Swarm Precise Orbit Determination and ionospheric signatures

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Low Earth Orbiters (LEOs)

CHAMP

GRACE

GOCE

CHAllenging Minisatellite Payload
Gravity Recovery And Climate Experiment
Gravity and steady-state Ocean Circulation Explorer

But there are many more missions equipped with GPS receivers …

Jason  Jason-2  MetOp-A  Icesat  COSMIC

Kühlungborn, 29.01. - 31.01.2018
LEO Constellations

TanDEM-X  
Swarm  
Sentinel

and of course, in the future

GRACE-FO Mission
LISA Technology  
Sheds Light on Climate Change

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Global Navigation Satellite Systems (GNSS)

GPS

visible sat = 12

Galileo

Empfangene Satelliten: 10

Other GNSS are already existing (GLONASS) or being built up (Galileo, Beidou), but there are no multi-GNSS spaceborne receivers (with open data policy) in LEO orbit yet.
GPS Signals

Signals driven by an **atomic clock**

Two **carrier signals** (sine waves):
- \( L_1 \): \( f = 1575.43 \text{ MHz} \), \( \lambda = 19 \text{ cm} \)
- \( L_2 \): \( f = 1227.60 \text{ MHz} \), \( \lambda = 24 \text{ cm} \)

Bits encoded on carrier by phase modulation:
- **C/A-code** (Clear Access / Coarse Acquisition)
- **P-code** (Protected / Precise)
- **Broadcast/Navigation Message**

(Ref: Blewitt, 1997)
Improved Observation Equation

\[ L_i^k = \rho_i^k - c \cdot \Delta t^k + c \cdot \Delta t_i + X_i^k + \lambda \cdot N_i^k + \Delta_{rel} - c \cdot b^k + c \cdot b_i + m_i^k + \epsilon_i^k \]

- \( \rho_i^k \): Distance between satellite and receiver
- \( \Delta t^k \): Satellite clock offset wrt GPS time
- \( \Delta t_i \): Receiver clock offset wrt GPS time
- \( T_i^k \): Tropospheric delay
- \( I_i^k \): Ionospheric delay
- \( N_i^k \): Phase ambiguity
- \( \Delta_{rel} \): Relativistic corrections
- \( b^k \): Delays in satellite (cables, electronics)
- \( b_i \): Delays in receiver and antenna
- \( m_i^k \): Multipath, scattering, bending effects
- \( \epsilon_i^k \): Measurement error

Satellite positions and clocks are known from ACs or IGS.
Not existent for LEOs.
Cancels out (first order only) when forming the ionosphere-free linear combination:

\[ L_c = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 \]
Geometric Distance

Geometric distance $\rho_{\text{leo}}^k$ is given by:

$$\rho_{\text{leo}}^k = |r_{\text{leo}}(t_{\text{leo}}) - r^k(t_{\text{leo}} - \tau_{\text{leo}}^k)|$$

- $r_{\text{leo}}$ Inertial position of LEO antenna phase center at reception time
- $r^k$ Inertial position of GPS antenna phase center of satellite $k$ at emission time
- $\tau_{\text{leo}}^k$ Signal traveling time between the two phase center positions

Different ways to represent $r_{\text{leo}}$:

- **Kinematic** orbit representation
- **Dynamic** or **reduced-dynamic** orbit representation
Kinematic Orbit Representation (1)

Satellite position $r_{leo}(t_{leo})$ (in inertial frame) is given by:

$$r_{leo}(t_{leo}) = R(t_{leo}) \cdot (r_{leo,e,0}(t_{leo}) + \delta r_{leo,e,ant}(t_{leo}))$$

- $R$ Transformation matrix from Earth-fixed to inertial frame
- $r_{leo,e,0}$ LEO center of mass position in Earth-fixed frame
- $\delta r_{leo,e,ant}$ LEO antenna phase center offset in Earth-fixed frame

Kinematic positions $r_{leo,e,0}$ are estimated for each measurement epoch:

- Measurement epochs need not to be identical with nominal epochs
- Positions are independent of models describing the LEO dynamics.
  Velocities cannot be provided
A kinematic orbit is an ephemeris at **discrete** measurement epochs. Kinematic positions are **fully independent** on the force models used for LEO orbit determination.

(Švehla and Rothacher, 2004)
## Kinematic Orbit Representation (3)

**Measurement epochs**  
(in GPS time)

<table>
<thead>
<tr>
<th>Year</th>
<th>Month</th>
<th>Day</th>
<th>Position (km) (Earth-fixed)</th>
<th>Clock correction to nominal epoch (μs), e.g., to epoch 00:00:20</th>
</tr>
</thead>
<tbody>
<tr>
<td>2016</td>
<td>6</td>
<td>1</td>
<td>0 0 17.99999974</td>
<td>0.262691</td>
</tr>
<tr>
<td>2016</td>
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<td>1</td>
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<tr>
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<td>1</td>
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<tr>
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<td>1</td>
<td>0 0 22.99999979</td>
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<tr>
<td>2016</td>
<td>6</td>
<td>1</td>
<td>0 0 23.99999984</td>
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<tr>
<td>2016</td>
<td>6</td>
<td>1</td>
<td>0 0 24.99999979</td>
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<tr>
<td>2016</td>
<td>6</td>
<td>1</td>
<td>0 0 26.99999979</td>
<td></td>
</tr>
</tbody>
</table>

**Excerpt of kinematic Swarm-C positions at begin of 1 June, 2016**

The kinematic orbits may be downloaded at ftp://ftp.unibe.ch/aiub/LEO_ORBITS/

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Dynamic Orbit Representation (1)

Satellite position \( r_{\text{leo}}(t_{\text{leo}}) \) (in inertial frame) is given by:

\[
    r_{\text{leo}}(t_{\text{leo}}) = r_{\text{leo},0}(t_{\text{leo}}; a, e, i, \Omega, \omega, u_0; Q_1, \ldots, Q_d) + \delta r_{\text{leo,ant}}(t_{\text{leo}})
\]

- \( r_{\text{leo},0} \) LEO center of mass position
- \( \delta r_{\text{leo,ant}} \) LEO antenna phase center offset
- \( a, e, i, \Omega, \omega, u_0 \) LEO initial osculating orbital elements
- \( Q_1, \ldots, Q_d \) LEO dynamical parameters

Satellite trajectory \( r_{\text{leo},0} \) is a particular solution of an equation of motion

- One set of initial conditions (orbital elements) is estimated per arc. Dynamical parameters of the force model on request
Dynamic orbit positions may be computed at any epoch within the arc.

Dynamic positions are fully dependent on the force models used, e.g., on the gravity field model.
Reduced-Dynamic Orbit Representation (1)

**Equation of motion** (in inertial frame) is given by:

\[
\ddot{r} = -GM \frac{r}{r^3} + f_1(t, r, \dot{r}, Q_1, \ldots, Q_d, P_1, \ldots, P_s)
\]

\(P_1, \ldots, P_s\) Pseudo-stochastic parameters

**Pseudo-stochastic** parameters are:

- additional empirical parameters characterized by a priori known **statistical properties**, e.g., by expectation values and a priori variances

- useful to **compensate** for deficiencies in dynamic models, e.g., deficiencies in models describing non-gravitational accelerations

- often set up as **piecewise constant accelerations** to ensure that satellite trajectories are continuous and differentiable at any epoch
Reduced-dynamic orbits are well suited to compute LEO orbits of highest quality (Jäggi et al., 2006; Jäggi, 2007)

Reduced-dynamic orbits heavily depend on the force models used, e.g., on the gravity field model.
Reduced-dynamic Orbit Representation (3)

Position epochs
(in GPS time)

<table>
<thead>
<tr>
<th>2016</th>
<th>6</th>
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<th>Positions (km) &amp; Velocities (dm/s) (Earth-fixed)</th>
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<tbody>
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<td>VL49</td>
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<td>-43931.500489</td>
</tr>
</tbody>
</table>

Excerpt of reduced-dynamic Swarm-C positions at begin of 1 June, 2016

Clock corrections are not provided
Orbit Differences KIN-RD (Swarm-C)

Differences at epochs of kin. positions

- **radial**
  - 0.7 ± 12.0 mm

- **along-track**
  - 1.0 ± 9.7 mm

- **cross-track**
  - -2.0 ± 7.5 mm

**Hours of 1 June, 2016**
Swarm A Gravity Field

Geoid height differences, static GRACE gravity field AIUB-GRACE03S - Swarm A gravity field, November 2014
Orbit Differences KIN-RD and Plasmadensity

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Orbit Differences KIN-RD and Plasmadensity

Swarm A, 14-296, 45.6°E, 20:37LT

Altitude in [km]

Plasmadensity in [N_e/m^3]
Previous Studies: AIUB-Cutoff

Swarm-A, Nov 2014, 400km Gauss, unweighted

Swarm A, Nov 2014, AIUB RINEX screening

Original Datapoints

Number of Points

AIUB Cutoff

Number of Points
Previous Studies: Graz-ROT\(I\)

\[
ROT\(I\) = \sqrt{\left< \Delta TEC^2 \right> - \left< \Delta TEC \right>^2 dt^2}
\]

- Applied to \(\Delta TEC\), moving window, 31s
- \(\sigma = 10 \cdot ROTI \cdot \sigma_0\) if \(ROTI > 0.1\)
- Developed for GOCE
Phase Residuals and Time derivatives \textit{L4}

\begin{itemize}
  \item L3-Residuals, \text{radial, in [m]}
  \item D1L4, \text{in [m/s]}
  \item D2L4, \text{in [m/s}^2]\end{itemize}

\text{2014/10/29, UT in [h]}

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Elevations and azimuth of affected GPS satellites

<table>
<thead>
<tr>
<th>Group</th>
<th>MLAT(UP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-35 → -25</td>
</tr>
<tr>
<td>2</td>
<td>-25 → -10</td>
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<tr>
<td>3</td>
<td>-10 → 10</td>
</tr>
<tr>
<td>4</td>
<td>10 → 25</td>
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<tr>
<td>5</td>
<td>25 → 35</td>
</tr>
</tbody>
</table>

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Elevations and azimut of affected GPS satellites

Swarm-A, Nov 2014, 400km Gauss, unweighted

Swarm A, Nov 2014, AIUB RINEX screening

Swarm-A, Nov 2014, 400km Gauss, d2L4/dt^2

Swarm-A, Nov 2014, 400km Gauss, Model LS

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Elevations and azimuth of affected GPS satellites

Swarm-A, Nov 2014, 400km Gauss, unweighted

Swarm-A, Nov 2014, 400km Gauss, dL4/dt

Swarm-A, Nov 2014, 400km Gauss, d2L4/dt^2

Swarm-A, Nov 2014, 400km Gauss, d3L4/dt^2
Evaluation of methods

SLR-validation of reduced dynamic orbits:

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean [mm]</th>
<th>Std [mm]</th>
<th>RMS [mm]</th>
</tr>
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<tbody>
<tr>
<td>Original</td>
<td>2.6</td>
<td>16.6</td>
<td>16.8</td>
</tr>
<tr>
<td>ROTI</td>
<td>2.7</td>
<td>17.3</td>
<td>17.5</td>
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<tr>
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<tr>
<td>Model</td>
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<tr>
<td>D1L4</td>
<td>0.3</td>
<td>22.1</td>
<td>22.1</td>
</tr>
<tr>
<td>D2L4</td>
<td>2.8</td>
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<td>16.1</td>
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<tr>
<td>D3L4</td>
<td>4.1</td>
<td>13.4</td>
<td>14.4</td>
</tr>
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</table>
Thank you for your attention