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ORIGINAL ARTICLE

Determination of GNSS pseudo-absolute code biases and their long-term combination

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Abstract

With the modernization of GPS and the establishment of additional global navigation satellite system (GNSS) constellations, such as Galileo, Beidou, and QZSS, more and more GNSS satellites are available transmitting on various frequencies with multiple signal modulations. In order to cope with the increasing number of observation types, the commonly used differential approach becomes more and more difficult regarding book-keeping. The actually processed original observation types have to be known in advance to define a linearly independent set of differential signal biases (DSB) while processing GNSS data. An alternative treatment of code biases is the usage of observable-specific signal biases (OSB) where the setup and correction of biases become trivial. Potential dependencies of the bias parameters can be considered after the setup of normal equations (NEQs), e.g., immediately before it is inverted. The code bias results are retrieved on a daily basis and their NEQs stored. This allows to combine bias results from various sources (or analysis lines) and different time periods. By combining all daily bias NEQs, we have generated a coherent multi-year bias solution from 2000 to 2017 with one common datum. If absolute receiver calibrations are available, the multi-year solution could be aligned to those receivers and thus could lead to an absolute estimation of the code biases. Finally, the estimated satellite OSBs are used for the receiver compatibility grouping testing which receivers are compatible with which bias sets. This may be achieved by solving for so-called OSB multipliers.

Keywords Code biases · Multi-GNSS code biases · Observable-specific bias (OSB) parametrization · Code bias multiplier

1 Introduction

Pseudo-range observations are obtained by measuring the time difference between the emission of the signal from the satellite antenna and the reception in the receiver antenna. The emission time from the satellite is encoded onto the signal before it gets transmitted by the antenna. The time difference between the signal emission and the related clock reading is not negligible and has to be addressed when processing pseudo-range data. The same mechanism is valid on the receiver side. There is a time difference between the reception time of the signal in the antenna and actual linking to the receiver clock. These hardware delays, addressed in this paper as code biases, are schematically shown in Fig. 1. The code biases for satellites and for receivers depend on the signal frequency and on the signal modulation. As a result of that, each specific observable type has an individual delay and needs to be carefully parameterized.

It is common to treat code biases in a differential mode as differential signal biases (DSB, also known as DCB) and ionosphere-free linear combination (IF) signal biases (ISB). The ISB is also known under the term inter-system bias and is relevant for the estimation of multi-GNSS clock products. DSBs are important for the ionosphere estimation relying on the geometry-free linear combination (GF).

With the modernization of GPS and the uprising GNSS, such as Galileo and Beidou, the handling of code biases becomes more and more complex as they have to be introduced between each involved GNSS and individual observable types. Within the IGS community, code biases are treated commonly in a differential mode (Montenbruck et al. 2012; Guo et al. 2015; Wang et al. 2016; Håkansson et al. 2017). When using a DSB parametrization, the observation types need to be known before the processing in order to define a set of linearly independent differential biases. Those are then used when setting up the observation equations while processing GNSS data. The observable-specific

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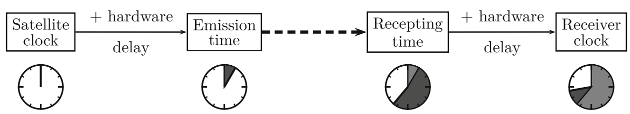


Fig. 1 Schematic description of the code biases (simplified)

signal bias (OSB) parametrization assigns each observable type involved an individual code bias. This gives full flexibility for later bias parameter transformations. OSBs are straight forward on how to apply them and are, for instance, used in the real-time GNSS community (RTCM 2016). However, the usage of the OSB parametrization for post-processing of GNSS data is not yet common practice. Furthermore, the OSB parameterization allows the generation of corresponding normal equations (NEQs) and to impose bias datum definitions according to the user's requirements. In the differential case, this is not possible because the datum definition is implicitly given by the selection of the DSB parametrization.

The current observation data interface is described in the RINEX 3.03 format description (RINEX 3.03 2015). The RINEX standards are crucial for the description of the various observable types and the associated biases (OSBs). A three-figure code is used to indicate a particular observable (e.g., C1C, C1W, C2W for GPS and C1C, C5Q for Galileo). It is worth mentioning that groups of RINEX 3 observable types (e.g., GPS: C2S, C2L) may have very similar code bias characteristics.

In order to verify the compatibility among the code bias characteristics of various receivers, the DCB multiplier approach was developed at the Center for Orbit Determination (CODE) (IGSMAIL-3887). This method has been generalized to the OSB representation and multiple GNSS. The extended OSB multiplier approach was tested in a dedicated study.

In June 2016, the code bias parametrization was changed at CODE from the DSB to a OSB representation, affecting all of our IGS analysis products. This implies that the Bernese GNSS Software (Dach et al. 2015), used for all analyses presented here, was updated accordingly. With the update of the new bias SINEX format from 0.01 to 1.00, the exchange of OSBs has been enabled (Schaer 2017).

Section 2 will give an overview on how observable-specific observation biases are estimated. Further, their relation with the classical differential approach is shown for validation purposes. Section 3 addresses their estimation and will give a detailed overview of a dedicated bias estimation effort for the year 2016. The Multi-GNSS extension (Montenbruck et al. 2017, MGEX) products from CODE (Prange et al. 2017a, b) and IGS data (Dow et al. 2009) were used to generate a full set of satellite OSB results for GPS, GLONASS, Galileo, and Beidou. Section 4 gives a short overview of a long-term bias estimation covering 18 years (2000 to 2017) for GPS and GLONASS. Our long-term combination is finally compared with GPS broadcast biases (group delays) (Wilson et al. 1999) in order to assess their long-term stability. Section 5 is dedicated to our OSB multiplier approach extension to verify the GNSS code bias characteristics of receivers.

2 Estimation of code biases

Receivers reveal for each tracked satellite and for each observable type an individual code bias value which needs to be accounted for when analyzing pseudo-range data. Since code biases mainly depend on the frequency and signal modulation, the satellite components may be assumed to be common for all receivers. As a consequence of this, the biases can be split up into a receiver and a satellite component (for each observable type) provided that the corresponding GNSS is based on code division multiple access (CDMA) (Meurer and Antreich 2017). Since GLONASS is using frequency division multiple access (FDMA), the situation is different and addressed in Sect. 2.4.

In this section, the theoretical aspects for the code bias estimation are introduced, specifically for IF, GF, and for the combination of bias results corresponding to both linear combinations. The observation equations for a pseudo-range observation between satellite i and receiver k are

$$\begin{aligned} P^i_{1k} &= \varrho^i_k + I^i_k + T^i_k + c(\Delta\rho_k + B_{1k}) - c(\Delta\rho^i + B^i_1) \\ P^i_{2k} &= \varrho^i_k + \frac{f_1^2}{f_2^2} I^i_k + T^i_k + c(\Delta\rho_k + B_{2k}) - c(\Delta\rho^i + B^i_2) \end{aligned} \tag{1}$$

where P^i_{1k} and P^i_{2k} are the pseudo-range observations for the two frequencies, ϱ^i_k is the geometrical distance between receiver k and satellite i, I^i_k is the ionospheric correction, T^i_k the troposphere delay, $\Delta\rho_k$ and $\Delta\rho^i$ the receiver and satellite clock corrections with respect to a common time system (e.g., GPS system time), B_{1k}, B_{2k}, B^i_1, and B^i_2 the receiver- and satellite-related code OSBs for the first and second frequencies which are intended to absorb the hardware delays according to Fig. 1, c the speed of light, and f_1 and f_2 are the frequencies of the corresponding signals.

2.1 Ionosphere-free linear combination for clock analysis

According to Hauschild (2017), the ionosphere-free linear combination (IF), P_{IF}, is

$$P^i_{IFk} = \kappa_1 P^i_{1k} + \kappa_2 P^i_{2k} \tag{2}$$

$\kappa_1 = \frac{f_1^2}{f_1^2-f_2^2}$ is the pre-factor for the first frequency and $\kappa_2 = -\frac{f_2^2}{f_1^2-f_2^2}$ for the second frequency. Inserting Eq. (1) into (2) leads to

$$P^i_{IFk} = \varrho^i_k + T^i_k + c(\Delta\rho_k + \underbrace{(\kappa_1 B_{1k} + \kappa_2 B_{2k})}_{\mathrm{ISB}_k})$$
$$- c(\Delta\rho^i + \underbrace{(\kappa_1 B^i_1 + \kappa_2 B^i_2)}_{\mathrm{ISB}^i}) \tag{3}$$

where ISB_k and ISB^i are the IF signal biases associated with the receiver k and the satellite i. For the IF, the ionosphere term I cancels out and the remaining terms ϱ, T, and $\Delta\rho$ remain (as $\kappa_1 + \kappa_2 = 1$). Equation (3) shows that it is not possible to distinguish between the clock corrections $\Delta\rho$ and the involved bias terms. There is a one-to-one correlation between clock and bias terms.

Let us explain the relationship between differential bias parameters and our OSB parameters for an example consisting of one receiver (k) tracking two GPS satellites (i, j) and one Galileo satellite (e). For a receiver tracking C1W/C2W for satellite i, C1C/C2W for satellite j, and C1C/C5Q for satellite e, this would lead to the following observation equations focusing just on the receiver bias contributions:

$$\begin{aligned}
P^i_{IFk} &= c(\Delta\rho_k + \underbrace{\kappa_1 B_{1k,G/C1W} + \kappa_2 B_{2k,G/C2W}}_{\mathrm{ISB}_{k,G,C1W/C2W}}) + R^i_k \\
P^j_{IFk} &= c(\Delta\rho_k + \underbrace{\kappa_1 B_{1k,G/C1C} + \kappa_2 B_{2k,G/C2W}}_{\mathrm{DSB}_{k,G,C1C/C1W}+\mathrm{ISB}_{k,G,C1W/C2W}}) + R^j_k \\
P^e_{IFk} &= c(\Delta\rho_k + \underbrace{\epsilon_1 B_{1k,E/C1C} + \epsilon_2 B_{2k,E/C5Q}}_{\mathrm{ISB}_{k,E,C1C/C5Q}}) + R^e_k
\end{aligned} \tag{4}$$

where R represents all non-receiver-dependent clock and bias parameters ($R^i_k = \varrho^i_k + T^i_k - c(\Delta\rho^i + \kappa_1 B^i_1 + \kappa_2 B^i_2)$) of Eq. (3). ϵ_1 and ϵ_2 represent the pre-factors for the first and second frequencies of the Galileo IF.

Let us discuss first the differential case. There are three receiver bias parameters involved: $\mathrm{ISB}_{k,G,C1W/C2W}$, $\mathrm{DSB}_{k,G,C1C/C1W}$, and $\mathrm{ISB}_{k,E,C1C/C5Q}$. The receiver clock correction $\Delta\rho_k$ is assumed to be common to all observations of a particular epoch (and all GNSS). Due to the one-to-one correlation between the clock and the ISB parameters, the condition $\mathrm{ISB}_{k,G,C1W/C2W} = 0$ is commonly imposed to fulfill the IGS clock convention (IGSMAIL-2734). This implies that the receiver clock correction is consistent to GPS C1W/C2W observation data. In this way, we end up with $3 - 1 = 2$ free DSB/ISB parameters.

In the OSB case, five OSB parameters are considered: $B_{1k,G/C1W}$, $B_{2k,G/C2W}$, $B_{1k,G/C1C}$, $B_{1k,E/C1C}$, and $B_{2k,E/C5Q}$). Three conditions are necessary for regularization of the bias parametrization. In order to be consistent with the differential case discussed above, the following conditions need to be applied:

$$\begin{aligned}
&\kappa_1 B_{1k,G/C1W} + \kappa_2 B_{2k,G/C2W} = 0 \\
&B_{1k,G/C1W} - B_{2k,G/C2W} = 0 \\
&B_{1k,E/C1C} - B_{2k,E/C5Q} = 0
\end{aligned} \tag{5}$$

The second and third equations are based on the GF and define the relation between the ISBs and the OSBs. As a result of the first two conditions, $B_{1k,G/C1W}$ and $B_{2k,G/C2W}$ become zero (in principal, these two condition could be substituted by: $B_{1k,G/C1W} = 0$ and $B_{2k,G/C2W} = 0$). Finally, because $\epsilon_1 + \epsilon_2 = 1$, this is equivalent to $\mathrm{ISB}_{k,E,C1C/C5Q} = B_{1,k,E/C1C} = B_{1,k,E/C5Q}$. With five OSB parameters and three conditions, we obtain again to $5 - 3 = 2$ free bias parameters.

The interested reader may recognize that the second and third equations reflect the, at this stage, missing GF bias contributions which will be discussed in Sect. 2.2 and finally included in our combined bias product (Sect 2.3).

The same principle may also be applied to satellite clocks and related bias parameters. In this context, we should mention that the satellite and receiver bias parameters B_i and B^k are correlated. Therefore, additional constraints are needed to account for these additional rank deficiencies. We impose a zero-mean condition with respect to each observable type and GNSS to account for that establishing an adequate bias datum definition.

2.2 Geometry-free linear combination for ionosphere analysis

For the estimation of the ionosphere, the geometry-free (GF) is used:

$$P^i_{\mathrm{GF}k} = P^i_{1k} - P^i_{2k} \tag{6}$$

with $\kappa_1 = 1$ and $\kappa_2 = -1$. Inserting Eq. (1) into (6) leads to

$$P^i_{\mathrm{GF}k} = \left(1 - \frac{f_1^2}{f_2^2}\right) I^i_k + c\,\underbrace{(B_{1k} - B_{2k})}_{\mathrm{DSB}_k} - c\,\underbrace{(B^i_1 - B^i_2)}_{\mathrm{DSB}^i}. \tag{7}$$

With the creation of the GF, the geometry part cancels out (receiver and satellite clocks $\Delta\rho$, troposphere delay T, and geometrical distance ϱ) leaving only the ionosphere and bias terms left in the observation equations. As the GF is constructed by differentiating two observations of different frequencies, only differential code biases can be estimated. In the DSB case, the corresponding code biases are introduced and estimated. When using observable-specific parameterization, the over parameterization needs to be eliminated introducing additional relative constrains. These constrains are identical for the receiver and satellite biases.

$$\frac{f_1^2}{(f_1^2 - f_2^2)} B_1 - \frac{f_2^2}{(f_1^2 - f_2^2)} B_2 = 0 \tag{8}$$

with f_1 and f_2 as the frequency of the first and second signals leading to compatible results between the OSB and DSB parametrization.

If we use the same receiver–satellite scenario as in Sect. 2.1, the corresponding equations using the GF can be written:

$$\begin{aligned}
P^i_{\mathrm{GF}k} &= c\,\underbrace{(B_{1k,G/C1W} - B_{2k,G/C2W})}_{\mathrm{DSB}_{k,G,C1W/C2W}} + S^i_k \\
P^j_{\mathrm{GF}k} &= c\,\underbrace{(B_{1k,G/C1C} - B_{2k,G/C2W})}_{\mathrm{DSB}_{k,G,C1C/C1W}+\mathrm{DSB}_{k,G,C1W/C2W}} + S^j_k \\
P^e_{\mathrm{GF}k} &= c\,\underbrace{(B_{1k,E/C1C} - B_{2k,E/C5Q})}_{\mathrm{DSB}_{k,E,C1C/C5Q}} + S^e_k
\end{aligned} \tag{9}$$

with S representing all non-receiver bias-related parameters (ionosphere and satellite bias parameters).

In the differential case, we have a total of three DSB parameters, $\mathrm{DSB}_{k,G,C1W/C2W}$, $\mathrm{DSB}_{k,G,C1C/C1W}$, and $\mathrm{DSB}_{k,E,C1C/C5Q}$, and three observations. So, we end up with a $3 - 3 = 0$ degree of freedom for the DSB parameters.

In the OSB case, we have (instead of three) five bias parameters: $B_{1k,G/C1W}$, $B_{2k,G/C2W}$, $B_{1k,G/C1C}$, $B_{1k,E/C1C}$, and $B_{2k,E/C5Q}$). With a degree of freedom of $3 - 5 = -2$, the system requires two additional constraints for regularization. In order to be consistent with the IF parametrization, a set of constraints is imposed in analogy to Eq. (8) to $B_{1k,G/C1W}$, $B_{2k,G/C2W}$ and $B_{1k,E/C1C}$, $B_{2k,E/C5Q}$.

Finally, a condition has to be introduced for each GNSS (in both cases) in order to define the bias datum. Typically, this is achieved by a zero-mean condition over all satellite bias parameters (referring to a GNSS).

2.3 Combination of clock and ionosphere analysis code biases

One of the advantages of the proposed OSB approach is the potential to create NEQs without any fixed bias datum definitions. This enables to combine the corresponding NEQs easily, even if they originate from different processing lines (e.g., clock or ionosphere analysis) and contain different subsets of observation types and signal types. This implies that combination of NEQs from different days, but also the combination of IF and GF NEQs is possible. In particular the latter scenario, where bias NEQs from ionosphere-free and geometry-free analyses are combined, is of interest. The combination of IF and GF bias results allows to extract pseudo-absolute OSB values that are consistent to both clock and ionosphere analysis.

Let us come back to the observation example as introduced in Eqs. (4) and (9). The combined observation equations lead to an equation system consisting of $2 \cdot 3 = 6$ observations, 1 receiver clock parameter, and 5 receiver OSBs. The two GF conditions given in Eq. (5) for the IF bias contribution are obsolete due to the GF bias contribution. The two IF conditions that were necessary in the GF-only case are substituted by the IF contribution. There is just one condition that remains to be considered: the first condition listed in Eq. (5). This particular condition actually represents the clock convention that should be used. According to the IGS clock convention, GPS satellite clock values have to be consistent to (the IF of) C1W and C2W. Note that a list of all necessary constraints for satellite and receiver biases is provided in Table 2.

Another advantage of the OSB approach is the flexibility to change the clock convention at NEQ level. By redefining the condition defining the clock convention, it is possible to change the bias datum in such a way that satellite clock parameters become consistent to, e.g., GPS C1C/C2L.

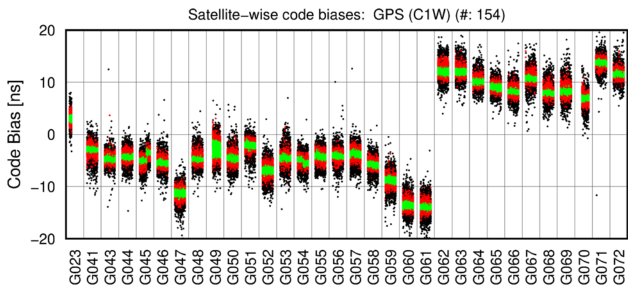
2.4 GNSS using FDMA

GLONASS has, compared to the other analyzed GNSS, a different modulation technique using FDMA (instead of CDMA). FDMA uses, in contrast to CDMA, different frequencies for each satellite. As a consequence, the assumption that all GNSS and observables have a common bias on the receiver side is no longer valid. Nevertheless, GLONASS observation biases tend to reveal a certain satellite-specific component as shown in Fig. 2.

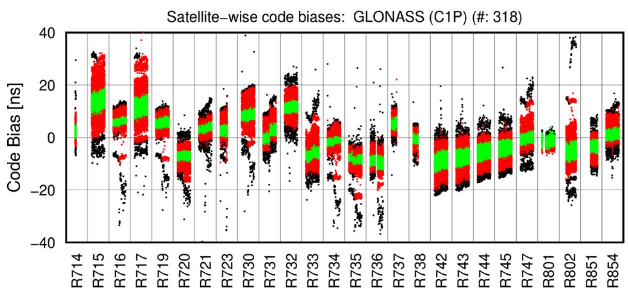
Figure 2 shows the result of a dedicated study on the impact of the FDMA compared to the CDMA when using receiver-satellite bias parametrization. Therefore, for each receiver and satellite pair an individual bias parameter is estimated. This is typically used for GLONASS (FDMA). It is worth mentioning that we use to perform such a receiver-satellite bias monitoring not only for GLONASS, but also for GPS on a regular basis at CODE. We took a data set of the first 120 days of the year 2016 and subtracted a mean value for each day, receiver, and GNSS. This mean value can be interpreted as the receiver bias (one per observable type and GNSS).

In the ideal case, the satellite biases should be identical for all stations. For the GPS satellites (Fig. 2, top), 68% (1 σ) of the C1W biases are within a range of ± 0.5 ns (G048) and ± 1.6 ns (G049) around the mean value. Note that G049 is set unhealthy and only poorly tracked by the IGS network.

For GLONASS, due to the FDMA modulation, the separation into receiver and satellite bias components is not feasible, because receiver biases strongly depend on the frequencies.



(a) GPS (C1W)



(b) GLONASS (C1C)

Fig. 2 Satellite-wise code biases extracted from receiver-satellite bias retrieval, accumulated for the first 120 days of the year 2016. The scale ranges from − 20 to + 20 ns for GPS and − 40 to + 40 ns for GLONASS. Note that green dots represent biases between the 0.16 and 0.84 quantile (1 σ) and red dots between 0.025 and 0.975 quantile (2 σ)

68% (1 σ) of the C1P satellite biases are within a band of ± 1.0 ns (R736) to ± 2.9 ns (R717) around the mean value. This is roughly by a factor of two larger compared to GPS. The satellite R801 (slot number 26) has been ignored as it was only observed by few stations. The stations have an average scattering over the 120 days between 0.5 ns and 1.6 ns for GPS. The scattering is higher and ranges between 1.0 ns and 2.9 ns for GLONASS, with one exception (8.7 ns). The 1-sigma approximation shows that 68% of the stations are within a 1 ns (or 2 ns for GLONASS) band. However, the 95% band is by a factor of 3 wider (in the case of normal distribution, it would be 2, indicated by red dots in Fig. 2).

2.5 Consistency between clock, ionosphere, and combined solution

The approach presented in Sect. 2.3 shows how a combined IF/GF OSB set can be determined. The presented constraints for the combined OSB determination ensure the compatibility with the DSB approach used by the IGS community. A numerical example is given in order to show the consistency between the three OSB solutions, clock (CLK), ionosphere (ION), and combination (CMB) in Table 1. For the CLK solution, C1W and C2W are constrained to zero. The ISB are zero for all solutions, which is in accordance to the IGS C1W-C2W clock convention. The DSB values agree at a level of 0.01 ns between the ION and CMB solution.

Table 1 Numerical example of satellite OSB values (for satellites G01, G02, and G03) on January 1, 2016, from the clock, ionosphere analysis, and their combination

	*G*01 (*ns*)	G02 (ns)	G03 (ns)
Clock analysis			
C1C	– 1.379	1.253	– 1.466
C1W	0.000	0.000	0.000
C2W	0.000	0.000	0.000
C1W-C1C	1.379	– 1.253	1.466
Ionosphere analysis			
C1C	10.477	– 12.432	– 6.721
C1W	11.854	–13.766	– 8.355
C2W	19.522	– 22.671	– 13.761
C1W-C1C	1.377	– 1.334	1.634
C1W-C2W	–7.669	8.906	– 5.405
ISB	0.000	0.000	0.000
Combination			
C1C	10.455	– 12.468	– 6.812
C1W	11.841	– 13.783	– 8.402
C2W	19.502	– 22.700	13.838
C1W-C1C	1.386	– 1.316	1.590
C1W-C2W	– 7.661	8.917	– 5.436
ISB	0.000	0.000	0.000

In addition, the DSB between C1W and C1C, C1W and C2W, and the corresponding ISB are derived from the OSB estimates

The above example shows that the ISB/DSB extracted from the OSB value of the IF, GF, or the IF/GF-combined solutions are consistent with respect to each other.

2.6 Summary

An overview of the OSB parametrization for the various modes (CLK, ION, CMB), for three systems (G, E, R), is given in Table 2. For GPS, more than two observation types are included (C1C, C1W, C2W, C2L). In addition, the situation is given when using a differential approach. The needed regularization conditions are indicated using different symbols.

The usage of an observable-specific bias (OSB) parameterization allows for a flexible handling of the biases in the processing scheme. Moreover, the datum definition can be applied after the generation of the NEQs. This allows to analyze the content of the NEQs to easily determine an adequate datum definition.

3 Multi-GNSS pseudo-absolute code biases

With the implementation of the OSB approach in the Bernese GNSS Software (Dach et al. 2015), the estimation of com-

Table 2 GNSS receiver code bias parameterization using a differential or an equivalent pseudo-absolute parameterization

Sys	Obs. type	Differential		Absolute		
		CLK	ION	CLK	ION	CMB
G	C1C			OSB	OSB	OSB
		DSB	DSB			
G	C1W			0	OSB	OSB
		0	DSB		∩	∩
G	C2W			0	OSB	OSB
		DSB	DSB			
G	C2L			OSB	OSB	OSB
E	C1C			OSB	OSB	OSB
		ISB	DSB	=	∩	
E	C5Q			OSB	OSB	OSB
R01	C1P			OSB	OSB	OSB
		ISB	DSB	=	∩	
R01	C2P			OSB	OSB	OSB

DSB differential signal biases, *ISB* ionosphere-free linear combination code bias, *OSB* observable-specific code bias, ”=”: parameter constrained to be equal; ”0”: parameter constrained to zero value; ”∩” indicates a relative IF constraint between the upper and lower observable type

Table 3 RINEX 3 observable types used for analyzes (and C2 from RINEX 2 as it could not be mapped unambiguously to RINEX 3)

System	RINEX 3 observable type
GPS	`C1C C1W C2L C2S C2 C2W C5Q C5X`
GLONASS	`C1C C1P C2C C2P`
Galileo	`C1C C1X C5Q C5X C7Q C8Q`
Beidou	`C2I C6I C7I`

plete sets of code biases for all observed GNSS became feasible. To study multi-GNSS OSB results, a dedicated reprocessing effort has been carried out for the year 2016. For the bias reprocessing effort about 260 IGS and MGEX stations have been used. RINEX 2 files have only been used if no RINEX 3 files were available (Gurtner and Lou 2012). RINEX 2 observation codes have been mapped to match the RINEX 3 standards. The mapping between the old and the new RINEX version is: P1 → C1W, P2 → C2W or C2D (depending on the receiver type), C1 → C1C, and C2 → C2. Note that the C2 could not be mapped to the RINEX 3 standards as it could be either C2L or C2S. Because of this, it is labeled as C2. The processed code observables are listed in Table 3.

The determination of the code biases is done by combining the NEQs from GF and IF analysis to obtain a complete set. The detailed work flow is shown in Fig. 3.

GF OSB NEQ results were produced using station-specific ionosphere modeling. Additionally, in order to cover all available observable types, the GF analysis was repeated for different observable type selections. The ionosphere modeling results from the initial GF analyzes were used for the subsequent GF analyzes steps for consistency reasons.

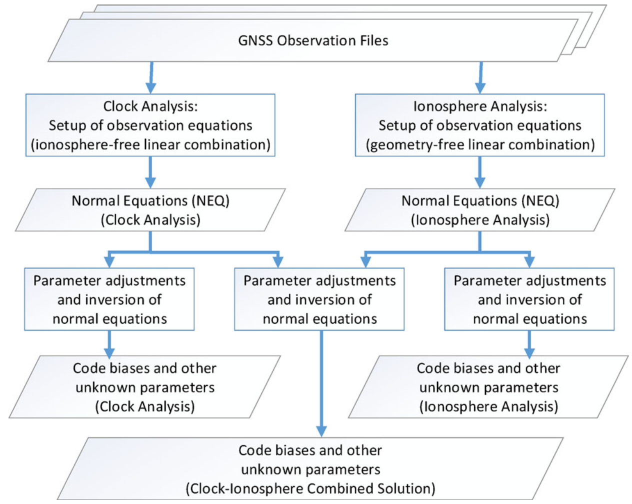


Fig. 3 Processing scheme for a combined OSB solution from clock and ionosphere procedures

The clock analysis (IF) was done for one frequency pair only. The combination of GF and IF NEQ results finally includes OSBs for all available observable types (as the GF results already covered all of them). In this way, a complete set of code biases for all processed GNSS (GPS, GLONASS, Galileo, and Beidou) is obtained.

In order to obtain bias results compatible to the IGS conventions, a constraining scheme according to Table 2 was applied to the OSB parameters. The following conditions are needed for datum definitions:

For ionosphere analysis:

– zero-mean conditions for each satellite OSB type and GNSS.

For clock and combined analysis:

– zero-mean conditions per satellite OSB type and GNSS;
– zero-mean condition with respect to the sum of all IF receiver OSB parameters combinations defining the reference clock;
– in case of receiver-satellite OSBs: zero-mean condition for each satellite and analyzed IF combination of the OSBs (only in case of GLONASS).

3.1 Multi-GNSS code bias results

The alignment of the originally produced OSB time series was done independent for each daily solution (by zero-mean

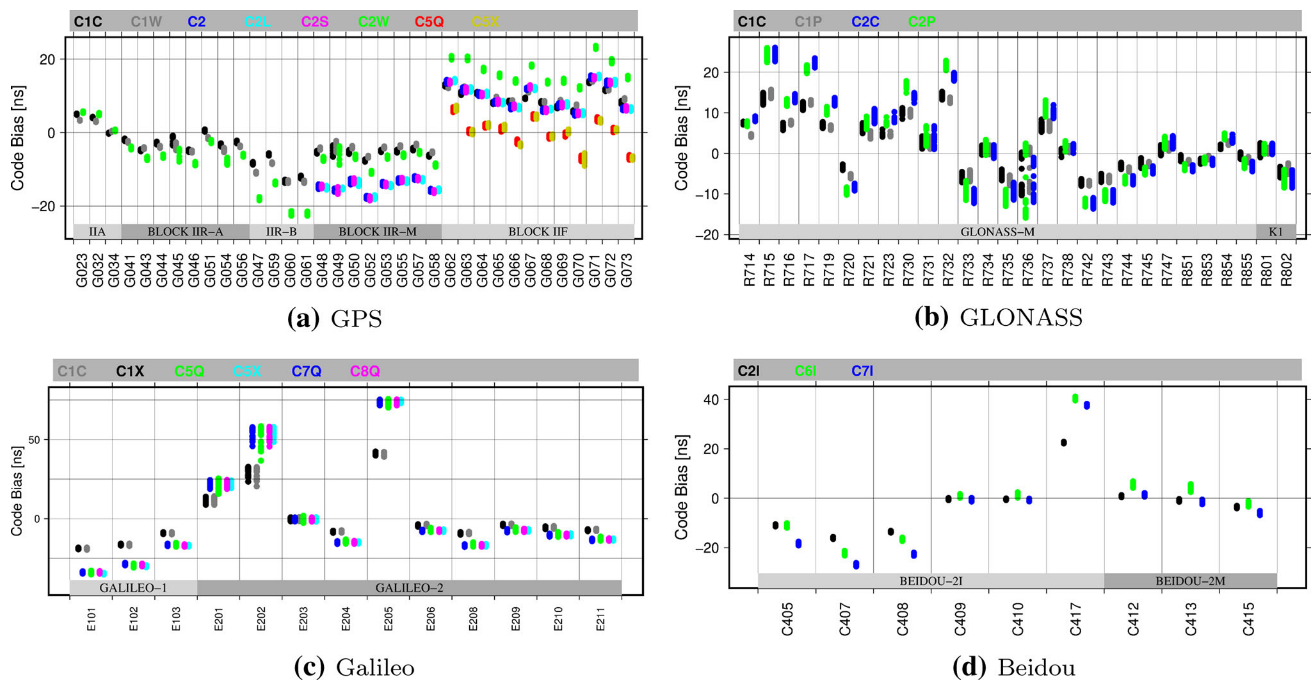


Fig. 4 Daily re-aligned OSB values from ionosphere and clock analysis combination in 2016. The OSBs have been re-aligned to the mean value over 2016

conditions). In order to achieve a common alignment for the entire time series, the daily solutions need to be re-aligned accordingly:

1. Combination of all GF/IF combined daily NEQs to generate a mean code bias values for the entire combination period (year 2016).
2. Re-alignment of all GF/IF combined daily NEQs with respect to the mean values obtained in step 1.

These daily OSB values, in the following addressed as re-aligned, are shown in Fig. 4. They allow to retrieve their daily scattering and stability over the observed time period.

The OSB values for GPS satellites in Fig. 4a reveal a scattering between 0.2 and 3 ns. They depend on the satellite generation (Block). In this study, group delay variations (GDV) are not considered (Wanninger et al. 2017). This could be a reason why individual satellites have a higher scattering than others. Additional signals C2L, C2S became available with Block IIR-M. These biases agree well with each other. Block IIF satellites introduced a third frequency (C5Q, C5X), which is tracked by many MGEX stations. Their OSB estimation shows that both signals have a similar bias pattern. Processing GF combinations also for L1 and L5 allowed to extract consistent biases for the third frequency.

The results for GLONASS are presented in Fig. 4b. The daily scattering of the re-aligned OSBs is between 1 and 5 ns (except for R736 which reveals a much higher variation). The more pronounced scattering compared to GPS can be explained by the frequency-dependent modulation of the GLONASS satellites. The GLONASS biases are just separated into receiver and satellite components (as for GPS, see Sect. 2.4). Due to this simplified parametrization, a higher scattering may be expected. Also the OSBs vary among the individual GLONASS-M satellites, and groups of satellites may be found revealing similar values.

GNSS receivers tracking Galileo may be divided into two groups with different tracking technologies. The first group of receivers, namely Javad and Trimble, tracks the X signal (e.g., C1X/C5X) whereas, the second group tracks the C and Q signal (C1C/C5Q).

In case of separate IF or GF OSB determination, the problem of having two subsets with no common observable types may appear. This can be resolved by introducing an additional IF constraint (see Eq. 8) to the C1X/C5X OSBs. In case of a IF/GF-combined solution, this problem is no longer an issue (see Sect. 2.3).

The OSB values for the combined GF and IF solution are presented in Fig. 4c. The scattering for Galileo is between 1 and 5 ns (except for E201 and E202) and is almost comparable to the much better observed GPS satellites. Satellite E201 and E202 are in an eccentric orbit and show a significant higher scattering than the rest. The biases for E205 are quite different to those of the other GALILEO-2 satellites.

OSBs for Beidou satellites in Fig. 4d may be divided into the MEO (Beidou-2M) and the IGSO (BEIDOU-2I) satellites. Their scattering is, with 1–3 ns, at a similar range as GPS.

3.2 GNSS satellite bias stability for the year 2016

The datum definition has to be consistent over the complete time interval. This is essential to analyze their evolution in time. The OSB values are referred to the space vehicle number (SVN) numbers to ensure that they are combined even if a satellite has been reassigned to a different pseudorandom noise sequence (PRN, Gold codes) number.

Figure 5a presents the stability of the GPS OSBs for the year 2016. Significant discontinuities could be detected for satellite G045 (early 2016). Figure 5a presents the stability of the GPS OSBs for the year 2016. Significant discontinuities could be detected for satellite G045 (early 2016).

For GLONASS, the situation in Fig. 5c is similar to that of GPS. However, OSBs for individual satellites show a more pronounced scattering (also due to the simplification mentioned in Sect. 2.4). The following significant discontinuities for GLONASS satellites could be detected: R721 (April 1), R731 (February 27, and May 30).

Two Galileo satellites E201 and E202 have been excluded from the datum definition due to higher scattering. Including these poorly behaving satellites would degrade the re-aligned bias results. Disregarding these two satellites from the datum definition leads to the OSB results presented in Fig. 5d. It is

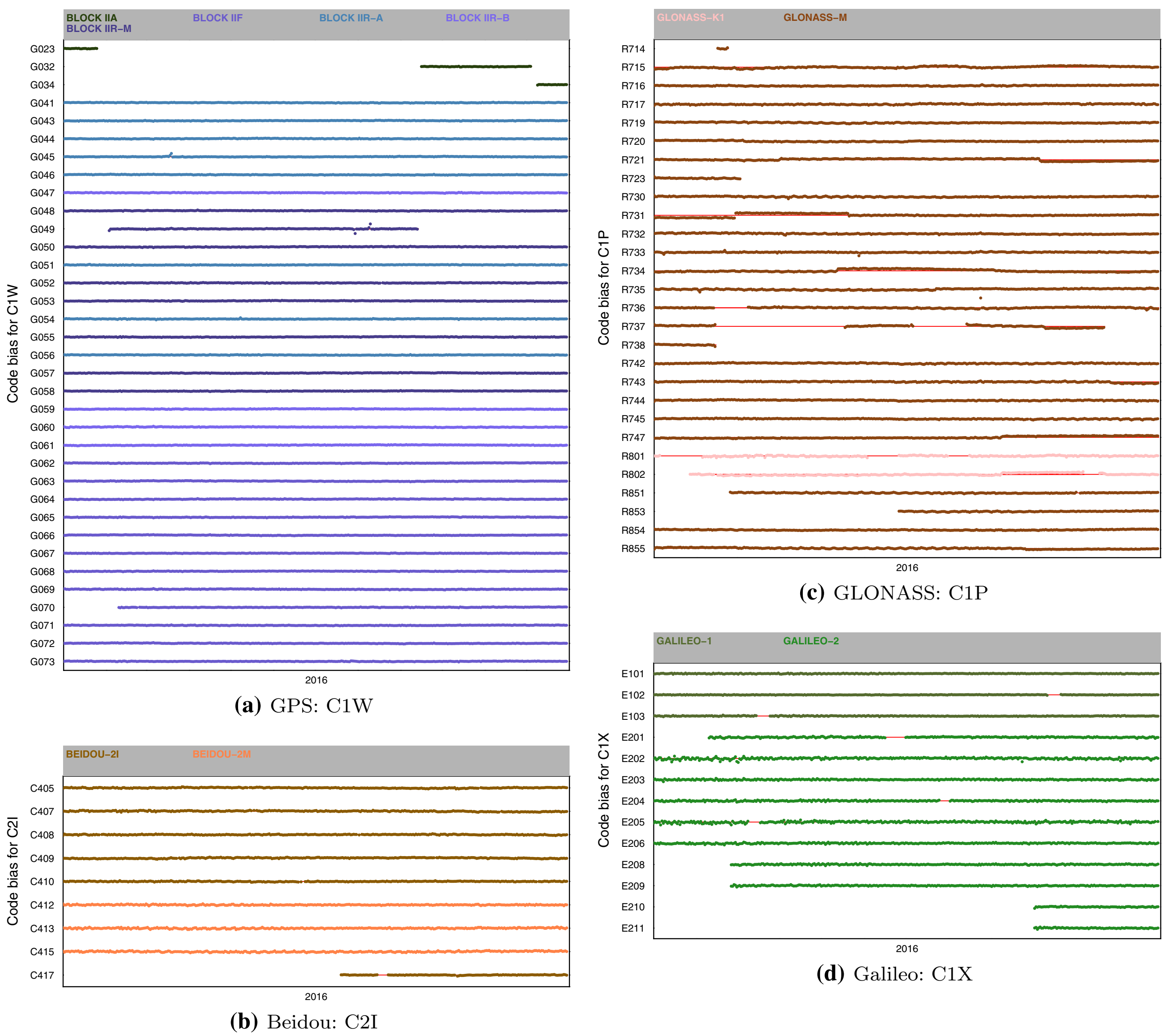


Fig. 5 The daily values are aligned to the combined solution for 2016. The yearly mean is subtracted showing only the difference to the average. The individual satellites are shifted by 5 ns

obvious that the selection of well-behaving satellites with stable code biases is crucial for the quality of a coherent long-term bias product.

For Beidou, shown in Fig. 5b, the OSBs behave similar to GPS. The time series of individual Beidou satellites tend to show small variations.

The OSB characteristics for all four constellations are quite similar. The re-aligned OSB values show a day-to-day repeatability at a level of 0.15 ns in best cases for GPS satellites. This is true not only for the presented OSB types but also valid for all other OSB types.

3.3 Comparison with DSB estimations from DLR and from JPL

The Jet Propulsion Laboratory (JPL) provides with their ionosphere solutions GPS "C1W-C2W" DSBs in their IONEX submission to the IGS. In addition, the German Aerospace Center (DLR) has been estimating complete sets of DSB for all GNSS in the framework of the MGEX project (Montenbruck et al. 2014; MGEX 2017). As both groups are estimating differential biases, our OSB values were transformed into DSBs for the purpose of comparison. In order to have the same datum definition for DSB sets, a daily shift was taken into account.

The mean differences, for GPS C1W-C2W, of satellites which have been observed during the entire year, are listed in Table 4.

The comparison for a selected subset of DSBs types over the year 2016 is presented in Table 5. The comparison reveals that there are systematic differences between our OSB solution and the DLR DSB estimations of up to 1.2 ns. Note that for GPS C1W-C2W DSB our solution is closer to JPL than DLR. It has to be mentioned that our (and JPL) bias estimation comes from a consistent ionosphere and bias estimation, whereas the DLR solutions are derived using global ionosphere maps (GIM) from an external source. Our OSB results show an excellent agreement with the solution from JPL. Due to the different processing method, the values from DLR show a minor deviation with respect to those from CODE and JPL.

Table 4 Average (median) offset between C1W-C2W code biases between CODE, JPL, and DLR for 2016

Difference	Min (ns)	SVN	Max (ns)	SVN	RMS (ns)
CODE-JPL	− 0.24	G032	0.12	G070	0.11
CODE-DLR	− 0.33	G047	0.47	G062	0.22
DLR-JPL	− 0.43	G062	0.38	G047	0.23

Table 5 Average (median) offset between DSBs from CODE and DLR for 2016

GNSS	DSB	Min (ns)	SVN	Max (ns)	SVN	RMS (ns)
G	C1C-C1W	− 0.50	G057	0.41	G047	0.22
G	C1C-C5X	− 0.15	G067	0.15	G073	0.20
R	C1P-C2P	− 1.20	R736	0.85	R855	0.55
R	C1C-C1P	− 0.23	R735	0.62	R717	0.88
E	C1C-C5Q	− 0.15	E103	0.15	E201	0.24
E	C1X-C5X	− 0.17	E101	0.16	E202	0.18
C	C2I-C7I	− 0.19	C409	0.16	C410	0.18
C	C7I-C6I	− 0.38	C410	0.33	C412	0.30

Both solutions are aligned on a daily basis. Outliers bigger than 1 ns are removed (up to 4%) for the RMS computation of Galileo and Beidou (in case of C7I-C6I 18%)

4 Long-time OSB combination

4.1 Estimation of a consistent code bias product

At CODE an effort was made to reprocess a selection of more than 300 IGS stations dedicated to the estimation of OSBs and ionosphere parameters, specifically from 2000 until the end of 2017. The outcome of the reprocessing was daily NEQs including GPS and GLONASS satellite bias parameters. In order to align all OSBs over the whole time period the NEQs were combined, pre-eliminating all receiver biases on a daily basis, thus finally leading to a long-term bias product. The time series were analyzed and discontinuities taken into account. Many discontinuities could be related to specific GPS Notice Advisory to Navstar Users (NANU) messages. After the estimation of the long-term bias product, the daily OSBs have been re-aligned to the previously generated long-term OSB set, thus leading to time series of daily OSBs with one common datum over the entire time span of 18 years. Figure 6 shows the original (aligned daily using zero-mean conditions) and re-aligned GPS C1W OSB time series. Both time series are plotted with respect to our long-term combination C1W OSB set. The need of a datum definition that is common over the entire period is clearly visible. The variations (induced by changes in the GPS constellation) as seen for the "non-aligned" time series are eliminated in the re-aligned time series. The used datum definition reveals already a remarkable stability over time. Nevertheless, the datum definition could be further fine-tuned by analyzing the OSB characteristics of individual satellites (i.e., by using just the most stable satellites for the datum definition) or consider stable receiver OSBs for datum definition. With absolute bias calibrations for particular receivers, one could think about an absolute alignment.

Our re-aligned OSB time series now allows to analyze the time evolution of satellite (and receiver) OSBs. In this

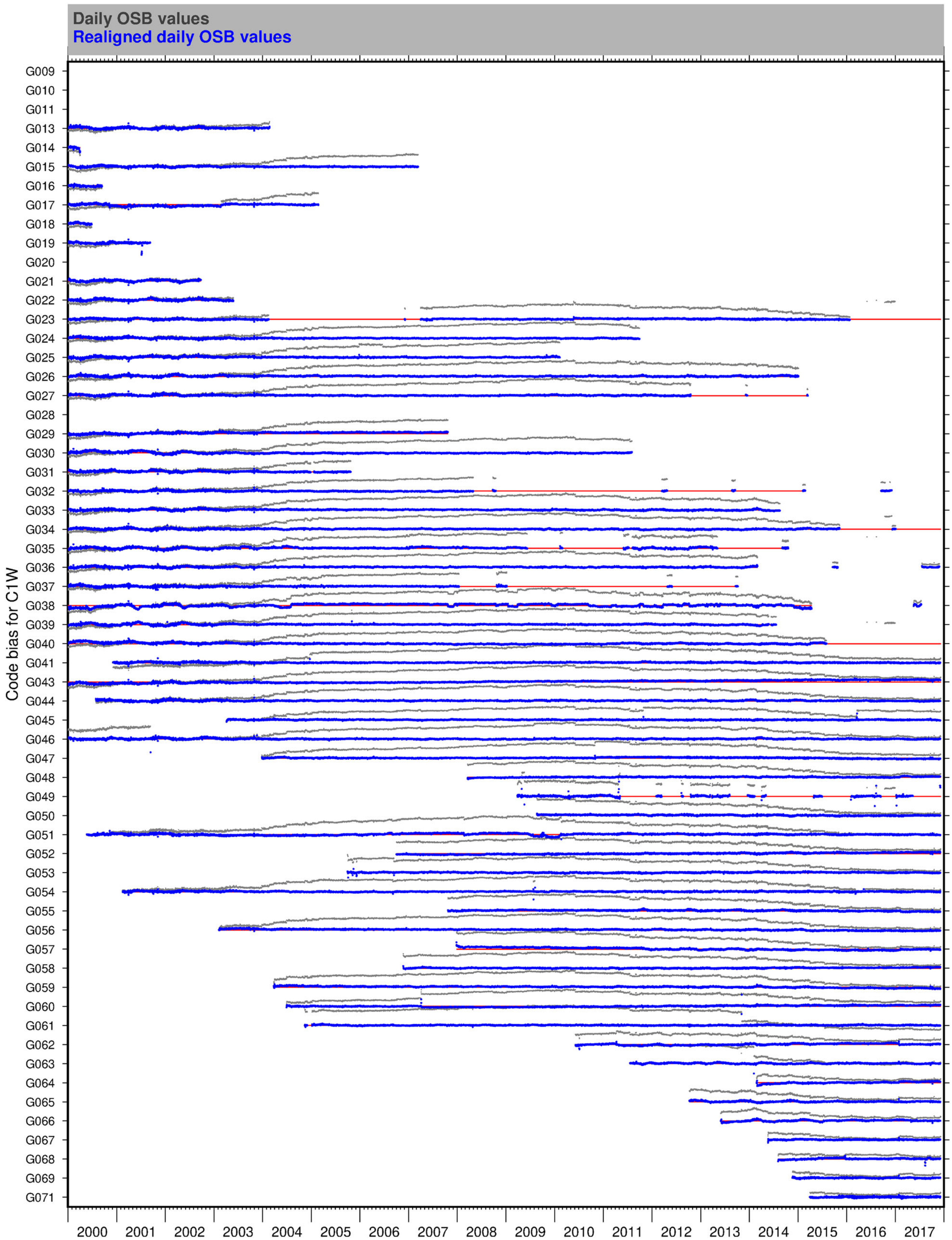


Fig. 6 Daily GPS OSB estimates for C1W. The original daily estimates are shown in light grey. The re-aligned values in blue. All estimates are plotted with respect with the obtained multi-year OSB combination. Note that a scale of ±2.5 ns around the mean bias levels was chosen in order to visualize the impact of the alignment

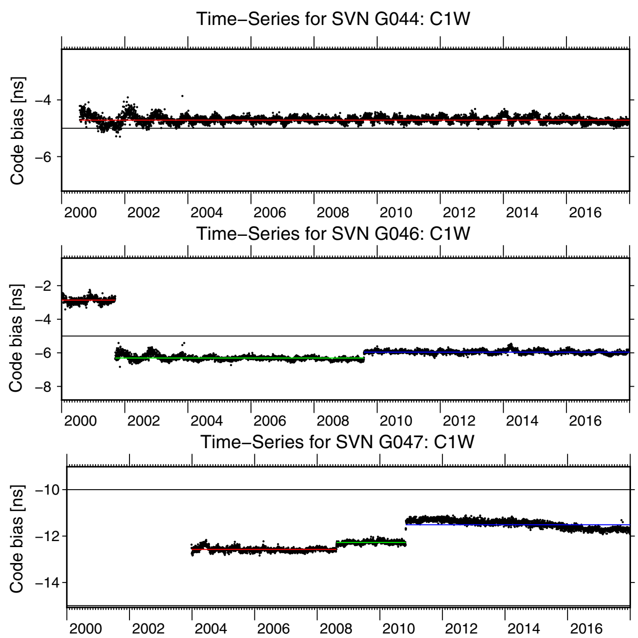


Fig. 7 Aligned C1W OSB time series for GPS G044, G046, and G047

context, it is worth mentioning that for satellites that have been reactivated the OSB values are in most cases consistent with the previous values and, therefore, typically remain unchanged. That is why the OSBs are assigned to individual satellites (SVN) and not PRN numbers.

Figure 7 shows the re-aligned C1W time series for three representative satellites: G044, G046, and G047. The discontinuities in the time series now become visible as the mean value has not been subtracted. The time series for satellite G044 shows a bias value that is remarkably stable over the very long lifetime. Satellite G046 shows two discontinuities: the first one was around the 13 September 2001 and may be attributed to NANU message 2001120: UNUSABLE; the second discontinuity occurred around 1 August 2009. Discontinuities for satellite G047 can be connected to NANU messages (4 August 2008: NANU 2008082: MAINTENANCE and 31 October 2010: NANU 2010134: MAINTENANCE 304-305). Some of the satellites show variations and drifts (e.g., G047, the third example in Fig. 7).

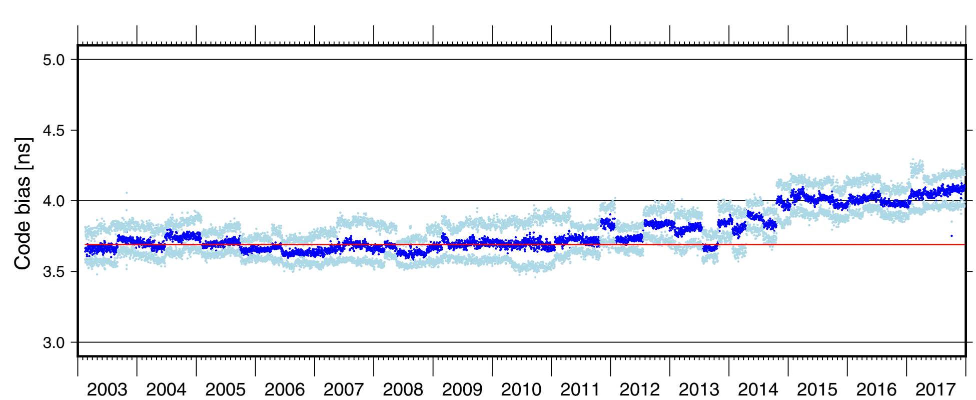
4.2 Comparison with GPS Broadcast Group Delay (GD) Values

In order to assess the stability of the estimated time series, a comparison with the broadcast GPS group delays (GD) was made. GPS broadcast GD values are quarterly updated on the basis of GPS C1W-C2W bias results provided by the JPL ionosphere analysis group (Wilson et al. 1999). They are declared to keep an absolute bias alignment with respect to absolute receiver calibrations from the past. Please note that broadcast GD values are of discrete nature ($2^{-31}\text{s} = 0.47\text{ns}$) due to the limited number of bits in the GPS navigation message (GPS interface specification IS-GPS-200J (2018)). Figure 8 shows daily differences between our re-aligned daily GPS C1W-C2W OSB estimation and the GPS reference values (generated by JPL) converted according to

$$\mathrm{DSB}_{C1W-C2W} = -\frac{f_1^2 - f_2^2}{f_2^2} \cdot \tau_{\mathrm{GD}} \tag{10}$$

with τ_{GD} as the group delay. Both time series, re-aligned OSBs and GPS GD, are expected to refer to a common bias datum over the entire time period shown (approx. 15 years). Our time series does not rely on any absolute calibrations and, therefore, has to be addressed as pseudo-absolute (referring to an arbitrary level). This is indicated by a common shift of about +3.7 ns (JPL minus CODE). The differences are stable over the first decade at a level of ± 0.1 ns (up to approx. 2012). After 2012, the differences seem to drift apart from each other. Our re-aligned daily OSBs refer to one common bias datum as given by our coherent multi-year bias combination. The stability of our re-aligned OSB time series is at the level of 0.15 ns on a long-term scale for most of the GPS satellites. Examples for three satellites, shown in Fig. 7, do not show any obvious drift around 2012. Possible explanations for the deviation could be missing absolute calibrations in the recent years or a change in the alignment scheme used to transfer

Fig. 8 Mean difference (median) between the C1W-C2W DSB computed from our re-aligned OSB estimation and their counterpart derived from the broadcast group delays (GD) originally provided by JPL (blue). 0.25 and 0.75 quantiles are indicated with light blue. The red line indicates the mean difference, before 2012, between both solutions (3.7 ns)



the alignment for JPL. Therefore, we are not aware of any reason causing such a noticeable drift.

5 GNSS receiver compatibility grouping

5.1 Method

GNSS code biases may differ between the various observable types. The old RINEX 2 format only allowed to distinguish between observations labeled with C1, P1, C2, and P2. It was therefore unknown which signal was actually tracked when P2 was indicated (P2 or X2 signal). In order to retrieve the actual observation type, CODE introduced the so-called code bias multiplier approach to determine observation types based on known satellite code bias values (Dach et al. 2015). Nowadays, more and more receivers are storing their measurements using the newer RINEX 3 standards that allows to indicate the actually sampled signals. The RINEX 3 format description lists all possible observation types. After extending the DSB multiplier approach to OSB, we are able to test the compatibility of the indicated observation types with respect to a given set of satellite OSB values.

The OSB multipliers indicate high correlation of the individual satellite bias pattern in the pseudo-range observations for a receiver to be tested. The multipliers are computed with respect to a given set of satellite patterns for all available observation types. Therefore, the satellite OSBs have to be known in advance. This multiplier can be estimated either using the IF or the GF linear combination. For this study, the IF has been used as it allows to distinguish between more observation types, in particular C2D for GPS.

The observation equations for the IF case are analogue to Eq. (3). Instead of introducing the satellite code biases as unknown parameters, they are introduced with an unknown multiplier ($m_{G,C1W}$, for each observation type) leading to:

$$
\begin{aligned}
P^i_{IFk} = {} & \varrho^i_k + T^i_k + c(\Delta\rho_k + (\kappa_1 B_{1k} + \kappa_2 B_{2k})) \\
& - c(\Delta\rho^i + (m_{G,C1C} B^i_{G,C1C} + m_{G,C1W} B^i_{G,C1W} \\
& + m_{G,C2} B^i_{G,C2} + \cdots))
\end{aligned} \quad (11)
$$

$$
\begin{aligned}
P^e_{IFk} = {} & \varrho^e_k + T^e_k + c(\Delta\rho_k + (\kappa_1 B_{1k} + \kappa_2 B_{2k})) \\
& - c(\Delta\rho^i + (m_{E,C1C} B^e_{E,C1C} + m_{E,C5Q} B^e_{E,C5Q} \\
& + m_{E,C6Q} B^e_{E,C6Q} + \cdots))
\end{aligned} \quad (12)
$$

This method requires OSB satellite patterns that are distinguishable for the individual observable types. In the ideal case, estimates for these multipliers are obtained that are close to the applied factors of the linear combination ($m_{G,C1C} = \kappa_1$ and $m_{G,C2} = \kappa_2$ for the two used signal types). If a station samples different observable types for two or more satellite groups, deviating multiplier estimates may be expected as only a subset of the satellite pattern (only those from the consistently observed satellites) can be used to estimate the associated multipliers.

In case of IF analysis, these code bias multiplier estimates should be consistent to the following expectation values for GPS: 2.55 for C1W and − 1.55 for C2W.

Table 6 The estimated OSB multipliers for each GNSS

GNSS	OSB multipliers considered for				
GPS	C1C	C1W	C2	C2W	C5Q
Galileo	C1C	C5Q	C5X	C7Q	
Beidou	C2I	C6I	C7I		

Multipliers depend on the differences between the satellite OSBs. Therefore, C2S and C2L for GPS (similar to C2) and C1X (similar to C1C) and C8Q (similar to C7Q) could not been estimated

Table 7 Averaged multipliers for GPS receivers according to their sampled signal types

Obs. type	# rec.	C1C	C1W	C2	C2W	C5Q
C1W/C2W	143	0.03	2.57	− 0.12	− 1.56	0.00
C1C/C2W	141	2.59	− 0.01	− 0.12	− 1.56	−0.00
C1C/C2	2	2.97	− 0.34	− 1.93	0.29	0.01
C1C/C2D	3	0.71	1.71	0.23	− 1.50	0.06

The standard deviations of the multipliers are between 0.1 (many receivers) and 0.5 (few receivers)

5.2 Results

For a reliable estimation of multipliers, satellite OSB patterns that are different between each involved observation types are mandatory. As a consequence of this, patterns referring to observation types that are similar to each other have been assigned to one multiplier. The reduced list of observable types is listed in Table 6. For instance, GPS C2S, C2L, and C2 have very similar OSB values and, therefore, only C2 is introduced. Note that C2 was introduced to represent the C2 observations in RINEX 2 as they could not be mapped unambiguously to RINEX 3 (they may be either C2S or C2L). The following results are based on the first seven days of 2016.

5.2.1 GPS

A summary of the estimated multipliers for GPS is listed in Table 7. The multiplier estimation showed that the indicated observation types were compatible with the estimated satellite OSBs. In case of C1C/C2D (C2D = C1C + (C2W - C1W)) receivers, the expected multiplier values should correspond to: C1C=1.00, C1W = 1.55, and C2W = − 1.55. The estimates are fairly close to the expected values.

As long as the number of satellites for a subgroup is not too small, our receiver grouping assigned the multiplier to the consistent observation types. It is obvious that the situation becomes less comfortable when a subgroup consists of few satellites. On average 2–4 sites of more than 280 per day have not been assigned as expected. Most of these cases were due to data issues (considerably reduced number of observations) or due to a subset consisting of few satellites. The following three sites showed severe problems: UNTR (day 001 of 2016) and HRAO (day 002 of 2016). Moreover, the site HARB sampled a subset with C1C/C2X that resulted in multiplier estimates varying from day to day between C2 and C2W making a clear assignment difficult.

5.2.2 Other GNSS

The number of consistently sampled satellites is the limiting factor to get reliable estimates for our GNSS OSB multipliers. During the test period (day 001 to 007 of 2016), only a limited number of satellites were available for Galileo (8) and Beidou (8). This number did not yet reach a level to apply our GNSS receiver compatibility grouping approach successfully. Meanwhile, both constellations reached a status where reliable multiplier estimates may be expected.

5.3 Summary

The GNSS receiver compatibility grouping based on the presented OSB multiplier approach is an extension of the differential multiplier approach. It depends on given satellite code biases. These satellite code biases are the outcome of the OSB estimation as discussed in the previous sections. We may summarize that the presented approach allows a reliable testing of the consistency of collected code observation data with respect to given satellite code bias values. The number of satellites is crucial for the success rate. Therefore, it works well for GPS. For Galileo and Beidou, the situation is not yet optimal and will be improved once their full constellations are reached. For GLONASS, the situation is different as it uses frequency-dependent signal modulation (FDMA) and the estimation of satellite bias components is possible, but much noisier (compared to the constellations using CDMA). That is the reason why no GLONASS multiplier estimation was performed.

Our OSB multiplier approach can be used to detect anomalies in the classification of the observable types as it could be demonstrated in the past using the DSB multiplier method (IGSMAIL-3887). We could not detect any anomalies in this sense for the limited test data set.

6 Summary and conclusions

The estimation and treatment of code biases is not only essential for clock and ionosphere analysis but also for ambiguity resolution when relying on the pseudo-range method (Melbourne-Wübbena). This is true for the double-difference as well for the undifferenced case (e.g., ambiguity-fixed precise point positioning). The presented OSB parametrization is well suited for this purpose due to the flexible and straightforward way of treating biases. It allows to map biases to each individual observation type involved. Furthermore, they may be applied to any linear combination when OSB results are generated from a combined clock and ionosphere bias analysis. Finally, our approach would even work in case of single frequency data only.

One main strength of our approach is the possibility to combine bias NEQs from different sources (ionosphere-free (IF) or geometry-free (GF)) considering the complete variance–covariance information to produce a common set of biases which can be used for any application. In addition, combining datum-free NEQs over long time periods can be used to define a common datum over the analyzed period. This is crucial for achieving an adequately aligned bias product which is not affected by satellite constellation changes (e.g., appearing or disappearing satellites directly influencing zero-mean conditions).

Based on 17 years of IGS observations (2000–2017), a long-term bias product was generated. With an adequate handling of discontinuities in the OSB time series, a stable long-term bias product was achieved. A comparison of specific code bias differences (GPS C1W-C2W) with GPS broadcast group delays (GD) confirmed an excellent agreement (± 0.1 ns) when allowing for one common shift for our pseudo-absolute time series. This shift (GPS GD minus CODE) is about +3.7 ns and might be used for an absolute alignment of our code bias series. The list of discontinuities is an important by-product of our effort. Many of these discontinuities may be attributed to specific NANU messages. The bias product consistently aligned to a long-term datum, allows to analyze the long-term behavior of GNSS satellites as well as receiver code biases. It is remarkable that OSBs from reactivated satellites often remain unchanged (same satellite/SVN reappearing with reassigned PRN). Receiver code biases are not addressed in this study. An absolute alignment could be achieved by inclusion of receiver data for which absolute receiver bias calibrations (referring to arbitrary epochs) are available. This implies that receiver bias parameters could be included to refine the long-term datum definition. Coherent time series of receiver OSB results are of relevance for precise timing applications where measurements are made at different epochs.

The one-year study on multi-GNSS OSBs confirmed the results as obtained from the 2000–2017 GPS/ GLONASS code bias analyses also for the additional GNSS: Galileo and Beidou. In particular, for the multi-GNSS case the OSB parametrization turned out to be an efficient tool to handle code biases.

The OSB multiplier approach allowed us to test the compatibility between our estimated satellite bias sets and the tracked observations for each receiver. This approach works as long as the majority of the considered receiver label their measurement types correctly. The more satellites are available for a GNSS, the more robust is the observation type assignment. In case of similar code bias patterns (for different sets of observation types), the separation between the observation types within a such a set (of similar pattern) is not possible.

In 2016, CODE switched from a differential to an observable-specific code bias representation. All our analysis procedures, including those used for generating the IGS contributions, use the OSB the parametrization as presented in this paper. Our DSB products, e.g., the monthly mean files, are based on our GF/IF combined OSB analysis performed at NEQ level and subsequently transformed into the DSB representation.

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Author contributions A.V., S.S., and R.D. designed the research; A.V. performed the research and wrote the paper; A.V. and S.S. analyzed the data; R.D., L.P., and A.S. contributed to the data analyzes; S.S, R.D, L.P., A.S., and A.J. gave helpful suggestions during the internal reviewing process.

Data availibility All the used data, including orbits and observation data, are publicly available from CODE (orbits) and the IGS (observation data). The OSB results from our multi-GNSS processing are publicly available as BIAS-SINEX files. The long-term bias analysis data are not yet publicly available due to ongoing research.

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