$T\bar{T}$ deformations with $\mathcal{N}=(0,2)$ supersymmetry

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We investigate the behavior of two-dimensional quantum field theories with $\mathcal{N}=(0,2)$ supersymmetry under a deformation induced by the "$T\bar{T}$" composite operator. We show that the deforming operator can be defined by a point-splitting regularization in such a way as to preserve $\mathcal{N}=(0,2)$ supersymmetry. As an example of this construction, we work out the deformation of a free $\mathcal{N}=(0,2)$ theory, compare to that induced by the Noether stress-energy tensor and argue that, despite their apparent difference, they are equivalent on shell. Finally, we show that the $\mathcal{N}=(0,2)$ supersymmetric deformed action actually possesses $\mathcal{N}=(2,2)$ symmetry, half of which is nonlinearly realized.

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I. INTRODUCTION AND SUMMARY

A fruitful way to gain new insight on quantum field theories (QFTs), and indeed on many physical theories, is to start by studying a particularly simple theory—for instance one that can be solved completely by virtue of its symmetries—and *deform it*. Generally, of course such deformations can be only understood in some perturbative or even formal expansion in a small parameter. In rare cases it is possible to treat the perturbation exactly, at least for the purpose of computing certain observables. Important examples of this type arise for instance for two-dimensional QFTs as *marginal* deformations, which preserve the scaling invariance of a two-dimensional relativistic conformal field theory (CFT), and (relevant) *integrable* deformations, which introduce a mass scale while preserving an infinite set of symmetries of the original (conformal) QFT. A recent addition to these two classes is that of so-called $T\bar{T}$ deformations of two-dimensional QFTs. They arise by deforming any Poincaré-invariant theory by a composite operator built as the determinant of the stress-energy tensor [1], leading to an *irrelevant* deformation. Not only does this deformation preserve many of the symmetries of the underlying theory (which makes it very interesting to study $T\bar{T}$ deformations of integrable QFTs and CFTs) but it modifies the theory's spectrum in a simple, "solvable" way [2,3], whose classical action can be often constructed in closed form [3,4].[1]

Following these observations, several other interesting properties of such deformations have recently been analyzed. In the case of integrable theories these deformations can be understood [2,3] as a modification of the factorized S matrix by a universal Castillejo-Dalitz-Dyson ("CDD") [10] factor. Such a CDD factor is rather peculiar, because it deforms the large-energy *ultraviolet* properties of the original theory—in accord with the fact that $T\bar{T}$ deformations are "irrelevant"—rather than introducing poles as it is usually the case for integrable deformations. Interestingly, the "$T\bar{T}$" CDD factor had also appeared in the study of the S matrix on the world sheet of flat-space strings [3,11–16], strongly suggesting that flat-space strings are the $T\bar{T}$ deformation of a *free* theory (a fact that can also be substantiated from a Lagrangian or Hamiltonian analysis [3,15,16]). Indeed $T\bar{T}$ deformations are naturally related [15,16] to the "uniform" light-cone gauge which is quite natural for the study of integrable string theories [17–19], and the $T\bar{T}$ CDD factor also describes the scattering on more general backgrounds such as AdS_3 Wess-Zumino-Witten backgrounds [15,20,21].[2] A separate but equally interesting link between $T\bar{T}$ deformations and AdS_3 strings appears in the context of holography [27–34], where the

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[1]A number of generalizations of $T\bar{T}$, involving other conserved currents, have also been considered in the literature [5–9].

[2]Such backgrounds are supported by Neveu-Schwarz-Neveu-Schwarz fluxes only; the world sheet scattering for AdS_3 backgrounds involving Ramond-Ramond fluxes is substantially more involved [22–24], as it may also be understood by world sheet–CFT considerations [25]; see e.g., Ref. [26].

deformed two-dimensional theory is *the holographic dual* of some (AdS_3) gravity or string theory rather than being a world sheet theory.

The irrelevant nature of $T\bar{T}$ deformations leads to a number of very peculiar consequences. It turns out that $T\bar{T}$-deformed QFTs can also be understood as a gravitational theory and more specifically [35–38] as coupling the original two-dimensional theory to Jackiw-Teitelboim gravity [39,40]. For a generic theory, furthermore, the $T\bar{T}$ flow induces a singular behavior of some of the energy levels for a finite value of the deformation parameter in units of the theory's volume [2,3,41]. One exception to this scenario are supersymmetric QFTs which admit a well-defined flow. This is in good accord with the fact that some superstring theories can be described as $T\bar{T}$ deformations; interestingly, this setup seems to be well defined even when supersymmetry is nonlinearly realized [21]. Such musings, together with the intrinsic interest in supersymmetric CFTs, spurred a systematic investigation of the relation between $T\bar{T}$ deformation and supersymmetry [16,42]. So far, this investigation focused on theories with $\mathcal{N}=(0,1)$ and $\mathcal{N}=(1,1)$ supersymmetry.

The aim of this note is to extend such considerations to the case of extended $\mathcal{N}=(0,2)$ supersymmetry. The conclusion is that, indeed, $T\bar{T}$ deformations can be defined in such a way as to manifestly preserve the extended supersymmetry. In particular, the deforming operator can be thought of as the supersymmetric descendant of some suitable composite operator. Still, there are a few technical complications and conceptual subtleties with respect to the cases known in the literature which we find worth addressing in some detail.

We begin our note by introducing the $\mathcal{N}=(0,2)$ framework in Secs. II and III. After reviewing in Sec. II the structure of the $\mathcal{N}=(0,2)$ supercurrent multiplet, in Sec. III A we construct the $T\bar{T}$ operator as the supersymmetric descendant of a suitably defined composite operator, which is constructed out of the coincident-point limit of a quadratic combination of the supercurrents. Here we encounter a new feature: the primary operator does not take the form discussed by Smirnov and Zamolodchikov [2], yet it is possible to show that the coincident-point limit is free of short-distance singularities. Interestingly, the well-definedness arguments that we develop in this paper apply also for $T\bar{T}$ deformations of $\mathcal{N}=(2,2)$ supersymmetric QFTs and $J\bar{T}/T\bar{J}$ deformations. Also in these cases, as we will report in the near future [43,44], the primary operators are not of Smirnov-Zamolodchikov type, yet they are well defined.

As an example of this construction, in Sec. IV we work out the supersymmetric $T\bar{T}$ deformation of a free $\mathcal{N}=(0,2)$ action and compare it with the one constructed out of the Noether $T\bar{T}$ operator. The latter is most easily obtained from the Green-Schwarz string; see Ref. [16]. We conclude that, while apparently different, the two deformations are identical on shell, giving rise to equivalent flow equations for the spectrum—as expected from the lower supersymmetric cases [16]. Finally, in Sec. V we show that the deformation of the free theory also describes the partial supersymmetry breaking from $\mathcal{N}=(2,2)$ to $\mathcal{N}=(0,2)$. In fact, such a $T\bar{T}$-deformed action is equivalent to the model of partial supersymmetry breaking in two dimensions by Hughes and Polchinski [45]; see also Ref. [46], which describes a $\mathcal{N}=(0,2)$ extension of a 4D Nambu-Goto superstring action. The action also shares several analogies with the four-dimensional Bagger-Galperin action describing the partial supersymmetry breaking from $\mathcal{N}=2$ to $\mathcal{N}=1$ in four dimensions [47]. This is a further example of how $T\bar{T}$-deformed theories possess nonlinearly realized (super)symmetries, which is something that would be interesting to explore in greater detail. Some first results will appear in the near future [43,48]. In particular, it can be shown that the $T\bar{T}$ deformation of free $\mathcal{N}=(2,2)$ theories also describes supersymmetric extensions of a 4D Nambu-Goto superstring action possessing extra $\mathcal{N}=(2,2)$ nonlinearly realized supersymmetry. In the $\mathcal{N}=(2,2)$ case, the resulting actions can in fact be recast in forms that are formally identical to the 4D Bagger-Galperin action [47] for the supersymmetric extension of the Dirac-Born-Infeld (DBI) action, hinting at a $T\bar{T}$ structure of the latter. It was already been shown in [38] that the classical bosonic 4D DBI action satisfies a peculiar $T\bar{T}$-flow equation. The same property generalizes to the supersymmetric case [48].

II. SUPERCURRENT MULTIPLET IN (0,2)

Let us review the structure of the supercurrent $\mathcal{S}$ multiplet of two-dimensional $\mathcal{N}=(0,2)$ supersymmetric field theories that we will need in our paper. In this section we follow the paper of Dumitrescu and Seiberg [49], including their conventions.

In light-cone coordinates, a flat 2D $\mathcal{N}=(0,2)$ superspace is parametrized by

$$\zeta^M = (\sigma^{++}, \sigma^{--}, \theta^+, \bar{\theta}^+), \tag{1}$$

with θ^+ a complex Grassmann coordinate and $\bar{\theta}^+$ its complex conjugate. The spinor covariant derivatives and supercharges are given by

$$\mathcal{D}_+ = \frac{\partial}{\partial\theta^+} - \frac{\mathrm{i}}{2}\bar{\theta}^+\partial_{++}, \quad \bar{\mathcal{D}}_+ = -\frac{\partial}{\partial\bar{\theta}^+} + \frac{\mathrm{i}}{2}\theta^+\partial_{++},$$
$$\mathcal{Q}_+ = \frac{\partial}{\partial\theta^+} + \frac{\mathrm{i}}{2}\bar{\theta}^+\partial_{++}, \quad \bar{\mathcal{Q}}_+ = -\frac{\partial}{\partial\bar{\theta}^+} - \frac{\mathrm{i}}{2}\theta^+\partial_{++} \tag{2}$$

and obey the anticommutation relations

$$\{\mathcal{D}_+, \bar{\mathcal{D}}_+\} = \mathrm{i}\partial_{++}, \qquad \{\mathcal{Q}_+, \bar{\mathcal{Q}}_+\} = -\mathrm{i}\partial_{++}, \tag{3}$$

with all the other (anti)commutators between $\mathcal{D}$'s, $\mathcal{Q}$'s, and $\partial_{\pm\pm}$ being identically zero. Given an $\mathcal{N}=(0,2)$ superfield[3] $\mathcal{F}(\zeta)=\mathcal{F}(\sigma,\theta)$, its supersymmetry transformations are given by

$$\delta_Q\mathcal{F} := \mathrm{i}\epsilon^+\mathcal{Q}_+\mathcal{F}(\sigma,\theta) - \mathrm{i}\bar{\epsilon}^+\bar{\mathcal{Q}}_+\mathcal{F}(\sigma,\theta). \tag{4}$$

Here ϵ^+ and its complex conjugate $\bar{\epsilon}^+$ are the complex fermionic supersymmetry transformation parameters. If $F(\sigma)$ is the operator defined as the $\theta^+=\bar{\theta}^+=0$ component of the superfield $\mathcal{F}(\zeta)$, $F(\sigma) := \mathcal{F}(\sigma,\theta)|_{\theta=0}$, then its supersymmetry transformations are such that

$$\begin{aligned}\delta_Q F(\sigma) &= [(\epsilon^+Q_+ - \bar{\epsilon}^+\bar{Q}_+), F(\sigma)]\\ &= [\mathrm{i}\epsilon^+\mathcal{Q}_+\mathcal{F}(\sigma,\theta) - \mathrm{i}\bar{\epsilon}^+\bar{\mathcal{Q}}_+\mathcal{F}(\sigma,\theta)]|_{\theta=0}\\ &= [\mathrm{i}\epsilon^+\mathcal{D}_+\mathcal{F}(\sigma,\theta) - \mathrm{i}\bar{\epsilon}^+\bar{\mathcal{D}}_+\mathcal{F}(\sigma,\theta)]|_{\theta=0}.\end{aligned} \tag{5}$$

We will indicate by Q_+ and $\bar{Q}_+$ the supersymmetry generator *acting on a component operator* and distinguish them from $\mathcal{Q}_+$ and $\bar{\mathcal{Q}}_+$, which are linear differential operators *acting on superfields*.

In two-dimensional $\mathcal{N}=(0,2)$ supersymmetric field theories, the supercurrent $\mathcal{S}$ multiplet is defined by the following constraints [49]:

$$\partial_{--}\mathcal{S}_{++} = \mathcal{D}_+\mathcal{W}_- - \bar{\mathcal{D}}_+\bar{\mathcal{W}}_-, \tag{6a}$$

$$\mathcal{D}_+\mathcal{T}_{----} = \frac{1}{2}\partial_{--}\bar{\mathcal{W}}_-, \tag{6b}$$

$$\bar{\mathcal{D}}_+\mathcal{T}_{----} = \frac{1}{2}\partial_{--}\mathcal{W}_-, \tag{6c}$$

$$\bar{\mathcal{D}}_+\mathcal{W}_- = C, \tag{6d}$$

$$\mathcal{D}_+\bar{\mathcal{W}}_- = -\bar{C}, \tag{6e}$$

where the complex constant C is associated with a space-time brane current. Since this term leads to symmetry breaking [45,49], in this paper for simplicity we will set it to zero, $C=0$.

In components, the supercurrent $\mathcal{S}$ multiplet is given by

$$\begin{aligned}\mathcal{S}_{++} &= j_{++} - \mathrm{i}\theta^+S_{+++} - \mathrm{i}\bar{\theta}^+\bar{S}_{+++} - \theta^+\bar{\theta}^+T_{++++},\\ \mathcal{W}_- &= -\bar{S}_{+--} - \mathrm{i}\theta^+\left(T_{++--} + \frac{\mathrm{i}}{2}\partial_{--}j_{++}\right) + \frac{\mathrm{i}}{2}\theta^+\bar{\theta}^+\partial_{++}\bar{S}_{+--},\\ \mathcal{T}_{----} &= T_{----} - \frac{1}{2}\theta^+\partial_{--}S_{+--} + \frac{1}{2}\bar{\theta}^+\partial_{--}\bar{S}_{+--} + \frac{1}{4}\theta^+\bar{\theta}^+\partial_{--}^2 j_{++}.\end{aligned} \tag{7}$$

The $j_{++}(\sigma)$, $S_{+\pm\pm}(\sigma)$, $\bar{S}_{+\pm\pm}(\sigma)$, $T_{\pm\pm\pm\pm}(\sigma)$, and $T_{++--}(\sigma)$ fields arise as the lowest, $\theta^+=\bar{\theta}^+=0$, components of the superfields $\mathcal{S}_{++}(\zeta)$, $\mathcal{W}_-(\zeta)$, $\bar{\mathcal{W}}_-(\zeta)$, and $\mathcal{T}_{----}(\zeta)$ together with their descendants,

$$\begin{aligned}&\mathcal{S}_{+++}(\zeta) := \mathrm{i}\mathcal{D}_+\mathcal{S}_{++}(\zeta), \qquad \bar{\mathcal{S}}_{+++}(\zeta) := -\mathrm{i}\bar{\mathcal{D}}_+\mathcal{S}_{++}(\zeta),\\ &\mathcal{T}_{++++}(\zeta) := \frac{1}{2}[\mathcal{D}_+,\bar{\mathcal{D}}_+]\mathcal{S}_{++}(\zeta), \qquad \mathcal{T}_{++--}(\zeta) := \frac{\mathrm{i}}{2}(\mathcal{D}_+\mathcal{W}_-(\zeta) + \bar{\mathcal{D}}_+\bar{\mathcal{W}}_-(\zeta)).\end{aligned} \tag{8}$$

In this paper we will also use the definitions

$$\begin{aligned}\mathcal{T}(\zeta) &:= \mathcal{T}_{++--}(\zeta) \equiv \mathcal{T}_{--++}(\zeta),\\ \Theta(\sigma) &:= \mathcal{T}|_{\theta=0} = T_{++--}(\sigma) \equiv T_{--++}(\sigma).\end{aligned} \tag{9}$$

From the supercurrent equations (6) together with the definitions (8), one can derive the conservation equations

$$\begin{aligned}\partial_{++}\mathcal{S}_{+--}(\zeta) &= -\partial_{--}\mathcal{S}_{+++}(\zeta),\\ \partial_{++}\mathcal{T}_{----}(\zeta) &= -\partial_{--}\mathcal{T}(\zeta),\\ \partial_{++}\mathcal{T}(\zeta) &= -\partial_{--}\mathcal{T}_{++++}(\zeta).\end{aligned} \tag{10}$$

These imply that the supersymmetry currents $S_{+\pm\pm}$ and $\bar{S}_{+\pm\pm}$ are conserved while the energy-momentum tensor $T_{\mu\nu}$ is real, symmetric and conserved.

It is possible to modify the $\mathcal{S}$ multiplet by a class of "improvement terms" without changing its defining constraint equations (6). This is analogous to how the energy-momentum tensor can be modified by improvement terms that do not affect its conservation equations. The improvement transformations that leave invariant the $\mathcal{S}$-multiplet conservation equations are

$$\begin{aligned}\mathcal{S}_{++} \to \tilde{\mathcal{S}}_{++} &= \mathcal{S}_{++} + 2[\mathcal{D}_+,\bar{\mathcal{D}}_+]\mathcal{U},\\ \mathcal{W}_- \to \tilde{\mathcal{W}}_- &= \mathcal{W}_- + 2\partial_{--}\bar{\mathcal{D}}_+\mathcal{U},\\ \mathcal{T}_{----} \to \tilde{\mathcal{T}}_{----} &= \mathcal{T}_{----} + \partial_{--}^2\mathcal{U},\end{aligned} \tag{11}$$

[3]For convenience we will equivalently use the notations $\mathcal{F}(\zeta)=\mathcal{F}(\sigma,\theta)=\mathcal{F}(\sigma^{++},\sigma^{--},\theta^+,\bar{\theta}^+)$; in particular, we will often indicate collectively by θ the dependence on both θ^+ and $\bar{\theta}^+$.

where $\mathcal{U}(\zeta)$ is a real scalar superfield with lowest component field $U(\sigma) := \mathcal{U}(\zeta)|_{\theta=0}$. The improvement transformations of the energy-momentum tensor induced by (11) are

$$\begin{aligned} T_{++++} &\to \tilde{T}_{++++} = T_{++++} + \partial_{++}^2 U, \\ T_{----} &\to \tilde{T}_{----} = T_{----} + \partial_{--}^2 U, \\ T_{++--} &\to \tilde{T}_{++--} = T_{++--} - \partial_{++}\partial_{--} U. \end{aligned} \tag{12}$$

It is clear that $\tilde{T}_{\mu\nu}$ is also real, symmetric and conserved.

The $\mathcal{S}$ multiplet described above is the most general supercurrent multiplet for a Lorentz invariant and $\mathcal{N} = (0,2)$ supersymmetric quantum field theory in two spacetime dimensions. In some cases, the multiplet is decomposable and the currents can be improved. A case that will play a central role in our paper is the $\mathcal{R}$ multiplet. This arises when $C = 0$, which is indeed the case under our consideration, and when there is a well-defined real superfield $\mathcal{R}_{--}(\zeta)$ resolving the chirality constraint of $\mathcal{W}_-(\zeta)$ as

$$\mathcal{W}_- = \mathrm{i}\bar{\mathcal{D}}_+ \mathcal{R}_{--}. \tag{13}$$

The defining conservation equations for the $\mathcal{R}$ multiplet can then be written as[4]

$$\partial_{--}\mathcal{R}_{++} + \partial_{++}\mathcal{R}_{--} = 0, \tag{14a}$$

$$\mathcal{D}_+\left(\mathcal{T}_{----} + \frac{\mathrm{i}}{2}\partial_{--}\mathcal{R}_{--}\right) = 0, \tag{14b}$$

$$\bar{\mathcal{D}}_+\left(\mathcal{T}_{----} - \frac{\mathrm{i}}{2}\partial_{--}\mathcal{R}_{--}\right) = 0, \tag{14c}$$

where $\mathcal{R}_{++} \equiv \mathcal{S}_{++}$. The main consequence of the extra constraints imposed on the $\mathcal{R}$ multiplet is the existence of an extra conserved vector current $j_{\pm\pm}(\sigma)$:

$$\partial_{++}j_{--} + \partial_{--}j_{++} = 0, \tag{15}$$

with $j_{--} := \mathcal{R}_{--}|_{\theta=0}$. This current is associated to 2D $\mathcal{N} = (0,2)$ theories possessing a $U(1)_R$ R symmetry. As described in Appendix B, the $\mathcal{R}$ multiplet naturally arises from $\mathcal{N} = (0,2)$ Poincaré supergravity. In the explicit examples of 2D $\mathcal{N} = (0,2)$ theories which we will consider in our paper, we will always compute the supercurrent multiplet by means of coupling the theory to supergravity with a procedure that mimics the calculation of the Hilbert stress-energy tensor from gravity. This approach will guarantee that the resulting supercurrent multiplet will be an $\mathcal{R}$ multiplet.

We conclude this section by mentioning that in a 2D $\mathcal{N} = (0,2)$ superconformal field theory (SCFT) the $\mathcal{S}$ multiplet can be further simplified. In fact, for a SCFT with $\mathcal{N} = (0,2)$ supersymmetry, $C = 0$ and $\mathcal{W}_-$ can be set to zero by an improvement transformation. Then the $\mathcal{S}$ multiplet only contains a right-moving superfield current $\mathcal{S}_{++}(\zeta)$, $\partial_{--}\mathcal{S}_{++} = 0$ and a left-moving antichiral superfield $\mathcal{T}_{----}(\zeta)$, $\mathcal{D}_+\mathcal{T}_{----}(\zeta) = 0$. This leads to a set of left- and right-moving currents in components.

III. THE $T\bar{T}$ OPERATOR AND $\mathcal{N} = (0,2)$ SUPERSYMMETRY

After having described in the previous section the structure of the $\mathcal{S}$ multiplet, we are ready to prove that the $T\bar{T}$ operator [1]

$$O(\sigma) = T_{++++}(\sigma)T_{----}(\sigma) - [\Theta(\sigma)]^2 \tag{16}$$

is a supersymmetric descendant, in complete analogy to the $\mathcal{N} = (0,1)$ and $\mathcal{N} = (1,1)$ cases first studied in [16,42].

A. The $T\bar{T}$ primary operator

We propose the $\mathcal{N} = (0,2)$ supersymmetric primary $T\bar{T}$ operator to be given by the following combination of the $\mathcal{S}$-multiplet superfields:

$$\mathcal{O}_{--}(\zeta) := \mathcal{T}_{----}(\zeta)\mathcal{S}_{++}(\zeta) - \bar{\mathcal{W}}_-(\zeta)\mathcal{W}_-(\zeta). \tag{17}$$

In fact, it is a straightforward exercise to show that the following relation holds:

$$\begin{aligned} \mathcal{D}_+\bar{\mathcal{D}}_+\mathcal{O}_{--}(\zeta) = {} & \mathcal{T}_{----}(\zeta)\mathcal{T}_{++++}(\zeta) - [\mathcal{T}(\zeta)]^2 \\ & + \partial_{--}\left[\frac{1}{4}\mathcal{S}_{++}(\zeta)\partial_{--}\mathcal{S}_{++}(\zeta) - \frac{1}{2}\mathcal{W}_-(\zeta)\mathcal{D}_+\mathcal{S}_{++}(\zeta) + \frac{1}{2}\bar{\mathcal{W}}_-(\zeta)\bar{\mathcal{D}}_+\mathcal{S}_{++}(\zeta)\right] \\ & + \partial_{++}\left[\frac{\mathrm{i}}{2}\mathcal{T}_{----}(\zeta)\mathcal{S}_{++}(\zeta) - \frac{\mathrm{i}}{2}\bar{\mathcal{W}}_-(\zeta)\mathcal{W}_-(\zeta)\right] \\ & + \mathrm{EOMs}, \end{aligned} \tag{18}$$

[4]The $\mathcal{R}$-multiplet conservation equations are also derived from supergravity in the Appendix B.

where with "EOMs" we mean terms that are identically zero when the $\mathcal{S}$-multiplet conservation equations (6) are used. Since the conservation equations classically hold only when the equations of motion are satisfied, the previous results show that, on shell and up to total derivatives, the previous descendant is equivalent to the $T\bar{T}$ operator (16). Quantum mechanically, the same statement is true for the corresponding operators since conservation equations (Ward identities) hold in correlation functions (up to contact terms).

If we now define

$$O_{--}(\sigma) \equiv \mathcal{O}_{--}(\zeta)|_{\theta=0}, \tag{19}$$

by using the previous results, up to total derivatives and EOMs, the operator $O(\sigma)$ [Eq. (16)] satisfies

$$\begin{aligned} O(\sigma) &= \int \mathrm{d}\bar{\theta}^{+}\mathrm{d}\theta^{+}\mathcal{O}_{--}(\zeta) = \mathcal{D}_{+}\bar{\mathcal{D}}_{+}\mathcal{O}_{--}(\zeta)|_{\theta=0} \\ &= \{Q_{+},[\bar{Q}_{+},O_{--}(\sigma)]\}. \end{aligned} \tag{20}$$

Then, $O_{--}(\sigma)$ is the supersymmetric primary operator of the multiplet containing $O(\sigma)$ as its bottom component. Hence the $T\bar{T}$ deformation for an $\mathcal{N}=(0,2)$ supersymmetric quantum field theory is manifestly supersymmetric since Eq. (20) implies

$$\left[Q_{+},\int \mathrm{d}^2\sigma O(\sigma)\right] = \left[\bar{Q}_{+},\int \mathrm{d}^2\sigma O(\sigma)\right] = 0. \tag{21}$$

The $T\bar{T}$ primary operator (17) is defined uniquely by the requirement that $O(\sigma)$ is its descendant, up to conservation equations and total derivatives. Another virtue enjoyed by $\mathcal{O}_{--}(\zeta)$ is that its form, up to total derivatives, is invariant under the improvement transformation (11):

$$\begin{aligned} \mathcal{O}_{--} \to \tilde{\mathcal{O}}_{--} &= \tilde{\mathcal{S}}_{++}\tilde{\mathcal{T}}_{----} - \tilde{\bar{\mathcal{W}}}_{-}\tilde{\mathcal{W}}_{-} \\ &= \mathcal{O}_{--} + \text{total derivatives}. \end{aligned} \tag{22}$$

This is not too surprising since the combination $(T_{----}T_{++++} - \Theta^2)$ is invariant under the improvement transformations (12).

B. Point splitting and well definedness

As shown by Zamolodchikov in his seminal work [1], one of the main properties of the $T\bar{T}$ operator $O(\sigma)$ is to be free of short-distance divergences and hence to be a well-defined, though irrelevant, composite local operator. More in general, it was later shown by Smirnov and Zamolodchikov in [2] that given two pairs of conserved currents (A_s, B_{s+2}) and $(A'_{s'}, B'_{s'-2})$ such that

$$\partial_{++}A_s = -\partial_{--}B_{s+2}, \qquad \partial_{--}A'_{s'} = -\partial_{++}B'_{s'-2}, \tag{23}$$

then the bilocal operator $[A_s(\sigma)A'_{s'}(\sigma') - B_{s+2}(\sigma)B_{s'-2}(\sigma')]$ is free of short-distance divergences and, up to total derivative terms, independent of the separation $(\sigma-\sigma')$. Here s and s' label spins. Hence, any composite local operator of the "Smirnov-Zamolodchikov" type,

$$\int \mathrm{d}^2\sigma[A_s(\sigma)A'_{s'}(\sigma) - B_{s+2}(\sigma)B_{s'-2}(\sigma)], \tag{24}$$

is well defined. This is indeed the case of the bosonic $T\bar{T}$ operator $O(\sigma)$. In the case of $\mathcal{N}=(0,1)$ and $\mathcal{N}=(1,1)$ supersymmetric $T\bar{T}$ deformations, the primary $T\bar{T}$ superfield operators are still of Smirnov-Zamolodchikov type [16,42], hence well defined and leading to a whole multiplet of well-defined composite operators.

In the $\mathcal{N}=(0,2)$ case, the $T\bar{T}$ primary operator $O_{--}(\sigma) = \mathcal{O}_{--}(\zeta)|_{\theta=0}$ from Eq. (17) is not of Smirnov-Zamolodchikov type. It is then natural to wonder whether the primary $O_{--}(\sigma)$ can also be defined by a similar point-splitting procedure without incurring in short-distance singularities. Were this not the case, we would have an apparent clash between supersymmetry and the structure of the $T\bar{T}$ deformation at the quantum level. We shall see below that, owing to supersymmetry, $\mathcal{O}_{--}(\sigma,\theta)$ can be indeed defined in superspace by a point-splitting procedure *of the bosonic coordinate*[5] σ in analogy with the arguments of [1,2].

Let us consider a point-split version of the $\mathcal{N}=(0,2)$ primary $T\bar{T}$ operator:

$$\begin{aligned} \mathcal{O}_{--}(\sigma,\sigma',\theta) \coloneqq\ & \mathcal{T}_{----}(\sigma,\theta)\mathcal{S}_{++}(\sigma',\theta) \\ & - \bar{\mathcal{W}}_{-}(\sigma,\theta)\mathcal{W}_{-}(\sigma',\theta). \end{aligned} \tag{25}$$

We want to show that the previous bilocal superfield is free of short-distance divergences in the limit $\sigma\to\sigma'$. Following Refs. [1,2], let us compute $\partial_{\pm\pm}\mathcal{O}_{--}(\sigma,\sigma',\theta)$. We start by defining

$$\mathcal{O}_{--}(\zeta,\zeta') \coloneqq [\mathcal{T}_{----}(\zeta)\mathcal{S}_{++}(\zeta') - \bar{\mathcal{W}}_{-}(\zeta)\mathcal{W}_{-}(\zeta')], \tag{26}$$

which is the fully superspace point-split version of $\mathcal{O}_{--}(\zeta)$—from that, we will easily extract $\mathcal{O}_{--}(\sigma,\sigma',\theta) = \mathcal{O}_{--}(\zeta,\zeta')|_{\theta=\theta'}$. Let us compute $\partial_{\pm\pm}\mathcal{O}_{--}(\zeta,\zeta')$. After some straightforward algebraic manipulation, and by using the fact that $\partial_{++} = -\mathrm{i}(\mathcal{D}_{+}\bar{\mathcal{D}}_{+} + \bar{\mathcal{D}}_{+}\mathcal{D}_{+})$, it is possible to derive the following result:

$$\begin{aligned} \partial_{\pm\pm}\mathcal{O}_{--}(\zeta,\zeta') = 0 + \text{EOMs} + (\partial+\partial')[\cdots] \\ + (\mathcal{D}+\mathcal{D}')[\cdots]. \end{aligned} \tag{27}$$

Here with EOMs we again refer to terms that are identically zero once the conservation equations (6) for the $\mathcal{S}$ multiplet are used while with the last two terms in (27) we indicate terms that are superspace total derivatives, such as for

[5] As for the θ's, we can straightforwardly set them to be equal since no divergence of the form e.g., $1/(\theta-\theta')$ arises for the Grassmann coordinates. See Sec. 6 of [50] for examples of point-splitting techniques in superspace.

example the vector derivatives $(\partial_{\pm\pm} + \partial'_{\pm\pm})$ or the spinor derivatives $(\mathcal{D}_+ + \mathcal{D}'_+)$, acting on bilocal operators. The precise expressions for (27) are given by Eqs. (A1a) and (A1b) in Appendix A, where we collect some technical results and explanations that support the analysis of this subsection. When we consider the coincident limit $\theta = \theta'$ in the Grassmann coordinates, thanks to (A2), Eq. (27) can be rewritten in the following useful form:

$$\partial_{\pm\pm}\mathcal{O}_{--}(\sigma,\sigma',\theta) = 0 + \{\text{EOMs} + (\partial + \partial')[\cdots] + (\mathcal{Q} + \mathcal{Q}')[\cdots]\}|_{\theta=\theta'}, \tag{28}$$

where the supersymmetry generators appear instead of the covariant spinor derivatives. In the previous expression $(\partial + \partial')$ generates translations in the σ and σ' coordinates while schematically $(\mathcal{Q} + \mathcal{Q}')$ generates supersymmetry transformations of the bilocal operators they act upon.[6] The results presented above are reminiscent of Zamolodchikov's argument to prove the well definedness of the bosonic $T\bar{T}$ operator, as well as to the arguments used in the $\mathcal{N} = (0,1)$ and $\mathcal{N} = (1,1)$ cases [16,42]. A new feature with respect to those cases is the supersymmetry-transformation terms, which represent a natural generalization of the translation contribution. Still, Zamolodchikov's operator product expansion (OPE) argument of [1] can be used almost identically here in the $\theta = \theta'$ limit, which is sufficient to probe the short-distance singularities in $\sigma \to \sigma'$. Let us briefly review it.

By setting to zero the EOM's terms, the left-hand side of (28) has an OPE expansion of the form

$$\sum_I \partial_{\pm\pm}F_I(\sigma - \sigma')\mathcal{O}_I(\sigma',\theta), \tag{29}$$

with $\{\mathcal{O}_I(\zeta')\}$ a complete set of local superfield operators, depending on the Grassmann coordinate $\theta' = \theta$. Similarly, the right-hand side of (28) will schematically be of the form

$$\sum_I A_I(\sigma - \sigma')\mathcal{Q}'\mathcal{O}_I(\sigma',\theta) + \sum_I B_I(\sigma - \sigma')\partial'\mathcal{O}_I(\sigma',\theta), \tag{30}$$

which is equivalent to

$$\sum_I A_I(\sigma - \sigma')\mathcal{D}'\mathcal{O}_I(\sigma',\theta) + \sum_I C_I(\sigma - \sigma')\partial'\mathcal{O}_I(\sigma',\theta). \tag{31}$$

Hence the OPE of $\partial_{\pm\pm}\mathcal{O}_{--}(\sigma,\sigma',\theta)$ involves only derivatives and supercovariant derivatives of local operators. This means that the OPE of $\mathcal{O}_{--}$ involves only such derivatives, or terms $F_I(\sigma - \sigma')\mathcal{O}_I(\sigma',\theta)$ such that the coefficients F_I are actually *constant* (so that $\partial_{\pm\pm}F_I = 0$), i.e., regular terms. Then, the point-split superfield operator leads to the definition of the composite $\mathcal{N} = (0,2)$ $T\bar{T}$ primary:

$$\mathcal{O}_{--}(\sigma,\sigma',\theta') = \mathcal{O}_{--}(\zeta') + \text{derivative terms}, \tag{32}$$

arising from the regular, nonderivative part of the OPE—precisely as for the purely bosonic $T\bar{T}$ operator of [1]. When considering the integral of $\mathcal{O}_{--}(\zeta)$ in superspace, only the regular terms in the OPE would contribute. As a result the integrated operator

$$S_{\mathcal{O}} = \int d^2\sigma d\bar{\theta}^+ d\theta^+ \lim_{\varepsilon\to 0}\mathcal{O}_{--}(\sigma,\sigma+\varepsilon,\theta) = \int d^2\sigma d\bar{\theta}^+ d\theta^+ :\mathcal{O}_{--}(\sigma,\sigma,\theta): \tag{33}$$

is free of any short-distance divergence and well defined.

As a further evidence of the consistency of the previous point-splitting argument with supersymmetry, one can consider the point-split version of Eq. (18) and show that

$$(\mathcal{D}_+ + \mathcal{D}'_+)(\bar{\mathcal{D}}_+ + \bar{\mathcal{D}}'_+)\mathcal{O}_{--}(\zeta,\zeta') = \mathcal{T}_{----}(\zeta)\mathcal{T}_{++++}(\zeta') - \mathcal{T}(\zeta)\mathcal{T}(\zeta') + \text{EOMs} + (\partial + \partial')[\cdots], \tag{34}$$

with the terms in the ellipsis being a simple point-split generalization of the total derivatives appearing in Eq. (18). This shows explicitly that the descendant of the point-split primary $T\bar{T}$ operator is equivalent, up to Ward identities and total vector derivatives, to the point-split version of the descendant (standard) $T\bar{T}$ operator.

IV. DEFORMING THE FREE SUPERSYMMETRIC ACTION

After having described some general properties of the $\mathcal{N} = (0,2)$ $T\bar{T}$ operator, we are ready to study $T\bar{T}$ deformations. We will focus our attention for the rest of the paper on the simplest possible case: the $T\bar{T}$ deformation of a free action with $\mathcal{N} = (0,2)$ supersymmetry. Though simple, we will see that a detailed analysis of this model is nontrivial and rich.[7]

[6] Note that, thanks to the super-Leibniz rule satisfied by $\frac{\partial}{\partial\theta^+}$ and $\frac{\partial}{\partial\theta^+}$, the operation of taking the $\theta = \theta'$ limit and acting on bilocal superfields with a Grassmann-dependent differential operator, such as $(\mathcal{D} + \mathcal{D}')$ or $(\mathcal{Q} + \mathcal{Q}')$, commutes. Hence (28) is well defined. See Appendix A for more comments on this point.

[7] Note that in our paper the definition of the $T\bar{T}$ flow is purely field theoretical and follows in spirit the original prescription of Zamolodchikov [1]. Alternative descriptions based on the relation with two-dimensional gravitational theories were pursued in [35–38]. It would be very interesting to extend these results to the supersymmetric case. Chiral supersymmetric theories, as $\mathcal{N} = (0,2)$, once coupled to supergravity are typically plagued by gravitational anomalies and it would be important to understand the role of the anomalies in $T\bar{T}$ deformation interpreted in terms of 2D quantum gravity. This is also an important issue for some nonsupersymmetric $T\bar{T}$ deformations, like the deformation of systems of chiral fermions.

Before turning to the supersymmetric analysis, let us briefly recall the form of the $T\bar{T}$ deformation of a *bosonic* action for a complex boson whose free action is

$$S_{0,\mathrm{bos}}=\frac{1}{4}\int \mathrm{d}^2\sigma[\partial_{++}\phi\partial_{--}\bar{\phi}+\partial_{++}\bar{\phi}\partial_{--}\phi]. \tag{35}$$

The aim of our analysis is to consider the supersymmetric extension of this simple model and derive its integrated deformation. The $T\bar{T}$-deformed action of the above free scalar can be compactly written as [3]

$$S_{\alpha,\mathrm{bos}}=\int \mathrm{d}^2\sigma\frac{\sqrt{1+2\alpha x+\alpha^2y^2}-1}{4\alpha}, \tag{36}$$

where we have introduced the shorthand notation

$$x=\partial_{++}\phi\partial_{--}\bar{\phi}+\partial_{++}\bar{\phi}\partial_{--}\phi,$$
$$y=\partial_{++}\phi\partial_{--}\bar{\phi}-\partial_{++}\bar{\phi}\partial_{--}\phi. \tag{37}$$

It is such that

$$\frac{\partial S_{\alpha,\mathrm{bos}}}{\partial\alpha}=-\frac{1}{2}\int \mathrm{d}^2\sigma O(\sigma), \tag{38}$$

where $O(\sigma)=\det[T_{\mu\nu}(\alpha)]$ with the stress-energy tensor of (36) given by

$$T_{\pm\pm,\pm\pm}=-\frac{\partial_{\pm\pm}\phi\partial_{\pm\pm}\bar{\phi}}{\sqrt{1+2\alpha x+\alpha^2y^2}},$$
$$T_{\pm\pm,\mp\mp}=\frac{1+\alpha x-\sqrt{1+2\alpha x+\alpha^2y^2}}{2\alpha\sqrt{1+2\alpha x+\alpha^2y^2}}. \tag{39}$$

It is not difficult to make an educated guess for the $\mathcal{N}=(0,2)$ supersymmetric extension of such a bosonic action, by requiring firstly that the action is manifestly supersymmetric and secondly that its bosonic part is given by Eq. (36). We can easily take care of the former requirement by working in superspace; as for the latter, let us note that the bosonic action can be recast in the form

$$S_{\alpha,\mathrm{bos}}=-\int \mathrm{d}^2\sigma\left(-\frac{x}{4}+\alpha\frac{\partial_{++}\phi\partial_{--}\phi\partial_{++}\bar{\phi}\partial_{--}\bar{\phi}}{1+\alpha x+\sqrt{1+2\alpha x+\alpha^2y^2}}\right). \tag{40}$$

This immediately suggests the following manifestly off-shell supersymmetric action:

$$S_\alpha=-\int \mathrm{d}^2\sigma\mathrm{d}\theta^+\mathrm{d}\bar{\theta}^+\left(-\frac{\mathrm{i}}{2}\bar{\Phi}\partial_{--}\Phi+\alpha\frac{\mathcal{D}_+\Phi\bar{\mathcal{D}}_+\bar{\Phi}\partial_{--}\Phi\partial_{--}\bar{\Phi}}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}\right), \tag{41}$$

which we have written in terms of the chiral and antichiral superfields Φ and $\bar{\Phi}$. They satisfy $\mathcal{D}_+\bar{\Phi}=\bar{\mathcal{D}}_+\Phi=0$ and are given by the following expansion in component fields:

$$\Phi=\phi+\theta^+\psi_+-\frac{\mathrm{i}}{2}\theta^+\bar{\theta}^+\partial_{++}\phi,$$
$$\bar{\Phi}=\bar{\phi}-\bar{\theta}^+\bar{\psi}_++\frac{\mathrm{i}}{2}\theta^+\bar{\theta}^+\partial_{++}\bar{\phi}. \tag{42}$$

We have also introduced the bilinear combinations

$$\mathcal{X}=\partial_{++}\Phi\partial_{--}\bar{\Phi}+\partial_{++}\bar{\Phi}\partial_{--}\Phi,$$
$$\mathcal{Y}=\partial_{++}\Phi\partial_{--}\bar{\Phi}-\partial_{++}\bar{\Phi}\partial_{--}\Phi. \tag{43}$$

Notice that they are simply related to the shorthand x, y of Eq. (37) as

$$x=\mathcal{X}|_{\theta=0},\qquad y=\mathcal{Y}|_{\theta=0}. \tag{44}$$

Actually, there is a very natural reason why the previous educated guess should work. The action (40) describes the Nambu-Goto action for a four-dimensional string in a uniform light-cone gauge. Its $\mathcal{N}=(0,2)$ superstring extension has been studied long ago by Hughes and Polchinski in one of the seminal works on partial supersymmetry breaking [45]. In fact, their results lead to an action equivalent to (41) which in turn admit a nonlinearly realized extra $\mathcal{N}=(2,0)$ supersymmetry. See Ref. [46] for a more recent analysis that we will follow quite closely in Sec. V when we review and extend the results on partial supersymmetry breaking of (41). Considering that the action (41) was already known to be an $\mathcal{N}=(0,2)$ extension of (40), it is absolutely natural to guess that it describes the supersymmetric $T\bar{T}$ flow. Let us now validate this guess.

A. Some limits of the deformed action

As a first sanity check of our proposal we consider it in some limits, starting from $\alpha\to 0$. In that case, it is manifest that only the first summand in (41) survives, so that we find

$$S_0=\frac{\mathrm{i}}{2}\int \mathrm{d}^2\sigma\mathrm{d}\theta^+\mathrm{d}\bar{\theta}^+\bar{\Phi}\partial_{--}\Phi$$
$$=\frac{1}{4}\int \mathrm{d}^2\sigma(\partial_{++}\phi\partial_{--}\bar{\phi}+\partial_{++}\bar{\phi}\partial_{--}\phi+2\mathrm{i}\bar{\psi}_+\partial_{--}\psi_+), \tag{45}$$

which is indeed the free action for the supersymmetric extension of (35).

Furthermore, we can check which form the action (41) takes when setting some of its fields to zero. To this end, it is useful to note that

$$\mathcal{D}_{+}\Phi=\psi_{+}-\mathrm{i}\bar{\theta}^{+}\partial_{++}\phi+\frac{\mathrm{i}}{2}\theta^{+}\bar{\theta}^{+}\partial_{++}\psi_{+},\qquad\bar{\mathcal{D}}_{+}\bar{\Phi}=\bar{\psi}_{+}+\mathrm{i}\theta^{+}\partial_{++}\bar{\phi}-\frac{\mathrm{i}}{2}\theta^{+}\bar{\theta}^{+}\partial_{++}\bar{\psi}_{+}. \tag{46}$$

Setting now all $\psi=\bar{\psi}=0$ we find that the action takes the form

$$\begin{aligned}S_{\alpha,\mathrm{bos}}&=-\int\mathrm{d}^2\sigma\mathrm{d}\theta^{+}\mathrm{d}\bar{\theta}^{+}\left(-\frac{\mathrm{i}}{2}\bar{\Phi}\partial_{--}\Phi+\alpha\frac{(-\mathrm{i}\bar{\theta}^{+}\partial_{++}\phi)(\mathrm{i}\theta^{+}\partial_{++}\bar{\phi})\partial_{--}\Phi\partial_{--}\bar{\Phi}}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}\right)\\&=\int\mathrm{d}^2\sigma\left(\frac{1}{4}x-\alpha\frac{\partial_{++}\phi\partial_{++}\bar{\phi}\partial_{--}\phi\partial_{--}\bar{\phi}}{1+\alpha x+\sqrt{1+2\alpha x+\alpha^2y^2}}\right),\end{aligned} \tag{47}$$

as expected. Conversely, keeping track of the fermions but setting the bosons to zero, $\phi=\bar{\phi}=0$, we have

$$\begin{aligned}S_{\alpha,\mathrm{ferm}}&=-\int\mathrm{d}^2\sigma\mathrm{d}\theta^{+}\mathrm{d}\bar{\theta}^{+}\left(-\frac{\mathrm{i}}{2}\bar{\Phi}\partial_{--}\Phi+\alpha\psi_{+}\bar{\psi}_{+}(\theta^{+}\partial_{--}\psi_{+})(-\bar{\theta}^{+}\partial_{--}\bar{\psi}_{+})\right)\\&=\int\mathrm{d}^2\sigma\left(\frac{\mathrm{i}}{2}\bar{\psi}_{+}\partial_{--}\psi_{+}+\alpha\psi_{+}\bar{\psi}_{+}\partial_{--}\psi_{+}\partial_{--}\bar{\psi}_{+}\right).\end{aligned} \tag{48}$$

This is indeed the $T\bar{T}$ deformation of a complex free-fermion action.

B. Constructing the deforming operator

If the action (41) satisfies the supersymmetric $T\bar{T}$ flow equation, then the following equation must be satisfied:

$$\partial_\alpha S_\alpha=-\frac{1}{2}\int\mathrm{d}^2\sigma\mathrm{d}\bar{\theta}^{+}\mathrm{d}\theta^{+}(\mathcal{S}_{++}\mathcal{T}_{----}-\bar{\mathcal{W}}_{-}\mathcal{W}_{-}). \tag{49}$$

It is easy to compute the left-hand side of this equation:

$$\partial_\alpha S_\alpha=\int\mathrm{d}^2\sigma\mathrm{d}\theta^{+}\mathrm{d}\bar{\theta}^{+}\frac{\mathcal{D}_{+}\Phi\bar{\mathcal{D}}_{+}\bar{\Phi}\partial_{--}\Phi\partial_{--}\bar{\Phi}}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}\frac{1}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}. \tag{50}$$

As for the right-hand side of Eq. (49), we can find the supercurrents by coupling the theory to supergravity—in analogy to how the Hilbert stress-energy tensor is computed by coupling the theory to a metric. For this task, we can use off-shell supergravity techniques developed in the 1980s; see Appendix B for detail and references. With this analysis at hand, it is straightforward, though lengthy, to derive the supercurrent $\mathcal{R}$ multiplet of the action (41). The details of the supercurrent computation, that might be in principle used in the future also for more complicated models, are relegated to Appendix B. The results of our analysis are as follows. We find

$$\begin{aligned}\mathcal{S}_{++}=\mathcal{R}_{++}&=-\frac{\mathcal{D}_{+}\Phi\bar{\mathcal{D}}_{+}\bar{\Phi}}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}},\\\mathcal{R}_{--}&=\frac{2\alpha}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}\frac{\mathcal{D}_{+}\Phi\bar{\mathcal{D}}_{+}\bar{\Phi}\partial_{--}\bar{\Phi}\partial_{--}\Phi}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}},\\\mathcal{T}_{----}&=-\frac{\partial_{--}\Phi\partial_{--}\bar{\Phi}}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}+(\cdots)\mathcal{D}_{+}\Phi+(\cdots)\bar{\mathcal{D}}_{+}\bar{\Phi},\end{aligned} \tag{51}$$

where we indicated with ellipses terms which will not play a role in our computation. Indeed when considering the product $\mathcal{S}_{++}\mathcal{T}_{----}$ such terms vanish identically due to their Grassmann-odd nature and the fact that $\mathcal{S}_{++}$ is proportional to $(\mathcal{D}_{+}\Phi\bar{\mathcal{D}}_{+}\bar{\Phi})$. Furthermore when truncating to the bosonic part of these supercurrents, one can check that the stress-energy tensor superfields (8) satisfy

$$\mathcal{T}_{\mu\nu}|_{\mathrm{bos},\theta=0}=T_{\mu\nu}, \tag{52}$$

where $\mu,\nu=++,--$ and the right-hand side is the bosonic stress-energy tensor in Eq. (39).[8]

[8]Note that for the bosonic part of $\mathcal{T}_{----}$ to match T_{----} we need the fermionic terms involving the ellipses to vanish. We have verified that this is indeed the case on shell, at leading order in α.

From these expressions and Eq. (13) it follows that

$$\bar{\mathcal{W}}_- = \frac{-2\alpha}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}} \frac{\partial_{++}\bar{\Phi}\mathcal{D}_+\Phi\partial_{--}\bar{\Phi}\partial_{--}\Phi}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}} + (\cdots)\mathcal{D}_+\Phi\mathcal{D}_+\bar{\Phi},$$
$$\mathcal{W}_- = \frac{-2\alpha}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}} \frac{\partial_{++}\Phi\bar{\mathcal{D}}_+\bar{\Phi}\partial_{--}\bar{\Phi}\partial_{--}\Phi}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}} + (\cdots)\mathcal{D}_+\Phi\mathcal{D}_+\bar{\Phi}, \tag{53}$$

where, once again, the terms in the ellipses are irrelevant when considering the product $\bar{\mathcal{W}}_-\mathcal{W}_-$. It is now a matter of algebra to find

$$\begin{aligned}\mathcal{O}_{--} &= \mathcal{S}_{++}\mathcal{T}_{----} - \bar{\mathcal{W}}_-\mathcal{W}_- \\ &= 2\frac{\mathcal{D}_+\Phi\bar{\mathcal{D}}_+\bar{\Phi}\partial_{--}\Phi\partial_{--}\bar{\Phi}}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}} \\ &\quad\times\frac{1}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}},\end{aligned} \tag{54}$$

which shows that indeed Eq. (49) holds.

C. Expression in components and comparison with the "Noether" deformation

It is instructive to rewrite the deformed action (41) explicitly in components. By using the definition of the superfields (42) and performing some integration by parts we can recast the action in the form

$$\begin{aligned}S_{\mathrm{susy}} = \int \mathrm{d}^2\sigma[&A(x,y) + B(x,y)\Psi \\ &+ C(x,y)\partial_{--}\phi\partial_{--}\bar{\phi}\Psi_{++++} + D_{--}(x,y)\bar{\psi}_+\psi_+ \\ &+ E(x,y)(\Psi)^2 + F(x,y)\partial_{--}\phi\partial_{--}\bar{\phi}\Psi\Psi_{++++} \\ &+ G(x,y)(\partial_{--}\phi\partial_{--}\bar{\phi})^2(\Psi_{++++})^2],\end{aligned} \tag{55}$$

where the subscript "susy" emphasizes that the action was obtained from our manifestly supersymmetric construction. Note that we introduced a shorthand notation for the fermion bilinears:

$$\begin{aligned}\Psi &= \bar{\psi}_+\partial_{--}\psi_+ + \psi_+\partial_{--}\bar{\psi}_+, \\ \Psi_{++++} &= \bar{\psi}_+\partial_{++}\psi_+ + \psi_+\partial_{++}\bar{\psi}_+.\end{aligned} \tag{56}$$

The coefficient $A(x,y)$, $B(x,y)$, etc., depend on the bilinear combinations of the bosonic fields x, y of Eq. (37) and on the deformation parameter α; they are given in Appendix C. Without delving too deep in their specific form, we simply note that all these coefficients are nonvanishing. We wish now to compare the form of this action with that of a $T\bar{T}$ deformation built out of the Noether energy-momentum tensor. As emphasised in Ref. [16], we should not expect the two actions to be identical—indeed that was found *not* to be the case already for deformations of an $\mathcal{N}=(0,1)$ Lagrangian. Again in Ref. [16], the deformation of a free supersymmetric action of eight $\mathcal{N}=(1,1)$ multiplets was constructed by exploiting a connection with light-cone gauge-fixed strings [16]. For the reader's convenience, let us copy that result—which is given in Eq. (4.18) there—specializing to the case where the undeformed action takes the form (45) corresponding to an $\mathcal{N}=(0,2)$ theory. We have

$$S_{\mathrm{Noether}} = \int \mathrm{d}^2\sigma \frac{1}{2\alpha}\left[-1+2\mathrm{i}\alpha\Psi+\sqrt{1+2\alpha x+\alpha^2y^2+\mathrm{i}\alpha(4-\alpha x)\Psi-4\alpha^2(\Psi)^2-\mathrm{i}\alpha^2(\partial_{--}\phi\partial_{--}\bar{\phi})\Psi_{++++}}\right]. \tag{57}$$

The action can be readily expanded in powers of the fermion bilinears Ψ and Ψ_{++++}, and indeed it truncates at quadratic order. It is easy to see that no term without derivatives on the fermions—such as the one multiplying $D_{--}(x,y)$ in the supersymmetric action (55)—may be generated in this expansion.[9] Despite such a substantial difference, the two deforming operators, which may be found from the two actions by taking the partial α derivative, should coincide on shell.[10] It is easy to verify that this is the case in the $\mathcal{N}=(0,1)$ case where $\psi_+\equiv\bar{\psi}_+$; then both actions are linear in Ψ and Ψ_{++++}, and the $\bar{\psi}_+\psi_+$ term vanishes identically. As discussed at some length in Ref. [16], the fermion equations of motions for the supersymmetric and Noether action then coincide, which is

[9] Indeed such a term cannot even be generated by partial integration, as that would introduce new fermion bilinears of the form $\bar{\psi}_+\partial_{\pm\pm}\psi_+ - \psi_+\partial_{\pm\pm}\bar{\psi}_+$.

[10] Equivalently, the two theories should be the same up to (nonlinear) field redefinitions. In particular, while the Noether-deformed action is not invariant under the free supersymmetry variations, it should be invariant under suitably modified supersymmetry variations (whose form is induced by the nonlinear field redefinition).

sufficient to show that the two deforming operators coincide on shell *at all orders in* α. In the full $\mathcal{N}=(0,2)$ case, however, the fermion equations of motions are different and in order to check the on-shell equivalence of the two deforming operators it is necessary to use both the fermion and boson equations of motion which makes things rather less transparent. We have verified that the two deforming operators coincide on shell up to order $O(\alpha^3)$ and total derivatives and we expect these results to hold at all order in α. Hence these two seemingly different deformations should give rise to the same deformed theory, at least as long as the spectrum is concerned. A more constructive check would be to explicitly produce the field redefinition relating the two actions. This would be particularly helpful in order to study the contact terms arising in deformed correlation functions. However, this is a relatively difficult task. A simpler starting point would be the $\mathcal{N}=(0,1)$ case [16,42] before moving to the $\mathcal{N}=(0,2)$ case. We leave both these interesting studies, that fall beyond the scope of this paper, for future research.

V. PARTIAL SUPERSYMMETRY BREAKING $\mathcal{N}=(2,2)\to(0,2)$

In the previous section, we have found the action for the supersymmetric $T\bar{T}$ deformation of a $\mathcal{N}=(0,2)$ free theory, Eq. (41). This action, which is equivalent to the one originally studied in [45], as shown in [46], resembles the four-dimensional Bagger-Galperin action which describes the partial supersymmetry breaking from $\mathcal{N}=2$ to $\mathcal{N}=1$ in four dimensions [47]. Interestingly, the $T\bar{T}$-deformed free model is related to partial supersymmetry breaking in two dimensions.[11] Indeed, following [46], we will show that exactly the same action describes a model of partial supersymmetry breaking from $\mathcal{N}=(2,2)\to\mathcal{N}=(0,2)$ in two dimensions [46].

In light-cone coordinates, a flat 2D $\mathcal{N}=(2,2)$ superspace is parametrized by

$$\boldsymbol{\zeta}^M=(\sigma^{\pm\pm},\theta^\pm,\bar{\theta}^\pm), \tag{58}$$

and spinor covariant derivatives and supercharges are given, respectively, by

$$\mathcal{D}_\pm=\frac{\partial}{\partial\theta^\pm}-\frac{\mathrm{i}}{2}\bar{\theta}^\pm\partial_{\pm\pm},\qquad \mathcal{Q}_\pm=\frac{\partial}{\partial\theta^\pm}+\frac{\mathrm{i}}{2}\bar{\theta}^\pm\partial_{\pm\pm}, \tag{59}$$

together with their complex conjugates. They obey the anticommutation relations

$$\{\mathcal{D}_\pm,\bar{\mathcal{D}}_\pm\}=\mathrm{i}\partial_{\pm\pm},\qquad \{\mathcal{Q}_\pm,\bar{\mathcal{Q}}_\pm\}=-\mathrm{i}\partial_{\pm\pm}, \tag{60}$$

with all the other (anti)commutators between $\mathcal{D}$'s, $\mathcal{Q}$'s, and $\partial_{\pm\pm}$ being identically zero. Given an $\mathcal{N}=(2,2)$ superfield $\mathcal{F}(\boldsymbol{\zeta})=\mathcal{F}(\sigma^{\pm\pm},\theta^\pm,\bar{\theta}^\pm)$ its supersymmetry transformations are given by

$$\delta_Q\mathcal{F}\coloneqq\mathrm{i}\epsilon^+\mathcal{Q}_+\mathcal{F}+\mathrm{i}\eta^-\mathcal{Q}_-\mathcal{F}-\mathrm{i}\bar{\epsilon}^+\bar{\mathcal{Q}}_+\mathcal{F}-\mathrm{i}\bar{\eta}^-\bar{\mathcal{Q}}_-\mathcal{F}. \tag{61}$$

To discuss the partial supersymmetry breaking, we begin introducing the simplest $\mathcal{N}=(2,2)$ scalar multiplet. This is described by an $\mathcal{N}=(2,2)$ chiral superfield $\Upsilon(\boldsymbol{\zeta})$ satisfying

$$\bar{\mathcal{D}}_+\Upsilon=\bar{\mathcal{D}}_-\Upsilon=0. \tag{62}$$

In general, $\Upsilon(\boldsymbol{\zeta})$ can efficiently be decomposed into $\mathcal{N}=(0,2)$ multiplets expanding in the θ^- and $\bar{\theta}^-$ coordinates

$$\begin{aligned}\Upsilon(\boldsymbol{\zeta})&=\Phi(\zeta)+\theta^-\Psi^+(\zeta)-\frac{\mathrm{i}}{2}\theta^-\bar{\theta}^-\partial_{--}\Phi(\zeta),\\ \Phi(\zeta)&\coloneqq\Upsilon(\boldsymbol{\zeta})|_{\theta^-=\bar{\theta}^-=0},\\ \Psi^+(\zeta)&\coloneqq\mathcal{D}_-\Upsilon(\boldsymbol{\zeta})|_{\theta^-=\bar{\theta}^-=0}.\end{aligned} \tag{63}$$

Here Φ and Ψ^+ are $\mathcal{N}=(0,2)$ scalar and Fermi chiral multiplets, respectively, satisfying

$$\bar{\mathcal{D}}_+\Phi=\bar{\mathcal{D}}_+\Psi^+=0. \tag{64}$$

Since we are interested in partial supersymmetry breaking, we only consider the transformation under $\mathcal{Q}_-$, $\bar{\mathcal{Q}}_-$ and thus set $\epsilon^+=\bar{\epsilon}^+=0$. The ϵ^+, $\bar{\epsilon}^+$ transformations will have preserved off-shell supersymmetry while the η^-, $\bar{\eta}^-$ transformations will be the ones spontaneously broken. The supersymmetry transformation rules can straightforwardly be read from (61) and, in particular, the two $\mathcal{N}=(0,2)$ superfields transform under the left supersymmetry as

$$\delta_\eta\Phi=\mathrm{i}\eta^-\Psi^+,\qquad \delta_\eta\Psi^+=\bar{\eta}^-\partial_{--}\Phi. \tag{65}$$

To realize the partial supersymmetry breaking from $\mathcal{N}=(2,2)$ to $\mathcal{N}=(0,2)$, one needs to deform the above transformation rules to

$$\tilde{\delta}_\eta\Phi=\mathrm{i}\eta^-(\kappa\theta^++\Psi^+),\qquad \tilde{\delta}_\eta\Psi^+=\bar{\eta}^-\partial_{--}\Phi, \tag{66}$$

where κ has mass dimension 1 and represents the supersymmetry-breaking scale. As described in detail in [45,46], the extra $\kappa\eta^-\theta^+$ term is linked to a central charge deformation of the supersymmetry algebra which is necessary to have partial supersymmetry breaking. In fact, if we define $\Xi_+\coloneqq\mathcal{D}_+\Phi$, by using the previous transformations it follows

[11] Though it was not discussed in Refs. [16,42], it is simple to show that the previously considered $T\bar{T}$ deformations of $\mathcal{N}=(0,1)$ and $\mathcal{N}=(1,1)$ free models of [16,42] also possess additional, nonlinearly realized supersymmetry [$\mathcal{N}=(1,0)$ and $\mathcal{N}=(1,1)$, respectively], as their actions are equivalent to those first studied in Ref. [46].

$$\tilde{\delta}_\eta \Xi_+ = -\mathrm{i}\eta^-(\kappa + \mathcal{D}_+\Psi^+). \qquad (67)$$

As in [51], these transformations can be converted into standard nonlinearly realized supersymmetry transformations by defining

$$\tilde{\Xi}_+ = e^{\tilde{\delta}_\eta}\Xi_+|_{\eta=-\frac{1}{\kappa}\lambda}, \qquad \tilde{\Psi}^+ = e^{\tilde{\delta}_\eta}\Psi^+|_{\eta=-\frac{1}{\kappa}\lambda}, \qquad (68)$$

where λ, which transforms as

$$\tilde{\delta}_\eta \lambda^- = \kappa\eta^- - \frac{\mathrm{i}}{2\kappa}(\eta^-\bar{\lambda}^- + \bar{\eta}^-\lambda^-)\partial_{--}\lambda^-, \qquad (69)$$

is the complex Goldstino associated to the $\mathcal{N}=(2,2)\to(0,2)$ partial supersymmetry breaking. One can then show that $\tilde{\Xi}_+$ and $\tilde{\Psi}^+$ transform homogeneously as

$$\tilde{\delta}_\eta \tilde{\Xi}^+ = -\frac{\mathrm{i}}{2\kappa}(\eta^-\bar{\lambda}^- + \bar{\eta}^-\lambda^-)\partial_{--}\tilde{\Xi}^+,$$
$$\tilde{\delta}_\eta \tilde{\Psi}^+ = -\frac{\mathrm{i}}{2\kappa}(\eta^-\bar{\lambda}^- + \bar{\eta}^-\lambda^-)\partial_{--}\tilde{\Psi}^+. \qquad (70)$$

This enables one to impose the supersymmetric invariant constraints $\tilde{\Xi}_+ = \tilde{\Psi}^+ = 0$ that are solved by

$$\Psi^+ = \mathrm{i}\frac{\bar{\mathcal{D}}_+\bar{\Phi}\partial_{--}\Phi}{\kappa - \bar{\mathcal{D}}_+\bar{\Psi}^+}, \qquad \bar{\Psi}^+ = -\mathrm{i}\frac{\mathcal{D}_+\Phi\partial_{--}\bar{\Phi}}{\kappa + \mathcal{D}_+\Psi^+},$$
$$(\Psi^+)^2 = (\bar{\Psi}^+)^2 = 0. \qquad (71)$$

The previous result can also be rewritten as

$$\Psi^+ = \frac{1}{\kappa}\bar{\mathcal{D}}_+[\mathrm{i}\bar{\Phi}\partial_{--}\Phi - \Psi^+\bar{\Psi}^+]$$
$$= \frac{1}{\kappa}\bar{\mathcal{D}}_+\left[\mathrm{i}\bar{\Phi}\partial_{--}\Phi - \frac{\bar{\mathcal{D}}_+\bar{\Phi}\mathcal{D}_+\Phi\partial_{--}\Phi\partial_{--}\bar{\Phi}}{(\kappa - \bar{\mathcal{D}}_+\bar{\Psi}^+)(\kappa + \mathcal{D}_+\Psi^+)}\right], \qquad (72)$$

together with its complex conjugates. By using a standard trick [46,47], the denominator $(\kappa + \mathcal{D}_+\Psi^+)$ cannot contribute terms like $\bar{\mathcal{D}}_+\bar{\Phi}$, since the same fermionic terms appear already in the numerator. Hence the $(\kappa + \mathcal{D}_+\Psi^+)$ term only appears effectively as

$$(\kappa + \mathcal{D}_+\Psi^+)_{\mathrm{eff}} = \left(\kappa + \mathcal{D}_+\frac{\mathrm{i}\bar{\mathcal{D}}_+\bar{\Phi}\partial_{--}\Phi}{\kappa - \bar{\mathcal{D}}_+\bar{\Psi}^+}\right)_{\mathrm{eff}}$$
$$= \kappa - \frac{\partial_{++}\bar{\Phi}\partial_{--}\Phi}{\kappa - (\bar{\mathcal{D}}_+\bar{\Psi}^+)_{\mathrm{eff}}}, \qquad (73)$$

which leads to

$$(\mathcal{D}_+\Psi^+)_{\mathrm{eff}} = -\frac{\partial_{++}\bar{\Phi}\partial_{--}\Phi}{\kappa - (\bar{\mathcal{D}}_+\bar{\Psi}^+)_{\mathrm{eff}}},$$
$$(\bar{\mathcal{D}}_+\bar{\Psi}^+)_{\mathrm{eff}} = \frac{\partial_{++}\Phi\partial_{--}\bar{\Phi}}{\kappa + (\mathcal{D}_+\Psi^+)_{\mathrm{eff}}}. \qquad (74)$$

Their solution gives

$$(\mathcal{D}_+\Psi^+)_{\mathrm{eff}} = \frac{1}{2\kappa}\Big(B - \bar{B} - \kappa^2 + \sqrt{\kappa^4 - 2\kappa^2(B+\bar{B}) + (B-\bar{B})^2}\Big),$$
$$B \coloneqq \partial_{++}\Phi\partial_{--}\bar{\Phi},$$
$$\bar{B} \coloneqq \partial_{++}\bar{\Phi}\partial_{--}\Phi. \qquad (75)$$

Substituting back into (71), one can express Ψ^+ in term of $\Phi, \bar{\Phi}$ and their derivatives:

$$\Psi^+ = \frac{1}{\kappa}\bar{\mathcal{D}}_+\left[\mathrm{i}\bar{\Phi}\partial_{--}\Phi - \frac{2\bar{\mathcal{D}}_+\bar{\Phi}\mathcal{D}_+\Phi\partial_{--}\Phi\partial_{--}\bar{\Phi}}{\kappa^2 - \mathcal{X} + \sqrt{\kappa^4 - 2\kappa^2\mathcal{X} + \mathcal{Y}^2}}\right]. \qquad (76)$$

By construction, the two $\mathcal{N}=(0,2)$ superfields $\Phi(\zeta)$ and $\Psi^+(\zeta)$ also possess a hidden nonlinearly realized $\mathcal{N}=(2,0)$ supersymmetry (66).

Now we can construct the following full superspace action:

$$S_\kappa = \frac{1}{2}\int \mathrm{d}^2\sigma \mathrm{d}\theta^+\mathrm{d}\bar{\theta}^+\mathrm{d}\theta^-\mathrm{d}\bar{\theta}^-\Upsilon\bar{\Upsilon} \qquad (77a)$$

$$= \frac{1}{2}\int \mathrm{d}^2\sigma \mathrm{d}\theta^+\mathrm{d}\bar{\theta}^+\left[\frac{\mathrm{i}}{2}(\bar{\Phi}\partial_{--}\Phi - \Phi\partial_{--}\bar{\Phi}) - \Psi^+\bar{\Psi}^+\right]. \qquad (77b)$$

Alternatively, since Υ is chiral, one can also consider the following supersymmetric action integrating over half superspace:

$$S_\kappa = -\frac{\kappa}{4}\int \mathrm{d}^2\sigma \mathrm{d}\theta^+\mathrm{d}\theta^-\Upsilon + \frac{\kappa}{4}\int \mathrm{d}^2\sigma \mathrm{d}\bar{\theta}^+\mathrm{d}\bar{\theta}^-\bar{\Upsilon} \qquad (78a)$$

$$= -\frac{\kappa}{4}\int \mathrm{d}^2\sigma \mathrm{d}\theta^+\Psi^+ - \frac{\kappa}{4}\int \mathrm{d}^2\sigma \mathrm{d}\bar{\theta}^+\bar{\Psi}^+. \qquad (78b)$$

Note that here $\Upsilon(\boldsymbol{\zeta})$ is defined as in (63) but with $\Phi(\zeta)$ and $\Psi^+(\zeta)$ now transforming as in (66). Then the supersymmetry $\mathcal{N}=(2,2)$ transformations of $\Upsilon(\boldsymbol{\zeta})$ gets modified to

$$\tilde{\delta}_Q\Upsilon = \delta_Q\Upsilon + \mathrm{i}\kappa\eta^-\theta^+, \qquad (79)$$

with $\delta_Q\Upsilon$ as in (61). Despite the deformation of the $\mathcal{N}=(2,0)$ supersymmetry one can still explicitly verify that the $\mathcal{N}=(2,0)$ supersymmetric variations of the integrands in (77b) and (78b) are total derivatives [46]. Together with their manifest $\mathcal{N}=(0,2)$ supersymmetry, one finds that (77b) and (78b) are supersymmetric under $\mathcal{N}=(2,2)$. This also justifies the manifest $\mathcal{N}=(2,2)$ superspace formulation of the actions in (77a) and (78a).

Using (72), it can be even shown that the above two actions with $\mathcal{N}=(2,2)$ supersymmetry are equivalent:

$$S_\kappa = \frac{1}{2}\int \mathrm{d}^2\sigma \mathrm{d}\theta^+ \mathrm{d}\bar{\theta}^+ \mathrm{d}\theta^- \mathrm{d}\bar{\theta}^- \Upsilon\bar{\Upsilon} = -\frac{\kappa}{4}\int \mathrm{d}^2\sigma \mathrm{d}\theta^+ \mathrm{d}\theta^- \Upsilon + \frac{\kappa}{4}\int \mathrm{d}^2\sigma \mathrm{d}\bar{\theta}^+ \mathrm{d}\bar{\theta}^- \bar{\Upsilon}. \quad (80)$$

Inserting the explicit solution (76), one can explicitly write down the action as

$$S_\kappa = -\frac{\kappa}{2}\int \mathrm{d}^2\sigma \mathrm{d}\theta^+ \Psi^+ = \int \mathrm{d}^2\sigma \mathrm{d}\theta^+ \mathrm{d}\bar{\theta}^+ \times \left[\frac{\mathrm{i}}{2}\bar{\Phi}\partial_{--}\Phi + \frac{\mathcal{D}_+\Phi\bar{\mathcal{D}}_+\bar{\Phi}\partial_{--}\Phi\partial_{--}\bar{\Phi}}{\kappa^2 - \mathcal{X} + \sqrt{\kappa^4 - 2\kappa^2\mathcal{X} + \mathcal{Y}^2}}\right]. \quad (81)$$

Once we identify

$$\alpha = -\frac{1}{\kappa^2}, \quad (82)$$

it is obvious that (81) gives exactly our previous $T\bar{T}$-deformed action S_α in Eq. (41), showing that S_α, besides being manifestly $\mathcal{N}=(0,2)$ supersymmetric, is also invariant under the extra spontaneously broken $\mathcal{N}=(2,0)$ supersymmetry (66).

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APPENDIX A: USEFUL RESULTS FOR SEC. III B

Here we collect some useful technical results used in Sec. III B. Equation (27) is

$$\begin{aligned}\partial_{--}\mathcal{O}_{--}(\zeta,\zeta') = {} & -\mathcal{T}_{----}(\zeta)[\partial'_{--}\mathcal{S}_{++}(\zeta') - \mathcal{D}'_+\mathcal{W}_-(\zeta') + \bar{\mathcal{D}}'_+\bar{\mathcal{W}}_-(\zeta')] \\ & + \left[\mathcal{D}_+\mathcal{T}_{----}(\zeta) - \frac{1}{2}\partial_{--}\bar{\mathcal{W}}_-(\zeta)\right]\mathcal{W}_-(\zeta') - \left[\bar{\mathcal{D}}_+\mathcal{T}_{----}(\zeta) - \frac{1}{2}\partial_{--}\mathcal{W}_-(\zeta)\right]\bar{\mathcal{W}}_-(\zeta') \\ & + (\partial_{--} + \partial'_{--})[\mathcal{T}_{----}(\zeta)\mathcal{S}_{++}(\zeta')] - (\mathcal{D}_+ + \mathcal{D}'_+)[\mathcal{T}_{----}(\zeta)\mathcal{W}_-(\zeta')] \\ & + (\bar{\mathcal{D}}_+ + \bar{\mathcal{D}}'_+)[\mathcal{T}_{----}(\zeta)\bar{\mathcal{W}}_-(\zeta')] \end{aligned} \quad \text{(A1a)}$$

and

$$\begin{aligned}\partial_{++}\mathcal{O}_{--}(\zeta,\zeta') = {} & -\mathrm{i}\left[\mathcal{D}_+\left(\bar{\mathcal{D}}_+\mathcal{T}_{----}(\zeta) - \frac{1}{2}\partial_{--}\mathcal{W}_-(\zeta)\right)\right]\mathcal{S}_{++}(\zeta') \\ & - \mathrm{i}\left[\bar{\mathcal{D}}_+\left(\mathcal{D}_+\mathcal{T}_{----}(\zeta) - \frac{1}{2}\partial_{--}\bar{\mathcal{W}}_-(\zeta)\right)\right]\mathcal{S}_{++}(\zeta') \\ & + \frac{\mathrm{i}}{2}[\mathcal{D}_+\mathcal{W}_-(\zeta) + \bar{\mathcal{D}}_+\bar{\mathcal{W}}_-(\zeta)][\partial'_{--}\mathcal{S}_{++}(\zeta') - \mathcal{D}'_+\mathcal{W}_-(\zeta') + \bar{\mathcal{D}}'_+\bar{\mathcal{W}}_-(\zeta')] \\ & - \frac{\mathrm{i}}{2}(\partial_{--} + \partial'_{--})[\mathcal{D}_+\mathcal{W}_-(\zeta)\mathcal{S}_{++}(\zeta') + \bar{\mathcal{D}}_+\bar{\mathcal{W}}_-(\zeta)\mathcal{S}_{++}(\zeta')] \\ & - \frac{\mathrm{i}}{2}(\bar{\mathcal{D}}_+ + \bar{\mathcal{D}}'_+)[(\mathcal{D}_+\mathcal{W}_-(\zeta))\bar{\mathcal{W}}_-(\zeta')] + \frac{\mathrm{i}}{2}(\mathcal{D}_+ + \mathcal{D}'_+)[(\bar{\mathcal{D}}_+\bar{\mathcal{W}}_-(\zeta))\mathcal{W}_-(\zeta')] \\ & + \frac{\mathrm{i}}{2}(\mathcal{D}_+ + \mathcal{D}'_+)[\mathcal{W}_-(\zeta)\mathcal{D}'_+\mathcal{W}_-(\zeta')] - \frac{\mathrm{i}}{2}(\bar{\mathcal{D}}_+ + \bar{\mathcal{D}}'_+)[\bar{\mathcal{W}}_-(\zeta)\bar{\mathcal{D}}'_+\bar{\mathcal{W}}_-(\zeta')], \end{aligned} \quad \text{(A1b)}$$

where the unprimed superspace covariant derivatives, $\mathcal{D}_A=(\partial_{\pm\pm},\mathcal{D}_+,\bar{\mathcal{D}}_+)$, act only on the superspace coordinates ζ while the primed derivatives $\mathcal{D}'_A$ act only on ζ'. To emphasize the difference between (27), (A1a), (A1b) and (28), before and after the $\theta=\theta'$ limit, it is also useful to point out that the following equations hold:

$$(\mathcal{D}_+ + \mathcal{D}'_+) = (\mathcal{Q}_+ + \mathcal{Q}'_+) - \mathrm{i}\bar{\theta}^+(\partial_{++}+\partial'_{++}) + \mathrm{i}(\bar{\theta}^+ - \bar{\theta}'^+)\partial'_{++}, \tag{A2a}$$

$$(\mathcal{D}_+ + \mathcal{D}'_+) = (\mathcal{Q}_+ + \mathcal{Q}'_+) - \mathrm{i}\bar{\theta}'^+(\partial_{++}+\partial'_{++}) - \mathrm{i}(\bar{\theta}^+ - \bar{\theta}'^+)\partial_{++}, \tag{A2b}$$

and

$$(\bar{\mathcal{D}}_+ + \bar{\mathcal{D}}'_+) = (\bar{\mathcal{Q}}_+ + \bar{\mathcal{Q}}'_+) + \mathrm{i}\theta^+(\partial_{++}+\partial'_{++}) - \mathrm{i}(\theta^+ - \theta'^+)\partial'_{++}, \tag{A3a}$$

$$(\bar{\mathcal{D}}_+ + \bar{\mathcal{D}}'_+) = (\bar{\mathcal{Q}}_+ + \bar{\mathcal{Q}}'_+) + \mathrm{i}\theta'^+(\partial_{++}+\partial'_{++}) + \mathrm{i}(\theta^+ - \theta'^+)\partial_{++}. \tag{A3b}$$

The last terms, that are functions of the distances in the Grassmannian directions, $(\theta^+ - \theta'^+)$ and $(\bar{\theta}^+ - \bar{\theta}'^+)$, are not multiplying a generator of (super)translations. For this reason, in general, they do not annihilate superspace OPE coefficients, which is necessary for the argument in Sec. III B to go through. Since these terms disappear when $\theta=\theta'$, it is enough to consider the coincident Grassmannian limit for (28), which derives from (A1a)–(A3), to be true. This in the end suffices to show the well definedness of the composite operator $\mathcal{O}_{--}(\zeta)$. To make more clear how to properly read Eq. (28) let us elaborate further on how to interpret expressions where the $\theta=\theta'$ limit is taken.

Given two superfields $\mathcal{U}^1(\zeta)$ and $\mathcal{U}^2(\zeta')$ we consider the superspace point-split bilocal operator $\mathcal{O}(\zeta,\zeta')=\mathcal{O}(\sigma,\theta;\sigma',\theta')$ defined as

$$\mathcal{O}(\zeta,\zeta') = \mathcal{U}^1(\zeta)\mathcal{U}^2(\zeta'). \tag{A4}$$

Its $\theta=\theta'$ limit, $\mathcal{O}(\sigma,\sigma',\theta) \coloneqq \mathcal{O}(\sigma,\theta;\sigma',\theta)$, is

$$\mathcal{O}(\sigma,\sigma',\theta) = \mathcal{U}^1(\sigma,\theta)\mathcal{U}^2(\sigma',\theta) \tag{A5}$$

and represents a point-split version in the bosonic coordinates σ and σ' of the composite operator $\mathcal{O}(\zeta) \coloneqq \mathcal{U}^1(\zeta)\mathcal{U}^2(\zeta)$. We define the following differential operators:

$$\hat{\mathcal{D}}_+ \coloneqq \frac{\partial}{\partial\theta^+} - \frac{\mathrm{i}}{2}\bar{\theta}^+\hat{\partial}_{++}, \qquad \bar{\hat{\mathcal{D}}}_+ = -\frac{\partial}{\partial\bar{\theta}^+} + \frac{\mathrm{i}}{2}\theta^+\hat{\partial}_{++}, \tag{A6a}$$

$$\hat{\mathcal{Q}}_+ \coloneqq \frac{\partial}{\partial\theta^+} + \frac{\mathrm{i}}{2}\bar{\theta}^+\hat{\partial}_{++}, \qquad \bar{\hat{\mathcal{Q}}}_+ = -\frac{\partial}{\partial\bar{\theta}^+} - \frac{\mathrm{i}}{2}\theta^+\hat{\partial}_{++}, \tag{A6b}$$

with

$$\hat{\partial}_{\pm\pm} \coloneqq \frac{\partial}{\partial\sigma^{\pm\pm}} + \frac{\partial}{\partial\sigma'^{\pm\pm}}. \tag{A7}$$

These satisfy the same algebra as the unhatted covariant derivatives and supercharges, e.g., $\{\hat{\mathcal{D}}_+,\bar{\hat{\mathcal{D}}}_+\}=\mathrm{i}\hat{\partial}_{++}$, etc.

It is then clear that, thanks to the Leibniz rule of the spinor derivatives $\frac{\partial}{\partial\theta^+}$ and $\frac{\partial}{\partial\bar{\theta}^+}$, it holds

$$\hat{\mathcal{D}}_+\mathcal{O}(\sigma,\sigma',\theta) = \{(\mathcal{D}_+ + \mathcal{D}'_+)\mathcal{O}(\zeta,\zeta')\}|_{\theta=\theta'}, \tag{A8a}$$

$$\hat{\mathcal{Q}}_+\mathcal{O}(\sigma,\sigma',\theta) = \{(\mathcal{Q}_+ + \mathcal{Q}'_+)\mathcal{O}(\zeta,\zeta')\}|_{\theta=\theta'}, \tag{A8b}$$

and similar expressions for their complex conjugates. The convenience to have introduced the hatted operators becomes clear when we consider how supersymmetry transformations act on $\mathcal{O}(\sigma,\sigma',\theta)$. By using (4) for $\delta_Q\mathcal{U}^1(\zeta)$ and $\delta_Q\mathcal{U}^2(\zeta)$, it follows

$$\begin{aligned}\delta_Q\mathcal{O}(\sigma,\sigma',\theta) &= [\delta_Q\mathcal{U}^1(\sigma,\theta)]\mathcal{U}^2(\sigma',\theta) + \mathcal{U}^1(\sigma,\theta)[\delta_Q\mathcal{U}^1(\sigma',\theta)] \\ &= \{[\mathrm{i}\epsilon^+(\mathcal{Q}_+ + \mathcal{Q}'_+) - \mathrm{i}\bar{\epsilon}^+(\bar{\mathcal{Q}}_+ + \bar{\mathcal{Q}}'_+)]\mathcal{O}(\zeta,\zeta')\}|_{\theta=\theta'}\end{aligned} \tag{A9a}$$

$$\begin{aligned}&= \{[\mathrm{i}\epsilon^+(\mathcal{D}_+ + \mathcal{D}'_+) - \mathrm{i}\bar{\epsilon}^+(\bar{\mathcal{D}}_+ + \bar{\mathcal{D}}'_+)]\mathcal{O}(\zeta,\zeta')\}|_{\theta=\theta'} \\ &\quad + (\epsilon^+\bar{\theta}^+ + \bar{\epsilon}^+\theta^+)(\partial_{++}+\partial'_{++})\mathcal{O}(\sigma,\sigma',\theta),\end{aligned} \tag{A9b}$$

which can be equivalently represented as

$$\delta_Q\mathcal{O}(\sigma,\sigma',\theta) = [\mathrm{i}\epsilon^+\hat{\mathcal{Q}}_+ - \mathrm{i}\bar{\epsilon}^+\bar{\hat{\mathcal{Q}}}_+]\mathcal{O}(\sigma,\sigma',\theta) \tag{A10a}$$

$$\begin{aligned}&= [\mathrm{i}\epsilon^+\hat{\mathcal{D}}_+ - \mathrm{i}\bar{\epsilon}^+\bar{\hat{\mathcal{D}}}_+]\mathcal{O}(\sigma,\sigma',\theta) \\ &\quad + (\epsilon^+\bar{\theta}^+ + \bar{\epsilon}^+\theta^+)\hat{\partial}_{++}\mathcal{O}(\sigma,\sigma',\theta).\end{aligned} \tag{A10b}$$

Then the hatted operators are the ones generating translations and $\mathcal{N}=(0,2)$ supersymmetry transformations of bilocal operators such as $\mathcal{O}(\sigma,\sigma',\theta)$. These are the operators appearing in (28). Moreover, the results above make it evident that taking the $\theta=\theta'$ limit and acting on bilocal superfields with Grassmann-dependent differential operators are two commuting operations.

APPENDIX B: COMPUTATION OF THE SUPERCURRENT

In this Appendix, we calculate the supercurrent of the action (41) in order to verify that it is indeed arising form the $T\bar{T}$ deformation of the free theory (45). The strategy to compute the supercurrent is to couple the model to supergravity in superspace and then take functional derivates with respect to the gravitational superfield prepotentials. This procedure is the superspace analog of the calculation of the Hilbert stress-energy tensor in a generic QFT.

The study of 2D $\mathcal{N}=(0,2)$ supergravity in superspace was largely developed in the 1980s and we refer the reader to the following works and references therein for details [52–59]. In particular, we refer to [54,59] that we will closely follow including their notations. The covariant derivatives are defined as

$$\mathcal{D}_+ = \frac{\partial}{\partial\theta^+} + \mathrm{i}\bar{\theta}^+\partial_{++}, \qquad \bar{\mathcal{D}}_+ = \frac{\partial}{\partial\bar{\theta}^+} + \mathrm{i}\theta^+\partial_{++}, \tag{B1}$$

satisfying

$$\mathcal{D}_+^2 = \bar{\mathcal{D}}_+^2 = 0, \qquad \{\mathcal{D}_+, \bar{\mathcal{D}}_+\} = 2\mathrm{i}\partial_{++},$$
$$\{\mathcal{D}_+, \partial_{\pm\pm}\} = \{\bar{\mathcal{D}}_+, \partial_{\pm\pm}\} = 0. \tag{B2}$$

Due to the covariant properties of these derivatives and the isomorphism of different representations of the superalgebra, all the expressions in different notations should take the same form except for the coefficients. When translating among different notations, the coefficients can be fixed unambiguously by comparing the component expression. For our supercurrent, the coefficients can be fixed by considering the component of the supercurrent which are related to the energy-momentum tensor.

1. $\mathcal{N}=(0,2)$ supergravity

In this section, we review the 2D $\mathcal{N}=(0,2)$ supergravity following Refs. [54,59]. The superspace geometry we consider is based on a structure group based on the 2D Lorentz group. The covariant derivatives include the super-Vielbein $E_M{}^A$ and its inverse $E_A{}^M$, together with the Lorentz connection superfield ω_M, and take in general the form[12] $\nabla_A = (\nabla_{\pm\pm}, \nabla_+, \bar{\nabla}_{\mp})$:

$$\nabla_A = E_A{}^M\partial_M + \omega_A\mathcal{M}, \qquad \partial_M = \left(\frac{\partial}{\partial\sigma^{\pm\pm}}, \frac{\partial}{\partial\theta^+}, \frac{\partial}{\partial\bar{\theta}^{\mp}}\right). \tag{B3}$$

They satisfy an algebra of the form

$$[\nabla_A, \nabla_B\} = T_{AB}{}^C\nabla_C + R_{AB}\mathcal{M}. \tag{B4}$$

The torsion $T_{AB}{}^C$ and curvature R_{AB} superfields represent highly reducible representations of local supersymmetry and they are in general constrained to appropriately describe the multiplet of $\mathcal{N}=(0,2)$ Poincaré supergravity off shell. We refer to [54,59] for a detailed analysis of the constraints for the torsion and curvature tensors and the Bianchi identities they satisfy. For the purpose of computing the supercurrent it is enough to describe how the constraints are solved at the linear order in terms of a set of unconstrained "prepotential" superfields that play the role of the metric in the context of superfield supergravity; see [50,60] for pedagogical reviews. At linearized order the covariant derivatives can be expanded about a flat background as

$$\nabla_A = \mathcal{D}_A - H_A{}^M\mathcal{D}_M + \omega_A(H)\mathcal{M}, \tag{B5}$$

where the superconnection ω_A is completely determined in terms of $H_A{}^M$. To linear order, the constraints can be solved in terms of three independent prepotential superfields: $H_{--}{}^{--}$, $H_{--}{}^{++}$, and H^{--}. All the other components of $H_A{}^M$ can be expressed in terms of the prepotentials. The expressions used in our paper are[13]

$$H_{--}{}^{+} = \frac{1}{2\mathrm{i}}\bar{\mathcal{D}}_+H_{--}{}^{++}, \tag{B6a}$$

$$H_{--}{}^{\bar{+}} = -\frac{1}{2\mathrm{i}}\mathcal{D}_+H_{--}{}^{++}, \tag{B6b}$$

$$H_{++}{}^{+} = \frac{1}{2\mathrm{i}}\bar{\mathcal{D}}_+H_{--}{}^{--}, \tag{B6c}$$

$$H_{++}{}^{\bar{+}} = -\frac{1}{2\mathrm{i}}\mathcal{D}_+H_{--}{}^{--}, \tag{B6d}$$

$$H_{++}{}^{++} = H_{--}{}^{--}, \tag{B6e}$$

$$H_{++}{}^{--} = -\frac{1}{2}[\mathcal{D}_+, \bar{\mathcal{D}}_+]H^{--}, \tag{B6f}$$

$$H_+{}^{\bar{+}} = H_+{}^{++} = H_{\bar{+}}{}^{+} = H_{\bar{+}}{}^{++} = 0, \tag{B6g}$$

$$H_+{}^{+} + H_{\bar{+}}{}^{\bar{+}} = H_{--}{}^{--}, \tag{B6h}$$

$$H_+{}^{--} = \mathrm{i}\mathcal{D}_+H^{--}, \tag{B6i}$$

$$H_{\bar{+}}{}^{--} = -\mathrm{i}\bar{\mathcal{D}}_+H^{--}. \tag{B6j}$$

The various components of the supergravity multiplet can be obtained from the prepotentials through projections. In particular, the linearized metric fluctuations are

[12] The $\bar{+}$ notation is used for convenience only to keep track of barred and unbarred terms.

[13] The complex conjugate relation is $(H_A{}^M)^* = H_{\bar{A}}{}^{\bar{M}}(-)^{|A|+|M|}$.

$$h_{--}{}^{++}=-H_{--}{}^{++}|_{\theta=0}, \tag{B7a}$$

$$h_{--}{}^{--}=h_{++}{}^{++}=-H_{--}{}^{--}|_{\theta=0}, \tag{B7b}$$

$$h_{++}{}^{--}=\frac{1}{2}[\mathcal{D}_+,\bar{\mathcal{D}}_+]H^{--}|_{\theta=0}. \tag{B7c}$$

After solving the constraints, the linearized supergravity transformations of the prepotentials turn out to be [54,59]

$$\delta H^{--}=\mathrm{i}(\Lambda_{++}-\bar{\Lambda}_{++}), \tag{B8a}$$

$$\delta H_{--}{}^{--}=-\frac{1}{2}\partial_{--}(\Lambda_{++}+\bar{\Lambda}_{++})-\frac{1}{2}\partial_{++}K_{--}, \tag{B8b}$$

$$\delta H_{--}{}^{++}=-\partial_{--}K_{--}, \tag{B8c}$$

where K_{--} is real, while $\Lambda_{++},\bar{\Lambda}_{++}$ are chiral and antichiral, respectively:

$$K_{--}=\bar{K}_{--},\qquad \bar{\mathcal{D}}_+\Lambda_{++}=0,\qquad \mathcal{D}_+\bar{\Lambda}_{++}=0. \tag{B9}$$

2. $\mathcal{R}$ multiplet from supergravity

Consider a general Lorentz invariant matter system coupled to $\mathcal{N}=(0,2)$ supergravity. Its action expanded to first order in the supergravity prepotential is

$$S_{\mathrm{int}}=-\frac{1}{4}\int \mathrm{d}^2\sigma\mathrm{d}\theta^+\mathrm{d}\bar{\theta}^+(2H^{--}\mathcal{T}_{----}+2H_{--}{}^{--}\mathcal{R}_{--}+H_{--}{}^{++}\mathcal{R}_{++}), \tag{B10}$$

which leads to

$$S_{\mathrm{int}}=-\frac{1}{2}\int \mathrm{d}^2\sigma(h_{--}{}^{++}T_{++++}+2h_{--}{}^{--}T_{++--}+h_{++}{}^{--}T_{----})+\cdots, \tag{B11}$$

where the ellipsis represents the fermionic contributions.

Assuming that the equations of motion for the matter are satisfied, the variation of the action under arbitrary supergravity gauge transformations (B9) takes the form

$$\delta S_{\mathrm{int}}=-\frac{1}{4}\int \mathrm{d}^2\sigma\mathrm{d}\theta^+\mathrm{d}\bar{\theta}^+\Big\{2\mathrm{i}\Lambda_{++}\Big(\mathcal{T}_{----}-\frac{\mathrm{i}}{2}\partial_{--}\mathcal{R}_{--}\Big)-2\mathrm{i}\bar{\Lambda}_{++}\Big(\mathcal{T}_{----}+\frac{\mathrm{i}}{2}\partial_{--}\mathcal{R}_{--}\Big)$$
$$+K_{--}(\partial_{++}\mathcal{R}_{--}+\partial_{--}\mathcal{R}_{++})\Big\}. \tag{B12}$$

The invariance of the action then dictates the following conservation equations:

$$\bar{\mathcal{D}}_+\Big(\mathcal{T}_{----}-\frac{\mathrm{i}}{2}\partial_{--}\mathcal{R}_{--}\Big)=0, \tag{B13a}$$

$$\mathcal{D}_+\Big(\mathcal{T}_{----}+\frac{\mathrm{i}}{2}\partial_{--}\mathcal{R}_{--}\Big)=0, \tag{B13b}$$

$$\partial_{++}\mathcal{R}_{--}+\partial_{--}\mathcal{R}_{++}=0. \tag{B13c}$$

These, are exactly the conservation law for the $\mathcal{R}$ multiplet (14).

3. Computing the supercurrent

Next we are going to derive the supercurrent multiplet for the models of interest in our paper. To do that, we first need to covariantize the actions.

Since we are dealing with scalar multiplets, the Lorentz connection ω_A will be irrelevant for our calculations. Then the covariant derivatives will always be

$$\nabla_A=\mathcal{D}_A-H_A{}^M\mathcal{D}_M. \tag{B14}$$

The superdensity is expanded at the linear order in terms of the prepotentials as

$$E^{-1}=1+\mathrm{Str}H_A{}^M=1+H_{++}{}^{++}+H_{--}{}^{--}-H_+{}^+$$
$$-H_-{}^-=1+H_{--}{}^{--}. \tag{B15}$$

We also need to define the covariantly chiral and antichiral superfields in supergravity:

$$\nabla_+\hat{\bar{\Phi}}=(\mathcal{D}_+-H_+{}^+\mathcal{D}_+-\mathrm{i}\mathcal{D}_+H^{--}\partial_{--})\hat{\bar{\Phi}}=0. \tag{B16}$$

To linearized order, one finds the following expression for a covariantly (anti)chiral superfield ($\hat{\bar{\Phi}}$) $\hat{\Phi}$, in terms of a standard (anti)chiral superfield ($\bar{\Phi}$) Φ:

$$\hat{\Phi}=(1-\mathrm{i}H^{--}\partial_{--})\Phi,\qquad \hat{\bar{\Phi}}=(1+\mathrm{i}H^{--}\partial_{--})\bar{\Phi}. \tag{B17}$$

To covariantize a matter action coupled to supergravity, we replace all the quantities with covariant ones: $\mathcal{D}_A \to \nabla_A$, $\Phi \to \hat{\Phi}, \bar{\Phi} \to \hat{\bar{\Phi}}$. The superdensity E^{-1} should also be taken into account in the superspace measure.

a. Free theory

To illustrate the strategy of computing the supercurrent, let us first consider the free theory. The action is

$$S_0 = -\frac{i}{4}\int d^2\sigma d\theta^+ d\bar{\theta}^+ \bar{\Phi}\partial_{--}\Phi. \tag{B18}$$

In the supergravity case the action takes the form

$$S_0 = -\frac{i}{4}\int d^2\sigma d\theta^+ d\bar{\theta}^+ E^{-1}\hat{\bar{\Phi}}\nabla_{--}\hat{\Phi}. \tag{B19}$$

The integrand can be computed explicitly. To linear order in the prepotentials, it holds

$$\begin{aligned} &E^{-1}\hat{\bar{\Phi}}\nabla_{--}\hat{\Phi} \\ &= \bar{\Phi}\partial_{--}\Phi + H_{--}{}^{--}(\bar{\Phi}\partial_{--}\Phi - \bar{\Phi}\partial_{--}\Phi) \\ &\quad + \left(-H_{--}{}^{++}\bar{\Phi}\partial_{++}\Phi + \frac{i}{2}\bar{\mathcal{D}}_+ H_{--}{}^{++}\bar{\Phi}\mathcal{D}_+\Phi\right) \\ &\quad + (iH^{--}(\partial_{--}\bar{\Phi}\partial_{--}\Phi - \bar{\Phi}\partial_{--}^2\Phi) - i\bar{\Phi}\partial_{--}H^{--}\partial_{--}\Phi). \end{aligned} \tag{B20}$$

Plugging this back into the action (B19) and integrating by parts, we get

$$\begin{aligned} S_0 = -\frac{1}{4}\int d^2\sigma d\theta^+ d\bar{\theta}^+ (&i\bar{\Phi}\partial_{--}\Phi + 2H^{--}\mathcal{T}_{----} \\ &+ 2H_{--}{}^{--}\mathcal{R}_{--} + H_{--}{}^{++}\mathcal{R}_{++}), \end{aligned} \tag{B21}$$

where

$$\mathcal{R}_{--} = 0, \tag{B22a}$$

$$\mathcal{R}_{++} = -\frac{1}{2}\mathcal{D}_+\Phi\bar{\mathcal{D}}_+\bar{\Phi}, \tag{B22b}$$

$$\mathcal{T}_{----} = -\partial_{--}\Phi\partial_{--}\bar{\Phi}. \tag{B22c}$$

This is indeed the correct supercurrent $\mathcal{R}$ multiplet for a massless free theory.

b. $T\bar{T}$-deformed action

Now, we switch to our $T\bar{T}$-deformed action (41). In terms of the notation employed in this Appendix it is given by

$$\begin{aligned} S_\alpha = \frac{1}{4}\int d^2\sigma d\theta^+ d\bar{\theta}^+ \\ \times \left(-i\bar{\Phi}\partial_{--}\Phi + \alpha\frac{\mathcal{D}_+\Phi\bar{\mathcal{D}}_+\bar{\Phi}\partial_{--}\Phi\partial_{--}\bar{\Phi}}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}\right). \end{aligned} \tag{B23}$$

Covariantizing it, we get

$$\begin{aligned} S_\alpha = \frac{1}{4}\int d^2\sigma d\theta^+ d\bar{\theta}^+ E^{-1} \\ \times \left(-i\hat{\bar{\Phi}}\nabla_{--}\hat{\Phi} + \alpha\frac{\nabla_+\hat{\Phi}\bar{\nabla}_+\hat{\bar{\Phi}}\nabla_{--}\hat{\Phi}\nabla_{--}\hat{\bar{\Phi}}}{1+\alpha\hat{\mathcal{X}}+\sqrt{1+2\alpha\hat{\mathcal{X}}+\alpha^2\hat{\mathcal{Y}}^2}}\right), \end{aligned} \tag{B24}$$

where

$$\begin{aligned} \hat{\mathcal{X}} &= \nabla_{++}\hat{\Phi}\nabla_{--}\hat{\bar{\Phi}} + \nabla_{++}\hat{\bar{\Phi}}\nabla_{--}\hat{\Phi}, \\ \hat{\mathcal{Y}} &= \nabla_{++}\hat{\Phi}\nabla_{--}\hat{\bar{\Phi}} - \nabla_{++}\hat{\bar{\Phi}}\nabla_{--}\hat{\Phi}. \end{aligned} \tag{B25}$$

Since the free part has been computed, we now focus on the second term, the nonlinear part. The three currents are considered separately.

Computation of $\mathcal{R}_{++}$.—The simplest current is $\mathcal{R}_{++}$ which can be computed by turning on only $H_{--}{}^{++}$ while setting the other prepotentials to zero $H_{--} = H_{--}{}^{--} = 0$. In this case, to leading order, the covariant derivatives are given by

$$\nabla_{++} = \partial_{++}, \qquad \nabla_+ = \mathcal{D}_+, \qquad \bar{\nabla}_+ = \bar{\mathcal{D}}_+, \tag{B26a}$$

$$\begin{aligned} \nabla_{--} = \partial_{--} - H_{--}{}^{++}\partial_{++} + \frac{i}{2}\bar{\mathcal{D}}_+ H_{--}{}^{++}\mathcal{D}_+ \\ - \frac{i}{2}\mathcal{D}_+ H_{--}{}^{++}\bar{\mathcal{D}}_+. \end{aligned} \tag{B26b}$$

The superdensity is $E^{-1} = 1$ and $\hat{\Phi} = \Phi, \hat{\bar{\Phi}} = \bar{\Phi}$. The numerator of the nonlinear part is

$$\nabla_+\hat{\Phi}\bar{\nabla}_+\hat{\bar{\Phi}} = \mathcal{D}_+\Phi_+\bar{\mathcal{D}}_+\bar{\Phi}_+. \tag{B27}$$

Due to the fermionic nature of this term, the denominators of the nonlinear part cannot have terms like $\mathcal{D}_+\Phi_+, \bar{\mathcal{D}}_+\bar{\Phi}_+$. So effectively, we can use

$$\nabla_{--} = \partial_{--} - H_{--}{}^{++}\partial_{++}. \tag{B28}$$

Ultimately, it is easy to find, to linear order in the prepotentials

$$\frac{\nabla_{--}\hat{\Phi}\nabla_{--}\hat{\bar{\Phi}}}{1+\alpha\hat{\mathcal{X}}+\sqrt{1+2\alpha\hat{\mathcal{X}}+\alpha^2\hat{\mathcal{Y}}^2}} = \frac{\partial_{--}\Phi\partial_{--}\bar{\Phi}}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}} + \frac{H_{--}{}^{++}}{2\alpha}\left(\frac{1}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}-1\right). \quad (B29)$$

Then, it is straightforward to extract the $\mathcal{R}_{++}$ supercurrent which takes the form

$$\mathcal{R}_{++} = -\frac{\mathcal{D}_+\Phi\bar{\mathcal{D}}_+\bar{\Phi}}{2\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}, \quad (B30)$$

where the free part contribution has also been included.

Computation of $\mathcal{R}_{--}$.—Now we set $H_{--}=H_{--}{}^{++}=0$ and keep only $H_{--}{}^{--}$. Then, the covariant derivatives become

$$\nabla_+ = \mathcal{D}_+ - H_+{}^+\mathcal{D}_+, \quad (B31a)$$

$$\bar{\nabla}_+ = \bar{\mathcal{D}}_+ - H_{\bar{+}}{}^{\bar{+}}\bar{\mathcal{D}}_+. \quad (B31b)$$

The superdensity is $E^{-1}=(1+H_{--}{}^{--})$ and $\hat{\Phi}=\Phi$, $\hat{\bar{\Phi}}=\bar{\Phi}$. In this case the numerator of (B24), at the linear order, takes the form

$$\nabla_+\hat{\Phi}\bar{\nabla}_+\hat{\bar{\Phi}} = (1-H_{--}{}^{--})\mathcal{D}_+\Phi_+\bar{\mathcal{D}}_+\bar{\Phi}_+. \quad (B32)$$

Using similar arguments as in the last subsection, effectively we have

$$\nabla_{--} = \partial_{--} - H_{--}{}^{--}\partial_{--}, \quad (B33a)$$

$$\nabla_{++} = \partial_{++} - H_{--}{}^{--}\partial_{++}. \quad (B33b)$$

Plugging these results back into the action and expanding to linear order in the prepotentials, one finds the supercurrent

$$\mathcal{R}_{--} = \alpha\frac{\mathcal{D}_+\Phi\bar{\mathcal{D}}_+\bar{\Phi}}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}} \times \frac{\partial_{--}\bar{\Phi}\partial_{--}\Phi}{1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}}. \quad (B34)$$

Computation of $\mathcal{T}_{----}$.—Finally, we are going to calculate $\mathcal{T}_{----}$ by just turning on H^{--} and by setting $H_{--}{}^{--}=H_{--}{}^{++}=0$. The covariant derivatives are

$$\nabla_+ = \mathcal{D}_+ - \mathrm{i}\mathcal{D}_+H^{--}\partial_{--}, \quad (B35a)$$

$$\bar{\nabla}_+ = \bar{\mathcal{D}}_+ + \mathrm{i}\bar{\mathcal{D}}_+H^{--}\partial_{--}, \quad (B35b)$$

$$\nabla_{++} = \partial_{++} + \frac{1}{2}[\mathcal{D}_+,\bar{\mathcal{D}}_+]H^{--}\partial_{--}, \quad (B35c)$$

$$\nabla_{--} = \partial_{--}. \quad (B35d)$$

The superdensity is simply given by $E^{-1}=1$, but the covariantly chiral superfield and its covariant derivative have a nontrivial dependence upon H^{--}:

$$\hat{\Phi} = (1-\mathrm{i}H^{--}\partial_{--})\Phi, \quad \nabla_+\hat{\Phi} = (\mathcal{D}_+ - 2\mathrm{i}\mathcal{D}_+H^{--}\partial_{--})\Phi. \quad (B36)$$

The extra pieces here pose an obstruction to further simplifying the covariant derivatives as we did before. This makes the calculation of $\mathcal{T}_{----}$ more complicated. Collecting all the results together, we get the covariantized action expanded to first order in H^{--}:

$$S_{\rm int} = \frac{1}{4}\int d^2\sigma d\theta^+ d\bar{\theta}^+ \Big[\alpha\mathcal{D}_+\Phi\bar{\mathcal{D}}_+\bar{\Phi}\cdot[\mathcal{D}_+,\bar{\mathcal{D}}_+]H^{--}(-\alpha)(\partial_{--}\Phi\partial_{--}\bar{\Phi})^2\frac{1+V}{VZ^2} + 2\mathrm{i}\alpha(\mathcal{D}_+\Phi\cdot\bar{\mathcal{D}}_+H^{--}\partial_{--}\bar{\Phi} + \bar{\mathcal{D}}_+\bar{\Phi}\cdot\mathcal{D}_+H^{--}\partial_{--}\Phi)\frac{\partial_{--}\Phi\partial_{--}\bar{\Phi}}{Z}\Big], \quad (B37)$$

where (we also introduce $\tilde{Z}$ for later convenience)

$$V = \sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}, \quad (B38a)$$

$$Z = 1+\alpha\mathcal{X}+\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}, \quad (B38b)$$

$$\tilde{Z} = 1+\alpha\mathcal{X}-\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}. \quad (B38c)$$

To simplify the calculation, from now on we focus on terms contributing to the supercurrent which have no bare $\mathcal{D}_+\Phi$ and $\bar{\mathcal{D}}_+\bar{\Phi}$ terms. The reason is that, when we consider the $T\bar{T}$ primary operator, the contribution involving $\mathcal{T}_{----}$ will appear in the product $\mathcal{T}_{----}\mathcal{R}_{++}$. From the explicit expression of $\mathcal{R}_{++}$ (B30), we immediately see that any $\mathcal{D}_+\Phi, \bar{\mathcal{D}}_+\bar{\Phi}$ part in $\mathcal{T}_{----}$ has no contributions due to its fermionic nature. After some integration by parts in (B37) we obtain

$$S_{\rm int} = \frac{1}{4}\int \mathrm{d}^2\sigma \mathrm{d}\theta^+ \mathrm{d}\bar{\theta}^+ 2H^{--}(\partial_{--}\Phi\partial_{--}\bar{\Phi}) \times \left[\alpha^2(\mathcal{X}^2-\mathcal{Y}^2)\frac{1+V}{VZ^2} - 2\alpha\frac{\mathcal{X}}{Z} + \cdots\right] = \frac{1}{4}\int \mathrm{d}^2\sigma \mathrm{d}\theta^+ \mathrm{d}\bar{\theta}^+ 2H^{--}(\partial_{--}\Phi\partial_{--}\bar{\Phi})\left[\frac{1}{V} - 1 + \cdots\right], \tag{B39}$$

where the ellipses represent terms proportional to $\mathcal{D}_+\Phi$ or $\bar{\mathcal{D}}_+\bar{\Phi}$, and we used the relations

$$\alpha^2(\mathcal{X}^2-\mathcal{Y}^2) = Z\tilde{Z}, \qquad \frac{1}{V} - \frac{\tilde{Z}}{VZ} = \frac{2}{Z}. \tag{B40}$$

Once we add the contribution form the free action (B22c) we obtain

$$\mathcal{T}_{----} = -\frac{\partial_{--}\Phi\partial_{--}\bar{\Phi}}{\sqrt{1+2\alpha\mathcal{X}+\alpha^2\mathcal{Y}^2}} + \mathcal{D}_+\Phi\cdot\# + \bar{\mathcal{D}}_+\bar{\Phi}\cdot\#. \tag{B41}$$

It is easy to verify that these supercurrents give rise to the correct energy-momentum tensor in the pure bosonic case. This also enables us to translate the results here into the notation of the main body of the paper.

APPENDIX C: DEFORMATION OF THE FREE ACTION IN COMPONENTS

We have given schematically the component expression of the supersymmetric $T\bar{T}$ deformation of a free $\mathcal{N}=(0,2)$ action in Eq. (55). Here we give the explicit form of the coefficients, in terms of the variable x, y introduced in Eq. (37):

$$\begin{aligned}
A(x,y) &= \frac{1}{4\alpha}\left(-1+\sqrt{1+2\alpha x+\alpha^2y^2}\right),\\
B(x,y) &= \frac{\mathrm{i}}{4\sqrt{1+2\alpha x+\alpha^2y^2}},\\
C(x,y) &= \frac{\mathrm{i}}{2\alpha(x^2-y^2)}\left(1-\frac{1+\alpha x}{\sqrt{1+2\alpha x+\alpha^2y^2}}\right),\\
D_{--}(x,y) &= \frac{\mathrm{i}\alpha}{4\sqrt{1+2\alpha x+\alpha^2y^2}}[(1+\alpha(x+2y))(\partial_{--}\partial_{--}\phi\partial_{++}\bar{\phi}+\partial_{--}\partial_{++}\bar{\phi}\partial_{--}\phi)] + \mathrm{i}\partial_{--}\phi(\partial_{--}\bar{\phi})^2\partial_{++}^2\phi\\
&\quad\times\left[\frac{\sqrt{1+2\alpha x+\alpha^2y^2}}{\alpha(x-y)(x+y^2)} + \frac{3}{2\alpha(x-y)^2(x+y)} + \frac{1+3\alpha^2x^2-\alpha^3(2x^3-x^2y-4xy^2+y^3)}{2\alpha(x-y)^2(x+y)(1+2\alpha x+\alpha^2y^2)}\right] + \mathrm{c.c.},\\
E(x,y) &= -\frac{\alpha}{4}\frac{1-\alpha x}{(1+2\alpha x+\alpha^2y^2)^{3/2}},\\
F(x,y) &= \frac{\alpha^2}{(1+2\alpha x+\alpha^2y^2)^{3/2}},\\
G(x,y) &= \frac{4}{\alpha(x^2-y^2)^2}\left(1-\frac{(1+\alpha x)}{\sqrt{1+2\alpha x+\alpha^2y^2}} + \frac{x^2-y^2}{2}\frac{\alpha^2}{(1+2\alpha x+\alpha^2y^2)^{3/2}}\right).
\end{aligned} \tag{C1}$$

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