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GOCO05c: A New Combined Gravity Field Model Based on Full Normal Equations and Regionally Varying Weighting

T. Fecher[1] · R. Pail[1] · T. Gruber[1] · the GOCO Consortium

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Abstract GOCO05c is a gravity field model computed as a combined solution of a satellite-only model and a global data set of gravity anomalies. It is resolved up to degree and order 720. It is the first model applying regionally varying weighting. Since this causes strong correlations among all gravity field parameters, the resulting full normal equation system with a size of 2 TB had to be solved rigorously by applying high-performance computing. GOCO05c is the first combined gravity field model independent of EGM2008 that contains GOCE data of the whole mission period. The performance of GOCO05c is externally validated by GNSS–levelling comparisons, orbit tests, and computation of the mean dynamic topography, achieving at least the quality of existing high-resolution models. Results show that the additional GOCE information is highly beneficial in insufficiently observed areas, and that due to the weighting scheme of individual data the spectral and spatial consistency of the model is significantly improved. Due to usage of fill-in data in specific regions, the model cannot be used for physical interpretations in these regions.

Keywords Gravity · Combined gravity field model · Full normal equation systems · High-performance computing · Stochastic model

1 Introduction

The Earth's gravity field is of fundamental importance for a variety of scientific applications, such as geodesy, oceanography, hydrology, cryospheric sciences, and geophysics. As GNSS–levelling in future will become the primary technique for the determination of

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The names of members of the GOCO Consortium are given in Appendix

T. Fecher
fecher@tum.de

[1] Institute of Astronomical and Physical Geodesy, Technical University of Munich, Arcisstraße 21, 80333 Munich, Germany

physical (orthometric or normal) heights, a global consistent high-resolution gravity field model, providing via Bruns formula the geoid as vertical datum (reference surface), will become more and more important (Pavlis et al. 2012). The vertical datum also plays a crucial role for a global unification of national height systems, which is essential for large engineering projects and sea level analysis. This is shown, for example, by Gruber et al. (2012), Rummel (2012), and Woodworth et al. (2012). Also in oceanographic applications such as the modelling of mean dynamic ocean topography (MDT) and the calculation of geostrophic ocean currents, the geoid serves as physical reference surface, as has been demonstrated, for instance, in Knudsen et al. (2011), Bingham et al. (2011), and Siegismund (2013). In geophysics, the gravity field serves as a boundary value and constraint for lithospheric modelling and determining Moho depth. This is shown, for example, in van der Meijde et al. (2013) and McKenzie et al. (2014).

Numerous global gravity field models have been published over the past 50 years. For an overview, refer to http://icgem.gfz-potsdam.de. It is expected that due to the availability of new observation sources and types, as well as developments in computational techniques, further advancements in gravity field modelling will continue well on into the future. The currently most important observation types for up-to-date combined gravity field models are data from satellite gravity missions, altimetry data, as well as data from terrestrial and airborne campaigns. Regarding the satellite gravity field missions, particular attention is given to Gravity Recovery And Climate Experiment (GRACE; Tapley et al. 2004) and Gravity field and steady-state Ocean Circulation Explorer (GOCE; Drinkwater et al. 2003). These two missions complement each other spectrally such that the long-wavelength part of the gravity field can be mapped with high accuracy up to about spherical harmonic degree and order (d/o) 220, corresponding to a spatial half-wavelength of 90 km. Higher-resolution gravity field information is contained in altimetry data over the oceans and terrestrial/airborne observations.

The currently most frequently used gravity field model is EGM2008 (Pavlis et al. 2012, 2013). EGM2008 is based on possibly the best global—but not freely accessible—$5' \times 5'$ data set of terrestrial observations and is parameterized up to degree 2190 and order 2159. Since the model was released before the launch of the GOCE mission, it does not contain any GOCE observations. A newer high-resolution model that contains GOCE data is EIGEN-6C4 (Förste et al. 2014). However, it contains prior information in land areas taken from the EGM2008.

In this study, the combined gravity field model GOCO05c is computed. It utilizes the best possible freely available gravity field information from GRACE, GOCE, altimetry, and terrestrial measurements. It is parameterized by the coefficients of a spherical harmonic series expansion up to d/o 720. GOCO05c is the first combined gravity field model that is combined on the basis of full normal equation systems up to full resolution, and therefore uses regionally varying weighting based on the varying quality of the terrestrial/altimetry data. This requires the use of high-performance computing systems, but it does allow for stochastic modelling without preconditions, which is an advantage that will be explained in the context of this paper.

The benefits of this model are that in contrast to EGM2008 it contains GOCE data, and unlike EIGEN-6C4 it is independent of EGM2008 (independent in terms of not using band-limited gravity anomalies computed from EGM2008 over land areas). This fact is not only highly valuable for external validation. Such an independent realization of a high-resolution gravity model, particularly in all land areas, might provide an interesting alternative to users. In contrast, when synthesizing gravity information from EGM2008 and incorporating it in a new combined model together with satellite data, GRACE information that is already contained in EGM2008 would be used twice, and the flexibility and benefits of regionally dependent weighting of satellite and terrestrial data would be lost.

The paper is structured as follows: Sect. 2 gives an overview on the data sets used for generating GOCO05c. In Sect. 3, the functional and stochastic models and the combination method are described. The resulting model GOCO05c is described and validated using external data in Sect. 4. Finally, in Sect. 5, the main conclusion is drawn and an outlook to future work is given.

2 Data Sets

2.1 GOCO05s

The gravity field model GOCO05s (Mayer-Gürr and the GOCO Team 2015) resolved to d/o 280 is the first component of the combination model GOCO05c. This pure satellite-only model was combined based on full normal equations. The methodology of the combination is described in Pail et al. (2010). The model's two main components are gravity information of GRACE and GOCE. The GRACE part comes from the GRACE-only model ITSG-GRACE2014s (Mayer-Gürr et al. 2014), which is resolved to d/o 220 and was calculated based on the integral equation approach (Schneider 1967; Mayer-Gürr 2006), with the observation period from February 2003 to December 2013. The GOCE component corresponds to GOCE-TIM5 (Brockmann et al. 2014) and uses observations from September 2009 to October 2013. In addition, GOCO05s contains information from kinematic orbits of other low Earth orbiting satellites (LEOs) and Satellite Laser Ranging (SLR) to support the low harmonic degrees. A non-regularized version of GOCO05s is applied for the combined solution GOCO05c.

2.2 Altimetric Gravity Anomalies

The fact that two-thirds of Earth's surface is covered with water highlights the importance of altimetric gravity anomalies. Moreover, an essential feature of altimetric gravity anomalies is their consistency and their generally high accuracy (with the exception of coastal regions), because they result from a consistent multi-mission processing where the specific accuracies of the altimeter data have been taken into account. In contrast, terrestrial gravity anomaly data sets are resulting from various surveys with different and usually unknown precision. The altimetric gravity anomalies of DTU2013 are used (Andersen et al. 2010, 2013). They are provided as a global grid, but the values over land are derived from EGM2008. The original DTU2013 gravity anomaly grid consists of $1' \times 1'$ cells, which are averaged for our purposes to $15' \times 15'$ area means. Considering that GOCO05c should be independent of EGM2008, only gravity anomalies derived from altimetry over the oceans are applied. However, it should be considered that in theory a small dependency on EGM2008 remains, as EGM2008 was used as a reference field in the remove/restore process for the calculation of DTU2013. As will be shown later, this dependency is limited to degree 140 and higher because the resulting GOCO05c model for the lower degrees up to 140 is completely determined from satellite information.

2.3 Terrestrial Gravity Anomalies

Terrestrial gravity anomalies constitute only one-third of the total data required for global coverage. However, there are regions on Earth where accurate terrestrial gravitational data do not exist, or are subject to restrictions and thus are not freely available. Various terrestrial data sets are used for computing GOCO05c. The National Geospatial-Intelligence Agency (NGA) has made available data for gravity anomalies covering South America (spatial resolution $15' \times 15'$ area means) and the contiguous USA (CONUS, $15' \times 15'$). The gravity anomalies from ArcGP (Forsberg and Kenyon 2004, $05' \times 05'$) are used for the Arctic land regions. ArcGP anomalies are not only available over land but also over the oceans, but, as described above, the newer DTU data were prioritized here. Gravity anomaly data sets from Curtin University (for Australia, $01' \times 01'$) and the Institute of Geodesy (IfE) at Leibniz University Hannover (for Europe, $15' \times 15'$) have been made available and are the basis for the regional geoid solutions of AusGeoid09 (Featherstone et al. 2011) and EGG08 (Denker et al. 2008), respectively. Furthermore, a data set for the gravity anomalies in Canada was provided by National Resources Canada (NRCan, $02' \times 02'$). As the spatial resolution of these data sets is at least $15' \times 15'$, the maximum degree of resolution of GOCO05c was chosen to be d/o 720, as all coefficients up to d/o 720, but the coefficients $\bar{C}_{720,0}$ and $\bar{C}_{720,720}$ can be determined from a $15' \times 15'$ observation grid (Colombo 1981; Fecher 2015). All data sets with a spatial resolution higher than $15' \times 15'$ were averaged to $15' \times 15'$ area means. The two remaining coefficients are introduced as prior information by applying regularization techniques (Koch and Kusche 2002). As prior information, the corresponding coefficients of EGM2008 were used. The coverage of terrestrial gravity anomalies constitutes 12.4% of the globe, which together with the altimetric anomalies results in almost 80% global coverage of data, which are appropriate for applications requiring high accuracy such as gravity field modelling (cf. Table 1; Fig. 1).

2.4 Fill-In Data sets

In order to ensure the stability of the normal equation system, the observation data grid has to be a complete $15' \times 15'$ grid. To fulfil this condition and to cover also the remaining 20% of the Earth's surface, fill-in data sets such as the NIMA96 data set (Kenyon and Pavlis 1996) of the Defense Mapping Agency (DMA, today: NGA) and the Goddard Space Flight Center (GSFC) have been used. It formed the basis for EGM96 (Lemoine et al. 1998), which was the best gravity field model of its time. However, by today's standards, this model has several shortcomings. Besides data points with poor accuracy, the data set

Table 1 Gravity anomaly data sets composing a global $15' \times 15'$ data grid included in GOCO05c

Region	Source	Number of data cells (percentage of global coverage)
Arctic	ArcGP Group	44,522 (4.3%)
Australia	Curtin University	11,170 (1.1%)
Canada	NRCan	19,259 (1.9%)
Europe	IfE	15,625 (1.5%)
Oceans	DTU	691,818 (66.7%)
South America	NGA	24,818 (2.4%)
USA	NGA	12,895 (1.2%)

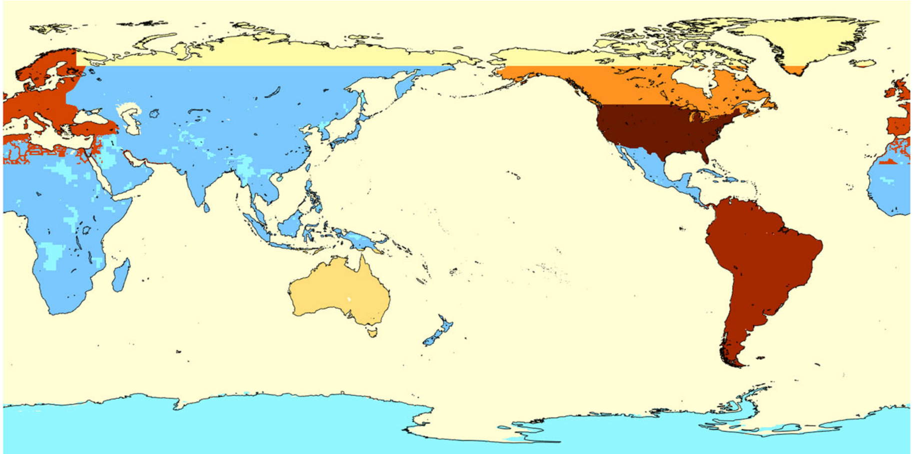

Fig. 1 Regional data sets constituting a global gravity anomaly grid according to Table 1 (*warm colours*; 79.1%) and Table 2 (*cold colours*; 20.9%, thereof Antarctica and various gaps GOCO05s, RWI_TOIS used in all fill-in regions)

Table 2 Gravity anomaly fill-in data sets

Data set	Source	Number of data cells (percentage of global coverage)
NIMA96	DMA/GSFC	110,594 (10.7%)
GOCO05s	GOCO Group	106,099 (10.2%)
RWI_TOIS2012	KIT	117,737 (11.4%)

has a spatial resolution of only $30' \times 30'$, which corresponds to a spectral resolution of d/o 360. Moreover, the NIMA96 data set does not cover all unobserved regions as shown in Fig. 1, but rather only about 50% of them (10% global). The remaining regions were filled with band-limited gravity anomalies computed from GOCO05s up to d/o 220.

Since the $30' \times 30'$ NIMA96 anomalies and the anomalies computed from GOCO05s are limited to d/o 360 and d/o 220, respectively, gravity anomalies synthesized from a topography model have been used in order to model the signal up to the target resolution of d/o 720. The topographic anomalies of GOCO05c are calculated from the model RWI_TOIS2012 (Grombein et al. 2013) up to d/o 720. Of course, it has to be kept in mind that topographic anomalies do not give the complete picture of the gravity field (Pavlis et al. 2012, 2013). However, Hirt and Kuhn (2014) and Fecher (2015) show that with this approach at least about 60–75% of the gravitational signal can be represented, while the remaining part is mainly due to non-modelled density anomalies.

For this reason, the resulting gravity field model in the affected regions—mainly Africa, Asia, and Central America—does not meet the highest accuracy requirements and, therefore, it should not be used to geophysical applications in these regions, because its high-frequency part resulted from simple synthetic numeric forward modelling of topography information. Nevertheless, this fill-in strategy is very valuable to reduce omission errors of pure satellite-only models, which is important, for example, for height system definition and unification (Rummel 2012). A summary of the fill-in data sets is provided in Table 2 and Fig. 1.

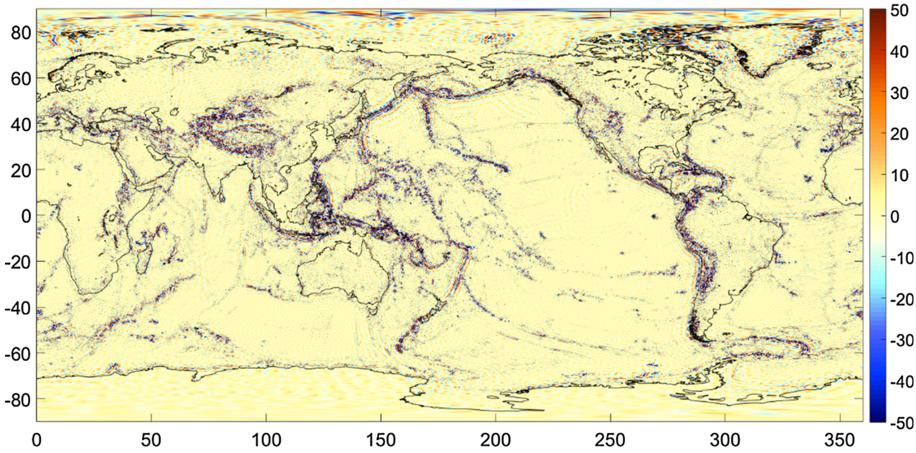

Fig. 2 Information from ground data which actually entered the combination model [mGal]

Finally, Fig. 2 shows the residual part of ground gravity data (all data sets from 2.3 to 2.5) which has entered the combination model GOCO05c. This information is derived from the diagonal matrix of the product of the GOCO05s normal equation and the covariance matrix of the combination model, as this diagonal matrix represents the relative contribution of GOCO05s to the combined model.

2.5 Considerations on Consistency

In principle, data sets must be pre-processed to be consistent among each other before they can be combined. That means, for example, the satellite data must be defined in the same tide system as the terrestrial and altimetric gravity anomalies, or that a series of reductions should be applied in an identical manner to all gravity anomaly data sets. Such reductions are the effects of permanent tides (Ekman 1989), atmosphere (Moritz 1980; Wenzel 1985), directional derivatives (Moritz 1980; Pavlis 1988; Rapp and Pavlis 1990), and normal gravity gradients of higher order (Torge and Müller 2012). In reality, however, it is not possible to apply correctly these reductions and corrections, because for many data sets background information on their pre-processing is not available. In fact, some data sets only contain information related to the station position (latitude, longitude, height) and the gravity value. For this reason, Fecher (2015) estimated the maximum absolute amplitude of each of the corrections. Adding up all of them, the geoid error is larger than 0.35 m (global standard deviation) for a solution purely based on altimetric and terrestrial gravity anomalies (with extreme values around 1.0 m), but with dominant long-wavelength characteristics. Since in this spectral region the combined solution is dominated largely by the satellite part (GRACE, GOCE), possible inconsistencies in the terrestrial/altimeter component are reduced to a standard deviation of only 0.002 m globally, with extreme values of up to 0.02 m in areas with rough terrain. Therefore, effectively the above listed reductions and corrections can be neglected when the associated data are combined with satellite data, as the resulting error occurs mainly in those wavelengths, which are strongly dominated by the satellite information. In particular, the tide system of the satellite solution is preserved in the combined model.

3 Modelling and Combination Methods

The gravitational potential V is parameterized by the spherical harmonic series expansion

$$V(\theta,\lambda,r)=\frac{GM}{R}\sum_{l}^{\infty}\left(\frac{R}{r}\right)^{l+1}\sum_{m}^{l}\left[\bar{Y}_{lm}^{C}(\theta,\lambda)\,\bar{C}_{lm}+\bar{Y}_{lm}^{S}(\theta,\lambda)\,\bar{S}_{lm}\right], \tag{1}$$

where l and m are the spherical harmonic degree and order , $\left[\bar{Y}_{lm}^{C},\bar{Y}_{lm}^{S}\right]$ the fully normalized spherical harmonic base functions, and $[\bar{C}_{lm},\bar{S}_{lm}]$ the corresponding spherical harmonic coefficients. The constant GM is the product of the gravitational constant and Earth mass (in the case of GOCO05c: in SI units $GM = 0.39860044150D + 15$), R the Earth radius (GOCO05c: $0.63781363000D + 07$), and $[\theta,\ \lambda,\ r]$ the spherical coordinate triplet. The aim of gravity field determination is the estimation of the unknown spherical harmonic coefficients $[\bar{C}_{lm},\bar{S}_{lm}]$. Functionals of V serve as observations. The spherical harmonic base functions establish the functional relationship between unknowns and observations. In order to estimate the unknowns, least squares technique based on a Gauss–Markov model is used. A normal equation system can be constructed for each data set. Estimates of the unknowns $\hat{\mathbf{x}}=\left[\hat{\bar{C}}_{00},\ldots,\hat{\bar{C}}_{lm},\ldots,\hat{\bar{S}}_{00},\ldots,\hat{\bar{S}}_{lm},\ldots\right]$ are given by

$$\hat{\mathbf{x}}=\left(\frac{1}{\hat{\sigma}_{01}^{2}}\mathbf{A}_{1}^{T}\mathbf{P}_{1}\mathbf{A}_{1}+\cdots+\frac{1}{\hat{\sigma}_{0\mu}^{2}}\mathbf{P}_{\mu}\right)^{-1}\cdot\left(\frac{1}{\hat{\sigma}_{01}^{2}}\mathbf{A}_{1}^{T}\mathbf{P}_{1}\mathbf{l}_{1}+\cdots+\frac{1}{\hat{\sigma}_{0\mu}^{2}}\mathbf{P}_{\mu}\mathbf{x}_{\mu}\right). \tag{2}$$

Here, $\mathbf{A}_i$ represents the design matrices that contain the spherical harmonics, $\mathbf{P}_i$ are the weight matrices (assumed to be diagonal for ground data sets), $\mathbf{l}_i$ are the observation vectors, and $\hat{\sigma}_{0i}^{2}$ are the variance components with respect to i data sets. The i data sets are assumed to be uncorrelated; thus, $\hat{\sigma}_{0i}\hat{\sigma}_{0j}=0$. The satellite information is introduced as prior information $\mathbf{x}_\mu$ (Koch and Kusche 2002) and weighted by their normal equation $\mathbf{P}_\mu$. A detailed description of the estimation method used here can be found in Fecher et al. (2015).

Due to the fact that for estimation of GOCO05c individual weighting of observations is performed, which can be expressed by the diagonal elements of $\mathbf{P}_i$, the parameters to be estimated are highly correlated. Since this impedes the applicability of block-diagonal techniques (Gruber 2001), full normal equation systems up to the maximum d/o are used. In the following, the derivation of an empirical stochastic model for a regional terrestrial data set is demonstrated by the example of the South American region. We will also analyse the improvements in the quality of the combined solution when such a spatially dependent stochastic model is applied.

Since it can be assumed that a pure satellite gravity field model (such as GOCO05s) achieves high accuracy for the long wavelengths, it can be used in this spectral region for the validation and the quantification of the accuracy of terrestrial data. For this purpose, we develop the terrestrial anomalies into spherical harmonics, so that we can use these spherical harmonics to compute the differences between the terrestrial gravity anomalies and GOCO05s in the specific degree range that is covered by the satellite-only model. Here the implicit assumption is that the data quality of a terrestrial observation is already reflected in its long-wavelength component. These band-limited (absolute) differences are displayed in Fig. 3 (left) at d/o 200 for South America. It is shown that the quality of terrestrial data is very good for the east coast of South America. In contrast, the differences

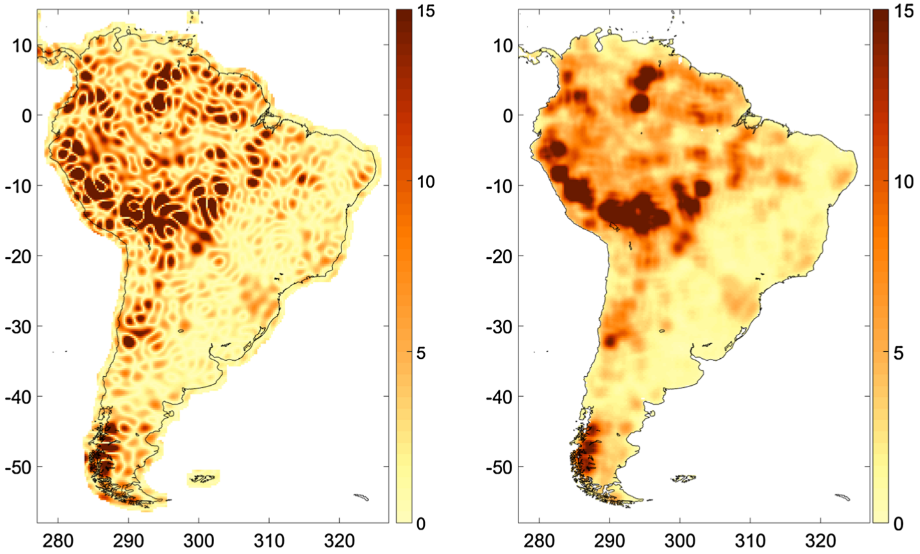

Fig. 3 Absolute differences [mGal] between band-limited terrestrial gravity anomalies and GOCO05s at d/o 200: (*left*) unfiltered; (*right*) filtered

in the Amazon region and the Andes are significantly larger. Since we can safely assume a largely homogeneous accuracy of the satellite solution, this demonstrates that the terrestrial data in these regions are less reliable. In principle, these absolute differences provide a basis for deriving an empirical stochastic model. There are, however, transition areas between (originally) negative and positive regions with null values that are not representative for the real accuracy. In order to overcome this problem, the absolute differences are low-pass filtered applying a 2D box filter. The result (Fig. 3 right) is used to describe the error standard deviations of individual grid cells.

As a next step, two combination models are computed. The first combination model uses the South American data set together with the empirically derived stochastic model, while the second one assumes unit weight for all grid cells. The first case is applied for GOCO05c with full normal equation systems, whereas the second case corresponds to the approach when using block-diagonal techniques. The difference between both combined solutions and GOCO05s (by itself a component of the combined solution), analysed up to d/o 200, is shown in Fig. 4. The result can be considered as good if the differences are small, because in this case the high quality of the satellite information in this spectral range is not deteriorated by the combination with the terrestrial data. It is clearly visible that large differences occur in the block-diagonal approach, which in turn indicates that the spectral transition from satellite to terrestrial data occurs too early in regions with less accurate terrestrial data. In the case of stochastic modelling and the use of full normal equation systems, the difference is much smaller. This clearly demonstrates the advantage of spatially dependent individual weighting, because in this case the relative weighting of satellite and terrestrial data can be varied for any grid cell, depending on the accuracy of the underlying terrestrial gravity data.

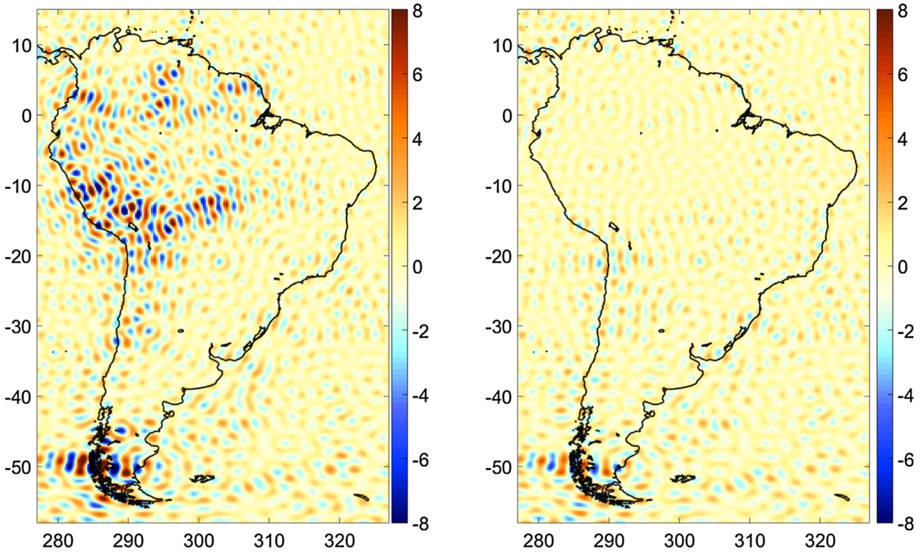

Fig. 4 Gravity anomaly difference [mGal] between the block-diagonal solution (*left*, mean: 0.03 mGal, std: 1.43 mGal, min: −12.70 mGal, max: 14.93 mGal) and the solution based on full normal equation systems (*right*, mean: 0.01 mGal, std: 0.88 mGal, min: −5.76 mGal, max: 6.41 mGal) and GOCO05s up to d/o 200

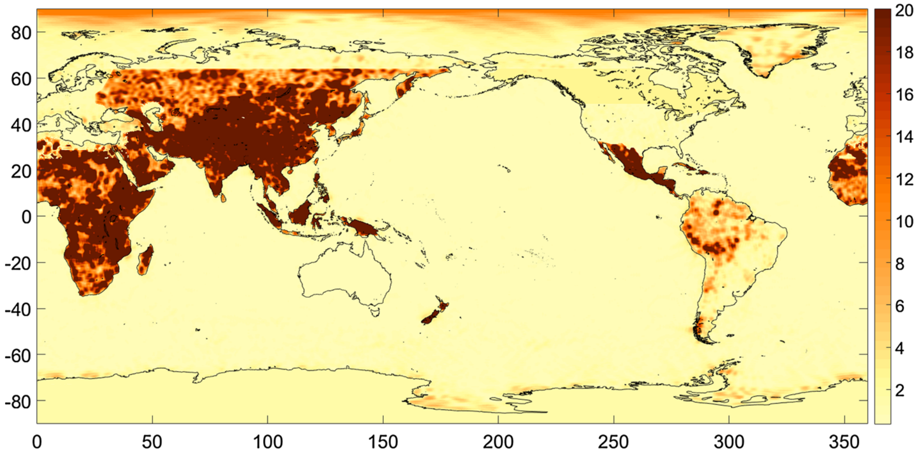

Fig. 5 Stochastic model of ground data [mGal]

The stochastic model for the different gravity anomaly data sets has been derived by analogy with the procedure described above when using the South American data set. It is displayed in Fig. 5.

4 The Combination Model GOCO05c

The GOCO05c combination model is computed by a rigorous solution of the fully occupied normal equation system up to d/o 720 (which corresponds to almost 520,000 parameters). The stochastic models for the terrestrial/altimetric anomalies are derived using the methodology discussed in Sect. 3 and shown in Fig. 3 (right) for the example of South America. In general, as it can be concluded from the description of the data sets in Sect. 2, the standard deviations in the stochastic model of the terrestrial/altimetric data are quite high in the fill-in regions, while they are clearly smaller over the oceans or the high-quality terrestrial gravity areas such as CONUS. The relative weighting between the satellite and terrestrial/altimetric information is carried out in the way that regions of bad

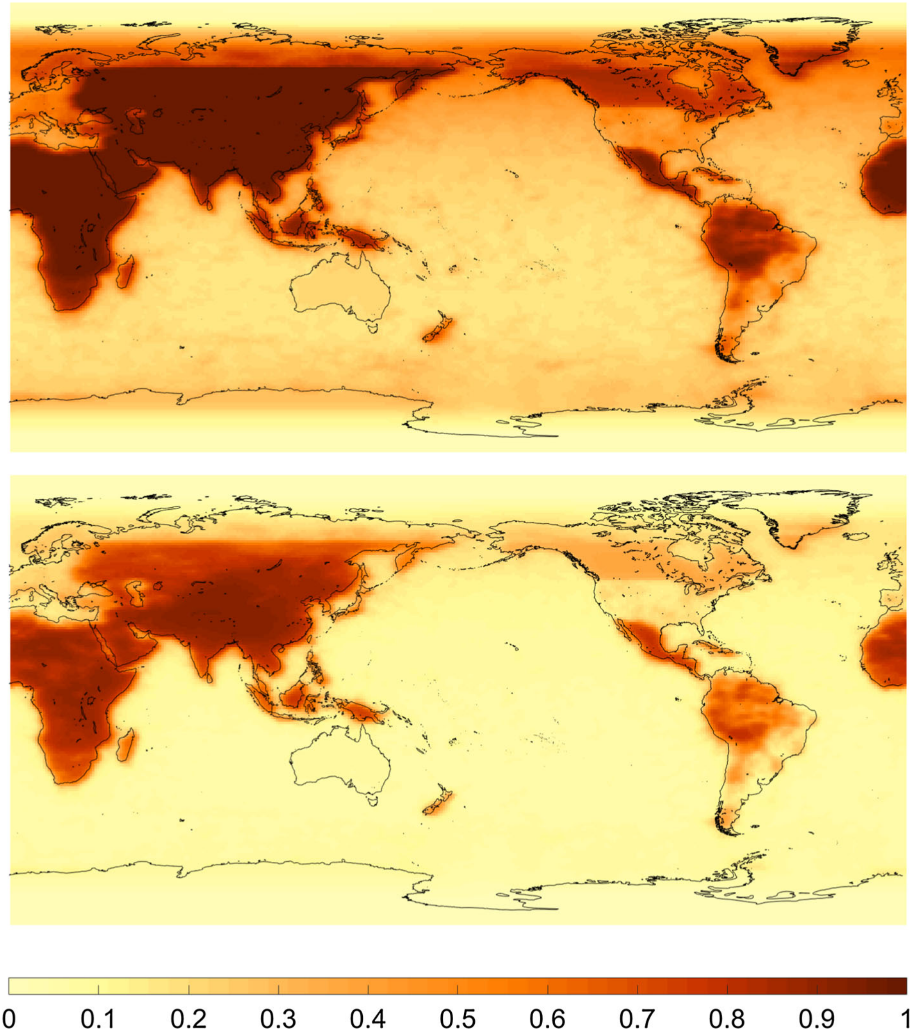

Fig. 6 Relative contribution of GOCO05s to GOCO05c for degree 200 (*top*) and degree 220 (*bottom*) [0 = 0%, 1 = 100%]

terrestrial data quality depend largely on satellite information at d/o 200, as pure GOCE models can be fully trusted at least until d/o 200 (Brockmann et al. 2014). The transition between satellite information and terrestrial/altimetric information takes place between degree 140 and degree 240, depending on the stochastic modelling. This also implies that any a priori MDT information has no influence on the combined model up to degree 140.

The consequence of this relative weighting scheme for the solution is demonstrated in Fig. 6, illustrating the relative contribution of GOCO05s to GOCO05c for two degrees 200 and 220. It was computed by covariance propagations to geoid heights based on full parameter variance–covariance matrices. The ratio between the propagation result of the satellite information and the combination model gives the relative contribution. The impact of satellite data, even for higher degrees, is especially visible in areas where fill-in data have been applied.

Degree variances of GOCO05c, their differences to GOCO05s, and the ground data information part (GOCO05t) as well as the corresponding formal errors are displayed in Fig. 7. The benefit of the stochastic modelling is clearly visible, as the formal errors of GOCO05s fit perfectly to the difference between GOCO05c and GOCO05s in the medium wavelength part. Also the formal errors of GOCO05t fit quite well to the difference of GOCO05c and GOCO05t for the long wavelengths. The small differences can be explained by the inconsistencies and offsets between the ground data sets which have not been modelled.

To evaluate the quality of GOCO05c, it is compared to established gravity field models. As a first test, a simple comparison in terms of geoid heights is performed. Figure 8 shows the geoid height differences of GOCO05c to EGM2008 (a) and EIGEN-6C4 (b). Since EGM2008 does not contain GOCE information and the spectral transition between satellite data and terrestrial/altimetric information in EGM2008 occurs very early (<d/o 100), the

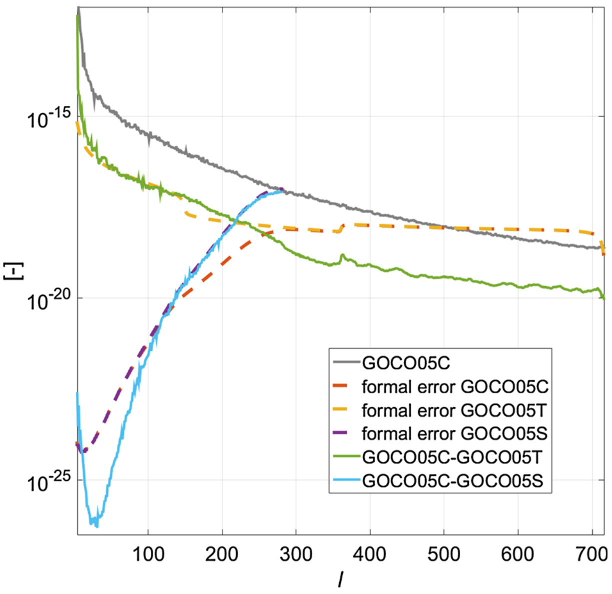

Fig. 7 Degree variances of GOCO05c and formal errors of its components

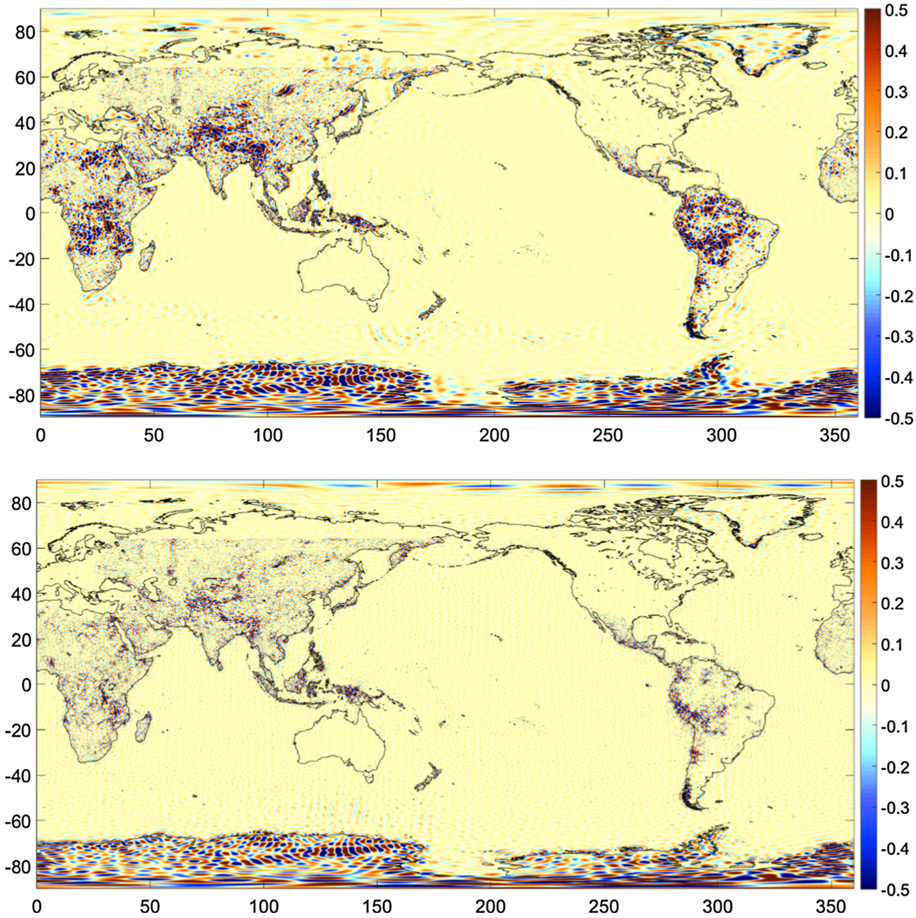

Fig. 8 Geoid height difference (m) of GOCO05c to EGM2008 (*top*, mean: 0.00 m, std: 0.18 m, min: −3.83 m, max: 5.42 m) and EIGEN-6C4 (*bottom*, mean: 0.00 m, std: 0.15 m, min: −3.18 m, max: 2.62 m) at degree 720

comparison with EGM2008 clearly indicates those areas where new gravity field information has been measured by GOCE, such as in the Amazon or large parts of Africa and Asia (Pail et al. 2011). Compared to EIGEN-6C4, much smaller differences are visible, also predominantly in regions with low terrestrial data accuracy. Since EIGEN-6C4 contains similar satellite information as GOCO05c, they can be explained by a different transition between satellite and terrestrial information, because in EIGEN-6C4 spatially dependent weighting was not applied.

In the next step, orbit tests were carried out in order to evaluate the quality of GOCO05c mainly in the long to medium wavelengths. For this purpose, 12-h GOCE reduced-dynamic orbit arcs were computed by numerical integration based on various gravity field models with gravity field information up to d/o 200, and the RMS of the differences to independently determined kinematic 3D positions was analysed (refer to Gruber et al. 2011 for the general procedure). The comparison was carried out for a four-month period from January to April 2013. The result shows that the satellite information contained in GOCO05c (on average 15.84 cm RMS compared to 15.87 cm for GOCO05s) is not deteriorated by the combination with terrestrial data. In fact, the result is even slightly better than the one related to GOCO05s. In this test, GOCO05c also performs slightly better than EIGEN-6C4 (RMS average 15.98 cm) and each model that includes GOCE information outperforms EGM2008 (RMS average 17.03 cm) significantly, thereby demonstrating the benefit of GOCE.

Another external validation step is GNSS–levelling comparisons, which were made according to the procedure described in Gruber et al. (2011). Geoid heights are calculated from gravity field models and are compared with geoid heights resulting from independent GNSS and levelling measurements. Models were truncated and analysed at various degrees. Since geoid heights derived from GNSS–levelling always contain the full spectral signal content, in order to avoid omission errors, the gravity field models are complemented by information from EGM2008 beyond the cut-off degree and by topographic information beyond the resolution of EGM2008. Figure 9 shows the result for three different test regions USA, Brazil, and Australia.

In the case of EGM2008, which serves as a reference, the curve of the RMS values is always a straight line, because the omission error is calculated from EGM2008, and thus, for EGM2008, always the full model is used [cf. Gruber et al. (2011)]. A curve below EGM2008 means that there is an improvement with respect to EGM2008. Evidently, both EIGEN-6C4 and GOCO05c show such improvements, which are the result of the additional GOCE information, again demonstrating the benefit of GOCE. It is also shown that the GOCO05c curve in most cases stays below the EIGEN-6C4 model, which is a hint to a better spectral combination of the different data sources.

In a final validation step, the mean dynamic topography (MDT), the difference between mean sea surface and geoid height (cf. Bingham et al. 2008), is calculated. The mean sea surface DTU13MSS (Anderson et al. 2015) is used and was for this purpose expanded into spherical harmonics (up to degree 2000), so that the mean sea surface and the geoid heights can be compared at the same spectral range. For the comparison, always the full resolution of each gravity field model was chosen. This validation step is not suitable to derive conclusions about the absolute accuracy, but it delivers a quick assessment, if a reasonable

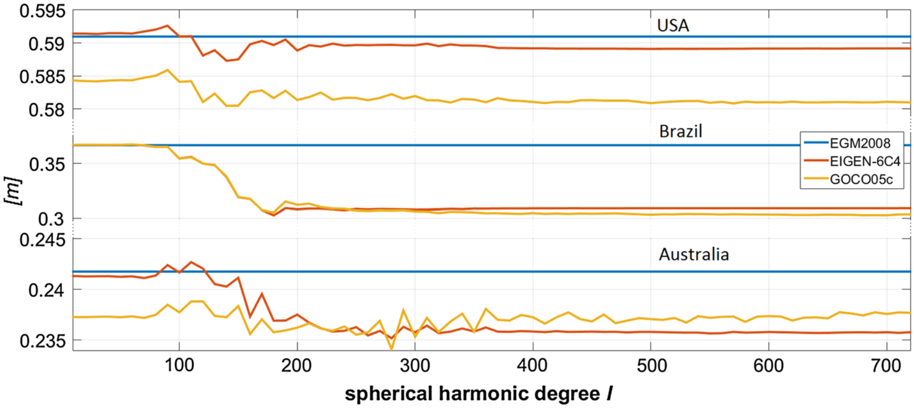

Fig. 9 RMS of geoid height differences between gravity field models and GNSS–levelling in Australia, Brazil, and the USA evaluated at different spherical harmonic degrees

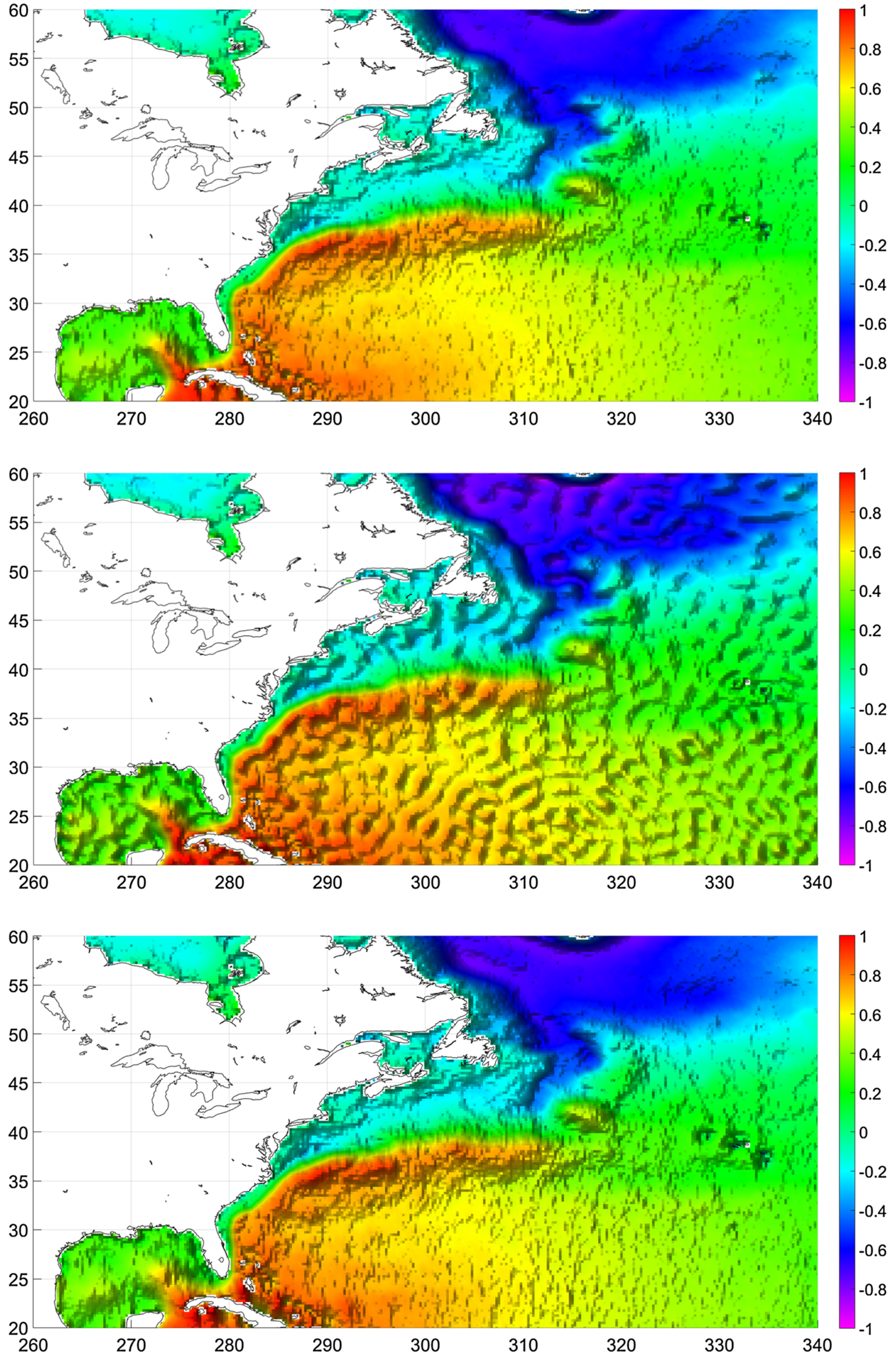

Fig. 10 MDT (m) in the Gulf Stream area derived from the differences between DTU13MSS to GOCO05c (*top*), EIGEN-6C4 (*middle*), and EGM2008 (*bottom*)

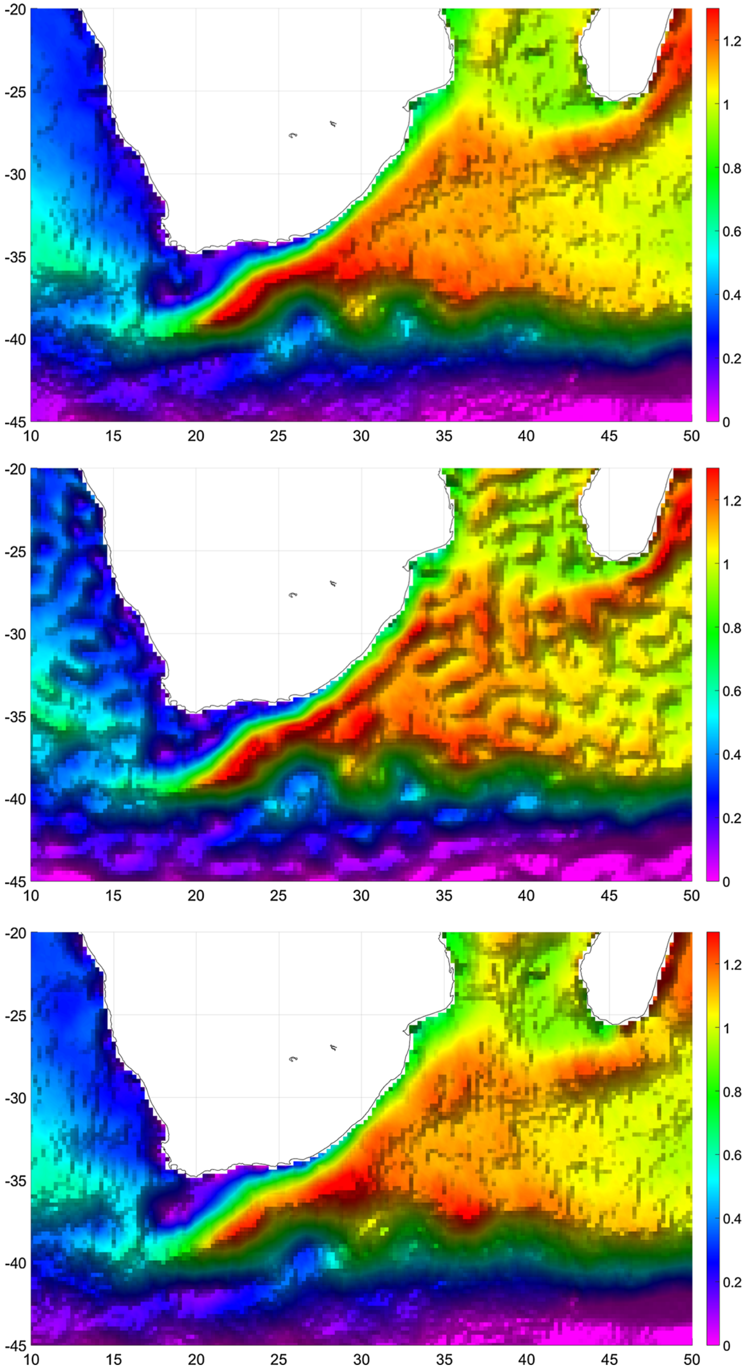

Fig. 11 MDT [m] in the Agulhas area derived from the differences between DTU13MSS to GOCO05c (*top*), EIGEN-6C4 (*middle*), and EGM2008 (*bottom*)

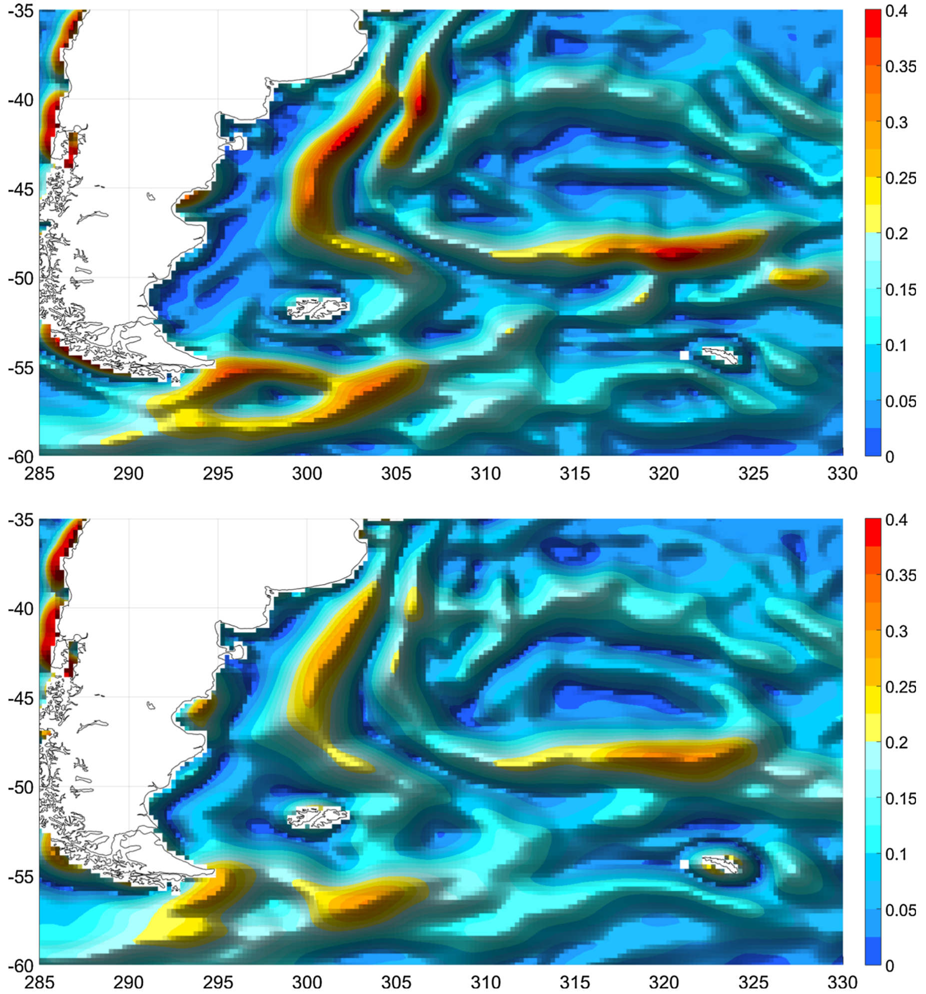

Fig. 12 Geostrophic velocities [m/s] of ocean currents in the Drake/Falkland area derived from the MDT of DTU13MSS and GOCO05c (*top*), respective EGM2008 (*bottom*)

MDT can be computed. Figures 10 and 11 indicate that a meaningful MDT can be derived from GOCO05c, which fits well to the EGM2008 result. In contrast to EIGEN-6C4, no bubble-like structures appear, which could be an indicator for a non-optimum combination procedure. The MDT of GOCO05c shows slight improvements compared to EGM2008 in the observed areas. The shape of the currents is more distinct at several spots, for example in the Gulf of Mexico. This becomes even more obvious, when geostrophic velocities are derived from the MDTs. They are displayed in Fig. 12 for GOCO05c and EGM2008. Here the added values of GOCE in GOCO05c are obvious, as the features of the currents are clearly accentuated. These accentuated features fit quite well to currents derived from the 'Maximenko MDT' (Maximenko et al. 2009), as, for example, displayed in Knudsen et al. (2011). Another MDT comparison between EGM2008 and GOCO05c implies that around the coast of South America (Fig. 13), GOCO05c achieves a smoother result in coastal

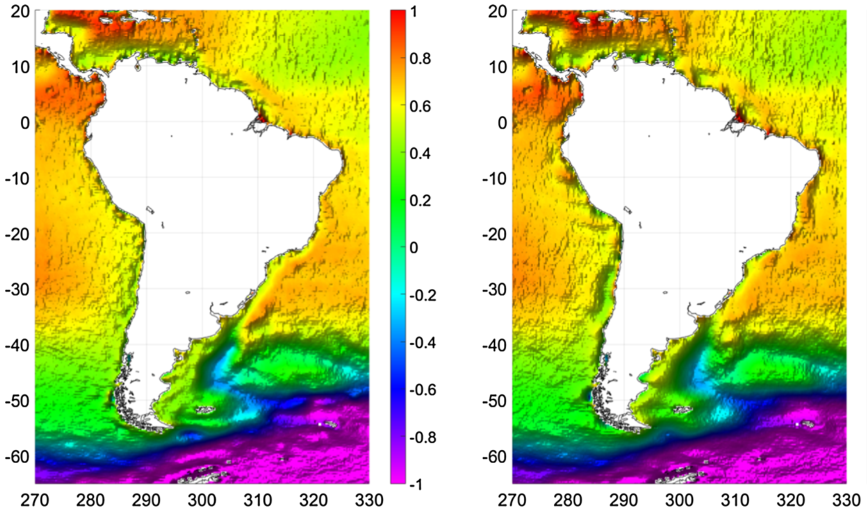

Fig. 13 MDT (m) in near-coastal regions around South America derived from the differences between DTU13MSS to EGM2008 (*left*) and GOCO05c (*right*)

regions. This follows from the improved quality of near-coastal altimetry data, in specific the improvements and the individual weighting of terrestrial data in the Andes. In these regions, the limited quality of the terrestrial data propagated into the neighbouring coastal regions.

5 Conclusions

The combination model GOCO05c is based on the solution of full normal equation systems and not (partially) on block-diagonal approximation. The advantage of full normal equation systems, enabling realistic stochastic modelling for all sub-components, especially regionally dependent weighting strategies to account for different quality of terrestrial/altimetric input data, was demonstrated. Such a weighting scheme, which was applied for the first time, results in a regionally different relative weighting of satellite and ground data. Correspondingly, in GOCO05c the transition from satellite to terrestrial data takes place between degrees 140 and 240, depending on the quality of the terrestrial data.

The resulting GOCO05c model is the only combined gravity field model that is independent of EGM2008, with the exception of the two coefficients $\bar{C}_{720,0}$ and $\bar{C}_{720,720}$, which have been used for regularization. Compared to EGM2008, the benefit of including GOCE data of the complete mission period could be shown, especially in the frame of orbit and GNSS–levelling tests. In general, the results of the validation with external data show that GOCO05c can at least achieve the level of accuracy of existing high-resolution models. However, it should be noted that GOCO05c should not be applied for geophysical interpretations in regions where fill-in data have been used.

From a computational point of view, the approach outlined here can be used to achieve considerably higher spectral resolution than d/o 720. The current limitation is related to available data material for several regions with a limited spatial resolution of $15' \times 15'$. The present-day computing performance of supercomputer systems is high enough to calculate a model such as GOCO05c in just a few hours, without coming close to the limits of available computing power. Moreover, much higher spectral resolutions can be achieved with hybrid approaches—a combination of full (resolved to the maximum possible extent) and block-diagonal systems.

GOCO05c is available via the International Centre for Global Earth Models (ICGEM, http://icgem.gfz-potsdam.de/ICGEM/).

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Appendix

GOCO Consortium Comprises: W-D Schuh, J Kusche, JM Brockmann, I Loth, S Müller, A Eicker, J Schall (University of Bonn, Institute of Geodesy and Geoinformation), T Mayer-Gürr, A Kvas, B Klinger, D Rieser, N Zehentner (Graz University of Technology, Institute of Geodesy), O Baur, E Höck, S Krauss (Austrian Academy of Sciences, Space Research Institute), A Jäggi, U Meyer, L Prange, A Maier (University of Bern, Astronomical Institute).

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