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Vector-scalar mixing to all orders
for an arbitrary gauge model in the generic linear gauge

Adrian Lewandowski [1]

Albert Einstein Center for Fundamental Physics,
Institute for Theoretical Physics, University of Bern,
Sidlerstrasse 5, CH-3012 Bern, Switzerland

Abstract

I give explicit fromulae for full propagators of vector and scalar fields in a generic spin-1 gauge model quantized in an arbitrary linear covariant gauge. The propagators, expressed in terms of all-order one-particle-irreducible correlation functions, have a remarkably simple form because of constraints originating from Slavnov-Taylor identities of Becchi-Rouet-Stora symmetry. I also determine the behavior of the propagators in the neighborhood of the poles, and give a simple prescription for the coefficients that generalize (to the case with an arbitrary vector-scalar mixing) the standard $\sqrt{\mathcal{Z}}$ factors of Lehmann, Symanzik and Zimmermann. So obtained generalized $\sqrt{\mathcal{Z}}$ factors, are indispensable to the correct extraction of physical amplitudes from the amputated correlation functions in the presence of mixing.

The standard R_ξ guauges form a particularly important subclass of gauges considered in this paper. While the tree-level vector-scalar mixing is, by construction, absent in R_ξ gauges, it unavoidably reappears at higher orders. Therefore the prescription for the generalized $\sqrt{\mathcal{Z}}$ factors given in this paper is directly relevant for the extraction of amplitudes in R_ξ gauges.

[1] Email: lewandowski.a.j@gmail.com

1 Introduction

Bosonic fields can mix with each other, unless the symmetries tell us otherwise. In the Standard Model (SM) of particle physics (see e.g. [1]), there are four neutral elementary bosonic fields: scalar h_0, vector γ_μ (photon), vector Z_μ and (in renormalizable gauges) the scalar would-be Goldstone field G_Z associated with Z_μ. While the mixing between h_0 and the other fields requires a transfer of CP-violation from the quark sector (and therefore cannot appear at low orders of perturbation theory), there are dozens of SM extension in which Z_μ is mixed with physical scalars already at one-loop; the singlet Majoron model [2] is perhaps the simplest extension of this sort. [2] Therefore, it would be welcomed to have a simple prescription that gives, in the presence of a generic mixing, the physical amplitudes directly it terms of amputated correlation functions. After all, in the case without mixing, there is a standard textbook algorithm, based on the $\sqrt{\mathcal{Z}}$ factors of Lehmann-Symanzik-Zimmermann (LSZ), which does the job in a simple and elegant manner, see e.g. [5] for a nice description (with a derivation) in English.

The standard first step in the LSZ-reduction [5] is to study the behavior of propagators with resummed quantum corrections, and $\sqrt{\mathcal{Z}}$'s are square roots of the residues (up to the i factor) of resummed propagators at the poles. However, despite many beautiful papers devoted to the study of physical (and unphysical) states in covariant gauges, of which I specifically mention here Refs. [6, 7], the complete and concise expressions for the resummed propagators of bosonic fields, in a generic gauge model and a generic linear covariant gauge, were not given in the literature, to the best of my knowledge. The present papers fills that gap, and completes the LSZ algorithm.

In fact, the present paper is a completion of my two earlier papers [3, 4], where the reader may find a more comprehensive list of relevant references. Inspired by the analysis of 2-by-2 and 3-by-3 mixing of Majorana fermions in [8], and almost generic (though complicated) analyses of n-by-n fermionic mixing in [9] and [10], I gave in [3] a simple prescription for handling mixed fermions in a completely generic case, with no dependence whatsoever on particular renormalization conditions, including situations in which multiple states are associated with a single pole. A generic prescription for dealing with mixed scalars in non-gauge theories was also given in [3]. Next, in [4] I provided a prescription for handling mixed vector-scalar systems in the Landau gauge.

Before generalizing the results of [4] to an arbitrary linear covariant gauge,

[2] In fact, already in the SM there are nontrivial one-loop corrections to the photon-Z_μ mixing. The Landau-gauge description of this mixing (in the formalism used in this paper) can be found in [4].

in order to make the present paper as self-contained as possible, I will recapitulate here the generalized LSZ algorithm in purely scalar theories. Suppose that $\{\phi^j\}$ is a set of renormalized (in some convenient renormalization scheme) scalar fields. Without loss of generality I assume that ϕ^j are Hermitian and have vanishing vacuum expectation values (VEVs). The renormalized one-particle-irreducible (1PI) correlation functions of scalars (with resummed quantum corrections) can be parametrized in the following way

$$\widetilde{\Gamma}_{kj}(-p,p) = S(p^2)_{kj} = \Big[p^2\mathbb{1} - M_S^2(p^2)\Big]_{kj}\,, \tag{1}$$

where $M_S^2(s) = M_S^2(s)^\top$ is a symmetric matrix. I emphasize here that all the relevant objects have been already renormalized, only because the procedure described below has, in principle, nothing to do with renormalization. In fact, the minimal subtraction schemes are by far the most popular ones these days, and (in this class of schemes) one always faces the problem of extracting physical results from renormalized correlation functions.

Inverting the matrix in Eq. (1) we get the (connected) propagator

$$\widetilde{G}^{kj}(p,-p) = i\Big[\big(p^2\mathbb{1} - M_S^2(p^2)\big)^{-1}\Big]^{kj}\,, \tag{2}$$

that has, as proved in [3, 4], the following behavior about the poles

$$\widetilde{G}^{kj}(p,-p) = \sum_\ell \sum_r \tilde{\zeta}^k_{S[\ell_r]}\,\frac{i}{p^2 - m^2_{S(\ell)}}\,\tilde{\zeta}^j_{S[\ell_r]} + [\text{non-pole part}]\,. \tag{3}$$

It is clear that the pole masses $m^2_{S(\ell)}$ are solutions to the following equation

$$\det(s\mathbb{1} - M_S^2(s))\big|_{s=m^2_{S(\ell)}} = 0\,. \tag{4}$$

It is also easy to believe that the coefficients $\tilde{\zeta}_{S[\ell_1]}, \tilde{\zeta}_{S[\ell_2]}, \ldots,$ form the basis of the corresponding eigenspace (in this notation $m^2_{S(\ell)} \neq m^2_{S(\ell')}$ for $\ell \neq \ell'$)

$$M_S^2(m^2_{S(\ell)})\,\tilde{\zeta}_{S[\ell_r]} = m^2_{S(\ell)}\,\tilde{\zeta}_{S[\ell_r]}\,. \tag{5}$$

It turns out [3, 4] that to prove the behavior (3), one just need to find eigenvectors satisfying the following normalization/orthogonality conditions

$$\tilde{\zeta}^\top_{S[\ell_r]}\,\big[\mathbb{1} - M_S^{2\prime}(m^2_{S(\ell)})\big]\,\tilde{\zeta}_{S[\ell_q]} = \delta_{rq}\,, \tag{6}$$

where $M_S^{2\prime}(s) \equiv \mathrm{d}M_S^2(s)/\mathrm{d}s$. [3] Such eigenvectors always exist (in perturbation theory) for real and complex poles [3, 4]. Moreover, for real poles, one can always find eigenvectors $\tilde{\zeta}^j_{S[\ell_q]}$ that obey (6) and are themselves real.[4]

From the representation (3) of the propagator it is clear how to generalize the LSZ algorithm to the case of (purely scalar) mixing. Indeed, Eq. (3) immediately shows that the asymptotic field $\boldsymbol{\phi}^j$ associated with the renormalized field ϕ^j has the following form

$$\boldsymbol{\phi}^j = {\sum_\ell}' \sum_r \tilde{\zeta}^j_{S[\ell_r]} \Phi^{\ell_r} \, , \tag{7}$$

where Φ^{ℓ_r} are free Hermitian scalar fields of mass $m_{S(\ell)}$ with canonically normalized propagators, and such that states created/annihilated by Φ^{ℓ_r} and $\Phi^{\ell'_{r'}} \neq \Phi^{\ell_r}$ are orthogonal to each other. (Strictly speaking, asymptotic fields Φ^{ℓ_r} exist only for *real* pole masses $m_{S(\ell)}$, and therefore I put the prime on the first sum in (7).) In other words, to obtain the correctly normalized (i.e. consistent with unitarity) amplitude of the process involving a particle corresponding to Φ^{ℓ_r}, one has to contract the eigenvector $\tilde{\zeta}^j_{S[\ell_r]}$ with the amputated correlation functions $\mathcal{A}_{j\ldots}(p,\ldots)$ of the renormalized scalar fields ϕ^j for $p^2 = m^2_{S(\ell)}$.

Now I will explain how to generalize this algorithm to the mixing in gauge theories, beyond the Landau gauge, what is the main subject of this paper.

2 Vector-scalar mixing in the generic case

As before, I can assume that vector (A^α_μ) and scalar (ϕ^j) fields are Hermitian. The gauge fixing Lagrangian in a generic linear covariant gauge has the following form [6] (see also [11, 1])

$$\mathcal{L}_{\text{gauge-fix}} = h_\alpha f^\alpha + \frac{1}{2}\xi^{\alpha\beta} h_\alpha h_\beta \, , \qquad f^\alpha = -\partial^\mu A^\alpha_\mu - \mathcal{U}^\alpha{}_j(\phi^j + v^j), \tag{8}$$

[3] Note that the eigenvectors corresponding to different pole masses are, in general, not orthogonal to each other beyond the tree-level.

[4] Of course, Eq. (6) implicitly assumes that infrared divergences are absent, so that the derivative $M_S^{2\prime}(m^2_{S(\ell)})$ is finite. It is however worth saying that, even if certain matrix elements of $M_S^{2\prime}(m^2_{S(\ell)})$ are IR-divergent, one can still use the normalization conditions (6) by replacing $M_S^{2\prime}(m^2_{S(\ell)}) \mapsto M_S^{2\prime}(q^2)$ and taking the $q^2 \to m^2_{S(\ell)}$ limit, as long as limit of the left-hand-side of (6) is finite (see Ref. [4]). Such IR-divergences are therefore spurious and do not change the structure of the asymptotic states, in contrast to the "physical" IR-divergences which are usually handled by introducing an IR-cutoff.

where h_α are Nakanishi-Lautrup fields (see e.g. [1]), [5] while $\xi^{\alpha\beta}$ and $\mathcal{U}^\alpha{}_j$ are matrices of gauge fixing parameters ($\xi^{\alpha\beta}$ is symmetric). As before I assume that ϕ^j has a vanishing VEV, and therefore v^j is an all-order VEV of the field $\phi^j + v^j$ in the "symmetric phase". Note that (as is clear from, for instance, the path-integral representation), the quantum corrected propagators in the scalar-vector sector are independent of whether one decides to keep Nakanishi-Lautrup fields, or integrates them out. However, the corresponding 1PI functions do depend on this decision. Nonetheless, it is well known that there are no quantum corrections to the gauge fixing terms in linear gauges (see e.g. [11]); in other words, the renormalized 1PI generating functional $\Gamma[\ ]$ to all orders of perturbation theory depends on h_α only through (the integral of) the tree-level gauge-fixing Lagrangian (8). Thus, the "effective" 1PI two-point functions in the vector-scalar sector, obtained by integrating out h_α's, differ from the "primordial" ones only by the terms originating from the "effective" gauge-fixing Lagrangian

$$\mathcal{L}^{eff}_{\mathrm{gauge-fix}} = -\frac{1}{2}(\xi^{-1})_{\alpha\beta} f^\alpha f^\beta \,.$$

It turns out that the "primordial" 1PI two-point functions are much closer to the physical reality (this fact is not particularly surprising, as they are independent of gauge-fixing parameters at the tree level), and therefore, in this paper, the h_α's are *not* integrated out. The additional advantage of such an approach is that one has, at every intermediate stage, a nonsingular transition to the Landau gauge ($\xi = 0 = \mathcal{U}$).

I use the following convention

$$\frac{\delta}{\delta\hat{\phi}^j(p)}\frac{\delta}{\delta\hat{\phi}^k(p')}\,\Gamma\,[\phi,\ldots]\bigg|_0 = (2\pi)^4\delta^{(4)}(p'+p)\,\widetilde{\Gamma}_{kj}(p',p)\,, \tag{9}$$

where $\mathcal{F}(x) \equiv \int \mathrm{d}^4p\, e^{-ipx}\hat{\mathcal{F}}(p)$, and 0 denotes the stationary point. I have chosen the following parametrization of the complete 1PI 2-point functions in the A^α_μ, ϕ^j and h_α sector; for vectors:

$$\widetilde{\Gamma}^{\mu\nu}_{\alpha\beta}(-q,q) \equiv -\eta^{\mu\nu}\Big[q^2\mathbb{1} - M^2_V(q^2)\Big]_{\alpha\beta} + q^\mu q^\nu \mathscr{L}_{\alpha\beta}(q^2)\,, \tag{10}$$

for the scalar-scalar two point function I employ once again Eq. (1), and parametrize the mixed vector-scalar correlation functions as follows

$$\widetilde{\Gamma}^\mu_{\alpha j}(-q,q) \equiv i\,q^\mu\,P_{\alpha j}(q^2) = -\widetilde{\Gamma}\,^\mu_{j\alpha}(-q,q)\,. \tag{11}$$

[5] I apologize for a potentially confusing notation: h_α's have nothing in common with the Higgs field h_0 mentioned earlier.

Finally, the functions involving Nakanishi-Lautrup multipliers have, to all orders of perturbation theory, the tree-level form

$$\widetilde{\Gamma}^{\alpha\nu}{}_{\beta}(-q,q) \equiv i\,\delta^{\alpha}{}_{\beta}\,q^{\nu} = -\widetilde{\Gamma}^{\nu\alpha}{}_{\beta}(-q,q)\,,$$
$$\widetilde{\Gamma}^{\alpha}{}_{j}(-q,q) = -\,\mathcal{U}^{\alpha}{}_{j} = \widetilde{\Gamma}_{j}{}^{\alpha}(-q,q)\,,$$
$$\widetilde{\Gamma}^{\alpha\beta}(-q,q) \equiv \xi^{\alpha\beta}\,. \tag{12}$$

For the reader's convenience, I list also the tree-level approximations to the full 1PI 2-point functions:

$$\mathscr{L}_{\alpha\beta}(q^2) = \delta_{\alpha\beta} + \mathcal{O}(\hbar)\,, \qquad M_V^2(q^2)_{\alpha\beta} = (\mathcal{T}_\alpha v)^\top(\mathcal{T}_\beta v) + \mathcal{O}(\hbar)\,,$$
$$P_{\alpha i}(q^2) = -\delta_{ij}(\mathcal{T}_\alpha v)^j + \mathcal{O}(\hbar)\,, \qquad M_S^2(q^2)_{ij} = \partial_{\varphi^i}\partial_{\varphi^j}V^{GI}_{tree}(\varphi)|_{\varphi=v} + \mathcal{O}(\hbar)\,,$$

where $\mathcal{T}_\alpha$ are real antisymmetric matrices forming the representation of the gauge Lie algebra on scalar fields ϕ^j (note that, in this notation, the matrix elements of $\mathcal{T}_\alpha$ contain the renormalized gauge coupling constants), v is the all-order VEV of the scalar field $\varphi = \phi + v$ in the symmetric phase (of course, to the leading order, v can be replaced with its $\hbar^0$ term), and $V^{GI}_{tree}(\varphi)$ is the gauge-invariant ($(\mathcal{T}_\alpha\varphi)^j\partial_{\varphi^j}V^{GI}_{tree}(\varphi) \equiv 0$) and independent of gauge-fixing parameters tree-level potential of Hermitian scalar fields.

To find the connected propagators with (resummed) quantum corrections, one needs to solve the equation

$$\widetilde{\Gamma}_{IJ}(-p,p)\,\widetilde{G}^{JK}(p,-p) = i\,\delta_I{}^K\,, \tag{13}$$

where I, J and K run over components of bosonic fields ϕ^n, A^α_μ and h_β. I have chosen the following parametrization of $\widetilde{G}^{JK}(p,-p)$; in the vector-vector block:

$$\widetilde{G}^{\beta\delta}_{\nu\rho}(q,-q) = -i\left[\eta_{\nu\rho} - \frac{q_\nu q_\rho}{q^2}\right]\left[V(q^2)^{-1}\right]^{\beta\delta} + i\,\frac{q_\nu q_\rho}{q^2}\mathcal{A}(q^2)^{\beta\delta}\,, \tag{14}$$

in scalar-scalar and vector-scalar blocks:

$$\widetilde{G}^{jn}(q,-q) = i\,H^{jn}(q^2)\,, \qquad \widetilde{G}^{\beta n}_{\nu}(q,-q) = q_\nu E^{\beta n}(q^2) = -\widetilde{G}^{n\beta}{}_{\nu}(q,-q)\,, \tag{15}$$

the block that mixes the Nakanishi-Lautrup fields h_β with vectors is written as

$$\widetilde{G}_{\beta}{}^{\delta}{}_{\rho}(q,-q) = -q_\rho J^{\delta}{}_{\beta}(q^2) = -\widetilde{G}^{\delta}{}_{\rho\beta}(q,-q)\,, \tag{16}$$

while the h_β-ϕ^n and h_β-h_γ blocks can be parametrized as

$$\widetilde{G}_\beta{}^n(q,-q) = i\,\mathcal{I}(q^2)^n{}_\beta = \widetilde{G}^n{}_\beta(q,-q)\,, \qquad \widetilde{G}_{\beta\gamma}(q,-q) = i\,K_{\beta\gamma}(q^2)\,. \tag{17}$$

Before discussing relations between 1PI two-functions originating from the Slavnov-Taylor identities of Becchi-Rouet-Stora symmetry (STids) [12, 13], I should stress that those relations are valid only at the stationary points of the 1PI generating functional $\Gamma[\ ]$. However, in certain applications, e.g. in order to calculate the effective potential of scalar fields, [6] one needs the propagators in the presence of an arbitrary constant scalar background $\overline{\varphi}^j$ which differs from the VEV v^j. For this reason, I give also (in Appendix A) formulae for propagators which are completely generic solutions to Eqs. (13), and do not rely on the STids; those formulae are not needed in what follows – it is in fact much simpler to solve (13) once again in the presence of STids, instead of trying to simplify the results from the Appendix. Nevertheless, there are two relations, satisfied by a *generic* solution to Eqs. (13), which are important in what follows [7]

$$\mathcal{A}(q^2)^{\alpha\delta} = \mathcal{U}^\alpha{}_j E^{\delta j}(q^2) - \xi^{\alpha\beta} J^\delta{}_\beta(q^2)\,, \tag{18}$$

$$q^2\, E^{\gamma n}(q^2) = \mathcal{U}^\gamma{}_j H^{jn}(q^2) - \xi^{\gamma\alpha}\mathcal{I}(q^2)^n{}_\alpha\,. \tag{19}$$

Additionally, the transverse part of (14) is also (*independently* of STIds) easy to express in terms of the 1PI two-point function (10)

$$V(q^2) = q^2\mathbb{1} - M_V^2(q^2)\,. \tag{20}$$

From now on, I assume that we are at the (physical – see below) stationary point of the 1PI generating functional, so that STids [12, 13] (see also [11] for an introduction, and [4] for a derivation in the present notation [8]) yield relations between the 1PI 2-point functions; namely there exists matrices $B(q^2)^j{}_\gamma$ and $\Omega(q^2)^\alpha{}_\gamma$ such that form-factors of 1PI 2-point functions satisfy the following constraints

$$P_{\beta j}(q^2)\,B(q^2)^j{}_\gamma = \left\{q^2\,\mathscr{L}_{\alpha\beta}(q^2) + [M_V^2(q^2) - q^2\mathbb{1}]_{\alpha\beta}\right\}\Omega(q^2)^\alpha{}_\gamma\,, \tag{21}$$

[6]See e.g. [15] for a recent determination of the 2-loop effective potential for an arbitrary renormalizable model in a subclass of gauges considered in the present paper.

[7]It is, perhaps, also worth saying that even though (18)-(19) follow from (13), they can be also obtained independently of (13), as they express the non-renormalization theorem for gauge-fixing terms in the form appropriate for connected (rather that 1PI) generating functional.

[8]Since I keep the h_α fields, the STids have exactly the same form as in the Landau gauge, thus the derivation from [4] applies in the present context.

and

$$q^2\, P_{\alpha j}(q^2)\, \Omega(q^2)^{\alpha}{}_{\gamma} = S(q^2)_{ij}\, B(q^2)^{i}{}_{\gamma}\,. \tag{22}$$

In fact, there is a concrete prescription for calculating $\Omega(q^2)$ and $B(q^2)$ in perturbation theory: let $\mathcal{L}^s$ be the BRS-exact part of the (tree-level) Lagrangian [11]

$$\mathcal{L}^s = s\big\{\overline{\omega}_\alpha[f^\alpha + \xi^{\alpha\beta} h_\beta/2]\big\} + L_\alpha\, s(\omega^\alpha) + K_i\, s(\phi^i) + \bar{K}_a\, s(\psi^a) + K^\mu_\alpha\, s(A^\alpha_\mu)\,,$$

where s is the nilpotent BRS-differential [11], the first term on the right-hand-side contains (in addition to gauge-fixing terms from Eq.(8)) the vertices involving ghost ω^α and antighost $\overline{\omega}_\alpha$ fields, ψ^a represents Majorana fields (i.e. natural counterparts of Hermitian bosonic fields), while K_i, $\bar{K}_a$, K^μ_α and L_α are external sources (antifields) controlling the quantum corrections to BRS transformations [12, 13, 11]. Then, the matrices $B(q^2)^{j}{}_{\gamma}$ and $\Omega(q^2)^{\alpha}{}_{\gamma}$ are given by the following derivatives at the stationary point:

$$\left.\frac{\delta}{\delta\hat{\omega}^\gamma(p)}\frac{\delta}{\delta\hat{K}_i(q)}\Gamma\right|_0 = (2\pi)^4\delta^{(4)}(q+p)\, B(q^2)^{i}{}_{\gamma}\ , \tag{23}$$

$$\left.\frac{\delta}{\delta\hat{\omega}^\gamma(p)}\frac{\delta}{\delta\hat{K}^\mu_\alpha(q)}\Gamma\right|_0 = (2\pi)^4\delta^{(4)}(q+p)\,\left\{i\, q_\mu\, \Omega(q^2)^{\alpha}{}_{\gamma}\right\}\ . \tag{24}$$

At the lowest order one finds

$$B(q^2)^{i}{}_{\gamma} = (\mathcal{T}_\gamma v)^i + \mathcal{O}(\hbar)\,, \qquad \Omega(q^2)^{\alpha}{}_{\gamma} = -\delta^{\alpha}{}_{\gamma} + \mathcal{O}(\hbar)\,, \tag{25}$$

in particular STids can be easily verified at the tree-level. Of course, at higher orders, the calculation of diagrams with external lines of K^μ_α, etc. is no different from calculations of diagrams involving the propagating fields A^μ_α, etc. (apart from simplifications due to vanishing propagators).

Strictly speaking, STids (21)-(22) are satisfied only if $\langle h_\alpha\rangle = 0$ at the stationary point of $\Gamma[\ ]$; however in practice one can always find stationary points obeying this condition (see e.g. [14] and references therein), in particular equation $\langle h_\alpha\rangle = 0$ can be interpreted as the Dashen's vacuum realignment condition (see e.g. [1]) for the effective potential $V_{eff}(\phi, h)$ that exhibits explicit breaking of the symmetry under global gauge transformations. In fact, the stationary points violating $\langle h_\alpha\rangle = 0$ are inherently unphysical, as they lead to spontaneous breaking of the BRS symmetry and thus violate the quartet mechanism of Kugo and Ojima [7], that is necessary for decoupling of unphysical modes from the physical S matrix.

I need just one additional identity to invert the matrix of 1PI 2-point functions; in linear gauges the ghost-antighost 1PI 2-point function is not independent of matrices $B(q^2)^i{}_\gamma$ and $\Omega(q^2)^\alpha{}_\gamma$ appearing in the STids (see e.g. [11]); in particular, the ghost-antighost propagator $\widetilde{\mathcal{G}}(q^2)^\beta{}_\alpha$ satisfies (to all orders)

$$i[\widetilde{\mathcal{G}}(q^2)^{-1}]^\alpha{}_\beta = -q^2\Omega(q^2)^\alpha{}_\beta + \mathcal{U}^\alpha{}_j B(q^2)^j_\beta \,. \tag{26}$$

Clearly, $\widetilde{\mathcal{G}}(q^2)$ has only unphysical poles, and therefore one expects that it can be useful for extraction of unphysical poles from propagators of bosonic fields. In fact, as the first STids-triggered simplification, one finds that the mixed propagators involving the h_β fields are simply given by (cf. Eqs. (16)-(17))

$$J^\beta{}_\delta(q^2) = i\,\Omega(q^2)^\beta{}_\gamma\,\widetilde{\mathcal{G}}(q^2)^\gamma{}_\delta\,, \qquad\qquad \mathcal{I}^j{}_\delta(q^2) = i\,B(q^2)^j{}_\gamma\,\widetilde{\mathcal{G}}(q^2)^\gamma{}_\delta\,, \tag{27}$$

and that $K_{\alpha\beta}(q^2) = 0$, i.e. the h_α-h_β propagator vanishes. Thus, the asymptotic field associated with h_α describes zero-norm states, which have non-vanishing scalar products only with unphysical bosonic states [7].

To express the scalar-scalar propagator $H^{jn}(q^2)$ in a simple form, I need to define the following (non-symmetric!) matrix (cf. Eq. (1))

$$T(q^2)_{ij} = S(q^2)_{ij} - P_{\alpha i}(q^2)\,\mathcal{U}^\alpha{}_j\,, \tag{28}$$

then (using the matrix-multiplication)

$$H(q^2) = T(q^2)^{-1}\,S(q^2)\,[T(q^2)^{-1}]^\top - \xi^{\alpha\delta}\,\mathcal{I}_\alpha(q^2)\,\mathcal{I}_\delta(q^2)^\top\,, \tag{29}$$

where $\mathcal{I}_\delta$ is a vector of $\mathcal{I}^j{}_\delta$ propagators from Eq. (27). Note that $H(q^2)$ is explicitly symmetric; nonetheless the STids allows us to rewrite $H(q^2)$ in a form that is even more useful for extraction of physical poles. Using the relation

$$T(q^2) = \frac{1}{q^2}S(q^2)\sigma(q^2)\,, \quad \text{where} \quad \sigma(q^2)^i{}_j \equiv q^2\,\delta^i_j - C^i{}_\alpha(q^2)\,\mathcal{U}^\alpha{}_j\,, \tag{30}$$

with

$$C^j{}_\gamma(q^2) \equiv B(q^2)^j{}_\beta\,[\Omega(q^2)^{-1}]^\beta{}_\gamma\,, \tag{31}$$

one gets

$$H(q^2) = (q^2)^2\,\sigma(q^2)^{-1}\,S(q^2)^{-1}\,[\sigma(q^2)^{-1}]^\top - \xi^{\alpha\delta}\,\mathcal{I}_\alpha(q^2)\,\mathcal{I}_\delta(q^2)^\top\,. \tag{32}$$

Thus every massive root of $\det(S(q^2)) = 0$ yields a singularity of the scalar-scalar propagator, what justifies my earlier claim that the 1PI correlation functions in the presence of h_α fields are more physical than their counterparts obtained by integrating h_α's out. I will have more to say about the structure of poles later on, but first I will complete the list of propagators. The mixed vector-scalar propagator has the form

$$E^{\delta n}(q^2) = \mathcal{W}(q^2)^{\gamma}{}_{i}\,[T(q^2)^{-1}]^{ni}\,J^{\delta}{}_{\gamma}(q^2)\,, \tag{33}$$

where

$$\mathcal{W}(q^2)^{\gamma}{}_{j} \equiv \mathcal{U}^{\gamma}{}_{j} - \xi^{\gamma\alpha}P_{\alpha j}(q^2)\,, \tag{34}$$

while $J^{\delta}{}_{\gamma}$ is the propagator from Eq. (27). Note that $\mathcal{W}(q^2)$ is momentum independent *only* at the tree-level; the standard R_ξ gauges (see e.g. [1]) are defined by the 't Hooft's condition $\mathcal{W}(q^2) = \mathcal{O}(\hbar)$, and thus the scalar-vector mixing reappears in the R_ξ gauges at the quantum level.

Last but not least, one gets the scalar part of the vector-vector propagator (14) (recall that the transverse part is given by (20) *independently* of STids)

$$\mathcal{A}(s)^{\alpha\delta} = \frac{1}{s}\mathcal{U}^{\alpha}{}_{j}\mathcal{U}^{\delta}{}_{n}\,H^{nj}(s) + \frac{1}{s}\xi^{\alpha\delta} - \xi^{\delta\beta}\Omega(s)^{\alpha}{}_{\epsilon}\,i\,\widetilde{\mathcal{G}}(s)^{\epsilon}{}_{\beta} - \xi^{\alpha\beta}\Omega(s)^{\delta}{}_{\epsilon}\,i\,\widetilde{\mathcal{G}}(s)^{\epsilon}{}_{\beta}\,. \tag{35}$$

where $s \equiv q^2$ while $\widetilde{\mathcal{G}}(s)$ is the ghost-antighost propagator.

The above formulae are valid to all orders of perturbation theory; nonetheless they have an amusing consequence at the tree-level: all the tree-level propagators, in contrast to the R_ξ gauges, are first order polynomials in ξ. Indeed, masses of unphysical modes are produced entirely by the $\mathcal{U}^{\alpha}{}_{i}$ parameter which (in the R_ξ gauges) becomes a function of ξ, once the 't Hooft's fine-tuning condition $\mathcal{W}(q^2) = \mathcal{O}(\hbar)$ is imposed. In particular, the $\mathcal{U}^{\alpha}{}_{i}$ parameter cures the $1/q^4$ IR-singularities which appear in the second term of the scalar-scalar propagator (32) for $\mathcal{U}^{\alpha}{}_{i} = 0 \neq \xi^{\alpha\beta}$ (of course, since $\mathcal{I}_\alpha = 0$ in the unbroken phase, these singularities exists only in spontaneously broken gauge theories).

The behavior of the scalar-scalar propagator $i\,H^{jn}(q^2)$ about the massive roots $q^2 = m^2_{S(\ell)} \neq 0$ of $\det(S(q^2)) = 0$ follows immediately from Eq. (32). I use the parametrization of $S(q^2)$ given in (1). Suppose that we have the eigenvectors $\tilde{\zeta}^k_{S[\ell_r]}$ obeying Eqs. (5)-(6); defining

$$\zeta^k_{S[\ell_r]} = m^2_{S(\ell)}\,[\sigma(m^2_{S(\ell)})^{-1}]^k{}_j\,\tilde{\zeta}^j_{S[\ell_r]}\,, \tag{36}$$

one gets (cf. Eq. (3))

$$\widetilde{G}^{kj}(q,-q) \approx \sum_r \zeta^k_{S[\ell_r]}\,\frac{i}{q^2 - m^2_{S(\ell)}}\,\zeta^j_{S[\ell_r]}\,, \qquad \text{for} \qquad q^2 \approx m^2_{S(\ell)} \neq 0\,. \tag{37}$$

In theories without physical massless scalars, all the remaining poles of the scalar-scalar propagator are unphysical (see the discussion in Appendix B).

The only naturally massless scalars are Goldstone bosons of nonlinearly realized exact global symmetries; since these days such symmetries are quite commonly considered as inconsistent with quantum gravity, I will not give here the generic prescription for the $\zeta^k_{S[\ell_r]}$-like vectors associated with them; nonetheless such a prescription can be obtained by studying the first term in Eq. (32) more carefully, once a concrete model is chosen. It is, however, worth saying that a simple and *generic* prescription for $\zeta^k_{S[\ell_r]}$ vectors corresponding to the physical Goldstone bosons exists in the Landau gauge [4], and shows that their directions are fixed by the spontaneously broken gauge symmetry rather than the underlying global symmetry.

Thus, excluding theories with *physical* massless scalars, the asymptotic field $\boldsymbol{\phi}^j$ corresponding to the renormalized scalar field ϕ^j has the form analogous to (7)

$$\boldsymbol{\phi}^j = \sum_\ell{}' \sum_r \zeta^j_{S[\ell_r]} \Phi^{\ell_r} + \dots\,, \tag{38}$$

where Φ^{ℓ_r} are canonically normalized free scalar fields corresponding to real pole masses (as indicated by the prime on the sum over ℓ), while the ellipsis indicates the contributions of unphysical asymptotic states. (As is clear from the discussion above, even in the presence of physical massless scalars, Eqs. (38) and (36) give the contributions of physical *massive* scalars to the asymptotic scalar fields $\boldsymbol{\phi}^j$.)

The reader might be worried that the prescription (36) for the $\zeta^j_{S[\ell_r]}$ coefficient depends, through the $\sigma(m^2_{S(\ell)})$ matrix, on the 1PI correlation functions of antifields (23)-(24). I could argue that, in principle, the $\sigma(q^2)$ function can be expressed in terms of $S(q^2)$ and the mixed scalar-vector 1PI two-point function $P_{\alpha i}(q^2)$, cf. Eqs. (30) and (28), although obtaining $\sigma(m^2_{S(\ell)})$ in this way necessarily involves taking the limit $q^2 \to m^2_{S(\ell)}$. In practice, however, one should notice that the number of diagrams contributing to the correlation functions of antifields is always significantly smaller than the number of diagrams contributing to $P_{\alpha i}(q^2)$. Therefore the form (36) of the prescription for $\zeta^j_{S[\ell_r]}$ is actually quite convenient, as it is (owing to the STids!) completely independent of $P_{\alpha i}(q^2)$, as well as of the scalar part $\mathcal{A}^{\alpha\delta}(q^2)$ of the vector-vector two point functions. In fact, the same is true for the prescription that gives us the asymptotic vector fields (see below). Thus, the functions $P_{\alpha i}(q^2)$ and $\mathcal{A}^{\alpha\delta}(q^2)$ are not needed in practice.

Using the simple form of the propagators listed above, it is also easy to find the asymptotic vector fields $\mathbf{A}^\alpha_\mu$ associated with the renormalized vector fields A^α_μ

$$\mathbf{A}^\alpha_\mu = \sum_\lambda{}' \sum_r \zeta^\alpha_{V[\lambda_r]} \mathbb{A}^{\lambda_r}_\mu + \mathcal{U}^\alpha{}_j \sum_\ell{}' \sum_r \frac{1}{m^2_{S(\ell)}} \zeta^j_{S[\ell_r]} \partial_\mu \Phi^{\ell_r} + \ldots\,, \tag{39}$$

as before the ellipsis represents the contributions of unphysical asymptotic states. The first term in (39) represents the spin-1 operators inferred from the transverse part of the vector-vector propagator (14); in particular the $\zeta^\alpha_{V[\lambda_r]}$ coefficients and the pole masses $m^2_{V(\lambda)}$ are obtained by the following replacements in Eqs. (4)-(6)

$$M^2_S(q^2) \mapsto M^2_V(q^2)\,, \qquad m^2_{S(\ell)} \mapsto m^2_{V(\lambda)}\,, \qquad \tilde{\zeta}_{S[\ell_r]} \mapsto \zeta^\alpha_{V[\lambda_r]},$$

where $M^2_V(q^2)$ parametrizes the 1PI two-point function of vector fields (10). The coefficient $\mathbb{A}^{\lambda_r}_\mu$ in Eq. (39) is a free Hermitian vector field of mass $m_{V(\lambda)}$ (in the unitarity gauge for $m_{V(\lambda)} \neq 0$, or the Coulomb gauge for $m_{V(\lambda)} = 0$) with canonically normalized propagator; the states created/annihilated by $\mathbb{A}^{\lambda_r}_\mu$ and $\mathbb{A}^{\lambda'_{r'}}_\mu \neq \mathbb{A}^{\lambda_r}_\mu$ are orthogonal to each other, and the prime on the sum over λ has the same meaning as for scalars. Note that the form of the second term in (39), which involves only on the objects present already in Eq. (38), follows unambiguously from the representation (19) of the vector-scalar propagator, as well as (up to a sign) from the scalar part (35) of the vector-vector propagator. [9] Using the generic form (given in [4]) of (unphysical) quantum fields having propagators with higher-order poles, one can also verify that unphysical states in Eqs. (38)-(39) form (together with their Faddeev-Popov counterparts) the standard quartet representations of the asymptotic BRS operator [7], [10] thereby ensuring unitarity of transition amplitudes between physical states.

Eq. (39) shows, in particular, that the amputated correlation functions of renormalized vector fields A^α_μ produce nontrivial contributions to amplitudes of scalar particles created by Φ^{ℓ_r}, just as the correlation functions of scalars ϕ^j do. It is also easy to check directly from Eq. (36) that, in the special case of R_ξ gauges, the coefficient $\mathcal{U}^\alpha{}_j \zeta^j_{S[\ell_r]}$ vanishes at the tree level. [11] Nonetheless, even in the R_ξ gauges, non-vanishing contributions to $\mathcal{U}^\alpha{}_j \zeta^j_{S[\ell_r]}$ are generated

[9] To arrive at this conclusion it is enough to realize that all the terms in Eqs. (19) and (35) except the first ones do not have physical poles (cf. Eqs. (27)).

[10] This can be done in an analogous way to the Landau gauge case, for which a detailed and completely generic proof can be found in [4].

[11] In the R_ξ gauges, $\mathcal{U}^\alpha{}_j$ is a (transposition of) null eigenvector of the tree-level contribution to $M^2_S(0)$, as follows from the STids; thus $\mathcal{U}^\alpha{}_j$ is orthogonal to the eigenvector $\tilde{\zeta}^j_{S[\ell_r]}$ corresponding to a nonzero mass, and then Eq. (36) shows that $\zeta^j_{S[\ell_r]} = \tilde{\zeta}^j_{S[\ell_r]} + \mathcal{O}(\hbar)$. .

by the quantum corrections. Therefore Eqs. (39) and (36), may come in handy even in this special case.

3 Conclusions

I gave explicit formulae for the propagators of a generic spin-1 gauge field theory model, valid to all orders of perturbation theory in the presence of mixing between vectors and scalars. Due to relations originating from the BRS symmetry, the propagators take a marvelously simple form, which is a convenient starting point for the study of their behavior about the physical (as well as unphysical) poles.

This way, I have arrived at Eqs. (38) and (39) which, accompanied by the explicit prescription for the relevant coefficients, in particular Eq. (36), generalize the Lehmann-Symanzik-Zimmermann algorithm to the case of an arbitrary mixing between bosonic fields in gauge theories. The treatment of spin-1/2 fermions in this framework can be found in [3]

A Generic solution to Eq. (13)

In this section I give, for completeness, the generic solution to Eqs. (13), which does not rely on the STids and therefore is valid not only at the stationary points of the 1PI effective action $\Gamma[\,]$, but also, for instance, in the presence of an arbitrary constant scalar background. To this end, I need (in addition to (28) and (34)) the following combinations of form-factors of 1PI 2-point functions ,

$$\mathscr{X}_{\alpha\beta}(q^2) = q^2 \mathscr{L}_{\alpha\beta}(q^2) + M_V^2(q^2)_{\alpha\beta} - q^2 \delta_{\alpha\beta}\,, \tag{40}$$

$$Q_{\gamma i}(q^2) = q^2 P_{\gamma i}(q^2) - \mathcal{U}^{\beta}{}_{i}\, \mathscr{X}_{\beta\gamma}(q^2)\,, \tag{41}$$

and

$$\Delta^{\alpha}{}_{\beta}(q^2) = q^2\, \delta^{\alpha}{}_{\beta} - \xi^{\alpha\gamma}\, \mathscr{X}_{\gamma\beta}(q^2)\,. \tag{42}$$

The scalar-scalar propagator $H(q^2)^{ij}$ is given by the inverse of the following matrix (the order of indices matters!)

$$[H(q^2)^{-1}]_{ij} = T(q^2)_{ji} - Q_{\gamma i}(q^2)[\Delta(q^2)^{-1}]^{\gamma}{}_{\alpha}\, \mathcal{W}^{\alpha}{}_{j}(q^2)\,, \tag{43}$$

the vector-scalar propagator reads

$$E^{\beta n}(q^2) = [\Delta(q^2)^{-1}]^{\beta}{}_{\alpha}\, \mathcal{W}^{\alpha}{}_{j}(q^2) H(q^2)^{jn}\,, \tag{44}$$

the form-factor of Eq. (16) reads

$$J^{\beta}{}_{\delta}(q^2) = [\Delta(q^2)^{-1}]^{\beta}{}_{\delta} + Q_{\kappa j}(q^2) E^{\beta j}(q^2) [\Delta(q^2)^{-1}]^{\kappa}{}_{\delta} \,, \tag{45}$$

the scalar part $\mathcal{A}(q^2)^{\alpha\delta}$ of the vector-vector propagator is now a linear combination of $J^{\delta}{}_{\beta}(q^2)$ and $E^{\beta n}(q^2)$ as given by Eq. (18), while the transverse part is given by (20). For completeness, I give here also the remaining two propagators that involve Nakanishi-Lautrup fields

$$\mathcal{I}(q^2)^{j}{}_{\epsilon} = P_{\epsilon i}(q^2) H^{ij}(q^2) - \mathscr{X}_{\epsilon\gamma}(q^2) E^{\gamma j}(q^2) \,, \tag{46}$$

and

$$K_{\beta\alpha}(q^2) = P_{\beta j}(q^2) \mathcal{I}(q^2)^{j}{}_{\alpha} - \mathscr{X}_{\beta\gamma}(q^2) J^{\gamma}{}_{\alpha}(q^2) \,. \tag{47}$$

Two comments are in order. Firstly, matrices $K_{\beta\gamma}(q^2)$, $\mathcal{A}^{\beta\gamma}(q^2)$ and $H^{ij}(q^2)$ are indeed symmetric, even though it requires some patience to verify this. Secondly, as is clear from the formulae given in the main part of the paper, *none* of the propagators corresponding to the actual stationary point of $\Gamma[\,]$ has a pole at q^2 such that $\det(\Delta(q^2)) = 0$.

B Unphysical poles

It is worth to take a closer look at the structure of the unphysical poles in the propagators of vector and scalar fields (see also [6]). To this end, in addition to STids (21)-(22), it is important to keep in mind that 1PI correlation functions in 4 dimensions do not have poles at finite orders of perturbation theory. Therefore, Eqs. (27) show that the form-factors $J(q^2)$ and $\mathcal{I}(q^2)$ have only poles for $q^2 = m^2_{gh(\lambda)}$ where $m^2_{gh(\lambda)}$ is one of the poles of the ghost-antighost propagator. Then Eq. (29) shows that, apart from poles at $q^2 = m^2_{gh(\lambda)}$, the scalar-scalar propagator has only poles for q^2 such that $\det(T(q^2)) = 0$. Finally, Eqs. (33) and (18) show that the same it true for the scalar-vector propagator and the scalar part of the vector-vector propagator. In other words, it is enough to study $T(q^2)$.

At the tree-level

$$T(q^2) = q^2 \mathbb{1} - \tau + \mathcal{O}(\hbar) \,, \tag{48}$$

with a momentum independent (but non-symmetric) matrix τ; τ has at most N_S eigenvectors, where N_S is the number of scalar fields ϕ^j (real non-symmetric matrices are not diagonalizable in general). Thus, in perturbation theory, $T(q^2)$ has at most N_S poles (where a pole with degeneracy is counted as multiple poles, as usual). Let us try to find as many eigenvectors of τ as possible. Eq. (30) implies that every vector (36) obeys $T(m^2_{S(\ell)})\zeta_{S[\ell_r]} = 0$, as long as $m^2_{S(\ell)} \neq 0$, to all orders of perturbation theory. This gives us $N_S - N_{\rm ker}$ eigenvectors of τ at the tree-level, where $N_{\rm ker}$ is the number of null eigenvectors of $M^2_S(0)$ or (equivalently) $S(0)$. The STid (22) shows that, to all orders of perturbation theory, every vector $C^j{}_\gamma(0)$, cf. Eq (31), belongs to the kernel of $S(0)$. Of course, not all of $C^j{}_\gamma(0)$ are linearly independent, but STids yield also the following equivalence (at every finite order of perturbation theory)

$$M^2_V(0)_{\alpha\beta}\Lambda^\beta = 0 \qquad \Leftrightarrow \qquad C^i{}_\beta(0)\,\Lambda^\beta(0) = 0\,, \tag{49}$$

(see [4], Eq. (150)), i.e. the number of linearly independent vectors $C^i{}_\beta(0)$ is the same as the number of non-massless gauge bosons (what is obvious at the tree-level). Therefore, in models without physical massless scalars, $N_{\rm ker}$ is imply the number of massive gauge bosons. It is interesting to note that Eq. (26) now implies that [12]

$$[\widetilde{\mathcal{G}}(0)^{-1}]^\alpha{}_\beta\,[\Omega(0)^{-1}]^\beta{}_\gamma\Lambda^\gamma = 0\,, \tag{50}$$

where Λ^γ obeys one of the conditions in (49), and thus for every massless gauge boson there is a massless ghost field (recall that $\Omega(q^2)$ is invertible at the tree-level and therefore it is also invertible at every finite order). Of course, the inverse theorem is, in general, not true (all ghosts are massless if $\mathcal{U}^\alpha{}_i = 0$). Nonetheless, since the Landau-gauge case was studied in details in [4] and since non-Landau gauges with massless ghosts corresponding to broken gauge symmetries have severe IR divergences (cf. the second term in (29)), it is reasonable to assume that $\mathcal{U}^\alpha{}_i$ has been chosen in such a way that all massless ghosts correspond to massless gauge bosons. Additionally, I will assume the total number of independent vectors $\Theta_{gh(\lambda_r)}$ obeying

$$[\widetilde{\mathcal{G}}(m^2_{gh(\lambda)})^{-1}]\,[\Omega(m^2_{gh(\lambda)})^{-1}]\,\Theta_{gh(\lambda_r)} = 0\,, \quad \text{with} \quad m^2_{gh(\lambda)} \neq 0\,, \tag{51}$$

is equal to the number of massive ghosts (and thus, to the number of massive gauge bosons). [13] This ensures that the propagator $\widetilde{\mathcal{G}}(q^2)$ has only simple poles in perturbation theory (note that the tree-level mass matrix of ghosts is not symmetric, and therefore this is not guaranteed even at the lowest order).

Under these assumptions it is easy to find the missing $N_{\rm ker}$ eigenvectors of τ from (48). Comparing (26) with (30) one gets (to all orders)

$$\sigma(m^2_{gh(\lambda)})^i{}_j C^j{}_\gamma(m^2_{gh(\lambda)})\Theta^\gamma_{gh(\lambda_r)} = 0, \tag{52}$$

[12] In what follows, with a little abuse of notation, $[\widetilde{\mathcal{G}}(q^2)^{-1}]$ represents the 1PI 2-point functions of ghosts, regardless of whether it is an invertible matrix or not.

[13] This is merely a technical assumptions: if it is not satisfied, then one has to study the relations between the generalized (rather than plain) eigenvectors of $[\widetilde{\mathcal{G}}(m^2_{gh(\lambda)})^{-1}]$ and $T(m^2_{gh(\lambda)})$, see e.g. [3] and references therein.

and thus $T(m^2_{gh(\lambda)})_{ij}C^j{}_\gamma(m^2_{gh(\lambda)})\Theta^\gamma_{gh(\lambda_r)} = 0$ if $m^2_{gh(\lambda)} \neq 0$. Let's suppose that a linear combination of $C^j{}_\gamma(m^2_{gh(\lambda)})\Theta^\gamma_{gh(\lambda_r)}$ vanishes:

$$\sum_\lambda \sum_r X^{(\lambda_r)} C^j{}_\gamma(m^2_{gh(\lambda)})\Theta^\gamma_{gh(\lambda_r)} = 0. \tag{53}$$

Contracting the above equation with $\mathcal{U}^\alpha{}_j$, using (51) and the explicit form (26) of the ghost-antighost 2-point function we now get

$$\sum_\lambda \sum_r X^{(\lambda_r)} m^2_{gh(\lambda)}\Theta^\gamma_{gh(\lambda_r)} = 0. \tag{54}$$

Under the assumptions listed above, $\Theta^\gamma_{gh(\lambda_r)}$ are linearly independent and correspond to $m^2_{gh(\lambda)} \neq 0$, thus (54) implies $X^{(\lambda_r)} = 0$, what proves that $C^j{}_\gamma(m^2_{gh(\lambda)})\Theta^\gamma_{gh(\lambda_r)}$ form the system of N_{ker} linearly independent vectors obeying $T(m^2_{gh(\lambda)})_{ij}C^j{}_\gamma(m^2_{gh(\lambda)})\Theta^\gamma_{gh(\lambda_r)} = 0$.

To summarize, in theories without physical massless scalars, under a natural assumption that $\mathcal{U}^\alpha{}_i$ produces masses for all ghost that do not correspond to massless gauge bosons, the $T(q^2)^{-1}$ matrix has two kinds of poles: (1) physical poles at $q^2 = m^2_{S(\ell)} \neq 0$ where $m^2_{S(\ell)}$ is a massive pole of $S(q^2)^{-1}$, and (2) unphysical poles at $q^2 = m^2_{gh(\lambda)} \neq 0$, where $m_{gh(\lambda)}$ is a pole mass of a *massive* ghost. This statement is valid at every finite order of perturbation theory, not just in the tree-level approximation.

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