Combination of monthly gravity fields on Normal Equation Level

6 Combination on Solution Level

After quality control (tests on signal strength and outlier screening) the monthly gravity fields are first combined on solution level applying field-wise weights derived by Variance Component Estimation (VCE). Therefore simple observation equations are set up and the number of observations \( n_i \) equal to the number of unknowns \( m_i \). Consequently
\[
N_i = A_i^T P_i A_i = I
\]
with the simplest possible stochastic model \( P_i = I \) Stacking several sets of observations we can compute partial redundancies
\[
\nu_i = n_i - \text{Tr}(N_i N_i^{-1}) = n_i \left(1 - \sum_{i=1}^{N} \nu_i \right)
\]
with \( N = \sum_{i=1}^{N} \nu_i \).

The square sum of residuals is
\[
\nu_i^T P_i \nu_i = \sum_{i=1}^{N} (x_i - \bar{x}_i)^2
\]
and the iterated weights consequently become
\[
\bar{w}_i = \frac{\nu_i^T P_i \nu_i}{\sum_{i=1}^{N} (x_i - \bar{x}_i)^2}
\]
applying the standard formulas of VCE.

7 Combination of Normal Equations

In the final combination of NEQs, the relative weights derived by noise analysis on solution level are applied. Prior to this combination the impact of each NEQ \( N_j \) on the combination is equalized by empirical weights \( w_j \) based on the analysis of pair-wise combinations:
\[
N_{\text{comb}} = \left( \sum_{j=1}^{N} w_j N_j \right)^{-1}
\]
The impact of an individual contribution on the combination is computed as the RMS of all differences between the harmonic coefficients \( K_{m} \) of the combined and the individual gravity fields:
\[
\text{RMS}_{\text{comb}} = \sqrt{\sum_{j=1}^{N} (K_{0,m} - K_{0,m}^j)^2}
\]

8 Combination Results

The years 2006 and 2007 were chosen to develop and test the combination strategy and to validate the results. In case of heterogeneous quality the combined fields reach at least the quality of the best individual contribution. In case of more homogeneous quality both, the combinations on solution level and on NEQ level, are clearly superior to the individual contributions in terms of noise. The combination on NEQ level is slightly more robust against artifacts in an individual contribution.

References: Jean Y, Meyer U, Jäggi A (2017): Combination of GRACE monthly gravity field solutions from different processing strategies. Submitted to JGR.

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