

# Bringing the Margins Back In. Using the *M*-Index for the Analysis of Social Mobility

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## Abstract

Analysis of intergenerational social mobility with respect to educational attainment and class position has been dominated for several decades by log-linear models, most prominently the so-called Unidiff a.k.a. Log Multiplicative Layer Effect model. In this paper we argue that these models are not ideal when the goal is to evaluate how the general relevance of social origin changed in a society over time or differs between countries, because these models neglect important aspects of the marginal distribution. We propose an alternative methodology based on the Mutual Information Index (*M*-Index), that solves this problem and, at the same time, equips researchers with a much more flexible framework for modelling effects of social origin. Our arguments are illustrated by two brief empirical applications.

## 1 Introduction

Sociological research on intergenerational mobility faces specific methodological challenges that are due to the categorical nature of one of its key variables of interest – class position. Consequentially, methodological advancements have played an important role in the literature on social mobility (Hout 1983: 7; Erikson and Goldthorpe 1992: 54). A main driver of these methodological advancement was the quest for an overall measure of social mobility that satisfies the criterion of being “margin-free” (Boudon 1973), i.e., that is unaffected by changes in the marginal distribution of individuals’ and parents’ social positions. In this paper, we argue that for certain research questions this quest has gone too far.

Widely used summary measures of odds ratios, such as the so-called unidiff parameters (Breen and Jonsson 2005: 234–5), have been employed in partly problematic ways. As we will discuss in this paper, odds ratios in the context of social mobility research can be seen as

measurements of class barriers. Consequently, (unweighted) summaries of odds ratios say something about the “average” rigidity of the class barriers in a given society. As we will argue, however, this does not necessarily equal the “average” effects of social origin, that, according to our understanding, would quantify the overall relevance of social origin for individual status attainment in a given society.

Whether a given measure is appropriate depends on the research question at hand, as the concepts behind the former should be paralleled by the concepts behind the latter. The aim of this paper is to clarify these concepts – in both substantial and technical terms – and to justify the choice of the Mutual Information Index as a promising methodology for the analysis of the relevance of social origin in a society. In Section 2 we will explore how we can conceptually think about the general relevance of social origin for an individual’s social standing and how far this concept is mirrored by traditional measures of social mobility. We then introducing the basic ideas behind the Mutual Information Index in Section 3 and go into some technical details in Section 4. In Sections 5 and 6 we provide two empirical applications that illustrate the usefulness of the proposed methodology for the analysis of social mobility.

## **2 How can societies’ social fluidity be quantified?**

Societies in which social origin has little influence on an individual’s social position are said to be “socially fluid” (Breen and Jonsson 2005). To approach a more concrete definition of social fluidity, a first noteworthy observation is that social fluidity usually comes with comparably high rates of observed intergenerational social mobility. In the literature, observed intergenerational social mobility describes the fact that the observed social position of a person differs from the social position of her or his family of origin (Ganzeboom et al. 1991). In other words, observed mobility is mobility in its manifest sense. Early research on social mobility (Sorokin 1927/1959) and early tests of the modernization thesis (Lipset and Zetterberg 1959) have investigated this immediate form of social mobility.

While social fluidity usually comes with comparably high rates of observed mobility, it is not necessarily the case that socially fluid societies have high mobility rates; nor is it the case that high

mobility rates mean high levels of social fluidity. A socially fluid society can have low rates of observed mobility if a large part of the population is concentrated within one (or very few) social positions. Consider a society with the three social classes A, B, and C, with B being by far the largest class (consisting of 90% of the population). In such a society, observed mobility is necessarily low, as individuals originating from class B will likely also belong to class B – not necessarily because their origin influences their destination, but certainly because there is not much choice other than belonging to class B. To assess the influence of social origin, we therefore need to compare the observed immobility with the marginal distribution of the classes. Low mobility rates point to a high influence of social origin if and only if an individual's likelihood of entering the class of their parents surpasses the likelihood that can be expected from chance alone, given the marginal distribution of the classes. Therefore, we could adjust the observed mobility rate by subtracting the number of individuals than can be expected to be immobile by chance alone from the total number of immobile individuals.

However, focusing only on mobility rates can be misleading, even when applying such corrections, as social origin can be of relevance for an individual's social class even if they are socially mobile and do not belong to the same social class as their parents. This is of special importance if the class structure changes from one generation to the next. For example, during rapid industrialization, the working class will grow from one generation to the next, while other classes, such as the class of farmers, will shrink. In such a situation, many descendants of a non-industrial class will be "forced" to be socially mobile, because there are not enough non-industrial positions within the class structure of their own generation. In the literature, this forced mobility is often labeled "structural mobility" (Boudon 1973: 17). If such structural changes from one generation to the next enforce mobility, but an individual's position in the new social stratification depends heavily on their parents' position in the old stratification, mobility rates are high despite the strong effects of social origin. Most people would agree that such a society cannot be called open respecting social origin, because the chances of individuals attaining a certain social position depend on their social origin and are not equal.

One way to deal with this problem is to measure social positions on a continuous scale and to use the correlation between the parents' and the individual's positions as an indicator for the importance of social origin. Blau and Duncan (1967) went even a step further and analyzed the status attainment process using a path model reflecting the idea that the parents' social status and education affects an individual's social status partly indirectly via the individual's education. Such models allow us to assess the degree of a society's openness by the ratio of paths related to social origin (ascriptive paths) and those that point to achievement (Ganzeboom et al. 1991: 283–4). Analyzing continuous scales of social status by means of path models may be helpful for revealing mechanisms within the process of status attainment, but fails to deliver detailed descriptions of mobility patterns (Hauser 1978). For example, continuous scales make it difficult to see who goes where or stays within their class of origin and to reveal boundaries and affinities between classes. Furthermore, researchers such as Erikson et al. (1979) insist that important barriers between social positions cannot be captured by a purely hierarchical ordering (see also Erikson and Goldthorpe 2009; Chan and Goldthorpe 2007). Finally, log-linear models, made popular among researchers of social mobility by Hauser (1978), allow for the modeling of specific patterns of mobility while applying a confirmatory approach to the mobility table and perfectly separating the effects of social origin from the effects of the marginal class distributions. The latter means that these models provide measures of social origin unaffected by changes in the marginal distribution ("margin-free"), a property researchers have been demanding for a long time (e.g., Boudon 1973).

The concept behind these models is called "relative mobility", which sometimes is simply understood as a synonym for "social fluidity" (e.g., Breen and Jonsson 2005). The idea behind this concept is that inequality of opportunities is something "inherently comparative" (Marshall and Swift 1996: 376). In other words, it involves comparing the opportunities of a person with those of another person. According to this definition, relative social mobility is high if the odds of attaining a certain position are similar for all social origins. Consequently, high relative mobility

implies that the odds ratio of a person from origin  $i$  compared to a person from origin  $j$  of attaining position  $k$  instead of  $l$  is close to one for all possible combinations of social positions  $i, j, k$ , and  $l$ :

$$OR_{ijk}^{high\ relative\ mobility} = \frac{\frac{p(y = k|x = i)}{p(y = l|x = i)}}{\frac{p(y = k|x = j)}{p(y = l|x = j)}} \cong 1, \quad \forall i, j, k, l \in K \quad 1$$

This definition of social mobility relies on distinct groups of social positions and is, therefore, often used to analyze social mobility between social classes. In this view, uneven odds for reaching certain classes of destination by class of origin (odds ratios deviating substantively from unity) indicate class barriers that are difficult to cross from one generation to the next.

Log-linear models, such as the ones proposed by Hauser (1978), model and describe a set of mobility tables – cross-tabulations of the current class of the child (often called “destination”) and the class of one or both parents (often called “origin”). They do so by means of a set of parameters representing the marginal distributions and all possible (or a selection of) odds ratios. Hauser (1978) promoted these kinds of models as a tool for describing patterns of social origin and the variations of these patterns between different periods or geographical areas. Revealing patterns and important class barriers (or, conversely, affinities between certain classes) is really the domain where these models excel. Most prominently, Erikson and Goldthorpe (1992) have arrived at a so-called “core model”, a description of a mobility regime shared by many countries, which also allows researchers to detect deviations from this pattern.

As class barriers distinguish socially open from socially closed societies, revealing such barriers is important to understand in what sense a society can be said to be “open” or “closed”. When comparing different societies, however, it is not straightforward to determine from such class barriers which society is more fluid or more open, simply because there is no obvious rule regarding how to aggregate these class barriers to the unidimensional measure necessary to rank such “openness”. Models such as the “core model” collapse certain barriers to a few meaningful dimensions, such as inheritance effects, hierarchical effects, sector effects, and affinity effects. Nonetheless, multiple dimensions remain, and the multidimensionality of this class barrier-based

concept of openness makes it difficult to answer research questions that rely on ranking – for example, whether a given society has become more open over the course of modernization.

In 1992, two independent publications proposed a rather technical solution to this problem. The so-called “unidiff model” (Erikson and Goldthorpe 1992) or the “log multiplicative layer effects model” (Xie 1992) distinguishes between the association pattern (which indicates the barriers between the classes) and the “strength” of these associations. While the pattern is common to all mobility tables analyzed, it is allowed to vary uniformly in strength between them. The so-called unidiff parameters of these models are factors that indicate how many times more strongly this pattern works in a given table compared to a reference table. As long as the uniformity assumption holds, these models can be used to compare (for example) the strength of the class barriers in one birth cohort to the class barriers in another birth cohort. Technically, the unidiff model offers an elegant and parsimonious way to model a set of mobility tables that differ in magnitude but not (much) in the pattern of the odds ratios describing the origin–destination association. In many empirical applications, the parsimonious unidiff model fits the data almost as well as a saturated model that allows the association parameters to vary freely between the tables (examples are: Erikson and Goldthorpe 1992; Breen 2004a; Jacot 2013; Hertel 2017).

While individual class barriers can only be revealed by “margin-free” measures, such as odds-ratios, we will argue that such measures may not be best suited for measuring and comparing the general importance of social origin for an individual’s status attainment, or for measuring the general openness of a society respecting the family of origin. Exactly for this purpose, however, many authors have used these models. For example, Erikson and Goldthorpe (1992) draw a direct line of argument between the margin-free “level of the pattern of relative mobility chances” (p. 24) and the “openness” of a society. Accordingly, a society with a mobility table showing a pronounced pattern of class barriers indicated by a high unidiff parameter is said to be more socially fluid than one with a lower parameter (Breen 2004b).

What is potentially problematic about this approach is that unidiff models model cells of mobility tables, and not the societies described by these mobility tables. In a mobility table, each

origin–destination combination always concerns exactly one cell; when comparing the chances of entering a given destination class between two classes of origin, this always concerns two rows in such a table. Depending on the research question, this can be perfectly fine. Odds ratios based on mobility tables can answer the question posed by a working-class girl about how much better her chances would be of reaching the upper service class if she had been born as a child of a manager. These are the sorts of questions Marshall and Swift (1996: 376) refer to when they characterize equality of opportunity as something that is “inherently comparative”, and in this case, it is indeed a one-by-one comparison.

However, this is not the research question when researchers are primarily interested in comparing the general level of origin effects over time or between countries. For answering such questions related to the general degree of a society’s openness, we need to generalize from particular class barriers to the society as a whole. Studies applying unidiff models do so by applying the aggregation-rule technically built into the model – less open societies have less pronounced patterns of class barriers, while each barrier receives the same weight irrespective of the proportion of the society that faces the barrier. This conforms with the paradigm according to which a good measure of social mobility should not be affected by changes in marginal class distribution, but there is ground for the argument that not considering changes in the class distribution at all can be misleading.

For example, when studying industrialization or modernization processes, the diminishing weight of the farming classes is of special importance, as it is a defining (or at least a characteristic) feature of these processes (Treiman 1970; Kuznets 1955). Marginalization of agriculture could mean that the size of the farming class approaches zero, for example because of the complete urbanization of an area; Singapore (Fields 1994) could serve as an almost perfect real world example. This extreme case is helpful for illustrating why ignoring changes in class distribution can produce misleading results when analyzing the changing effects of social origin. For illustrative purpose, we assume the farming class to be the only source of social origin effects. More specifically, the odds ratios between two non-farming classes are thought to equal one. By

contrast, the odds ratios between descendants of farmers and individuals with a non-farming background are very uneven in this example. Additionally, we assume that none of these class barriers changes over time. In other words, we assume that nothing changes except the shrinking proportion of the farming class and the proportional growth of the other classes. In this example, it is obvious that a purely margin-free, odds ratio-based concept of an open society leads to a paradoxical result: while this society would be called completely open without the farming class, it retains the exact same level of openness while the proportion of the farming class decreases. Thus, if we apply the aggregation rule built into the unidiff model for making substantive generalizations from individual class barriers to the overall openness of a society, we accept that a large farming class makes the same contribution to a society's social rigidity as an almost disappeared farming class – while a farming class that has completely disappeared contributes nothing. We conclude that giving each origin–destination combination the exact same weight may not be appropriate, and may lead to paradoxical results.

If we accept this conclusion, the question remains of what properties an appropriate measure of the importance of social origin should have. With an odds ratio-based approach, such a measure should clearly share the ability to detect complete independence of origin and destination. Moreover, and in contrast to observed mobility, an existing association between origin and destination should not be masked by structural mobility. In other words, if a part of the population is forced to leave the class of their parents because of structural changes, this should only affect our measure if the structural changes go hand in hand with changes in the relevance of social origin for the status attainment of individuals. Contrary to purely “margin-free” measures, however, the measure should take into account changes in the marginal distribution that affect the relevance of existing dependencies between origin and destination – either because these changes affect the influence of the margins, or because these changes increase (or decrease) the proportion of the society affected by strong origin–destination associations. For analytical purposes, it would nevertheless be valuable to decompose changes in the measure into a part stemming from changes in the marginal distribution and a part originating from changes in the



dependence structure between origin and destination. A different form of decomposability is also important: to assess the importance of certain class barriers, it should be possible to decompose the measurement into the contributions of different destinations (or origins) to the overall measurement. Such a decomposition would make it possible to identify those classes for which social origin is of particular importance, and to reveal class barriers.

Finally, the measure should allow researchers to respect the multi-faceted nature of social origin. The terms “social origin” or “family background” do not refer to a single aspect of the social reality in which a person was born and raised. Instead, it refers to the whole package of origin family resources that potentially affect her or his future social standing. The question of “the composition of family background” (Buis 2013) had already been discussed when Blau and Duncan (1967: 175) proposed their seminal “model of status attainment”. Only relatively recently, however, has it reappeared prominently in the literature on social stratification and the effects of social origin (Bukodi and Goldthorpe 2013; Buis 2013; Hällsten and Pfeffer 2017; Mood 2017). It is difficult to capture the joint influence of several of these dimensions within the framework of odds ratios. Capturing the joint influence of several dimensions of origin would be a valuable feature of an alternative measure for the relevance of social origin. In other words, such a measure should make it possible to estimate the overall importance of social origin, even if we assume multiple dimensions of social origin to be relevant, such as the highest level of education of each of the parents plus both parents’ occupational status.

### **3 The Index of Mutual Information as a measure for the relevance of social origin**

To approach a measure that matches the wish list sketched out above, we might need to reconsider the kinds of questions we want to answer with a given measure. As has been pointed out, odds ratios can answer the question of a working-class girl who wants to know how her chances of reaching the upper service class compare to those of a manager’s child. While this question is perfectly relevant, we have argued that it is not straightforward to extend it to society as a whole. Instead of comparing two odds of reaching a certain class, we might instead ask how

much we learn about her destination class by becoming aware of her working-class origin. This question too directly relates to the relevance of a working-class background. In addition, it is easily generalizable to the whole society by asking how much we can learn on average about a person's social standing by knowing his or her social origin. Because origin can only carry significant amounts of information on destination if origin is relevant for destination, the answer to this question is also an answer to the question on the importance of social origin for an individual's own social position.

When introducing logistic regression to sociologists, Theil (1970) concluded with a section on the measurement of "the degree to which the determining factors of our relations account for the phenomenon which they serve to explain" (p. 125). This is exactly what we are interested in when analyzing the relevance of social origin for an individual's class affiliation: we estimate associations, but in the end we are interested in whether the degree social origin (measured by one or multiple variables) determines the class an individual belongs to. Theil's (1970) approach for measuring this degree of determination operates along the lines sketched out above, by asking how much information one can gain on the phenomenon at stake by learning these determining factors.

Information theory, which goes back to Shannon (1948) and has been introduced to economics and the social sciences by Theil (1967, 1972), deals with these kinds of questions by turning them around: the more information I have about something, the less information I will gain by actually observing it. The question "How much can I learn about  $Y$  by learning  $X$ ?" can thus be answered by the difference between the *a priori* and the *a posteriori* information gain. Here, the *a priori* information gain measures how much one can learn by observing  $X$  if one only knows the unconditional distribution of  $X$ . Similarly, the *a posteriori* information gain measures how much one can learn by observing  $X$  if one knows the distribution of  $X$  conditional on  $Y$ . The former takes into account the "steering power" of the marginal distribution, while the latter additionally includes the influence of origin. If the difference between the two is large, we can conclude that social origin is important for an individual's own social position as it carries an important amount

of information on this person's class over and above the information included in the distribution of social positions. Because of this, information theory can serve as a conceptual framework for analyzing linkages between two entities: between district and race, in the case of residential segregation by race (Mora and Ruiz-Castillo 2011); between fields of study and occupational positions, in the case of school-to-work linkages (DiPrete et al. 2017); or between the social class of parents and their children, in the case of social mobility (Silber and Spadaro 2011)<sup>1</sup>.

We propose using the Mutual Information Index (*M*-index) as a measurement for the linkage between origin and destination, which is based on entropy, the measure of information available before and after learning the class of origin. As we will see in more detail in the next section, the *M*-index is given by

$$\begin{aligned}
 M &= \sum_{j=1}^J \sum_{k=1}^K p(x_j, y_k) \{ [-\ln(p(y_k))] - [-\ln(p(y_k|x_j))] \} \\
 &= \sum_{j=1}^J \sum_{k=1}^K p(x_j, y_k) \ln \left( \frac{p(y_k|x_j)}{p(y_k)} \right),
 \end{aligned}
 \tag{2}$$

where  $p(y_k)$  is an individual's unconditional probability of belonging to class  $k$  (the basis for the *a priori* information gain),  $p(y_k|x_j)$  is her/his probability of belonging to class  $k$  conditional on her/his parent's class  $j$  (the basis for the *a posteriori* information gain), while  $p(y_k, x_j)$  refers to the joint distribution of individual's and parents' class (used for weighting the difference between the two information gains).

The *M*-index is not the only possible measure based on information theory that could be used to measure linkages between origin and destination. Mora and Ruiz-Castillo (2011) discuss several entropy-based measures for measuring segregation – all of those measures could also be used for studying social mobility. The measure they denote *H*, for example, has been used for analyzing social mobility in Switzerland (Jann and Combet 2012; Jann and Seiler 2014) and can

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<sup>1</sup> To the best of our knowledge, this is the only publication that makes use of the *M*-index for analyzing social mobility.

be calculated by normalizing the  $M$ -index by the *a priori* information available on the destination class. The main advantage of this latter measure lies in its somewhat more intuitive “Proportional Reduction of Error (PRE)”-interpretation – a proportion might be easier to grasp than the difference of two abstract entropies. However, the normalization destroys some of the desired properties of the measure. If the destination class is already strongly determined by the marginal distribution, we want to take this into account when measuring the relevance of social origin for an individual’s class affiliation. Mora and Ruiz-Castillo (2009a: 188–90) show that the way this is done when calculating the  $M$ -index leads to more consistent results than the  $H$ -index. These authors also point out that only the  $M$ -index has strong decomposability properties respecting the contributions of several subgroups to the overall linkage (Mora and Ruiz-Castillo 2011: 173–84). For example, the  $M$ -index can be decomposed into local  $M$ -indices for each class of origin without introducing any ambiguities. More specifically, the local  $M$ -index for those with farming parents will tell us how strongly predetermined the class of daughters and sons of farmers is because of the fact that their parents were farmers. An additional, more mundane reason for choosing the  $M$ -index instead of the  $H$ -index relates to the fact that the normalization necessary to calculate the  $H$ -index can be difficult to implement in the case of more complex applications. Choosing a simpler measure thus prepares the ground for an easy implementation of future features, allowing new insights into the mobility process. Finally, the  $M$ -index has been chosen by other researchers for studying conceptually similar social phenomena (Mora and Ruiz-Castillo 2011; DiPrete et al. 2017; Forster and Bol 2018), mostly because it can be perfectly decomposed into local  $M$ -indices. Therefore, choosing the  $M$ -index instead of another measure grounded into information theory provides a better integration into the existing literature.<sup>2</sup>

Before formalizing the  $M$ -index in the next section, we would like to illustrate the basic idea using two somewhat exaggerated examples. First, imagine you were visiting a large building and

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<sup>2</sup> A counterexample is the very recent study by Ferguson and Koning (2018) on segregation within firms using  $H$ . It is unfortunate that they do not justify their choice of measure within the family of entropy-based measures, as some of the arguments against the use of  $H$  discussed by Mora and Ruiz-Castillo (2011) seem to apply here.

you were guessing the sex of the next person to come around the corner. Your surprise to meet a man will be limited, as the odds are about even. Nevertheless, you would be even less surprised if you knew you were in a monastery. The second example is closer to the framework of social mobility. Imagine you were meeting an unknown woman and guessing her social status. You are very surprised to learn you are talking to a princess, as the odds of doing so are very small. However, you would not be surprised at all if you knew her mother is a queen.

In both cases, the difference between the *a priori* and the *a posteriori* information gain is large because the additional information largely determines the outcome at hand. Monasteries and convents are segregated by sex and a mother being a queen usually makes the daughter a princess. The mutual information between the outcome (male or princess, respectively) and the context information (monastery or mother being a queen, respectively) thus reflects the strong link between them. Given the different marginal distributions, however, the mutual information will be much lower in the case of the monk than in case of the princess. This reflects the fact that for a randomly chosen woman being a princess, it is much more relevant to have a queen as a mother than it is to live in a monastery for a randomly chosen person to be male.

In the first case, mutual information can be seen as a measure for residential segregation by gender, in the second for the effects of social origin on social position. In both cases, however, the examples tell little about segregation or the importance of social origin in general, because in most societies both monks and queens make up only a very small fraction of the population. The *M*-index considers this by weighting the local mutual information measure by its respective demographic proportion. The fact that *M*-index can be additively combined from weighted local measures means, in reverse, that *M*-index is directly decomposable into sub-group specific local linkages. The advantage of this decomposability is two-fold. First, a decomposition into origin specific linkages allows measuring the influence of a specific origin. Analogously, the decomposition into destination specific linkages allows answering the question how relevant social origin is for entering a certain destination class. Second, the overall *M*-index can be split into

linkages by macro-units – for example by birth cohort or country. Differences in macro-unit specific linkages can then be explained by macro-level characteristics.

The main strength of such a measure of the relevance of social origin stems from the flexibility in the specification of what one initially knows about an individual's social position and of what makes up the additional information on social origin. In other words, what is known *a priori* about  $X$  is not necessarily limited to the unconditional distribution of  $X$ , but could include information on some control variables  $V$ . For example, the data used in our second empirical example below stem from different sources. In such a case,  $V$  could include a variable indicating the data sources, which controls for differences in the marginal distribution of  $X$  between surveys.  $V$  could also contain mediators between  $Y$  and  $X$ . For example, if  $V$  includes an individual's own education,  $M$  is a measure of the direct effect of social origin, net of education. Obviously,  $V$  also needs to be part of the *a posteriori* information to make the two comparable. Finally, researchers applying this approach are also free to specify the additional information that makes up (together with the *a priori* information) the *a posteriori* information. More specifically, the added information is not limited to one measure of social origin (such as the father's class), but could include both parents' own occupational status plus the highest educational level attained by each of them. In this case, the  $M$ -index represents what one learns about an individual's own status by becoming aware of all these characteristics of her or his social origin.

What remains on the list of desirable features of a measure for the general relevance of social origin discussed above is the ability to separate changes in the influence of social origin from the influence of the marginal distribution. As we will see in the next sub-section, two different decomposition methods have been proposed to approach this goal. As discussed above, the  $M$ -index considers the marginal distribution in two respects: first, for comparing the influence of social origin to the one of the margins; and, second, when weighting the information gain for each origin–destination-combination with its population share. In other words, the  $M$ -index can be thought to consist of these three elements: it increases with the influence of social origin, it decreases with the influence of the margins, and it increases with share of the population for

which origin is of high relevance. Mora and Ruiz-Castillo (2009a, 2011) proposed a method that makes it possible to decompose differences in the  $M$ -index between two groups (e.g., between two birth cohorts) at least partially into these elements. It separates the part originating from differences of the influence of one of the margins, the part originating from differences in the weights according to the other marginal distribution, and residual differences net of the other two differences. As the residual part still contains an unknown part stemming from other differences in the marginal distribution, this is not a perfect decomposition for obtaining a margin-free measure; it nevertheless provides valuable insights into the differences in the stratification system of the two groups compared. However, Deutsch et al. (2006) proposed a method that directly aims at separating the differences stemming from the marginal distribution from the differences stemming from the “internal structure” (i.e., the pattern of associations between the classes of the two generations). Their thinking is that when comparing two mobility tables, one might change the one into the other by taking two steps: first by changing the margins, and second by changing the internal structure (or the other way around). The proposed method allows retracing these steps; this makes it possible to obtain the portions changed in each step, and thus to determine the contribution of differences in the associations to the overall difference in the  $M$ -index separately from the contribution of the differences in the margins. In other words, this complete decomposition of any change or difference in the  $M$ -index yields both the counterfactual change in the  $M$ -index if only the margins changed, and the part if only the associations had changed, but not the marginal distribution – together, they add up to the factual difference in the  $M$ -index. Unfortunately, this decomposition is not yet available for more advanced uses of the  $M$ -index and can only be applied when analyzing and comparing the overall association between two categorical variables without any further variables involved.

## **4 Methodological and technical aspects of the $M$ -index**

### **Basic definition**

As discussed above, the linkages between the class of parents and their children can be approached by measuring the amount of information on a child’s class  $y$  that can be gained by

learning the class of her or his parents  $x$ . In this section, we give a more technical overview. While the basic concepts are presented based on the literature (Theil 1970; Theil and Finizza 1971; Mora and Ruiz-Castillo 2009a; Frankel and Volij 2011; DiPrete et al. 2017), we also provide some original contributions when it comes to extensions specifically designed for studying questions of social mobility.

The  $M$ -index is an entropy-based measure, as entropy measures the amount of information available about  $y$ . The index measures the mutual information shared by the class of parents and their children, and can be obtained by comparing the *a posteriori* entropy (after learning the parents' class) to the *a priori* entropy, which measures the information available on  $y$  before learning the parents' class and is a function of the marginal distribution of the classes of destination. The *a priori* entropy is given by

$$T(P_Y) = - \sum_{k=1}^K p(y_k) \ln(p(y_k)). \quad 3$$

If everyone belongs to the same class and all other classes are empty, we know everything on  $y_i$  just by being aware of the distribution of  $Y$ . In this case, the entropy is zero,<sup>3</sup> because we learn nothing by actually observing  $y_k$ . By contrast, if all classes are equally distributed, it is much harder to guess  $y_k$ , and the information gained by actually observing it is much greater. In this case,  $T(P_Y)$  reaches its theoretical maximum, which is  $\log(K)$  (Theil 1970).

The analog logic applies for calculating the *a posteriori* entropy, which measures the information on  $y$  after learning  $x$ , the class of the parents. Once we know the parents' class, the relevant distribution is no longer the marginal distribution of  $Y$  but the distribution of  $Y|x_j$ , that is the distribution of classes among the descendants of the class  $x_j$  (the class of the parents). For this class, the entropy is

$$T(P_Y|x_j) = - \sum_{k=1}^K p(y_k|x_j) \ln(p(y_k|x_j)) \quad 4$$

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<sup>3</sup>  $0 \ln(0)$  is treated as 0 here (Theil 1972: 5).



and the weighted average over all classes of origin yields the overall *a posteriori* entropy:

$$T(P_{Y|X}) = - \sum_{j=i}^J p(x_j) \sum_{k=1}^K p(y_k|x_j) \ln(p(y_k|x_j)). \quad 5$$

The *M*-index measuring the intergenerational class linkage is then given by the difference between equations 5 and 4, which can be simplified to equation 2 given above:

$$\begin{aligned} M &= T(P_Y) - T(P_{Y|X}) \\ &= \left[ - \sum_{k=1}^K p(y_k) \ln(p(y_k)) \right] - \left[ - \sum_{j=i}^J p(x_j) \sum_{k=1}^K p(y_k|x_j) \ln(p(y_k|x_j)) \right] \\ &= \left[ - \sum_{j=1}^J \sum_{k=1}^K p(x_j, y_k) \ln(p(y_k)) \right] - \left[ - \sum_{j=i}^J \sum_{k=1}^K p(x_j, y_k) \ln(p(y_k|x_j)) \right] \quad 6 \\ &= \sum_{j=1}^J \sum_{k=1}^K p(x_j, y_k) \ln \left( \frac{p(y_k|x_j)}{p(y_k)} \right). \end{aligned}$$

So far, the *M*-index has been presented in its basic form. To exploit the full flexibility of the *M*-index, the definition needs to be generalized. However, continuing based on equation 6 would require a completely different notation (see Mora and Ruiz-Castillo 2009b; Stone 2016) that would add little to the understanding of the *M*-index in the way we like to apply it. Rather, we can rewrite the *M*-index with elements calculated at the individual level. We do so by defining an individual-level  $m_i$  in a way such that the expected value of  $m_i$ , i.e.  $E(m_i)$ , equals the macro-level *M*. This condition is satisfied if

$$m_i = \ln \left( \frac{\Pr(Y_i = k|X_i)}{\Pr(Y_i = k)} \right), \quad 7$$

where  $\Pr(Y_i = k)$  is the probability that the destination class is the one observed in case  $i$ , while  $\Pr(Y_i = k|X_i)$  is the probability that the destination class is the one observed in case  $i$  conditional on the observed class of origin.  $E(m_i)$  equals *M* because

$$\begin{aligned}
E(m_i) &= \frac{1}{N} \sum_{i=1}^N \ln \left( \frac{\Pr(Y_i = k | X_i)}{\Pr(Y_i = k)} \right) \\
&= \sum_{j=1}^J \sum_{k=1}^K \sum_{i=1}^N \frac{1}{N} [X_i = j, Y_i = k] \ln \left( \frac{\Pr(Y_i = k | X_i)}{\Pr(Y_i = k)} \right) \\
&= \sum_{j=1}^J \sum_{k=1}^K p(x_j, y_k) \ln \left( \frac{p(y_k | x_j)}{p(y_k)} \right) = M.
\end{aligned} \tag{8}$$

The second step in equation 8 may not be necessary, but it illustrates the fact that we can replace the cell-based weighting of the term  $\ln \left( \frac{p(y_k | x_j)}{p(y_k)} \right)$  by averaging over the sample: running through all cases in the sample, counting those satisfying the condition  $[X = x_j, Y = y_k]$ , and dividing the result by  $N$  yields  $p(x_j, y_k)$ .

With an  $M$ -index definition based on individual-level elements at hand, the generalization of the index is straightforward. As already noted by Theil (1970), the  $M$ -index cannot only be used to measure the information gain (or the reduction of entropy in  $Y$ ) between the state zero, when only the marginal distribution of  $Y$  is known, and the state one, when the distribution of  $Y|X$  is known (i.e.,  $T(P_Y) - T(P_{Y|X})$ ). Rather, it can also be used for measuring the (partial) entropy reduction due to learning any set of variables  $\mathbf{X}$ , be they categorical or continuous, over and above the entropy reduction due to the set of variables  $\mathbf{V}$ , where  $\mathbf{V}$  can (but does not necessarily need to) be empty. In this more general form, the  $M$ -index is defined by

$$M^* = T(P_{Y|\mathbf{V}}) - T(P_{Y|(\mathbf{X},\mathbf{V})}), \tag{9}$$

where  $\mathbf{X}$  is a vector of variables measuring social origin and  $\mathbf{V}$  is an optional set of (control) variables. Note that  $M$ , as defined in equation 2, is a special case of  $M^*$ , where  $\mathbf{V}$  is empty and  $\mathbf{X}$  includes only a single (categorical) variable.  $M^*$  can be obtained by combining 9 and 8:

$$M^* = T(P_{Y|\mathbf{V}}) - T(P_{Y|(\mathbf{X},\mathbf{V})}) \tag{10}$$

$$= \frac{1}{N} \sum_{i=1}^N \ln \left( \frac{\Pr(Y_i = k | X_i, V_i)}{\Pr(Y_i = k | V_i)} \right) = E(m_i^*).$$

### Implementation and statistical inference

There are two distinct technical approaches for obtaining the  $M$ -index. The first is to calculate it directly on the basis of contingency tables; the second is a model-based approach using multinomial logistic regression as a basis for predicting the (conditional) probabilities.

The first approach is straightforward and very fast in terms of computation time. However, its implementation quickly becomes unfeasible when more than three variables (one each for origin, destination, and birth cohort) are involved. For exploiting the full flexibility offered by measuring the linkage between origin and destination by the  $M$ -index, we use the second approach based on multinomial logistic regression models for predicting the probabilities to be plugged into equation 10 (any other appropriate statistical model that allows predicting the required probabilities would do). This can be done separately for distinct birth cohorts by estimating the models used to predict both  $\widehat{\Pr}(Y_i = k | V_i)$  and  $\widehat{\Pr}(Y_i = k | X_i, V_i)$  separately for each cohort. Alternatively, one can fully interact the variables  $\mathbf{X}$  and  $\mathbf{V}$  with  $\mathbf{Z}$ , where the last variable can be (for example) a set of dummy variables measuring birth cohorts or a linear or quadratic parametrization of time using the respondent's year of birth.

From a practical point of view, our procedure implements four steps:

1. Estimate the restricted model, which does not include the variables  $\mathbf{X}$ :

$$\widehat{\Pr}(Y_i = k | \mathbf{V}, \mathbf{Z}, \mathbf{VZ}) = \frac{\exp(\alpha_k + \boldsymbol{\varphi}'_k \mathbf{V}_i + \boldsymbol{\gamma}'_k \mathbf{Z}_i + \boldsymbol{\delta}'_k \mathbf{V}_i \mathbf{Z}_i)}{1 + \sum_{h=1}^{J-1} \exp(\alpha_h + \boldsymbol{\varphi}'_h \mathbf{X}_i + \boldsymbol{\gamma}'_h \mathbf{Z}_i + \boldsymbol{\delta}'_h \mathbf{V}_i \mathbf{Z}_i)} \quad 11$$

where  $\alpha_h$  are the outcome specific constants of the multinomial logistic regression model and  $\boldsymbol{\varphi}'_h$ ,  $\boldsymbol{\gamma}'_h$ , and  $\boldsymbol{\delta}'_h$  are outcome specific coefficient vectors for the variables  $\mathbf{V}$ ,  $\mathbf{Z}$  and their interaction, respectively.

2. Estimate the unrestricted model, which does include the variables  $\mathbf{X}$ :

$$\begin{aligned} & \widehat{\Pr}(Y_i = k | \mathbf{V}, \mathbf{X}, \mathbf{Z}, \mathbf{V}\mathbf{X}, \mathbf{V}\mathbf{Z}) \\ &= \frac{\exp(\alpha_k + \boldsymbol{\beta}'_k \mathbf{X}_i + \boldsymbol{\varphi}'_k \mathbf{V}_i + \boldsymbol{\gamma}'_k \mathbf{Z}_i + \boldsymbol{\delta}^{X'}_k \mathbf{X}_i \mathbf{Z}_i + \boldsymbol{\delta}^{V'}_k \mathbf{V}_i \mathbf{Z}_i)}{1 + \sum_{h=1}^{J-1} \exp(\alpha_h + \boldsymbol{\beta}'_h \mathbf{X}_i + \boldsymbol{\varphi}'_h \mathbf{V}_i + \boldsymbol{\gamma}'_h \mathbf{Z}_i + \boldsymbol{\delta}^{X'}_h \mathbf{X}_i \mathbf{Z}_i + \boldsymbol{\delta}^{V'}_h \mathbf{V}_i \mathbf{Z}_i)} \end{aligned} \quad 12$$

where  $\boldsymbol{\beta}'_k$  and  $\boldsymbol{\delta}^{X'}_k$  are additional outcome specific coefficient vectors for the variables  $\mathbf{X}$ , and the interaction of the  $\mathbf{X}$  with  $\mathbf{Z}$ .

3. Calculate  $m_i$  based on the predictions under the models estimated in step 1 and 2:

$$m_i = \log \left( \frac{\widehat{\Pr}(Y_i = k | \mathbf{V}, \mathbf{X}, \mathbf{Z}, \mathbf{V}\mathbf{X}, \mathbf{V}\mathbf{Z})}{\widehat{\Pr}(Y_i = k | \mathbf{V}, \mathbf{Z}, \mathbf{V}\mathbf{Z})} \right) \quad 13$$

4. Estimate the expected value of  $m_i$  to obtain the  $M$ -index separately for various values of  $\mathbf{Z}^*$  (and optional controls for  $\mathbf{V}^*$ ) by using an ordinary least square regression model, where  $\mathbf{Z}^*$  and  $\mathbf{V}^*$  are subsets of the sets of variables  $\mathbf{Z}$  and  $\mathbf{V}$ , respectively,  $\boldsymbol{\beta}'$  and  $\boldsymbol{\gamma}'$  are the corresponding coefficient vectors and  $\alpha$  is the constant:<sup>4</sup>

$$M(\mathbf{V}, \mathbf{Z}) = E(m_i | \mathbf{V}, \mathbf{Z}) = \widehat{m}_i = \alpha + \boldsymbol{\beta}' \mathbf{V}_i^* + \boldsymbol{\gamma}' \mathbf{Z}_i^*, \quad \mathbf{V}_i^* \subseteq \mathbf{V}_i, \mathbf{Z}_i^* \subseteq \mathbf{Z}_i \quad 14$$

For the examples in this paper, we have written an estimator based on the Generalized Method of Moments (GMM), which allows to complete the above four steps simultaneously while taking into account that the probabilities used for calculating  $m_i$  are estimated and not observed (Greene 2012; Drukker 2014). Not taking this into account will result in biased standard errors produced by the regression model in equation 14.

### Counterfactual decomposition

As pointed out in section 3, two counterfactual decomposition methods are available that make it possible to assess the contributions of changes in the marginal distribution to changes in the  $M$ -index. Both provide pairwise decompositions that allow for the decomposition of the difference in the  $M$ -index between two birth cohorts into counterfactual portions. In other words, they

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<sup>4</sup> There are scenarios in which  $\mathbf{Z}^*$  is not strictly a subset of  $\mathbf{Z}$ . For example, if  $\mathbf{Z}$  contains dummy variables for distinct groups such as cohorts or countries,  $\mathbf{Z}^*$  could include macro-variables measuring characteristics of  $\mathbf{Z}$  that explain differences between these groups.

answer questions like “What would this difference look like if only A, but not B and C, had changed between the two birth cohorts?”

A first decomposition, proposed by Mora and Ruiz-Castillo (2009a; DiPrete et al. 2017), allows us to decompose the difference between cohort  $k$  and  $k'$  into  $\Delta O$ , the difference in the entropy of parents' class distribution,  $\Delta D$ , the difference in the distributions of the classes of destination, and  $\Delta N$ , the residual change, net of these differences. For the technical details on this decomposition method, see Mora and Ruiz-Castillo (2009a) or DiPrete et al. (2017).

The second decomposition of pairwise differences in the  $M$ -index was proposed for the study of social mobility by Silber and Spadaro (2011), based on work by Deutsch et al. (2006; partly inspired by Karmel and Maclachlan 1988), and makes it possible to perfectly separate the change in the  $M$ -index due to changes in the association patterns from the own caused by changes in the marginal distribution.

The decomposition consists of a conceptual and a technical part. The conceptual part starts with the idea that the difference in the  $M$ -index between two cohorts  $k$  and  $k'$  is the result of two contributions:  $C_{marg}$ , stemming from differences in the marginal distributions; and  $C_{int}$ , stemming from differences in the internal structures of the two mobility tables. Therefore,

$$M_k - M_{k'} = \Delta M(\Delta marg, \Delta int) = C_{marg} + C_{int}. \quad 15$$

There are two equivalent ways to identify the contribution that comes only from differences in the internal structures of the mobility tables. First, we can either calculate directly  $\Delta M(\Delta int)$  from the two tables  $\mathbf{K}$ , which is the factual mobility tables for cohort  $k$ , and  $\mathbf{K}'_{cf}$ , which is the counterfactual table for cohort  $k'$ , with a factual internal structure but counterfactual marginal distributions. Alternatively, we can calculate the factual difference  $\Delta M(\Delta marg, \Delta int)$ , then subtracting  $\Delta M(\Delta marg)$ , based on  $\mathbf{K}$  and  $\mathbf{K}_{cf}$ . Because both ways are equally justified, we average between them for obtaining  $C_{int}$ :

$$C_{int} = \frac{1}{2}(\Delta M(\Delta marg, \Delta int) - \Delta M(\Delta marg)) + \frac{1}{2}(\Delta M(\Delta int)) \quad 16$$

The same Shapley decomposition procedure (Chantreuil and Trannoy 1999, 2013) can then be analogously applied for obtaining  $C_{marg}$ .

The technical part of this decomposition method consists in the use of the raking procedure first proposed by Deming and Stephan (1940). Let  $\pi_{kij}$  be the cell proportions,  $\pi_{k\cdot j}$  the marginal distribution of the classes of origin, and  $\pi_{ki\cdot}$  marginal distribution of the destination classes for the cohort  $k$ , while  $\pi_{k'ij}$ ,  $\pi_{k'\cdot j}$ , and  $\pi_{k'i\cdot}$  are the equivalents for cohort  $k'$ .  $\mathbf{K}_{cf}$ , which has the internal structure of  $\mathbf{K}$  but the margins of  $\mathbf{K}'$ , can then be obtained by iteratively re-weighting  $\pi_{kij}$  with  $w_{row} = \frac{\pi_{k'\cdot j}}{\pi_{k\cdot j}}$  and  $w_{col} = \frac{\pi_{k'i\cdot}}{\pi_{ki\cdot}}$ . After a few iterations, the resulting table converges to  $\mathbf{K}_{cf}$ . The resulting tables can then be used for calculating the elements of equations 16:

$$\begin{aligned}\Delta M(\Delta marg, \Delta int) &= M(\mathbf{K}) - M(\mathbf{K}') \\ \Delta M(\Delta int) &= M(\mathbf{K}) - M(\mathbf{K}'_{cf}) \\ \Delta M(\Delta marg) &= M(\mathbf{K}) - M(\mathbf{K}_{cf})\end{aligned}\tag{17}$$

## 5 Example application 1: Reanalysis of Long and Ferrie (2013)

A brief reanalysis of the data analyzed by Long and Ferrie (2013a) highlights the usefulness of the  $M$ -index as a measure of the overall level of social fluidity in general and the decomposition proposed by Deutsch et al. (2006) in particular. Long and Ferrie (2013a) analyzed social mobility in Great Britain and the United States after 1850. Their most controversial conclusion was that the US was more open in the 19<sup>th</sup> than in the 20<sup>th</sup> century. Both their own measure and the unidiff parameters they estimated suggested so.

When reanalyzing their data using a unidiff model for 1880, 1900 (the reference table), and 1973 (Long and Ferrie 2013a: Tables 1 & 3), we can confirm their conclusion: class barriers became more rigid from 1880 to 1900 and again from 1900 to 1973.<sup>5</sup> In the counterfactual case

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<sup>5</sup> Compared to 1990 the unidiff parameter is estimated to be lower for 1880 (-0.219,  $p < 0.001$ ) and for 1973 (0.199,  $p = 0.036$ ); we follow Long and Ferrie (2013a) in estimating separate models for each pairwise contrast.

(that the margins in 1880 and 1973 had been the same as in 1900; applying the decomposition proposed by Deutsch et al. (2006)), the *M*-index leads to the same conclusion. Compared to 1900, the *M*-index would be lower in 1880 ( $-0.036, p=0.001$ ) and higher in 1973 ( $0.036, p=0.009$ ; both *p*-values based on bootstrapped standard errors with 1,000 replications).

These results, indicating a consistent increase from 1880 to 1900 and from 1900 to 1973, are surprising and were disputed when they were first published by Long and Ferrie (2013a). Both Hout and Guest (2013) and Xie and Killewald (2013) criticized the results as driven only by the (increasingly) strong rate of farmers recruited among sons of farmers, while at the same time the proportion of farmers among the US's population had decreased dramatically – something that had already been highlighted by Long and Ferrie (2013a). Using the *M*-index, we can take this shrinking proportion of farmers into account by weighting each origin–destination combination by its relative population weight, which yields the factual (not decomposed) *M*-index. This tells a different story from the margin-free measures. If we are analyzing the general relevance of social origin for an individual's class affiliation, we see that origin has indeed become more important between 1880 and 1900 (the *M*-index rose from 0.073 to 0.107,  $p=0.002$ ), but between 1900 and 1973 the relevance of social origin returned to about the level of 1880 (the *M*-index decreased from 0.107 to 0.070,  $p=0.002$ ). Finally, using the *M*-index, it is also straightforward to reassess the role of farmers in this process, which had led to such divergent results (Xie and Killewald 2013; Hout and Guest 2013). If we calculate the *M*-index locally for each destination class, we see a stable trend for the white-collar class from 1880 to 1900 and a clear decrease between 1900 and 1973, while for both the skilled/semiskilled and for the unskilled working classes the relevance of social origin did not vary significantly. In contrast to the classes where the relevance remained stable or decreased, the class of origin increased dramatically in relevance for becoming a farmer: the local *M*-index rose from 0.081 in 1880 to 0.173 in 1900, then rose to an excessive 1.069 in 1973 (because a four-fold classification scheme is used, the theoretical maximum is  $\log(4) = 1.386$ ). However, as the proportion of farming sons decreased equally strongly (1880: 43.9%, 1900:

31.5%, and 1973: 2.5%), this increased relevance of social origin for becoming a farmer is of little importance for the overall *M*-index in 1973.

Following this approach, the *M*-index confirms the hypothesis that the increasing importance of social origin between 1900 and 1973 found by Long and Ferrie (2013a) was only driven by farmers. However, instead of “glossing over” (Long and Ferrie 2013b) the problem of dominant origin–destination combinations by simply ignoring the main-diagonal of the mobility table (i.e., by ignoring class immobility; Long and Ferrie 2013b; Xie and Killewald 2013), the *M*-index weights each of these combinations according to their population weight. Because of this weighting, the *M*-index properly counterbalances the increasing relevance of social origin for becoming a farmer by the shrinking importance of this class for an assessment of the level of origin effects in the whole population.

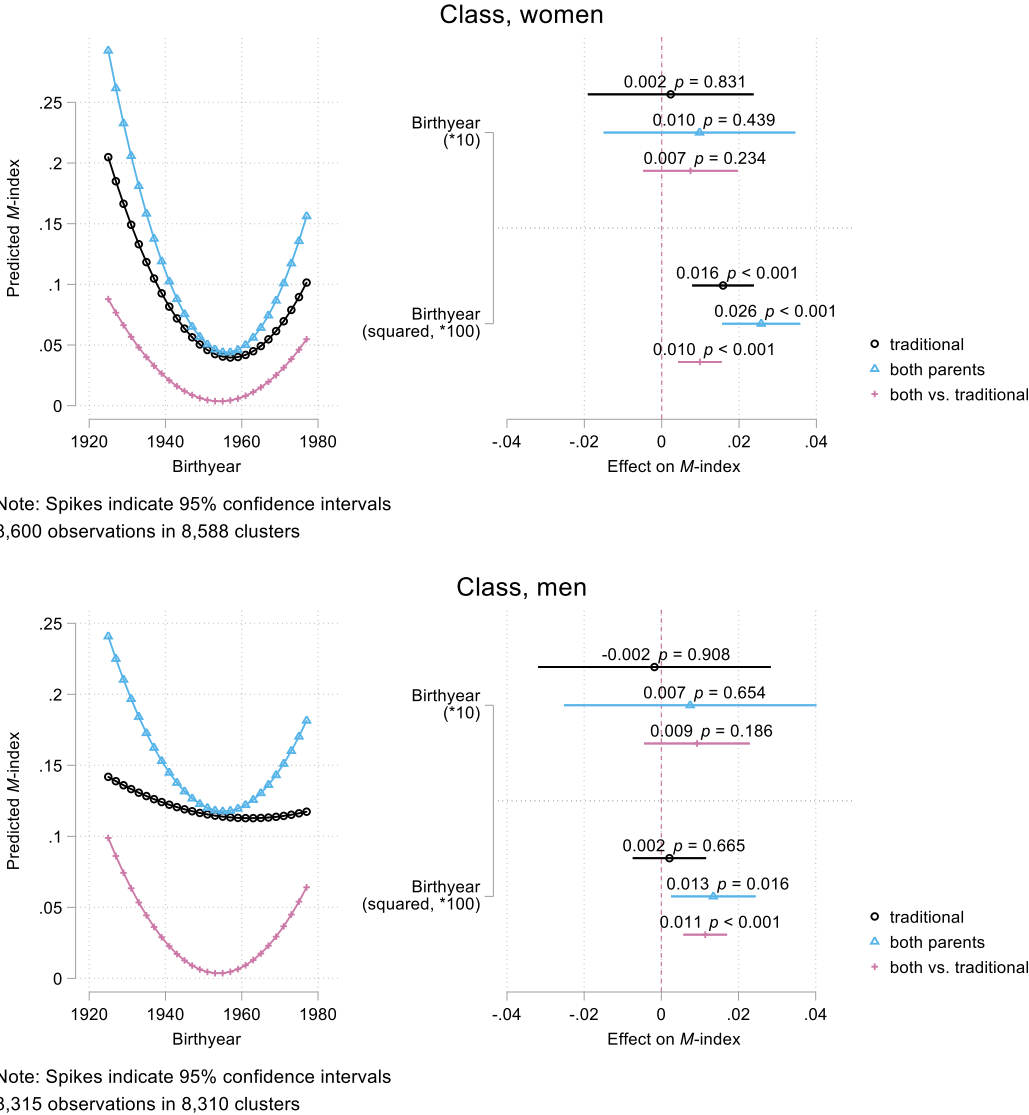
## **6 Example Application 2: Social mobility in 20<sup>th</sup> century Switzerland**

As discussed above, the *M*-index can easily be used for quantifying the joint relevance of multiple variables measuring social origin. This makes it possible to study the effects of different components of social origin. Such questions have been neglected as status attainment models have gone out of fashion, and have only recently regained attention (Buis 2013; Bukodi and Goldthorpe 2013). For example, for a long time, research on class mobility considered only either father’s class or the class of the parent with the highest status (dominance approach, Erikson 1984), which conforms with the conventional view on social origin, according to which it is the household as an entity that is relevant. Only relatively recently did this view lose its dominance (Beller 2009), although arguments and evidence against it have been around for a while (Sorensen 1994; Korupp et al. 2002). Using data from Switzerland, this section showcases the importance of considering multiple measurements of social origin based on fathers and mothers, which is straightforward using the *M*-index.

We use a harmonized dataset that has, in part, operated as the basis of studies undertaken by Jann and Combet (2012) and Jann and Seiler (2014). It includes 10 surveys with a total of about 24,000 observations in 20 waves that all include information on the educational attainment and



social position of parents and respondents (see Jann and Seiler 2014). In our analysis we include all cases for which information on educational attainment and social class is available for the respondents and both parents. To avoid strong age effects, we restrict the sample to respondents aged between 35 and 69. For social class, we use a slightly simplified EGP scheme (Erikson et al. 1983: 307) with seven classes (again see Jann and Seiler 2014), but to utilize the full richness of the data we include *homemaker* as an eighth class in the case of mothers.



**Figure 1.** Intergenerational class linkage: adding information on mothers

Note: Includes controls for survey- and age-effects.

Following the traditional approach for measuring social origin (father’s class only), **Figure 1** shows an U-shaped, curvilinear time trend for women: While the linear ten-year coefficient (centered around 1958) is close to zero, the quadratic term is significantly positive. Thus, the

relevance of father's class for daughter's class decreased in the beginning of the observed period (slope in 1925:  $-0.010$ ,  $p < 0.001$ ) reached a minimum around 1957.3 (95% CI [1950.6, 1964.0]) and re-increased later-on (slope in 1978:  $0.007$ ,  $p = 0.002$ ). Further including mother's class for measuring social origin reveals that the additional relevance of the mother's social class over and above that of the father's social class follows a pronounced U-shaped pattern. While mothers mattered strongly at the beginning and at the end of the observed period, this was not the case in the middle part. In fact, for both women and men born in the 1940s and 1950s, there is no evidence that their mothers' occupation was relevant for the occupational class to which the respondents belonged. Because the additional information gained from learning the mother's class follows the same pattern as the relevance of father's class, the resulting time trend is even clearer in its particular shape. Specifically, the relevance of parental class was high at the beginning of the observed period, but clearly decreased (with a slope at 1925:  $-0.016$ ,  $p < 0.001$ ), reached a minimum estimated at 1956.1 (95% CI [1951.3, 1960.9]), and increased again thereafter (with a slope at 1978:  $+0.011$ ,  $p < 0.001$ ).

For men, explicitly considering mothers radically changes our conclusion on temporal changes in the relevance of parent's social class for men's class affiliation. When only considering father's occupational class, we do not find any time trend (neither coefficient for the linear nor the one for the quadratic term for time differ significantly from zero); however, when taking into account each parent's class independently, we find a U-shaped trend for men too – a result of the clear curvilinear pattern of the mother's relevance. Accordingly, we find a pattern that is very similar to that for women, albeit somewhat less pronounced (with a slope at 1925:  $-0.008$ ,  $p = 0.028$ , a minimum at 1955.2 (95% CI [1943.4, 1967.1]), and a slope at 1978:  $+0.006$ ,  $p = 0.045$ ).

We could continue our analysis by including further social origin variables, such as father's and mother's education. Using the *M*-index, such an extension is as simple as adding further predictors to the right-hand side of a regression model.

## 7 Conclusion

In this paper, we argued that the “margin-free” approach of log-linear models may be well suited for analyzing specific class barriers and other patterns in the mobility regimes. However, when it comes to measuring and comparing the overall relevance of social origin in a society, methods such as the unidiff model may not be an ideal choice. In particular, it seems conceptually questionable whether it is reasonable to quantify the general origin effects in a society at the analytical level of the cells of a mobility table, without taking the prevalence into account with which certain classes exist in a society. Furthermore, an important limitation of log-linear models is that it is difficult to deal with situations in which multiple independent characteristics of family of origin, such as each parents’ educational attainment and social class, should be taken into account. To overcome these issues, we proposed the use of the *M*-index – which measures “the degree to which the determining factors of our relations account for the phenomenon which they serve to explain” (Theil 1970: 125) – as a valid measure of the overall relevance of social origin for individual’s status attainment. We showed that the *M*-index can be a flexible tool for analyzing questions of social mobility, which includes considering multiple dimensions of social origin, disaggregation by origins or destinations, controls for confounding variables, and more. Most importantly, however, the *M*-index avoids false conclusions on the development of social fluidity at the societal level by bringing the margins back in.

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