Finally, McGinn's inquiry into the five logical properties assumes an uncommon methodological stance which, arguably, is dictated by the realist view of these properties. In general, since truth conditions by themselves underdetermine semantics, our conception of the semantic properties of terms should dictate the assignments of truth-conditions to the sentences in which the terms occur, not the other way around. Hence McGinn's contention that the quantifier is vastly over-rated as a tool of logical and linguistic analysis. We should not be seduced by the success of the reduction of identity, existential, and modal sentences to quantifications over properties, things, and possible worlds, respectively. Even if these reductions fix the truth-conditions for identity, existence, and modal sentences, the appeal to quantification distorts the semantic facts about '=', 'exists', and 'possibly' because it fails to yield adequate analyses of the concepts of identity, existence, and modality. More generally, McGinn believes that there are substantive facts about the semantic functioning of the syntactic correlates of the five logical properties, and that these facts are epistemically accessible to us. McGinn gets to these semantic facts by first doing metaphysics—getting an account of the nature of the logical properties—and then using the results to develop an account of the semantic functioning of the corresponding terms.

Thus, semantics cannot be the royal road to metaphysics, because the correct semantics is constrained by the correct metaphysics. Surely, on some level this is correct. We can't argue against the Parmenidean by claiming that there exists a counterexample to the argument: some are eternal, so all are eternal. Since the counterexample requires the possibility of there being more than one thing, the required semantics of the quantifiers presupposes that Parmenidean metaphysics is wrong. McGinn's work highlights the extent to which metaphysics penetrates and permeates the conceptual framework of logic. Inquiry into the central concepts of logic cannot ignore metaphysics. Even if one does not agree with all the conclusions of the book, one has to grant that *Logical Properties* makes a good case for bringing, as McGinn puts it, philosophy back into philosophical logic.

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SERGEI TUPAILO. *Realization of analysis into explicit mathematics*. ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 1848–1864.

The paper under review explores realizability interpretations of various intuitionistic subsystems of second-order arithmetic into Feferman's explicit mathematics. For the latter systems, this therefore provides an alternative method to establish their lower proof-theoretic bounds.

Explicit mathematics was introduced by Feferman (*A language and axioms for explicit mathematics*, JSL XLIX 308(1); *Constructive theories of functions and classes*, JSL LXIX 308(2)) in the seventies. Beyond its original aim to provide a basis for Bishop-style constructivism, the explicit framework has gained considerable importance in proof theory in connection with the proof-theoretic analysis of subsystems of second-order arithmetic and set theory. The most famous example in this connection is the reduction of the classical subsystem of second-order arithmetic based on Δ^1_2 comprehension and bar induction to the most prominent framework of explicit mathematics, T_0, achieved by Jäger (*A well-ordering proof for Feferman's theory T_0*, ***Archiv für mathematische Logik und Grundlagenforschung***, vol. 23 (1983), pp. 65–77) and Jäger and Pohlers (*Eine beweistheoretische Untersuchung von* $(\Delta^1_2\text{-}CA) + (BI)$ *und verwandter Systeme*, ***Sitzungsberichte der Bayerischen Akademie der Wissenschaften, Mathematisch-naturwissenschaftliche Klasse***, 1982, pp. 1–28).

The language of explicit mathematics, more precisely its operational or applicative core, allows for a very smooth and elegant formulation of abstract Kleene-style notions of realizability. Moreover, even realizability for second-order languages finds its natural place, since

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collections of (first-order) individuals, so-called classes or types, are directly named or represented by objects of first order, which in turn can act as arguments of abstract operations in the applicative basis of explicit mathematics.

The subsystems of intuitionistic second-order arithmetic that are shown in this article to be realizable into the system T_0 and its subsystems are based on schemes of arithmetic comprehension, choice, and replacement, as well as a principle of inductive generation. The strongest of these systems is the theory IARI introduced by Griffor and Rathjen (*The strength of some Martin-Löf type theories*, ***Archive for Mathematical Logic***, vol. 33 (1994), pp. 347–385), which has the same proof-theoretic strength as Δ^1_2 comprehension plus bar induction, hence exhausting all of T_0.

The present paper explores realizability in explicit mathematics of the relatively simple language of second-order arithmetic. This keeps the amount of technical details at a minimum and demonstrates the method most distinctly. The paper can be seen as an important preparatory work for two subsequent papers by Tupailo, dealing with a much more involved realizability interpretation of constructive set theory CZF, possibly augmented by large set axioms, into explicit mathematics (*Realization of constructive set theory into explicit mathematics: a lower bound for impredicative Mahlo universe*, ***Annals of Pure and Applied Logic***, to appear; *On non-wellfounded constructive set theory*, to appear in the proceedings of the Ninth Annual CSLI Colloquium on Logic, Language and Computation; both papers are available at `http://greta.cs.ioc.ee/~sergei/publ.html`). The former of these two papers provides a relative lower bound of the impredicative Mahlo universe in explicit mathematics.

In summary, the author offers an interesting, concisely written paper, which directly relates strong subsystems of intuitionistic analysis to explicit mathematics, without going through ordinal-theoretic proof theory.

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F. A. MULLER. *Sets, classes, and categories*. ***British Journal for the Philosophy of Science***, vol. 52 (2001), pp. 539–573.

Muller skillfully expounds the challenge for set theoretic foundations for category theory: Simple categorical methods, with evidently low proof theoretic strength, get formalized by using very large sets. The standard device, a *Grothendieck universe*, is a set U which itself satisfies all the ZF axioms. The category **Top** of all topological spaces, for example, is taken as the category of all topological spaces that exist in U. That is a set, though not in U, and there is a set of all functors $\mathbf{F}\colon \mathbf{Top} \to \mathbf{Set}$ and much more. The categorical theorems used in practice survive almost entire.

As Muller says, this is "not just a bit abundant [it is] mindbogglingly, flabbergastingly abundant"(550). A universe amounts to an inaccessible cardinal. Muller offers a more elegant approach using his strengthened form of Ackermann's theory of sets and classes.

He motivates his theory and proves numerous theorems on its expressive power. It has classes of every finite rank over the class of all sets, and so is more agile than Gödel-Bernays. Muller argues that his sets and classes reflect the true difference between logical and combinatorial collections, and his sets are sharply delineated in Cantor's sense while his classes are not (though they have extensionality). He shows his theory equiconsistent with ZF.

As to ZF-style foundations, the reviewer proposes a related strategy within ZF, defining 'small' sets as elements of $V(\omega+\omega)$. All usual mathematics apart from category theory is done in small sets. Define the category **Top** as all small topological spaces and continuous maps and similarly for similar categories. Both approaches are decidedly deflationary compared to

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