Two Continuous-Time Assignment-Based Models for the Multi-Mode Resource-Constrained Project Scheduling Problem

Abstract

In the multi-mode resource-constrained project scheduling problem, a set of precedence-related project activities and, for each activity, a set of alternative execution modes are given. Each activity requires some time and some scarce resources during execution; these requirements depend on the selected execution mode. Sought is a project schedule, i.e, a start time and an execution mode for each activity, such that the project makespan is minimized. In the literature, beside a large variety of specific solution approaches, several Mixed-Integer Linear Programming (MILP) models have been proposed for this problem. We present two novel MILP models that are based on mode-selection, resource-assignment and sequencing variables; we enhance the performance of the models by eliminating some symmetric solutions from the search space and by adding some redundant sequencing constraints for pairs and for triples of activities that cannot be processed in parallel. In a comparison with reference models from the literature, it turned out that the advantages of the novel models are a simple structure, an enhanced flexibility, and a superior performance when the range of the activities' durations is relatively large.
Keywords: Operations Research, Mixed-Integer Linear Programming, Multi-Mode Resource-Constrained Project Scheduling

1. Introduction

A project consists of a set of activities that are interrelated by precedence relationships and require time and scarce resources for their execution (cf., e.g., Brucker et al. 1999). Often there is a trade-off between the duration and the resource requirements of the project activities; this trade-off can be represented by alternative execution modes. Determining the start times and execution modes for the activities and allocating the scarce resources over time to the execution of the activities such that the project makespan is minimized represents a challenging combinatorial optimization problem.

We consider the multi-mode resource-constrained project scheduling problem (MRCPSP), which can be described as follows (cf., e.g., Mika et al. 2015). Given are a set of project activities that require time and scarce resources for their execution, and a set of completion-start precedence relationships among the project activities. Three different types of resources are distinguished (cf. Słowiński 1980): renewable, non-renewable and doubly constrained resources. Renewable resources, e.g., manpower, are limited over each time period and are renewed from one time period to the next. For non-renewable resources, e.g., raw materials, the total usage over the entire project duration is limited. Doubly constrained resources represent a combination of a renewable and a non-renewable resource (cf. Talbot 1982). Furthermore, for each activity, a set of alternative execution modes is given, which differ in the duration and the resource requirements of the activity. Sought is a project schedule, i.e., a start time and an execution mode for each activity, such that all precedence relationships are respected, the total required quantity of each renewable and each non-renewable resource does not exceed its prescribed capacity at any point in time, and the project makespan is minimized. The MRCPSP is a generalization of the widely studied single-mode resource-constrained project scheduling problem (RCPSP) and has been applied to the scheduling of, e.g., table-tennis leagues (cf. Knust 2010), construction projects (cf. Xu & Zeng 2015), and automotive R&D projects (cf. Bartels & Zimmermann 2015).

In the literature, in addition to many problem-specific heuristic and exact solution approaches, several Mixed-Integer Linear Programming (MILP) models have been proposed for the MRCPSP (cf., e.g., Mika et al. 2015 for an overview). The models can be classified into discrete-time (DT) models and continuous-time (CT) models. In DT models, the planning horizon is divided into a set of equal-length time intervals, and the activities can start only at the beginning of each of these intervals. In general, DT models involve time-indexed binary variables (cf., e.g., Talbot 1982, Maniezzo & Mingozzi 1999, Zhu et al. 2006); hence, the number of binary variables increases with the number of time intervals considered, which constitutes a potential drawback for projects that consist of activities with long durations, i.e., projects with a long planning horizon. By contrast, in CT models (cf., e.g., Kyriakidis et al. 2012), activities can start at any point in time over the planning horizon. In the known models, however, the formulation of the resource-capacity constraints requires a computation of the resource utilization based on the activities' start times, which is rather cumbersome. Further MILP models have been proposed for problems that extend the MRCPSP by, e.g., mode dependent time lags (cf. Sabzehparvar & Seyed-Hosseini 2008), multi-project scheduling (cf. Zapata et al. 2008) or non-preemptive activity splitting (cf. Cheng et al. 2015). In general, two major advantages of MILP models are their flexibility with respect to modifications of the planning situation and the possibility to apply standard solver software such as Gurobi or CPLEX (cf. Vielma 2015). The performance of an MILP-based solution approach for a specific planning problem, however, depends on the MILP model used and should be evaluated in an experimental analysis; for other scheduling problems, such analyzes have been performed by, e.g., Keha et al. (2009), Baker & Keller (2010) or Unlu & Mason (2010).

In this paper, we present two novel CT models for the MRCPSP. In both models, similar to the approach proposed in Trautmann et al. (2018) for the single-mode RCPSP, we use two types of binary variables to formulate the resource-capacity constraints: assignment variables specify which individual renewable-resource units are used for the execution of each activity, and sequencing variables specify the order in which pairs of activities that are assigned to the same renewable-resource unit are processed. In the second model, similar to the idea presented in Gnägi et al. (2018b), we use some additional auxiliary resource-overlap variables to identify these pairs of activities. To enhance the performance of the two novel models, we eliminate some symmetric solutions from the search space and add some redundant constraints for pairs and for triples of activities that cannot be processed in parallel due to the resource capacities. In a comparative analysis, we have applied the novel models and three reference models from the literature to two standard test sets and two novel test sets. Our computational results indicate a superior performance of the novel models when the range of the activities' durations is relatively large.

The remainder of this paper is structured as follows. In Section 2, we describe the MRCPSP in detail. In Section 3, we give an overview of the DT and CT models that we use as reference models. In Section 4, we present the novel MILP models for the MRCPSP. In Section 5, we report the computational results. In Section 6, we provide some concluding remarks and an outlook on future research.

2. Planning problem

In this section, we describe the MRCPSP in detail (cf. Subsection 2.1), and we illustrate the planning problem by means of an illustrative example (cf. Subsection 2.2). In the remainder of this paper, we use the sets and parameters listed in Table 1.

2.1. Multi-mode resource-constrained project scheduling problem

We assume that the project consists of a set V of activities. For each activity $i \in V$, a set M_i of alternative execution modes is given. The duration of activity $i \in V$ when executed in mode $m \in M_i$ is denoted as p_{im}. Furthermore, a set R of renewable resources and a set N of non-renewable resources are given. For the renewable resources $k \in R$, the resource capacities are denoted as R_k, and for the non-renewable resources $k \in N$, the resource capacities are denoted as W_k. Furthermore, we denote the resource requirements for the activities $i \in V$ when executed in mode $m \in M_i$ for the renewable resources $k \in R$ as r_{ikm} and for the non-renewable resources $k \in N$ as w_{ikm}. The set V of activities contains n real activities and two dummy activities 0 and $n+1$ representing the start and the completion of the project, respectively; both have a single execution mode with duration zero and no resource requirements. Furthermore, some completion-start precedence relationships are given among some pairs of activities $(i, j) \in V \times V$; these pairs of activities form the set E. The activity-on-node graph G depicts the activities $i \in V$ as the set of nodes and the completion-start precedence relationships $(i, j) \in E$ as the set of directed arcs between the nodes. The set TE consists of all pairs of activities $(i, j) \in V \times V$ for which the graph G contains a path from i to j or from j to i.

The planning horizon T is calculated as the sum of the maximum durations of all activities, i.e., $T = \sum_{i \in V} p_i^{max}$, where p_i^{max} denotes the maximum duration of activity $i \in V$, i.e., $p_i^{max} = \max_{m \in M_i} \{p_{im}\}$. The earliest start time ES_i and the latest start time LS_i of the activities $i \in V$ are determined by forward and backward pass calculations (cf. Demeulemeester & Herroelen 2002), respectively; for each activity, the mode with the shortest duration is used for both passes (cf. Talbot 1982).

2.2. Illustration of the planning problem

We illustrate the MRCPSP by means of an illustrative example. Given are four real activities, i.e., $V = \{0, 1, \ldots, 4, 5\}$; two execution modes are given for the activities $i \in \{1, 3\}$, and one execution mode is given for the activities $i \in \{0, 2, 4, 5\}$. The activities require one renewable resource with a capacity of two units, i.e., $R = \{1\}$ and $R_1 = 2$, and one non-renewable resource with a capacity of eight units, i.e., $N = \{1\}$ and $W_1 = 8$, for their execution. Figure 1 shows the activity-on-node graph G that depicts the completion-start precedence relationships among the activities and, below each node, the mode-dependent durations and resource requirements for each activity. The earliest and the latest start times are shown in Table 2; they are calculated based on the given precedence relationships and the planning horizon $T = 14$. In Figure 2, an optimal project schedule is shown; the usage for the renewable resource $r_1(t)$ is depicted as a function of the time t, and the selected execution modes are

Table 1: Sets and parameters.

Set	Description
V	Set of all activities ($V = \{0, 1, \dots, n, n+1\}$)
M_i	Set of all execution modes of activity $i \in V$
R	Set of all renewable resources
N	Set of all non-renewable resources
E	Set of all completion-start precedence relationships
G	Activity-on-node graph
TE	Set of all pairs of activities that cannot be executed in parallel due to their precedence relationships
Parameter	**Description**
p_{im}	Duration of activity $i \in V$ when executed in mode $m \in M_i$
R_k	Capacity of renewable resource $k \in R$
W_k	Capacity of non-renewable resource $k \in N$
r_{ikm}	Requirement of activity $i \in V$ of renewable resource $k \in R$ per period when executed in mode $m \in M_i$
w_{ikm}	Requirement of activity $i \in V$ of non-renewable resource $k \in N$ when executed in mode $m \in M_i$
p_i^{max}	Maximum duration of activity $i \in V$
T	Planning horizon
ES_i	Earliest start time of activity $i \in V$
LS_i	Latest start time of activity $i \in V$

Figure 1: Illustrative example: completion-start precedence relationships, durations and resource requirements.

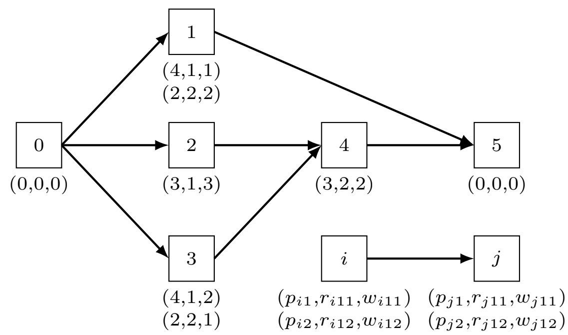


Table 2: Illustrative example: earliest and latest start times.

Activity i	0	1	2	3	4	5
Earliest start time ES_i	0	0	0	0	3	6
Latest start time LS_i	8	12	8	9	11	14

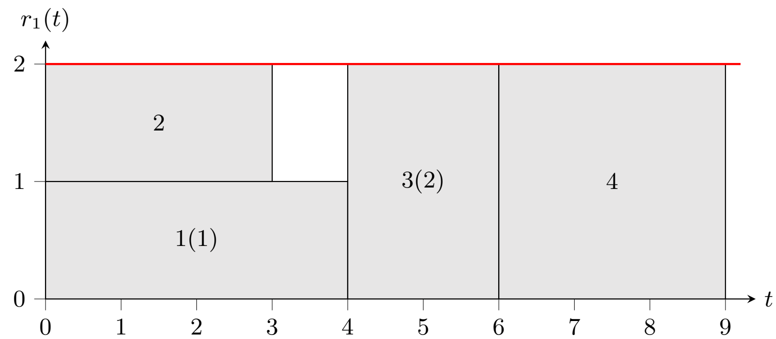
indicated in brackets. The minimal project makespan is nine time periods.

3. MILP models from the literature

In this section, we describe the types of variables used in the DT and CT models that we use as reference models. We selected the model of Talbot (1982), because it is the most commonly used DT model (cf., e.g., Mika et al. 2015), and the two CT models of Kyriakidis et al. (2012), because they are, to the best of our knowledge, the only known CT models for the MRCPSP.

The DT model introduced by Talbot (1982) is an extension of the well-known model proposed by Pritsker et al. (1969) for the single-mode RCPSP. The planning horizon is divided into a set of equal-length time intervals. The model employs some time-indexed variables that indicate whether an activity starts at the beginning of a specific time interval. Furthermore, each of these variables has an additional index that indicates the selected execution mode for the activity. In the following, we will refer to the model of Talbot (1982) as

Figure 2: Illustrative example: optimal project schedule.



Tal82.

In the two CT models presented by Kyriakidis et al. (2012), the planning horizon is divided into a set of time intervals with variable length. The first model (referred to as MMRTN1) involves two types of binary variables that are used to indicate whether an activity starts at the beginning of a specific time interval and whether an activity spans over multiple consecutive time intervals. Furthermore, three types of continuous variables are used to model the variable length of the planning horizon, the variable length of the time intervals and the resource availability at the beginning of each time interval. The second model (referred to as MMRTN2) employs similar types of variables as the first model; furthermore, two additional types of variables are introduced to indicate for each activity the resource usage at the beginning and the completion of the activity and to express the selected execution mode for each activity. In the following, we will refer to the first and the second model of Kyriakidis et al. (2012) as KyrKopGeo12-1 and KyrKopGeo12-2, respectively.

4. Novel MILP models for the MRCPSP

In this section, we present the two novel continuous-time assignment-based models for the MRCPSP. In Subsection 4.1, we present the model without auxiliary resource-overlap variables and in Subsection 4.2 the model with auxiliary resource-overlap variables. In Subsection 4.3, we present some supplements for the two models which shall improve their performance.

4.1. Continuous-time assignment-based model without auxiliary variables

The continuous-time assignment-based model, hereafter referred to as the MCTAB model, is based on the four types of variables listed in Table 3; a preliminary version of this model has been presented in Gnägi et al. (2018a). The model employs the start-time variables S_i $(i \in V)$ and the sequencing variables y_{ij} $(i,j \in V : i \neq j, (i,j) \notin TE)$; similar variables have been used in the model proposed by Artigues et al. (2003) for the single-mode RCPSP. The sequencing variables y_{ij} are only defined for all pairs of activities (i,j) with $i \neq j$ which can be executed simultaneously with respect to the precedence relationships, i.e., $(i,j) \notin TE$. Furthermore, the model includes the resource-assignment variables r_{ik}^{l} $(i \in V;\ k \in R;\ l = 1,\dots,R_k)$ that explicitly assign activities to individual renewable-resource units. Similar variables are used in Correia et al. (2012) to model the multi-skill project scheduling problem and in Rihm et al. (2018) to model an assessment center planning problem; both of these problems are extensions of the single-mode RCPSP. Finally, the binary mode-selection variables x_{im} $(i \in V;\ m \in M_i)$ are used to indicate the selected execution mode for each activity. We illustrate the types of variables used in the MCTAB model in Figure 3 by means of our illustrative example.

The objective is to minimize the project makespan, i.e. the start time of the dummy activity $n+1$:

$$\text{Min. } S_{n+1}$$

Constraints (1) link the resource-assignment variables to the sequencing variables. If two activities i and j are assigned to the same resource unit, these activities must be scheduled sequentially, i.e, either $y_{ij} = 1$ or $y_{ji} = 1$.

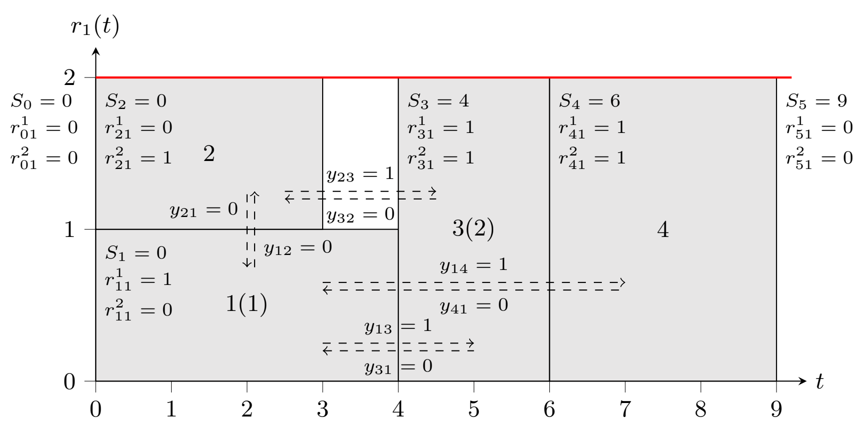
$$r_{ik}^{l} + r_{jk}^{l} \leq 1 + y_{ij} + y_{ji} \quad (i,j \in V;\ k \in R;\ l = 1,\dots,R_k : i < j, (i,j) \notin TE) \quad (1)$$

Constraints (2) indicate that for each activity the number of assigned renewable-resource units is equal to the number required by the activity in its selected

Table 3: Variables used in the MCTAB model.

Variable	Description
S_i	Start time of activity i
y_{ij}	$\begin{cases} = 1, & \text{if activity } i \text{ must be completed before the start of activity } j \\ = 0, & \text{otherwise} \end{cases}$
r_{ik}^{l}	$\begin{cases} = 1, & \text{if activity } i \text{ is assigned to unit } l \text{ of renewable resource } k \\ = 0, & \text{otherwise} \end{cases}$
x_{im}	$\begin{cases} = 1, & \text{if activity } i \text{ is executed in mode } m \\ = 0, & \text{otherwise} \end{cases}$

Figure 3: Types of variables used in the MCTAB model.



mode.

$$\sum_{l=1}^{R_k} r_{ik}^{l} = \sum_{m \in M_i} r_{ikm} x_{im} \quad (i \in V;\; k \in R) \tag{2}$$

Constraints (3) ensure that the capacities of the non-renewable resources are not exceeded.

$$\sum_{i \in V} \sum_{m \in M_i} w_{ikm} x_{im} \le W_k \quad (k \in N) \tag{3}$$

Constraints (4) represent the precedence relationships among the activities.

$$S_i + \sum_{m \in M_i} p_{im} x_{im} \le S_j \quad ((i,j) \in E) \tag{4}$$

Constraints (5) link the sequencing variables and the start-time variables. If two activities i and j are assigned to the same resource unit and are therefore forced to be scheduled sequentially by the constraints (1), then either activity i must be completed before the start of activity j, i.e, $y_{ij} = 1$, or activity j must be completed before the start of activity i, i.e, $y_{ji} = 1$.

$$S_i + \sum_{m \in M_i} p_{im} x_{im} \leq S_j + (LS_i + p_i^{max} - ES_j)(1 - y_{ij}) \qquad (i, j \in V : i \neq j, (i,j) \notin TE) \tag{5}$$

Constraints (6) assure that each activity is executed in exactly one mode.

$$\sum_{m \in M_i} x_{im} = 1 \quad (i \in V) \tag{6}$$

The model reads as follows.

$$\text{(MCTAB)} \begin{cases} \text{Min. } S_{n+1} \\ \quad \text{s.t. } (1) - (6) \\ \qquad S_i \geq 0 \quad (i \in V) \\ \qquad y_{ij} \in \{0,1\} \quad (i, j \in V : i \neq j, (i,j) \notin TE) \\ \qquad r_{ik}^l \in \{0,1\} \quad (i \in V;\ k \in R;\ l \in \{1, \ldots, R_k\}) \\ \qquad x_{im} \in \{0,1\} \quad (i \in V;\ m \in M_i) \end{cases}$$

4.2. Model with auxiliary resource-overlap variables

The model with auxiliary resource-overlap variables, hereafter referred to as the MCTABO model, is based on the five types of variables listed in Table 4. The start-time variables, the resource-assignment variables and the mode-selection variables are used analogously to the MCTAB model. In contrast, the sequencing variables y_{ij} are used such that $y_{ij} = 1$ if activity i starts before or at the same time as activity j and $y_{ij} = 0$ if activity j starts before or at the same time as activity i. Furthermore, the auxiliary resource-overlap variables z_{ij} $(i, j \in V : i < j, (i,j) \notin TE)$ are used to indicate a possible overlap between the activities i and j with regard to at least one renewable resource. The sequencing variables y_{ij} and the auxiliary resource-overlap variables z_{ij} are only defined for

Table 4: Variables used in the MCTABO model.

Variable	Description
S_i	Start time of activity i
y_{ij}	$\begin{cases} = 1, \text{ if activity } i \text{ starts before or at the same time as activity } j \\ = 0, \text{ if activity } j \text{ starts before or at the same time as activity } i \end{cases}$
r_{ik}^{l}	$\begin{cases} = 1, \text{ if activity } i \text{ is assigned to unit } l \text{ of renewable resource } k \\ = 0, \text{ otherwise} \end{cases}$
z_{ij}	$\begin{cases} = 1, \text{ if activities } i \text{ and } j \text{ use the same renewable-resource unit} \\ = 0, \text{ otherwise} \end{cases}$
x_{im}	$\begin{cases} = 1, \text{ if activity } i \text{ is executed in mode } m \\ = 0, \text{ otherwise} \end{cases}$

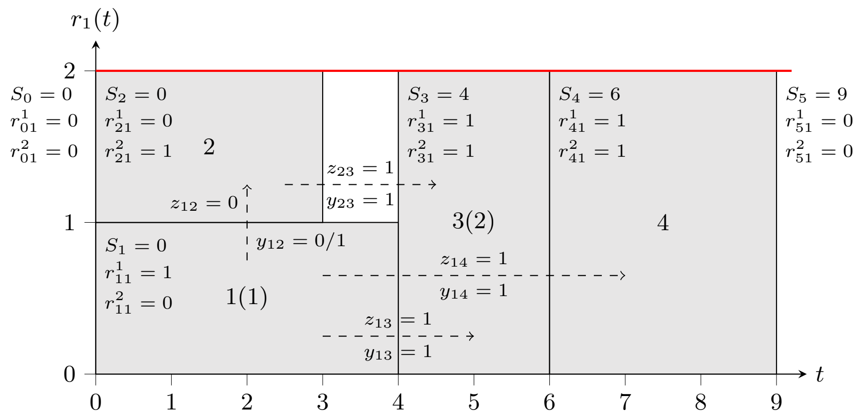
all pairs of activities (i, j) with $i < j$ which can be executed simultaneously with respect to the precedence relationships, i.e., $(i, j) \notin TE$. We illustrate the types of variables used in the MCTABO model in Figure 4 by means of our illustrative example.

With regard to the constraints, the differences between the MCTAB model and the MCTABO model are as follows. Constraints (1) are replaced by constraints (7). These constraints link the resource-assignment variables to the auxiliary resource-overlap variables. If two activities i and j are assigned to the same resource unit, then the corresponding auxiliary resource-overlap variable is forced to be one, i.e., $z_{ij} = 1$.

$$r_{ik}^{l} + r_{jk}^{l} \leq 1 + z_{ij} \quad (i, j \in V;\ k \in R;\ l = 1, \ldots, R_k : i < j, (i, j) \notin TE) \tag{7}$$

Constraints (5) are replaced by constraints (8) and (9). These constraints link the start-time variables, the sequencing variables and the auxiliary resource-overlap variables. If two activities i and j are assigned to the same resource unit, i.e., $z_{ij} = 1$, then these activities must be scheduled sequentially; then, either activity i must be completed before the start of activity j, i.e, $y_{ij} = 1$, or activity j must be completed before the start of activity i, i.e, $y_{ij} = 0$. If there is no resource overlap between the activities i and j, i.e., $z_{ij} = 0$, then either

Figure 4: Types of variables used in the MCTABO model.



activity i must start before or at the same time as activity j, i.e, $y_{ij} = 1$, or activity j must start before or at the same time as activity i, i.e, $y_{ij} = 0$.

$$S_i + \sum_{m \in M_i} p_{im} x_{im} - p_i^{max}(1 - z_{ij}) \leq S_j + (LS_i + p_i^{max} - ES_j)(1 - y_{ij})$$

$$(i, j \in V : i < j, (i, j) \notin TE) \quad (8)$$

$$S_j + \sum_{m \in M_j} p_{jm} x_{jm} - p_j^{max}(1 - z_{ij}) \leq S_i + (LS_j + p_j^{max} - ES_i) y_{ij}$$

$$(i, j \in V : i < j, (i, j) \notin TE) \quad (9)$$

The model reads as follows.

$$(\text{MCTABO}) \begin{cases} \text{Min.} & S_{n+1} \\ \text{s.t.} & (2) - (4), (6) - (9) \\ & S_i \geq 0 \quad (i \in V) \\ & y_{ij} \in \{0, 1\} \quad (i, j \in V : i < j, (i, j) \notin TE) \\ & r^l_{ik} \in \{0, 1\} \quad (i \in V;\ k \in R;\ l \in \{1, \ldots, R_k\}) \\ & z_{ij} \in \{0, 1\} \quad (i, j \in V : i < j, (i, j) \notin TE) \\ & x_{im} \in \{0, 1\} \quad (i \in V;\ m \in M_i) \end{cases}$$

4.3. Model supplements

In this subsection, we present various supplements for the models MCTAB and MCTABO. These supplements shall improve the performance of the models by eliminating some symmetric solutions from the search space and by adding some redundant constraints that explicitly enforce the sequential scheduling of pairs or triples of activities that cannot be processed in parallel due to the resource capacities.

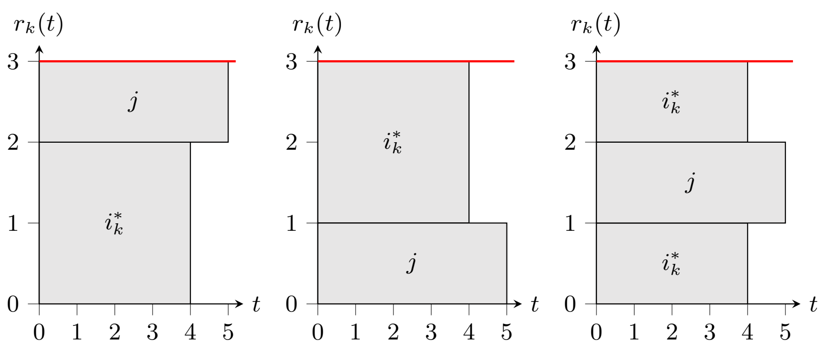
First, to eliminate some symmetric solutions from the search space w.l.o.g., we can assign ex ante some units of each renewable resource to an arbitrary activity, because all individual units of a renewable resource are identical. For each renewable resource $k \in R$, we select an activity i_k^* with the largest minimum requirement $r_{ik}^{min} = \min_{m \in M_i}\{r_{ikm}\}$ for this resource, and we add the constraints (10) to both models MCTAB and MCTABO; for each renewable resource $k \in R$, these constraints assign the first $r_{i_k^*,k}^{min}$ renewable-resource units to the execution of activity i_k^*.

$$r_{i_k^*,k}^l = 1 \quad (k \in R, l \in \{1, \dots, r_{i_k^*,k}^{min}\}) \tag{10}$$

Figure 5 illustrates the constraints (10); by explicitly assigning the first and the second renewable-resource unit to activity i_k^*, the two symmetric resource-unit assignments in the middle and on the right-hand side are eliminated from the search space.

Second, we analyze all pairs of activity-mode combinations $((i, m_i), (j, m_j))$ with $i, j \in V$, $m_i \in M_i$ and $m_j \in M_j$ for which the requirement for some renewable resource exceeds the resource capacity, i.e., $r_{ikm_i} + r_{jkm_j} > R_k$ for some renewable resource $k \in R$; let the set V^2 contain all these pairs of activity-mode combinations with $i < j$, excluding all pairs of activity-mode combinations that contain pairs of activities that are in TE. For all pairs of activity-mode combinations $((i, m_i), (j, m_j)) \in V^2$, if activities i and j are executed in modes m_i and m_j, respectively, in each feasible solution, at least one unit of resource k will be assigned to both activities i and j. Thus, the activities i and j must

Figure 5: Elimination of some symmetric resource-unit assignments.



be scheduled sequentially; therefore, we add the (redundant) constraints

$$y_{ij} + y_{ji} \geq x_{im_i} + x_{jm_j} - 1 \quad (((i, m_i), (j, m_j)) \in V^2) \tag{11}$$

to the MCTAB model and the (redundant) constraints

$$z_{ij} \geq x_{im_i} + x_{jm_j} - 1 \quad (((i, m_i), (j, m_j)) \in V^2) \tag{12}$$

to the MCTABO model. Analogously, we analyze all triples of activity-mode combinations $((i, m_i), (j, m_j), (h, m_h))$, where $i, j, h \in V$, $m_i \in M_i$, $m_j \in M_j$ and $m_h \in M_h$, with $r_{ikm_i} + r_{jkm_j} + r_{hkm_h} > R_k$ for some renewable resource $k \in R$; let the set V^3 contain all these triples of activity-mode combinations with $i < j < h$, excluding all triples of activity-mode combinations that contain pairs of activities that are in TE or pairs of activity-mode combinations that are in V^2. For all triples of activity-mode combinations $((i, m_i), (j, m_j), (h, m_h)) \in V^3$, we add the (redundant) constraints (13) and (14) to the models MCTAB and MCTABO, respectively.

$$y_{ij} + y_{ji} + y_{ih} + y_{hi} + y_{jh} + y_{hj} \geq x_{im_i} + x_{jm_j} + x_{hm_h} - 2$$
$$(((i, m_i), (j, m_j), (h, m_h)) \in V^3) \tag{13}$$

$$z_{ij} + z_{ih} + z_{jh} \geq x_{im_i} + x_{jm_j} + x_{hm_h} - 2$$
$$(((i, m_i), (j, m_j), (h, m_h)) \in V^3) \tag{14}$$

The extended models read as follows.

$$
\text{(MCTAB extended)}\begin{cases}
\text{Min.} & S_{n+1} \\
\text{s.t.} & (1)-(6),(10)-(11),(13) \\
& S_i \geq 0 \quad (i \in V) \\
& y_{ij} \in \{0,1\} \quad (i,j \in V : i \neq j, (i,j) \notin TE) \\
& r_{ik}^l \in \{0,1\} \quad (i \in V;\ k \in R;\ l \in \{1,\ldots,R_k\}) \\
& x_{im} \in \{0,1\} \quad (i \in V;\ m \in M_i)
\end{cases}
$$

$$
\text{(MCTABO extended)}\begin{cases}
\text{Min.} & S_{n+1} \\
\text{s.t.} & (2)-(4),(6)-(10),(12),(14) \\
& S_i \geq 0 \quad (i \in V) \\
& y_{ij} \in \{0,1\} \quad (i,j \in V : i < j, (i,j) \notin TE) \\
& r_{ik}^l \in \{0,1\} \quad (i \in V;\ k \in R;\ l \in \{1,\ldots,R_k\}) \\
& z_{ij} \in \{0,1\} \quad (i,j \in V : i < j, (i,j) \notin TE) \\
& x_{im} \in \{0,1\} \quad (i \in V;\ m \in M_i)
\end{cases}
$$

5. Computational results

In this section, we present the design (cf. Subsection 5.1) and the numerical results (cf. Subsection 5.2) of the experimental performance analysis.

5.1. Test design

We compare the performance of the novel MILP models with the performance of the DT model of Talbot (1982) and the two CT models of Kyriakidis et al. (2012). We implemented these models in AMPL, and we used the Gurobi Optimizer 8.1 with the default solver settings. All computations were performed on a workstation with two 8-core Intel Xeon E5-2687W CPUs (3.1 GHz) and 128 GB RAM. We set a time limit of 300 seconds per instance, and we limited the number of threads to four.

For the comparative analysis, we used the test sets J20 and J30 from the PSPLIB (cf. Kolisch & Sprecher 1996). Each of these test sets contains 640 instances that consist of 20 and 30 real activities, respectively, with three alternative execution modes each. Furthermore, the activities require two renewable and two non-renewable resources for their execution. For 86 instances of the set J20 and for 88 instances of the set J30, no feasible solution exists; we excluded these instances from our comparative analysis. The instances of the sets J20 and J30 consist of activities with relatively short durations ranging from one to ten time units; this could be an advantage for time-indexed models such as the model of Talbot (1982). Therefore, to broaden our comparative analysis, we generated two novel test sets that are based on the instances of the sets J20 and J30, but consist of instances with a relatively large range of the activities' durations. We generated these instances by adopting the procedure that has been proposed by Koné et al. (2011) for single-mode RCPSP instances. For each of the sets J20 and J30, we randomly selected α real activities. For these activities, we multiplied the duration for each mode by $\beta + \varepsilon$ with ε being a uniformly distributed random number between zero and one and rounded the resulting duration to the nearest integer. Analogously to Koné et al. (2011), we set $\alpha = n/2$, i.e., $\alpha = 10$ for set J20 and $\alpha = 15$ for set J30, and we set $\beta = 25$ resulting in two novel sets of instances consisting of activities with durations ranging from one to up to 260 time units. In the following, we will refer to these novel test sets as D20 and D30.

5.2. Numerical results

We use the following metrics to evaluate the performance of the tested models:

- Feas (%): Fraction of instances for which a feasible schedule has been found within the prescribed time limit.
- Opt (%): Fraction of instances for which a feasible schedule has been found and proven to be optimal within the prescribed time limit.
- Best (%): Fraction of instances for which, within the prescribed time limit, the best schedule among those found with all tested models has

been found.

- Gap$^{\text{LB}}$ (%): Average relative deviation between the objective function value of the best schedule returned by the solver (OFV) and the best lower bound returned by the solver (LB), calculated as $(OFV - LB)/LB$.

- Gap$^{\text{CP}}$ (%): Average relative deviation between OFV and the critical-path based lower bound (CP), calculated as $(OFV - CP)/CP$.

- Gap$^{\text{BEST}}$ (%): Average relative deviation between OFV and the objective function value of the best schedule ($BEST$) found among those found with all tested models, calculated as $(OFV - BEST)/BEST$.

- # Cons: Average number of constraints.

- # Vars: Average number of variables.

The results of the comparative analysis are summarized in Tables 5 and 6; bold values indicate the best results among all tested models.

For the sets J20 and J30, all tested models except the two models proposed by Kyriakidis et al. (2012) obtain at least a feasible schedule for all considered instances. The model of Talbot (1982) performs best in terms of the number of instances that are proven to be solved to optimality and of the average deviation from the best lower bound returned by the solver; this may be attributed to the relatively strong linear programming relaxations of models with time-indexed binary variables (cf. Artigues et al. 2015). The novel models, and especially those including the proposed supplements, perform second best for the sets J20 and J30; they even achieve a matchable performance in terms of the average deviation from the critical-path based lower bound.

For the sets D20 and D30, the novel models clearly outperform the other tested models regarding all reported metrics; this might be attributed to the strong increase of the average number of variables used for the DT model of Talbot (1982). In contrast, the average number of variables used for the presented novel CT models is relatively small and remains constant also for the sets D20 and D30. For the CT models proposed by Kyriakidis et al. (2012), the average number of variables used is also constant but very large; both models are clearly outperformed by the other tested models. In Table 7, we provide the results of the comparative analysis for all considered instances of the sets D20 and D30 for which both the model of Talbot (1982) and the novel models yield at least a feasible solution; the results for the models of Kyriakidis et al. (2012) are omitted because they do not obtain a feasible solution for many instances, especially for the set D30. Also with respect to these results, the novel models outperform the model of Talbot (1982).

Without the model supplements, both models MCTAB and MCTABO achieve a comparable performance for all test sets. For both models, the proposed supplements result in a considerable performance improvement with regard to all reported metrics. The supplements proposed for the MCTAB model, however, tend to be more effective and thus lead to slightly larger improvements.

In Table 8, the results for further supplements of the novel models are summarized, representatively for the set J30. We tested the performance of the models MCTAB extended and MCTABO extended considering pairs of activity-mode combinations that cannot be process in parallel due their resource requirements only, i.e., without the constraints (13) and (14), respectively. Furthermore, we also tested the performance of the models MCTAB extended and MCTABO extended with analogous additional constraints for quadruples of activity-mode combinations that cannot be process in parallel due their resource requirements. The models MCTAB extended and MCTABO extended perform best among all tested models; this may be attributed to the relatively small number of redundant constraints when considering pairs of activity-mode combinations only, while the large number of redundant constraints for the models also considering quadruples of activity-mode combinations seems to slow down the solution process.

Table 5: Computational results: all considered instances of the sets J20 and J30.

Set	Model	Feas (%)	Opt (%)	Best (%)	Gap^{LB} (%)	Gap^{CP} (%)	Gap^{BEST} (%)	# Cons	# Vars
J20	Tal82	**100.00**	**97.47**	**99.10**	**0.22**	**17.13**	**0.05**	**382**	8,676
	KyrKopGeo12-1	96.75	0.00	8.84	607.97	45.55	24.19	6,159	3,513
	KyrKopGeo12-2	99.64	5.96	43.68	118.48	25.19	6.31	22,229	18,623
	MCTAB	**100.00**	81.41	88.27	3.05	17.90	0.59	4,310	**1,223**
	MCTAB extended	**100.00**	88.99	96.21	1.71	17.30	0.17	4,841	**1,223**
	MCTABO	**100.00**	80.69	89.35	3.20	17.91	0.59	4,310	**1,223**
	MCTABO extended	**100.00**	85.74	92.06	2.47	17.53	0.33	4,841	**1,223**
J30	Tal82	**100.00**	**88.41**	**97.64**	**2.50**	**15.35**	**0.81**	**570**	19,859
	KyrKopGeo12-1	9.24	0.00	0.00	1,558.85	110.03	81.19	13,993	7,881
	KyrKopGeo12-2	47.28	0.00	5.62	383.02	34.23	27.98	64,939	57,229
	MCTAB	**100.00**	74.09	77.54	8.06	15.95	1.79	13,495	**2,328**
	MCTAB extended	**100.00**	76.09	79.89	7.85	15.77	1.71	15,306	**2,328**
	MCTABO	**100.00**	73.55	78.26	8.15	15.93	1.80	13,495	**2,328**
	MCTABO extended	**100.00**	75.54	78.99	7.87	15.75	1.67	15,306	**2,328**

Table 6: Computational results: all considered instances of the sets D20 and D30.

Set	Model	Feas (%)	Opt (%)	Best (%)	Gap^{LB} (%)	Gap^{CP} (%)	Gap^{BEST} (%)	# Cons	# Vars
D20	Tal82	87.36	73.83	74.19	34.64	42.34	28.30	**4,282**	113,440
	KyrKopGeo12-1	73.10	0.00	9.57	580.52	38.66	25.51	6,159	3,513
	KyrKopGeo12-2	**100.00**	2.71	65.34	71.54	12.99	2.02	22,229	18,623
	MCTAB	**100.00**	95.49	99.46	0.16	10.37	0.01	4,310	**1,223**
	MCTAB extended	**100.00**	**99.82**	**100.00**	**0.01**	**10.36**	**0.00**	4,841	**1,223**
	MCTABO	**100.00**	93.50	98.01	0.32	10.38	0.02	4,310	**1,223**
	MCTABO extended	**100.00**	99.28	99.82	0.10	**10.36**	**0.00**	4,841	**1,223**
D30	Tal82	71.92	63.04	63.04	45.03	47.06	39.40	**6,434**	260,446
	KyrKopGeo12-1	3.99	0.00	0.00	857.53	43.33	41.67	13,993	7,881
	KyrKopGeo12-2	23.91	0.00	4.17	209.71	19.76	17.88	64,939	57,229
	MCTAB	**100.00**	84.06	87.86	3.48	8.38	0.48	13,495	**2,328**
	MCTAB extended	**100.00**	**87.14**	**93.30**	**2.69**	**7.96**	**0.16**	15,306	**2,328**
	MCTABO	**100.00**	84.42	89.13	3.51	8.21	0.34	13,495	**2,328**
	MCTABO extended	**100.00**	86.96	92.75	2.81	8.00	0.19	15,306	**2,328**

Table 7: Computational results: instances of the sets D20 and D30 with at least a feasible solution obtained by all considered models (Tal82 and novel models).

Set	Model	Feas (%)	Opt (%)	Best (%)	Gap^{LB} (%)	Gap^{CP} (%)	Gap^{BEST} (%)	# Cons	# Vars
D20	Tal82	**100.00**	84.50	84.92	34.64	42.34	28.30	**4,280**	113,377
	MCTAB	**100.00**	96.49	99.59	0.12	6.26	0.01	4,460	**1,257**
	MCTAB extended	**100.00**	**100.00**	**100.00**	**0.00**	**6.25**	**0.00**	4,889	**1,257**
	MCTABO	**100.00**	96.28	99.17	0.24	6.27	0.01	4,460	**1,257**
	MCTABO extended	**100.00**	99.79	99.79	0.05	**6.25**	**0.00**	4,889	**1,257**
D30	Tal82	**100.00**	87.66	87.66	45.03	47.06	39.40	**6,426**	259,873
	MCTAB	**100.00**	94.96	95.97	0.89	1.93	0.16	14,263	**2,426**
	MCTAB extended	**100.00**	**96.22**	97.23	**0.63**	**1.80**	**0.05**	15,084	**2,426**
	MCTABO	**100.00**	94.96	97.23	0.84	1.82	0.08	14,263	**2,426**
	MCTABO extended	**100.00**	95.97	**98.24**	0.66	**1.80**	**0.05**	15,084	**2,426**

Table 8: Computational results: models MCTAB and MCTABO with further model supplements.

Set	Model	Feas (%)	Opt (%)	Best (%)	Gap^{LB} (%)	Gap^{CP} (%)	Gap^{BEST} (%)	# Cons	# Vars
J30	MCTAB	**100.00**	74.09	82.79	8.06	15.95	0.94	**13,495**	**2,328**
	MCTAB (pairs)	**100.00**	74.64	**84.60**	**7.58**	**15.69**	**0.80**	13,554	**2,328**
	MCTAB extended	**100.00**	**76.09**	84.24	7.85	15.77	0.85	15,306	**2,328**
	MCTAB (quadruples)	**100.00**	75.54	80.62	9.81	17.62	1.97	24,559	**2,328**
	MCTABO	**100.00**	73.55	83.51	8.15	15.93	0.95	**13,495**	**2,328**
	MCTABO (pairs)	**100.00**	73.73	83.70	**7.75**	15.78	0.87	13,554	**2,328**
	MCTABO extended	**100.00**	**75.54**	**84.42**	7.87	**15.75**	**0.81**	15,306	**2,328**
	MCTABO (quadruples)	99.82	75.18	82.61	8.23	16.06	1.16	24,559	**2,328**

6. Conclusions

In this paper, we have proposed two novel continuous-time MILP models for the multi-mode resource-constrained project scheduling problem MRCPSP. The models are based on continuous variables that represent the start times of the activities and binary variables that represent the assignment of the project activities to the individual resource units, the sequential relationships between activities that are assigned to at least one identical resource unit, and the selection of an execution mode for each activity. Compared to the continuous-time models known from the literature, the novel models have a simpler structure. Our computational results indicate that the novel models outperform all reference models when the range of the activities' durations is relatively high.

In future research, the efficient elimination of additional symmetric solutions from the search space should be investigated. Furthermore, analogous models for related project scheduling problems such as, e.g., the resource-constrained project scheduling problem with minimum and maximum time lags, should be analyzed.

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