Published for SISSA by Springer

Received: *October 22, 2018*
Revised: *February 13, 2019*
Accepted: *March 8, 2019*
Published: *March 20, 2019*

JHEP03(2019)113

Confinement in 3d $\mathcal{N} = 2$ Spin(N) gauge theories with vector and spinor matters

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Abstract: We present various confinement phases in three-dimensional $\mathcal{N} = 2$ Spin(N) gauge theories with vector and spinor matters. The quantum Coulomb branch in the moduli space of vacua is drastically modified when the rank of the gauge group and the matter contents are changed. In many examples, the Coulomb branch is one- or two-dimensional but its interpretation varies. In some examples, the Coulomb branch becomes three-dimensional and we need to introduce a "dressed" Coulomb branch operator.

Keywords: Confinement, Field Theories in Lower Dimensions, Supersymmetric Gauge Theory

ArXiv ePrint: 1810.06618

https://doi.org/10.1007/JHEP03(2019)113

Contents

1 Introduction

Strongly-coupled gauge theories exhibit various phases depending on the gauge group, matter contents, spacetime dimensions, and so on. When we increase the number of dynamical matters, the theory flows to an IR-free phase. On the other hand, when reducing dynamical matters, the theory becomes strongly-coupled and non-perturbative. Among various strongly-coupled phases, the confinement phase is a most fascinating one since our world is described by QCD which is actually confining. The low-energy dynamics of the confining gauge theories is described by mesons and baryons and exhibits chiral symmetry breaking. We cannot see dynamical quarks as low-energy asymptotic states.

In supersymmetric gauge theories, there is a very special class of the confinement phases, which is known as "s-confinement". Usually, confinement appears, being accompanied by some symmetry breaking, such as chiral symmetry breaking. However, the SUSY gauge theories sometimes show confinement without any symmetry breaking at the origin of the moduli space of vacua. This is called "s-confinement" [1]. In addition to this special property, supersymmetry allows us to exactly study the non-perturbative dynamics of the gauge theory because of non-renormalization theorems and holomorphy [2, 3]. In 4d, the s-confinement phases are classified in [1, 4] for classical and exceptional gauge groups while the corresponding 3d analysis is not completely performed.

In this paper, we study the s-confinement phases of the 3d $\mathcal{N}=2$ supersymmetric Spin(N) gauge theory with vector matters and spinor matters. The 3d SUSY gauge theories contain Higgs and Coulomb branches in the moduli spaces of vacua. In general, the Coulomb branch is drastically modified and different from the classical picture. In [5], we studied the 3d $\mathcal{N}=2$ Spin(7) gauge theory with vector and spinor matters. We found that the Coulomb moduli space is one- or two-dimensional depending on the matter contents and also found various s-confinement phases. These phases were beautifully connected to the quantum-deformed moduli space of the 4d $\mathcal{N}=1$ Spin(7) gauge theory via a non-perturbative superpotential which is generated by the twisted monopoles. In this paper, we will find the similar confinement phases for the Spin(N) $(N>7)$ cases and argue that the Coulomb moduli space is more complicated and in some cases we need three coordinates for describing it. We will give a systematic way of studying the quantum Coulomb branch and the 3d s-confinement phases. These confinement phases are also connected to the 4d quantum-deformed moduli spaces [6, 7].

The rest of this paper is organized as follows. In section 2, we briefly review the Coulomb branch operators which were studied in [8, 9]. In sections 3, 4, 5, 6, 7, 8 and 9, we study the 3d $\mathcal{N}=2$ Spin(N) $(8\le N\le 14)$ gauge theory with vector and spinor matters. We will give a detailed analysis of the quantum Coulomb branch for each rank. In section 10, we will summarize our results and comment on future directions.

2 Coulomb branch in Spin(N) theories

In this section, we will briefly review some Coulomb branch operators in the 3d $\mathcal{N}=2$ Spin(N) gauge theory. These were studied in [8, 9] for the cases where the theory contains

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only vector matters. In [5], we studied these operators in the 3d $\mathcal{N}=2$ Spin(7) theory with vector and spinor matters. In these examples, we found that almost all the classical Coulomb branches are lifted and the quantum Coulomb moduli space is described by only a few operators. Here we review these operators and explain why these directions can remain massless.

For theories with only vector matters, the classical Coulomb branch whose expectation value breaks the gauge group as

$$so(N) \to so(N-2) \times u(1), \tag{2.1}$$

can remain exactly massless and the other directions are all lifted [8]. We denote this operator as Y in this paper. Along this branch, the spinor matters are all massive and integrated out while the vector matters reduce to the massless vector representations of the unbroken Spin$(N-2)$ group. When the number of the vector representations of Spin$(N-2)$ is less than $N-4$, there is no stable supersymmetric vacuum [8] due to the runaway superpotential

$$W_{\text{eff}} \sim \left(\frac{1}{Y^2_{\mathrm{SO}(N-2)} \det\, M_{QQ}}\right)^{\frac{1}{N-N_v-4}} \quad (N_v < N-4). \tag{2.2}$$

Therefore, for $N_v < N-4$, this direction cannot be flat. The theories with $N_v \geq N-4$ vector matters can have this Coulomb branch operator. The Spin(N) theory only with spinor matters also cannot have this branch since the low-energy Spin$(N-2)$ theory has no dynamical matter and its vacuum is unstable due to the monopole superpotential [8]. This observation is consistent with the semi-classical analysis of the Coulomb branch. For concreteness, let us take $N=2n+1$. The classical Coulomb branch is described by the fundamental monopole creating operators $Y_i\,(i=1,\cdots,n)$. In the presence of vector matters, these monopoles generate a non-perturbative superpotential except for Y_n:

$$W = \sum_{i=1}^{n-1} \frac{1}{Y_i} \tag{2.3}$$

The monopole Y_n has too many fermion zero-modes from the vector matters and cannot contribute to the superpotential. As a result, only a one-dimensional direction $Y \sim Y_1^2 Y_2^2 \cdots Y_{n-1}^2 Y_n$ can survive the non-perturbative effects and become exactly massless.

The second Coulomb branch denoted as Z appears when the Spin(N) theory includes spinor matters or when we put the 4d $\mathcal{N}=1$ Spin(N) theory on a circle [5, 8–11]. This operator corresponds to the gauge symmetry breaking

$$so(N) \to so(N-4) \times su(2) \times u(1). \tag{2.4}$$

Along this breaking, the remaining massless components of the spinor representations are charged under the Spin$(N-4) \times$ SU(2) and chargeless under the U(1). Therefore, the low-energy Spin$(N-4) \times$ SU(2) theory may have a stable SUSY vacuum because of the massless dynamical quarks. If we consider this branch for the theory only with vector matters, the

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low-energy SU(2) theory has no massless charged field and the supersymmetry is broken by the monopole superpotential (similar to (2.3)) of the SU(2) sector. As a result, this branch Z is available only for the theories with spinor matters. When we consider the 4d theory on a circle, the twisted monopole corresponds to this operator.

In the following sections, we will study the 3d $\mathcal{N}=2$ Spin(N) gauge theories with $7 < N < 15$, where we will find that the quantum Coulomb branch becomes more richer and we need additional operators to parametrize those additional Coulomb branches. Since the corresponding breaking patterns depend on the rank of the gauge group, we will give a case-by-case analysis in what follows. See [12–15] for various branching rules of Spin(N).

3 Spin(8) theories

We start with the 3d $\mathcal{N}=2$ Spin(8) gauge theories with N_v vectors, N_s spinors and N_c conjugate spinors. The corresponding 4d theories were studied in [16, 17]. There are three **8** dimensional representations in a Spin(8) group, which are denoted as $\mathbf{8}_v, \mathbf{8}_s$ and $\mathbf{8}_c$. Those are related by triality, outer automorphism of the D_4 Dynkin diagram. For the purpose of listing up all the s-confinement phases, it is sufficient to consider the six cases which will be discussed in the following subsections.

When the Coulomb branch Y obtains a non-zero expectation value, the gauge group is spontaneously broken as

$$so(8) \to so(6) \times u(1) \tag{3.1}$$
$$\mathbf{8}_v \to \mathbf{6}_0 + \mathbf{1}_2 + \mathbf{1}_{-2} \tag{3.2}$$
$$\mathbf{8}_s \to \mathbf{4}_1 + \overline{\mathbf{4}}_{-1} \tag{3.3}$$
$$\mathbf{8}_c \to \mathbf{4}_{-1} + \overline{\mathbf{4}}_1 . \tag{3.4}$$

All the components of the spinor matters are charged under the unbroken U(1) gauge subgroup. Hence, they are all massive and integrated out from the low-energy spectrum. In order to obtain a stable SUSY vacuum along the Y direction, the low-energy SO(6) theory also must have a stable SUSY vacuum. This is possible only for $N_v \geq 4$ [8]. Therefore, the Spin(8) theory only with spinor matters generates the monopole potential (2.3) along the Y-branch and does not need this operator.

The second Coulomb branch Z corresponds to the breaking

$$so(8) \to so(4) \times su(2) \times u(1) \tag{3.5}$$
$$\mathbf{8}_v \to (\mathbf{4},\mathbf{1})_0 + (\mathbf{1},\mathbf{2})_{\pm 1} \tag{3.6}$$
$$\mathbf{8}_s \to (\mathbf{2},\mathbf{2})_0 + (\mathbf{2}^*,\mathbf{1})_{\pm 1} \tag{3.7}$$
$$\mathbf{8}_c \to (\mathbf{2},\mathbf{1})_{\pm 1} + (\mathbf{2}^*,\mathbf{2})_0 . \tag{3.8}$$

Notice that the vector representation does not contain any massless field charged under the SU(2) subgroup and cannot make the SU(2) vacuum of the low-energy theory stable. Therefore, this branch exists only for the theory with spinor matters. When there is only a single spinor, the low-energy SU(2) theory has a deformed moduli space $M_{SS}Y_{\mathrm{SU}(2)} \sim 1$ [18, 19]

	Spin(8)	SU(5)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	$\mathbf{8}_v$	$\square$	1	0	R_v
S	$\mathbf{8}_s$	1	0	1	R_s
$\eta = \Lambda^b_{N_v,N_s,N_c}$	1	1	10	2	$10(R_v-1)+2(R_s-1)+12=10R_v+2R_s$
$M_{QQ}:=QQ$	1	$\square\square$	2	0	$2R_v$
$M_{SS}:=SS$	1	1	0	2	$2R_s$
$P_4:=SQ^4S$	1	$\overline{\square}$	4	2	$4R_v+2R_s$
$Y:=Y_1^2Y_2^2Y_3Y_4$	1	1	-10	-4	$-12-10(R_v-1)-4(R_s-1)=2-10R_v-4R_s$

Table 1. 3d $\mathcal{N}=2$ Spin(8) theory with $(N_v,N_s,N_c)=(5,1,0)$.

and thus the origin of the moduli space is excluded from the quantum moduli space. In order that the Z-branch with all the matter fields turned off can be a flat direction, the theory must contain at least two spinors.

3.1 $(N_v, N_s, N_c) = (5, 1, 0)$

The first example is the 3d $\mathcal{N} = 2$ Spin(8) gauge theory with five vectors and one spinor. In this case, the Y-branch is allowed since the low-energy theory contains a 3d $\mathcal{N} = 2$ SO(6) gauge theory with five vectors, which has a supersymmetric vacuum and we can safely take the low-energy limit at this point. On the other hand, the Z-branch, where $\langle Z \rangle$ acquires a vev and all the matter fields are turned off, is not allowed. Consequently, we expect that there is only a single Coulomb branch parametrized by Y.

The low-energy dynamics is described by M_{QQ}, M_{SS}, P_4 and Y. The confining superpotential is constrained by the global symmetries listed in table 1 and we find

$$W = Y\left[M_{SS}^2 \det M_{QQ} + P_4^2 M_{QQ}\right] + \eta Y M_{SS}, \tag{3.9}$$

where the last term appears when we put the 4d Spin(8) theory on $\mathbb{S}_1 \times \mathbb{R}_3$. η is a dynamical scale of the 4d gauge interaction. By integrating out the Coulomb branch operator, we can go up to the 4d $\mathcal{N} = 1$ Spin(8) theory with five vectors and one spinor and reproduce a deformed moduli space [16].

3.2 $(N_v, N_s, N_c) = (4, 2, 0)$

The second example is the 3d $\mathcal{N} = 2$ Spin(8) gauge theory with four vectors and two spinors. As in the previous case, the low-energy SO(6) dynamics along the Y-direction is made stable by four vector matters. Along the Z-direction, the low-energy theory includes an SU(2) gauge theory with four fundamentals, which has a stable SUSY vacuum. Therefore, we need to introduce the two Coulomb branch coordinates, Y and Z.

The low-energy dynamics is described by the Higgs branch operators M_{QQ}, M_{SS}, P_2, P_4 defined in table 2 and the two Coulomb branch coordinates. The confining superpotential becomes

$$W = Z\left[M_{SS}^2 \det M_{QQ} + M_{QQ}^2 P_2^2 + P_4^2\right] + Y(P_2^2 + M_{SS}P_4), \tag{3.10}$$

which is consistent with all the symmetries in table 2.

	Spin(8)	SU(4)	SU(2)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	$\mathbf{8}_v$	$\square$	1	0	0	R_v
S	$\mathbf{8}_s$	1	$\square$	0	1	R_s
$M_{QQ} := QQ$	1	$\square\square$	1	2	0	$2R_v$
$M_{SS} := SS$	1	1	$\square\square$	0	2	$2R_s$
$P_2 := SQ^2S$	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	2	2	$2R_v+2R_s$
$P_4 := SQ^4S$	1	1	$\square\square$	4	2	$4R_v+2R_s$
$Z := Y_1Y_2^2Y_3Y_4$	1	1	1	-8	-4	$-10-8(R_v-1)-4(R_s-1)=2-8R_v-4R_s$
$Y := \sqrt{Y_1^2Y_2^2Y_3Y_4}$	1	1	1	-4	-4	$-6-4(R_v-1)-4(R_s-1)=2-4R_v-4R_s$

Table 2. 3d $\mathcal{N}=2$ Spin(8) theory with $(N_v,N_s,N_c)=(4,2,0)$.

3.3 $(N_v, N_s, N_c) = (4, 1, 1)$

Let us study the case where we introduce both spinor and conjugate spinor matters. The s-confinement phase appears in the 3d $\mathcal{N}=2$ Spin(8) gauge theory with four vectors, one spinor and one conjugate spinor. The corresponding 4d theory was studied in [17]. The Higgs branch is identical to the 4d case and parametrized by three mesons $M_{QQ}, M_{SS}, M_{S'S'}$ and four vector-spinor composites P_1, P_3, P_4, P'_4. These are defined in table 3.

The Coulomb branch Y is allowed since the four vectors $\mathbf{6}_0 \in \mathbf{8}_v$ can make this direction stable. The Z-direction is also allowed due to the two spinors. The matter content and their quantum numbers are summarized in table 3 which includes the dynamical scale of the gauge interaction in the corresponding 4d $\mathcal{N}=1$ Spin(8) theory. The superpotential becomes

$$\begin{aligned} W &= Z\left[M_{SS}M_{S'S'}\det M_{QQ} + M_{QQ}^3P_1^2 + P_3M_{QQ}P_3 + P_4P'_4\right] \\ &\quad + Y\left[P_1P_3 + M_{SS}P'_4 + M_{S'S'}P_4\right] + \eta Z, \end{aligned} \tag{3.11}$$

where the last term appears only when we put the 4d theory on $\mathbb{S}^1 \times \mathbb{R}^3$. By integrating out the Coulomb branch operators, we can reproduce the deformed and un-deformed constraints of the 4d theory [17]. When Y obtains a non-zero vev, the composite operators containing the spinor fields become massive. This is consistent with our definition of the Coulomb branch Y along which the spinor matters become massive.

3.4 $(N_v, N_s, N_c) = (3, 3, 0)$

Let us consider the 3d $\mathcal{N}=2$ Spin(8) gauge theory with three vectors and three spinors. The Y direction is not allowed since the low-energy SO(6) gauge theory contains only three vectors and there is no stable SUSY vacuum. Along the semi-classical region of the Y-branch, the runaway potential (2.2) is generated. Along the Z-branch, the resulting $\mathrm{SO}(4)\times\mathrm{SU}(2)$ gauge theory obtains a stable SUSY vacuum due to a sufficient number of matter fields to stabilize the vacuum.

The low-energy dynamics is described by the four chiral superfields M_{QQ}, M_{SS}, P_2 and Z, which are defined in table 4. By using the symmetries listed in table 4, the confining superpotential is determined as

$$W = Z\left[\det M_{QQ}\det M_{SS} + M_{QQ}M_{SS}P_2^2\right]. \tag{3.12}$$

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	Spin(8)	SU(4)	U(1)$_v$	U(1)$_s$	U(1)$_c$	U(1)$_R$
Q	$\mathbf{8}_v$	$\square$	1	0	0	R_v
S	$\mathbf{8}_s$	1	0	1	0	R_s
S'	$\mathbf{8}_c$	1	0	0	1	R_c
$\eta=\Lambda^b_{N_v,N_s,N_c}$	1	1	8	2	2	$8(R_v-1)+2(R_s-1)+2(R_c-1)+12=8R_v+2R_s+2R_c$
$M_{QQ}:=QQ$	1	$\square\!\square$	2	0	0	$2R_v$
$M_{SS}:=SS$	1	1	0	2	0	$2R_s$
$M_{S'S'}:=S'S'$	1	1	0	0	2	$2R_c$
$P_1:=SQS'$	1	$\square$	1	1	1	$R_v+R_s+R_c$
$P_3:=SQ^3S'$	1	$\overline{\square}$	3	1	1	$3R_v+R_s+R_c$
$P_4:=SQ^4S$	1	1	4	2	0	$4R_v+2R_s$
$P'_4:=S'Q^4S'$	1	1	4	0	2	$4R_v+2R_c$
$\det M_{QQ}$	1	1	8	0	0	$8R_v$
$M_{QQ}P_3^2$	1	1	8	2	2	$8R_v+2R_s+2R_c$
$M^3_{QQ}P_1^2$	1	1	8	2	2	$8R_v+2R_s+2R_c$
P_1P_3	1	1	4	2	2	$4R_v+2R_s+2R_c$
$Z:=Y_1Y_2^2Y_3Y_4$	1	1	-8	-2	-2	$2-8R_v-2R_s-2R_c$
$Y:=\sqrt{Y_1^2Y_2^2Y_3Y_4}$	1	1	-4	-2	-2	$2-4R_v-2R_s-2R_c$

Table 3. 3d $\mathcal{N}=2$ Spin(8) theory with $(N_v,N_s,N_c)=(4,1,1)$.

	Spin(8)	SU(3)	SU(3)	U(1)$_v$	U(1)$_s$	U(1)$_R$
Q	$\mathbf{8}_v$	$\square$	1	1	0	R_v
S	$\mathbf{8}_s$	1	$\square$	0	1	R_s
$M_{QQ}:=QQ$	1	$\square\!\square$	1	2	0	$2R_v$
$M_{SS}:=SS$	1	1	$\square\!\square$	0	2	$2R_s$
$P_2:=SQ^2S$	1	$\overline{\square}$	$\overline{\square}$	2	2	$2R_v+2R_s$
$Z:=Y_1Y_2^2Y_3Y_4$	1	1	1	-6	-6	$-10-6(R_v-1)-6(R_s-1)=2-6R_v-6R_s$

Table 4. 3d $\mathcal{N}=2$ Spin(8) theory with $(N_v,N_s,N_c)=(3,3,0)$.

3.5 $(N_v,N_s,N_c)=(3,2,1)$

The next example is the 3d $\mathcal{N}=2$ Spin(8) gauge theory with three vectors, two spinors and one conjugate spinor. The analysis of the Coulomb branch is the same as the previous example. Since the number of the vector matters is less than four, the Y-branch cannot be a stable vacuum due to the dynamically generated runaway potential (2.2). Along the Z-direction, there are plenty of matter fields charged under the $so(4)\times su(2)$ and the Z-direction can be made stable and supersymmetric.

The Higgs branch is described by the six composite operators, $M_{QQ}, M_{SS}, M_{S'S'}, P_1, P_2$ and P_3, which are defined in table 5. Table 5 summarizes the quantum numbers of the moduli coordinates. The confining superpotential takes

$$W=Z\left[\det M_{QQ}\det M_{SS}M_{S'S'}+P_1P_2P_3+M_{QQ}P_2^2M_{S'S'}+M_{SS}P_3^2+M_{QQ}^2M_{SS}P_1^2\right]. \tag{3.13}$$

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	Spin(8)	SU(3)	SU(2)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_c$	$\mathrm{U}(1)_R$
Q	$\mathbf{8}_v$	$\square$	1	1	0	0	R_v
S	$\mathbf{8}_s$	1	$\square$	0	1	0	R_s
S'	$\mathbf{8}_c$	1	1	0	0	1	R_c
$M_{QQ} := QQ$	1	$\square\!\square$	1	2	0	0	$2R_v$
$M_{SS} := SS$	1	1	$\square\!\square$	0	2	0	$2R_s$
$M_{S'S'} := S'S'$	1	1	1	0	0	2	$2R_c$
$P_1 := SQS'$	1	$\square$	$\square$	1	1	1	$R_v+R_s+R_c$
$P_2 := SQ^2S$	1	$\overline{\square}$	1	2	2	0	$2R_v+2R_s$
$P_3 := SQ^3S'$	1	1	$\square$	3	1	1	$3R_v+R_s+R_c$
$Z := Y_1Y_2^2Y_3Y_4$	1	1	1	-6	-4	-2	$2-6R_v-4R_s-2R_c$

Table 5. 3d $\mathcal{N}=2$ Spin(8) theory with $(N_v,N_s,N_c)=(3,2,1)$.

	Spin(8)	SU(2)	SU(2)	SU(2)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_c$	$\mathrm{U}(1)_R$
Q	$\mathbf{8}_v$	$\square$	1	1	1	0	0	R_v
S	$\mathbf{8}_s$	1	$\square$	1	0	1	0	R_s
S'	$\mathbf{8}_c$	1	1	$\square$	0	0	1	R_c
$M_{QQ} := QQ$	1	$\square\!\square$	1	1	2	0	0	$2R_v$
$M_{SS} := SS$	1	1	$\square\!\square$	1	0	2	0	$2R_s$
$M_{S'S'} := S'S'$	1	1	1	$\square\!\square$	0	0	2	$2R_c$
$P_1 := SQS'$	1	$\square$	$\square$	$\square$	1	1	1	$R_v+R_s+R_c$
$P_2 := SQ^2S$	1	1	1	1	2	2	0	$2R_v+2R_s$
$P_2' := S'Q^2S'$	1	1	1	1	2	0	2	$2R_v+2R_c$
$B := S^2S'^2$	1	1	1	1	0	2	2	$2R_s+2R_c$
$F := S^2S'^2Q^2$	1	1	1	1	2	2	2	$2R_v+2R_s+2R_c$
$Z := Y_1Y_2^2Y_3Y_4$	1	1	1	1	-4	-4	-4	$2-4R_v-4R_s-4R_c$

Table 6. 3d $\mathcal{N}=2$ Spin(8) theory with $(N_v,N_s,N_c)=(2,2,2)$.

3.6 $(N_v, N_s, N_c) = (2, 2, 2)$

The final example of the Spin(8) s-confinement phases is the 3d $\mathcal{N} = 2$ Spin(8) gauge theory with two vectors, two spinors and two conjugate spinors. The theory has a one-dimensional Coulomb branch labeled by Z. The Y-branch is excluded from the moduli space of vacua since the low-energy SO(6) theory along this direction does not have enough vector matters to realize a stable supersymmetric vacuum. Along the semi-classical region of Y, the runaway superpotential (2.2) is generated.

The low-energy dynamics is described by $M_{QQ}, M_{SS}, M_{S'S'}, P_1, P_2, P_2', B, F$ and Z whose quantum numbers are summarized in table 6. The confining superpotential becomes

$$W = Z\left[M_{QQ}^2 M_{SS}^2 M_{S'S'}^2 + M_{QQ}^2 B^2 + M_{SS}^2 P_2'^2 + M_{S'S'}^2 P_2^2 + P_2 P_2' B + F^2\right]. \tag{3.14}$$

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4 Spin(9) theories

Let us move on to the 3d $\mathcal{N}=2$ Spin(9) gauge theories with N_v vectors and N_s spinors. When the Coulomb branch operator Y obtains a non-zero vacuum expectation value, the gauge group is broken as

$$so(9) \to so(7) \times u(1) \tag{4.1}$$
$$\mathbf{9} \to \mathbf{7}_0 + \mathbf{1}_2 + \mathbf{1}_{-2} \tag{4.2}$$
$$\mathbf{16} \to \mathbf{8}_1 + \mathbf{8}_{-1}. \tag{4.3}$$

The spinor matters are all massive and integrated out while the vector matters reduce to the massless $\mathbf{7}$ fields. For the theories only with spinors, this branch is not allowed since the low-energy SO(7) pure SYM has no stable SUSY vacuum due to the monopole potential (2.3). For the theories with $N_v(\geq 5)$ vectors, on the other hand, the low-energy SO(7) SQCD can have a stable SUSY vacuum at the origin of moduli space. Therefore, for $N_v \geq 5$, we need to introduce this coordinate.

The second Coulomb branch is denoted as Z and its expectation value breaks the gauge group as

$$so(9) \to so(5) \times su(2) \times u(1) \tag{4.4}$$
$$\mathbf{9} \to (\mathbf{5},\mathbf{1})_0 + (\mathbf{1},\mathbf{2})_{\pm 1} \tag{4.5}$$
$$\mathbf{16} \to (\mathbf{4},\mathbf{2})_0 + (\mathbf{4},\mathbf{1})_{\pm 1}. \tag{4.6}$$

For the theories only with vectors, this branch is not allowed since the low-energy SU(2) gauge theory has no dynamical field and its vacuum becomes runaway, $W_{\text{eff}} = \frac{1}{Y_{\text{SU}(2)}}$. When the theory includes the spinor matters, the low-energy $\text{SO}(5) \times \text{SU}(2)$ theory can obtain a stable SUSY vacuum due to the presence of $(\mathbf{4},\mathbf{2})_0$. Therefore, we need to introduce a Z coordinate for the theories with spinors.

When $N_s \geq 4$, there could be an additional Coulomb branch V which corresponds to the gauge symmetry breaking

$$so(9) \to su(4) \times u(1) \tag{4.7}$$
$$\mathbf{9} \to \mathbf{1}_0 + \mathbf{4}_1 + \overline{\mathbf{4}}_{-1} \tag{4.8}$$
$$\mathbf{16} \to \mathbf{6}_0 + \mathbf{4}_{-1} + \overline{\mathbf{4}}_1 + \mathbf{1}_2 + \mathbf{1}_{-2}. \tag{4.9}$$

Almost all the components of the vector matter are massive and reduce to a singlet. The spinor matter reduces to $\mathbf{6}_0$ and the dynamics of the $\text{SO}(6) \simeq \text{SU}(4)$ theory has a stable SUSY vacuum for $N_s \geq 4$. In the following subsection, we will only consider the theories with $N_s \leq 3$ spinors, where the runaway superpotential (2.2) is dynamically generated. Therefore, this operator does not appear for $N_s \leq 3$.

4.1 $(N_v, N_s) = (5, 1)$

The first example of the Spin(9) s-confinement is the 3d $\mathcal{N}=2$ Spin(9) gauge theory with five vectors and one spinor. In this case, we need to introduce the two Coulomb branch

	Spin(9)	SU(5)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	$\mathbf{9}$	$\square$	1	0	R_v
S	$\mathbf{16}$	1	0	1	R_s
$\eta = \Lambda^b_{N_v,N_s}$	1	1	10	4	$10R_v + 4R_s$
$M_{QQ} := QQ$	1	$\square\square$	2	0	$2R_v$
$M_{SS} := SS$	1	1	0	2	$2R_s$
$P_1 := SQS$	1	$\square$	1	2	$R_v + 2R_s$
$P_4 := SQ^4S$	1	$\overline{\square}$	4	2	$4R_v + 2R_s$
$P_5 := SQ^5S$	1	1	5	2	$5R_v + 2R_s$
$Z := Y_1Y_2^2Y_3^2Y_4$	1	1	-10	-4	$2 - 10R_v - 4R_s$
$Y := \sqrt{Y_1^2Y_2^2Y_3^2Y_4}$	1	1	-5	-4	$2 - 5R_v - 4R_s$

Table 7. 3d $\mathcal{N} = 2$ Spin(9) theory with $(N_v, N_s) = (5, 1)$.

coordinates Z and Y. The Higgs branch is described by the five composite operators, M_{QQ}, M_{SS}, P_1, P_4 and P_5 (defined in table 7). The confining superpotential becomes

$$W = Z\left[M_{SS}^2 \det M_{QQ} + M_{QQ}^4 P_1^2 + M_{QQ}P_4^2 + P_5^2\right] + Y\left[P_1P_4 + M_{SS}P_5\right] + \eta Z, \qquad (4.10)$$

where the last term appears when we consider the corresponding 4d theory on a circle. By integrating out the Coulomb branches, we can reproduce the quantum-deformed moduli space of the 4d theory [17]. From the superpotential above, we find that the composite operators including the spinor fields become massive along the Y directions.

4.2 $(N_v, N_s) = (3, 2)$

The second example is the 3d $\mathcal{N} = 2$ Spin(9) gauge theories with three vectors and two spinors. In this case, we need not introduce the Coulomb branch coordinate Y since the number of the vector matters is less than five and the runaway potential (2.2) is generated. The Coulomb branch is one-dimensional and parametrized by Z. The Higgs branch operators are listed in table 8. The confining superpotential is determined from table 8 as follows.

$$\begin{aligned} W = Z\Big[&M_{QQ}^3(M_{SS}^2 + B)^2 + M_{QQ}^2P_1^2(M_{SS}^2 + B) + M_{QQ}P_2^2(M_{SS}^2 + B) \\ &+ M_{SS}P_1P_2P_3 + (P_1P_2)^2 + P_3^2(M_{SS}^2 + B) + N^2\Big] \end{aligned} \qquad (4.11)$$

4.3 $(N_v, N_s) = (1, 3)$

The final example is the 3d $\mathcal{N} = 2$ Spin(9) gauge theory with one vector and three spinors. In this case, the Coulomb branch is again one-dimensional and parametrized by Z. The Higgs branch is described by the five composite operators defined in table 9. The confining superpotential is determined as

$$W = Z\left[M_{QQ}(M_{SS}^6 + B^3 + M_{SS}^2B^2) + M_{SS}^4P_1^2 + (P_1B)^2 + (B + M_{SS}^2)N^2\right]. \qquad (4.12)$$

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	Spin(9)	SU(3)	SU(2)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	$\mathbf{9}$	$\square$	1	1	0	R_v
S	$\mathbf{16}$	1	$\square$	0	1	R_s
$M_{QQ} := QQ$	1	$\square\square$	1	2	0	$2R_v$
$M_{SS} := SS$	1	1	$\square\square$	0	2	$2R_s$
$P_1 := SQS$	1	$\square$	$\square\square$	1	2	$R_v + 2R_s$
$P_2 := SQ^2S$	1	$\overline{\square}$	1	2	2	$2R_v + 2R_s$
$P_3 := SQ^3S$	1	1	1	3	2	$3R_v + 2R_s$
$N := S^4Q^3$	1	1	1	3	4	$3R_v + 4R_s$
$B := S^4$	1	1	1	0	4	$4R_s$
$Z := Y_1 Y_2^2 Y_3^2 Y_4$	1	1	1	-6	-8	$2 - 6R_v - 8R_s$

Table 8. 3d $\mathcal{N} = 2$ Spin(9) theory with $(N_v, N_s) = (3, 2)$.

	Spin(9)	SU(3)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	$\mathbf{9}$	1	1	0	R_v
S	$\mathbf{16}$	$\square$	0	1	R_s
$M_{QQ} := QQ$	1	1	2	0	$2R_v$
$M_{SS} := SS$	1	$\square\square$	0	2	$2R_s$
$P_1 := SQS$	1	$\square\square$	1	2	$R_v + 2R_s$
$B := S^4$	1	$\overline{\square\square}$	0	4	$4R_s$
$N := S^4Q$	1	$\square$	1	4	$R_v + 4R_s$
$Z := Y_1 Y_2^2 Y_3^2 Y_4$	1	1	-2	-12	$2 - 2R_v - 12R_s$

Table 9. 3d $\mathcal{N} = 2$ Spin(9) theory with $(N_v, N_s) = (1, 3)$.

5 Spin(10) theories

Next, we move on to the 3d $\mathcal{N} = 2$ Spin(10) theory with N_v vectors, N_s spinors and $N_{s'}$ (complex) conjugate spinors. This case will be very special since we have to introduce a dressed Coulomb branch operator. There are three Coulomb branches where vector and spinor representations supply massless fields charged under the unbroken gauge group. The first Coulomb branch Y leads to the following breaking pattern

$$so(10) \to so(8) \times u(1) \tag{5.1}$$

$$\mathbf{10} \to \mathbf{8}_{v,0} + \mathbf{1}_2 + \mathbf{1}_{-2} \tag{5.2}$$

$$\mathbf{16} \to \mathbf{8}_{c,-1} + \mathbf{8}_{s,1} \tag{5.3}$$

$$\overline{\mathbf{16}} \to \mathbf{8}_{c,1} + \mathbf{8}_{s,-1}. \tag{5.4}$$

The spinor fields are all massive and integrated out. In order to make the low-energy SO(8) dynamics stable and evade the runaway potential (2.2), we can use $\mathbf{8}_{v,0}$ from the vector

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representation. Since the 3d $\mathcal{N}=2$ SO(8) theory with N_v vectors has a stable SUSY vacuum for $N_v \geq 6$, the Y-branch is available for $N_v \geq 6$.

The second Coulomb branch Z leads to the breaking

$$so(10) \to so(6) \times su(2) \times u(1) \tag{5.5}$$
$$\mathbf{10} \to (\mathbf{6},\mathbf{1})_0 + (\mathbf{1},\mathbf{2})_{\pm 1} \tag{5.6}$$
$$\mathbf{16} \to (\mathbf{4},\mathbf{1})_{\pm 1} + (\overline{\mathbf{4}},\mathbf{2})_0 \tag{5.7}$$
$$\overline{\mathbf{16}} \to (\overline{\mathbf{4}},\mathbf{1})_{\pm 1} + (\mathbf{4},\mathbf{2})_0. \tag{5.8}$$

In order that this branch becomes a flat direction, the vacuum of the low-energy SO(6) $\times$ SU(2) theory must have a stable SUSY vacuum. The SU(2) part is made stable by $(\overline{\mathbf{4}},\mathbf{2})_0 \in \mathbf{16}$ or $(\mathbf{4},\mathbf{2})_0 \in \overline{\mathbf{16}}$. The SO(6) part is made stable by both vector and spinor matters.

The third Coulomb branch X needs a special care. This operator corresponds to the gauge symmetry breaking

$$so(10) \to su(4) \times so(2) \times u(1) \tag{5.9}$$
$$\mathbf{10} \to \mathbf{4}_{0,-1} + \overline{\mathbf{4}}_{0,-1} + \mathbf{1}_{2,0} + \mathbf{1}_{-2,0} \tag{5.10}$$
$$\mathbf{16} \to \mathbf{4}_{-1,-1} + \overline{\mathbf{4}}_{-1,1} + \mathbf{6}_{1,0} + \mathbf{1}_{1,2} + \mathbf{1}_{1,-2} \tag{5.11}$$
$$\overline{\mathbf{16}} \to \mathbf{4}_{1,-1} + \overline{\mathbf{4}}_{1,1} + \mathbf{6}_{-1,0} + \mathbf{1}_{-1,2} + \mathbf{1}_{-1,-2}. \tag{5.12}$$

Notice that there are two U(1) factors and the Coulomb branch is related to the second U(1) factor. Along this branch, the effective Chern-Simons level between $so(2)$ and $u(1)$ is introduced, which is calculated as

$$k_{\text{eff}}^{so(2),u(1)} = -N_s + N_{s'}. \tag{5.13}$$

Therefore, the bare Coulomb branch X is not gauge invariant and its $so(2)$ charge is $N_s - N_{s'}$. In order to construct a gauge invariant coordinate, we can use $\mathbf{6}_{\pm 1,0}$ from the spinor representation or $\mathbf{1}_{2,0}$ from the vector representation. The vacuum of the low-energy SU(4) theory can be made stable only by spinor matters.

5.1 $(N_v, N_s, N_{s'}) = (6, 1, 0)$

The first example is the 3d $\mathcal{N}=2$ Spin(10) theory with six vectors and one spinor. The corresponding 4d theory was studied in [20, 21]. The Higgs branch is described by three composite operators M_{QQ}, P_1 and P_5 which are defined in table 10. The Coulomb moduli are two-dimensional, which are parametrized by Y and Z. The Coulomb branch operator X now has an SO(2) $\simeq$ U(1) charge 2 and cannot be made gauge invariant. The confining superpotential becomes

$$W = Z\left[M_{QQ}P_5^2 + M_{QQ}^5 P_1^2\right] + YP_1P_5, \tag{5.14}$$

which is consistent with the 4d result [20, 21]. Notice that when Y obtains a non-zero vev, P_1 and P_2 become massive. This is consistent with the fact that the spinor fields are massive along the Y-branch.

	Spin(10)	SU(6)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	**10**	$\square$	1	0	R_v
S	**16**	1	0	1	R_s
$M_{QQ} := QQ$	1	$\square\!\square$	2	0	$2R_v$
$P_1 := SQS$	1	$\square$	1	2	$R_v + 2R_s$
$P_5 := SQ^5S$	1	$\overline{\square}$	5	2	$5R_v + 2R_s$
$Z := Y_1 Y_2^2 Y_3^2 Y_4 Y_5$	1	1	-12	-4	$2 - 12R_v - 4R_s$
$Y := \sqrt{Y_1^2 Y_2^2 Y_3^2 Y_4 Y_5}$	1	1	-6	-4	$2 - 6R_v - 4R_s$

Table 10. 3d $\mathcal{N}=2$ Spin(10) theory with $(N_v, N_s, N_{s'}) = (6, 1, 0)$.

	Spin(10)	SU(4)	SU(2)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	**10**	$\square$	1	1	0	R_v
S	**16**	1	$\square$	0	1	R_s
$M_{QQ} := QQ$	1	$\square\!\square$	1	2	0	$2R_v$
$P_1 := SQS$	1	$\square$	$\square\!\square$	1	2	$R_v + 2R_s$
$P_3 := SQ^3S$	1	$\overline{\square}$	1	3	2	$3R_v + 2R_s$
$B := S^4$	1	1	1	0	4	$4R_s$
$R := S^4Q^4$	1	1	1	4	4	$4R_v + 4R_s$
$Z := Y_1 Y_2^2 Y_3^2 Y_4 Y_5$	1	1	1	-8	-8	$2 - 8R_v - 8R_s$

Table 11. 3d $\mathcal{N}=2$ Spin(10) theory with $(N_v, N_s, N_{s'}) = (4, 2, 0)$.

5.2 $(N_v, N_s, N_{s'}) = (4, 2, 0)$

The second example is the 3d $\mathcal{N}=2$ Spin(10) theory with four vectors and two spinors. The corresponding 4d theory was studied in [22, 23]. The Coulomb branch Y is not available since the low-energy SO(8) theory with four vectors has no stable SUSY vacuum (remember (2.2)). The X-branch is also not allowed in the same manner. As a result, the Coulomb branch is one-dimensional, which is described by Z. Table 11 shows the moduli coordinates and their quantum numbers. The confining superpotential becomes

$$W = Z\left[B^2 \det M_{QQ} + M_{QQ}^3 P_1^2 B + M_{QQ}^2 P_1^4 + M_{QQ} P_3 P_1^3 + B M_{QQ} P_3^2 + (P_1 P_3)^2 + R^2\right], \tag{5.15}$$

which is consistent with all the symmetries in table 11 and the 4d result [22].

5.3 $(N_v, N_s, N_{s'}) = (4, 1, 1)$

Let us move on to the 3d $\mathcal{N}=2$ Spin(10) theory with four vectors, one spinor and one (complex) conjugate spinor. The Coulomb branch Y is not allowed for the same reason as the previous example. The operator X is lifted since the low-energy $\mathrm{SO}(6) \simeq \mathrm{SU}(4)$ theory only has two massless vectors and its vacuum is unstable due to the runaway potential (2.2). Consequently, the Coulomb branch is one-dimensional and described by Z. The confining

	Spin(10)	SU(4)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_{s'}$	$\mathrm{U}(1)_R$
Q	**10**	$\square$	1	0	0	R_v
S	**16**	1	0	1	0	R_s
$\overline{S}$	$\overline{\mathbf{16}}$	1	0	0	1	$R_{s'}$
$M_{QQ} := QQ$	1	$\square\!\square$	2	0	0	$2R_v$
$M_{S\overline{S}} := S\overline{S}$	1	1	0	1	1	$R_s+R_{s'}$
$P_1 := SQS$	1	$\square$	1	2	0	R_v+2R_s
$\overline{P}_1 := \overline{S}Q\overline{S}$	1	$\square$	1	0	2	$R_v+2R_{s'}$
$R_2 := SQ^2\overline{S}$	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	2	1	1	$2R_v+R_s+R_{s'}$
$R_4 := SQ^4\overline{S}$	1	1	4	1	1	$4R_v+R_s+R_{s'}$
$T_0 := S^2\overline{S}^2$	1	1	0	2	2	$2R_s+2R_{s'}$
$T_2 := S^2\overline{S}^2Q^4$	1	1	4	2	2	$4R_v+2R_s+2R_{s'}$
$Z := Y_1Y_2^2Y_3^2Y_4Y_5$	1	1	-8	-4	-4	$2-8R_v-4R_s-4R_{s'}$

Table 12. 3d $\mathcal{N}=2$ Spin(10) theory with $(N_v, N_s, N_{s'}) = (4,1,1)$.

superpotential becomes

$$\begin{aligned} W = Z\Big[&(M_{S\overline{S}}^4 + M_{S\overline{S}}^2 T_0 + T_0^2)\det M_{QQ} + M_{QQ}^3 P_1\overline{P}_1(T_0 + M_{S\overline{S}}^2) \\ &+ M_{QQ}^2(P_1^2\overline{P}_1^2 + R_2^2(T_0 + M_{S\overline{S}}^2)) + R_2^2(R_2^2 + T_4) + R_4^2(T_0 + M_{S\overline{S}}^2) + (R_4 M_{S\overline{S}} + B_4)^2\Big], \end{aligned} \tag{5.16}$$

which is consistent with all the symmetries in table 12.

5.4 $(N_v, N_s, N_{s'}) = (2,3,0)$

Let us consider the 3d $\mathcal{N}=2$ Spin(10) theory with two vectors and three spinors. The Coulomb branch Y is not allowed since the number of the vector matters is less than six. The operator X is not available since the low-energy SO(6) theory with three vectors has no stable SUSY vacuum. In this case, only the Z-branch is available. The confined degrees of freedom are summarized in table 13. The confining superpotential becomes

$$W = Z\left[\det M_{QQ}\det B + M_{QQ}(P_1 B)^2 + BR^2\right]. \tag{5.17}$$

5.5 $(N_v, N_s, N_{s'}) = (2,2,1)$

The next example is the 3d $\mathcal{N}=2$ Spin(10) theory with two vectors, two spinors and one conjugate spinor. As in the previous case, the Coulomb branch is described by the single operator Z. The Coulomb branch Y is not available since N_v is less than six. The Coulomb branch X is not allowed since the low-energy SO(6) theory with three vectors has a runaway potential (2.2). The moduli coordinates and their quantum numbers are summarized in table 14. We will not explicitly write down the confining superpotential.

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	Spin(10)	SU(2)	SU(3)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	$\mathbf{10}$	$\square$	1	1	0	R_v
S	$\mathbf{16}$	1	$\square$	0	1	R_s
$M_{QQ} := QQ$	1	$\square\square$	1	2	0	$2R_v$
$P_1 := SQS$	1	$\square$	$\square\square$	1	2	$R_v + 2R_s$
$B := S^4$	1	1	$\overline{\square\square}$	0	4	$4R_s$
$R := S^4Q^2$	1	1	$\square$	2	4	$2R_v + 4R_s$
$Z := Y_1Y_2^2Y_3^2Y_4Y_5$	1	1	1	-4	-12	$2 - 4R_v - 12R_s$

Table 13. 3d $\mathcal{N} = 2$ Spin(10) theory with $(N_v, N_s, N_{s'}) = (2, 3, 0)$.

	Spin(10)	SU(2)	SU(2)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_{s'}$	$\mathrm{U}(1)_R$
Q	$\mathbf{10}$	$\square$	1	1	0	0	R_v
S	$\mathbf{16}$	1	$\square$	0	1	0	R_s
$\overline{S}$	$\overline{\mathbf{16}}$	1	1	0	0	1	R_s
$M_{QQ} := QQ$	1	$\square\square$	1	2	0	0	$2R_v$
$M_{S\overline{S}} := S\overline{S}$	1	1	$\square$	0	1	1	$R_s + R_{s'}$
$M_{2,S\overline{S}} := SQ^2\overline{S}$	1	1	$\square$	2	1	1	$2R_v + R_s + R_{s'}$
$P_1 := SQS$	1	$\square$	$\square\square$	1	2	0	$R_v + 2R_s$
$\overline{P}_1 := \overline{S}Q\overline{S}$	1	$\square$	1	1	0	2	$R_v + 2R_{s'}$
$B := S^4$	1	1	1	0	4	0	$4R_s$
$F := S^2\overline{S}^2$	1	1	$\square\square$	0	2	2	$2R_s + 2R_{s'}$
$R_1 := S^3\overline{S}Q$	1	$\square$	$\square$	1	3	2	$R_v + 3R_s + R_{s'}$
$R_2 := S^2\overline{S}^2Q^2$	1	1	1	2	2	2	$2R_v + 2R_s + 2R_{s'}$
$Z := Y_1Y_2^2Y_3^2Y_4Y_5$	1	1	1	-4	-8	-4	$2 - 4R_v - 8R_s - 4R_{s'}$

Table 14. 3d $\mathcal{N} = 2$ Spin(10) theory with $(N_v, N_s, N_{s'}) = (2, 2, 1)$.

5.6 $(N_v, N_s, N_{s'}) = (0, 4, 0)$

Next, we move on to the theories with spinor matters and without a vector. The first example of the s-confinement is the 3d $\mathcal{N} = 2$ Spin(10) gauge theory with four spinors. Since the theory does not include the vector matters, the Coulomb branch Y is not available. The direction Z can be made stable by the component $(\overline{\mathbf{4}}, \mathbf{2})_0 \in \mathbf{16}$. The Coulomb branch X is now charged under the $so(2)$ subgroup and cannot be made gauge invariant since there is no complex conjugate spinor $(\overline{\mathbf{16}})$ in the theory. As a result, the Coulomb branch is one-dimensional and described by Z. The confining superpotential becomes

$$W = ZB^4, \tag{5.18}$$

which is consistent with all the symmetries in table 15.

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	Spin(10)	SU(4)	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
S	$\mathbf{16}$	$\square$	1	R_s
$B := S^4$	1	$\begin{smallmatrix}\square & \square \\ \square & \square\end{smallmatrix}$	4	$4R_s$
$Z := Y_1 Y_2^2 Y_3^2 Y_4 Y_5$	1	1	-16	$2 - 16R_s$

Table 15. 3d $\mathcal{N} = 2$ Spin(10) theory with $(N_v, N_s, N_{s'}) = (0, 4, 0)$.

	Spin(10)	SU(3)	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_{s'}$	$\mathrm{U}(1)_R$
S	$\mathbf{16}$	$\square$	1	0	R_s
$\overline{S}$	$\overline{\mathbf{16}}$	1	0	1	$R_{s'}$
$M_{S\overline{S}} := S\overline{S}$	1	$\square$	1	1	$R_s + R_{s'}$
$F_2 := S^2\overline{S}^2$	1	$\square\square$	2	2	$2R_s + 2R_{s'}$
$B := S^4$	1	$\overline{\square\square}$	4	0	$4R_s$
$C := S^5\overline{S}$	1	$\square\square$	5	1	$5R_s + R_{s'}$
$Z := Y_1 Y_2^2 Y_3^2 Y_4 Y_5$	1	1	-12	-4	$2 - 12R_s - 4R_{s'}$
$X^{\text{dressed}} := \overline{S}^2 \sqrt{Y_1 Y_2^2 Y_3^3 Y_4^2 Y_5^2}$	1	1	-9	-1	$2 - 9R_s - R_{s'}$

Table 16. 3d $\mathcal{N} = 2$ Spin(10) theory with $(N_v, N_s, N_{s'}) = (0, 3, 1)$.

5.7 $(N_v, N_s, N_{s'}) = (0, 3, 1)$

Let us consider the 3d $\mathcal{N} = 2$ Spin(10) theory with three spinors and a single (complex) conjugate spinor. The Coulomb branch Z is available since the low-energy SU(4) theory with two fundamentals and six anti-fundamentals has a stable SUSY vacuum [24]. Similarly, the Coulomb branch X is allowed although it is not gauge invariant. Therefore, we need to introduce the dressed operator

$$X^{\text{dressed}} := X\overline{S}^2. \tag{5.19}$$

The moduli coordinates and their quantum numbers are summarized in table 16. The confining superpotential becomes

$$W = Z\left[B^2(F_2 + M_{S\overline{S}}^2)^2 + C^2(F_2 + M_{S\overline{S}}^2)\right] + X^{\text{dressed}} BC. \tag{5.20}$$

5.8 $(N_v, N_s, N_{s'}) = (0, 2, 2)$

The final example is the 3d $\mathcal{N} = 2$ Spin(10) theory with two spinors and two (complex) conjugate spinors. The theory is "vector-like" in the sense that there are equal numbers of spinors and conjugate spinors. Since the theory is now "vector-like", the bare Coulomb branch operator X is gauge invariant and does not need "dressing". The low-energy $\mathrm{SU}(4) \simeq \mathrm{SO}(6)$ theory along $\langle X \rangle \neq 0$ contains four vector matters and hence its low-energy vacuum is stable and supersymmetric. The Coulomb branch Z is also allowed since the low-energy SU(4) theory with four fundamental flavors has a stable SUSY vacuum. Table 17 summarizes the quantum numbers of the moduli coordinates.

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	Spin(10)	$\mathrm{SU}(2)_s$	$\mathrm{SU}(2)_{s'}$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_{s'}$	$\mathrm{U}(1)_R$
S	$\mathbf{16}$	$\square$	1	1	0	R_s
$\overline{S}$	$\overline{\mathbf{16}}$	1	$\square$	0	1	$R_{s'}$
$M_{S\overline{S}} := S\overline{S}$	1	$\square$	$\square$	1	1	$R_s + R_{s'}$
$B := S^4$	1	1	1	4	0	$4R_s$
$\overline{B} := \overline{S}^4$	1	1	1	0	4	$4R_{s'}$
$F_2 := S^2\overline{S}^2$	1	$\square\square$	$\square\square$	2	2	$2R_s + 2R_{s'}$
$F_3 := S^3\overline{S}^3$	1	$\square$	$\square$	3	3	$3R_s + 3R_{s'}$
$C_{6,2} = S^6\overline{S}^2$	1	1	1	6	2	$6R_s + 2R_{s'}$
$C_{2,6} := S^2\overline{S}^6$	1	1	1	2	6	$2R_s + 6R_{s'}$
$Z := Y_1 Y_2^2 Y_3^2 Y_4 Y_5$	1	1	1	-8	-8	$2 - 8R_s - 8R_{s'}$
$X := \sqrt{Y_1 Y_2^2 Y_3^3 Y_4^2 Y_5^2}$	1	1	1	-6	-6	$2 - 6R_s - 6R_{s'}$

Table 17. 3d $\mathcal{N} = 2$ Spin(10) theory with $(N_v, N_s, N_{s'}) = (0, 2, 2)$.

6 Spin(11) theories

Here, we consider the 3d $\mathcal{N} = 2$ Spin(11) theory with N_v vectors and N_s spinors. The correponding 4d theory was studied in [25]. As will be explained in the following subsections, the s-confinement phases appear in $(N_v, N_s) = (5, 1)$ and $(N_v, N_s) = (1, 2)$. There are three Coulomb branches whose branching rules include the fields neutral under the unbroken U(1) subgroup but charged under the non-abelian subgroups. The first Coulomb branch Y corresponds to the breaking

$$so(11) \to so(9) \times u(1) \tag{6.1}$$

$$\mathbf{11} \to \mathbf{9}_0 + \mathbf{1}_2 + \mathbf{1}_{-2} \tag{6.2}$$

$$\mathbf{32} \to \mathbf{16}_1 + \mathbf{16}_{-1}, \tag{6.3}$$

where all the components of the spinor representation are massive and those masses are proportional to the U(1) charges. The vector field reduces to the massless **9** representation. When the Spin(11) theory has more than six vectors, the vacuum of the low-energy SO(9) theory can be stable and supersymmetric due to the sufficient number of **9** vectors. In the s-confining examples which will be discussed in the following subsections, the theory contains $N_v \leq 5$ vectors and generates a runaway potential (2.2). Therefore, this branch does not appear in what follows. See [8], where the 3d $\mathcal{N} = 2$ SO(11) theory with N_v vectors is studied and this operator is introduced.

When the second Coulomb branch Z obtains an expectation value, the gauge group is broken as

$$so(11) \to so(7) \times su(2) \times u(1) \tag{6.4}$$

$$\mathbf{11} \to (\mathbf{7}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{2})_{\pm 1} \tag{6.5}$$

$$\mathbf{32} \to (\mathbf{8}, \mathbf{2})_0 + (\mathbf{8}, \mathbf{1})_{\pm 1}. \tag{6.6}$$

Along this direction, the Spin(11) theory must have at least one spinor so that the vacuum of the low-energy SU(2) theory has a stable supersymmetric vacuum. Otherwise, this direction is quantum-mechanically lifted and excluded from the chiral ring due to the SU(2) monopole potential $W_{\text{eff}} \sim \frac{1}{Y_{\text{SU}(2)}}$. In order to make the vacuum of the low-energy SO(7) theory stable, we have to take (N_v, N_s) above the s-confinement bound of the Spin(7) theory, which was studied in [5].

The third Coulomb branch X corresponds to the breaking

$$so(11) \rightarrow so(3) \times su(4) \times u(1) \tag{6.7}$$

$$\mathbf{11} \rightarrow (\mathbf{3}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{4})_1 + (\mathbf{1}, \overline{\mathbf{4}})_{-1} \tag{6.8}$$

$$\mathbf{32} \rightarrow (\mathbf{2}, \mathbf{6})_0 + (\mathbf{2}, \mathbf{1})_{-2} + (\mathbf{2}, \mathbf{1})_{-2} + (\mathbf{2}, \mathbf{4})_{-1} + (\mathbf{2}, \overline{\mathbf{4}})_1. \tag{6.9}$$

When there are two spinor matters, the low-energy SU(4) dynamics is stable by the two massless components $(\mathbf{2}, \mathbf{6})_0$. The SO(3) vacuum can be made stable by $(\mathbf{3}, \mathbf{1})_0$ or $(\mathbf{2}, \mathbf{6})_0$. Therefore, the Spin(11) theory with more than one spinor includes this branch.

6.1 $(N_v, N_s) = (5, 1)$

The first s-confining example is the 3d $\mathcal{N} = 2$ Spin(11) gauge theory with five vectors and one spinor. The corresponding 4d theory was studied in [25]. Since the number of the vector matters is less than seven, the Coulomb branch Y is not available. The X-branch is also not required since a single spinor $(\mathbf{2}, \mathbf{6})_0 \in \mathbf{16}$ cannot make the low-energy SU(4) $\simeq$ SO(6) vacuum stable, where the runaway potential (2.2) is generated. As a result, there is a one-dimensional Coulomb branch parametrized by Z.

The low-energy dynamics is dual to a non-gauge theory with the Higgs branch fields M_{QQ}, B, P_1, P_2, R and the Coulomb branch field Z. Table 18 shows the quantum numbers of these moduli fields. The confining superpotential takes

$$\begin{aligned} W = Z \big[& B^2 \det M_{QQ} + B M_{QQ}^4 P_1^2 + B M_{QQ}^3 P_2^2 \\ & + M_{QQ}^2 P_1^2 P_2^2 + M_{QQ} P_2^4 + P_1 P_2^2 P_5 + B P_5^2 + R^2 \big] . \end{aligned} \tag{6.10}$$

When we put the 4d theory on $\mathbb{S}^1 \times \mathbb{R}^3$, an additional non-perturbative superpotential $\Delta W = \eta Z$ is added to the above superpotential. By integrating out the Coulomb branch operator, we can reproduce the quantum-mechanically deformed moduli space in the 4d $\mathcal{N} = 1$ Spin(11) theory with five vectors and one spinor [25].

6.2 $(N_v, N_s) = (1, 2)$

The second example is the 3d $\mathcal{N} = 2$ Spin(11) gauge theory with one vector and two spinors. The Y-branch is not available since there is only a single vector which is insufficient for the stable SO(9) vacuum. The Z-branch is required since the SO(7) vacuum is made stable by $(\mathbf{7}, \mathbf{1})_0$ and two $(\mathbf{8}, \mathbf{2})_0$. In addition to Z, the X-branch can be now turned on since the SO(6) $\simeq$ SU(4) theory with four vectors $\mathbf{6}$ can have a stable SUSY vacuum. The Coulomb branch is two-dimensional and the Higgs branch is described by the fields listed in table 19. We will not explicitly write down the confining potential but one can construct it from table 19.

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	Spin(11)	SU(5)	U(1)$_v$	U(1)$_s$	U(1)$_R$
Q	**11**	$\square$	1	0	R_v
S	**32**	1	0	1	R_s
$M_{QQ} := QQ$	1	$\square\square$	2	0	$2R_v$
$B := S^4$	1	1	0	4	$4R_s$
$P_1 := SQS$	1	$\square$	1	2	$R_v + 2R_s$
$P_2 := SQ^2S$	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	2	2	$2R_v + 2R_s$
$P_5 := SQ^5S$	1	1	5	2	$5R_v + 2R_s$
$R := S^4Q^5$	1	1	5	4	$5R_v + 4R_s$
$Z := Y_1Y_2^2Y_3^2Y_4^2Y_5$	1	1	-10	-8	$2 - 10R_v - 8R_s$

Table 18. 3d $\mathcal{N} = 2$ Spin(11) theory with $(N_v, N_s) = (5, 1)$.

	Spin(11)	SU(2)	U(1)$_v$	U(1)$_s$	U(1)$_R$
Q	**11**	1	1	0	R_v
S	**32**	$\square$	0	1	R_s
$M_{QQ} := QQ$	1	1	2	0	$2R_v$
$M_{SS} := SS$	1	1	0	2	$2R_s$
$B := S^4$	1	$\square\square\square\square$	0	4	$4R_s$
$B' := S^4$	1	1	0	4	$4R_s$
$P_1 := SQS$	1	$\square\square$	1	2	$R_v + 2R_s$
$F_1 := S^4Q$	1	$\square\square$	1	4	$R_v + 4R_s$
$F_1' := S^4Q$	1	1	1	4	$R_v + 4R_s$
$F_2 := S^4Q^2$	1	1	2	4	$2R_v + 4R_s$
$T_0 := S^6$	1	1	0	6	$6R_s$
$T_1 := S^6Q$	1	$\square\square$	1	6	$R_v + 6R_s$
$U_0 := S^8$	1	1	0	8	$8R_s$
$U_1 := S^8Q$	1	1	1	8	$R_v + 8R_s$
$Z := Y_1Y_2^2Y_3^2Y_4^2Y_5$	1	1	-2	-16	$2 - 2R_v - 16R_s$
$X := \sqrt{Y_1Y_2^2Y_3^3Y_4^4Y_5^2}$	1	1	-2	-12	$2 - 2R_v - 12R_s$

Table 19. 3d $\mathcal{N} = 2$ Spin(11) theory with $(N_v, N_s) = (1, 2)$.

7 Spin(12) theories

Let us move on to the 3d $\mathcal{N} = 2$ Spin(12) theory with N_v vectors, N_s (Weyl) spinors and $N_{s'}$ conjugate (another Weyl) spinors. The correponding 4d theory was studied, for instance, in [26]. We will find three s-confinement examples for $(N_v, N_s, N_{s'}) = (6, 1, 0), (2, 2, 0)$ and $(2, 1, 0)$. In this case, various directions of the classical Coulomb branches can be stable and survive quantum corrections since we have two inequivalent spinors and the branching

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rules of these spinors are different. We start with the Y direction whose expectation value leads to the breaking

$$so(12) \to so(10) \times u(1) \tag{7.1}$$
$$\mathbf{12} \to \mathbf{10}_0 + \mathbf{1}_2 + \mathbf{1}_{-2} \tag{7.2}$$
$$\mathbf{32} \to \mathbf{16}_{-1} + \overline{\mathbf{16}}_1 \tag{7.3}$$
$$\mathbf{32}' \to \mathbf{16}_1 + \overline{\mathbf{16}}_{-1}. \tag{7.4}$$

Since the spinor matters are massive along this direction, the Spin(12) theory with only spinors cannot have this branch as a flat direction due to the monopole potential. In order to make the vacuum of the low-energy SO(10) theory stable, the theory must have $N_v \geq 8$ vector matters. In this section, we will consider the cases with $N_v \leq 6$ and then this operator does not appear in the following discussion.

The second Coulomb branch Z corresponds to the breaking

$$so(12) \to so(8) \times su(2) \times u(1) \tag{7.5}$$
$$\mathbf{12} \to (\mathbf{8}_v, \mathbf{1})_0 + (\mathbf{1}, \mathbf{2})_{\pm 1} \tag{7.6}$$
$$\mathbf{32} \to (\mathbf{8}_s, \mathbf{1})_{\pm 1} + (\mathbf{8}_c, \mathbf{2})_0 \tag{7.7}$$
$$\mathbf{32}' \to (\mathbf{8}_c, \mathbf{1})_{\pm 1} + (\mathbf{8}_s, \mathbf{2})_0 \tag{7.8}$$

The SU(2) dynamics can be made stable and supersymmetric by the components $(\mathbf{8}_c, \mathbf{2})_0$ or $(\mathbf{8}_s, \mathbf{2})_0$. The SO(8) vacuum can be made stable by $(\mathbf{8}_v, \mathbf{1})_0$, $(\mathbf{8}_c, \mathbf{2})_0$ or $(\mathbf{8}_s, \mathbf{2})_0$. In all the s-confinement examples which we discuss in the following subsections, there are enough **8** dimensional representations so that this branch becomes a quantum moduli operator.

The third Coulomb branch X corresponds to the following breaking

$$so(12) \to so(4) \times su(4) \times u(1) \tag{7.9}$$
$$\mathbf{12} \to (\mathbf{4}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{4})_1 + (\mathbf{1}, \overline{\mathbf{4}})_{-1} \tag{7.10}$$
$$\mathbf{32} \to (\mathbf{2}, \mathbf{6})_0 + (\mathbf{2}, \mathbf{1})_2 + (\mathbf{2}, \mathbf{1})_{-2} + (\mathbf{2}^*, \mathbf{4})_{-1} + (\mathbf{2}^*, \overline{\mathbf{4}})_1 \tag{7.11}$$
$$\mathbf{32}' \to (\mathbf{2}^*, \mathbf{6})_0 + (\mathbf{2}^*, \mathbf{1})_2 + (\mathbf{2}^*, \mathbf{1})_{-2} + (\mathbf{2}, \mathbf{4})_{-1} + (\mathbf{2}, \overline{\mathbf{4}})_1. \tag{7.12}$$

The vacuum of the SO(4) dynamics can be made stable by the first components of the above branching rules, which are neutral under the U(1) subgroup and hence massless. In order to have a stable SUSY vacuum of the SU(4) $\sim$ SO(6) sector, we need at least four **6** representations. Therefore, the Spin(12) theories with two or more spinors will contain the X operator in their spectrum of the chiral ring.

The final Coulomb branch V corresponds to the breaking

$$so(12) \to su(6) \times u(1) \tag{7.13}$$
$$\mathbf{12} \to \mathbf{6}_1 + \overline{\mathbf{6}}_{-1} \tag{7.14}$$
$$\mathbf{32} \to \mathbf{20}_0 + \mathbf{6}_{-2} + \overline{\mathbf{6}}_2 \tag{7.15}$$
$$\mathbf{32}' \to \mathbf{15}_{-1} + \overline{\mathbf{15}}_1 + \mathbf{1}_3 + \mathbf{1}_{-3}. \tag{7.16}$$

	Spin(12)	SU(6)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	**12**	$\square$	1	0	R_v
S	**32**	1	0	1	R_s
$M_{QQ} := QQ$	1	$\square\square$	2	0	$2R_v$
$P_2 := SQ^2S$	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	2	2	$2R_v + 2R_s$
$P_6 := SQ^6S$	1	1	6	2	$6R_v + 2R_s$
$B := S^4$	1	1	0	4	$4R_s$
$F := S^4Q^6$	1	1	6	4	$6R_v + 4R_s$
$Z := Y_1Y_2^2Y_3^2Y_4^2Y_5Y_6$	1	1	-12	-8	$2 - 12R_v - 8R_s$

Table 20. 3d $\mathcal{N} = 2$ Spin(12) theory with $(N_v, N_s, N_{s'}) = (6, 1, 0)$.

Almost all the components are massive while the spinor field leads to a massless third-order antisymmetric tensor of the unbroken SU(6), which can make the SU(6) vacuum stable. As studied in [27], the SU(6) theory with a single three-index matter cannot have a stable vacuum, which will lead to a runaway potential. Therefore, the Spin(12) theory with more than one spinor can have this direction as a quantum flat direction.

7.1 $(N_v, N_s, N_{s'}) = (6, 1, 0)$

The first s-confinement example is the 3d $\mathcal{N} = 2$ Spin(12) theory with six vectors and one spinor. The Y operator is not allowed since the low-energy SO(10) theory along this direction contains only six **10** representations, which generates a runaway potential (2.2) and this vacuum is unstable. Along the Z direction, the low-energy SO(8) dynamics is made stable by $(\mathbf{8}_v, \mathbf{1})_0 \in \mathbf{12}$ while the SU(2) dynamics is also made stable by $(\mathbf{8}_c, \mathbf{2})_0 \in \mathbf{32}$. The X direction is unstable since the SU(4) $\sim$ SO(6) theory only contains two **6** representations, which is insufficient for a stable supersymmetric vacuum. The V direction is also excluded due to the similar reason. The confinement phase is described by the five Higgs branch operators defined in table 20 and a single Coulomb branch Z. The superpotential becomes

$$W = Z\left[B^2 \det M_{QQ} + P_6 \mathrm{Pf}\, P_2 + M_{QQ}^4 P_2^2 B + M_{QQ}^2 P_2^4 + BP_6^2 + F^2\right]. \tag{7.17}$$

The quantum numbers of the moduli operators are summarized in table 20. The corresponding 4d theory was studied in [26] and (7.17) is consistent with the 4d result where we have a quantum-deformed constraint.

7.2 $(N_v, N_s, N_{s'}) = (2, 2, 0)$

The next s-confinement example is the 3d $\mathcal{N} = 2$ Spin(12) theory with two vectors and two spinors. In this case, the Y branch is not allowed as in the previous case. The Coulomb branch Z becomes stable since the low-energy Spin(8) theory has two vectors and four spinors and it leads to a stable vacuum. Along the X-branch, the low-energy SU(4) $\simeq$ SO(6) theory contains four vectors and its vacuum is stable and supersymmetric. The V direction is also allowed since the low-energy SU(6) theory contains two third-order antisymmetric matters and becomes stable. As a result, the Coulomb branch is now

	Spin(12)	SU(2)	SU(2)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	$\mathbf{12}$	$\square$	1	1	0	R_v
S	$\mathbf{32}$	1	$\square$	0	1	R_s
$M_{QQ} := QQ$	1	$\square\!\square$	1	2	0	$2R_v$
$M_{SS} := SS$	1	1	1	0	2	$2R_v$
$P_2 := SQ^2S$	1	1	$\square\!\square$	2	2	$2R_v{+}2R_s$
$B_0 := S^4$	1	1	$\square\!\square\!\square\!\square$	0	4	$4R_s$
$B_2 := S^4Q^2$	1	1	$\square\!\square$	2	4	$2R_v{+}4R_s$
$B_2' := S^4Q^2$	1	$\square\!\square$	1	2	4	$2R_v{+}4R_s$
$F_0 := S^6$	1	1	1	0	6	$6R_s$
$F_2 := S^6Q^2$	1	1	$\square\!\square$	2	6	$2R_v{+}6R_s$
$T_2 := S^8Q^2$	1	$\square\!\square$	1	2	8	$2R_v{+}8R_s$
$Z := Y_1Y_2^2Y_3^2Y_4^2Y_5Y_6$	1	1	1	-4	-16	$2{-}4R_v{-}16R_s$
$X := \sqrt{Y_1Y_2^2Y_3^3Y_4^4Y_5^2Y_6^2}$	1	1	1	-4	-12	$2{-}4R_v{-}12R_s$
$V := (Y_1Y_2^2Y_3^3Y_4^4Y_5^2Y_6^3)^{\frac{1}{3}}$	1	1	1	-4	-8	$2{-}4R_v{-}8R_s$

Table 21. 3d $\mathcal{N}=2$ Spin(12) theory with $(N_v, N_s, N_{s'}) = (2, 2, 0)$.

three-dimensional and described by Z, X and V. We will not explicitly show the confining superpotential. Table 21 shows the moduli fields and their quantum numbers. One can write down the superpotential from table 21.

7.3 $(N_v, N_s, N_{s'}) = (2, 1, 1)$

The third example is the 3d $\mathcal{N}=2$ Spin(12) theory with two vectors, one spinor and one conjugate spinor. The Coulomb branch Y is not allowed since the low-energy SO(10) theory with two vectors generates a runaway potential (2.2) and its vacuum is unstable. The Coulomb branch V cannot be turned on since the stability of this branch at least requires two third-order anti-symmetric tensors $\mathbf{20}_0 \in \mathbf{32}$. The Coulomb branch Z is stable since the SU(2) dynamics is made stable by the massless components of the two spinors and since the SO(8) dynamics is also stable and supersymmetric by two vectors and two spinors. The Coulomb branch X is also available since the low-energy $\mathrm{SO}(4) \times \mathrm{SU}(4)$ dynamics can be stable due to $(\mathbf{2}, \mathbf{6})_0$ and $(\mathbf{2}^*, \mathbf{6})_0$. Table 22 shows the moduli fields and their quantum numbers. We will not explicitly write down the superpotential, but one can do it from table 22.

8 Spin(13) theories

Let us study the Coulomb branch of the 3d $\mathcal{N}=2$ Spin(13) gauge theory with N_v vectors and N_s spinors whose dimension is $\mathbf{64}$. There are a lot of classical Coulomb branches corresponding to the fundamental monopoles $Y_i\,(i = 1, \cdots, 6)$. However, most of them are quantum-mechanically excluded from the quantum moduli space via the monopole

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	Spin(12)	SU(2)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_{s'}$	$\mathrm{U}(1)_R$
Q	**12**	$\square$	1	0	0	R_v
S	**32**	1	0	1	0	R_s
S'	**32**$'$	1	0	0	1	$R_{s'}$
$M_{QQ} := QQ$	1	$\square\square$	2	0	0	$2R_v$
$P_2 := SQ^2S$	1	1	2	2	0	$2R_v+2R_s$
$P'_2 := S'Q^2S'$	1	1	2	0	2	$2R_v+2R_{s'}$
$M_{1,SS'} := SQS'$	1	$\square$	1	2	2	$R_v+R_s+R_{s'}$
$B := S^4$	1	1	0	4	0	R_s
$B' := S'^4$	1	1	0	0	4	$4R_{s'}$
$F_0 := S^2S'^2$	1	1	0	2	2	$2R_s+2R_{s'}$
$F_2 := S^2S'^2Q^2$	1	$\square\square$	2	2	2	$2R_v+2R_s+2R_{s'}$
$F'_2 := S^2S'^2Q^2$	1	1	2	2	2	$2R_v+2R_s+2R_{s'}$
$C := S^3S'Q$	1	$\square$	1	3	1	$R_v+3R_s+R_{s'}$
$C := SS'^3Q$	1	$\square$	1	1	3	$R_v+3R_s+R_{s'}$
$T := S^3S'^3Q$	1	$\square$	1	3	3	$R_v+3R_s+3R_{s'}$
$D := S^4S'^2Q^2$	1	1	2	4	2	$2R_v+4R_s+2R_{s'}$
$D' := S2S'^4Q^2$	1	1	2	2	4	$2R_v+2R_s+4R_{s'}$
$U_0 := S^4S'^4$	1	1	0	4	2	$4R_s+4R_{s'}$
$U_2 := S^4S'^4Q^2$	1	1	2	4	4	$2R_v+4R_s+4R_{s'}$
$Z := Y_1Y_2^2Y_3^2Y_4^2Y_5Y_6$	1	1	-4	-8	-8	$2-4R_v-8R_s-8R_{s'}$
$X := \sqrt{Y_1Y_2^2Y_3^3Y_4^4Y_5^2Y_6^2}$	1	1	-4	-6	-6	$2-4R_v-6R_s-6R_{s'}$

Table 22. 3d $\mathcal{N}=2$ Spin(12) theory with $(N_v, N_s, N_{s'}) = (2,1,1)$.

potential such as (2.3) since almost all the components of the matter fields are massive along those directions and we will obtain a pure SYM or an SQCD as a low-energy description, which will not have enough charged matters to make the supersymmetric vacuum stable. Therefore, we are left with a few Coulomb branch directions.

The first candidate denoted as Y corresponds to the breaking

$$so(13) \to so(11) \times u(1) \tag{8.1}$$

$$\mathbf{13} \to \mathbf{11}_0 + \mathbf{1}_2 + \mathbf{1}_{-2} \tag{8.2}$$

$$\mathbf{64} \to \mathbf{32}_1 + \mathbf{32}_{-1}. \tag{8.3}$$

All the components of the spinor representations are massive and integrated out along this branch while the vector matter reduces to a massless vector $\mathbf{11}_0$. Therefore, the moduli space of the Spin(13) theory only with spinors cannot have this operator. In order to make the low-energy SO(11) vacuum stable, there must be more than eight vector matters.

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The second Coulomb branch Z breaks the gauge group as

$$so(13) \to so(9) \times su(2) \times u(1) \tag{8.4}$$

$$\mathbf{13} \to (\mathbf{9}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{2})_{\pm 1} \tag{8.5}$$

$$\mathbf{64} \to (\mathbf{16}, \mathbf{2})_0 + (\mathbf{16}, \mathbf{1})_{\pm 1}. \tag{8.6}$$

The vacuum of the low-energy SU(2) theory is made stable by the massless component $(\mathbf{16}, \mathbf{2})_0 \in \mathbf{64}$ while the SO(9) part can have a stable SUSY vacuum via $(\mathbf{9}, \mathbf{1})_0 \in \mathbf{13}$ or $(\mathbf{16}, \mathbf{2})_0 \in \mathbf{64}$. Therefore, the Spin(13) theory with spinor matters includes this branch.

The third candidate denoted as X corresponds to the breaking

$$so(13) \to so(5) \times su(4) \times u(1) \tag{8.7}$$

$$\mathbf{13} \to (\mathbf{5}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{4})_1 + (\mathbf{1}, \overline{\mathbf{4}})_{-1} \tag{8.8}$$

$$\mathbf{64} \to (\mathbf{4}, \mathbf{6})_0 + (\mathbf{4}, \mathbf{1})_2 + (\mathbf{4}, \mathbf{1})_{-2} + (\mathbf{4}, \mathbf{4})_{-1} + (\mathbf{4}, \overline{\mathbf{4}})_1. \tag{8.9}$$

The vector field cannot make the low-energy SU(4) vacuum stable since there is no massless component charged under the SU(4) subgroup. When the theory has at least one spinor, the component $(\mathbf{4}, \mathbf{6})_0 \in \mathbf{64}$ makes the SO(5) $\times$ SU(4) dynamics stable and keeps it supersymmetric. Therefore, the Spin(13) theory with spinor matters also includes this operator.

Finally, we mention that there could be an additional Coulomb branch operator V which induces the gauge symmetry breaking

$$so(13) \to su(6) \times u(1) \tag{8.10}$$

$$\mathbf{13} \to \mathbf{1}_0 + \mathbf{6}_1 + \overline{\mathbf{6}}_{-1} \tag{8.11}$$

$$\mathbf{64} \to \mathbf{20}_0 + \mathbf{6}_{-2} + \overline{\mathbf{6}}_2 + \mathbf{15}_{-1} + \overline{\mathbf{15}}_1 + \mathbf{1}_3 + \mathbf{1}_{-3}. \tag{8.12}$$

Along this direction, the massless components $\mathbf{20}_0 \in \mathbf{64}$ can make the SU(6) vacuum stable. However, this is only possible when there are two spinors in the theory [27]. In what follows, we will only consider the Spin(13) theory with a single spinor and this operator is not necessary.

8.1 $(N_v, N_s) = (3, 1)$

The 3d $\mathcal{N} = 2$ Spin(13) theory with three vectors and one spinor exhibits s-confinement. The Higgs branch in the moduli space of vacua is described by eleven composite operators $M_{QQ}, P_2, P_3, R_0, R_1, R_2, R_3, T_2, T_3, U_0$ and U_3, which are defined in table 23. The Coulomb branch is two-dimensional. These are described by Z and X which are defined above. We will not show a confining superpotential since the explicit form is cumbersome. Table 23 summarizes the quantum numbers of the moduli operators.

	Spin(13)	SU(3)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	**13**	$\square$	1	0	R_v
S	**64**	1	0	1	R_s
$M_{QQ} := QQ$	1	$\square\!\square$	2	0	$2R_v$
$P_2 := SQ^2S$	1	$\overline{\square}$	2	2	$2R_v + 2R_s$
$P_3 := SQ^3S$	1	1	3	2	$3R_v + 2R_s$
$R_0 := S^4$	1	1	0	4	$4R_s$
$R_1 := S^4Q$	1	$\square$	1	4	$R_v + 4R_s$
$R_2 := S^4Q^2$	1	$\square\!\square$	2	4	$2R_v + 4R_s$
$R_3 := S^4Q^3$	1	1	3	4	$3R_v + 4R_s$
$T_2 := S^6Q^2$	1	$\overline{\square}$	2	6	$2R_v + 6R_s$
$T_3 := S^6Q^3$	1	1	3	6	$3R_v + 6R_s$
$U_0 := S^8$	1	1	0	8	$8R_s$
$U_3 := S^8Q^3$	1	1	3	8	$3R_v + 8R_s$
$Z := Y_1Y_2^2Y_3^2Y_4^2Y_5^2Y_6$	1	1	-6	-16	$2 - 6R_v - 16R_s$
$X := \sqrt{Y_1Y_2^2Y_3^3Y_4^4Y_5^4Y_6^2}$	1	1	-6	-12	$2 - 6R_v - 12R_s$

Table 23. 3d $\mathcal{N} = 2$ Spin(13) theory with $(N_v, N_s) = (3, 1)$.

9 Spin(14) theories

The final example is the 3d $\mathcal{N} = 2$ Spin(14) gauge theory with vector (**14**) matters and one spinor (**64**). Since the dimension of the spinor representation is huge, the theory with more than one spinor (**64** or $\overline{\mathbf{64}}$) will exhibit a conformal window or a non-abelian Coulomb phase. Since we are now interested in the s-confinement phases of the Spin(N) gauge theories, we focus on the Spin(14) theory with one spinor and some vectors.

There are two Coulomb branches which we have to take into account. The non-zero vev of the first coordinate Z corresponds to the breaking

$$so(14) \to so(10) \times su(2) \times u(1) \tag{9.1}$$

$$\mathbf{14} \to (\mathbf{10}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{2})_{\pm 1} \tag{9.2}$$

$$\mathbf{64} \to (\mathbf{16}, \mathbf{1})_{\pm 1} + (\overline{\mathbf{16}}, \mathbf{2})_0. \tag{9.3}$$

The Chern-Simons term for U(1) is not introduced as it should be. This is a necessary condition that the Coulomb branch Z can be a flat direction. The resulting low-energy theory contains the 3d $\mathcal{N} = 2$ SO(10) $\times$ SU(2) SQCD with massless chiral superfields in fundamental and spinor representations of SO(10) and SU(2), respectively. In order that the coordinate Z can be a stable vacuum, there must be enough matters charged under the SO(10) $\times$ SU(2). For example, the theories without a spinor matter cannot have this flat direction since there is no massless field charged under the SU(2) and then the SU(2) vacuum is unstable due to the monopole potential $W_{\text{eff}} \sim \frac{1}{Y_{\mathrm{SU}(2)}}$.

The second Coulomb branch X corresponds to the breaking

$$\begin{aligned} so(14) &\to so(6)\times su(4)\times u(1) && (9.4)\\ \mathbf{14} &\to (\mathbf{6},\mathbf{1})_0+(\mathbf{1},\mathbf{4})_1+(\mathbf{1},\overline{\mathbf{4}})_{-1} && (9.5)\\ \mathbf{64} &\to (\mathbf{4},\mathbf{4})_{-1}+(\mathbf{4},\overline{\mathbf{4}})_1+(\overline{\mathbf{4}},\mathbf{6})_0+(\overline{\mathbf{4}},\mathbf{1})_2+(\overline{\mathbf{4}},\mathbf{1})_{-2}. && (9.6)\end{aligned}$$

This vacuum can be stable by massless components $(\mathbf{6},\mathbf{1})_0$ or $(\overline{\mathbf{4}},\mathbf{6})_0$. The theory only with vector matters cannot include this operator since the low-energy SU(4) dynamics is unstable due to the SU(4) monopole potential (similar to (2.3)). For the theory with a spinor matter, each gauge dynamics can be stable due to $(\overline{\mathbf{4}},\mathbf{6})_0$.

Notice that when we introduce more general matter contents (N_v vectors, N_s spinors and $N_{s'}$ complex conjugate spinors), there may be additional Coulomb branches. For instance, the classical Coulomb branch will include the following direction

$$\begin{aligned} so(14) &\to su(6)\times so(2)\times u(1) && (9.7)\\ \mathbf{14} &\to \mathbf{6}_{0,1}+\overline{\mathbf{6}}_{0,-1}+\mathbf{1}_{2,0}+\mathbf{1}_{-2,0} && (9.8)\\ \mathbf{64} &\to \mathbf{6}_{1,-2}+\mathbf{20}_{1,0}+\overline{\mathbf{6}}_{1,2}+\mathbf{15}_{-1,-1}+\overline{\mathbf{15}}_{-1,1}+\mathbf{1}_{-1,3}+\mathbf{1}_{-1,-3} && (9.9)\\ \overline{\mathbf{64}} &\to \mathbf{6}_{-1,-2}+\mathbf{20}_{-1,0}+\overline{\mathbf{6}}_{-1,2}+\mathbf{15}_{1,-1}+\overline{\mathbf{15}}_{1,1}+\mathbf{1}_{-1,3}+\mathbf{1}_{1,-3}, && (9.10)\end{aligned}$$

where the Coulomb branch operator corresponds to the second U(1) factor. We can use two massless components $\mathbf{20}_{\pm 1,0}$ in order to have a stable vacuum of the low-energy SU(6) gauge theory. When the theory has $N_v \geq 10$ vector matters, there is another Coulomb branch Y which corresponds to the breaking

$$\begin{aligned} so(14) &\to so(12)\times u(1) && (9.11)\\ \mathbf{14} &\to \mathbf{12}_0+\mathbf{1}_2+\mathbf{1}_{-2} && (9.12)\\ \mathbf{64} &\to \mathbf{32}_1+\mathbf{32}'_{-1}. && (9.13)\end{aligned}$$

All the components of the spinor are massive and only the vector matters can make the low-energy SO(12) theory stable.

9.1 $(N_v, N_s, N_{s'}) = (4, 1, 0)$

The s-confinement phase appears only in the 3d $\mathcal{N}=2$ Spin(14) gauge theory with four vectors and one spinor (or four vectors and one conjugate spinor). The Higgs branch is described by seven composites: $M_{QQ}, P_3, B_{4,2}, B_{4,4}, B_{6,3}, B_{8,0}$ and $B_{8,4}$. These are defined in table 24. As explained above, there are two Coulomb branch coordinates Z and X. The superpotential takes

$$\begin{aligned} W = Z\,&\big[B_{8,0}^2 \det M_{QQ} + \det B_{4,2} + B_{8,0}(M_{QQ}^2 B_{4,2}^2 + M_{QQ}P_3B_{6,3} + B_{4,2}P_3^2)\\ &+B_{4,2}B_{6,3}^2 + B_{4,4}^2B_{8,0} + B_{8,4}^2\big]\\ &+X\left[(M_{QQ}^3B_{4,2}+M_{QQ}P_3^2)B_{8,0} + M_{QQ}(B_{4,2}^3+B_{6,3}^2)+P_3B_{6,3}B_{4,2}+B_{4,4}B_{8,4}\right]. \end{aligned} \quad (9.14)$$

	Spin(14)	SU(4)	$\mathrm{U}(1)_v$	$\mathrm{U}(1)_s$	$\mathrm{U}(1)_R$
Q	**14**	$\square$	1	0	R_v
S	**64**	1	0	1	R_s
$M_{QQ} := QQ$	1	$\square\square$	2	0	$2R_v$
$P_3 := SQ^3S$	1	$\overline{\square}$	3	2	$3R_v + 2R_s$
$B_{4,2} := S^4Q^2$	1	$\square\square$	2	4	$2R_v + 4R_s$
$B_{4,4} := S^4Q^4$	1	1	4	4	$4R_v + 4R_s$
$B_{6,3} := S^6Q^3$	1	$\overline{\square}$	3	6	$3R_v + 6R_s$
$B_{8,0} := S^8$	1	1	0	8	$8R_s$
$B_{8,4} := S^8Q^4$	1	1	4	8	$4R_v + 8R_s$
$Z := Y_1Y_2^2Y_3^2Y_4^2Y_5^2Y_6Y_7$	1	1	-8	-16	$2 - 8R_v - 16R_s$
$X := \sqrt{Y_1Y_2^2Y_3^3Y_4^4Y_5^4Y_6^2Y_7^2}$	1	1	-8	-12	$2 - 8R_v - 12R_s$

Table 24. 3d $\mathcal{N} = 2$ Spin(14) theory with $(N_v, N_s, N_{s'}) = (4, 1, 0)$.

10 Summary

In this paper, we investigated the various s-confinement phases in the 3d $\mathcal{N} = 2$ Spin(N) gauge theories with vector matters and spinor matters. We found that the 3d s-confinement is connected to the (quantum-deformed) moduli space of the corresponding 4d $\mathcal{N} = 1$ Spin(N) gauge theories via the twisted-monopole superpotential [6, 7]. Naively, one might consider that almost all the classical Coulomb branches are quantum-mechanically lifted since the matter fields are massive and the non-perturbative superpotential (such as (2.3)) lifts those flat directions. However, we pointed out that the Spin(N) theory with vectors and spinors can have the additional Coulomb branches. Along these new branches, some components of the spinor representations can remain massless and they can make these flat directions stable and supersymmetric.

We gave a systematic study of the Coulomb branch and the s-confinement phases for the 3d $\mathcal{N} = 2$ Spin(N) gauge theories. Although the analysis of the Coulomb branch was systematic, the resulting Coulomb branch structure was drastically changed, depending on the rank of the gauge group. For example, the Spin(10) theory with three spinors and one conjugate spinor was very special, where we need to introduce the "dressed" Coulomb branch operator. This was because there are two unbroken U(1) subgroups along the Coulomb branch and the mixed Chern-Simons term is introduced. As another example, the Spin(12) theory with two vectors and two spinors exhibited the three-dimensional Coulomb branch while, in most other cases, the Coulomb branch was one- or two-dimensional.

Since we are interested in the s-confinement phases, the number of the spinor matters is highly restricted especially in the case of large Spin(N) gauge groups where the dimensions of the spinors are huge. When there are more spinor matters, we could define the additional Coulomb branches which survive quantum corrections. Remember the two examples, Spin(13) and Spin(14), where we claimed that the additional Coulomb branch will be necessary when there are more than one spinor. It is important to check the validity of this analysis, for instance, by computing the superconformal indices [28, 29]. This is a hard and challenging problem since the rank of the gauge group is large and the calculation would be quite heavy.

In this paper, we focused on the s-confinement phases of the 3d Spin(N) gauge theory and proposed various confining phases. It is important to test our proposal, for instance, by computing the superconformal indices [28, 29]. The dual descriptions are given by the non-gauge theories presented here. It is also very important to study different phases by introducing more vector and spinor matters. For example, the Seiberg dualities of the 4d Spin(N) theories were studied in [16, 17, 20–22, 30, 31]. One can, in principle, derive the corresponding 3d dualities from the 4d ones by following the argument in [9, 32]. We will soon come back to this problem elsewhere.

Acknowledgments

I would like to thank Prof. Jan de Boer for helpful discussion. This work is supported by the Swiss National Science Foundation (SNF) under grant number PP00P2_157571/1 and by "The Mathematics of Physics" (SwissMAP) under grant number NCCR 51NF40-141869.

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