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On axions and the chiral anomaly at finite temperature

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Abstract. A brief introduction to a plenary discussion session at the Lattice'17 conference on axions and the chiral anomaly at finite temperature is provided.

1 Introduction

It is an intriguing prospect that microscopic quantum-mechanical phenomena could lead to macroscopic consequences in the Early Universe. The very high temperatures and densities appearing there guarantee that processes which are rare today, could have proceeded with a fast rate in all patches of spacetime. For instance, CP violating reactions could have produced the matter over antimatter excess observed in astronomy, and particles interacting feebly enough to have evaded detection at the LHC might have been produced so efficiently that they could account for all of dark matter.

Among the remarkable features of gauge field theories is the existence of anomalies, i.e. quantum-mechanical violations of symmetries that would be exact on the classical level. The axial anomaly, in particular, states that the divergence of the flavour singlet axial vector current has an additional quantum-mechanical source term. Roughly speaking, this implies that the "singlet" η' meson is heavier than the "octet" of π, K and η mesons, even though in other systems (such as the hydrogen atom) states transforming in lowest-dimensional representations tend to be the lightest ones.

In vacuum, the non-singlet part of the chiral symmetry is spontaneously broken, $\mathrm{SU_L}(N_\mathrm{f}) \times \mathrm{SU_R}(N_\mathrm{f}) \to \mathrm{SU_V}(N_\mathrm{f})$. This symmetry gets restored at a temperature of a few hundred MeV, in a transition which can be of first order, second order, or smooth, depending on the explicit breaking of the symmetry by quark masses. It has been an age-old "dream" that this transition, whose properties can be investigated both on the lattice and in heavy ion collision experiments, could have had cosmological significance [1, 2]. Unfortunately, convincing evidence in this direction is hard to come by. One reason is that lattice simulations suggest the transition to be a smooth crossover rather than a first order transition, which could have led to more prominent consequences.

The question at the center of the current discussion session is whether the anomalous $\mathrm{U_A}(1)$ part of chiral symmetry also experiences some "transition" at a finite temperature [3], and whether this could have cosmological significance. Even though well-posed within QCD, the issue is often embedded in an extension of the Standard Model involving a weakly interacting axion field. Then issues related to $\mathrm{U_A}(1)$ could have ramifications not only for the QCD transition, but also as a dynamical solution to the strong CP problem [4–6] and for the nature of dark matter [7–9].

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It turns out to be difficult to resolve the fate of $U_A(1)$ on the lattice. Obviously, respecting chiral symmetry faithfully in the presence of a lattice regularization is a challenge by itself. Moreover, going to finite temperature ($T > 0$), the imaginary-time extent of the lattice gets squeezed. If we want to have several points in the temporal direction in order to avoid discretization artifacts, we are soon driven to very fine lattice spacings. This leads to problems of its own (see below), and furthermore requires very large spatial volumes for avoiding finite-volume effects in a region where correlation lengths are becoming substantial. Therefore, systematic uncertainties are hard to get under control.

The purpose of the session was to underline the physics motivation for eventually resolving these issues; to shed light on the existing lattice results; and to clarify the reasons for the large discrepancies that currently exist. In the remainder of the present writeup, a few more words are offered on the basic context (sec. 2) and on the potential physics significance of the axial anomaly at finite temperature (sec. 3). This hopefully serves as a motivation for the reader to study the actual contributions, summarized in sec. 4. A previous review from a Lattice conference [10] is also worth consulting.

2 Common knowledge on the axial anomaly at $T > 0$

There should be no doubt that the basic form of the anomaly equation, interpreted as an expectation value of the divergence of the axial current in a gauge field background, and written here for simplicity for a smooth gauge field and in continuum notation, stays intact at finite temperature [11]:

$$\Big\langle \partial_\mu [\bar\psi \gamma_\mu \gamma_5 \psi] \Big\rangle_A \;=\; \Big\langle 2m\bar\psi\gamma_5\psi \Big\rangle_A + \frac{N_f}{32\pi^2} F^a_{\mu\nu} \widetilde{F}^a_{\mu\nu} \,. \tag{1}$$

The integral of this equation over the space-time volume, with the left-hand side vanishing and the first term on the right-hand side saturated by zero modes (if any exist), gives in turn the index theorem [12]. However, the anomaly equation and the index theorem by themselves say nothing about how frequently zero modes (or topologically non-trivial configurations) appear in the ensemble average.

Now, the likelihood of non-trivial configurations is expected to decrease dramatically as the temperature increases. The average value of the topological charge is zero, and the width of its distribution is characterized by the topological susceptibility. At very high T, only topologically trivial configurations contribute. One way to understand this is that because of antiperiodic boundary conditions in the temporal direction, the Dirac equation has no zero modes in the free theory. At high temperature the effective coupling is weak and perturbation theory works, so that this intuition should also apply at finite coupling. In another language, the short temporal extent of the lattice prevents the existence of large instantons, and therefore of the configurations that would allow transitions to non-trivial topological sectors [13]. These effects can be estimated in an instanton calculus (see e.g. ref. [14] and references therein), and lead to a power-like decrease of the topological susceptibility at high T.

Despite the decrease of the topological susceptibility, it can be argued that there are quantities which do display remnants of the chiral anomaly at any temperature. Consider for instance the singlet vector and axial vector currents. The vector current is conserved, the axial one is not, which should lead to different screening masses, measured in a spatial direction [15, 16]. The demonstration of this difference necessitates the inclusion of singlet contractions: the screening mass in the axial channel is expected [15] to correspond to a particular (gauge independent) Debye mass of gluonic modes [17].

To summarize, there is an unambiguous expectation that the topological susceptibility, and therefore any topological "activity", should be very small at high temperatures. Alas, on the lattice, there is an additional reason why this small number is hard to resolve: as mentioned above one is driven to a fine lattice spacing. This leads to the problem of topological freezing [18], which then needs to be excluded as a possible wrong reason for the small value of the measurement.

3 Possible physics effects from the axial anomaly at $T > 0$

Suppose that the topological susceptibility can be measured, and that it is small but non-zero at high temperatures. What would this imply?

Much of the recent excitement is due to the idea that if the axion field couples linearly to the topological charge density, its thermal mass squared is proportional to the Euclidean topological susceptibility. Here an essential part of the argument is that the axion field has no bare mass parameter, or that it is small compared with the thermal correction. The absence of any bare mass would be consistent with the fact that the Euclidean topological susceptibility is ultraviolet finite [19], however a symmetry argument is needed for a rigorous statement. Indeed, the axion field is generally viewed as a Goldstone mode, which would become exactly massless in the chiral limit.

As a side remark, it should be kept in mind that any field getting a thermal mass from interactions with a plasma generally also gets a thermal friction coefficient affecting its evolution. The friction coefficient can be shown to be related to a Minkowskian version of the topological susceptibility [20]. The latter is a genuine real-time quantity ("transport coefficient"), whose determination represents another great challenge for the lattice, similar to that of viscosities. Fortunately in the cosmological context its contribution is overshadowed by a "Hubble friction" appearing in the equation of motion of the axion field, originating from the dilution of the energy density caused by the expansion of the Universe (i.e. redshift). The order of magnitude of the friction coefficient has been estimated in the weak-coupling limit, through the use of so-called classical lattice gauge theory simulations [21].

Apart from axion physics, a reduced anomaly could have consequences purely within QCD. If the nature of the chiral symmetry that gets restored is changed, this could influence the universality class and phase diagram of the chiral transition [3]. Moreover, if the axial-vector current is almost conserved, both because quark masses are small compared with the temperature and because the anomalous part is inefficient, chiral degrees of freedom should be included in the effective low-energy description of the non-equilibrium evolution of the plasma generated in heavy ion collision experiments. Such "anomalous hydrodynamics", of interest also to other systems such as neutron stars and the cosmological evolution of primordial magnetic fields, has turned into a large field of its own in recent years (see e.g. ref. [22] and references therein).

4 Session contributions

The session started with a phenomenological overview of axion cosmology and the role that lattice QCD can play in supporting or ruling out this scenario [23]. Focal points were a discussion of the numerical simulation of axion string networks and an estimate of the temperature range that appears to be most important for practical applications (540 – 1150 MeV). Subsequently, various measurements of the topological susceptibility in unquenched QCD were presented. The first outlined a framework aimed at controlling systematic uncertainties [24]; the second a somewhat controversial position according to which the thermal topological susceptibility vanishes exactly below a certain critical quark mass [25]; and a third a measurement which, with the price of a substantial "reweighting step", has come up with results widely appreciated by cosmologists [26].

The presentations were followed by a lively discussion session. Not surprisingly, it was questioned whether systematic errors are under control or in principle controllable in all of the approaches.

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