effective action of several independent WZW models for G all at level k, perturbed by current bilinears mixing the different WZW models with nearest neighbour-type interactions. These models are also canonically equivalent to a sum of single λ-deformed models with appropriate couplings. Furthermore, we will argue that the Hamiltonian of

* Corresponding author.
E-mail addresses: georgiou@inp.demokritos.gr (G. Georgiou), ksfetsos@phys.uoa.gr (K. Sfetsos), siampos@itp.unibe.ch (K. Siampos).

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these new models maps to itself under an inversion of all couplings $\lambda_i \mapsto \lambda_i^{-1}$, $i = 1, \dots, n$ accompanied generically by non-local redefinitions of the group elements involved when $n = 3, 4, \dots$. This symmetry, which in the special cases where $n = 1, 2$ simplifies to the one reviewed in section 1, is in accordance with the fact that the β-functions and anomalous dimensions of currents are again given by the same expressions as in the case of the single λ-deformed model.

1. Review and canonical equivalence

1.1. Single λ-deformed σ-models

The construction of the single λ-deformed σ-model starts by considering the sum of a gauged WZW and a PCM for a group G, defined with group elements g and $\tilde{g}$, respectively and next gauging the global symmetry [2]

$$g \mapsto \Lambda^{-1} g \Lambda\,, \quad \tilde{g} \mapsto \Lambda^{-1} \tilde{g}\,.$$

This is done by introducing gauge fields $A_\pm$ in the Lie-algebra of G transforming as

$$A_\pm \mapsto \Lambda^{-1} A_\pm \Lambda - \partial_\pm \Lambda\,.$$

The choice $\tilde{g} = \mathbb{I}$ completely fixes the gauge and the gauged fixed action reads

$$\begin{aligned} S_{k,\lambda}(g; A_\pm) = S_k(g) + \frac{k}{\pi} \int \mathrm{d}^2\sigma \, \mathrm{Tr}\Big(& A_- \partial_+ g g^{-1} - A_+ g^{-1} \partial_- g \\ & + A_- g A_+ g^{-1} - A_+ \lambda^{-1} A_- \Big)\,, \end{aligned} \tag{1.1}$$

where $S_k(g)$ is the WZW model. The $A_\pm$'s are non-dynamical and their equations of motion read

$$\nabla_+ g\, g^{-1} = (\lambda^{-T} - \mathbb{I}) A_+\,, \quad g^{-1} \nabla_- g = -(\lambda^{-1} - \mathbb{I}) A_-\,, \tag{1.2}$$

with $\nabla_\pm g = \partial_\pm g - [A_\pm, g]$. Solving them in terms of the gauge fields we find

$$A_+ = i \left(\lambda^{-T} - D\right)^{-1} J_+\,, \quad A_- = -i \left(\lambda^{-1} - D^T\right)^{-1} J_-\,, \tag{1.3}$$

where

$$\begin{aligned} & J_+^a = -i\, \mathrm{Tr}(t_a \partial_+ g g^{-1})\,, \qquad J_-^a = -i\, \mathrm{Tr}(t_a g^{-1} \partial_- g)\,. \\ & D_{ab} = \mathrm{Tr}(t_a g t_b g^{-1})\,, \end{aligned} \tag{1.4}$$

where t_a's are Hermitian representation matrices obeying $[t_a, t_b] = i f_{abc} t_c$, so that the structure constants f_{abc} are real. We choose the normalization such that $\mathrm{Tr}(t_a t_b) = \delta_{ab}$.

Using (1.3) into (1.1) one finds the action [2]

$$S_{k,\lambda}(g) = S_k(g) + \frac{k}{\pi} \int \mathrm{d}^2\sigma \, \mathrm{Tr}\left(J_+ (\lambda^{-1} - D^T)^{-1} J_- \right)\,. \tag{1.5}$$

For small elements of the matrix λ this action becomes

$$S_{k,\lambda}(g) = S_k(g) + \frac{k}{\pi} \int \mathrm{d}^2\sigma \, \mathrm{Tr}\,(J_+ \lambda J_-) + \cdots\,.$$

Hence (1.5) represents the effective action of self-interacting current bilinears of a single WZW model. The action (1.5) has the remarkable *non-perturbative* symmetry [6,9]

$$k \mapsto -k\,, \quad \lambda \mapsto \lambda^{-1}\,, \quad g \mapsto g^{-1}\,. \tag{1.6}$$

As in the case of gauged WZW models [23], we define the currents $\mathcal{J}_\pm$

$$\mathcal{J}_+ = \nabla_+ g g^{-1} + A_+ - A_-\,, \quad \mathcal{J}_- = -g^{-1} \nabla_- g + A_- - A_+\,, \tag{1.7}$$

The above form for the $\mathcal{J}_\pm^a$'s when rewritten in terms of phase space variables of the σ-model action, assumes the same form as the currents $J_\pm^a$ of the WZW action. Hence, they satisfy two commuting current algebras as in [24]

$$\{\mathcal{J}_\pm^a, \mathcal{J}_\pm^b\} = \frac{2}{k} f_{abc} \mathcal{J}_\pm^c \delta_{\sigma\sigma'} \pm \frac{2}{k} \delta_{ab} \delta'_{\sigma\sigma'}\,, \quad \delta_{\sigma\sigma'} = \delta(\sigma - \sigma')\,. \tag{1.8}$$

Moreover using (1.2) we can rewrite (1.7) as

$$\mathcal{J}_+ = \lambda^{-T} A_+ - A_-\,, \quad \mathcal{J}_- = \lambda^{-1} A_- - A_+\,. \tag{1.9}$$

Inversely

$$\begin{aligned} & A_+ = h^{-1} \lambda^T (\mathcal{J}_+ + \lambda \mathcal{J}_-)\,, \quad A_- = \tilde{h}^{-1} \lambda (\mathcal{J}_- + \lambda^T \mathcal{J}_+)\,, \\ & h = \mathbb{I} - \lambda^T \lambda\,, \quad \tilde{h} = \mathbb{I} - \lambda \lambda^T\,, \end{aligned} \tag{1.10}$$

assuming that the matrix λ is such that $h, \tilde{h}$ are positive-definite matrices. To obtain the Poisson algebra in the base of $A_\pm$ we use (1.8), (1.9) and (1.10).

To study the Hamiltonian structure of the problem we need to define its phase space [3,4]. This is given in terms of the currents $\mathcal{J}_\pm$, the gauge fields $A_\pm$ and the associated momenta $P_\pm$ to $A_\pm$. The $\mathcal{J}_\pm$ obey two commuting current algebras (1.8) and have vanishing Poisson brackets with $A_\pm$ and $P_\pm$

$$\{P_\pm^a(\sigma), A_\mp^b(\sigma')\} = \delta^{ab} \delta(\sigma - \sigma')\,.$$

Furthermore, since the $A_\pm$'s are non-dynamical their associated momenta $P_\pm$ vanish. This introduces two primary constraints

$$\varphi_1 = P_+ \approx 0\,, \quad \varphi_2 = P_- \approx 0\,.$$

Their time-evolution gives rise to the secondary constraints

$$\varphi_3 = \mathcal{J}_+ - \lambda^{-T} A_+ + A_- \approx 0\,, \quad \varphi_4 = \mathcal{J}_- - \lambda^{-1} A_- + A_+ \approx 0\,.$$

Time evolution generates no further constraints. The φ_i's with $i = 1, 2, 3, 4$, turn out to be second class constraints, since the matrix of their Poisson brackets is invertible in the deformed case. Finally, the Hamiltonian density of the single λ-deformed model before integrating out the gauge fields takes the form [2,23]

$$\begin{aligned} \mathcal{H}_{\text{single}} = \frac{k}{4\pi} \mathrm{Tr}\,\Big(& \mathcal{J}_+ \mathcal{J}_+ + \mathcal{J}_- \mathcal{J}_- + 4(\mathcal{J}_+ A_- + \mathcal{J}_- A_+) \\ & + 2(A_+ - A_-)(A_+ - A_-) - 4 A_+ (\lambda_1^{-1} - \mathbb{I}) A_- \Big)\,, \end{aligned}$$

or equivalently through (1.9), in terms of $A_\pm$'s

$$\boxed{\mathcal{H}_{\text{single}} = \frac{k}{4\pi} \mathrm{Tr}\left(A_+ \left(\lambda^{-1} \tilde{h} \lambda^{-T}\right) A_+ + A_- \left(\lambda^{-T} h \lambda^{-1}\right) A_- \right)}\,. \tag{1.11}$$

1.2. Doubly λ-deformed σ-models

The action defining the doubly deformed models depends on two group elements $g_i \in G$, $i = 1, 2$ and is given by the deformation of the sum of two WZW models $S_k(g_1)$ and $S_k(g_2)$ as [1]

$$\begin{aligned} S_{k,\lambda_1,\lambda_2}(g_1, g_2) = & \, S_k(g_1) + S_k(g_2) \\ & + \frac{k}{\pi} \int \mathrm{d}^2\sigma \, \mathrm{Tr}\Bigg\{ (J_{1+} \;\; J_{2+}) \begin{pmatrix} \Lambda_{21} \lambda_1 D_2^T \lambda_2 & \Lambda_{21} \lambda_1 \\ \Lambda_{12} \lambda_2 & \Lambda_{12} \lambda_2 D_1^T \lambda_1 \end{pmatrix} \\ & \times \begin{pmatrix} J_{1-} \\ J_{2-} \end{pmatrix} \Bigg\}\,, \end{aligned} \tag{1.12}$$

where

$$\Lambda_{12} = (\mathbb{I} - \lambda_2 D_1^T \lambda_1 D_2^T)^{-1}\,, \quad \Lambda_{21} = (\mathbb{I} - \lambda_1 D_2^T \lambda_2 D_1^T)^{-1}\,. \quad (1.13)$$

The matrices D_{ab} and the currents $J_\pm^a$ are defined in (1.4). When a current or the matrix D has the extra index 1 or 2 this means that one should use the corresponding group element in its definition. The action (1.12) has the *non-perturbative* symmetry [1]

$$k \mapsto -k\,, \quad \lambda_1 \mapsto \lambda_1^{-1}\,, \quad \lambda_2 \mapsto \lambda_2^{-1}\,, \quad g_1 \mapsto g_2^{-1}\,, \quad g_2 \mapsto g_1^{-1}\,, \quad (1.14)$$

which is similar to (1.6). For small elements of the matrices λ_i's the action (1.12) becomes

$$S_{k,\lambda_1,\lambda_2}(g_1,g_2) = S_k(g_1) + S_k(g_2) + \frac{k}{\pi}\int d^2\sigma\, \mathrm{Tr}(J_{1+}\lambda_1 J_{2-} + J_{2+}\lambda_2 J_{1-}) + \cdots\,.$$

Hence (1.12) represents the effective action of two WZW models mutually interacting via current bilinears. Similarly to (1.7) we define the currents[1,2]

$$\begin{aligned}
\mathcal{J}_+^{(1)} &= \nabla_+ g_1 g_1^{-1} + A_+^{(1)} - A_-^{(1)}\,,\\
\mathcal{J}_-^{(1)} &= -g_1^{-1}\nabla_- g_1 + A_-^{(2)} - A_+^{(2)}\,,\\
\mathcal{J}_+^{(2)} &= \nabla_+ g_2 g_2^{-1} + A_+^{(2)} - A_-^{(2)}\,,\\
\mathcal{J}_-^{(2)} &= -g_2^{-1}\nabla_- g_2 + A_-^{(1)} - A_+^{(1)}\,.
\end{aligned} \quad (1.15)$$

These currents obey two commuting copies of current algebras [1]

$$\{\mathcal{J}_\pm^{(i)a}, \mathcal{J}_\pm^{(i)b}\} = \frac{2}{k} f_{abc}\mathcal{J}_\pm^{(i)c}\delta_{\sigma\sigma'} \pm \frac{2}{k}\delta_{ab}\delta'_{\sigma\sigma'}\,, \quad i=1,2\,, \quad (1.16)$$

which encode the canonical structure of the theory. The action does not depend on derivatives of $A_\pm^{(i)}, i=1,2$, so that as in subsection 1.1, their equations of motion are second class constraints [1]

$$\begin{aligned}
&\nabla_+ g_1\, g_1^{-1} = (\lambda_1^{-T} - \mathbb{I})A_+^{(1)}\,, \quad g_1^{-1}\nabla_- g_1 = -(\lambda_2^{-1} - \mathbb{I})A_-^{(2)}\,,\\
&\nabla_+ g_2\, g_2^{-1} = (\lambda_2^{-T} - \mathbb{I})A_+^{(2)}\,, \quad g_2^{-1}\nabla_- g_2 = -(\lambda_1^{-1} - \mathbb{I})A_-^{(1)}\,,
\end{aligned} \quad (1.17)$$

determining the gauge fields in terms of the group elements similarly to (1.3) (for the precise expressions we refer to [1]). Then (1.15) rewrites as

$$\begin{aligned}
&\mathcal{J}_+^{(1)} = \lambda_1^{-T}A_+^{(1)} - A_-^{(1)}\,, \quad \mathcal{J}_-^{(1)} = \lambda_2^{-1}A_-^{(2)} - A_+^{(2)}\,,\\
&\mathcal{J}_+^{(2)} = \lambda_2^{-T}A_+^{(2)} - A_-^{(2)}\,, \quad \mathcal{J}_-^{(2)} = \lambda_1^{-1}A_-^{(1)} - A_+^{(1)}\,.
\end{aligned} \quad (1.18)$$

Equivalently the gauge fields in terms of the dressed currents are given by

$$\begin{aligned}
&A_+^{(1)} = h_1^{-1}\lambda_1^T(\mathcal{J}_+^{(1)} + \lambda_1\mathcal{J}_-^{(2)})\,, \quad A_-^{(1)} = \tilde{h}_1^{-1}\lambda_1(\mathcal{J}_-^{(2)} + \lambda_1^T\mathcal{J}_+^{(1)})\,,\\
&A_+^{(2)} = h_2^{-1}\lambda_2^T(\mathcal{J}_+^{(2)} + \lambda_2\mathcal{J}_-^{(1)})\,, \quad A_-^{(2)} = \tilde{h}_2^{-1}\lambda_2(\mathcal{J}_-^{(1)} + \lambda_2^T\mathcal{J}_+^{(2)})\,,\\
&h_i = \mathbb{I} - \lambda_i^T\lambda_i\,, \quad \tilde{h}_i = \mathbb{I} - \lambda_i\lambda_i^T\,, \quad i=1,2\,.
\end{aligned} \quad (1.19)$$

To obtain the Poisson algebra in the base of $A_\pm^{(1)}$ and $A_\pm^{(2)}$ we use (1.16), (1.18) and (1.19). As a corollary one can easily show that $\{A_\pm^{(1)}, A_\pm^{(2)}\} = 0$, for all choices of signs and for generic coupling matrices $\lambda_{1,2}$. The Hamiltonian density of our system before integrating out the gauge fields takes the form [1]

$$\begin{aligned}
\mathcal{H}_{\text{doubly}} = \frac{k}{4\pi}\mathrm{Tr}\Big\{&\mathcal{J}_+^{(1)}\mathcal{J}_+^{(1)} + \mathcal{J}_-^{(1)}\mathcal{J}_-^{(1)} + \mathcal{J}_+^{(2)}\mathcal{J}_+^{(2)} + \mathcal{J}_-^{(2)}\mathcal{J}_-^{(2)}\\
&+4(\mathcal{J}_+^{(1)}A_-^{(1)} + \mathcal{J}_+^{(2)}A_-^{(2)} + \mathcal{J}_-^{(1)}A_+^{(2)} + \mathcal{J}_-^{(2)}A_+^{(1)})\\
&+2(A_+^{(1)} - A_-^{(1)})(A_+^{(1)} - A_-^{(1)})\\
&+2(A_+^{(2)} - A_-^{(2)})(A_+^{(2)} - A_-^{(2)})\\
&-4A_+^{(1)}(\lambda_1^{-1} - \mathbb{I})A_-^{(1)} - 4A_+^{(2)}(\lambda_2^{-1} - \mathbb{I})A_-^{(2)}\Big\}
\end{aligned}$$

and can be rewritten through (1.18) in terms of $A_\pm^{(i)}$ and λ_i as

$$\boxed{\begin{aligned}\mathcal{H}_{\text{doubly}} = \frac{k}{4\pi}\sum_{i=1}^{2}\mathrm{Tr}\Big(&A_+^{(i)}\left(\lambda_i^{-1}\tilde{h}_i\lambda_i^{-T}\right)A_+^{(i)}\\ &+ A_-^{(i)}\left(\lambda_i^{-T}h_i\lambda_i^{-1}\right)A_-^{(i)}\Big)\end{aligned}}\,. \quad (1.20)$$

The fact that the Hamiltonian density (1.20) is the sum of two terms one depending on $A_\pm^{(1)}$ and the other on $A_\pm^{(2)}$ combined with the fact that the currents $\mathcal{J}_\pm^{(i)}$, $i=1,2$, obey two commuting copies of the current algebra of the single λ-deformed model shows that the doubly deformed models are canonically equivalent to the sum of two single λ-deformed models, one with coupling λ_1 and the other with coupling λ_2. The relations defining the canonical transformation are given by

$$A_\pm^{(1)} = \tilde{A}_\pm^{(1)}\,, \quad A_\pm^{(2)} = \tilde{A}_\pm^{(2)}\,, \quad (1.21)$$

where the gauge fields without the tildes correspond to the doubly deformed models and depend on $(\lambda_1,\lambda_2; g_1, g_2)$, while the tilded gauge fields correspond to the canonically equivalent sum of two single λ-deformed models the first of which depends on $(\lambda_1;\tilde{g}_1)$ only while the second depends on $(\lambda_2;\tilde{g}_2)$.

Furthermore, the gauge fields of (1.21) should be considered as functions of the coordinates parametrising the group elements and their conjugate momenta. We may write relations involving world-sheet derivatives of the various group elements by using (1.3) and (1.17). As in all canonical transformation involving canonical variables as well as their momenta, the relation between the g_i's and the $\tilde{g}_i$'s is a non-local one.

A comment is in order concerning the η-deformed models [25–29] which are closely related to the single λ-deformed ones via Poisson–Lie T-duality [30] and an appropriate analytic continuation of the coordinates and the parameters [31–35]

$$\lambda \mapsto \frac{iE - \eta\mathbb{I}}{iE + \eta\mathbb{I}}\,,$$

where E is an arbitrary constant matrix. Poisson–Lie T-duality can also be formulated as a canonical transformation [16,17] and therefore there is a chain of canonical transformations from doubly λ-deformed, to two single λ-deformed and to η-deformed models. It would be interesting to formulate the canonical transformation (1.21) via a duality invariant action similarly perhaps to the case of Poisson–Lie T-duality in [36].

There is an important observation for further use in section 3. The Hamiltonian density (1.20) has the following *non-perturbative* symmetry

[1] To conform with the notation of the current work we have renamed the gauged fields $(A_\pm, B_\pm)$ of [1] by $(A_\pm^{(1)}, A_\pm^{(2)})$.

[2] The various covariant derivatives are defined according to the transformation properties of the object they act on. For instance

$$\nabla_\pm g_1 = \partial_\pm g_1 - A_\pm^{(1)} g_i + g_i A_\pm^{(2)}\,,$$

$$\nabla_\pm(\nabla_\mp g_1 g_1^{-1}) = \partial_\pm(\nabla_\mp g_1 g_1^{-1}) - [A_\pm^{(1)}, \nabla_\mp g_1 g_1^{-1}]\,.$$

.

$$k \mapsto -k, \quad \lambda_i \mapsto \lambda_i^{-1}, \quad A_+^{(i)} \mapsto \lambda_i^{-T} A_+^{(i)},$$
$$A_-^{(i)} \mapsto \lambda_i^{-1} A_-^{(i)}, \quad i = 1, 2. \tag{1.22}$$

In other words $\mathcal{H}_{\text{doubly}}$ maps to itself under (1.22). By using (1.18) this implies the following transformation for the group elements g_1 and g_2

$$\mathcal{J}_+^{(1)} \mapsto -\mathcal{J}_-^{(2)}, \quad \mathcal{J}_+^{(2)} \mapsto -\mathcal{J}_-^{(1)},$$
$$\mathcal{J}_-^{(1)} \mapsto -\mathcal{J}_+^{(2)}, \quad \mathcal{J}_-^{(2)} \mapsto -\mathcal{J}_+^{(1)}. \tag{1.23}$$

Since the currents $\mathcal{J}_\pm^{(i)}$, $i = 1, 2$, depend both on the group elements and their derivatives, the transformation (1.23) can be viewed as a non-local transformation at the level of the group elements. In the special cases of the single and doubly λ-deformed theories the symmetry (1.22) and (1.23) can be realized locally simply by a mapping of group elements, i.e. (1.6) and (1.14). Indeed, it is not difficult to check that (1.6) and (1.14) imply for the gauge fields the transformation (1.22). The situation is slightly different for the generic cyclic models constructed below in section 3 which can have arbitrarily many group elements.

2. Doubly-deformed models and non-Abelian T-duality

It is has been known that the action (1.5) admits the non-Abelian T-dual limit that involves taking $k \to \infty$, whereas simultaneously taking the matrix λ and the group element g to the identity [2]. Specifically, if we let

$$\lambda = \mathbb{I} - \frac{E}{k}, \qquad g = \mathbb{I} + i\frac{v}{k}, \qquad k \to \infty,$$

where E is a constant matrix and $v = v_a t^a$, then the action (1.5) becomes

$$S(v, E) = \frac{1}{\pi}\int d^2\sigma \, \text{Tr}\left(\partial_+ v (E + f)^{-1} \partial_- v\right),$$

where f is a matrix with elements $f_{ab} = f_{abc} v^c$. This is the non-Abelian T-dual of the PCM action with general coupling matrix E

$$S_{\text{PCM}}(g, E) = -\frac{1}{\pi}\int d^2\sigma \, \text{Tr}\left(g^{-1}\partial_+ g \, E \, g^{-1}\partial_- g\right),$$

with respect to the global symmetry $g \mapsto \Lambda g$, $\Lambda \in G$. The above limit is well defined when is taken on the β-function for λ, as well as on the anomalous dimensions of various operators in the theory. In the case of doubly λ or even multiple/cyclic λ-deformations (see section 3) we have shown in particular that, the β-functions and current anomalous dimensions are the same with those of two or more simple λ-deformations. Hence, it is expected that it should be possible to take a well defined non-Abelian type limit in the action (1.12). This is not necessarily simple since a suitable limit involves the two group elements.

In the following we focus on the most interesting case in which the matrices λ_i, $i = 1, 2$ are isotropic, i.e. $(\lambda_i)_{ab} = \lambda_i \delta_{ab}$. It is convenient to use the group element $\mathcal{G} = g_1 g_2$ and also rename g_2 by g. Then employing the Polyakov–Wiegmann identity [44], the action (1.12), using also (1.13), takes the form

$$\begin{aligned}
&S_{k,\lambda_1,\lambda_2}(\mathcal{G}, g) = S_k(\mathcal{G}) \\
&+\frac{k}{\pi}\int d^2\sigma \, \text{Tr}\Big((1-\lambda_2) g^{-1}\partial_+ g \, (\mathcal{D} - \lambda_1 \mathbb{I}) \, \Sigma \, g^{-1}\partial_- g \\
&-(1-\lambda_2) g^{-1}\partial_+ g \, \Sigma \, \partial_- \mathcal{G}\mathcal{G}^{-1} + \lambda_1(1-\lambda_2)\mathcal{G}^{-1}\partial_+\mathcal{G} \, \Sigma \, g^{-1}\partial_- g \\
&+\lambda_1\lambda_2 \, \mathcal{G}^{-1}\partial_+\mathcal{G} \, \Sigma \, \mathcal{G}^{-1}\partial_-\mathcal{G}\Big),
\end{aligned} \tag{2.1}$$

where: $\Sigma = (\lambda_1\lambda_2\mathbb{I} - \mathcal{D})^{-1}$ and $\mathcal{D} = D(\mathcal{G}) = D(g_1)D(g_2)$. Next we take the limit

$$\lambda_i = 1 - \frac{\kappa_i^2}{k}, \quad i = 1, 2, \qquad \mathcal{G} = \mathbb{I} + i\frac{v}{k}, \qquad k \to \infty. \tag{2.2}$$

After some algebra we find that (2.1) becomes

$$\begin{aligned}
S_{\kappa_1^2,\kappa_2^2}(v, g) = &-\frac{1}{\pi}\int d^2\sigma \, \text{Tr}\Big(\kappa_2^2 g^{-1}\partial_+ g g^{-1}\partial_- g \\
&+ \left(i\partial_+ v - \kappa_2^2 g^{-1}\partial_+ g\right)\left((\kappa_1^2 + \kappa_2^2)\mathbb{I} + f\right)^{-1} \\
&\times \left(i\partial_- v + \kappa_2^2 g^{-1}\partial_- g\right)\Big).
\end{aligned} \tag{2.3}$$

It can be shown that this action is the non-Abelian T-dual of

$$\begin{aligned}
S = -\frac{1}{\pi}\int d^2\sigma \, \text{Tr}\Big(&\kappa_1^2 \tilde{g}^{-1}\partial_+\tilde{g}\tilde{g}^{-1}\partial_-\tilde{g} \\
&+ \kappa_2^2 (g^{-1}\partial_+ g - \tilde{g}^{-1}\partial_+\tilde{g})(g^{-1}\partial_- g - \tilde{g}^{-1}\partial_-\tilde{g})\Big),
\end{aligned}$$

with respect to the global symmetry $\tilde{g} \mapsto \Lambda\tilde{g}$, $\Lambda \in G$. Note that, if we define the new group element $\widetilde{\mathcal{G}} = g\tilde{g}^{-1}$ one may write the previous action as

$$S = -\frac{1}{\pi}\int d^2\sigma \, \text{Tr}\Big(\kappa_1^2 \, \tilde{g}^{-1}\partial_+\tilde{g}\tilde{g}^{-1}\partial_-\tilde{g} + \kappa_2^2 \, \widetilde{\mathcal{G}}^{-1}\partial_+\widetilde{\mathcal{G}}\widetilde{\mathcal{G}}^{-1}\partial_-\widetilde{\mathcal{G}}\Big), \tag{2.4}$$

which is the sum of two independent PCM actions for a group G. The previous group element redefinition introduces interactions between them.

Finally consider a limit in which only λ_2 tends to one, whereas λ_1 stays inactive. Then, (2.2) has to be modified as

$$\lambda_2 = 1 - \frac{\kappa_2^2}{k}, \quad \mathcal{G} = \mathbb{I} + i\frac{v}{\sqrt{k}}, \quad k \to \infty,$$

in order for (2.1) to stay finite. In particular, this becomes

$$\begin{aligned}
S_{\kappa^2}(v, g) = &\frac{1}{2\pi}\frac{1+\lambda_1}{1-\lambda_1}\int d^2\sigma \, \text{Tr}(\partial_+ v \partial_- v) \\
&- \frac{\kappa_2^2}{\pi}\int d^2\sigma \, \text{Tr}(g^{-1}\partial_+ g g^{-1}\partial_- g),
\end{aligned} \tag{2.5}$$

representing $\dim G$ free bosons and a PCM model for a group G. This is consistent with the limit of the β-functions for λ_1 and λ_2 (see, eqs. (2.6) and (2.7) in [5]). In this limit, the constant λ_1 does not run since it can be absorbed into a redefinition of the v's. Also the coupling constant κ_2^2 obeys the same RG flow equation appropriate for the PCM model and its non-Abelian T-dual, since these models are canonically equivalent.

It would be very interesting to explore physical applications in an AdS/CFT context of this version of non-Abelian T-duality along the lines and developments of [37–43] (for a partial list of works in this direction). Prototype examples this can be applied are the backgrounds $AdS_3 \times S^3 \times S^3 \times S^1$ and $AdS_5 \times S^5$.

3. Cyclic λ-deformations

In this section we construct a class of multi-parameter deformations of conformal field theories of the WZW type Consider n WZW models and n PCMs for a group G, defined with group elements g_i and $\tilde{g}_i$, respectively. We would like to gauge the global symmetry

$$g_i \mapsto \Lambda_i^{-1} g_i \Lambda_{i+1}, \quad \tilde{g}_i \mapsto \Lambda_i^{-1}\tilde{g}_i, \quad i = 1, 2, \ldots, n,$$

with the periodicity condition $\Lambda_{n+1} = \Lambda_1$ implied. We introduce gauge fields $A_\pm^{(i)}$ in the Lie-algebra of G transforming as

$$A_\pm^{(i)} \mapsto \Lambda_i^{-1} A_\pm^{(i)} \Lambda_i - \Lambda_i^{-1}\partial_\pm \Lambda_i\,, \quad i = 1, 2, \ldots, n\,. \tag{3.1}$$

In this way we have a periodic chain of interacting models each one of which separately is gauge anomalous by a term independent of the group elements. The full model has no gauge anomaly since these cancel among themselves (the chain may be open as long as it is infinite long). The details are quite similar to those for the $n = 2$ case [1], so that we omit them here.

The choice $\tilde{g}_i = \mathbb{I}$, $i = 1, 2, \ldots, n$ completely fixes the gauge and is consistent with the equations of motion for the group elements $\tilde{g}_i$ of the PCMs which are automatically satisfied. Then, the gauged fixed action becomes

$$\begin{aligned} S_{k,\lambda_i}(\{g_i; A_\pm^{(i)}\}) &= \sum_{i=1}^{n} S_k(g_i) \\ &+ \frac{k}{\pi}\int d^2\sigma \sum_{i=1}^{n} \mathrm{Tr}\Big(A_-^{(i)}\partial_+ g_i g_i^{-1} - A_+^{(i+1)} g_i^{-1}\partial_- g_i \\ &+ A_-^{(i)} g_i A_+^{(i+1)} g_i^{-1} - A_+^{(i)}\lambda_i^{-1}A_-^{(i)}\Big)\,, \end{aligned} \tag{3.2}$$

where the index i is defined modulo n. The equations of motion with respect to the $A_\pm^{(i)}$'s are given by

$$\lambda_i^T D_i A_+^{(i+1)} - A_+^{(i)} = -i\lambda_i^T J_+^{(i)}\,, \quad \lambda_{i+1} D_i^T A_-^{(i)} - A_-^{(i+1)} = i\lambda_{i+1} J_-^{(i)}\,.$$

Solving them we find that

$$A_+^{(1)} = i(\mathbb{I} - x_1 x_2 \cdots x_n)^{-1}\sum_{i=1}^{n} x_1 x_2 \cdots x_{i-1}\lambda_i^T J_+^{(i)}\,, \quad x_i = \lambda_i^T D_i\,. \tag{3.3}$$

The rest can be obtained by cyclic permutations. Plugging the latter into (3.2) we find that the on-shell action reads

$$\begin{aligned} S_{k,\lambda_i}(\{g_i\}) &= \frac{k}{12\pi}\int \mathrm{Tr}(g_1^{-1}dg_1)^3 \\ &+ \frac{k}{\pi}\int d^2\sigma\, \mathrm{Tr}\Bigg(\frac{1}{2}J_+^{(1)} D_1 \frac{\mathbb{I} + x_1^T x_n^T x_{n-1}^T \cdots x_2^T}{\mathbb{I} - x_1^T x_n^T x_{n-1}^T \cdots x_2^T} J_-^{(1)} \\ &+ \sum_{i=2}^{n} J_+^{(i)}\lambda_i x_{i-1}^T \cdots x_2^T (\mathbb{I} - x_1^T x_n^T x_{n-1}^T \cdots x_2^T)^{-1} J_-^{(1)}\Bigg) \\ &+ \text{cyclic in } 1, 2, \ldots, n\,, \end{aligned} \tag{3.4}$$

where we have separated the Wess–Zumino term from the WZW model action. For small values of the matrices we have that

$$S_{k,\lambda_i}(\{g_i\}) = \sum_{i=1}^{n} S_k(g_i) + \frac{k}{\pi}\sum_{i=1}^{n}\int d^2\sigma\, \mathrm{Tr}\left(J_+^{(i+1)}\lambda_{i+1}J_-^{(i)}\right) + \mathcal{O}(\lambda^2)\,, \tag{3.5}$$

representing n distinct WZW models interacting by mutual current bilinears, for which (3.4) is the all loop, in the λ_i's, effective action.

We would like to stress that the $n = 2$ is significantly different with respect to higher n's. Firstly, the *non-perturbative* symmetry $\lambda_i \mapsto \lambda_i^{-1}$ and $k \mapsto -k$, is seemingly realized at a local level for the group elements only when $n = 2$, see (1.14) (also for $n = 1$, see (1.6)). For higher values of n the group elements need to be transformed non-locally by using $\mathcal{J}_\pm^{(i)} \mapsto -\mathcal{J}_\mp^{(i+1)}$, with $n + 1 \equiv 1$. There are exceptions to this. In particular, if all λ_i are equal and isotropic, i.e. $\lambda_i = \lambda\mathbb{I}$, then this duality-type symmetry is

$$\begin{aligned} &k \mapsto -k\,, \quad \lambda \mapsto \frac{1}{\lambda}\,, \quad g_1 \leftrightarrow g_2^{-1}\,, \\ &g_n \leftrightarrow g_3^{-1}\,, \quad g_{n-1} \leftrightarrow g_4^{-1}\,, \quad \text{etc.}\,, \end{aligned} \tag{3.6}$$

that is the group elements are paired up as above. For odd n one group element simply gets inverted. Despite the fact that the symmetry can not be realized locally for the generic case it is still powerful enough to constrain the β-functions and current correlation functions of the cyclic model to have the same values as those of the single λ-deformations.

A second remark concerns the form of the action (3.4) when one of the coupling matrices vanishes. Consider this action for $n = 2$ and $n = 3$ when $\lambda_1 = 0$ while the other coupling matrices stay general

$$S_{k,0,\lambda_2}(g_1, g_2) = \sum_{i=1}^{2} S_k(g_i) + \frac{k}{\pi}\int d^2\sigma\, \mathrm{Tr}\left(J_+^{(2)}\lambda_2 J_-^{(1)}\right)\,,$$

$$\begin{aligned} S_{k,0,\lambda_2,\lambda_3}(g_1, g_2, g_3) &= \sum_{i=1}^{3} S_k(g_i) + \frac{k}{\pi}\int d^2\sigma\, \mathrm{Tr}\Big(J_+^{(2)}\lambda_2 J_-^{(1)} \\ &+ J_+^{(3)}\lambda_3 J_-^{(2)} + J_+^{(2)}\lambda_3 D_2^T \lambda_2 J_-^{(1)}\Big)\,. \end{aligned}$$

When $n = 2$ the exact expression matches the approximate one in (3.5), while for $n = 3$ the last term couples the three WZW models and it is quadratic in the λ's.

3.1. Algebra and Hamiltonian

Here we provide the proof that the σ-model action (3.4) is integrable for specific choices of the matrices λ_i, $i = 1, 2, \ldots, n$. In particular, we will show that it is integrable for all choices of the deformation matrices λ_i which, separately, give an integrable λ-deformed model. These include the isotropic λ for semi-simple group and symmetric coset, the anisotropic $SU(2)$ and the λ-deformed Yang–Baxter model [2–4,33,45].

It is equivalent and more convenient to work with the gauged fixed action before integrating out the gauge fields. Varying the gauged fixed action with respect $A_-^{(i)}$ and $A_+^{(i+1)}$ we find the constraints

$$\nabla_+ g_i\, g_i^{-1} = (\lambda_i^{-T} - \mathbb{I})A_+^{(i)}\,, \quad g_i^{-1}\nabla_- g_i = -(\lambda_{i+1}^{-1} - \mathbb{I})A_-^{(i+1)}\,, \tag{3.7}$$

respectively. Varying with respect to g_i we obtain that

$$\nabla_-(\nabla_+ g_i g_i^{-1}) = F_{+-}^{(i)}\,, \quad \nabla_+(g_i^{-1}\nabla_- g_i) = F_{+-}^{(i+1)}\,, \tag{3.8}$$

which are in fact equivalent and where $F_{+-}^{(i)} = \partial_+ A_-^{(i)} - \partial_- A_+^{(i)} - [A_+^{(i)}, A_-^{(i)}]$.

Substituting (3.7) into (3.8) we obtain after some algebra that

$$\begin{aligned} &\partial_+ A_-^{(i)} - \lambda_i^{-T}\partial_- A_+^{(i)} = [\lambda_i^{-T} A_+^{(i)}, A_-^{(i)}]\,, \\ &\lambda_i^{-1}\partial_+ A_-^{(i)} - \partial_- A_+^{(i)} = [A_+^{(i)}, \lambda_i^{-1} A_-^{(i)}]\,. \end{aligned} \tag{3.9}$$

Hence the equations of motion split into n identical sets which are seemingly decoupled even though the $A_\pm^{(i)}$ depend on all group elements g_i and coupling matrices λ_i, $i = 1, 2, \ldots, n$. Moreover, each set is the same one that one would have obtained had we performed the corresponding analysis for the λ-deformed action (1.5). Working along the lines of subsection 1.2; Eqns. (1.15)–(1.20) we find (for $n = 2$ this was performed in detail in [1])

$$\begin{aligned}&\{\mathcal{J}_\pm^{(i)a}, \mathcal{J}_\pm^{(i)b}\} = \frac{2}{k}\, f_{abc}\mathcal{J}_\pm^{(i)c}\delta_{\sigma\sigma'} \pm \frac{2}{k}\delta_{ab}\,\delta'_{\sigma\sigma'}\,,\\ &\mathcal{J}_+^{(i)} = \lambda_i^{-T} A_+^{(i)} - A_-^{(i)}\,, \quad \mathcal{J}_-^{(i)} = \lambda_{i+1}^{-1} A_-^{(i+1)} - A_+^{(i+1)}\end{aligned} \tag{3.10}$$

and as a consequence $\{A_\pm^{(i)}, A_\pm^{(j)}\} = 0$, for $i \neq j$, for all choices of signs and for generic coupling matrices λ_i. Hence, all choices for matrices known to give rise to integrability for the λ-deformed models provide integrable models here as well with independent conserved changes. The Hamiltonian density of the system in terms of $A_\pm^{(i)}$ and λ_i is

$$\boxed{\begin{aligned}\mathcal{H}_{\text{cyclic}} = \frac{k}{4\pi}\sum_{i=1}^{n}\text{Tr}\Big(&A_+^{(i)}\left(\lambda_i^{-1}\tilde{h}_i\lambda_i^{-T}\right)A_+^{(i)}\\ &+ A_-^{(i)}\left(\lambda_i^{-T}h_i\lambda_i^{-1}\right)A_-^{(i)}\Big)\end{aligned}}\,. \tag{3.11}$$

Using the above we generalize the result of subsection 1.2, that the cyclic λ-deformed models are canonically equivalent to n single λ-deformed σ-model. The relations which define the canonical transformation are given by: $A_\pm^{(i)} = \tilde{A}_\pm^{(i)}$, $i = 1, 2, \dots, n$, where the gauge fields without the tildes correspond to the cyclic deformed models and depend on $(\lambda_1, \dots, \lambda_n; g_1, \dots, g_n)$, while those with tildes correspond to the canonically equivalent sum of n single λ-deformed models each one depending on $(\lambda_i; \tilde{g}_i)$.

3.2. RG flows and currents anomalous dimensions

Similar to the case with $n = 2$ considered in [5], the expression (3.5) can be used to argue that the RG flow equations of the n coupling matrices λ_i for the cyclic model (3.4) as well as the currents anomalous dimensions are the same with those obtained for the single λ-deformations model [6,9,46]. The basic reason is that the various interaction terms have regular OPE among themselves so that correlations functions involving currents factorize to those of n single λ-deformed models. This is also in agreement with the fact that the cyclic model is canonically equivalent to n single λ-deformations. Furthermore we mention without presenting any details that using the analysis performed in [5,46] we have explicitly checked the above claim for the cases of n isotropic couplings for general groups and symmetric spaces.

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