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Strategic communication with reporting costs

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Abstract
A decision maker relies on information of parties affected by her decision. These parties try to influence her decision by selective disclosure of facts. As is well known from the literature, competition between the informed parties constrains their ability to manipulate information. We depart from this literature by introducing a cost to communicate. Our parties trade off their reporting cost against the effect on the decision. Some information is never revealed. In contrast to setups without communication costs, our decision maker can benefit by ex ante committing to an ex post suboptimal decision rule. Moreover, committing ex ante not to listen to one of the parties may also be beneficial for the decision maker.

Keywords Disclosure · Persuasion · Active judging · Adversarial · Inquisitorial

1 Introduction

Decision makers must frequently rely on the information of parties who are affected by their decisions. Being interested, these parties will try to manipulate the decision maker's choice by, say, concealing facts or providing selective information. One possibility to counteract such manipulations is to solicit advice from parties with conflicting interests. Any piece of information will tend to favor one party or the other. Therefore, competition between the parties constrains their ability to selectively disclose facts,

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which, in turn, allows more appropriate decisions to be made.[1] Typically, this literature assumes that the interested parties incur no costs for making their reports.

By contrast, we look at the case where the interested parties bear a cost to communicate their information: conveying the facts intelligibly takes time and effort. Examples abound: A detailed report in written form may be necessary to convince the adjudicator; in a trial a party may have to testify in person in the courtroom. In both these examples, there is an implicit opportunity cost of preparing and communicating the information. The cost can also be an explicit one. In a malpractice suit, the facts may have to be certified by an independent physician. In a product liability case, the true state may have to be validated by a neutral lab. An external auditor may have to confirm the statements in a financial dispute. In a patent infringement proceeding, an expert may have to substantiate the claims.

With a cost to communicate the parties trade off their reporting cost against the effect on the decision. In equilibrium, they never reveal all information. In contrast to previous literature, the decision maker can benefit by ex ante committing to an ex post suboptimal decision rule. Moreover, committing ex ante not to listen to one of the parties may be beneficial for the decision maker.

Specifically, we consider the interaction between competitive advocacy and reporting costs in a simple persuasion game. Two informed parties (experts, litigants, managers) with opposed interests can influence a decision maker (adjudicator, judge, CEO). The principal must take a decision in a bounded continuous action space. Her payoff from the decision depends on the underlying state which is unknown to her: she wants to match the true state as closely as possible. The parties are informed about the state. They have conflicting preferences over the principal's decision. They can submit verifiable information disclosing the true state, thereby carrying the decision. Nevertheless, they incur a reporting cost when they do so. The informed parties, therefore, trade off their cost of reporting against the effect on the principal's decision. As a result, some facts are never disclosed.

Within this framework, we study two different procedures. First, we analyze unconstrained competition between the interested parties. The parties simultaneously decide whether they disclose the true state or not. The adjudicator is completely passive; once the parties have played, she updates her beliefs and makes a decision. This benchmark setup is motivated by the *adversarial procedure* in civil litigation of common law countries.[2] Moreover, it is the framework usually discussed in the persuasion game literature on advocacy.

In the next procedure, we endow the arbiter with the possibility to interfere, which we refer to as active adjudication. In its strong form, the adjudicator can commit to a decision in case of no reports. In its weak form, the adjudicator can bar one of the

[1] See Milgrom and Roberts (1986), Lipman and Seppi (1995), and Gentzkov and Kamenica (2017). At the end of the section, we review the literature related to our paper.

[2] Under the adversary system "it is for the parties to determine not only the issues which the court is to decide, but also the material on which the decision will be based. The evidence presented to the court will be that which the parties choose to present and none other. The judge may not require that a particular witness be summoned to give evidence or that a particular document be produced; he may not even question the witnesses himself except for the purpose of clarifying some doubt as to the meaning of what a witness has said under examination by counsel," Jolowicz (2000, p. 28).

parties from the influence game by announcing beforehand that she will not hear the party or read his report.[3] In this setup, the principal cannot commit to a decision that is not ex post optimal given her beliefs and the parties' strategies at equilibrium. Barring one sender creates, however, a crude form of commitment. It puts the onus or burden of proof on the other party to provide information. If this party opts to be silent, the sequentially rational decision will tend to favor the barred sender.

Next, we identify circumstances when "active adjudication" may be beneficial compared to "passive adjudication" where the principal merely updates her beliefs and decides once the parties have played. Active adjudication obviously has no useful role if the informed parties can report at no cost. Indeed, with zero reporting cost competition between opposed parties plays no useful role either: the decision maker's skeptical posture by assuming the worst induces full unraveling even from a single sender (Grossman and Hart 1980; Grossman 1981; Milgrom 1981).[4]

The unraveling argument is well known to fail, however, if the sender may have no hard information (Dye 1985; Shavell 1989; Shin 1994a). In this case, competition between multiple senders is useful (Shin 1994b, 1998). Within this framework of zero reporting costs and the possibility of no hard information, Bhattacharya and Mukherjee (2013) show that all facts are disclosed with positive probability and that the decision maker gains nothing from the ability to commit to a default decision in case of no report.[5] If commitment capacity has no value, actively barring one of the informed parties has no value either.

We show that these results need not hold with positive reporting costs. When communicating is costly, there are states of nature that are never disclosed by the parties. Under passive adjudication, these undisclosed states may be ex ante very likely. For example, when the arbiter has symmetrical and strictly unimodal priors over the state space and the informed parties have identical reporting costs, in the unique equilibrium undisclosed states are concentrated in the middle of the probability distribution. By committing to an extreme decision in case of no report or by barring one party from the persuasion game, the adjudicator shifts the burden of proof solely on one party. This moves the no-disclosure set to more extreme states which are ex ante unlikely and, therefore, matter less for appropriate decision making. Alternatively, it may be that this increases the probability that one party reports by an extent that more than offsets the loss from the other party's report. In our setup, the adjudicator usually does better if she is able to commit ex ante to a decision given no report that is not sequentially

[3] In the *inquisitorial procedure* of civil law countries, "it is for the judge to examine the witnesses, if any, it is for the judge to decide whether to summon the parties for interrogation and it is the judge who acts to obtain the assistance of an expert when required," Jolowicz (2000, p. 220). Our weak form of active adjudication is only one of the multiple instruments an inquisitorial judge has at hand.

[4] Matthis (2008) provides necessary and sufficient conditions, with respect to preferences and available messages, for full unraveling in one-sender games.

[5] The irrelevance of commitment is representative of a family of equivalence results between optimal mechanisms and equilibria in one-sender persuasion games; see Glazer and Rubinstein (2004, 2006), Sher (2011), and Hart et al. (2017). In the mechanism design approach, the decision maker moves first and commits to decision rules. The equivalence results state that she does as well when she moves second and plays a sequentially rational strategy. Bhattacharya and Mukherjee (2013) and Bhattacharya et al. (2018) show that the result extends to multiple senders, provided they have extreme agendas (either the same extreme agenda or diametrically opposed agendas).

rational. Barring one sender allows for a crude form of commitment that may help the adjudicator reach a better outcome.

The literature on reporting costs is relatively scant. It is well known from one-sender persuasion games that unraveling also depends on the assumption of costless disclosure. Costly disclosure typically yields "partial unraveling;" see Jovanovic (1982), Verrecchia (1983), Shavell, (1989), or Cheong and Kim (2004). We extend these results to a persuasion game with two opposed senders. More distantly related papers with reporting costs include Eső and Galambos (2013) who consider a cheap-talk game à la Crawford and Sobel (1982) where the sender also has the opportunity to disclose hard information. If disclosure is not too costly, the intervals of babbling types who do not separate shrink due to partial unraveling. Hedlund (2015) considers a persuasion game where the sender's communication costs are a continuous function of the message's observable precision, defined as the length of the interval containing the true state. Upon receiving a message, the uninformed party gets information about the true state and about the communication cost incurred by the sender. Despite communication costs, there are fully revealing separating equilibria involving a combination of disclosure and signaling. Kartik et al. (2017) consider the disclosure of hard signals when multiple senders incur no reporting costs but must invest in information acquisition. When senders have the same agenda, the parties' efforts in acquiring information are strategic substitutes. The decision maker may do better by soliciting advice from only one of them. They provide an example where the same result holds when the two informed parties have opposed preferences.[6]

We proceed as follows. Section 2 presents our basic setup which, abstracting from reporting costs, borrows much from Bhattacharya and Mukherjee (2013). Section 3 characterizes the equilibria under passive and active adjudication. Section 4 discusses circumstances where active strategies may be beneficial for the adjudicator. Section 5 concludes. All proofs are relegated to the Appendix.

2 Model

There are two informed parties, A and B, and an uninformed decision maker J who will be referred to as the adjudicator. The true state of the world is $x \in [0, 1]$. It is distributed according to the cdf $F(x)$ with full support on $[0, 1]$; $f(x)$ denotes the probability density function. The adjudicator's priors, as defined by F, are her beliefs given the information available before the informed parties take their actions.

The adjudicator must take an action $y \in [0, 1]$ which yields her a payoff $u_J(y, x)$ that depends on the underlying state. The payoff is expressed in terms of a symmetric loss function, $u_J(y, x) = -v(|y - x|)$ where $v(0) = v'(0) = 0$, $v' > 0$ and $v'' > 0$. The payoff is thus maximal when the decision matches the true state.

[6] Hay and Spier (1997) analyze the allocation of the burden of proof between plaintiff and defendant from the point of view of minimizing litigation costs. Because each party's submission cost is less than the stake, the trial outcome is always without error. However, litigation expenditures will differ depending on the burden of proof assignment. More distantly related strands of literature deal with costly acquisition of information by the senders or with communication through "fabricated evidence". See, e.g., Dewatripont and Tirole (1999), Gerardi and Yariv (2008), and Kim (2014) on the first issue; Kartik (2009) and Emons and Fluet (2009, 2019) on the second.

The parties A and B are concerned about the adjudicator's decision. They have state-independent utilities. Party i 's utility from the adjudicator's decision is $u_i(y)$ with $u_A(y) = -y$ and $u_B(y) = y$. Party A wants the decision to be as small as possible, equivalently as much to the left as possible; party B wants the decision to be as large or as much to the right as possible.[7]

Knowing the true state, they may report it to the adjudicator: either they provide hard information about the state or they communicate nothing. Formally, they send the message $m_i \in \{x, \emptyset\}$; $m_i = x$ means disclosure of the true state and $m_i = \emptyset$ denotes silence. Total payoff to an informed party is

$$U_i(y, m_i) = u_i(y) - C_i(m_i), \quad i = A, B,$$

where

$$C_i(m_i) = \begin{cases} 0, & \text{if } m_i = \emptyset, \\ c_i, & \text{if } m_i = x, \end{cases}$$

is the cost to the party of sending the message m_i with $c_i \geq 0$. Obviously, it could be that reporting costs are so high that both parties always remain silent. To rule out this possibility, we assume $c_A + c_B < 1$. The parties' reporting costs are common knowledge.

3 Equilibrium characterization

Let us now derive the equilibria when the adjudicator has no commitment power; her decision is always sequentially rational given her beliefs. We first consider the benchmark where the adjudicator is completely passive. The parties simultaneously decide whether or not to disclose the true state; once the parties have played, the adjudicator updates her beliefs and makes a decision. This benchmark is motivated by the adversarial procedure. Then, we turn to the case where, at a preliminary stage, the adjudicator can announce that a party is barred from reporting. The adjudicator now deals only with one sender. A judge has this possibility under the inquisitorial procedure.

Two-sender equilibrium The equilibrium concept is Perfect Bayesian (PBE). A strategy for the adjudicator is a function $y(m_A, m_B)$. Party i's strategy is a function $m_i(x)$, $i = A, B$.

The adjudicator's best response is to choose $y = x$ if one or both parties disclose the state. If the state is not disclosed, her best response is

$$y^* = \arg\max_y \mathbb{E}\left(u_J(y, x) \mid m_A = m_B = \emptyset\right). \tag{1}$$

The right-hand side is the adjudicator's expected payoff from decision y given her beliefs about x conditional on the event that both parties remain silent.

[7] Sher's (2011) condition for the irrelevance of commitment in one-sender games is that the decision maker's utility function is a concave transformation of the sender's utility function. This is satisfied here with respect to both senders with $u_J(y, x) = -v\left(|u_A(y) + x|\right) = -v\left(|u_B(y) - x|\right)$.

Party A wishes y to be as small as possible. It is, therefore, optimal for party A to remain silent when the true state satisfies $x + c_A \geq y^*$. To see this, observe that if the state is revealed by the other party, disclosure by A does not change the decision but imposes a reporting cost; conversely, if the state is not disclosed by the other party, A is better off with the adjudicator's default action y^* than by reporting at cost c_A and inducing the decision $y = x$. Similarly, and recalling that B wishes y to be as large as possible, it is an optimal strategy for party B to remain silent when the true state and his reporting cost satisfy $x - c_B \leq y^*$.

Conversely, it is an optimal strategy for party A to report if $x + c_A < y^*$ because A knows that, given the state x, the other party will not report. Similarly, it is optimal for B to report if $x - c_B > y^*$. Given the adjudicator's default action when she gets no reports, the no-disclosure event is therefore

$$N(y^*) = \{x : y^* - c_A \leq x \leq y^* + c_B, x \in [0, 1]\}. \tag{2}$$

Condition (1) can be rewritten as:

$$y^* = \arg\max_y \mathbb{E}\left(u_J(y, x) \mid N(y^*)\right). \tag{3}$$

An equilibrium is completely characterized by a solution y^* to (3).

Proposition 1 *A PBE for the two-sender game with passive adjudicator always exists. In any equilibrium* $y^* \in (0, 1)$. *The informed parties' strategies are*

$$m_A(x) = \begin{cases} x, & \text{if } x + c_A < y^*, \\ \emptyset, & \text{otherwise,} \end{cases} \tag{4}$$

$$m_B(x) = \begin{cases} x, & \text{if } x - c_B > y^*, \\ \emptyset, & \text{otherwise.} \end{cases} \tag{5}$$

The adjudicator's strategy is

$$y(m_A, m_B) = \begin{cases} x, & \text{if } m_A = x \text{ or } m_B = x, \\ y^*, & \text{otherwise,} \end{cases} \tag{6}$$

where

$$y^* = \arg\max_y \int_{\max(0, y^* - c_A)}^{\min(y^* + c_B, 1)} u_J(y, x) f(x)\, \mathrm{d}x. \tag{7}$$

Note that the right-hand side of (7) is not a conditional expectation. However, condition (7) is equivalent to

$$\begin{aligned} y^* &= \arg\max_y \int_{\max(0, y^* - c_A)}^{\min(y^* + c_B, 1)} u_J(y, x) f(x)\, \mathrm{d}x \Big/ \Pr(N(y^*)) \\ &= \arg\max_y \mathbb{E}\left(u_J(y, x) \mid N(y^*)\right). \end{aligned}$$

Because $y^* \in (0, 1)$, it also follows that it satisfies the first-order condition

$$\int_{\max(0, y^*-c_A)}^{\min(y^*+c_B, 1)} u'_J(y^*, x) f(x)\, \mathrm{d}x = 0, \tag{8}$$

with $u'_J(y, x) := \partial u_J(y, x)/\partial y$.

Qualitatively, the equilibrium is similar to the equilibrium in Bhattacharya and Mukherjee (2013). The crucial difference is that there is a wedge between the experts' disclosure sets in our model, meaning that some states are never disclosed. By contrast, in the setup of Bhattacharya and Mukherjee (2013), all states are disclosed with positive probability.

It is useful to make a distinction between an interior and a corner equilibrium. An equilibrium is *interior* if both parties disclose with positive probability. In a *corner* equilibrium, only one party discloses with positive probability and the other party is always silent. An outcome is referred to as a corner-i equilibrium if only party i is active.

Corollary 1 *In the game with passive adjudicator, at least one party discloses with positive probability.*

In an equilibrium with y^* as the adjudicator's default action, both parties remain silent for states in

$$S := [y^* - c_A, y^* + c_B] \cap [0, 1].$$

We refer to S as the no-disclosure set. In an interior equilibrium, S is in the interior of the state space as illustrated in Fig. 1a. Both parties then report with positive probability. Figure 1b illustrates a corner-A equilibrium. The no-disclosure set is now of the form $[y^* - c_A, 1]$. Party B is always silent because $x - c_B < y^*$ for all x in the state space. Let L denote the length of the no-disclosure set. Observe that in an interior equilibrium, $L = c_A + c_B$. In a corner equilibrium, $L \leq c_A + c_B$.

In a corner-A equilibrium, condition (7) reduces to

$$y^* = \arg\max_y \int_{y^*-c_A}^{1} u_J(y, x) f(x)\, \mathrm{d}x. \tag{9}$$

This condition is equivalent to

$$\begin{aligned} y^* &= \arg\max_y \int_{y^*-c_A}^{1} u_J(y, x) \left(\frac{f(x)}{1 - F(y^* - c_A)} \right) f(x)\, \mathrm{d}x \\ &= \arg\max_y \mathbb{E}\left(u_J(y, x)) \mid x > y^* - c_A \right). \end{aligned}$$

One-sender equilibrium As we show in the next section, competition between the informed parties to influence the adjudicator may not be desirable from her point of view. She may prefer not to be influenced by a party by refusing to listen to him. We modify the original game by adding an initial stage where the adjudicator announces

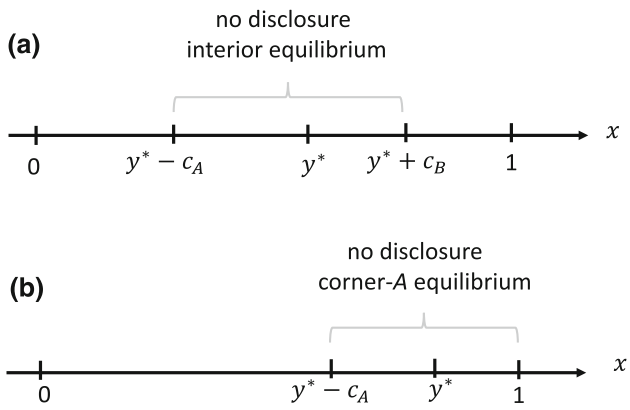

Fig. 1 Interior and corner equilibria

whether she listens to both parties or whether one party is barred from reporting (this would specify which one). In the latter case, the continuation game that ensues is referred to as a one-sender game.

Proposition 2 *A PBE for the one-sender game always exists. If only A is allowed to report, A's strategy is given by* (4) *and the adjudicator's strategy is*

$$y(m_A) = \begin{cases} x, & \text{if } m_A = x, \\ y^*, & \text{otherwise}, \end{cases} \tag{10}$$

where

$$y^* = \arg\max_y \int_{\max(0, y^* - c_A)}^{1} u_J(y, x) f(x)\, \mathrm{d}x. \tag{11}$$

If only B is allowed to report, B's strategy is given by (5) *and the adjudicator's strategy is*

$$y(m_B) = \begin{cases} x, & \text{if } m_B = x, \\ y^*, & \text{otherwise}, \end{cases} \tag{12}$$

where

$$y^* = \arg\max_y \int_{0}^{\min(y^* + c_B, 1)} u_J(y, x) f(x)\, \mathrm{d}x. \tag{13}$$

In either case $y^* \in (0, 1)$.

Allowing only one party to report forces a corner equilibrium. When only A is allowed, large values of x will never be reported but smaller values (those satisfying $x < y^* - c_A$) will be disclosed. Conversely, if B is the only one to report, small values of x will never be disclosed.

There is a distinction between one-sender and two-sender corner equilibria. The outcome, by force a corner equilibrium, of the one-sender game where only i can report need not be an equilibrium of the two-sender game. The reason is that competition between the informed parties may induce an interior equilibrium or the opposite corner equilibrium.

Corollary 2 *If the two-sender game has a corner-i equilibrium with default decision y^*, then the latter is also an equilibrium default in the one-sender game where only i is allowed to report. An equilibrium strategy profile of the one-sender game need not be part of an equilibrium strategy profile of the two-sender game.*

In a two-sender equilibrium $L \leq c_A + c_B$: in an interior equilibrium the equality holds and in a corner equilibrium the weak inequality holds. In a one-sender equilibrium $L \leq c_A + c_B$ if it is part of the equilibrium strategy profile of the two-sender game; if it is not part thereof, then $L > c_A + c_B$. This implies the following: If the arbiter forces a corner equilibrium that is not part of an equilibrium of the two-sender game, she increases the length of the no-disclosure set. She also changes, however, its location. Therefore, barring a sender creates a trade-off between the size and the location of the no-disclosure set. Or, put differently: The virtue of competition is to limit the length of the no-disclosure set to $L \leq c_A + c_B$. The problem of competition may be, however, the location of the no-disclosure set.

To conclude this section, let us discuss the possibility of multiple equilibria. It is well established that disclosure costs may lead to multiple equilibria in one-sender persuasion games. In games where the single sender wants the adjudicator's decision to be as large as possible, equivalent here to a sender-B game, log-concavity of the density f or of the cdf F ensures a unique equilibrium; see Bagnoli and Bergstrom (2005).[8] The former condition also guarantees a unique equilibrium in a sender-A game, i.e., when the single sender wants the adjudicator's decision to be as small as possible.[9] Nevertheless, in the two-sender game the log-concavity condition does not eliminate the possibility of multiple equilibria . We provide examples in Sect. 4. When multiple equilibria arise, we select the adjudicator-preferred equilibrium, henceforth referred to simply as the equilibrium.

4 Active versus passive adjudication

We now compare the arbiter's payoff under passive adjudication and two forms of active adjudication. Passive adjudication refers to the two-sender equilibria described in the preceding section. The parties decide whether they submit a report and the arbiter's decision is sequentially rational given the parties' submissions. Under the first form of active adjudication, the arbiter can fully commit to a default decision in case of no reports. We show that this is often strictly beneficial from the arbiter's point of view. Under the second form of active adjudication, the arbiter can only commit not to hear a party or read his report, i.e., the arbiter can discard a party from the persuasion game but her decision is sequentially rational given the remaining party's report. Her payoff is the same as in the one-sender equilibria described in the preceding section. We show that this form of active adjudication can also be beneficial.

8 Log-concavity of f implies log-concavity of F and is therefore a stronger condition.

9 A log-concave f implies that the hazard rate $f(x)/(1 - F(x))$ is increasing. This ensures uniqueness in the sender-A game. The proof is similar to the argument in Bagnoli and Bergstrom (2005).

Commitment to a default decision If the adjudicator has full commitment capacity, she announces the default action $\hat{y}$ maximizing

$$\overline{U}_J(\hat{y}) = \int_{\max(0,\hat{y}-c_A)}^{\min(\hat{y}+c_B,1)} u_J(\hat{y}, x) f(x)\, \mathrm{d}x. \tag{14}$$

The parties' strategies are as described in Proposition 1 with $\hat{y}$ substituted for y^*.

If the problem has an interior solution $\hat{y} \in (0, 1)$, it satisfies the first-order condition $\overline{U}'_J(\hat{y}) = 0$ where

$$\begin{aligned}\overline{U}'_J(\hat{y}) = &\int_{\max(0,\hat{y}-c_A)}^{\min(\hat{y}+c_B,1)} u'_J(\hat{y}, x) f(x)\, \mathrm{d}x \\ &+ u_J(\hat{y}, \min(\hat{y} + c_B, 1)) f(\min(\hat{y} + c_B, 1)) \frac{\mathrm{d} \min(\hat{y} + c_B, 1)}{\mathrm{d}\hat{y}} \\ &- u_J(\hat{y}, \max(0, \hat{y} - c_A)) f(\max(0, \hat{y} - c_A)) \frac{\mathrm{d} \max(0, \hat{y} - c_A)}{\mathrm{d}\hat{y}}.\end{aligned} \tag{15}$$

Evaluated at $\hat{y} = y^*$, where y^* is the equilibrium default decision in the two-sender game without commitment capacity, the first term in (15) is equal to zero; recall condition (8). Yet, the sum of the last two terms will in general differ from zero, implying that the adjudicator benefits from being able to commit to an action that differs from the ex post optimal decision.

Moreover, a default action satisfying $\overline{U}'_J(\hat{y}) = 0$ need not be the solution under commitment because the maximization problem is typically not concave. In particular, the solution may be the corner solution $\hat{y} = 0$ or $\hat{y} = 1$ implying that only one party is induced to report. Even when the optimal $\hat{y}$ is interior and satisfies (15), it may also induce disclosure by only one party. When $\hat{y} \le c_A$, only B reports. When $\hat{y} \ge 1 - c_B$, only A reports. Solutions satisfying $\hat{y} \notin (c_A, 1 - c_B)$ will be referred to as *one-sender outcomes*.[10] Such outcomes are of particular interest because commitment can be shown to be strictly beneficial as compared to passive adjudication. Furthermore, a one-sender outcome amounts to discarding one party, although this is accomplished here through the default decision under commitment. Note that in a one-sender outcome the length of the no-disclosure set $L < c_A + c_B$.

We now provide sufficient conditions for commitment to be beneficial as compared to passive adjudication. Throughout we assume that the density f is continuous and focus on the case where it is unimodal. This includes the case where f has a modal interval $[m_l, m_h]$, i.e., $f(x)$ is maximized at all x in this interval. When $m_l = m_h := m$, the density is strictly unimodal with the unique mode m. There are two possibilities to consider. Either the modal interval is at one of the boundaries of the state space or it is interior, i.e., $0 < m_l \le m_h < 1$.

[10] To clarify our jargon: If under commitment only one agent reports, we call it a one-sender outcome. Without commitment, when only one party sends a message, be it in the game where both or only one party is allowed to report, we speak of a corner equilibrium.

Proposition 3 *Commitment is strictly beneficial if it yields a one-sender outcome. When f is unimodal, any of the following conditions is sufficient for a one-sender outcome:*

(i) *f is monotonic;*
(ii) *f is symmetric and $c_A + c_B \leq \frac{1}{2}$;*
(iii) *f has an interior modal interval $[m_l, m_h]$ and $c_A + c_B \leq m_h - m_l$;*
(iv) *$f(x)$ is concave over the interval $[m - c_A - c_B, m + c_A + c_B] \cap [0, 1]$, for any mode m.*

The first claim in the proposition follows from the fact that the optimal default in a one-sender outcome under commitment necessarily differs from the equilibrium default without commitment (whether an interior or a corner equilibrium). To decipher condition (iv), suppose the density is strictly unimodal with mode m. If f is smooth, it must be concave in some neighborhood of m. The condition then holds if c_A and c_B are not too large. To give a second illustration, the condition also holds, without additional qualifications, if f is concave over the state space, e.g., as with the triangular and trapezoidal distributions. We provide examples further illustrating the one-sender outcomes.

Example 1 f is decreasing.

Borrowing from the proof of part (i) of Proposition 3, the adjudicator chooses $\hat{y} = 1$ if $c_A \leq c_B$ or if $c_A - c_B$ is positive and not too large; otherwise, she chooses some $\hat{y} \leq c_A$. In other words, the adjudicator trades off the size of the no-disclosure set which determines the error costs $v(|\hat{y} - x|)$ and the location of this set which determines the probability of the error costs. Setting $\hat{y} = 1$ yields the one-sender outcome where only A reports. The no-disclosure set is then $[1 - c_A, 1]$. When f is strictly decreasing, this set belongs to the region of the state space with the smallest density. However, when c_A becomes large, the error cost will also become large: the more so the more convex the function v. At some point, it is better to switch to the opposite one-sender solution where only B reports. The adjudicator then chooses some positive $\hat{y}$ less than c_A.[11] With a strictly decreasing f, this allows the more severe errors to arise with smaller probability than if she picks $\hat{y} = 0$. The B-sender solution is better because the no-disclosure set $[0, \hat{y} + c_B]$ is smaller than the no-disclosure set $[1 - c_A, 1]$. Note that $\hat{y} = 1$ or $\hat{y} \in (0, c_A]$ are not sequentially rational.

Example 2 f has a unique interior mode $m \leq \frac{1}{2}$ and $c_A + c_B$ is not too large.

We now always have a corner solution $\hat{y} = 0$ or $\hat{y} = 1$, but the adjudicator may again need to trade off the location of the no-disclosure set against its size. Sufficient conditions for $\hat{y} = 1$ are $c_A \leq c_B$ and $f(z) \geq f(1 - z)$ for $z \in [0, c_A]$. However, with a symmetric density, including the uniform distribution as a limiting case, the adjudicator simply picks the one-sender outcome where the lower-cost sender reports.

To summarize, under appropriate conditions the optimal policy under commitment puts the burden of proof fully on one party. Either only unlikely states remain undisclosed or the size of the no-disclosure set is small because the reporting party has

[11] With a strictly decreasing f, condition (19) in the proof implies $\overline{U}'_J(0) > 0$, so $\hat{y} = 0$ cannot be optimal.

lower costs. It remains to give some intuition for why the adjudicator would ever want to choose a two-sender outcome. This can arise when reporting costs are sufficiently large.

Example 3 an optimal two-sender outcome under commitment.

Let the density be symmetric so that the mode $m = 1/2$. Reporting costs are also symmetric and equal to c. Candidates for a solution are $\hat{y} = 0$, $\hat{y} = 1$, or possibly $\hat{y} = 1/2$. Let f be increasing and convex for $x < 1/2$ and decreasing and convex for $x > 1/2$. The density function has a spike at the mode, hence condition (iv) does not hold. For c sufficiently large, condition (ii) will not hold either. Then, for c large, $f(c) = f(1-c)$ will be 'large' while $f(1/2-c) = f(1/2+c)$ will be 'small'. With $\hat{y} = 1/2$, large errors, i.e., those close to $1/2 \pm c$ will occur with small probability; with $\hat{y} = 0$ or $\hat{y} = 1$, large errors, i.e., those close to c or $1 - c$ will have large probability. With a sufficiently convex error cost function, the two-sender outcome with default decision $\hat{y} = 1/2$ does better. Moreover, there is no benefit as compared to passive adjudication because $y^* = 1/2$ is the unique equilibrium of the two-sender game.

No-commitment Let us now examine the case where the adjudicator cannot commit to a default decision in case of no report. She can only bar a sender.[12] There are two possibilities for this limited form of active adjudication to yield higher payoff for the arbiter than passive adjudication. Either the two-sender equilibrium under passive adjudication is interior but the adjudicator does better by allowing only one party to report. Or the two-sender equilibrium is a corner equilibrium where one party is silent. The adjudicator does, however, better by allowing only that silent party to report: she prefers to force the opposite corner equilibrium.

The adjudicator's decision must be sequentially rational. As shown in the previous section, the optimal default under commitment is often a corner solution with $\hat{y} = 0$ or $\hat{y} = 1$. In the one-sender equilibrium where one party is barred from reporting, the equilibrium default satisfies $y^* \in (0, 1)$. Accordingly, no-commitment yields a worse outcome than commitment. The same is also true when the solution under commitment is some $\hat{y} \in (0, 1)$ yielding a one-sender outcome. As noted, $\hat{y}$ differs from the equilibrium y^* where the same party is the only one allowed to report. Recall that in a one-sender outcome under commitment the length of the no-disclosure set $L < c_A + c_B$. By contrast, it is strictly greater in a forced one-sender equilibrium. Thus, barring a sender may not be beneficial even though commitment yields a one-sender outcome that is better than the passive adjudication outcome. We illustrate the preceding observations for the case of the uniform distribution.

Example 4 the case of the uniform distribution.

The uniform distribution is consistent with condition (i) of Proposition 3. Under full commitment, $c_A = c_B = c$ implies $\hat{y} = 0$ or $\hat{y} = 1$. Under passive adjudication, the two-sender game has a continuum of payoff equivalent equilibria with $y^* \in [c, 1-c]$. Since the loss function is symmetric and all states are equally likely, any $y^* \in [c, 1-c]$

[12] It is obviously never in the adjudicator's interest to prohibit both parties from reporting.

is an equilibrium and these equilibria yield the adjudicator the same payoff. Given her uniform priors, she is indifferent as to the location of the no-disclosure interval.[13] In particular, $y^* = c$ and $y^* = 1 - c$ are corner equilibria which amount to discarding A or B, respectively. So active adjudication in the form of barring one party is not beneficial.

Similarly, under full commitment, $c_A < c_B$ implies $\hat{y} = 1$, i.e., only A reports. Under passive adjudication, the two-sender game has a unique corner-A equilibrium with $y^* = 1 - c_A$ and the no-disclosure interval is $[1 - 2c_A, 1]$. Active adjudication through barring B, therefore, yields the same outcome. Note that barring A forces the corner-B equilibrium $y^* = c_B$. This is detrimental because the no-disclosure interval $[0, 2c_B]$ of the corner-B equilibrium is larger than the no-disclosure interval of the corner-A equilibrium.

Consider now the case where the adjudicator has specific views concerning the likely values of the state. Intuitively, strong priors suggest that active adjudication may be beneficial. Suppose, for example, F is skewed with most of the probability mass concentrated on small values of x. The two-sender game may nevertheless have an interior equilibrium. If A's reporting cost is not too large, it may be beneficial to force a corner-A equilibrium by barring B. Alternatively, suppose F is strictly unimodal and symmetric. If the parties have the same reporting cost c, the two-sender game has a unique interior equilibrium with $y^* = 1/2$. The no-disclosure interval $[1/2 - c, 1/2 + c]$ is centered and may include most of the probability mass. It may be beneficial to force a corner equilibrium by barring one party: although the size of the no-disclosure set will be larger, no disclosure will occur for values of the state that are a priori unlikely. We now provide examples for both arguments.

We illustrate with the case where the adjudicator's priors are given by the Beta distribution $\mathcal{B}(a, b)$ with parameters $a > 0$ and $b > 0$. The mean is $\mu = a/(a + b)$. The mode is $m = (a - 1)/(a + b - 2)$ if $a > 1$ and $b > 1$; for $a < 1$ and $b \geq 1$, the density is everywhere decreasing so that $m = 0$. See Fig. 2. In all our examples, the distribution is unimodal and is either symmetric or skewed with mean and mode less than or equal to 1/2. The loss function is the square error. Costs are $c_A = 0.1$ and $c_B \in \{0.1, 0.15, 0.2\}$. In all cases, an adjudicator with full commitment capacity chooses $\hat{y} = 1$, i.e., the A-sender outcome. Accordingly, when the adjudicator cannot commit, she will often (but not always) do better by allowing only A to report.

The first case in Table 1 is $\mathcal{B}(0.5, 1)$ with $\mu = 1/3$ and mode $m = 0$.[14] When $c_A = c_B = 0.1$, the two-sender game has a unique corner-B equilibrium with $y^* = 0.05$. The adjudicator does, however, better by barring B, thus forcing a corner-A equilibrium.[15] When B's cost is $c_B = 0.15$, the two-sender game has three equilibria: a corner-A, a corner-B, and an interior equilibrium. The best outcome from the adjudicator's point of view is the corner-A equilibrium. This outcome arises 'naturally' under our adjudicator-preferred selection convention. Alternatively, it can be implemented by

[13] Note that the uniform density is log-concave, but nevertheless leads to multiple equilibria in the two-sender game.

[14] In Table 1, the type of equilibrium is denoted *c-A* for corner-A, *c-B* for corner-B, and *int* for interior.

[15] Due to the skewness of the distribution, the no-disclosure set of the corner- A equilibrium is about twice the size of the no-disclosure set of the corner-B equilibrium. Yet, the no-disclosure states in the corner-A equilibrium are ex ante very unlikely as compared to the corner-B equilibrium.

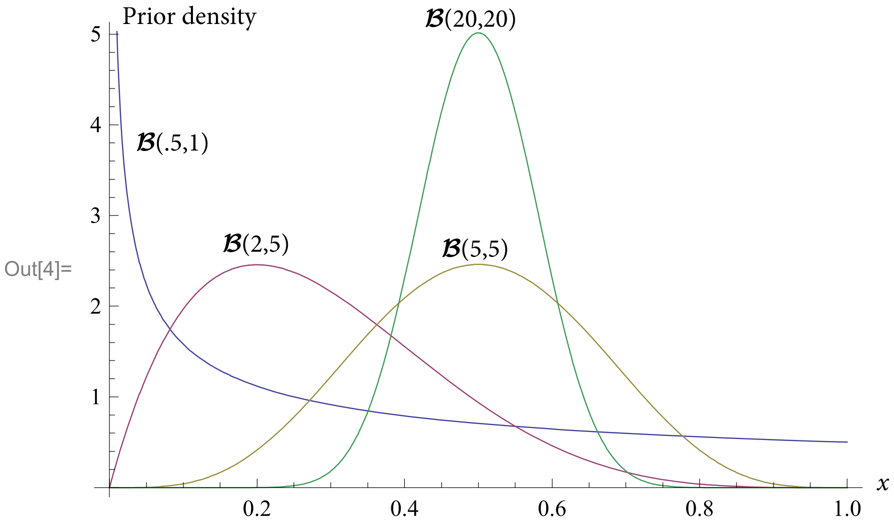

Fig. 2 Some of the beta distributions used in Table 1

discarding B . When B's cost is $c_B = 0.20$, the two-sender game has a unique corner-A equilibrium. There is then no need for the principal to actively adjudicate.

For $\mathcal{B}(0.5, 2)$, $\mathcal{B}(1, 5)$, and $\mathcal{B}(2, 5)$, a corner-A equilibrium is always the best option for the adjudicator. The two-sender game never has a corner-A equilibrium. Therefore, the adjudicator always has to bar B to enforce the corner-A equilibrium.

Next, consider the last two cases in Table 1 with symmetric distributions. In all cases, the two-sender game has a unique interior equilibrium which, with one exception, is also best for the adjudicator. Only for the concentrated distribution $\mathcal{B}(10, 10)$ and $c_B = 0.2$, a corner-A equilibrium is better. The intuition is straightforward. By barring one sender, the adjudicator has control over the location of the no-disclosure set. However, because she cannot commit to a default decision, she has no control over the size of this set. Consider the case where both parties have the same reporting cost $c_A = c_B = c = 0.1$. In the interior equilibrium, the length of the no-disclosure set is $2c = 0.2$. In the corner-A equilibrium, the no-disclosure set $[y^* - c, 1]$ is much larger. For the more concentrated distribution, $y^* = 0.571$ and the length of the no-disclosure set equals 0.53. This yields a much larger maximum error.

Table 2 provides additional examples with concentrated and symmetric densities and symmetric reporting costs. We now consider small costs. For the distribution $\mathcal{B}(10, 10)$, the arbiter does better with a corner equilibrium if the reporting cost is reduced to $c = 0.05$ or $c = 0.02$. With the more concentrated densities, a one-sender equilibrium remains better for the lower reporting cost $c = 0.02$. Even though the no-disclosure set is larger than the no-disclosure set of the interior equilibrium, silence is ex ante sufficiently unlikely in the corner equilibria as compared to the interior ones.[16]

[16] In our examples, the one-sender games always have unique equilibria. Recall that uniqueness obtains when the density $f(x)$ is log-concave, which is the case for $a \geq 1$ and $b \geq 1$.

Table 1 Square-error loss function and Beta priors

Beta		Commitment		One-sender A		Two-sender		
(a, b)	c_B	$\hat{y}$	MSE (×100)	y^*	MSE (×100)	y^*	MSE (×100)	Type
$(.5, 1)$	.10	1	.017	.896	.037*	.050	.077	c-B
						.075	.213	c-B
	.15	1	.017	.896	.037*	.116	.207	int
						.896	.037*	c-A
	.20	1	.017	.896	.037*	.896	.037*	c-A
$(.5, 2)$	.10	1	.002	.790	.021*	.046	.100	c-B
						.068	.260	c-B
	.15	1	.002	.790	.021*	.141	.219	int
						.736	.024	int
	.20	1	.002	.790	.021*	.087	.495	c-B
$(1, 5)$	.10	1	.000	.500	.055*	.077	.156	c-B
	.15	1	.000	.500	.055*	.168	.265	int
	.20	1	.000	.500	.055*	.405	.088	int
$(2, 5)$	.10	1	.000	.562	.097*	.207	.148	int
	.15	1	.000	.562	.097*	.426	.145	int
	.20	1	.000	.562	.097*	.519	.110	int
$(5, 5)$	.10	1	.001	.664	.188	.500	.148*	int
	.15	1	.001	.664	.188	.621	.175*	int
	.20	1	.001	.664	.188	.654	.186*	int
$(10, 10)$	.10	1	.000	.570	.296	.500	.189*	int
	.15	1	.000	.570	.296	.551	.256*	int
	.20	1	.000	.570	.296*	.566	.370	int

$c_A = 0.1$; a star identifies the smallest MSE over the one and two-sender equilibria

When priors are symmetric and reporting costs do not differ too much, two-sender equilibria are interior. The larger the c_B, the larger the y^*: the onus to disclose falls more heavily on the least cost party A. By contrast, when costs are symmetric and priors sufficiently favor A, y^* will favor A, putting the onus to disclose on B. Thus, in our examples the asymmetries in reporting costs and in prior beliefs move the no-disclosure set in opposite directions. The following rough qualitative generalizations emerge:

(i) When priors are symmetric and strictly unimodal, the two-sender game has an interior equilibrium if reporting costs do not differ much. It may be better to discard the high cost party.

(ii) When priors are skewed against party B, strictly unimodal, and reporting costs do not differ much, the outcome of the two-sender game is either an interior equilibrium or a corner equilibrium putting the onus on the a priori disfavored

Table 2 Symmetric Beta priors and identical costs

Beta (a,b)	c	Commitment $\hat{y}$	Commitment MSE (×100)	One-sender A y^*	One-sender A MSE (×100)	Two-sender y^*	Two-sender MSE (×100)	Two-sender Type
(10, 10)	.10	1	.00000	.571	.29632	.5	.18958*	*int*
	.05	1	.00000	.681	.02022*	.5	.02784	*int*
	.02	1	.00000	.832	.00003*	.5	.00186	*int*
(20, 20)	.10	1	.00000	.523	.31835	.5	.10880*	*int*
	.05	1	.00000	.589	04873	.5	.03731*	*int*
	.02	1	.00000	.730	.00009*	.5	.00262	*int*
(30, 30)	.10	1	.00000	.511	.27992	.5	.21277*	*int*
	.05	1	.00000	.554	.06710	.5	.04319*	*int*
	.02	1	.00000	.668	.00031*	.5	.00319	*int*

$c_A = c_B = c$; a star identifies the smallest MSE over the one- and two-sender equilibria

party B. This implies that ex ante relatively likely (smaller) states will not be disclosed. Discarding party B may be beneficial: it puts the burden of proof on A, thus shifting no-disclosure to the less likely high states.

Parties with unknown information endowment To conclude this section, let us discuss the possibility that the adjudicator does not observe whether or not the parties are informed about x, as is the case in, e.g., Bhattacharya and Mukherjee (2013). Party i observes the true state with probability p_i. Under passive adjudication, the equilibrium is a straightforward generalization of Proposition 1. The parties' equilibrium strategies are the same, except, of course, that they report nothing when uninformed. The adjudicator's equilibrium decision in case of no report is

$$\begin{aligned} y^* = \arg\max_y (1-p_A)\int_0^{\max(0,y^*-c_A)} & u_J(y,x)f(x)\,\mathrm{d}x \\ + \int_{\max(0,y^*-c_A)}^{\min(y^*+c_B,1)} & u_J(y,x)f(x)\,\mathrm{d}x \\ + (1-p_B)\int_{\min(y^*+c_B,1)}^{1} & u_J(y,x)f(x)\,\mathrm{d}x. \end{aligned} \tag{16}$$

As in our baseline setup, there is a wedge between the parties' disclosure sets due to the reporting costs. However, the adjudicator now also takes the possibility into account that parties do not report because they are uninformed. A similar formulation obtains in the case of one-sender equilibria.[17]

[17] Equation (16) combines Proposition 1 with the condition in Bhattacharya and Mukherjee (2013) for the case of two opposed experts. Setting $p_B = 0$ in (16) yields the equilibrium default decision in the one-sender equilibrium where only A is allowed to report.

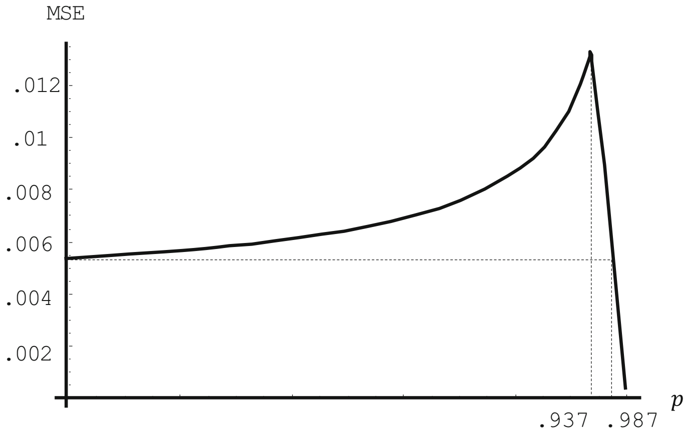

Fig. 3 Mean-square error in the two-sender game with uniform priors, $c_A = 0.2$, $c_B = 0$, $p_A = 1$, and $p_B = p$

Active adjudication may again be beneficial. Consider the following example with a uniform distribution. Party A is always informed, i.e., $p_A = 1$, and has reporting cost $c_A > 0$. Party B is informed with probability $p_B = p$ and incurs no reporting cost, i.e., $c_B = 0$. The loss function is the square error $v(|y - x|) = (y - x)^2$.

Under passive adjudication, the equilibrium default decision is

$$y^* = \begin{cases} 1 - c_A/\sqrt{1-p}, & \text{if } p < p_0, \\ (\sqrt{1-p} - 1 + p)/p, & \text{if } p \geq p_0, \end{cases}$$

where $p_0 = (1 - 2c_A)/(1 - c_A)^2$.

When $0 < p < p_0$, $y^* > c_A$ and we have an interior equilibrium where both parties report with some probability. When $p \geq p_0$, $y^* \leq c_A$ and the outcome is a corner-B equilibrium where only B reports.

Figure 3 depicts the adjudicator's expected loss (the mean square error) as a function of p for $c_A = 0.2$. When $p = 0$, B never discloses and the loss is the same as in the one-sender game where only A is allowed to report. A larger p decreases the default decision y^*: the more likely it is that B is informed, the more "no report" suggests a small value of x. A lower y^* , however, reduces the probability that A reports. It turns out that the loss from this reduction more than offsets the gain from B's higher reporting probability. The adjudicator's expected loss, therefore, increases in p for $p \leq p_0 = 0.937$ where the equilibrium switches from interior to corner-B. In the corner-B regime, the expected loss is decreasing in p. As p approaches unity, y^* tends to zero; B always discloses so that there is complete unraveling. Computations yield that for $p < 0.987$ the arbiter does better by barring B, thus forcing a corner-A equilibrium, rather than adjudicating passively and implementing an interior or a corner-B equilibrium.

An adjudicator with full commitment capacity picks $\hat{y} = 1$ except for p sufficiently close to unity; such a policy yields a mean square error equal to 0.0027. For p sufficiently close to unity (and above 0.987), the optimal full commitment default is $\hat{y} = (\sqrt{1-p} - 1 + p)/p$ and yields a corner-B outcome. This is the same as the equilibrium default in the corner-B equilibrium.[18]

5 Concluding remarks

We have discussed a simple setup showing that a sophisticated uninformed principal can often do better than merely wait passively for interested competing parties to decide whether or not they will provide information. When communication is costly, the decision maker usually does better if she is able to commit ex ante to a default decision in case of no report. This contrasts with some general results on persuasion games without disclosure costs. Active adjudication by barring one sender also often does better even though the decision maker has limited power, e.g., she can force no one and cannot commit to a default decision. Nevertheless, deciding who is allowed to take part in the persuasion game may be beneficial, compared with passive adjudication.

Our results do not mean, of course, that competition between interested parties is not generally useful, as in the environment studied by Milgrom and Roberts (1986). In Lipman and Seppi (1995) for instance, the issue is whether relatively simple strategies (e.g., "believe unless refuted") on the part of the decision maker can elicit information transmission through competition. However, our analysis shows that reporting costs introduce a wedge between purely passive adjudication and a more active stance. A decision maker may wish to make clear which interested party she will listen to.

Finally, we have evaluated passive versus active adjudication solely on the basis of error costs. A natural extension of our approach is to also take reporting costs into account.[19] If, for example, the arbiter gives a lot of weight to submission costs and little weight to errors on her part, it may be optimal to bar both parties from reporting. Abstracting from this extreme case, there is a natural congruence between reducing error costs and reducing reporting costs. When barring one party from the persuasion game reduces expected error costs, it also tends to reduce expected reporting costs.

Appendix

Proof of Proposition 1 Let the parties's strategies be as described in (4) and (5) for some y^*. The probability of the no-disclosure event is then

$$\Pr(N(y^*)) = \int_{\max(0, y^*-c_A)}^{\min(y^*+c_B, 1)} f(x)\,\mathrm{d}x.$$

Applying Bayes' rule, the adjudicator's expected payoff from decision y conditional on no disclosure is, therefore,

[18] In the corner-B equilibrium reporting costs play no role. Therefore, the optimal default decision under commitment equals the equilibrium default decision without commitment.

[19] See, e.g., Emons and Fluet (2009, 2019) for such a setup.

$$\mathbb{E}\left(u_J(y,x) \mid N(y^*)\right) = \int_{\max(0,y^*-c_A)}^{\min(y^*+c_B,1)} u_J(y,x) f(x)\,\mathrm{d}x / \Pr(N(y^*)).$$

Maximizing this expression with respect to y is, therefore, equivalent to maximizing the numerator. The necessary and sufficient condition is

$$\int_{\max(0,y^*-c_A)}^{\min(y^*+c_B,1)} u'_J(y,x) f(x)\,\mathrm{d}x = 0$$

where $u'_J(y,x) := \partial u_J(y,x)/\partial y$. Sufficiency follows from $u''_J(y,x) < 0$.

Define

$$Z(y) = \int_{\max(0,y-c_A)}^{\min(y+c_B,1)} u'_J(y,x) f(x)\,\mathrm{d}x$$

so that y is an equilibrium if $Z(y) = 0$. The function $Z(y)$ is easily seen to be continuous. Recall that $u'_J(y,x) > 0$ if $y < x$ and $u'_J(y,x) < 0$ if $y > x$. Therefore

$$Z(0) = \int_0^{c_B} u'_J(0,x) f(x)\,\mathrm{d}x > 0.$$

Similarly,

$$Z(1) = \int_{1-c_A}^{1} u'_J(1,x) f(x)\,\mathrm{d}x < 0.$$

Hence, there exists $y \in (0,1)$ such that $Z(y) = 0$. □

Proof of Corollary 1 Let y^* be the equilibrium decision under no disclosure. A reports with some probability if $y^* > c_A$ and B reports with some probability if $y^* < 1 - c_B$. At least one of these conditions holds. Suppose not. Then, $y^* - c_A \leq 0$ and $y^* + c_B \geq 1$ yielding $c_A + c_B \geq 1$, which contradicts our assumption on reporting costs. □

Proof of Proposition 2 Consider the one-sender continuation game where only A is allowed to report. A's strategy is as described in (10) for some y^*. Using the same argument as in the proof of Proposition 1, the adjudicator's best response when she gets no report is y satisfying

$$\int_{y^*-c_A}^{1} u'_J(y,x) f(x)\,\mathrm{d}x = 0.$$

Define

$$Z(y) = \int_{\max(0,y-c_A)}^{1} u'_J(y,x) f(x)\,\mathrm{d}x, \tag{17}$$

so that y is an equilibrium if $Z(y) = 0$. Then

$$Z(0) = \int_0^{1} u'_J(0,x) f(x)\,\mathrm{d}x > 0,$$

and

$$Z(1) = \int_{1-c_A}^{1} u'_J(1, x) f(x)\,\mathrm{d}x < 0.$$

Hence, there exists $y \in (0, 1)$ solving $Z(y) = 0$. □

Proof of Proposition 3 The solution $\hat{y}$ to the adjudicator's problem under commitment is a one-sender outcome if $\hat{y} \notin (c_A, 1 - c_B)$. Consider the case where $\hat{y} \leq c_A$. Then $\hat{y}$ maximizes

$$\overline{U}_J(\hat{y}) = \int_0^{\hat{y}+c_B} u_J(\hat{y}, x) f(x)\,\mathrm{d}x.$$

If the solution is $\hat{y} = 0$, this cannot be sequentially optimal. If the solution is interior, it satisfies the first-order condition

$$\overline{U}'_J(\hat{y}) = \int_0^{\hat{y}+c_B} u'_J(\hat{y}, x) f(x)\,\mathrm{d}x + u_J(\hat{y}, \hat{y} + c_B) f(\hat{y} + c_B) = 0.$$

Therefore, in either case, $\hat{y}$ differs from an equilibrium default decision without commitment. It follows that commitment does better. A similar argument holds if $\hat{y} \geq 1 - c_B$.

We now consider sufficient conditions for a one-sender outcome. The adjudicator's expected payoff is

$$\overline{U}_J(\hat{y}) = -V(\hat{y}) := -\int_{\max(0,\hat{y}-c_A)}^{\hat{y}} v(\hat{y} - x) f(x)\,\mathrm{d}x - \int_{\hat{y}}^{\min(\hat{y}+c_B,1)} v(x - \hat{y}) f(x)\,\mathrm{d}x, \tag{18}$$

where $V(\hat{y})$ is the expected error cost. In what follows, we use the fact that the derivative $f'(x)$ exists almost everywhere when the density is unimodal. Differentiating the expected error cost with respect to $\hat{y}$ and integrating by parts yields

$$V'(\hat{y}) = \begin{cases} \int_0^{\hat{y}+c_B} v(|\hat{y} - x|) f'(x)\,\mathrm{d}x \\ \quad +v(\hat{y}) f(0), & \text{if } \hat{y} < c_A; \\ \int_{\hat{y}-c_A}^{\hat{y}+c_B} v(|\hat{y} - x|) f'(x)\,\mathrm{d}x, & \text{if } c_A \leq \hat{y} \leq 1 - c_B; \\ \int_{\hat{y}-c_A}^{1} v(|\hat{y} - x|) f'(x)\,\mathrm{d}x \\ \quad -v(1 - \hat{y}) f(1), & \text{if } \hat{y} > 1 - c_B. \end{cases} \tag{19}$$

We show that $\hat{y} \in (c_A, 1 - c_B)$ cannot be optimal under the conditions of the proposition.

(i) Let $f(x)$ be non-increasing, i.e., $f'(x) \leq 0$. Then, from (19), $V'(\hat{y}) \leq 0$ for all $\hat{y} > c_A$ with strict inequality for $\hat{y} > 1 - c_B$. The strict inequality follows because either the density is strictly decreasing at some $x > 1 - c_B$ or $f(1) > 0$. Hence $V(1) < V(\hat{y})$ for $\hat{y} > c_A$. Therefore, the optimal default decision is either $\hat{y} = 1$ or some $\hat{y} \leq c_A$. A symmetrical argument applies when $f'(x) \geq 0$.

(ii) The case where $f(x)$ is constant is a subcase of (i). Abstracting from this case, either $f(x)$ has a strict mode at $\frac{1}{2}$ or an interior modal interval centered on $\frac{1}{2}$. Consider the strict mode. Then, $f'(x) \geq 0$ for $x < \frac{1}{2}$ and $f'(x) \leq 0$ for $x > \frac{1}{2}$

with strict inequalities around $\frac{1}{2}$. It is easily seen that the only possible candidates for a solution are $\hat{y} \in \{0, 1\}$ or some interior $\hat{y}$ such that $\frac{1}{2} \in [\hat{y} - c_A, \hat{y} + c_B]$. The latter possibility, together with the condition $c_A + c_B \leq \frac{1}{2}$, implies $\hat{y} - c_A \geq 0$ and $\hat{y} + c_B \leq 1$. We show that the solution cannot be interior. Suppose the contrary. Then, the expected error cost is,

$$V(\hat{y}) = \int_{\hat{y}-c_A}^{\hat{y}} v(\hat{y} - x) f(x)\, \mathrm{d}x + \int_{\hat{y}}^{\hat{y}+c_B} v(\hat{y} - x) f(x)\, \mathrm{d}x. \tag{20}$$

Without loss of generality, let $c := c_A \leq c_B$. Then

$$V(\hat{y}) \geq \int_{\hat{y}-c}^{\hat{y}} v(\hat{y} - x) f(x)\, \mathrm{d}x + \int_{\hat{y}}^{\hat{y}+c} v(x - \hat{y}) f(x)\, \mathrm{d}x. \tag{21}$$

Consider now the optimal policy when both parties have reporting cost c. Denote the expected error cost by $W(\overline{y})$ where $\overline{y}$ is the default decision. Candidates for a solution to this problem are $\overline{y} \in \{0, 1\}$ or $\overline{y} = \frac{1}{2}$. In the first case, by symmetry, both $\overline{y} = 0$ or $\overline{y} = 1$ yield the same error cost,

$$\begin{aligned} W(1) &= \int_{1-c}^{1} v(1 - x) f(x)\, \mathrm{d}x \\ &= \int_{0}^{c} v(z) f(1 - z)\, \mathrm{d}z. \end{aligned} \tag{22}$$

In the second case,

$$\begin{aligned} W\left(\tfrac{1}{2}\right) &= \int_{\frac{1}{2}-c}^{\frac{1}{2}} v\left(\tfrac{1}{2} - x\right) f(x)\, \mathrm{d}x + \int_{\frac{1}{2}}^{\frac{1}{2}+c} v\left(x - \tfrac{1}{2}\right) f(x)\, \mathrm{d}x \\ &= 2\int_{\frac{1}{2}}^{\frac{1}{2}+c} v\left(x - \tfrac{1}{2}\right) f(x)\, \mathrm{d}x \\ &= 2\int_{0}^{c} v(z) f\left(\tfrac{1}{2} + z\right)\, \mathrm{d}z, \end{aligned}$$

where the second equality follows from the symmetry of f. Therefore,

$$W\left(\tfrac{1}{2}\right) - W(1) = \int_{0}^{c} v(z) \left[2f\left(\tfrac{1}{2} + z\right) - f(1 - z)\right] \mathrm{d}z.$$

Now, $c_A + c_B \leq \frac{1}{2}$ implies $c \leq \frac{1}{4}$. Because $f'(x) \leq 0$ for all $x > \frac{1}{2}$, it follows that $f(\frac{1}{2} + z) \geq f(1 - z)$ for all $z \in [0, c]$. Therefore, $W(\frac{1}{2}) > W(1)$, so $\overline{y} = 1$ is a solution when both parties have the cost c. In the original problem where costs are c_A and c_B, respectively, with $c_A \leq c_B$, it then follows from (21) that $V(\hat{y}) > W(1)$ for any interior $\hat{y}$. The optimal default decision in the original problem is, therefore, $\hat{y} = 1$, yielding $V(1) = W(1)$. A similar argument applies in the case of an interior modal interval.

(iii) When the density has an interior modal interval, $f'(x) > 0$ for x in a right-neighborhood of 0 and $f'(x) < 0$ in a left-neighborhood of 1. We refer to this as Fact 1. If $c_A + c_B \leq m_h - m_l$, then $m_h - c_B > 0$. Because $f'(x) \geq 0$ for all $x \leq m_h$, it follows from (19) that $V'(\hat{y}) \geq 0$ for all $\hat{y} \leq m_h - c_B$. Moreover, using Fact 1, $V'(0) > 0$. Therefore, $V(\hat{y}) > V(0)$ for $\hat{y} \in (0, m_h - c_B]$. A similar argument shows that $V(\hat{y}) > V(1)$ for $\hat{y} \in [m_l + c_A, 1)$. It follows that $V(\hat{y}) > \min[V(0), V(1)]$ for $\hat{y} \in (0, 1)$ because $m_l + c_A \leq m_h - c_B$ implies $(0, m_h - c_B] \cup [m_l + c_A, 1) = (0, 1)$.

(iv) Suppose $c_A + c_B > m_h - m_l$, otherwise condition (iii) applies. It suffices to show that $V(\hat{y}) \geq \min[V(c_A), V(1 - c_B)]$ for $\hat{y} \in (c_A, 1 - c_B)$. In the present case,

$$\max(c_A, m_h - c_B) < \min(1 - c_B, m_l + c_A). \tag{23}$$

The inequality follows from $c_A + c_B > m_h - m_l$ and the assumption $c_A + c_B < 1$, given that we have an interior modal interval. If $c_A < m_h - c_B$, by an argument similar to the one used in (iii), it is easily seen that $V(\hat{y}) \geq V(c_A)$ for $\hat{y} \in (c_A, m_h - c_B]$. Similarly, if $1 - c_B > m_l + c_A$, then $V(\hat{y}) \geq V(1 - c_B)$ for $\hat{y} \in [m_l + c_A, 1 - c_B)$. Thus, it suffices to show that,

$$V(\hat{y}) \geq \min[V(\max(c_A, m_h - c_B)), V(\min(1 - c_B, m_l + c_A))], \tag{24}$$

for $\hat{y} \in D := (\max(c_A, m_h - c_B), \min(1 - c_B, m_l + c_A))$. Observe that the result holds if $V(\hat{y})$ is concave and, therefore, quasiconcave on D. We conclude the proof by showing that the latter is implied by the condition in the Proposition. For $\hat{y} \in D$,

$$\begin{aligned} V(\hat{y}) &= \int_{\hat{y}-c_A}^{\hat{y}} v(\hat{y} - x) f(x)\, \mathrm{d}x + \int_{\hat{y}}^{\hat{y}+c_B} v(x - \hat{y}) f(x)\, \mathrm{d}x \\ &= \int_0^{c_A} v(z) f(\hat{y} - z)\, \mathrm{d}z + \int_0^{c_B} v(z) f(\hat{y} + z)\, \mathrm{d}x. \end{aligned}$$

$V(\hat{y})$ is concave over D if, for $\hat{y} \in D$, $f(\hat{y} - z)$ is concave in $\hat{y}$ for all $z \leq c_A$ and $f(\hat{y} + z)$ is concave in $\hat{y}$ for all $z \leq c_B$. But this amounts to condition (iv) in the proposition. □

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