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Complete List of Authors:	Germann, Peter; University of Bern, Department of Geography
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Viscosity - the weak link between Darcy's law and Richards' capillary flow

P.F. Germann, Prof. em.

University of Bern, Geography Department, Bern (Switzerland)

pf.germann@bluewin.ch

Address: Haldenackerweg 21, CH-3065 Bolligen (Switzerland)

Dedication

I dedicate this contribution to Miroslav Kutilek (1927 to 2016) who inspired soil hydrology in many very personal ways.

Abstract

Preferential flow, a term that includes macropore flow, non-equilibrium flow, and finger flow, stands in well-known conflict with Richards (1931) capillary flow. Acoustic velocity experiments demonstrate that preferential flow moves independently from, faster than, and before capillary flow during gravity-driven infiltration. Viscous flow in permeable media is briefly introduced to the point where Richards' (1931) particular treatment of viscosity turns out as the hydro-mechanical bifurcation from general laminar flow. Preferential flow is expected during significant infiltration, however, spatio-temporarily limited according to the viscous-flow regime. Two ways of delineating capillary flow from viscous flow reveal minimum path widths of preferential flow in the range of about 20 µm.

Key words: Capillary flow, non-equilibrium flow, preferential flow, viscous flow.

Abbreviations: AWI: air-water interface; CF: capillary flow; PF: preferential flow; REV: representative elementary volume; SWI: soil-water interface; TDR: time-domain reflectometry; VF: viscous flow.

Running title: Independence of preferential flow in view of viscosity in Richards' (1931) capillary flow:

1 Introduction

In partially water-saturated soils and similar permeable media the expression preferential flow summarizes non-equilibrium flow, finger flow, and macropore flow according to Jarvis et al. (2016) that occurs during infiltration. Preferential flow (PF) suggests exceptions that pivot around the ordinary, equilibrium, and spatially well-behaved flow i.e., Richards' (1931) capillary flow (CF) in porous media. So far, PF has not gained the recognition as an independent hydro-mechanical flow process at the same level as CF. Moreover, PF is usually associated with macropores of some sorts (Beven and Germann, 1982 and 2013; Jarvis et al. 2016). In order to avoid any a priori delineation of macropores from meso- and micropores, the general expression of permeable media is here given preference over the term of porous media. Permeable media consist of solids that are penetrated by continuous voids permitting water to seep or flow through, regardless of the underpinned process, and of the dimensions and geometries of the voids.

Two recent reviews on the subject stake out quite well the breadth of paradigms covering preferential flow. Weiler (2017), on the one hand, considers too restrictive the application of laminar viscous flow, for example, according to Germann and Karlen (2016). He proposes an extension of the approach that may also include turbulent PF. There are no basic restrictions to the proposition because the associated Reynolds numbers indicate the degree of turbulence, thus the deviation from the theoretical requirements. Ultimately, only experimentations can demonstrate the upper limit of tolerable Reynolds numbers (see Eq.[8] below). Internal erosion may set a more practical limit to high Reynolds numbers. On the other hand, Jarvis et al. (2016) explicitly mistrust hydro-mechanical approaches to preferential flow that are based, for instance, on Hagen-Poiseuille flow (Poiseuille, 1846). Instead, they favor hydro-dynamical approaches that require investigations at the pore scale, using micro-image techniques and powerful computers. Whatsoever, there is an increasing number of papers expressing the malaise in soil hydro-mechanics. Alberti and Cey (2011), for instance, simulated with a version of the HYDRUS 2D/3D code tension infiltrometer data collected from experimenting with a macroporous soil. Their conclusion "*These findings indicate that a Darcy-based model may not adequately represent flow in macroporous systems. Research is needed to develop physically based models of the hydraulic functioning of macropores near saturation*." A major stumbling block to the development of such models is the ignorance of hydro-mechanical limitations in Richards' (1931) CF that requires the soil hydraulic property functions i.e., relationships among the volumetric water content θ (m^3m^{-3}), the hydraulic conductivity, K ($m\ s^{-1}$), and the capillary potential ψ (Pa). A hydro-mechanical approach to infiltration and drainage that is independent from Richards' CF may open new vistas on PF.

The statement of Beven and Germann (2013), still pivoting around CF, outlines the purpose of this contribution: "… *non-equilibrium capillary and preferential flows are important in heterogeneous field soils: an extreme, but to us rather attractive, view would be to suggest that much of soil physics (at least during significant infiltration) is predicted on the wrong experimental techniques as used by Richards in 1931."* The next section illustrates PF's independence from antecedent capillarity. The third section briefly reviews viscous flow (VF) in permeable media, while the fourth sheds light on the link between Darcy's (1856) law and Richards' (1931) capillary flow. The second to last section delineates CF from VF before the conclusions.

2 Preferential flow's independence from capillarity

Nimmo (2012), for instance, reports about PF occurring in unsaturated conditions i.e., independently from ψ-θ relationships, while this section provides experimental evidence. The sand castle effect qualitatively illustrates the relationship between the rigidity of a sand-water-air mélange in relation to its moisture content. An optimal water-content range is required for sand shaping. If the sand is too wet or too dry, the shapes creep or crumble away. Too high water contents cause high ψ (i.e., closer to atmospheric pressure) resulting in weak forces pulling the sand grains together despite the large contact area between the grains and the liquid. Too low water contents exert much stronger pulling forces (i.e., $\psi << 0$) from water onto the sand grains but along small contact areas between the liquid and the solid phases. Intermediate water contents provide optimal combinations of ψ and the extents of the contact areas that keep the sand shapes in place. Thus, ψ not only expresses the capillary potential in the water but also the rigidity of the grain-water-air composite. Conversely, the rigidity of an unsaturated granular medium in relation to θ is also implicitly measures ψ.

The velocity v_{ac} (m s^{-1}) of acoustic waves crossing the sand-water-air mélange depends on the mélange's rigidity as expressed with the P-wave modulus M_P (Pa) (i.e., the pressure exerted from the acoustic shock wave onto the mélange that were required to compress it to half its antecedent volume. Hence the huge approximate range of $10^8 < M_P < 10^9$ Pa in Fig. 2). Flammer et al. (2001) measured v_{ac} during infiltration into a column of an undisturbed Typic Hapludalf with the device depicted in Fig. 1, while Fig. 2 summarizes the results from three consecutive infiltration experiments with increasing initial θ. The figure also illustrates Brutsaert's (1964) approach of v_{ac} vs. θ that is based on Richards' (1931) ψ-θ relationship. The recorded θ-values from the three runs bulge considerably away from Brutsaert's (1964) expectations. Three stages are discernible (only shown for Run 1 in Fig. 2). Stage 1: Increase of θ with but miniscule decreases of v_{ac} and M_P. Stage 2: Decreases of v_{ac} and M_P

approximately parallel to Brutsaert's (1964) approach. Stage 3: decreases of θ, v_{ac}, and $M_{P,}$ presumably ending at the Triple-point that hints at the joining of Brutsaert's (1964) ψ-θ approach with the v_{ac}-data towards the end of Run 1 and at the beginning of Run 2. The bulges in the three runs, stages 1 to 3, indicate transient water content waves gliding across the medium during recording with the TDR-probe. Apparently, θ-increase did neither alter v_{ac} nor M_P and, hence, did not alter ψ during stages 1, whereas the waves' decreasing θ during stages 3 indicate decreases of v_{ac} and M_P that are due to the delayed ψ-increases. Thus, constant v_{ac} during infiltration indicates constant rigidity of the medium, even under increasing water contents. Moreover, ψ may have approached atmospheric pressure, as Germann and al Hagrey (2008) reported from the Kiel sand tank during infiltration. In conclusion, there was no impact of antecedent ψ on early infiltration while water moved freely. The deviations of the experiments of Flammer et al. (2001) from Brutsaert's (1964) approach demonstrate non-equilibrium during infiltration with regard to Richards' (1931) CF, hence, PF's independence from capillarity during infiltration. In addition, the section demonstrates that PF occurs prior to CF.

3 Review of viscous flow in permeable media

Newton (1729) introduced viscosity in order to apply to liquids his definition of a force being equal to the product of mass times acceleration. He noted that "*The resistance, arising from the want of lubricity in the parts of a fluid, is, caeteris paribus, proportional to the velocity with which the parts of the fluid are separated from each other.*" As illustrated with Fig. 3, the resulting law of shear for water

$$\varphi(f) = -\eta \cdot \rho \cdot \frac{\mathrm{d}\, v(f)}{\mathrm{d}\, f}\bigg|_f \qquad [1]$$

states that the shear force φ (N m^{-2}) in the water film at the distance f (m) from the supporting solid surface acting in the opposite direction of flow is proportional to the derivative of the velocity v (m s^{-1}) at f during laminar flow, where η ($\approx 10^{-6}$ $m^2 s^{-1}$) and ρ (=1000 kg m^{-3}) are the water's kinematic viscosity and the density, respectively. The term [ρ dv/df] expresses momentum dissipation at f in the direction towards the solid with η acting as the dissipation coefficient. Consider a water film with thickness F (m) between the solid-water interface (SWI) and the air-water interface (AWI) creeping as laminar flow along a vertical wall. The shear force $\varphi(f)$ at f balances the weight of the film between f and the AWI at F (m) i.e., $\varphi(f) = -\rho\ g\ (F-f)$, where g (= 9.81 m s^{-2}) is acceleration due to gravity. Thus, replacing the shear force in Eq. [1] with the weight, separating variables and integrating from the SWI to the AWI leads to the parabolic velocity profile within the film under the no-slip condition of $v(0) = 0$. Dividing the integral of the velocity profile by F yields the average velocity of the film that equals the velocity of the wetting shock front as

$$v_W = \frac{g}{3 \cdot \eta} \cdot F^2 \qquad [2]$$

(m s^{-1}). Consider further the specific contact length L (m m^{-2}) of the film with the solid per unit horizontal cross-sectional area A (m^2) of the permeable medium (Fig. 3). Additional integration of the product of the parabolic velocity profile times the differential volume flux density dq at f then leads to the volume flux density of the film as

$$q(F, L) = \frac{g}{3 \cdot \eta} \cdot L \cdot F^3 \qquad [3]$$

(m s^{-1}). The mobile water content of the film amounts to

$$w = F \cdot L \qquad [4]$$

(m^3 m^{-3}) with $w < \theta$. The combination of Eqs.[3] and [4] produce the simple relationship of

$$q = b \cdot w^3 \qquad [5]$$

with the conductance of

$$b = \frac{g}{3 \cdot \eta \cdot L^2} \qquad [6]$$

(m s^{-1}), indicating the dominance of L on q when considering the flow of w. The depth of the wetting front (positive vertically down from the surface, Fig. 3) is

$$z_W(t) = v_W \cdot (t - T_B) \qquad [7]$$

where T_B (s) is the point in time when the wetting front starts moving at the surface of the permeable medium due to adequate input, while t (s) is time and Eq. [2 to 7] are valid during constant infiltration. See also Germann and Karlen (2016) and Germann (2014) for a complete derivation of VF. Multiplication with $z_W(t)$ of both, L and A yields the predominately vertical area per unit volume of a permeable medium onto which momentum dissipates according to Eq. [1] in the depth range of $0 < z < z_W(t)$. Equations [1] to [7] require laminar flow as assessed with the Reynolds number i. e., the ratio of convective vs. dispersive momentum transport according to

$$Re = \frac{F \cdot v}{\eta} = \frac{F^3 \cdot g}{3 \cdot \eta^2} \qquad [8]$$

Strict laminar flow occurs when $Re \leq 1$ (Lamb, 1924), however, a broader range seems tolerable.

Representing shear flow with cylindrical coordinates, hydraulic engineer Hagen and physiologist Poiseuille (1846), independently from one another, presented laminar flow in a thin tube as

$$Q(R) = \frac{p_1 - p_2}{\Delta l} \cdot \frac{\pi}{8 \cdot \mu} \cdot R^4 \qquad [9]$$

where Q (m^3 s^{-1}) is volume flux, R (m) is the radius of the tube, $(p_1 - p_2)/\Delta l$ (Pa m^{-1}) is the pressure gradient acting along the axis of the tube, and the dynamic viscosity is $\mu = \eta \times \rho$ ($\approx 10^{-3}$ Pa s). The application of Eq. [9] to exclusively gravity driven flow requires the replacement of $(p_1 - p_2)/\Delta l$ by $[\rho \times g]$. Further comparison of Eq. [9] with Eq. [3] reveals the analogy of $[2R \times \pi]$ with $[L \times A]$ and the remaining R^3 with F^3.

In the quest of designing the water supplies of Dijon, Darcy (1856) introduced hydraulic conductivity K (m s^{-1}) to characterize flow in water-saturated granular media considered suitable for filter materials. Darcy's (1856) law

$$q = K \cdot \left(\frac{-\Delta H}{\Delta l} \right) \qquad [10]$$

states that the volume flux density in a saturated permeable medium is proportional to the gradient of the hydraulic head, [$\Delta H/\Delta l$]. Equation [10] is now applied to exclusively gravity-driven flow in a permeable medium that is water-saturated, where θ equals the porosity ε (m^3m^{-3}). Considering H as the water's potential energy of [$\rho\ g\ z$] a vertical distance z (m) above an arbitrary datum per the water's unit weight [$\rho\ g$] leads to [$\Delta H/\Delta z$ = -1]. Therefore, vertical and exclusively gravity-driven Darcy-flow becomes

$$q = K_{sat} = \frac{g}{3 \cdot \eta} \cdot \left(L \cdot F^3 \right)_{sat} \qquad [11]$$

where the index *sat* indicates the presumed maximum values of $(L \text{ x } F)_{sat}$ at ε. Thus, hydraulic conductivity K_{sat} expresses momentum dissipation related to $(L \text{ x } F)_{sat}$. To generally and simultaneously satisfy the complex dimensionalities of L and F in the basic VF-equations, Eqs. [2] to [6] - $w \propto (L^1, F^1)$, $v \propto (L^0, F^2)$, and $q \propto (L^1, F^3)$ - linearity seems the only plausible type of relationship in Eq. [10]. Consequently, $(L \text{ x } F)_{sat}$ should remain constant and independent from [$\Delta H/\Delta z \neq$ -1]. However, this is not a proof of linearity in Darcy's (1856) law but a reasonable argument in its favor.

Thus, under the restriction of Eq. [8], the law of Hagen-Poiseuille, Eq. [9], Darcy's law, Eq. [10], and VF in unsaturated permeable media, Eqs. [2 to 7], are members of the Newtonian family of laminar shear-flow, Eq. [1], in those viscosity opposes the respective driving forces in the mobile water. Lamb's (1924) comment on the theoretical derivation of Eq. [9] "*The formula … contains exactly the laws found experimentally by Poiseuille (1846) in his researches on the flow of water through capillary tubes* .." provides strong support for linking theory and experiment in shear flow. Lamb's (1924) support extends easily to the other shear-flow family members. The proportionality of $q \propto (L^1,\ F^{\,3})$ in all three cases relates volume flux density with mobile (i.e., moving) water (according to Fig.2) in the respective system while viscosity acts against the driving force.

4 Examination of viscosity in Richards' (1931) capillary flow

This section extensively quotes verbatim Richards (1931) with the intention of following his ideas as closely as possible. Expression numbers in {} refer to the equation numbers in his seminal paper.

Richards (1931) introduces with Eqs.{1,2} the forces affecting capillary action. Accordingly, flow may occur when a porous medium has been minimally wetted and the adhesive forces need no further consideration, thus *"The liquid lying outside the adsorbed films is free to move under the action of unbalanced forces."* Starting with Lamb's (1924) momentum balance, Eq.{3}, he presents the force balance as

$$\mathrm{d}v/\mathrm{d}t = F - \nabla p/\rho + (\mu/\rho)\cdot(\nabla\nabla \bullet v/3 + \nabla \bullet \nabla v) \qquad \{4\}$$

where "*F represents the external or body forces and for most capillary problems gravity is the only external force which need considered. The term* $-\nabla p/\rho$ *is the expression for the force due to the pressure gradient and the third term, being a function of the viscosity μ, and the velocity v, is the expression for the viscous retarding forces."* Richards' retarding forces correspond with Newton's (1729) "… *want of lubricity* …" and, hence, with momentum dissipation. Further, $\phi = g \cdot z$, where z is the height from a reference level, then $F = -\nabla\phi = -g$ and "*…the force represented by the term* $-\nabla p/\rho$ *may be expressed as the gradient of the potential* $\psi = \int dp/\rho$, *the integral being taken from an arbitrary chosen reference pressure* [i.e., atmospheric pressure later on, PG] *to the pressure at the point in question, Eq.{4} may be written*

$$\mathrm{d}v / \mathrm{d}t = -\nabla \Phi + (\mu / \rho) \cdot (\nabla \nabla \bullet v / 3 + \nabla \bullet \nabla v) \qquad \{6\}$$

where $\Phi = (\phi + \psi)$ *and* $-\nabla \Phi$ *is the total water-moving field or the total field tending to produce a motion of water."* Equation {6} still contains the term for the viscous retarding forces. Further, *"If the liquid in a porous medium is in contact with free water and is at equilibrium under gravity, then ... with height z above the flat water surface, we may write*

$$p_w = T_1 (1 / R_1 + 1 / R_2) = -g \cdot \rho \cdot z \qquad \{7\}$$

where p_w is the water pressure at the AWI with respect to the air pressure, T_1 is surface tension in the AWI, and $(1/R_1 + 1/R_2)$ is the total curvature of the surface. Richards relates the capillary potential with the properties of a porous medium with *"In order for the capillary water to attain the correct pressure for equilibrium ... the moisture content of the medium changes until the curvature* [of the menisci at the AWI, P.G.] *has the right value"*. His Figure 1 presents three *"Curves showing the relation between capillary potential and moisture content .."* that are known, among other specifications, as water release or water retention curves. Moreover*, "When the conditions for equilibrium under gravity, as expressed by Eq. {7}, are fulfilled, the velocity and acceleration of the capillary liquid are everywhere zero and Eq. {5}* [and Eq.{6} as well P.G.] *becomes,*

$$\nabla(\phi + \psi) = 0 \qquad \{8\}$$

which means that the force arising from the pressure gradient just balances gravity." Richards correctly also drops the terms for the retarding forces because momentum can only dissipate from moving water. However, he is not going to re-introduce viscous forces anymore in a hydro-mechanical fashion when he defines capillary flow with the paramount Eq. {8} as *"If this condition does not obtain there will be a resultant water-moving force and in general there will be capillary flow."* He continues with "*Because of the complex configuration of the capillary liquid it would be difficult to derive expressions for capillary flow from the general hydro-dynamical equations but it is possible to deduce generalizations* *from experimental data which enable us to set up mathematical relations between the flow and the factors causing flow. Fourier's law and Ohm's law are just such generalizations. An experimental law, quite analogous to these two, and attributed to Darcey, may be used in connection with the present problem."* Further, "*In view of the experimental data now available it is assumed that Darcey's law holds for the low velocities and pressure gradients dealt with in this paper. Mathematically Darcey's law may be expressed by the equation*

$$q = -K \cdot \nabla \Phi \qquad \{9\}$$

where q is the volume of water crossing unit area perpendicular to the flow, in unit time and K is a proportionality factor which for a medium whose pore spaces are filled with water will depend on the number and kind of pore spaces and the viscosity". Although in a lumped way, the proportionality factor K has to include viscous momentum dissipation. Moreover, Richards' *"From analogy with the thermal and the electrical cases it will here be called capillary conductivity",* more stresses *K*'s mathematical formalism rather than its hydro-mechanical importance, for instance, in the sense of Eq. [11].

On page 323 Richards states *"If there is a steady flow of liquid through a porous medium which is only partially saturated, then the larger pore spaces contain air and the effective cross-sectional area of the water conducting region is reduced."* Further, "*If these air spaces could in some way be filled with solid, the condition of the flow would be unchanged and the proportionality between the flow and the water-moving force would still hold because Darcey's law is independent of the size of particles or the state of packing. Hence the essential difference between flow through a porous medium which is saturated and flow through a medium which is unsaturated is that under this latter condition the pressure is determined by capillary forces and the conductivity depends on the moisture content of the medium."* Thus, $K(\theta)$ and subsequently $K(\psi)$. These functions assume that *".. the effective cross-sectional area of the water conducting region .."* is strongly related to $\theta(\psi)$ that expresses equilibrium. Continuously fulfilling the equilibrium condition implies that during infiltration any water flowing in "*.. the larger pore spaces containing air ..* " has instantaneously and mandatorily to be adsorbed by ψ.

No doubt, there is a capillary K for any steady flow in a non-saturated porous medium that approaches static θ-ψ-equilibrium while CF remains steady. Richards' Fig. 2 illustrates the experimental procedure to determine K with respect to Eq. {9}. Accordingly, "*Since the conductivity was to be measured at a constant film thickness* [static with respect to the AWI, P.G.] *or constant curvature it was necessary to maintain a constant difference between the capillary water pressure and atmospheric pressure*." Thus, smooth series of K-θ and K-ψ pairs emerge experimentally from a step-wise sequence of steady flows with monotonously either increasing or decreasing flow rates from step to step. Continuous functions of $K(\theta)$ and $K(\psi)$ can be constructed from these pairs, however, with but a mathematical meaning. From a strict hydro-mechanical point of view there is no gain when linking with static θ any property that describes momentum dissipation. However, relaxing on strict hydro-mechanical conditions, there might be a walk on the crest along the border between the hydro-static and the hydro-mechanic properties of flow in porous media. The crest is the locus of infinitesimal increases or decreases of q such that the ultimately static $K(\theta)$- or $K(\psi)$-functions, together with the equilibration function $\theta(\psi)$, are continuously quasi fulfilled. Thus, antecedent air-filled pores may sequentially and slowly fill with water during infiltration, starting with the tiniest air-filled void. Conversely, the widest voids start empting during drainage before the narrower ones can follow. Thus, the continuous adjustment among θ-ψ-K during flow of small transient steps requires a representative elementary volume, REV, as the spatial unit related to the miniscule variations.

During substantial infiltrations, however, away from gentle increases of volume flux densities, deviations from the walk on the crest release shocks of water content waves that move as avalanches. They brutally set back any subtle approach to a θ-ψ-K equilibration by temporarily connecting flowing water with atmospheric pressure, as Fig. 2 demonstrates.

Water sorption into finer pores is particularly active due to strong capillary gradients as Germann et al. (1984) have demonstrated with the uptake of bromide from macropores. Water content waves, Eqs. [1 to 11], during predominately VF are the culprit of the θ-ψ-K non-equilibria.

In conclusion, Richards (1931) prioritizes capillary flow and summarizes viscosity with capillary conductivity in a formal sense by referring to Fourier, Ohm, and Darcy. Thus, hydro-mechanical viscosity has sneaked out from Richards' derivations between the momentum balance equations, Eq. {3, 4} and the capillary flow equation, Eq. {9}. Here it has sneaked in again simply as capillary conductivity that is a lumped analogue to Darcy's hydraulic conductivity but without referring to further details as they are presented, for instance, in Eq. [11]. Furthermore, Richards vaguely assumes a relationship of capillary conductivity with the entire antecedent water content, hence $K(\theta)$. The contrast between Lamb's (1924) fundamental presentation of viscous momentum dissipation in Eq. {4} and Richards' (1931) formalistic view on it, as expressed with $K(\theta)$, unveils viscosity as the weak link between Richards' (1931) capillary flow and Darcy's (1856) law.

5 Delineation of capillary flow from viscous flow

The previous sections demonstrated that non-equilibrium flow is due to incomplete consideration of the momentum balance in Richards' (1931) CF. Short-circuiting of capillary potentials is a mandatory prerequisite for VF that is grossly limited to a period lasting 3/2 of the duration of input. During this period, the emerging wetting front moves with constant v_W, Eq. [2], that decelerates considerably thereafter because the water content wave flattens due to the cessation of input. The decreasing part of VF after infiltration is not demonstrated here, whereas Germann and Prasuhn (2017) present examples of decelerating v_W. The wetting front velocity depends on the structure of the permeable medium and the volume flux density of input. Thus, rate and duration of input together with the wetting front velocity constrain casewise the spatio-temporal extent of viscous flow and, hence, of non-equilibrium flow with respect to Richards'(1931) capillary flow.

Minimum widths of viscous flow paths are assessed from a theoretical and an experimental vantage point. Today's diffusivity is defined in the wake of Eq. {15} (the Richards equation of transient capillary flow, also referred to as Richards' continuity equation, that is not shown here) as

$$D(\psi) = \frac{K(\psi)}{A(\psi) \cdot \rho_S} \qquad [12]$$

($cm^2\ s^{-1}$), where in Richards' notation "*$A(\psi)$ is the rate of change of the moisture content with respect to the capillary potential and will be called the capillary capacity of the medium*" and ρ_s ($g\ cm^{-3}$) is bulk density. Based on the notion that D represents dissipation of any flow-driving potential, Germann et al. (1997) postulated the dominance of viscous flow whenever $D > \eta$ ($\approx 10^{-6}\ m^2\ s^{-1}$). For various soil textures they derived $D(\psi)$-functions with the model of Campbell (1974) using the coefficients provided by Clapp and Hornberger (1978). Depending on soil texture, the capillary potential at $D = \eta$ varied within $-20 \geq \psi \geq -70$ (kPa), resulting in a range of equivalent flow path widths from about 4 to 15 (µm) that are considered as minima of VF-conduits.

From 215 in-situ and column infiltration experiments Hincapié and Germann (2009) derived F from the application of Eq. [2] to TDR-data retrieved from horizontally installed wave guides. The frequency distribution was in the approximate range of $5 \leq F \leq 120$ (µm). The lower limit of the range is considered the minimum width of paths allowing for VF. Thus, D from Eq. [12] and η from Eq. [1] suggest a minimal flow path width between 5 and 20 (µm) as one prerequisite for VF.

Gravity exerts the exclusive force for driving VF. Therefore, the fraction of the gravitational potential in the total hydraulic gradient, Eqs. [10, 11], provides for another measure of the susceptibility of a permeable medium to non-equilibrium flow. The relative fraction of gravity in Eq.{9} amounts to $\nabla\Phi\cos(\alpha)$, where α (°) is the angle of the flow's deviation from the vertical-down direction, while cos(α) = 1, 0, -1, 0 for α = 0, 90, 180, 270 (°). Thus, Richards-flow completely dominates capillary rise in the direction opposite to gravity, while the determination of K-ψ-θ relationships in thin horizontal columns of porous media almost completely circumvents gravity (e.g., Hillel, 1998). Consequently, VF is by no means suited to deal with water redistribution after infiltration. In particular, water uptake by roots due to transpiration and capillary rise from ground water certainly remain in Richards' (1931) realm.

6 Summary and Conclusion

During infiltration VF occurs prior to and independently from CF as indicated in Fig. 2. Moreover, VF, for instance according to Germann and Karlen (2016), is one member of the Newtonian family of laminar shear flow among others, like Poiseuille (1846) flow, Eq.[9], and Darcy (1856) flow, Eq.[10]. Richards (1931) derives experimentally individual data pairs of the assumed smooth capillary conductivity function $K(\theta)$ according to his Fig. 2 and the associated descriptions, where each data pair is derived from steady state capillary flow. The aimed at smooth function follows from assumed sequential flow. His statement of setting the hydraulic property of an impermeable stone in a porous medium equal to an air-filled void of the same dimension illustrates well the restriction he implied on CF. In view of Figure 2, this restriction must ultimately lead to so called non-equilibrium flow during infiltration.

There are attempts to combine approaches of VF with those of CF. Germann and Beven (1985) added a sink term to an infiltrating kinematic wave, while Di Pietro et al. (2003) explored travelling-dispersive waves. However, the spatio-temporal scales of the two processes not only differ in their velocities but, more fundamentally, in their basic hydro-mechanical and mathematical structures. For example, in contrast to CF does VF not require a representative elementary volume, REV. Thus, it might be very difficult to develop a unified analytical approach that combines viscous and capillary flows, as Germann (2014) resumed. It occurs that the first step in a attempt to numerically model the entirety of infiltration has to deal with VF whose spatio-temporal extents are limited and strongly dependent on the intensity and duration of input (Germann and Prasuhn, 2017). The subsequent steps then would include numerical procedures concerning CF, using as initial conditions the water content distribution from fading VF.

In conclusion, the statement of Alberti and Cey (2011) " .. *that a Darcy-based model may not adequately represent flow in macroporous systems*" has to be rejected in view of these deliberations. First, Darcy-based models may indeed represent preferential flow if they are rooted in Newton's (1729) shear flow, Eq. [1]. Second, the functionally well-defined parameters F and L, Eqs. [2 to 4] of VF will eventually displace the ill-defined delineation between macropores and micropores. Moreover, the VF-approach to PF has matured to independence such that it does not require the subjugating status of a non-Richards-type flow.

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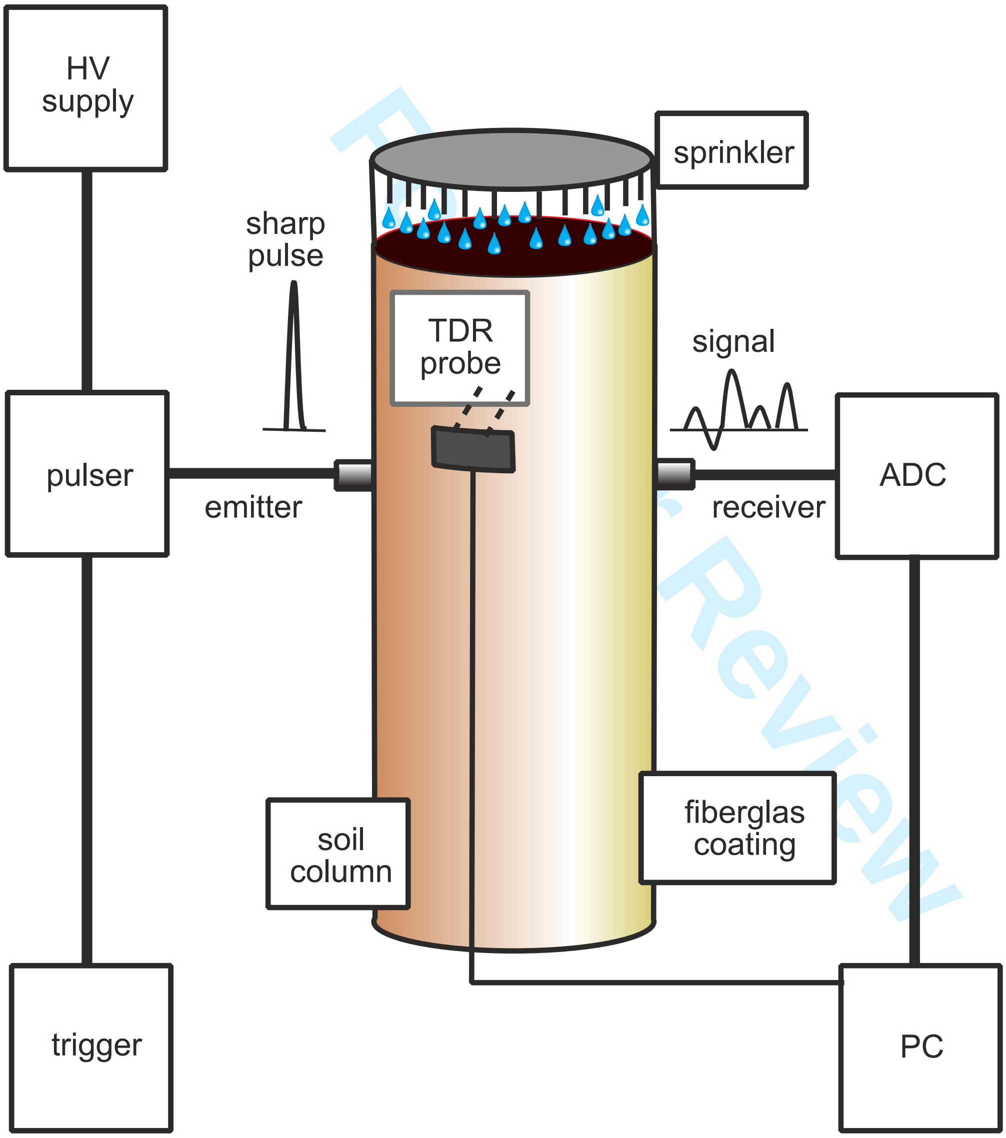

Figure 1: Experimental set-up to record acoustic velocities and volumetric water contents in a column of an undisturbed Typic Hapludalf. Diameter and height of the column are 0.3 and 0.8 (m). An acoustic emitter and an acoustic receiver are located on opposite ends of a diameter. The PC controls timing of the experiment, acquisition and analysis of data; the sprinkler applies water to the soil surface with preset intensity and time interval. HV: High voltage supply; ADC: Analog-digital-converter. (Flammer et al., 2001; from Germann, 2014, with permission).

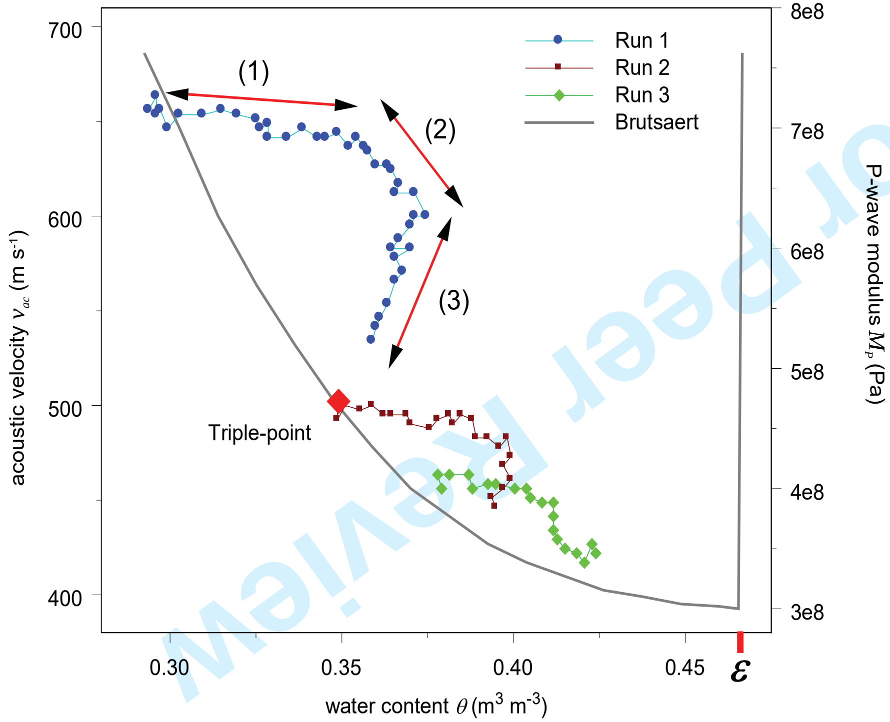

Figure 2: Pressure-wave modulus, M_p, and acoustic velocity, v_{ac}, vs. volumetric water content θ during three consecutive infiltration runs. Model according to Brutsaert (1964). The red arrows indicate Stages (1), (2), and (3) durint Run 1; the Tipple-point hints at the presumed gathering of Brutsaert's (1964) model with the data at the end of Run 1 and at the beginning of Run 2. At saturation, θ = ε, where ε is porosity, M_p increases sharply because the incompressible water supports the pores. (Flammer et al., 2001; from Germann, 2014, with permission).

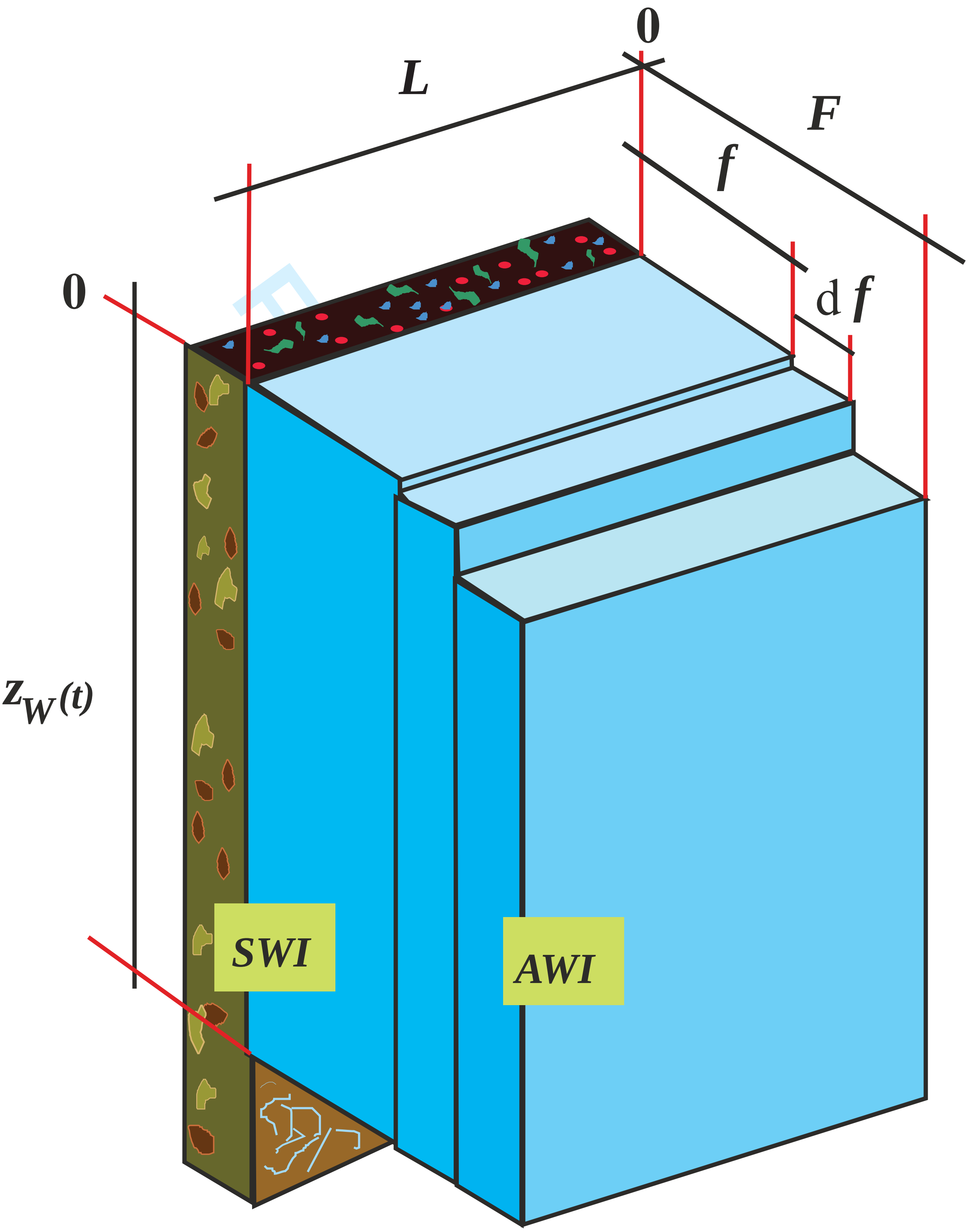

Figure 3: Schematic representation of film flow. *F, f*, and d*f* represent the film thickness, the film thickness variable ($0 \leq f \leq F$), and the thickness of a lamina; $z_w(t)$ is the time-dependent depth of the wetting front, *L* is the contact length per unit cross-section of the horizontal area *A*; $L \times z_w(t)$ is also the vertical surface area per unit volume $A \times z_w(t)$ of the permeable medium onto which momentum dissipates; *SWI* and *AWI* are the solid-water and the air-water interfaces of the film. (From Germann, 2014, with permission).