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Supersymmetric Liouville theory in AdS_2 and AdS/CFT

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Abstract: In a series of recent papers, a special kind of AdS_2/CFT_1 duality was observed: the boundary correlators of elementary fields that appear in the Lagrangian of a 2d conformal theory in rigid AdS_2 background are the same as the correlators of the corresponding primary operators in the chiral half of that 2d CFT in flat space restricted to the real line. The examples considered were: (i) the Liouville theory where the operator dual to the Liouville scalar in AdS_2 is the stress tensor; (ii) the abelian Toda theory where the operators dual to the Toda scalars are the $\mathcal{W}$-algebra generators; (iii) the non-abelian Toda theory where the Liouville field is dual to the stress tensor while the extra gauged WZW theory scalars are dual to non-abelian parafermionic operators. By direct Witten diagram computations in AdS_2 one can check that the structure of the boundary correlators is indeed consistent with the Virasoro (or higher) symmetry. Here we consider a supersymmetric generalization: the $\mathcal{N} = 1$ superconformal Liouville theory in AdS_2. We start with the super Liouville theory coupled to 2d supergravity and show that a consistent restriction to rigid AdS_2 background requires a non-zero value of the supergravity auxiliary field and thus a modification of the Liouville potential from its familiar flat-space form. We show that the Liouville scalar and its fermionic partner are dual to the chiral half of the stress tensor and the supercurrent of the super Liouville theory on the plane. We perform tests supporting the duality by explicitly computing AdS_2 Witten diagrams with bosonic and fermionic loops.

Keywords: AdS-CFT Correspondence, Supersymmetry and Duality

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Contents

1 Introduction and summary

Recent investigations of correlators of operators on 1/2 BPS Wilson loop at strong coupling using $AdS_5 \times S^5$ superstring action (see [1, 2] and refs. there) motivate the study of boundary correlators in conformal quantum field theories in AdS_2 space. While for a general QFT in AdS_2 one expects the boundary correlators to be covariant under the isometry of AdS_2 or the 1d conformal group $SO(2,1)$, in the case of a conformal (Weyl-covariant) theory one may expect this symmetry to be enhanced to the infinite-dimensional Virasoro symmetry (or reparametrizations of 1d boundary) putting strong constraints on the structure of the boundary correlators.

This was studied recently on the examples of the Liouville and Toda theories [3–6]. One remarkable feature is that while the S-matrix of elementary fields of 2d CFT in flat space is not well defined, its counterpart in AdS_2 (i.e. the set of boundary correlators) is. In general, a scalar with $m^2 = \Delta(\Delta-1)$ in AdS_2 with metric $ds^2 = \frac{1}{\mathsf{z}^2}(d\mathsf{t}^2 + d\mathsf{z}^2)$ having boundary asymptotics $\varphi(\mathsf{t},\mathsf{z})\big|_{\mathsf{z}\to 0} = \mathsf{z}^{\Delta}\,\Phi(\mathsf{t}) + \cdots$ should be dual to a 1d conformal field V_Δ with dimension Δ. In the case of the Liouville scalar in AdS_2 expanded near the constant vacuum (cf. [7–9]) one finds $m^2 = 2$ and thus $\Delta = 2$ which is the same as the dimension of the stress tensor $T = V_2$ in the 2d CFT in flat space. Then the expected Virasoro symmetry of the boundary scalar correlators implies that they should be proportional to the correlators of the holomorphic stress tensor of the original Liouville theory on the complex w-plane formally restricted to the real line (boundary of half-plane).[1] This correspondence (dubbed "$AdS_2/CFT_2^{1/2}$ duality") extends also to the case of higher spin $\mathcal{W}$-symmetry generators in Toda theories. In general, in the case of a theory with several fields

$$
\begin{aligned}
\langle \Phi_1(\mathsf{t}_1)\cdots\Phi_N(\mathsf{t}_N)\rangle &\equiv \lim_{\mathsf{z}_i\to 0}\prod_{i=1}^{N}\mathsf{z}_i^{-\Delta_i}\,\langle\varphi_1(\mathsf{t}_1,\mathsf{z}_1)\cdots\varphi_N(\mathsf{t}_N,\mathsf{z}_N)\rangle_{AdS_2} \\
&= \prod_{i=1}^{N}\kappa_{\Delta_i}\langle V_{\Delta_1}(w_1)\cdots V_{\Delta_N}(w_N)\rangle\Big|_{w_i\to\mathsf{t}_i}\,. \qquad (1.1)
\end{aligned}
$$

The proportionality coefficients κ_{Δ_i} are functions of the central charge c (or couplings) of the underlying CFT and are a non-trivial part of this correspondence (see [4–6] for details).

The aim of the present paper is to extend the analysis of the bosonic Liouville theory in [4] to the case of $\mathcal{N}=1$ super Liouville theory containing 2d fermions. One motivation is to learn how to systematically compute fermionic loops in AdS_2 in a way consistent with all relevant symmetries which should be important also in the $AdS_5 \times S^5$ superstring context [1, 2]. Below we shall summarize our main results.

[1] In that sense the boundary AdS_2 correlators provide another realization of the same Virasoro symmetry (with the same central charge) as the flat-space stress tensor correlators.

1.1 Bosonic Liouville theory

Let us first recall the main duality relations in the bosonic Liouville theory. The quantum Weyl-covariant Liouville action in a general curved 2d space is [10, 11]

$$\mathcal{S} = \frac{1}{4\pi}\int d^2x\,\sqrt{g}\,\left[\partial^\lambda\phi\partial_\lambda\phi+\mu^2\,e^{2\,b\,\phi}+Q\,R\,\phi\right], \qquad\qquad Q=\frac{1}{b}+b\,, \tag{1.2}$$

with the central charge being

$$c = 1+6\,Q^2\ . \tag{1.3}$$

Let us specify the background to be the Euclidean AdS_2 space with coordinates $x=(\mathsf{t},\mathsf{z})$ and the Poincaré metric $ds^2=\frac{1}{\mathsf{z}^2}(d\mathsf{t}^2+d\mathsf{z}^2)$ (with curvature $R=-2$). In this case a non-trivial constant vacuum value of the the scalar field is $\langle\phi\rangle=\phi_0=\frac{1}{2b}\log\frac{Q}{b\mu^2}$. Expanding near it, $\phi=\phi_0+\zeta$, one finds that the fluctuation ζ has classical mass given by $m^2=2$. Assuming Dirichlet boundary conditions for ζ, we have

$$\zeta(\mathsf{t},\mathsf{z})\big|_{\mathsf{z}\to 0}=\mathsf{z}^2\Phi(\mathsf{t})+\mathcal{O}(\mathsf{z}^3)\ . \tag{1.4}$$

Identifying the corresponding $\Delta=2$ boundary conformal field with the real-line restriction of the 2d holomorphic stress tensor $T(w)$ on the complex w-plane one can then verify that the key $\mathrm{AdS}_2/\mathrm{CFT}_2^{1/2}$ relation (1.1) is satisfied at the quantum level (i.e. for any coupling b)

$$\langle\prod_{i=1}^{N}\Phi(\mathsf{t}_i)\cdots\Phi_N(\mathsf{t}_N)\rangle=\kappa^N\ \langle\prod_{i=1}^{N}T(w_i)\rangle\Big|_{w_i\to\mathsf{t}_i}\ . \tag{1.5}$$

The all-order expression for $\kappa=\kappa(b)$ was determined in [4]

$$\kappa=-4\frac{Q}{c}=-\frac{4\,b(1+b^2)}{(3+2\,b^2)(2+3\,b^2)}=-\frac{2}{3}\,b+\frac{7}{9}\,b^3+\cdots\ . \tag{1.6}$$

The structure of the correlators of $T(w)$ is completely determined by the Virasoro algebra and then the same expressions should be found for the boundary correlators; this can be checked directly using small b (or large c) perturbation theory. For example, the well-known expression for the 4-point function of the stress-tensor ($w_{ij}=w_i-w_j$)

$$\begin{aligned}\langle T(w_1)\cdots T(w_4)\rangle=&\frac{c^2}{4}\left(\frac{1}{w_{12}^4\,w_{34}^4}+\frac{1}{w_{13}^4\,w_{24}^4}+\frac{1}{w_{14}^4\,w_{23}^4}\right)\\&+c\left(\frac{1}{w_{12}^2\,w_{23}^2\,w_{34}^2\,w_{14}^2}+\frac{1}{w_{13}^2\,w_{24}^2\,w_{14}^2\,w_{23}^2}+\frac{1}{w_{12}^2\,w_{24}^2\,w_{34}^2\,w_{13}^2}\right),\end{aligned} \tag{1.7}$$

corresponds to the decomposition of $\langle\Phi(\mathsf{t}_1)\cdots\Phi(\mathsf{t}_4)\rangle$ into the disconnected and connected parts

$$\langle\Phi(\mathsf{t}_1)\ \cdots\ \Phi(\mathsf{t}_4)\rangle=\langle\Phi(\mathsf{t}_1)\ \cdots\ \Phi(\mathsf{t}_4)\rangle_{\rm disc}+\langle\Phi(\mathsf{t}_1)\ \cdots\ \Phi(\mathsf{t}_4)\rangle_{\rm conn}, \tag{1.8}$$

$$\begin{aligned}\langle\Phi(\mathsf{t}_1)\cdots\Phi(\mathsf{t}_4)\rangle_{\rm disc}&=(C_{\Phi\Phi})^2\left(\frac{1}{\mathsf{t}_{12}^2\,\mathsf{t}_{34}^2}+\frac{1}{\mathsf{t}_{13}^2\,\mathsf{t}_{24}^2}+\frac{1}{\mathsf{t}_{14}^2\,\mathsf{t}_{23}^2}\right),\\ \langle\Phi(\mathsf{t}_1)\cdots\Phi(\mathsf{t}_4)\rangle_{\rm conn}&=C_{\Phi\Phi\Phi\Phi}\left(\frac{1}{\mathsf{t}_{12}^2\,\mathsf{t}_{23}^2\,\mathsf{t}_{34}^2\,\mathsf{t}_{14}^2}+\frac{1}{\mathsf{t}_{13}^2\,\mathsf{t}_{24}^2\,\mathsf{t}_{14}^2\,\mathsf{t}_{23}^2}+\frac{1}{\mathsf{t}_{12}^2\,\mathsf{t}_{24}^2\,\mathsf{t}_{34}^2\,\mathsf{t}_{13}^2}\right),\end{aligned} \tag{1.9}$$

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where

$$C_{\Phi\Phi} = \kappa^2 \frac{c}{2} = \frac{8\,(1+b^2)^2}{(3+2b^2)(2+4b^2)} = \frac{4}{3} - \frac{2}{9}\,b^2 + \frac{13}{27}\,b^4 + \cdots\,, \tag{1.10}$$

$$C_{\Phi\Phi\Phi\Phi} = \kappa^4\,c = \frac{256\,b^2\,(1+b^2)^4}{(3+2b^2)^3(2+3b^2)^3} = \frac{32}{27}\,b^2 - \frac{80}{27}\,b^4 + \cdots\,. \tag{1.11}$$

These relations were checked explicitly in [4] at the one-loop level. Similar relations are found for the 2-, 3-point and higher point boundary correlators.

1.2 $\mathcal{N}=1$ super Liouville theory

The action of $\mathcal{N}=1$ supersymmetric Liouville theory on a curved background can be found by constructing a locally supersymmetric generalization of (1.2), i.e. by coupling the Liouville matter multiplet to $\mathcal{N}=1$ 2d supergravity [12, 13]. The condition of Weyl invariance on a general curved background fixes the value of the analog of Q in (1.2) and thus the central charge to be

$$Q = \frac{1}{b} + \frac{b}{2}\,, \qquad\qquad c = \frac{3}{2} + 6Q^2\,. \tag{1.12}$$

Restricting to the AdS_2 background in a way preserving global AdS supersymmetry requires setting gravitino to zero and fixing the supergravity auxiliary field A to a non-zero constant value in terms of the AdS_2 curvature ($A = 2 = -R$). This leads to the following action in AdS_2 background (see section 3)

$$S = \frac{1}{4\pi}\int d^2x\sqrt{g}\,\Big(\partial^\lambda\phi\partial_\lambda\phi + \bar{\Psi}\not{D}\Psi + \mu^2 b^2 e^{2b\phi} - 2Q\phi - \mu(bQ-1)e^{b\phi} - \mu b^2 e^{b\phi}\bar{\Psi}\Psi\Big)\,, \tag{1.13}$$

where $\Psi = \begin{pmatrix}\psi \\ -i\bar{\psi}\end{pmatrix}$ is the 2-component Majorana fermion. Here the $-2Q\phi$ term is the same as $QR\phi$ in (1.2) restricted to AdS_2 while the (possibly unfamiliar) extra $\mu(bQ-1)e^{b\phi}$ potential term originates from the non-zero value of the supergravity auxiliary field and is thus necessary for the global AdS supersymmetry of the action (1.13). The final action (1.13) is a special case of the general (rigidly) supersymmetric Wess-Zumino actions on AdS_2 derived in [14, 15]. The special choice of the superpotential is dictated by the supergravity construction and assures superconformal invariance.

As in the bosonic case (1.2), there is again a constant minimum of the potential $\langle\phi\rangle = \phi_0$, $\langle\Psi\rangle = 0$, and expanding near it gives a massive bosonic fluctuation with $m_\zeta^2 = 2$ and a massive fermion with $m_\psi = 1$. The asymptotic behaviour of the bosonic fluctuation field ζ is again as in (1.4), while for the (upper component of) the fermion field we have

$$\psi(\mathsf{t},\mathsf{z})\big|_{\mathsf{z}\to 0} = \mathsf{z}^{3/2}\,\Psi(\mathsf{t}) + \mathcal{O}(\mathsf{z}^2)\,. \tag{1.14}$$

The associated conformal fields (with $\Delta_\zeta = 2$ and $\Delta_\psi = \frac{3}{2}$) appearing in (1.1) are to be identified with the stress tensor $T(w)$ and its partner the supercurrent $G(w)$ realizing the $\mathcal{N}=1$ superconformal algebra with the central charge c in (1.12).

The boundary correlators of ζ and ψ are defined as

$$\langle\prod_{i=1}^{N}\Phi(\mathsf{t}_i)\prod_{j=1}^{M}\Psi(\mathsf{t}'_j)\rangle \equiv \lim_{\mathsf{z}_i,\mathsf{z}'_j\to 0}\prod_{i=1}^{N}\mathsf{z}_i^{-2}\prod_{j=1}^{M}\mathsf{z}'^{-3/2}_j\,\langle\zeta(\mathsf{t}_i,\mathsf{z}_i)\,\psi(\mathsf{t}'_j,\mathsf{z}'_j)\rangle_{\mathrm{AdS}_2}. \tag{1.15}$$

The goal of this paper is to show that these correlators are expressed in terms of the correlators of the generators T, G of the $\mathcal{N}=1$ super-Virasoro algebra as in (1.1)

$$\langle\prod_{i=1}^{N}\Phi(\mathsf{t}_i)\prod_{j=1}^{M}\Psi(\mathsf{t}'_j)\rangle = (\kappa_\zeta)^N\,(\kappa_\psi)^M\,\langle\prod_{i=1}^{N}\langle T(w_i)\prod_{j=1}^{M}G(w'_j)\rangle\Big|_{w_i\to\mathsf{t}_i,w'_j\to\mathsf{t}'_j}. \tag{1.16}$$

As we will find below, the expression for $\kappa_\zeta(b)$ has the same form in terms of Q and c in (1.12) as in (1.6) while $\kappa_\psi(b)$ is proportional to it as a consequence of supersymmetry

$$\kappa_\zeta = -4\frac{Q}{c}, \qquad\qquad \kappa_\psi = \frac{3}{2\sqrt{2}}\,\kappa_\zeta = -3\sqrt{2}\,\frac{Q}{c}\ . \tag{1.17}$$

Below we shall demonstrate the validity of (1.16), (1.17) for various correlators at the tree and one-loop level.

The content of the rest of this paper is as follows. In section 2 we briefly review the superconformal Liouville theory in flat 2d space and the underlying $\mathcal{N}=1$ superconformal algebra.

In section 3 we discuss how to formulate the super Liouville theory on AdS_2 space, preserving both conformal invariance and global supersymmetry. We start in section 3.1 with a locally supersymmetric theory of a scalar multiplet coupled to the $\mathcal{N}=1$ 2d supergravity including both general superpotential and curvature coupling. We show how to restrict this theory to a rigid AdS_2 background getting a particular supersymmetric Wess-Zumino theory. In section 3.2 we find the special conformal superpotential that solves the condition of (super) Weyl invariance in a general 2d supergravity background.

In section 4 we present the main features of the $\mathrm{AdS}_2/\mathrm{CFT}_2^{1/2}$ correspondence for the super Liouville theory. In section 4.1 we show how the global AdS_2 supersymmetry is holographically projected to the $\mathfrak{osp}(1|2)$ superconformal symmetry on the boundary of AdS_2. In section 4.2 we determine the exact expressions for the two proportionality coefficients $\kappa_\zeta(b)$ and $\kappa_\psi(b)$ appearing in the key relation (1.16). Finally, in section 4.3 we summarize the precise predictions that follow from (1.16) for the 2-, 3- and 4-point boundary correlation functions of the super Liouville fields.

In section 5 we set up perturbation theory in AdS_2 and discuss in particular the explicit form of the fermionic propagator. Then in section 6 we compute the 2-point functions for the bosonic and fermionic fields in the one-loop approximation. This computation provides a non-trivial check of the duality predictions (4.40), (4.41).

Section 7 is devoted to a similar calculation for the 3-point scalar correlation function. Again, we confirm the predictions (4.42), (4.43) at the one-loop level. In section 8 we consider the non-vanishing 4-point boundary correlators. Starting from 4-point level the (super)conformal invariance alone does not a priori fix the structure of the correlators so their direct computation provides a check of the Virasoro symmetry. We check agreement with the duality (1.16) explicitly at the tree level, i.e. the leading order in the weak-coupling expansion.

There are several technical appendices. We recall our conventions in appendix A. In appendix B we summarize information about different forms of the fermionic propagator in AdS_{d+1} and specifically in AdS_2. In appendix C we present a lengthy calculation of the mixed 4-point function of two scalars and two fermions discussed in section 8.

2 Superconformal Liouville theory in flat space

Let us first briefly review the super Liouville theory in flat 2d space [11, 12, 16]. Its action is given by[2]

$$\mathcal{S} = \frac{1}{4\pi}\int d^2x \left(\partial^\lambda\phi\partial_\lambda\phi + \bar\Psi\partial\!\!\!/\Psi + \mu^2 b^2 e^{2b\phi} - \mu b^2 e^{b\phi}\bar\Psi\Psi\right). \tag{2.1}$$

This is a special case of the 2d Wess-Zumino theory for $\mathcal{N}=1$ scalar multiplet (ϕ,Ψ,F)

$$\mathcal{S} = \frac{1}{4\pi}\int d^2x \left[\partial^\lambda\phi\partial_\lambda\phi + \bar\Psi\partial\!\!\!/\Psi - F^2 + 2FW'(\phi) - W''(\phi)\bar\Psi\Psi\right], \tag{2.2}$$

with the exponential superpotential W

$$W(\phi) = \frac{\mu}{b}\, e^{b\phi}\,. \tag{2.3}$$

The action (2.2) is thus invariant under the standard global $\mathcal{N}=1$ supersymmetry transformation

$$\delta\phi = \overline{\mathcal{E}}\,\Psi, \qquad \delta\Psi = \partial\!\!\!/\phi\,\mathcal{E} + F\,\mathcal{E}, \qquad \delta F = \overline{\mathcal{E}}\,\partial\!\!\!/\Psi\,, \tag{2.4}$$

where $\mathcal{E}$ is a constant Majorana spinor. Eliminating F in (2.2) gives (2.1).

The holomorphic components of the conserved conformal stress tensor and the supercurrent admit the following free-field representation[3]

$$T = -(\partial\phi)^2 + Q\partial^2\phi - \frac{1}{2}\psi\partial\psi, \qquad G = i\sqrt{2}\,(\psi\partial\phi - Q\,\partial\psi)\,. \tag{2.5}$$

Q and c are related to the coupling b as in (1.12). The singular part of the OPE of T and G corresponds to the $\mathcal{N}=1$ superconformal algebra

$$T(w')T(w) \sim \frac{c}{2}\frac{1}{(w'-w)^4} + \frac{2\,T(w)}{(w'-w)^2} + \frac{\partial T(w)}{w'-w}, \tag{2.6}$$

$$T(w')G(w) \sim \frac{3}{2}\frac{G(w)}{(w-w')^2} + \frac{\partial G(w)}{w'-w}, \tag{2.7}$$

$$G(w')G(w) \sim \frac{2c}{3}\frac{1}{(w'-w)^3} + \frac{2T(w)}{w'-w}. \tag{2.8}$$

[2] Our conventions are discussed in appendix A. Note that compared, e.g., to [11] we rescale ϕ and Q by a factor of $\sqrt{2}$ for convenience.

[3] The free fields may be related to the fields in (2.1) by a Backlund transformation, cf. [17]. The improvement Q-terms may be either postulated to satisfy the superconformal algebra or derived from coupling the scalar multiplet to the supergravity background.

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Note that (2.7) can be rewritten as

$$G(w')T(w) \sim \frac{3}{2}\frac{G(w)}{(w'-w)^2} + \frac{1}{2}\frac{\partial G(w)}{w'-w}. \tag{2.9}$$

The exponential NS primary field and its superpartner in super Liouville theory are (α is a constant parameter)[4]

$$V_\alpha = e^{2\alpha\phi}, \qquad \Lambda_\alpha = -i\sqrt{2}\alpha\, e^{2\alpha\phi}\,\psi\,, \tag{2.10}$$

and their OPEs with T and G are

$$T(w')V_\alpha(w) \sim \frac{h_\alpha V_\alpha(w)}{(w'-w)^2} + \frac{\partial V_\alpha(w)}{w'-w}\,, \quad T(w')\Lambda_\alpha(w) \sim \frac{(h_\alpha+\frac{1}{2})\Lambda_\alpha(w)}{(w'-w)^2} + \frac{\partial\Lambda_\alpha(w)}{w'-w}\,, \tag{2.11}$$

$$G(w')V_\alpha(w) \sim \frac{\Lambda_\alpha(w)}{w'-w}\,, \qquad G(w')\Lambda_\alpha(w) \sim \frac{2h_\alpha V_\alpha}{(w'-w)^2} + \frac{\partial V_\alpha(w)}{w'-w}\,, \tag{2.12}$$

where the chiral dimension is $h_\alpha = \alpha(Q-\alpha)$.

3 $\mathcal{N}=1$ super Liouville theory in AdS_2 background

While putting bosonic Liouville theory in AdS_2 background is straightforward by just specifying the metric, to do the same for the super Liouville theory in a way consistent with global $\mathcal{N}=1$ supersymmetry in AdS space is more non-trivial.

One is to start with the general locally supersymmetric coupling of the WZ model for the scalar multiplet (2.2) to the $\mathcal{N}=1$ 2d supergravity multiplet (e^a_μ, χ_μ, A) [19–21], set the gravitino χ_μ to zero and the metric to be the AdS_2 one, and finally determine the value of the supergravity auxiliary field A from the integrability condition for existence of Killing spinors (i.e. from the vanishing of the gravitino supersymmetry variation).[5]

The resulting action for a WZ theory in AdS_2 was found in [14, 15, 25, 26] (and used also in [27, 28]). However, these papers did not include the possibility of the curvature coupling (or "dilaton term" in string context) which is crucial for the conformal invariance of the Liouville theory. The manifestly supersymmetric form of the $R\phi$ coupling in (1.2) on a general supergravity background was given in [13] (see also [29, 30]) but the restriction of the resulting theory to the AdS_2 background was not explicitly constructed in the past.[6]

Below we will close this gap, explaining the derivation of the action (1.13) of the $\mathcal{N}=1$ super Liouville action in AdS_2. We shall start with the manifestly locally supersymmetric form of the 2d scalar multiplet theory with generic superpotential W ("tachyon") and curvature coupling function U ("dilaton") on a general supergravity background and consider its supersymmetric reduction to AdS_2 background. We shall also determine the conditions on the functions W and U required for the (super) Weyl covariance of such model on a general background implying its superconformal invariance and then consider the resulting super Liouville theory in AdS_2.

[4]Here we consider a complex plane with independent holomorphic and antiholomorphic sectors. For a discussion of the super Liouville theory on a plane with boundary see [18].

[5]A similar approach is used in higher dimensions when putting a supersymmetric field theory on a specific curved background in way preserving some global supersymmetres (see [22–24] and refs. there).

[6]However, some related discussion appeared in [17].

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3.1 Scalar multiplet theory in 2d supergravity background and restriction to AdS_2

The general action for the scalar superfield

$$\Phi = \phi + i\theta\psi + i\bar{\theta}\bar{\psi} + i\theta\bar{\theta}F\,, \tag{3.1}$$

coupled to 2d $\mathcal{N}=1$ supergravity may be written as [21, 25, 29]

$$\mathcal{S} = -\frac{i}{4\pi}\int d^2Z\,E\left[i\mathcal{D}^\alpha\Phi\mathcal{D}_\alpha\Phi + 2W(\Phi) + \mathcal{R}\,U(\Phi)\right]\,, \tag{3.2}$$

where $d^2Z \equiv d^2x d\theta d\bar{\theta}$ and $E = \mathrm{sdet}\,E_M^A$. E_M^A is the supervielbein (containing the supergravity multiplet (e^a_μ, χ_μ, A)) and $\mathcal{R}$ is the supercurvature[7]

$$\mathcal{R} = A + i\theta\bar{\theta}\left(R + \frac{1}{2}A^2\right) + \dots\,, \qquad\qquad E = e\left(1 - \frac{i}{2}\theta\bar{\theta}A + \dots\right), \tag{3.3}$$

where R is the scalar curvature of the metric and dots stand for the terms depending on the gravitino. The bosonic part of (3.2) thus contains the terms (cf. (1.2), (2.2)) $\mathcal{S} = \frac{1}{4\pi}\int d^2x\big[-F^2 + 2FW'(\phi) + RU(\phi) + \dots\big]$.

To restrict this theory to AdS_2 preserving rigid supersymmetry we first set the gravitino to zero $\chi_\mu = 0$. Then under the supersymmetry variation $\delta e_\mu{}^a = \delta A = 0$ while the condition of the vanishing of the local supersymmetry variation of gravitino becomes

$$\delta\chi_\mu = 2\left(D_\mu - \frac{1}{4}\gamma_\mu A\right)\mathcal{E} = 0\,. \tag{3.4}$$

Specifying the metric to be the AdS_2 one (with unit radius)

$$ds^2 = \frac{d\mathsf{z}^2 + d\mathsf{t}^2}{\mathsf{z}^2}\,, \qquad\qquad R = -2\,, \tag{3.5}$$

to preserve the global $\mathrm{SO}(2,1)$ symmetry we also choose the background value of the auxiliary field A to be constant. Then the integrability condition of (3.4) gives

$$R + \frac{1}{2}A^2 = 0\,, \qquad\qquad A = 2a\,, \qquad\qquad a = \pm 1\,, \tag{3.6}$$

and (3.4) becomes equivalent to the AdS_2 Killing spinor equation

$$D_\mu\mathcal{E} = \frac{1}{2}\,a\,\gamma_\mu\mathcal{E}\,. \tag{3.7}$$

To find the form of the action (3.2) on this AdS_2 background we observe that (cf. (3.3), (3.6))

$$\int d\theta d\bar{\theta} E\Big[2W(\Phi) + \mathcal{R}\,U(\Phi)\Big] = ie\Big[2F\,W'(\phi) - 2a\,W(\phi) - 2U(\phi) + 2aF\,U'(\phi)\Big]\,. \tag{3.8}$$

[7] $\mathcal{R} = R_{+-}$ in the notation of [29]. We assume $\int d\theta d\bar{\theta}\,\theta\bar{\theta} = 1$.

Then the bosonic potential part of (3.2) may be written as

$$V(\phi, F) = -F^2 + 2F\hat{W}' - 2a\,\hat{W}(\phi)\,, \tag{3.9}$$
$$\hat{W} \equiv W(\phi) + a\,U(\phi)\ . \tag{3.10}$$

where $-F^2$ comes from the kinetic part of the action (3.2).

Including also the fermionic terms we finally get the following supersymmetric action on AdS_2

$$\mathcal{S} = \frac{1}{4\pi}\int d^2x\sqrt{g}\left(\partial^\lambda\phi\partial_\lambda\phi + \bar{\Psi}\not{D}\Psi - F^2 + 2F\hat{W}' - 2a\hat{W} - \hat{W}''\bar{\Psi}\Psi\right). \tag{3.11}$$

where $\int d^2x\sqrt{g} = \int \frac{\mathrm{d}t\,\mathrm{d}z}{\mathrm{z}^2}$ (cf. (3.5)). This action is invariant under the following supersymmetry transformations

$$\delta\phi = \overline{\mathcal{E}}\,\Psi, \qquad \delta\Psi = \not{\partial}\phi\,\mathcal{E} + F\,\mathcal{E}, \qquad \delta F = \overline{\mathcal{E}}\,\not{D}\Psi\,, \tag{3.12}$$

where $\mathcal{E}$ is the solution to the Killing spinor equation (3.7). Note that if we set formally the background value a of the auxiliary field A to zero and take the metric flat then (3.11) reduces to the flat-space WZ action (2.2).

Solving for the auxiliary field ($F = \hat{W}'$) we get from (3.11)

$$\mathcal{S} = \frac{1}{4\pi}\int d^2x\sqrt{g}\left(\partial^\lambda\phi\partial_\lambda\phi + \bar{\Psi}\not{D}\Psi + \hat{W}'^2 - 2a\hat{W} - \hat{W}''\bar{\Psi}\Psi\right). \tag{3.13}$$

The action (3.13) has the same form as the general supersymmetric WZ action in AdS_2 found in [14, 15]. The fact that the curvature coupling function $U(\phi)$ in (3.2) appears in (3.12), (3.13) only through the generalized superpotential function $\hat{W}$ is a check of consistency of this coupling with the AdS_2 supersymmetry.

3.2 Super Weyl invariance conditions

The action (3.11) has rigid AdS_2 supersymmetry but does not correspond to a superconformal theory for a generic $W(\phi)$. To determine the required conditions on W and U we need to go back to the action (3.2) defined on a general supergravity background and impose the condition of decoupling of the superconformal factor at the quantum level.

Let us first recall the analogous argument in the bosonic case starting with the single scalar action

$$\mathcal{S} = \frac{1}{4\,\pi}\int d^2x\sqrt{g}\Big[\partial^\lambda\phi\partial_\lambda\phi + R\,U(\phi) + T(\phi)\Big]\ , \tag{3.14}$$

with generic coupling functions U and T. Interpreting them as the "dilaton" and the "tachyon" in 1d target space we may readily write down the corresponding expressions for the Weyl anomaly coefficients (see, e.g., [31–33]; here $\alpha' = 1$)

$$\bar{\beta}^U = -\frac{1}{2}U'' + U'^2\,, \qquad \bar{\beta}^T = -2T - \frac{1}{2}T'' + U'T'\ . \tag{3.15}$$

Here the non-derivative term originates from the classical scaling dimension of the corresponding coupling, the second-derivative term is the quantum anomalous dimension (one-loop Weyl anomaly assuming reparametrization covariant regularization of the propagator) and the first-derivative terms originate (upon use of the equations of motion) from the classically non-invariant dilaton coupling. The solution of $\bar{\beta}^U = \text{const} = \frac{1}{6}(c-1)$ required to decouple the conformal factor is the linear dilaton $U = Q\phi$ and then the solution of $\bar{\beta}^T = 0$ is the exponential Liouville potential in (1.2) with $-2 - 2b^2 + 2Qb = 0$, i.e. (cf. (1.2))

$$T = \mu^2 e^{2b\phi}\,, \qquad\qquad Q = \frac{1}{b} + b\,. \tag{3.16}$$

A similar argument applies in the supersymmetric case of the action (3.2).[8] The equation for U is the same as in (3.15) fixing it again to be $U = Q\phi$. The equation for the "super-tachyon" or the superpotential W differs from the one for T in (3.15) only in first classical dimension term: W term in (3.2) has half dimension of the bosonic tachyon coupling, i.e.

$$\bar{\beta}^W = -W - \frac{1}{2}W'' + U'W'\,. \tag{3.17}$$

The condition of the vanishing of $\bar{\beta}^W$ gives again the exponential function $W = \mu e^{b\phi}$ with b now constrained by $-1 - \frac{1}{2}b^2 + Qb = 0$. Hence, we finish with

$$W = \mu\, e^{b\phi}\,, \qquad\qquad U = Q\,\phi\,, \qquad\qquad Q = \frac{1}{b} + \frac{b}{2}\,, \tag{3.18}$$

thus determining the structure of the super Liouville theory on a general curved background (cf. (1.12)).

3.3 Action for super Liouville theory in AdS$_2$

Let us now combine the discussion of the previous two sections and write down the explicit form of the action of the $\mathcal{N} = 1$ supersymmetric Liouville theory in AdS$_2$.[9]

Using (3.18) the generalized superpotential in (3.10) takes the form

$$\hat{W}(\phi) = \mu\, e^{b\phi} + a\, Q\, \phi\,. \tag{3.19}$$

Then solving for the auxiliary field F in (3.11) gives the following expression for the super Liouville theory on AdS$_2$

$$\mathcal{S} = \frac{1}{4\pi}\int d^2x\sqrt{g}\,\Big[\partial^\lambda\phi\partial_\lambda\phi + \bar{\Psi}\not{D}\Psi - 2Q\phi + 2a\mu(bQ-1)e^{b\phi} + \mu^2 b^2 e^{2b\phi} - \mu b^2 e^{b\phi}\bar{\Psi}\Psi\Big]\,. \tag{3.20}$$

We used that $a^2 = 1$ in (3.6) and dropped a constant Q^2 term. For $a = -1$ this is the action quoted in (1.13).

The condition for the extremum of the scalar potential in (3.13) $V = \hat{W}'^2 - 2a\hat{W}$ is $(\hat{W}'' - a)\hat{W}' = 0$. Since $F = \hat{W}'$, the constant vacuum $\phi = \phi_0$ preserving supersymmetry (cf. (3.12)) is the solution of $\hat{W}' = 0$, i.e.

$$\mu\, b\, e^{b\phi_0} + a\, Q = 0\,. \tag{3.21}$$

[8] One may repeat the bosonic derivation using covariantly regularized superpropagator, cf. [34].

[9] One may wonder why one needed separate conditions on W and U if the action (3.11) depends only on their sum $\hat{W}$ in (3.10). The answer is analogous to that in the flat space case: having fixed a background one is still to specify the stress tensor and its superpartner and their definition depends implicitly on the right choice of $U = Q\phi$ (cf. (2.5)).

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Expanding near this vacuum $\phi = \phi_0 + \zeta$ we get

$$\mathcal{S} = \frac{1}{4\pi}\int d^2x\sqrt{g}\left[\partial^\lambda\zeta\partial_\lambda\zeta + \bar{\Psi}\not{D}\Psi - 2Q\zeta - 2Q(Q-b^{-1})e^{b\zeta} + Q^2e^{2b\zeta} + a\,b\,Q e^{b\zeta}\bar{\Psi}\Psi\right]. \tag{3.22}$$

Thus $a = \pm 1$ determines the sign of the fermion mass term, the two options being equivalent and related by a chiral transformation; in what follows we shall choose

$$a = -1\,. \tag{3.23}$$

So far we did not use the explicit value of Q in (3.16) required for conformal invariance. Expanding (3.22) to quadratic order in the fields we find

$$\mathcal{S}^{(2)} = \frac{1}{4\pi}\int d^2x\sqrt{g}\left[\partial_\mu\zeta\partial^\mu\zeta + 2\left(1+\frac{1}{2}b^2\right)\left(1+\frac{1}{4}b^2\right)\zeta^2 + \bar{\Psi}\not{D}\Psi - \left(1+\frac{1}{2}b^2\right)\bar{\Psi}\Psi\right]. \tag{3.24}$$

Thus the classical ($b \to 0$) values of the bosonic and fermionic masses are $m_\zeta^2 = 2$ and $m_\psi = 1$.

4 Super Liouville theory: $\mathrm{AdS}_2/\mathrm{CFT}_2^{1/2}$ set up

Starting with the AdS_2 action (3.22), (3.23) let us now discuss the AdS/CFT duality. According to the standard $\mathrm{AdS}_2/\mathrm{CFT}_1$ rule, the scalar and fermion masses are related to the dimensions of the dual conformal operators as, respectively, $m^2 = \Delta(\Delta-1)$ and $m = \Delta - \frac{1}{2}$. Thus for $m_\zeta^2 = 2$ and $m_\psi = 1$ in (3.24) we get

$$\Delta_\zeta = 2\,, \qquad\qquad \Delta_\psi = \frac{3}{2}, \tag{4.1}$$

with the corresponding asymptotic expansions near the AdS_2 boundary given by (recall that in (3.22) we have $\Psi = \begin{pmatrix}\psi\\ -i\bar{\psi}\end{pmatrix}$)

$$\zeta(\mathsf{t},\mathsf{z}) = \mathsf{z}^2\,\Phi(\mathsf{t}) + \mathcal{O}(\mathsf{z}^3)\,, \tag{4.2}$$

$$\psi(\mathsf{t},\mathsf{z}) = \mathsf{z}^{3/2}\Psi(\mathsf{t}) + \mathsf{z}^{5/2}\tilde{\Psi}(\mathsf{t}) + \mathcal{O}(\mathsf{z}^{7/2})\,, \quad \bar{\psi}(\mathsf{t},\mathsf{z}) = \mathsf{z}^{3/2}\bar{\Psi}(\mathsf{t}) + \mathsf{z}^{5/2}\tilde{\bar{\Psi}}(\mathsf{t}) + +\mathcal{O}(\mathsf{z}^{7/2})\,. \tag{4.3}$$

To preserve supersymmetry we will later need to impose the boundary condition on Ψ at $\mathsf{z} = 0$:

$$\Psi = -\bar{\Psi}\,. \tag{4.4}$$

As we will see, this boundary condition is also compatible with the standard bulk-to-boundary propagators of the fermion (5.13).

The conformal dimensions (4.1) are the same as the dimensions of the generators T and G of the $\mathcal{N} = 1$ super-Virasoro algebra. As in the Liouville theory [2], we thus expect the $\mathrm{AdS}_2/\mathrm{CFT}_2^{1/2}$ correspondence (in the sense of the matching of the correlation functions in (1.16))[10]

$$\Phi(\mathsf{t}) \;\to\; \kappa_\zeta\,T(\mathsf{t})\,, \qquad\qquad \Psi(\mathsf{t}) \;\to \kappa_\psi\,G(\mathsf{t})\,, \tag{4.5}$$

where the b-dependent coefficients κ_ζ and κ_ψ are to be determined below.

[10] In view of (4.5) the boundary correlators involving $\bar{\psi}$ are determined by those with ψ.

4.1 From bulk supersymmetry to $\mathfrak{osp}(1|2)$ superconformal symmetry at the boundary

To establish the duality, it is necessary to show that the symmetries of two sides are the same. Let us first show that the boundary global superconformal symmetry follows indeed from the supersymmetry of the bulk super Liouville theory. This is an example of the familiar relation between the supersymmetry of a theory in AdS_{d+1} and global superconformal symmetry of the CFT_d. In the special case of the locally conformal (e.g. Liouville) theory in AdS_2 this global symmetry is expected to be further enhanced to the super-Virasoro symmetry.

The supersymmetry parameter $\mathcal{E}$ in (3.12) is the solution of the AdS_2 Killing spinor equation (3.7) which for $a=-1$ is given by (see appendix A for spinor conventions):

$$\mathcal{E}(\mathsf{t},\mathsf{z})=\begin{pmatrix}\epsilon\\ -i\bar{\epsilon}\end{pmatrix}=Z(\mathsf{t},\mathsf{z})\Lambda\,,\quad \Lambda=\begin{pmatrix}\lambda\\ -i\bar{\lambda}\end{pmatrix},\quad Z=\frac{1}{2\sqrt{\mathsf{z}}}\begin{pmatrix}1+i\mathsf{t}+\mathsf{z} & -i-\mathsf{t}+i\mathsf{z}\\ i-\mathsf{t}-i\mathsf{z} & 1-i\mathsf{t}+\mathsf{z}\end{pmatrix}. \tag{4.6}$$

Here Λ is a constant Majorana spinor and Z satisfies $\bar{Z}Z=1$ with $\bar{Z}=\mathcal{C}Z^T\mathcal{C}$.[11] Then the supersymmetry transformation that leaves invariant the super Liouville action (3.22) expanded near the supersymmetric vacuum (3.21) follows from (3.21), (3.12)

$$\delta\zeta=\bar{\Psi}\mathcal{E}\,,\qquad \delta\Psi=\partial\!\!\!/\zeta\mathcal{E}+F\mathcal{E}\,,\qquad F=Q\big(e^{b\zeta}-1\big)\,, \tag{4.7}$$

where $F=\hat{W}'$ is the "on-shell" value of the auxiliary field in (3.11) (that vanishes in the vacuum $\zeta=0$). Note that in the classical limit ($b\to 1,\ Q\to b^{-1}$) one has $F=\zeta+\mathcal{O}(b^2,\zeta^2)$.

Using the asymptotic expansion (4.2), (4.3) in the r.h.s. of (4.7) we learn that near the boundary the supersymmetry transformations (4.7) reduce to

$$\delta\zeta(\mathsf{t},\mathsf{z})=-\frac{1}{2}i\mathsf{z}\xi(\Psi+\bar{\Psi})+\frac{1}{2}\mathsf{z}^2\big[\partial_\mathsf{t}\xi(\Psi-\bar{\Psi})-i\xi(\tilde{\Psi}+\bar{\tilde{\Psi}})\big]+\mathcal{O}(\mathsf{z}^3) \tag{4.8}$$

$$\delta\psi(\mathsf{t},\mathsf{z})=\frac{3}{2}\mathsf{z}^{3/2}\,\xi\Phi+\frac{1}{2}i\mathsf{z}^{5/2}\partial_\mathsf{t}(\xi\Phi)+\mathcal{O}(\mathsf{z}^{7/2})\,,$$

$$\delta\bar{\psi}(\mathsf{t},\mathsf{z})=-\frac{3}{2}\mathsf{z}^{3/2}\,\xi\Phi+\frac{1}{2}i\mathsf{z}^{5/2}\partial_\mathsf{t}(\xi\Phi)+\mathcal{O}(\mathsf{z}^{7/2})\,,$$

$$\xi(\mathsf{t})=\lambda-\bar{\lambda}+i\mathsf{t}(\lambda+\bar{\lambda})\equiv\epsilon+\mathsf{t}\,\eta\,. \tag{4.9}$$

Comparing this with the variation of (4.2), (4.3) implies that we should set $\Psi+\bar{\Psi}=0$, i.e. assume the boundary condition (4.5). Then we also find that $\delta\Psi=-\delta\bar{\Psi}$, $\delta\tilde{\Psi}=\delta\bar{\tilde{\Psi}}$ and $\delta\tilde{\Psi}=\frac{i}{3}\delta\partial_\mathsf{t}\Psi$. As a result, we arrive at the following consistency conditions required for preservation of the supersymmetry near the boundary[12]

$$\Psi(\mathsf{t})=-\bar{\Psi}(\mathsf{t}),\qquad \tilde{\Psi}(\mathsf{t})=\bar{\tilde{\Psi}}(\mathsf{t}),\qquad \tilde{\Psi}(\mathsf{t})=\frac{i}{3}\partial_\mathsf{t}\Psi(\mathsf{t}) \tag{4.10}$$

The resulting supersymmetry transformation rules for the boundary fields are

$$\delta\Phi(\mathsf{t})=\partial_\mathsf{t}\xi\Psi(\mathsf{t})+\frac{1}{3}\xi\partial_\mathsf{t}\Psi(\mathsf{t})\,,\qquad \delta\Psi(\mathsf{t})=\frac{3}{2}\xi\Phi(\mathsf{t})\,. \tag{4.11}$$

[11] This guarantees that $\bar{\mathcal{E}}\mathcal{E}=\bar{\Lambda}\Lambda=\text{const}$, i.e. $\partial_\mu(\bar{\mathcal{E}}\mathcal{E})=0$, which follows from (3.7).

[12] Note that the fermion equation of motion $D\!\!\!\!/\,\Psi=\hat{W}''(\phi)\Psi$ following from (3.13) reduces to the last relation in (4.10) near the boundary (in the classical limit $b\to 0$).

Since here $\xi = \epsilon + \mathsf{t}\eta$ is not constant (cf. (4.9)) we thus get two independent global supersymmetry transformations with constant parameters

$$\delta_\epsilon \Phi(\mathsf{t}) = \frac{1}{3}\epsilon\partial_\mathsf{t}\Psi(\mathsf{t}), \qquad \delta_\epsilon\Psi(\mathsf{t}) = \frac{3}{2}\epsilon\Phi(\mathsf{t})\,, \tag{4.12}$$

$$\delta_\eta \Phi(\mathsf{t}) = \eta\Psi(\mathsf{t}) + \frac{1}{3}\eta\mathsf{t}\partial_\mathsf{t}\Psi(\mathsf{t}), \qquad \delta_\eta\Psi(\mathsf{t}) = \frac{3}{2}\eta\mathsf{t}\Phi(\mathsf{t})\ . \tag{4.13}$$

The transformation in (4.12) is the same as the standard Q-supersymmetry in flat 2d space (relating also T and G in (2.5), cf. (4.5)) restricted to the real line, i.e. with $w \to \mathsf{t}$. The transformation in (4.13) is the conformal S-supersymmetry for fields with conformal dimensions as in (4.1). The commutators of the two η-transformations are

$$[\delta_{\eta_2}, \delta_{\eta_1}]\Phi = \eta_1\eta_2\big(4\mathsf{t} + \mathsf{t}^2\partial_\mathsf{t}\big)\Phi, \qquad [\delta_{\eta_2}, \delta_{\eta_1}]\Psi = \eta_1\eta_2\big(3\mathsf{t} + \mathsf{t}^2\partial_\mathsf{t}\big)\Psi\,, \tag{4.14}$$

where the right-hand sides are recognized to be the special conformal K-transformations: $\{S,S\} = K$.[13] The generator K acting on a field φ with dimension Δ_φ gives

$$K\varphi = (2\Delta_\varphi\mathsf{t} + \mathsf{t}^2\partial_\mathsf{t})\varphi\,, \tag{4.15}$$

so here $\Delta_\Phi = 2$, $\Delta_\Psi = \frac{3}{2}$ (cf. (4.1)). Computing the commutators of Q- and S-supersymmetries

$$[\delta_\eta, \delta_\epsilon]\Phi = \epsilon\eta\big(2 + \mathsf{t}\,\partial_\mathsf{t}\big)\Phi, \qquad [\delta_\eta, \delta_\epsilon]\Psi = \epsilon\eta\left(\frac{3}{2} + \mathsf{t}\,\partial_\mathsf{t}\right)\Psi, \tag{4.16}$$

one checks also that $\{S,Q\} = D$ where D is the generator of dilations

$$D\varphi = (\Delta_\varphi + \mathsf{t}\partial_\mathsf{t})\varphi\ . \tag{4.17}$$

Finally, the commutator of two Q-supersymmetries gives $[\delta_{\epsilon_2}, \delta_{\epsilon_1}]\Phi = \epsilon_1\epsilon_2\partial_\mathsf{t}\Phi$, $[\delta_{\epsilon_2}, \delta_{\epsilon_1}]\Psi = \epsilon_1\epsilon_2\partial_\mathsf{t}\Psi$ i.e.

$$\{Q,Q\} = H, \qquad H = \partial_\mathsf{t}\ . \tag{4.18}$$

Thus the generators (Q,S,H,D,K) form the $\mathfrak{osp}(1|2)$ super Lie algebra (see, e.g., [35, 36]), i.e. the super-extension of the conformal algebra $so(2,1)$ in one dimension.

This is the finite-dimensional sub-algebra of the infinite dimensional super-Virasoro algebra (the same as generated by T and G in (2.5)). It is natural to expect that given a super Weyl invariant theory in the bulk the boundary global superconformal symmetry should get extended to the full super-Virasoro symmetry. To demonstrate this directly beyond the classical level will require a choice of a symmetry-preserving regularization.

[13] Here we use the standard notation

$$[\delta_{\eta_2}, \delta_{\eta_1}] \equiv [\eta_2 S, \eta_1 S] = \eta_1\eta_2\{S,S\}, \quad [\delta_\eta, \delta_\epsilon] \equiv [\eta S, \epsilon Q] = \epsilon\eta\{S,Q\}, \quad [\delta_{\epsilon_2}, \delta_{\epsilon_1}] \equiv [\epsilon_2 Q, \epsilon_1 Q] = \epsilon_1\epsilon_2\{Q,Q\}.$$

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4.2 Determination of the proportionality coefficients $\kappa_\zeta(b)$ and $\kappa_\psi(b)$

Since the supersymmetry relating the boundary fields Φ and Ψ is the same as the one relating the operators T and G one may expect that the ratio of the coefficients κ_ζ and κ_ψ in (4.5) should be universal, i.e. should not depend on b. To fix these coefficients we may follow the arguments in [4, 6].

Let us start with a heuristic semiclassical argument generalizing the one for the Liouville theory in [4]. Assuming $b \to 0$ (i.e. $Q \to b^{-1}$) the AdS$_2$ super Liouville action (3.22) may be related to the flat space action (2.1) by the special Weyl transformation

$$g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}, \qquad \phi \to \phi - b^{-1}\sigma, \qquad \Psi \to e^{-\frac{1}{2}\sigma}\Psi\,, \qquad \sigma = \log \mathsf{z} \tag{4.19}$$

Then using (4.2), (4.3) the boundary asymptotics of the corresponding fields on the flat half-plane is found to be

$$\phi(\mathsf{t},\mathsf{z})\big|_{\mathsf{z}\to 0} = \mathsf{z}^2\Phi(t) - b^{-1}\log\mathsf{z} + \ldots, \qquad \psi(\mathsf{t},\mathsf{z})\big|_{\mathsf{z}\to 0} = \mathsf{z}^{3/2}e^{-\frac{1}{2}\log\mathsf{z}}\Psi(\mathsf{t}) + \ldots = \mathsf{z}\Psi(\mathsf{t}) + \ldots\,. \tag{4.20}$$

Evaluating T and G in (2.5) on these fields and taking the boundary limit we get

$$T(\mathsf{t},\mathsf{z})\big|_{\mathsf{z}\to 0} = -\frac{3}{2b}\Phi(\mathsf{t}) + \ldots\,, \qquad\qquad G(\mathsf{t},\mathsf{z})\big|_{\mathsf{z}\to 0} = -\frac{\sqrt{2}}{b}\Psi(\mathsf{t}) + \ldots\,. \tag{4.21}$$

Comparing this with (4.5), we conclude that in the semiclassical limit

$$\kappa_\zeta = -\frac{2}{3}b + \mathcal{O}(b^2)\,, \qquad\qquad \kappa_\psi = -\frac{1}{\sqrt{2}}b + \mathcal{O}(b^2)\,, \tag{4.22}$$

in agreement with the leading asymptotics of the general expressions quoted in (1.17).

Let us now give another argument based on the OPE in the boundary CFT which leads to the exact expressions for κ_ζ and κ_ψ in (1.17). Let us first determine κ_ζ starting with the OPE of T with the primary field V_α in (2.10) following the discussion in [4] for the Liouville theory. Expanding the expression for TV_α in (2.11) to the leading order in small α gives

$$T(w')\,\phi(w) \;\sim\; \frac{Q}{2(w'-w)^2} + \frac{\partial\phi(w)}{w'-w}\,. \tag{4.23}$$

Setting $w' = \mathsf{t}', w = \mathsf{t} + i\mathsf{z}$ and using the asymptotic form of ϕ near the boundary (now we use Q in place of its classical value b^{-1} in (4.20))

$$\phi(\mathsf{t},\mathsf{z})\big|_{\mathsf{z}\to 0} = \mathsf{z}^2\Phi(\mathsf{t}) - Q\log\mathsf{z} + \cdots\,, \tag{4.24}$$

one finds (note that $\partial \equiv \partial_w = \frac{1}{2}(\partial_\mathsf{t} - i\mathsf{z})$)

$$T(\mathsf{t}')\phi(\mathsf{z},\mathsf{t}) \sim \frac{iQ}{2z(\mathsf{t}'-\mathsf{t})} + \mathsf{z}\left[\frac{iQ}{2(\mathsf{t}'-\mathsf{t})^3} - \frac{i\Phi(\mathsf{t})}{\mathsf{t}'-\mathsf{t}}\right] + \mathsf{z}^2\left[-\frac{Q}{(\mathsf{t}'-\mathsf{t})^4} + \frac{\Phi(\mathsf{t})}{(\mathsf{t}'-\mathsf{t})^2} + \frac{2\partial_\mathsf{t}\Phi(\mathsf{t})}{(\mathsf{t}'-\mathsf{t})^2}\right]. \tag{4.25}$$

Including the conjugate part to account for the real-line boundary (i.e. adding the expression with $\mathsf{z} \to -\mathsf{z}$), gives the OPE for the boundary fields

$$T(\mathsf{t}')\Phi(\mathsf{t}) \sim -\frac{2Q}{(\mathsf{t}'-\mathsf{t})^4} + \frac{2\Phi(\mathsf{t})}{(\mathsf{t}'-\mathsf{t})^2} + \frac{\partial_\mathsf{t}\Phi(\mathsf{t})}{(\mathsf{t}'-\mathsf{t})^2}\,. \tag{4.26}$$

Comparing this with the boundary limit of the OPE for $T(w')$ and $T(w)$ in (2.6) we conclude that the two relations have the same form provided we identify $\Phi(\mathsf{t}) \sim \kappa_\zeta T(\mathsf{t})$ as in (4.5) with κ_ζ given by

$$\kappa_\zeta = -\frac{4Q}{c} = -\frac{4\,b\,(2+b^2)}{3\,(1+b^2)(4+b^2)} = -\frac{2}{3}\,b + \frac{1}{2}\,b^3 + \cdots\,, \tag{4.27}$$

where we used the expressions (1.12) for Q and c in the super Liouville theory.

To determine κ_ψ let us start with the OPE of G and V_α in (2.12) and again expand in α, getting[14]

$$G(w')\,\phi(w) \sim -\frac{i}{\sqrt{2}}\,\frac{\psi(w)}{w'-w}\,. \tag{4.28}$$

Setting $w' = \mathsf{t}', w = \mathsf{t} + i\mathsf{z}$, and using the asymptotic near-boundary form of ϕ (4.24) and ψ (cf. (4.3), (4.19))

$$\psi(\mathsf{t},\mathsf{z})\big|_{\mathsf{z}\to 0} = e^{-\frac{1}{2}\log \mathsf{z}}\big[\mathsf{z}^{3/2}\Psi(\mathsf{t}) + \mathsf{z}^{5/2}\tilde{\Psi}(\mathsf{t}) + \cdots\big] = \mathsf{z}\Psi(\mathsf{t}) + \mathsf{z}^2\tilde{\Psi}(\mathsf{t}) + \cdots\,, \tag{4.29}$$

we arrive at

$$G(\mathsf{t}')\Phi(\mathsf{t}) \sim \frac{i\Psi(\mathsf{t})}{\sqrt{2}\mathsf{z}(\mathsf{t}'-\mathsf{t})} + \frac{\Psi(\mathsf{t})}{\sqrt{2}(\mathsf{t}'-\mathsf{t})^2} - i\frac{\tilde{\Psi}(\mathsf{t})}{\sqrt{2}(\mathsf{t}'-\mathsf{t})}\,. \tag{4.30}$$

Adding the $\mathsf{z} \to -\mathsf{z}$ conjugate expression to the r.h.s. gives the boundary OPE

$$G(\mathsf{t}')\,\Phi(\mathsf{t}) \sim \frac{\sqrt{2}\,\Psi(\mathsf{t})}{(\mathsf{t}'-\mathsf{t})^2} - i\frac{\sqrt{2}\,\tilde{\Psi}(\mathsf{t})}{\mathsf{t}'-\mathsf{t}}\,. \tag{4.31}$$

On the other hand, we may compare this with the boundary limit of the the OPE of G and T in (2.9); assuming the identification $\Phi(\mathsf{t}) \sim \kappa_\zeta\,T(\mathsf{t}),\ \Psi(\mathsf{t}) \sim \kappa_\psi\,G(\mathsf{t})$ we get from (2.9)

$$G(\mathsf{t}')\,\Phi(\mathsf{t}) \sim \frac{\kappa_\zeta}{\kappa_\psi}\left[\frac{3}{\sqrt{2}}\frac{\Psi(\mathsf{t})}{(\mathsf{t}'-\mathsf{t})^2} + \frac{1}{2}\frac{\partial_\mathsf{t}\Psi(\mathsf{t})}{\mathsf{t}'-\mathsf{t}}\right]\,. \tag{4.32}$$

The expressions in (4.31) and (4.32) match provided we identify

$$\frac{\kappa_\zeta}{\kappa_\psi} = \frac{2\sqrt{2}}{3}\,, \qquad\qquad \tilde{\Psi} = \frac{i}{3}\partial_\mathsf{t}\Psi\,. \tag{4.33}$$

Hence we get, in agreement with (1.17), (4.22),

$$\kappa_\psi = \frac{3}{2\sqrt{2}}\,\kappa_\zeta = -3\sqrt{2}\,\frac{Q}{c}\,, \tag{4.34}$$

The relation for $\tilde{\Psi}$ in (4.33) is the same as the one in (4.10) found from the condition of preservation of the supersymmetry near the boundary; this supports the consistency of the above argument.

[14]This OPE can be found also using the free field realization of the stress tensor T and supercurrent G in (2.5).

4.3 Duality predictions for the boundary correlation functions

The $\mathrm{AdS}_2/\mathrm{CFT}_2^{1/2}$ correspondence implies the relations (1.16) between the AdS_2 boundary correlators of the elementary super Liouville fields ζ,ψ and the flat-space CFT correlators of the super-Virasoro generators T and G restricted to the real line. While the matching of the coordinate dependence on the two sides of the correspondence should follow from the super-Virasoro symmetry, direct computation of the boundary correlators in small b (or large c) perturbation theory allows one to check the expressions (4.27) and (4.34) for the coefficients κ_ζ and κ_ψ.

The correlation functions of the chiral T and G operators on a plane are completely fixed by the super-Virasoro symmetry or the OPE relations (2.6)–(2.8). Explicitly, one finds (see, e.g., [37])

$$\langle T(w_1)T(w_2)\rangle = \frac{c}{2}\,\frac{1}{w_{12}^4}, \qquad \langle G(w_1)G(w_2)\rangle = \frac{2c}{3}\,\frac{1}{w_{12}^3}, \tag{4.35}$$

$$\langle T(w_1)T(w_2)T(w_3)\rangle = c\frac{1}{w_{12}^2\,w_{13}^2\,w_{23}^2}, \quad \langle T(w_1)G(w_2)G(w_3)\rangle = c\frac{1}{w_{12}^2\,w_{13}^2\,w_{23}}, \tag{4.36}$$

$$\langle T(w_1)T(w_2)T(w_3)T(w_4)\rangle = \frac{1}{w_{13}^4w_{24}^4}\left[\frac{c^2}{4}\left(\frac{1}{\chi^4}+\frac{1}{(\chi-1)^4}+1\right)+2c\frac{\chi^2-\chi+1}{(\chi-1)^2\chi^2}\right], \tag{4.37}$$

$$\langle G(w_1)T(w_2)G(w_3)T(w_4)\rangle = \frac{1}{w_{13}^3w_{24}^4}\left[\frac{c^2}{3}+\frac{c}{2}\frac{4\chi^2-4\chi+3}{\chi^2(\chi-1)^2}\right], \qquad \chi\equiv\frac{w_{12}w_{34}}{w_{13}w_{24}}, \tag{4.38}$$

$$\langle G(w_1)G(w_2)G(w_3)G(w_4)\rangle = \frac{1}{w_{13}^3w_{24}^3}\left[\frac{4c^2}{9}\left(\frac{1}{\chi^3}-1+\frac{1}{(1-\chi)^3}\right)+2\,c\,\frac{1}{\chi(1-\chi)}\right], \tag{4.39}$$

where χ is the conformally invariant cross ratio. As in (1.7)–(1.9) the order c^2 parts of the 4-point functions correspond to disconnected contributions, while the order c parts correspond to the connected contributions to the boundary correlators.

Then using the duality relation (1.16), we get the following predictions for 2-point boundary correlation functions:

$$\langle \Phi(\mathsf{t}_1)\Phi(\mathsf{t}_2)\rangle = \frac{C_{\Phi\Phi}}{\mathsf{t}_{12}^4}, \qquad \langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\rangle = \frac{C_{\Psi\Psi}}{\mathsf{t}_{12}^3}, \tag{4.40}$$

$$C_{\Phi\Phi} = \frac{1}{2}\kappa_\zeta^2\,c = \frac{4\,(2+b^2)^2}{3\,(1+b^2)(4+b^2)} = \frac{4}{3}-\frac{1}{3}b^2+\cdots,$$

$$C_{\Psi\Psi} = \frac{2}{3}\kappa_\psi^2\,c = \frac{3}{2}C_{\Phi\Phi} = 2-\frac{1}{2}b^2+\cdots. \tag{4.41}$$

The non-vanishing 3-point boundary correlators should be given by

$$\langle \Phi(\mathsf{t}_1)\Phi(\mathsf{t}_2)\Phi(\mathsf{t}_3)\rangle = \frac{C_{\Phi\Phi\Phi}}{\mathsf{t}_{12}^2\,\mathsf{t}_{13}^2\,\mathsf{t}_{23}^2}, \qquad \langle \Phi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\rangle = \frac{C_{\Phi\Psi\Psi}}{\mathsf{t}_{12}^2\,\mathsf{t}_{13}^2\,\mathsf{t}_{23}}, \tag{4.42}$$

$$C_{\Phi\Phi\Phi} = \kappa_\zeta^3\,c = -\frac{32b(2+b^2)^3}{9(1+b^2)^2(4+b^2)^2} = -\frac{16}{9}\,b+\frac{16}{9}\,b^3+\cdots, \tag{4.43}$$

$$C_{\Phi\Psi\Psi} = \kappa_\zeta\,\kappa_\psi^2\,c = \frac{9}{8}\,C_{\Phi\Phi\Phi} = -2\,b+2\,b^3+\cdots. \tag{4.44}$$

As the disconnected part of the 4-point correlators is determined by the 2-point functions, the non-trivial prediction follows only from the connected (order c) part of (4.37)–(4.22):

$$\langle \Phi(\mathrm{t}_1)\Phi(\mathrm{t}_2)\Phi(\mathrm{t}_3)\Phi(\mathrm{t}_4)\rangle_{\rm conn} = \frac{C_{\Phi\Phi\Phi\Phi}}{\mathrm{t}_{13}^4\mathrm{t}_{24}^4}\frac{2(1-\chi+\chi^2)}{\chi^2(1-\chi)^2}\,, \qquad \chi\equiv\frac{\mathrm{t}_{12}\mathrm{t}_{34}}{\mathrm{t}_{13}\mathrm{t}_{24}}\,, \tag{4.45}$$

$$\langle \Psi(\mathrm{t}_1)\Phi(\mathrm{t}_2)\Psi(\mathrm{t}_3)\Phi(\mathrm{t}_4)\rangle_{\rm conn} = \frac{C_{\Psi\Phi\Psi\Phi}}{\mathrm{t}_{13}^3\,\mathrm{t}_{24}^4}\frac{3-4\chi+4\chi^2}{2\,\chi^2(1-\chi)^2}\,, \tag{4.46}$$

$$\langle \Psi(\mathrm{t}_1)\Psi(\mathrm{t}_2)\Psi(\mathrm{t}_3)\Psi(\mathrm{t}_4)\rangle_{\rm conn} = \frac{C_{\Psi\Psi\Psi\Psi}}{\mathrm{t}_{13}^3\mathrm{t}_{24}^3}\frac{1}{\chi(1-\chi)}\,, \tag{4.47}$$

$$C_{\Phi\Phi\Phi\Phi} = \kappa_\zeta^4\, c = \frac{128\, b^2\,(2+b^2)^4}{27(1+b^2)^3(4+b^2)^3} = \frac{32}{27}\,b^2-\frac{56}{27}\,b^4+\cdots\,. \tag{4.48}$$

$$C_{\Psi\Phi\Psi\Phi} = \kappa_\zeta^2\kappa_\psi^2\, c = \frac{9}{8}C_{\Phi\Phi\Phi\Phi} = \frac{4}{3}\,b^2-\frac{7}{3}\,b^4+\cdots\,, \tag{4.49}$$

$$C_{\Psi\Psi\Psi\Psi} = 2\,\kappa_\psi^4\, c = \frac{81}{32}\,C_{\Phi\Phi\Phi\Phi} = 3\,b^2-\frac{21}{4}\,b^4+\cdots\,, \tag{4.50}$$

where, as in (4.44), we used the supersymmetry implied relation (4.34).

On the AdS_2 side the relation between the correlators with only bosons and correlators where some of the bosons are replaced by fermions should also follow from the supersymmetry (4.12). In more detail, the kinematical structure (i.e. t-dependence) of the 2- and 3-point correlators in (4.40), (4.42) is fixed by the global conformal symmetry while for the 4-point correlators in (4.45)–(4.47) it is fixed by the expected Virasoro symmetry. Then the relations between their coefficients, i.e.

$$C_{\Psi\Psi} = \frac{3}{2}\,C_{\Phi\Phi}\,, \qquad C_{\Phi\Psi\Psi} = \frac{9}{8}\,C_{\Phi\Phi\Phi}\,, \qquad C_{\Psi\Phi\Psi\Phi} = \frac{9}{8}C_{\Phi\Phi\Phi\Phi}\,,$$
$$C_{\Psi\Psi\Psi\Psi} = \frac{9}{4}C_{\Psi\Phi\Psi\Phi} = \frac{81}{32}\,C_{\Phi\Phi\Phi\Phi}\,, \tag{4.51}$$

follow from the Ward identities corresponding to the boundary supersymmetry (4.12). For example, considering the supersymmetry variation (4.12) of the vanishing correlator $\langle\Phi(\mathrm{t}_1)\Phi(\mathrm{t}_2)\Phi(\mathrm{t}_3)\Psi(\mathrm{t}_4)\rangle = 0$, one obtains

$$\frac{3}{2}\langle\Phi(\mathrm{t}_1)\Phi(\mathrm{t}_2)\Phi(\mathrm{t}_3)\Phi(\mathrm{t}_4)\rangle + \frac{1}{3}\Big[\partial_{\mathrm{t}_1}\langle\Psi(\mathrm{t}_1)\Phi(\mathrm{t}_2)\Phi(\mathrm{t}_3)\Psi(\mathrm{t}_4)\rangle + \partial_{\mathrm{t}_2}\langle\Phi(\mathrm{t}_1)\Psi(\mathrm{t}_2)\Phi(\mathrm{t}_3)\Psi(\mathrm{t}_4)\rangle$$
$$+\,\partial_{\mathrm{t}_3}\langle\Phi(\mathrm{t}_1)\Phi(\mathrm{t}_2)\Psi(\mathrm{t}_3)\Psi(\mathrm{t}_4)\rangle\Big] = 0\,. \tag{4.52}$$

As a result, using (4.45), (4.46) we find that $C_{\Psi\Phi\Psi\Phi} = \frac{9}{8}C_{\Phi\Phi\Phi\Phi}$. One can similarly show that $C_{\Psi\Psi\Psi\Psi} = \frac{9}{4}C_{\Psi\Phi\Psi\Phi}$ by starting with $\delta\langle\Phi\Psi\Psi\Psi\rangle = 0$.

Our aim below will be to check explicitly such symmetry-implied relations and also the expression for $\kappa_\zeta(b)$ in (4.27) (which does not follow just from symmetries). In particular, checking the supersymmetry-implied relations beyond tree level (where the derivation of the boundary supersymmetry in section 4.1 directly applies) will rest on a choice of a regularization prescription consistent with the underlying symmetry.

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5 Perturbation theory in AdS_2

The goal in the rest of the paper is to check the predictions summarized in the previous section starting from the AdS_2 super Liouville action (3.22) and using small b perturbation theory. Expanding (3.22) in powers of b using the value of Q in (3.18) and separating out the free part from interacting vertices (containing powers of b) we get

$$\begin{aligned}\mathcal{S} = \frac{1}{4\pi}\int d^2x\sqrt{g}\,\Big\{&\partial_\mu\zeta\partial^\mu\zeta + 2\zeta^2 + \bar\Psi\not{D}\Psi - \bar\Psi\Psi\\ &+ \big(3b^2 + \tfrac{1}{2}b^4\big)\frac{\zeta^2}{2!} + \big(8b + 7b^3 + \tfrac{3}{2}b^5\big)\frac{\zeta^3}{3!} + \big(16b^2 + 15b^4 + \tfrac{7}{2}b^6\big)\frac{\zeta^4}{4!} + \cdots\\ &- \frac{1}{2}\Big[b^2 + \big(2\,b + b^3\big)\,\zeta + \big(2\,b^2 + b^4\big)\frac{\zeta^2}{2!} + \big(2\,b^3 + b^5\big)\frac{\zeta^3}{3!} + \cdots\Big]\,\bar\Psi\Psi\Big\}\,. \end{aligned} \tag{5.1}$$

We will treat the b-dependent quadratic $\sim b^2\,\zeta^2$ and $\sim b^2\,\bar\Psi\Psi$ terms as interaction terms leading to self-energy insertions into diagrams.

To develop perturbation theory in powers of b we will need to know the explicit form of the free propagators for the massive fields ζ and Ψ. Various representations for the scalar and fermion propagator in AdS_{d+1} are reviewed in appendix B. Here we will specify them to the present AdS_2 case of the scalar ζ with mass $m_\zeta^2 = 2$ and a Majorana fermion Ψ with mass $m_\psi = 1$.

We will use the following sets of coordinates in Euclidean AdS_2

$$x^a = x_a = (x_0, x_1) = (\mathsf{z}, \mathsf{t})\,; \qquad w = \mathsf{t} + i\,\mathsf{z} = -i\,\frac{z+1}{z-1}\,; \qquad z = re^{i\theta}\,. \tag{5.2}$$

Here the complex variable w parametrizes the upper half of complex plane (with the AdS_2 metric in (3.5)), while complex z is a coordinate on a unit disc. From (5.2), the coordinate θ of the boundary of the disk $(r = |z| = 1)$ and the coordinate t of the boundary $(\mathsf{z} = 0)$ of the upper half plane are related by

$$\mathsf{t}(\theta) = -\cot\frac{\theta}{2}\,. \tag{5.3}$$

The AdS_2 measure is then given by

$$\mathsf{d}^2x \equiv \frac{d^2x}{4\pi\mathsf{z}^2} = \mathsf{d}^2w \equiv \frac{d^2w}{4\pi\mathsf{z}^2} = \mathsf{d}^2z \equiv \frac{d^2z}{\pi(1-|z|^2)^2}\,, \quad d^2w = d\mathsf{t}\,d\mathsf{z}\,, \quad d^2z = rdrd\theta\,. \tag{5.4}$$

The free propagator of the scalar ζ with $m^2 = 2$ (with Dirichlet boundary conditions, i.e. $\Delta_\zeta = 2$) is given by[15]

$$\langle\zeta(x)\,\zeta(x')\rangle_0 \equiv g(x,x') = \overset{x}{}\!\!\rule[0.5ex]{6em}{0.4pt}\!\!\overset{x'}{} = 2\pi G(x,x') = -\frac{1}{2}\left(\frac{1+\eta}{1-\eta}\log\eta + 2\right), \tag{5.5}$$

where

$$\eta = \left|\frac{z-z'}{1-z\bar z'}\right|^2 = \frac{u}{u+2}\,, \qquad u \equiv \frac{(\mathsf{z}-\mathsf{z}')^2 + (\mathsf{t}-\mathsf{t}')^2}{2\,\mathsf{z}\,\mathsf{z}'}. \tag{5.6}$$

[15] The propagator G corresponds to canonical normalization of the action $\frac{1}{2}\int d^2x\,\sqrt{g}\,(\partial_\mu\zeta\partial^\mu\zeta - 2\zeta^2)$.

The bulk-to-boundary propagator is then $(g(x,x') \equiv g(\mathsf{t},\mathsf{z};\mathsf{t}',\mathsf{z}') = g(w,w'))$

$$g_\partial(\mathsf{t};x') = g_\partial(\mathsf{t};\mathsf{t}',\mathsf{z}') \equiv \lim_{\mathsf{z}\to 0} \mathsf{z}^{-2} g(\mathsf{t},\mathsf{z};\mathsf{t}',\mathsf{z}') = \frac{4}{3}\left[\frac{\mathsf{z}'}{\mathsf{z}'^2 + (\mathsf{t}-\mathsf{t}')^2}\right]^2 . \tag{5.7}$$

For the fermion with mass $m_\psi = 1$ (and thus $\Delta_\psi = \frac{3}{2}$) one finds (see, e.g., [38] and appendix B)

$$\begin{aligned} \mathsf{S}(x,x') &= \overset{x}{}\!\!\longrightarrow\!\!\overset{x'}{} = 2\pi S(x,x') \\ &= -\frac{1}{2}\frac{1}{(u+2)^2}\frac{1}{\sqrt{\mathsf{z}\,\mathsf{z}'}}\Big[(x^a\,\Gamma_a\Gamma_{\mathsf{z}} + \Gamma_{\mathsf{z}}\,\Gamma_a\,x'^a)\,\mathsf{F}_1(u) - \Gamma^a(x-x')_a\,\mathsf{F}_2(u)\Big], \end{aligned} \tag{5.8}$$

where $\Gamma^a = \Gamma_a = (\Gamma_{\mathsf{t}}, \Gamma_{\mathsf{z}})$ are the flat-space 2d Dirac matrices (see appendix A) and

$$\mathsf{F}_1 = -\frac{u+2}{2}\left[2 + (u+2)\,\log\frac{u}{u+2}\right], \qquad \mathsf{F}_2 = \frac{(u+2)^2}{2\,u}\left[2 + u\,\log\frac{u}{u+2}\right] . \tag{5.9}$$

Here the propagator $S(x,x')$ corresponds to the canonically normalized kinetic term for a Majorana fermion: $\frac{1}{2}\int d^2x\,\sqrt{g}\,\bar{\Psi}(\not{D} - m)\bar{\Psi}$. It satisfies

$$(\not{D}_x - 1)\,\mathsf{S}(x,x') = \mathsf{S}(x,x')(-\overleftarrow{\not{D}}_{x'} - 1) = \frac{2\pi}{\sqrt{g}}\,\delta^{(2)}(x-x'). \tag{5.10}$$

The component form of (5.8) is

$$\begin{aligned} \mathsf{S}(x,x') = \langle\Psi(x)\bar{\Psi}(x')\rangle_0 &= \begin{pmatrix} i\langle\psi(x)\bar{\psi}(x')\rangle_0 & \langle\psi(x)\psi(x')\rangle_0 \\ \langle\bar{\psi}(x)\bar{\psi}(x')\rangle_0 & -i\langle\bar{\psi}(x)\psi(x')\rangle_0 \end{pmatrix} \equiv \begin{pmatrix} ig_{\psi\bar{\psi}}(x,x') & g_{\psi\psi}(x,x') \\ g_{\bar{\psi}\bar{\psi}}(x,x') & -ig_{\bar{\psi}\psi}(x,x') \end{pmatrix} \\ &= \frac{1}{2}\frac{1}{(u+2)^2}\frac{1}{\sqrt{\mathsf{z}\mathsf{z}'}}\begin{pmatrix} -i(\bar{w}-w')\mathsf{F}_1 & (\bar{w}-\bar{w}')\mathsf{F}_2 \\ (w-w')\mathsf{F}_2 & i(w-\bar{w}')\mathsf{F}_1 \end{pmatrix} . \end{aligned} \tag{5.11}$$

These component propagators are also independently computed in appendix B.2. Taking one leg to the boundary and adding a $\mathsf{z}^{-3/2}$ factor we get the bulk-to-boundary propagators $(x' = (\mathsf{t}',\mathsf{z}'),\ w' = \mathsf{t}' + i\mathsf{z}')$

$$\begin{aligned} \mathsf{S}_\partial(\mathsf{t};x') = \lim_{\mathsf{z}\to 0}\mathsf{z}^{-3/2}\mathsf{S}(x,x') &\equiv \begin{pmatrix} ig_{\partial\,\psi\bar{\psi}}(\mathsf{t};x') & g_{\partial\,\psi\psi}(\mathsf{t};x') \\ g_{\partial\,\bar{\psi}\bar{\psi}}(\mathsf{t};x') & -ig_{\partial\,\bar{\psi}\psi}(\mathsf{t};x') \end{pmatrix} \\ &= \frac{1}{2\mathsf{z}'^{3/2}}\begin{pmatrix} \frac{-i}{(\mathsf{t}-w')(\mathsf{t}-\bar{w}')^2} & \frac{1}{(\mathsf{t}-w')^2(\mathsf{t}-\bar{w}')} \\ \frac{1}{(\mathsf{t}-w')(\mathsf{t}-\bar{w}')^2} & \frac{i}{(\mathsf{t}-w')^2(\mathsf{t}-\bar{w}')} \end{pmatrix} . \end{aligned} \tag{5.12}$$

Especially, they satisfy

$$g_{\partial\,\psi\psi}(\mathsf{t};x') = -g_{\partial\,\bar{\psi}\psi}(\mathsf{t};x'), \qquad g_{\partial\,\psi\bar{\psi}}(\mathsf{t};x') = -g_{\partial\,\bar{\psi}\bar{\psi}}(\mathsf{t};x') . \tag{5.13}$$

This is consistent with the supersymmetric boundary condition $\Psi = -\bar{\Psi}$ in (4.4), (4.10) (cf. (4.3)) implying that in boundary correlators we can freely replace $\bar{\psi}$ with $-\psi$, i.e. it is enough to consider boundary correlators of ψ only.

The bulk-to-boundary fermion propagators may be expressed in terms of the bosonic propagator in (5.7) as follows

$$\mathsf{S}_\partial(x;\mathsf{t}') = \lim_{\mathsf{z}'\to 0} \mathsf{z}'^{-3/2}\mathsf{S}(x,x') = 3\,U(x;\mathsf{t}')\,\mathcal{P}_-\,g_\partial(\mathsf{t}';x)\,, \tag{5.14}$$

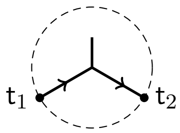
$$\mathsf{S}_\partial(\mathsf{t};x') = \lim_{\mathsf{z}\to 0} \mathsf{z}^{-3/2}\mathsf{S}(x,x') = -3\,\mathcal{P}_+\,U(x';\mathsf{t})\,g_\partial(\mathsf{t};x')\,, \tag{5.15}$$

$$U(x,\mathsf{t}') = \frac{1}{\sqrt{\mathsf{z}}}\Big[\mathsf{z}\Gamma^{\mathsf{z}} + (\mathsf{t}-\mathsf{t}')\Gamma^{\mathsf{t}}\Big]\,, \qquad \mathcal{P}_\pm = \frac{1}{2}(1\pm\Gamma^{\mathsf{z}})\,. \tag{5.16}$$

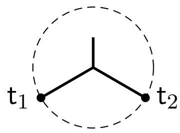
One can then prove the following useful relation:

$$\lim_{\mathsf{z}_1,\mathsf{z}_2\to 0} \mathsf{z}_1^{-3/2}\mathsf{z}_2^{-3/2}\mathsf{S}(x_1,x)\,\mathsf{S}(x,x_2) \equiv \mathsf{S}_\partial(\mathsf{t}_1;x)\,\mathsf{S}_\partial(x;\mathsf{t}_2) = -9\,\mathsf{t}_{12}\,\Gamma^{\mathsf{t}}\,\mathcal{P}_-\,g_\partial(\mathsf{t}_1;x)\,g_\partial(\mathsf{t}_2;x)\,. \tag{5.17}$$

In (5.5), (5.8) and below we will denote the scalar propagator by a line and the fermion propagator by a line with an arrow. The dashed line will stand for the boundary of AdS_2 (for convenience represented as a circle, i.e. a boundary in Poincare disc representation). For example, the relation (5.17) implies the following graphical equality of the fermion and scalar Witten diagrams (with one point in the bulk)



$$= -\frac{9}{2}\mathsf{t}_{12} \tag{5.18}$$



Here we assumed the projection ($\Gamma^{\mathsf{t}}\,\mathcal{P}_- \to \frac{1}{2}$) to the upper ψ component of Ψ assuming that the boundary field Ψ (cf. (4.3)) is inserted at $\mathsf{t}_1, \mathsf{t}_2$.

There is another useful representation for the free fermion propagator in terms of the bosonic one following from the Ward identity for AdS_2 supersymmetry [15, 39]. Consider the WZ model (3.11) (with $a=-1$) with the superpotential $\hat{W}(\phi) = \frac{1}{2}\phi^2$ which describes the AdS_2 supersymmetric system of the free scalar field ϕ with mass $m^2=2$ and the free fermion Ψ with $m=1$. The supersymmetric Ward identity similar to the one in (4.52) implies (here we use w, w' to denote the coordinates)

$$0 = \delta\big\langle\Psi(w)\,\phi(w')\big\rangle = \big\langle\Psi(w)\,\bar{\Psi}(w')\big\rangle Z(w')\Lambda + \big\langle(\partial\!\!\!/_w + 1)\phi(w)\,\phi(w')\big\rangle Z(w)\Lambda\,, \tag{5.19}$$

where we used the supersymmetry transformation (3.12) with the parameter $\mathcal{E}$ in (4.6) and $F = \hat{W}' = \phi$. Since this equation holds for an arbitrary constant Majorana spinor Λ, it should be also true as a matrix equation. Multiplying by $Z^{-1}(w')$ and using the explicit form of Z in (4.6) we arrive at

$$\mathsf{S}(w,w') = \langle\Psi(w)\bar{\Psi}(w')\rangle = -\Big[(\partial\!\!\!/_w + 1)\langle\phi(w)\,\phi(w')\rangle\Big] R(w,w')\,, \tag{5.20}$$

$$R(w,w') \equiv Z(w)\,Z^{-1}(w') = \frac{1}{2\sqrt{\mathsf{z}\mathsf{z}'}}\begin{pmatrix} i\,(\mathsf{t}-\mathsf{t}') + \mathsf{z} + \mathsf{z}' & -(\mathsf{t}-\mathsf{t}') + i\,(\mathsf{z}-\mathsf{z}') \\ -(\mathsf{t}-\mathsf{t}') - i\,(\mathsf{z}-\mathsf{z}') & -i\,(\mathsf{t}-\mathsf{t}') + \mathsf{z} + \mathsf{z}' \end{pmatrix}. \tag{5.21}$$

Thus we get the following representation for the fermion bulk-to-bulk propagator in terms of the bosonic one in (5.5)

$$\mathsf{S}(w,w') = -\Big[(\partial\!\!\!/_w + 1)\,g(w,w')\Big] R(w,w') = R(w,w')\,\Big[(\partial\!\!\!/_{w'} - 1)\,g(w,w')\Big]. \tag{5.22}$$

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Let us now comment on the issue of UV regularization. In the bosonic 2d Liouville theory the exponential potential is special being the eigen-function of the anomalous dimension operator (the second derivative term in (3.15)) and also the interaction term in the classical equation. As a result (using, e.g., the background field method) the UV divergences can be absorbed into a field redefinition, i.e. there is no need for the coupling b renormalization. This is equivalent to replacing the exponential interaction term in (1.2) by its normal ordered expression $e^{2b\phi} \to: e^{2b\phi}:$, i.e. to assuming that the subtracted value of the propagator at the coinciding points $g(x,x)$ is zero (or equal to a scheme-dependent constant). The condition $g(x,x)=0$ is natural in the present AdS_2 context where it can be achieved by using the covariant regularization $u \to u+\varepsilon$ or $\eta \to \eta + \varepsilon$ in (5.5), (5.6) and assuming minimal subtraction of the resulting logarithmic divergence.[16]

The discussion of UV divergences in the supersymmetric Liouville theory in flat space [17, 40] implies that the picture should be essentially the same. Assuming a covariant regularization and then minimal subtraction of the log divergence $g(x,x')$ and $\mathsf{S}(x,x')$ appear as components of a superpropagator that is vanishing (or constant) at $x=x'$.[17] We shall thus assume that all the divergences come only from the tadpole diagrams and thus can be eliminated by assuming that

$$g(x,x) = \mathsf{S}(x,x) = 0 \ . \tag{5.23}$$

Indeed, in all instances that are considered in the following sections, we shall confirm by explicit calculations that the vanishing tadpole condition (5.23) is enough to make finite all radiative corrections.

6 Two-point functions

We shall start with the computation of the one-loop corrections to the two-point boundary correlators of the elementary scalar ζ and fermion ψ with the aim of checking the order b^2 terms in the duality prediction in (4.40), (4.41). Note that the tree-level terms in (4.41) are correctly normalized as follows from (5.7) and (5.8), (5.11) or (5.14):

$$\langle \Phi(\mathsf{t}_1)\Phi(\mathsf{t}_2)\rangle_0 \equiv (\mathsf{zz}')^{-2}\langle\zeta(x)\zeta(x')\rangle_0\Big|_{\mathsf{z},\mathsf{z}'\to 0} = (\mathsf{zz}')^{-2} g(x,x')\Big|_{\mathsf{z},\mathsf{z}'\to 0} \to \frac{4}{3}\frac{1}{(\mathsf{t}-\mathsf{t}')^4}\,, \tag{6.1}$$

$$\langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\rangle_0 \equiv (\mathsf{zz}')^{-3/2}\langle\psi(x)\psi(x')\rangle_0\Big|_{\mathsf{z},\mathsf{z}'\to 0} = (\mathsf{zz}')^{-3/2}\mathsf{S}_{12}(x,x')\Big|_{\mathsf{z},\mathsf{z}'\to 0} \to \ 2\frac{1}{(\mathsf{t}-\mathsf{t}')^3}\,. \tag{6.2}$$

6.1 Scalar propagator correction

The one-loop correction to the scalar 2-point function in (4.40) is given by the boundary limit of the sum of the contributions of the three bulk diagrams. For the calculation of

[16] A detailed comparison with other approaches to regularization of the Liouville theory in AdS_2 can be found in [4].

[17] This is equivalent to normal ordering of the exponential interaction in the linear in $\hat{W}$ terms in (3.11), before integrating out the auxiliary field F.

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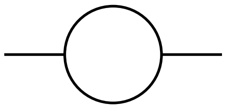
the Witten diagrams below it will be often useful to use the AdS_2 disk parametrization, cf. (5.2) and (5.4), and the Fourier representation of $g(x,x')$ derived in [41].

$$\begin{aligned}\langle\zeta(z_1)\zeta(z_2)\rangle &= g(z_1,z_2)+D_1(z_1,z_2)+\mathcal{O}(b^4)\,,\\ D_1(z_1,z_2) &= D_{\rm sc}(z_1,z_2)+D_{\rm ins}(z_1,z_2)+D_{\rm f}(z_1,z_2).\end{aligned} \tag{6.3}$$

Scalar loop. The first contribution in (6.3) is that of the boson bubble diagram (see (A.15) in [4])

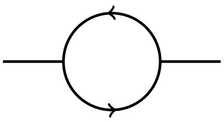
$$D_{\rm sc}(z_1,z_2) =$$

$$\begin{aligned} &= \frac{1}{2}\,(-8b)^2\int \mathsf{d}^2z'\mathsf{d}^2z''\, g(z_1,z')\,g(z',z'')^2\,g(z'',z_2)\\ &= 32\,b^2\,\mathrm{D}(z_1,z_2),\\ \mathrm{D}(z_1,z_2) &= \frac{\eta^2\log^2\eta}{64(\eta-1)^2}-\frac{4\eta+9}{96\;(\eta-1)}-\frac{\eta\log\eta}{48(\eta-1)}+\log(1-\eta)\,\left[\frac{1}{48}-\frac{(\eta+1)\log\eta}{48(\eta\;-1)}\right]\\ &\quad -\frac{(\eta+1)\mathrm{Li}_2(\eta)}{96(\eta-1)}-\frac{39+\pi^2}{576}\frac{1+\eta}{1-\eta}\,. \end{aligned} \tag{6.4}$$

Insertion diagram. The second contribution is due to the insertion of the $3\,b^2\,\frac{\zeta^2}{2!}$ vertex in (5.1) (see (A.16) in [4])

$$D_{\rm ins}(z_1,z_2) =$$

$$\begin{aligned} &= -3b^2\int \mathsf{d}^2z'\,g(z_1,z')\,g(z',z_2) = -3\,b^2\,\widehat{B}(z_1,z_2),\\ \widehat{B}(z_1,z_2) &= -\frac{\eta\log\eta}{6(\eta-1)}+\log(1-\eta)\left[\frac{1}{6}-\frac{(\eta+1)\log\eta}{12(\eta-1)}\right]-\frac{(\eta+1)\mathrm{Li}_2(1-\eta)}{6(\eta-1)}-\frac{1}{6}\,. \end{aligned} \tag{6.5}$$

Fermionic loop. The third contribution is that of the fermion loop diagram[18]

$$D_{\rm f}(z_1,z_2) =$$

$$= (2b)^2\frac{1}{2}(-1)\int \mathsf{d}^2z\mathsf{d}^2z'\,\mathrm{Tr}[\mathsf{S}(z,z')\mathsf{S}(z',z)]\,g(z_1,z)\,g(z',z_2). \tag{6.6}$$

To compute this integral it is convenient to first go back to the half-plane coordinates and use (5.22). Then

$$\begin{aligned}\mathrm{Tr}[\mathsf{S}(w,w')\mathsf{S}(w',w)] &= -\mathrm{Tr}\Big[(\partial\!\!\!/+1)g(w,w')R(w,w')R(w',w)(\partial\!\!\!/-1)g(w,w')\Big],\\ &= -\mathrm{Tr}\Big[(\partial\!\!\!/+1)g(w,w')(\partial\!\!\!/-1)g(w,w')\Big] = -2\left(\partial^\mu g\partial_\mu g-g^2\right),\end{aligned} \tag{6.7}$$

where all derivatives are over w. As a result,

$$\begin{aligned}\frac{1}{4b^2}\,D_{\rm f}(z_1,z_2) &= \int \mathsf{d}^2z\;\mathsf{d}^2z'\;g_2(z_1,z)\Big(\partial^\mu g(z,z')\partial_\mu g(z,z')-[g(z,z')]^2\Big)\,g(z',z_2)\\ &= -\mathrm{D}(z_1,z_2)+\int \mathsf{d}^2z\;g(z_1,z)B_{\partial\partial}(z,z_2),\end{aligned} \tag{6.8}$$

[18]For a detailed discussion of Feynman rules for Majorana fermion see for instance [42, 43].

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where the function $B_{\partial\partial}$ reads (see appendix D of [6])

$$B_{\partial\partial}(z_1, z_2) = -\frac{3}{8} + \frac{1+\eta}{4\,(\eta-1)}\log\eta - \frac{1+\eta^2}{16(\eta-1)^2}\log^2\eta. \tag{6.9}$$

This function was found in [6] by integrating by parts and assuming $\nabla^2 g \to 2g$, i.e. removing the δ-function consistently with the renormalization condition in (5.23). Using the method in appendix A of [4], a straightforward calculation then gives the (finite) expression for the disk integral

$$\begin{aligned}\int \mathsf{d}^2 z\; g(z_1, z)\, B_{\partial\partial}(z, z_2) = &-\frac{(\eta-2)\eta\log^2\eta}{64(\eta-1)^2} + \frac{\eta\log\eta}{48(\eta-1)} + \log(1-\eta)\left[\frac{(\eta+1)\log\eta}{96(\eta-1)} - \frac{1}{48}\right] \\ &- \frac{(\eta+1)\mathrm{Li}_2(1-\eta)}{96(\eta-1)} - \frac{11}{192}.\end{aligned} \tag{6.10}$$

Total one-loop correction. Combining the contributions (6.4), (6.5) and (6.6), we get for (6.3)

$$D_1 = b^2\left\{\frac{\pi^2(\eta+1)}{8(\eta-1)} + \frac{\eta(3\eta+1)\log^2\eta}{8(\eta-1)^2} - \frac{3(\eta+1)\log(1-\eta)\log\eta}{4(\eta-1)} - \frac{3(\eta+1)\mathrm{Li}_2(\eta)}{4(\eta-1)} + 1\right\}. \tag{6.11}$$

Taking the boundary limit, we find the 1-loop correction to the boundary two-point correlator

$$\langle \Phi(\mathsf{t}_1)\Phi(\mathsf{t}_2)\rangle_1 = \lim_{\mathsf{z}_1,\mathsf{z}_2\to 0} \mathsf{z}_1^{-2}\mathsf{z}_2^{-2}\, D_1(x_1, x_2) = -\frac{1}{3}b^2\frac{1}{(\mathsf{t}_1-\mathsf{t}_2)^4}\;. \tag{6.12}$$

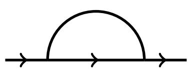
It agrees with the duality prediction in (4.40), (4.41).

6.2 Fermion propagator correction

Let us now check the one-loop b^2 term in the expression for the fermionic correlator in (4.40). This requires computing the boundary limit of the one-loop corrected $\langle\psi(x_1)\psi(x_2)\rangle$ (or S_{12}) component of the fermion propagator (cf. (5.11), (6.2)) given by the sum of the contributions of the scalar exchange with two $b\zeta\bar\Psi\Psi$ vertices from (5.1) and the insertion of the $b^2\bar\Psi\Psi$ vertex in (5.1)[19]

$$\begin{aligned}\langle\Psi(w_1)\bar\Psi(w_2)\rangle &= \mathsf{S}(w_1, w_2) + \mathsf{S}_1(w_1, w_2) + \mathcal{O}(b^4),\\ \mathsf{S}_1(w_1, w_2) &= \mathsf{S}_{\rm sc}(w_1, w_2) + \mathsf{S}_{\rm ins}(w_1, w_2)\;.\end{aligned} \tag{6.13}$$

Scalar exchange. The first contribution in (6.13) is given by

$$\mathsf{S}_{\rm sc}(w_1, w_2) =$$  $$= (2b)^2\, I(w_1, w_2)\,, \tag{6.14}$$

$$I(w_1, w_2) = \int \mathsf{d}^2 w\, \mathsf{d}^2 w'\; \mathsf{S}(w_1, w)\,\mathsf{S}(w, w')\,\mathsf{S}(w', w_2)\, g(w, w')\;. \tag{6.15}$$

[19] Here we will use the half-plane parametrization $w = \mathsf{t} + i\mathsf{z}$.

Using the representation (5.22), the integrand in I can be rewritten as

$$\begin{aligned} I(w_1,w_2) &= -\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}(w_1,w)\,[(\not{\partial}+1)g(w,w')]\,R(w,w')\mathsf{S}(w',w_2)\,g(w,w') \\ &= -\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}(w_1,w)\,(1+\frac{1}{4}\Gamma^{\mathsf{z}})\,R(w,w')\mathsf{S}(w',w_2)\,[g(w,w')]^2 \\ &\quad -\frac{1}{2}\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}(w_1,w)\,\not{D}[g(w,w')]^2\,R(w,w')\mathsf{S}(w',w_2)\,. \end{aligned} \tag{6.16}$$

The second term can be simplified using integration by parts: then the derivative can either act on $\mathsf{S}(w_1,w)$ and the measure d^2w on the left hand side, or on $R(w,w')$ on the right on side.[20] In view of (5.10) we thus find

$$\begin{aligned} I(w_1,w_2) &= -\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}(w_1,w)\left(1+\frac{1}{4}\Gamma^{\mathsf{z}}\right)R(w,w')\mathsf{S}(w',w_2)\,[g(w,w')]^2 \\ &\quad -\frac{1}{2}\int \mathsf{d}^2w\,\mathsf{d}^2w'\left(\mathsf{S}(w_1,w)+\frac{2\pi}{\sqrt{g(w)}}\delta^{(2)}(w-w_1)\right)[g(w,w')]^2\,R(w,w')\mathsf{S}(w',w_2) \\ &\quad -\frac{1}{2}\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}(w_1,w)\left(1-\frac{1}{2}\Gamma^{\mathsf{z}}\right)R(w,w')\mathsf{S}(w',w_2)[g(w,w')]^2 \\ &= -\frac{1}{2}\int \mathsf{d}^2w\,\mathsf{d}^2w'\left(4\mathsf{S}(w_1,w)+\frac{2\pi}{\sqrt{g(w)}}\delta^{(2)}(w-w_1)\right)[g(w,w')]^2\,R(w,w')\mathsf{S}(w',w_2) \\ &\equiv I^{(0)}+I^{(1)}\,, \end{aligned} \tag{6.17}$$

where we also used that $\not{\partial}R(w,w') = \left(\not{D}+\frac{1}{2}\Gamma^{\mathsf{z}}\right)R(w,w') = \left(-1+\frac{1}{2}\Gamma^{\mathsf{z}}\right)R(w,w')$. Thus the integral $I(w_1,w_2)$ reduces to the sum of two pieces where the first in (6.17) is the one without δ-function

$$I^{(0)}(w_1,w_2) = -2\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}(w_1,w)\,R(w,w')\mathsf{S}(w',w_2)\,[g(w,w')]^2\,. \tag{6.18}$$

Taking both end points to the boundary, we get

$$\begin{aligned} \mathcal{I}^{(0)}(\mathsf{t}_1,\mathsf{t}_2) &= \lim_{\mathsf{z}_1,\mathsf{z}_2\to 0}\mathsf{z}_1^{-3/2}\mathsf{z}_2^{-3/2}I^{(0)}(w_1,w_2) \\ &= -2\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}_\partial(\mathsf{t}_1;w)\,R(w,w')\mathsf{S}_\partial(w';\mathsf{t}_2)\,[g(w,w')]^2 \\ &= 18\mathsf{t}_{12}\Gamma^{\mathsf{t}}\mathcal{P}_-\int \mathsf{d}^2w\,\mathsf{d}^2w'\,g_\partial(\mathsf{t}_1;w)g_\partial(\mathsf{t}_2;w')[g(w,w')]^2\,, \end{aligned} \tag{6.19}$$

[20] Explicitly, using (C.11), we get:

$$\begin{aligned} I(w_1,w_2) &= -\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}(w_1,w)\left(1+\frac{1}{4}\Gamma^{\mathsf{z}}\right)R(w,w')\mathsf{S}(w',w_2)\,[g(w,w')]^2 \\ &\quad -\frac{1}{2}\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}(w_1,w)\,(-\overleftarrow{\not{D}})[g(w,w')]^2\,R(w,w')\mathsf{S}(w',w_2) \\ &\quad -\frac{1}{2}\int \mathsf{d}^2w\,\mathsf{d}^2w'\,\mathsf{S}(w_1,w)\,(-\not{\partial}R(w,w'))\mathsf{S}(w',w_2)[g(w,w')]^2\,. \end{aligned}$$

where S_∂ is the bulk-to-boundary propagator in (5.14), (5.15). It is easy to see that the integral here corresponds to the boundary limit of the scalar bubble diagram contribution in (6.4). The term in (6.17) that contains the δ-function is

$$I^{(1)}(w_1, w_2) = -\frac{1}{2}\int \mathsf{d}^2 w\, \mathsf{d}^2 w'\, \frac{2\pi}{\sqrt{g(w)}}\delta^{(2)}(w - w_1)[g(w, w')]^2\, R(w, w')\mathsf{S}(w', w_2)$$
$$= -\frac{1}{4}\int \mathsf{d}^2 w'\, [g(w_1, w')]^2\, R(w_1, w')\mathsf{S}(w', w_2)\ . \tag{6.20}$$

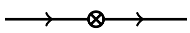
Using again (5.22) and computing the integral as in (A.14) in [4] we get

$$I^{(1)}(w_1, w_2) = -\frac{1}{4}\int \mathsf{d}^2 w'\, [g(w_1, w')]^2\, R(w_1, w')R(w', w_2)(\not{\partial}_{w_2} - 1)g(w', w_2)$$
$$= -\frac{1}{4}R(w_1, w_2)(\not{\partial}_{w_2} - 1)\int \mathsf{d}^2 w'\, [g(w_1, w')]^2\, g(w', w_2)$$
$$= -\frac{1}{32}R(w_1, w_2)(\not{\partial}_2 - 1)\left[1 - \frac{\eta \log^2 \eta}{(1-\eta)^2}\right], \qquad \eta \equiv \eta(w_1, w_2)\ . \tag{6.21}$$

Sending the legs to the boundary gives finally

$$\mathcal{I}^{(1)}(\mathsf{t}_1, \mathsf{t}_2) = \lim_{\mathsf{z}_1\to 0, \mathsf{z}_2\to 0} \mathsf{z}_1^{-3/2}\mathsf{z}_2^{-3/2} I^{(1)}(w_1, w_2) = -\frac{1}{8}\frac{1}{\mathsf{t}_{12}^3}\Gamma^{\mathsf{t}}\mathcal{P}_-\ . \tag{6.22}$$

Insertion diagram. The second contribution in (6.13) is given by

$$S_{\text{ins}}(z_1, z_2) = $$  $$= b^2\, J(w_1, w_2), \qquad J(w_1, w_2) = \int \mathsf{d}^2 w\ \mathsf{S}(w_1, w)\, \mathsf{S}(w, w_2)\ . \tag{6.23}$$

Taking the boundary limit and using the relation (5.17), we get

$$\mathcal{J}(\mathsf{t}_1, \mathsf{t}_2) = \lim_{\mathsf{z}_1, \mathsf{z}_2\to 0} \mathsf{z}_1^{-3/2}\mathsf{z}_2^{-3/2} J(w_1, w_2) = \int \mathsf{d}^2 w\ \mathsf{S}_\partial(\mathsf{t}_1; w)\, \mathsf{S}_\partial(w; \mathsf{t}_2)$$
$$= -9\, \mathsf{t}_{12}\, \Gamma^{\mathsf{t}}\, \mathcal{P}_- \int \mathsf{d}^2 w\ g_\partial(\mathsf{t}_1; x)\, g_\partial(\mathsf{t}_2; x)\ . \tag{6.24}$$

Total one-loop correction. Summing up the contributions in (6.19), (6.22) and (6.24) with appropriate coupling factors, we get the full one-loop correction to the boundary two-fermion correlator. Projecting to the relevant correlator $\langle\psi(x_1)\psi(x_2)\rangle$ (or 12 matrix element of the spinor matrix propagator) using $(\Gamma^{\mathsf{t}}\mathcal{P}_-)_{12} = \frac{1}{2}$ we find

$$\langle\Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\rangle_1 = \Big[(2b)^2(\mathcal{I}^{(0)} + \mathcal{I}^{(1)}) + b^2\mathcal{J}\Big]_{12} = 36\, b^2\, \mathsf{t}_{12}\, Y(\mathsf{t}_1, \mathsf{t}_2) - \frac{b^2}{4\mathsf{t}_{12}^3}\,, \tag{6.25}$$
$$Y = \int \mathsf{d}^2 w\, \mathsf{d}^2 w'\ g_\partial(\mathsf{t}_1; w)\, g(w, w')\, g_\partial(w'; \mathsf{t}_2) - \frac{1}{8}\int \mathsf{d}^2 w\, \mathsf{d}^2 w'\ g_\partial(\mathsf{t}_1; w)\, g_\partial(w; \mathsf{t}_2)\,. \tag{6.26}$$

The expression for the combination of the bosonic integrals in (6.26) can be found from the boundary limit of eq. (A17) in [4]:

$$Y = \frac{1}{32}\lim_{\mathsf{z}_1, \mathsf{z}_2\to 0}\frac{1}{\mathsf{z}_1^2\mathsf{z}_2^2}\left[\frac{3}{2} + \frac{\eta_{12}^2\, \log^2\eta_{12}}{2\,(1-\eta_{12})^2} - \frac{1+\eta_{12}}{1-\eta_{12}}\,\mathrm{Li}_2(1-\eta_{12})\right] = -\frac{1}{144}\frac{1}{\mathsf{t}_{12}^4}, \tag{6.27}$$

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Thus finally we get

$$\langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\rangle_1 = b^2 \left(-\frac{36}{144} - \frac{1}{4} \right) \frac{1}{\mathsf{t}_{12}^3} = -\frac{1}{2} b^2 \frac{1}{\mathsf{t}_{12}^3} , \tag{6.28}$$

which is again in agreement with the one-loop prediction for the fermionic correlator in (4.40), (4.41).

7 Three-point functions

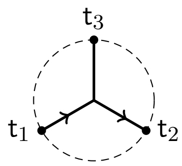
In this section we shall first discuss the tree level values of the non-vanishing three-point function of three scalars and three-point function of one scalar and two fermions, then compute the one-loop correction to the three-scalar correlator, verifying the duality predictions in (4.42), (4.43), (4.44).

7.1 Tree level

At the tree level, the three-point function $\langle \Phi\Phi\Phi \rangle$ is the same as in the bosonic Liouville theory, i.e. we have [4]

$$C_{\Phi\Phi\Phi} = -\frac{16}{9} b + \mathcal{O}(b^3) , \tag{7.1}$$

which agrees with (4.43). This follows from the fact that the $b\zeta^3$ term in the action (3.22), (5.1) is the same as in the Liouville theory. The tree-level correlator $\langle \Psi\Psi\Phi \rangle$ is given by the following Witten diagram (see (5.7), (5.13)–(5.15))



$$= 2b \int \mathsf{d}^2 w \, \mathsf{S}_\partial(\mathsf{t}_1; w) \mathsf{S}_\partial(w; \mathsf{t}_2) g_\partial(\mathsf{t}_3; w) \ . \tag{7.2}$$

Using (5.17) and picking up the $\psi\psi$ (i.e. 12) component gives (see (5.18))

$$\left[\int \mathsf{d}^2 w \, \mathsf{S}_\partial(\mathsf{t}_1; w) \mathsf{S}_\partial(w; \mathsf{t}_2) g_\partial(\mathsf{t}_3; w) \right]_{12} = -\frac{9}{2} \mathsf{t}_{12} \int \mathsf{d}^2 w \, g_\partial(\mathsf{t}_1; w) g_\partial(w; \mathsf{t}_2) g_\partial(\mathsf{t}_3; w) \ . \tag{7.3}$$

The integral here is the same as in the scalar three-point function [4] so that we get

$$\langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\Phi(\mathsf{t}_3)\rangle = 2b \cdot \left(-\frac{9}{2} \mathsf{t}_{12} \right) \cdot \frac{2}{9} \frac{1}{\mathsf{t}_{12}^2 \mathsf{t}_{13}^2 \mathsf{t}_{23}^2} = -2b \frac{1}{\mathsf{t}_{12} \mathsf{t}_{13}^2 \mathsf{t}_{23}^2} , \tag{7.4}$$

in agreement with (4.44).

7.2 One-loop correction to three-scalar correlator

According to the duality relation (4.43) the boundary correlator of three ζ fields should have the coefficient

$$C_{\Phi\Phi\Phi} = C_0 \, b + C_1 \, b^3 + \cdots , \qquad C_0 = -\frac{16}{9}, \qquad C_1 = \frac{16}{9}. \tag{7.5}$$

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The one-loop b^3 contribution comes from several diagrams. In addition to the bosonic (scalar) loop diagrams which are the same as in the Liouville theory computed in [4] and give

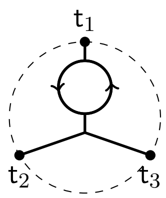
$$C_1^{(\mathrm{sc})} = \frac{64}{27}\,, \tag{7.6}$$

there are also diagrams with fermion loop and also the insertion diagram with the higher-order $b^3\zeta^3$ vertex in (5.1).[21] There will be four different contributions to this additional part of C_1 that we will denote with tilde, i.e.

$$\tilde{C}_1 \equiv C_1 - C_1^{(\mathrm{sc})} = \tilde{C}_1^{(a)} + \tilde{C}_1^{(b)} + \tilde{C}_1^{(c)} + \tilde{C}_1^{(d)} = -\frac{16}{27} = \frac{1}{3}C_0\,. \tag{7.7}$$

Here we gave the expected value of $\tilde{C}_1$ following from (7.5), (7.6) that we confirm below by computing the four contributions to it in turn.

a) Self-energy diagrams. The first type of one-loop diagrams contributing to $\tilde{C}_1$ is the fermionic loop correction to the scalar ζ field propagator (or self-energy correction)



(7.8)

It amounts to the following correction to the bulk to boundary scalar propagator in (5.7)

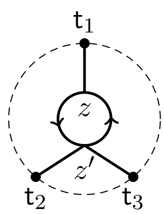
$$g_\partial(\mathsf{t}; z) \to \left(1 - \frac{1}{12}\,b^2 + \cdots\right) g_\partial(\mathsf{t}; z)\,. \tag{7.9}$$

It can be found from the value of $C_{\Phi\Phi}$ in (4.41) (confirmed in (6.12) in the previous section) after subtraction of the bosonic Liouville theory value: from (1.10) and (4.41) we get $\left(\frac{4}{3} - \frac{1}{3}b^2 + \cdots\right) / \left(\frac{4}{3} - \frac{2}{9}b^2 + \cdots\right) = 1 - \frac{1}{12}\,b^2 + \cdots$. Then from (7.9) it follows that

$$\tilde{C}_1^{(a)} = 3 \cdot \left(-\frac{1}{12}\right) \cdot C_0 = -\frac{1}{4}C_0\,, \tag{7.10}$$

where the factor 3 accounts for the fact that each of the three scalar propagators may get a fermion loop correction.

b) Quasi-self-energy diagrams. There are also fermion loop diagrams with one $\zeta\bar{\Psi}\Psi$ and one $\zeta^2\bar{\Psi}\Psi$ vertices in (5.1), i.e.



(7.11)

[21] It comes from the Q-dependent terms in the action (3.22) with subleading term in Q in super Liouvile theory (3.18) having extra 1/2 factor compared to the bosonic Liouville case.

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The contribution of this diagram contains the integral

$$H(z_1, z') = \int \mathsf{d}^2 z\, g(z_1, z)\, \mathrm{Tr}[\mathsf{S}(z, z')\, \mathsf{S}(z', z)] \ . \tag{7.12}$$

Using (6.7) we get

$$\begin{aligned} H(z_1, z') &= -2 \int \mathsf{d}^2 z\, g(z_1, z)\, \partial^\mu g(z, z') \partial_\mu g(z, z') + 2 \int \mathsf{d}^2 z\, g(z_1, z)\, [g(z, z')]^2 \\ &= -2\, B_{\partial\partial}(z_1, z') + 2\, B(z_1, z'), \end{aligned} \tag{7.13}$$

where $B(z_1, z_2) = \frac{1}{8} - \frac{\eta \log^2 \eta}{8(1-\eta)^2}$ is found from (A.14) of [4] and $B_{\partial\partial}$ (from appendix D of [6]) was given already in (6.9). Taking the boundary limit one finds

$$\lim_{\mathsf{z}_1 \to 0} \mathsf{z}_1^{-2} H(z_1, z_2) = \frac{1}{2} g_\partial(\mathsf{t}_1; z_2) \ . \tag{7.14}$$

As a consequence, the upper leg in diagram (7.11) may be effectively replaced by a free propagator so that (7.11) reduces to the tree-level diagram and we get

$$\tilde{C}_1^{(b)} = ((-8b)^{-1}\, bC_0) \cdot \frac{1}{2} \cdot (-1) \cdot (2b) \cdot b^2 \cdot \frac{1}{2!} \cdot 3b^{-3} = \frac{3}{8}\, C_0\,, \tag{7.15}$$

where (-1) is the fermion loop sign, $2b$, b^2 are the vertex couplings, $\frac{1}{2!}$ is a symmetry factor (fermions are Majorana), and we added extra factor of 3 because there are three diagrams like (7.8).

c) Insertion diagram. The cubic ζ^3 vertex in the AdS_2 Liouville action (1.2) expanded near the constant vacuum is multiplied by the factor $Qb = 1 + b^2$ [4]. In the super Liouville theory, the analogous factor is $1 + \frac{7}{8} b^2 + \cdots$, see (5.1). Hence, the extra contribution is simply

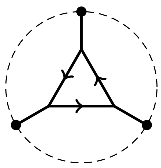
$$\tilde{C}_1^{(c)} = \left(\frac{7}{8} - 1\right) C_0 = -\frac{1}{8}\, C_0 \ . \tag{7.16}$$

Remarkably, the total contribution from the above three types of diagrams given by the sum of (7.10), (7.15) and (7.16) vanishes

$$\tilde{C}_1^{(a)} + \tilde{C}_1^{(b)} + \tilde{C}_1^{(c)} = \left(-\frac{1}{4} + \frac{3}{8} - \frac{1}{8}\right) C_0 = 0 \ . \tag{7.17}$$

d) Fermion loop correction to 3-vertex. Thus the non-zero value of $\tilde{C}_1$ in (7.7) should be solely due to the contribution of the remaining most non-trivial diagram:

$$b^3 \tilde{C}_1^{(d)} = \tag{7.18}$$



$$= \frac{(-1)(2b)^3}{\mathcal{K}_3(\boldsymbol{\theta})} \int \mathsf{d}^2 z_1 \mathsf{d}^2 z_2 \mathsf{d}^2 z_3\, \mathrm{Tr}[\mathsf{S}(z_1, z_2) \mathsf{S}(z_2, z_3) \mathsf{S}(z_3, z_1)] \prod_{i=1}^{3} g_\partial\big(\mathsf{t}(\theta_i); z_i\big) \ .$$

Here we divided by the kinematical prefactor $\mathcal{K}_3 = \mathsf{t}_{12}^{-2}\,\mathsf{t}_{13}^{-2}\,\mathsf{t}_{23}^{-2}$ in the three-point function in (4.42); expressed in terms of the angles on the circular (compactified) boundary of AdS_2 it is given by

$$\mathcal{K}_3(\boldsymbol{\theta}) \equiv \mathcal{K}(\theta_1,\theta_2,\theta_3) = \prod_{i<j}^{3} \big|\mathsf{t}(\theta_i) - \mathsf{t}(\theta_j)\big|^{-2}, \qquad \mathsf{t}(\theta) = -\cot\frac{\theta}{2}\,. \tag{7.19}$$

Computing the trace in (7.18) we get (using (5.8), (5.9) and $u_{ij} \equiv u(z_i, z_j)$)

$$\begin{aligned}
&\mathrm{Tr}[\mathsf{S}(z_1,z_2)\mathsf{S}(z_2,z_3)\mathsf{S}(z_3,z_1)] = \frac{1}{2\,(u_{12}+2)^2}\frac{1}{2\,(u_{13}+2)^2}\frac{1}{2\,(u_{23}+2)^2} \\
&\times \Big\{\mathsf{F}_2(u_{12})\Big[(u_{12}+u_{13}-u_{23})\mathsf{F}_1(u_{23})\mathsf{F}_2(u_{13}) + (u_{12}-u_{13}+u_{23})\mathsf{F}_1(u_{13})\mathsf{F}_2(u_{23})\Big] \\
&- \mathsf{F}_1(u_{12})\Big[(u_{12}+u_{13}+u_{23}+4)\mathsf{F}_1(u_{13})\mathsf{F}_1(u_{23}) + (u_{12}-u_{13}-u_{23})\mathsf{F}_2(u_{13})\mathsf{F}_2(u_{23})\Big]\Big\}\,.
\end{aligned} \tag{7.20}$$

To compute the remaining integral in (7.18) we use numerical integration that can be performed by the `Suave` routine of the `Cuba` library [44] as discussed also in [4, 6]. Our best estimate is

$$\tilde{C}_1^{(d)} = -0.590 \pm 0.002\,. \tag{7.21}$$

At the same time, according (7.7), (7.17) the contribution of this diagram should be

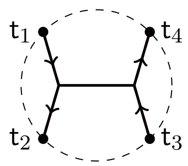
$$\tilde{C}_1^{(d)} = -\frac{16}{27} = -0.59259\ldots\,. \tag{7.22}$$

The Monte Carlo numerical integration (7.21) is compatible with the analytical prediction within the statistical uncertainty and with a deviation below 0.5 % that we consider satisfactory.

8 Four-point functions

Let us now consider the tree-level four-point correlators (4.45)–(4.47). The tree-level expression of the 4-scalar correlator is given in (4.45) and it is the same as in the bosonic Liouville theory [3, 4]. As discussed in section 4.3, the other two non-vanishing four-point correlators $\langle\Psi(\mathsf{t}_1)\Phi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\Phi(\mathsf{t}_4)\rangle$ and $\langle\Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\Psi(\mathsf{t}_4)\rangle$ have their kinematical structure fixed by the conformal symmetry and their coefficients related as in (4.51) to $C_{\Phi\Phi\Phi\Phi}$ in $\langle\Phi(\mathsf{t}_1)\Phi(\mathsf{t}_2)\Phi(\mathsf{t}_3)\Phi(\mathsf{t}_4)\rangle$ by the boundary supersymmetry (4.12). Thus their direct computation (with some details discussed in appendix C) is simply a check of consistency of our perturbation theory rules for the fermions.

For example, let consider the tree-level computation of the (connected part of) 4-fermion correlator $\langle\Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\Psi(\mathsf{t}_4)\rangle$. As follows from the structure of the action (5.1) here the only cointribution comes from the three scalar exchange diagrams with the s-channel one being



(8.1)

Using the relations (5.17), (5.18), the contribution of this diagram reduces to that of the four-scalar diagram with the scalar exchange:

$$
\begin{aligned}
\langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\Psi(\mathsf{t}_4)\rangle_s &= (2b)^2\cdot\frac{-9\mathsf{t}_{12}}{2}\cdot\frac{-9\mathsf{t}_{34}}{2} \\
&\quad\times\int \mathsf{d}^2w\,\mathsf{d}^2w'\, g_\partial(\mathsf{t}_1,w)g_\partial(\mathsf{t}_2,w)g(w,w')g_\partial(\mathsf{t}_3,w')g_\partial(\mathsf{t}_4,w') \\
&= 81\mathsf{t}_{12}\mathsf{t}_{34}b^2\cdot 2\pi\cdot(\frac{4}{3})^4\cdot\frac{1}{(4\pi)^2}\cdot \mathsf{t}_{34}^{-2}D_{2211}(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4) \qquad (8.2)\\
&= 3b^2\frac{\mathsf{t}_{12}\mathsf{t}_{34}}{\mathsf{t}_{13}^4\mathsf{t}_{24}^4}\bar{D}_{2211}(\chi)\,, \qquad (8.3)
\end{aligned}
$$

where $\chi = \frac{\mathsf{t}_{12}\mathsf{t}_{34}}{\mathsf{t}_{13}\mathsf{t}_{24}}$ and the D and $\bar{D}$ functions are defined as, e.g., in [2, 3]. The contributions of the t and u channels can be obtained by permuting the legs. Taking into account the fermi statistics, they are $\langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\Psi(\mathsf{t}_4)\rangle_t = -\langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_3)\Psi(\mathsf{t}_2)\Psi(\mathsf{t}_4)\rangle_s$, $\langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\Psi(\mathsf{t}_4)\rangle_u = -\langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_4)\Psi(\mathsf{t}_3)\Psi(\mathsf{t}_2)\rangle_s$. As a result, summing the three contributions we get the connected part of the tree-level correlator

$$
\langle \Psi(\mathsf{t}_1)\Psi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\Psi(\mathsf{t}_4)\rangle_{0,\mathrm{conn}} = 3b^2\frac{1}{\mathsf{t}_{13}^3\mathsf{t}_{24}^3}\,\frac{1}{\chi(1-\chi)}\,. \qquad (8.4)
$$

This is in agreement with the tree-level of the duality prediction in (4.47), (4.50).

We discuss the tree-level computation of the mixed $\langle\Psi\Phi\Psi\Phi\rangle$ correlator in appendix C. The result is again in agreement with the leading-order value in (4.46), (4.49), i.e.

$$
\langle \Psi(\mathsf{t}_1)\Phi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\Phi(\mathsf{t}_4)\rangle_{0,\mathrm{conn}} = \frac{4}{3}b^2\frac{1}{\mathsf{t}_{13}^3\mathsf{t}_{24}^4}\,\frac{3-4\chi+4\chi^2}{2\chi^2(1-\chi)^2}\,. \qquad (8.5)
$$

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A Majorana fermion action in Euclidean AdS$_2$

Let us list our conventions for the fermionic fields. We start with the flat Euclidean space with metric $ds^2 = dx^a dx^a \equiv d\mathsf{t}^2 + dz^2 = dw\,d\bar{w}$. We shall use the following representation for the Clifford algebra of Dirac matrices $\{\Gamma^a,\Gamma^b\} = 2\delta^{ab}$

$$
\Gamma^{\mathsf{t}} = \Gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Gamma^{\mathsf{z}} = \Gamma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad (A.1)
$$

with the chirality matrix Γ_* and the charge conjugation matrix $\mathcal{C}$ (satisfying $\mathcal{C}\,\Gamma^a\,\mathcal{C}^{-1} = -(\Gamma^a)^T$) defined as

$$
\Gamma_* = i\Gamma^1\Gamma^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \mathcal{C} = \Gamma^2, \qquad \mathcal{C} = \mathcal{C}^\dagger = \mathcal{C}^{-1}, \qquad (A.2)
$$

The Dirac spinor and its charge-conjugate then are

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \qquad \Psi^c = \mathcal{C}\,\Psi^* = \begin{pmatrix} -i\bar{\psi}_2 \\ i\bar{\psi}_1 \end{pmatrix}, \tag{A.3}$$

where ψ_1, ψ_2 are complex and $\bar{\psi}$ denotes complex conjugation of ψ.

Since $(\Psi^c)^c = -\Psi$, a Majorana fermion may be defined as satisfying $\Psi^c = \Gamma_* \Psi$; then $\psi_2 = -i\bar{\psi}_1$ (cf., e.g., appendix A in [45]). Hence, the explicit form of a Majorana fermion Ψ and its Majorana conjugate $\bar{\Psi}$ are

$$\Psi = \begin{pmatrix} \psi \\ -i\bar{\psi} \end{pmatrix}, \qquad \bar{\Psi} \equiv i\Psi^T \mathcal{C} = \begin{pmatrix} i\bar{\psi} & \psi \end{pmatrix} . \tag{A.4}$$

For any two arbitrary Majorana spinors Ψ_1 and Ψ_2, the product $\bar{\Psi}_1 \Psi_2$ is invariant under SO(2) rotation and the following identities hold

$$\bar{\Psi}_1 \Psi_2 = \bar{\Psi}_2 \Psi_1, \qquad \bar{\Psi}_1 \Gamma^a \Psi_2 = -\bar{\Psi}_2 \Gamma^a \Psi_1. \tag{A.5}$$

The action for a Majorana Ψ in flat 2d space is

$$\mathcal{S} = \int d^2x \bar{\Psi}(\Gamma^a \partial_a - m)\Psi = 2\int d^2x \Big(\psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi} - im\bar{\psi}\psi\Big), \tag{A.6}$$

where

$$w = \mathsf{t} + i\mathsf{z}, \qquad \partial \equiv \partial_w = \frac{1}{2}(\partial_{\mathsf{t}} - i\partial_{\mathsf{z}}), \qquad \bar{\partial} \equiv \partial_{\bar{w}} = \frac{1}{2}(\partial_{\mathsf{t}} + i\partial_{\mathsf{z}}) . \tag{A.7}$$

Let us now consider the AdS$_2$ space with metric

$$ds^2 = \frac{d\mathsf{t}^2 + d\mathsf{z}^2}{\mathsf{z}^2} . \tag{A.8}$$

The zweibein is $e^a_\mu = \frac{1}{\mathsf{z}}\,\delta^a_\mu$, $e^\mu_a = \mathsf{z}\,\delta^\mu_a$,[22] so that the spin-connection is $\omega^{ab}_\mu = \frac{1}{\mathsf{z}}(\delta^a_{\mathsf{z}}\delta^b_\mu - \delta^b_{\mathsf{z}}\delta^a_\mu)$. The Dirac operator is then

$$D_\mu = \partial_\mu + \frac{1}{2}\omega^{bc}_\mu \Omega_{bc}, \qquad \not{D} \equiv \gamma^\mu D_\mu = e^\mu_a \Gamma^a \left(\partial_\mu + \frac{1}{2}\omega^{bc}_\mu \Omega_{bc}\right) = \mathsf{z}\Gamma^a \partial_a - \frac{1}{2}\Gamma^{\mathsf{z}}, \tag{A.9}$$

where $\Omega_{ab} = \frac{1}{4}[\Gamma^a, \Gamma^b]$ and the "curved" γ_μ matrices are defined as

$$\gamma^\mu = e^\mu_a \Gamma^a, \qquad \frac{1}{\mathsf{z}}\gamma^{\mathsf{t}} = \mathsf{z}\,\gamma_{\mathsf{t}} = \Gamma^{\mathsf{t}}, \qquad \frac{1}{\mathsf{z}}\gamma^{\mathsf{z}} = \mathsf{z}\,\gamma_{\mathsf{z}} = \Gamma^{\mathsf{z}}. \tag{A.10}$$

The action for a Majorana spinor in AdS$_2$ is then (cf. (A.6))

$$\mathcal{S} = \int d^2x \sqrt{g}\,\bar{\Psi}\,(\not{D} - m)\,\Psi = 2\int \frac{d\mathsf{t}d\mathsf{z}}{\mathsf{z}^2}\,\big(\mathsf{z}\,\psi\bar{\partial}\psi + \mathsf{z}\,\bar{\psi}\partial\bar{\psi} - im\bar{\psi}\psi\big). \tag{A.11}$$

The spin connection term here dropped out because of the anti-commuting property of the fermions.

[22] We will use Greek letters to denote a curved index and Latin letters to denote a flat index.

B Propagators in AdS

B.1 Scalar and fermion propagators in AdS_{d+1}

Consider a Euclidean AdS_{d+1} with the metric

$$ds^2 = \frac{1}{z_0^2}\left(dz_0^2 + d\vec{z}^2\right) = \frac{1}{z_0^2}\left(dz_0^2 + \sum_{i=1}^{d} dz_i^2\right) . \tag{B.1}$$

Here we use subscript "0" to label the radial direction z_0 and $\vec{z} = (z_1, z_2, \ldots, z_d)$ (with $z_0 \equiv \mathsf{z}$ and $z_1 \equiv \mathsf{t}$ in AdS_2 case). The tangent-space Γ-matrices satisfy $\{\Gamma^a, \Gamma^b\} = 2\delta^{ab}$ with $a = 0, 1, \ldots, d$.

The bulk-to-bulk propagator G_Δ for a scalar field with mass m defined via

$$\big(-\nabla^2 + m^2\big) G_\Delta(z, z') = \frac{1}{\sqrt{g(z)}} \delta^{(d+1)}(z - z') , \tag{B.2}$$

is given by (see, e.g., [46])

$$G_\Delta(z, z') = G_\Delta(u(z, z')) = C_\Delta \left(\frac{2}{u}\right)^\Delta {}_2F_1\left(\Delta, \Delta - \frac{d}{2} + \frac{1}{2}; 2\Delta - d + 1; -\frac{2}{u}\right), \tag{B.3}$$

$$m^2 = \Delta(\Delta - d), \quad u(z, z') = \frac{(z_0 - z_0')^2 + (\vec{z} - \vec{z}\,')^2}{2 z_0 z_0'}, \quad C_\Delta = \frac{1}{2^{2\Delta+1}\pi^{d/2}} \frac{\Gamma(\Delta)}{\Gamma\left(\Delta - \frac{d}{2} + 1\right)} . \tag{B.4}$$

Its equivalent form is

$$G_\Delta(u) = (2\xi)^\Delta C_\Delta\, {}_2F_1\left(\frac{\Delta}{2}, \frac{\Delta + 1}{2}, \Delta - \frac{d}{2} + 1, \xi^2\right), \qquad \xi \equiv \frac{1}{u + 1} . \tag{B.5}$$

The bulk-to-boundary propagator is

$$K_\Delta(z, \vec{x}) = c_\Delta \tilde{K}_\Delta(z, \vec{x}), \quad \tilde{K}_\Delta(z, \vec{x}) = \left(\frac{z_0}{z_0^2 + (\vec{z} - \vec{x})^2}\right)^\Delta, \quad c_\Delta = \frac{\Gamma(\Delta)}{\pi^{d/2}\Gamma\left(\Delta - \frac{d}{2}\right)}. \tag{B.6}$$

Up to a normalization factor, it is the boundary limit of the bulk-to-bulk propagator

$$K_\Delta(z, \vec{x}) = \mathcal{N} \lim_{x_0 \to 0} x_0^{-\Delta} G_\Delta(z, x), \qquad \mathcal{N} = \frac{c_\Delta}{4^\Delta C_\Delta} = 2\Delta - d . \tag{B.7}$$

For a Dirac field (with mass parameter $m > 0$) with the action $\int d^{d+1}z \sqrt{g}\, \bar{\Psi}(\not{D} - m)\Psi$ the propagator satisfying[23]

$$(\not{D}_z - m)\, S(z, z') = S(z, z')(-\overleftarrow{\not{D}}_{z'} - m) = \frac{1}{\sqrt{g}}\, \delta^{(d+1)}(z - z'), \tag{B.8}$$

[23]Here $\not{D} f = z_0 \Gamma^a \partial_a f - \frac{d}{2}\Gamma^0 f$, $f \overleftarrow{\not{D}} = z_0 \partial_a f \Gamma^a - \frac{d}{2} f \Gamma^0$.

was presented in various equivalent forms in [38, 47–49], e.g.,

$$S(z,z') = -\frac{1}{\sqrt{z_0 z_0'}}\Big[(\not{z}\mathcal{P}_- - \mathcal{P}_+\not{z}')G'_{\Delta_-}(z,z') + (\not{z}\mathcal{P}_+ - \mathcal{P}_-\not{z}')G'_{\Delta_+}(z,z')\Big], \tag{B.9}$$

$$S(z,z') = -\Big(\not{D} + \Gamma^0 + m\Big)\Big[z_0^{-1/2}\Big(G_{\Delta_-}(z,z')\mathcal{P}_- + G_{\Delta_+}(z,z')\mathcal{P}_+\Big) z_0'^{1/2}\Big], \tag{B.10}$$

$$\begin{aligned} S(z,z') = &-\frac{1}{\pi^{d/2}\, 2^{m+(d+3)/2}} \frac{\Gamma\left(m + \frac{d+1}{2}\right)}{\Gamma\left(m+\frac{1}{2}\right)} \frac{1}{(u+2)^{m+(d+1)/2}} \frac{1}{\sqrt{z_0 z_0'}} \\ &\times \Big[\Big(z_a\Gamma^a\Gamma^0 + \Gamma^0\Gamma^a z_a'\Big)\mathsf{F}_1(u) - (z-z')_a\Gamma^a\mathsf{F}_2(u)\Big] . \end{aligned} \tag{B.11}$$

Here $\not{z} = \Gamma_a z^a$, $\Delta_\pm = m + \frac{d}{2} \pm \frac{1}{2}$, and

$$\mathcal{P}_\pm = \frac{1}{2}(1 \pm \Gamma^0), \qquad G'_\Delta(z,z') = \frac{dG_\Delta(u(z,z'))}{du}, \tag{B.12}$$

$$\begin{aligned} \mathsf{F}_1(u) &= {}_2F_1\left(m + \frac{d+1}{2}, m; 2m+1; \frac{2}{u+2}\right), \\ \mathsf{F}_2(u) &= {}_2F_1\left(m + \frac{d+1}{2}, m+1; 2m+1; \frac{2}{u+2}\right). \end{aligned} \tag{B.13}$$

The bulk-to-boundary fermionic propagator is given by [38]

$$\Sigma_\Delta(z,\vec{x}) = U(z,\vec{x})K_\Delta(z,\vec{x})\mathcal{P}_- \,, \tag{B.14}$$

$$\bar{\Sigma}_\Delta(z,\vec{x}) = \mathcal{P}_+ K_\Delta(z,\vec{x})U(z,\vec{x})\,, \qquad U(z,\vec{x}) = \frac{1}{\sqrt{z_0}}\Gamma^a(z_a - x_a)\Big|_{x_0=0}\,, \tag{B.15}$$

where $\Delta = \Delta_+ = m + \frac{d}{2} + \frac{1}{2}$. Its relation to the bulk-to-bulk propagator is

$$\Sigma(z,\vec{x}) = \lim_{x_0\to 0} x_0^{-\Delta_f} S(z,x), \qquad \bar{\Sigma}(z,\vec{x}) = \lim_{x_0\to 0} -x_0^{-\Delta_f} S(x,z)\,, \tag{B.16}$$

where $\Delta_f = \Delta_+ - \frac{1}{2} = m + \frac{d}{2}$ is the dimension of a CFT$_d$ operator dual to the fermion field.

B.2 Component form of fermion propagator in AdS$_2$

To check consistency of the above general expressions when applied to the Majorana fermion in AdS$_2$ case, here we shall explicitly determine the fermion propagator in AdS$_2$ by starting with the component action (A.6) for a Majorana fermion, i.e.

$$\begin{aligned} \mathcal{S} &= \int d^2x\sqrt{g}\bar{\Psi}(\not{D} - m)\Psi = 2\int \frac{d\mathsf{t}d\mathsf{z}}{\mathsf{z}^2}\Big(\mathsf{z}\psi\bar{\partial}\psi + \mathsf{z}\bar{\psi}\partial\bar{\psi} - im\bar{\psi}\psi\Big) \\ &= 2\int d\mathsf{t}d\mathsf{z}\Big(\eta\bar{\partial}\eta + \bar{\eta}\partial\bar{\eta} - \frac{im}{\mathsf{z}}\bar{\eta}\eta\Big), \qquad \psi = \mathsf{z}^{1/2}\eta, \qquad \bar{\psi} = \mathsf{z}^{1/2}\bar{\eta}\ . \end{aligned} \tag{B.17}$$

The equations of motion for the η fields are

$$\bar{\partial}\eta - \frac{m}{w-\bar{w}}\bar{\eta} = 0, \qquad \partial\bar{\eta} + \frac{m}{w-\bar{w}}\eta = 0, \tag{B.18}$$

implying, in particular,

$$\left[\partial\bar{\partial} + \frac{1}{w-\bar{w}}\bar{\partial} + \frac{m^2}{(w-\bar{w})^2}\right]\eta = 0\ . \tag{B.19}$$

Let us make the following Ansatz for the propagator, i.e. the free two-point correlator ($w = \mathsf{t} + i\mathsf{z}$),

$$\langle \eta(w,\bar{w})\eta(w',\bar{w}')\rangle = \frac{\bar{w}-\bar{w}'}{(w-\bar{w})(w'-\bar{w}')}F(u)\,, \tag{B.20}$$

$$u = \frac{(\mathsf{z}-\mathsf{z}')^2+(\mathsf{t}-\mathsf{t}')^2}{2\mathsf{z}\mathsf{z}'} = -2\frac{(w-w')(\bar{w}-\bar{w}')}{(w-\bar{w})(w'-\bar{w}')}\ . \tag{B.21}$$

Then using (B.19) we get the following differential equation for $F(u)$

$$\left[u(u+2)\frac{d^2}{du^2}+(4+3u)\frac{d}{du}+(1-m^2)\right]F(u)=0\ . \tag{B.22}$$

The most general solution to (B.22) is ($U \equiv \frac{2}{2+u}$)

$$F(u) = \mathsf{c}_1 U^{m+1}\,{}_2F_1(m+1,m+1;2m+1;U)+\mathsf{c}_2 U^{1-m}\,{}_2F_1(1-m,1-m;1-2m;U)\ . \tag{B.23}$$

To compute the mixed propagator $\langle \eta\bar{\eta}\rangle$ we may use (B.18) in order to write

$$\langle \bar{\eta}(w,\bar{w})\eta(w',\bar{w}')\rangle = \frac{w-\bar{w}}{m}\bar{\partial}\,\langle \eta(w,\bar{w})\eta(w',\bar{w}')\rangle\ . \tag{B.24}$$

From (B.20) and (B.23), we then obtain

$$\begin{aligned}\langle \bar{\eta}(w,\bar{w})\eta(w',\bar{w}')\rangle = \frac{w-\bar{w}'}{(w-\bar{w})(w'-\bar{w}')}\Big[&-\mathsf{c}_1 U^{1+m}\,{}_2F_1(m,m+1;2m+1;U)\\ &+\mathsf{c}_2 U^{1-m}\,{}_2F_1(1-m,-m;1-2m;U)\Big]\ .\end{aligned} \tag{B.25}$$

The correlators $\langle \bar{\eta}\bar{\eta}\rangle$ and $\langle \eta\bar{\eta}\rangle$ can be obtained by the complex conjugation. The coefficients $\mathsf{c}_1, \mathsf{c}_2$ in (B.23) may be determined by inspection of the short distance limit. Assuming $m > 0$, we find

$$F(u) = \frac{2\mathsf{c}_1}{u}\frac{\Gamma(2m+1)}{\Gamma(m+1)^2}+\cdots\,, \quad \langle \eta(w,\bar{w})\eta(w',\bar{w}')\rangle = -\mathsf{c}_1\frac{\Gamma(2m+1)}{\Gamma(m+1)^2}\frac{1}{w-w'}+\cdots\,, \quad w\to w'\,. \tag{B.26}$$

On the other hand, taking $u \to \infty$ in (B.23) we will get a divergence in $F(u) \sim \mathsf{c}_2 u^{m-1}$ (which also yields a divergence when setting one leg to the boundary); this implies we should set $\mathsf{c}_2 = 0$. The short-distance behavior of two-point function in AdS_2 should be the same as in flat space (in particular, it should be independent of the mass). Comparing to the flat space two-point function, we find c_1

$$\mathsf{c}_1 = -\frac{1}{4\pi}\frac{\Gamma(m+1)^2}{\Gamma(2m+1)} = -\frac{1}{\sqrt{\pi}2^{2+2m}}\frac{\Gamma(m+1)}{\Gamma\left(m+\frac{1}{2}\right)}\ . \tag{B.27}$$

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As a result, we find from (B.20), (B.23)

$$g_{\psi\psi}(w,w') = \langle\psi(w,\bar{w})\psi(w',\bar{w}')\rangle = C\frac{\bar{w}-\bar{w}'}{\sqrt{-(w-\bar{w})(w'-\bar{w}')}}\frac{\mathsf{F}_2(u)}{(2+u)^{m+1}}\,, \tag{B.28}$$

$$g_{\bar\psi\bar\psi}(w,w') = \langle\bar\psi(w,\bar{w})\bar\psi(w',\bar{w}')\rangle = C\frac{w-w'}{\sqrt{-(w-\bar{w})(w'-\bar{w}')}}\frac{\mathsf{F}_2(u)}{(2+u)^{m+1}}\,,$$

$$g_{\psi\bar\psi}(w,w') = \langle\psi(w,\bar{w})\bar\psi(w',\bar{w}')\rangle = C\frac{-(w-\bar{w}')}{\sqrt{-(w-\bar{w})(w'-\bar{w}')}}\frac{\mathsf{F}_1(u)}{(2+u)^{m+1}}\,,$$

$$g_{\bar\psi\psi}(w,w') = \langle\bar\psi(w,\bar{w})\psi(w',\bar{w}')\rangle = C\frac{-(\bar{w}-w')}{\sqrt{-(w-\bar{w})(w'-\bar{w}')}}\frac{\mathsf{F}_1(u)}{(2+u)^{m+1}}\,, \tag{B.29}$$

where

$$C \equiv \frac{1}{\sqrt{\pi}\,2^{2+m}}\frac{\Gamma(m+1)}{\Gamma\left(m+\frac{1}{2}\right)}, \tag{B.30}$$

and

$$\mathsf{F}_1(u) = {}_2F_1\left(m+1,m,2m+1,\frac{2}{u+2}\right), \quad \mathsf{F}_2(u) = {}_2F_1\left(m+1,m+1,2m+1,\frac{2}{u+2}\right). \tag{B.31}$$

It is easy to verify that the propagators in (B.29) agree with the $d=1$ limit of the general expression in (B.11) (up to a factor of 2 due to the Majorana condition assumed in (B.17)).

In the main text (5.11), we consider the action (B.17) with $m=1$ and an extra overall normalization factor $\frac{1}{4\pi}$, implying that the above propagators are to be multiplied by 4π.

Let us note that in the massless case $m=0$, we need to choose $\mathsf{c}_1 = \mathsf{c}_2$ to ensure $\langle\eta\bar\eta\rangle = 0$ as here the two fields decouple in (B.17). The resulting non-zero propagators are the same as on a flat half-plane (reflecting conformal invariance of the massless case)

$$\langle\psi(w,\bar{w})\psi(w',\bar{w}')\rangle = \frac{1}{8\pi}\frac{\sqrt{-(w-\bar{w})(w'-\bar{w}')}}{w-w'},$$

$$\langle\bar\psi(w,\bar{w})\bar\psi(w',\bar{w}')\rangle = \frac{1}{8\pi}\frac{\sqrt{-(w-\bar{w})(w'-\bar{w}')}}{\bar{w}-\bar{w}'}\,. \tag{B.32}$$

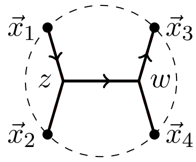
C Tree-level calculation of $\langle\Phi^2\Psi^2\rangle$ boundary correlation function

Here we describe the calculation of the tree level four-point correlator of two scalars and two fermions in the super Liouville theory in AdS_2. This requires to evaluate the contribution of the fermion exchange diagram. We did not find a direct way to exploit the supersymmetry relation (5.22) to reduce its calculation to that of a scalar exchange diagram (although in principle this should be possible). Instead, we will compute it following the same strategy as used in [38].

C.1 Fermion exchange diagram in AdS_{d+1}

Let us first compute the fermion exchange diagram in the general case of AdS_{d+1} by exploiting inversion transformations as in [38] and express then it in terms of differential operators acting on the scalar exchange contribution. Our notation in this subsection follows appendix B. Stripping out couplings, the contribution of this diagram to the mixed scalar-fermion boundary correlator is[24]

$$A(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4) = \tag{C.1}$$



$$= -\int d^{d+1}z\sqrt{g(z)}d^{d+1}w\sqrt{g(w)}\; K_{\Delta_2}(z,\vec{x}_2)\;\bar{\Sigma}_{\Delta_1}(z,\vec{x}_1)S(z,w)\Sigma_{\Delta_3}(w,\vec{x}_3)\;K_{\Delta_4}(w,\vec{x}_4)\;.$$

Here Δ_r are dimensions of the corresponding fields and the exchanged field has mass m. Using the boundary translational invariance, we can set $\vec{x}_3 = 0$, i.e.

$$A(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4) = A(\vec{x}_{13},\vec{x}_{23},0,\vec{x}_{43}), \qquad \vec{x}_{i3} = \vec{x}_i - \vec{x}_3\;. \tag{C.2}$$

Let us now consider the inversion transformation of a bulk and a boundary point

$$z^a \to \hat{z}^a = \frac{z^a}{|z|^2} = \frac{z^a}{z_0^2+\vec{z}^2}\,, \qquad \vec{x} \to \hat{\vec{x}} = \frac{\vec{x}}{|\vec{x}|^2}\,, \tag{C.3}$$

under which the AdS measure and the geodesic length are left invariant

$$\int d^{d+1}z\sqrt{g(z)} = \int d^{d+1}\hat{z}\sqrt{g(\hat{z})}, \qquad u(z,w) = u(\hat{z},\hat{w}), \tag{C.4}$$

while the bulk-to-boundary propagators transform according to

$$K_\Delta(\hat{z},\hat{\vec{x}}) = |\vec{x}|^{2\Delta}K_\Delta(z,\vec{x}), \qquad K_\Delta(\hat{w},\hat{\vec{x}}=0) = c_\Delta w_0^\Delta, \tag{C.5}$$

$$\bar{\Sigma}(\hat{z},\hat{\vec{x}}) = -\frac{\not{\vec{x}}}{|\vec{x}|^{2-2\Delta}}\mathcal{P}_-K_\Delta(z,\vec{x})U(z,\vec{x})\frac{\not{z}}{|z|}, \qquad \Sigma(\hat{w},\hat{\vec{x}}=0) = c_\Delta\frac{\not{w}}{|w|}w_0^{\Delta-1/2}\mathcal{P}_-\,. \tag{C.6}$$

The fermion bulk-to-bulk propagator in (B.10) changes as

$$S(\hat{z},\hat{w}) = \frac{\not{z}}{|z|}\Big(\not{D}+\Gamma^0-m\Big)\Big[z_0^{-1/2}\Big(G_{\Delta_-}(z,w)\mathcal{P}_+ + G_{\Delta_+}(z,w)\mathcal{P}_-\Big)w_0^{1/2}\frac{\not{w}}{|w|}\Big]\;. \tag{C.7}$$

Using these inversion transformation rules, we can express the arguments of propgators in (C.2), (C.2) in terms of the inverted coordinates $\hat{x}_{i3}$

$$\begin{aligned}
A(\vec{x}_1,\vec{x}_2,0,\vec{x}_4) &= -\int d^{d+1}z\sqrt{g(z)}d^{d+1}w\sqrt{g(w)}\;K_{\Delta_2}(z,\vec{x}_{23})\bar{\Sigma}_{\Delta_1}(z,\vec{x}_{13})S(z,w) \\
&\quad\times\Sigma_{\Delta_3}(w,0)K_{\Delta_4}(w,\vec{x}_{43}) \\
&= c_{\Delta_3}\frac{\hat{\not{\vec{x}}}_{13}}{|\hat{\vec{x}}_{13}|^{2-2\Delta_1}|\hat{\vec{x}}_{23}|^{-2\Delta_2}|\hat{\vec{x}}_{43}|^{-2\Delta_4}}\mathcal{P}_-\int d^{d+1}z\sqrt{g(z)}d^{d+1}w\sqrt{g(w)} \\
&\quad K_{\Delta_2}(\hat{z},\hat{\vec{x}}_{23})K_{\Delta_1}(\hat{z},\hat{\vec{x}}_{13})\;U(\hat{z},\hat{\vec{x}}_{13})\frac{\hat{\not{z}}}{|\hat{z}|}S(z,w)\frac{\hat{\not{w}}}{|\hat{w}|}\hat{w}_0^{\Delta_3-1/2}\;K_{\Delta_4}(\hat{w},\hat{\vec{x}}_{43})\mathcal{P}_- \\
&= c_{\Delta_3}\frac{\not{\vec{x}}_{13}}{|\vec{x}_{13}|^{2\Delta_1}|\vec{x}_{23}|^{2\Delta_2}|\vec{x}_{43}|^{2\Delta_4}}\mathcal{P}_-\int d^{d+1}\hat{z}\sqrt{g(\hat{z})}d^{d+1}\hat{w}\sqrt{g(\hat{w})} \\
&\quad K_{\Delta_2}(\hat{z},\hat{\vec{x}}_{23})K_{\Delta_1}(\hat{z},\hat{\vec{x}}_{13})\;U(\hat{z},\hat{\vec{x}}_{13})\frac{\hat{\not{z}}}{|\hat{z}|}S(\hat{z},\hat{w})\frac{\hat{\not{w}}}{|\hat{w}|}\hat{w}_0^{\Delta_3-1/2}\;K_{\Delta_4}(\hat{w},\hat{\vec{x}}_{43})\mathcal{P}_-,
\end{aligned} \tag{C.8}$$

[24] Here z and w denote generic bulk points in AdS_{d+1}.

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where we used that $\hat{\hat{z}} = z$, i.e. that the inversion squares to the identity. To simplify notation, we will use $\vec{y}_i = \hat{\vec{x}}_{i3}$ and make the replacements $\hat{z} \to z, \hat{w} \to w$. Then

$$A(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4) = c_{\Delta_3} \frac{\not{x}_{13}}{|\vec{x}_{13}|^{2\Delta_1} |\vec{x}_{23}|^{2\Delta_2} |\vec{x}_{43}|^{2\Delta_4}} B(\vec{y}_1, \vec{y}_2, \vec{y}_4)\,, \tag{C.9}$$

$$\begin{aligned} B(\vec{y}_1, \vec{y}_2, \vec{y}_4) &= \mathcal{P}_- \int d^{d+1}z\sqrt{g(z)}d^{d+1}w\sqrt{g(w)}\; K_{\Delta_2}(z, \vec{y}_2) K_{\Delta_1}(z, \vec{y}_1) \\ &\quad \times U(z, \vec{y}_1) \frac{\not{z}}{|z|} S(\hat{z}, \hat{w}) \frac{\not{w}}{|w|} w_0^{\Delta_3 - 1/2}\; K_{\Delta_4}(w, \vec{y}_4)\mathcal{P}_- \\ &= \mathcal{P}_- \int d^{d+1}z\sqrt{g(z)}d^{d+1}w\sqrt{g(w)}\; K_{\Delta_2}(z, \vec{y}_2) K_{\Delta_1}(z, \vec{y}_1)\; U(z, \vec{y}_1) \\ &\quad \times \Big(\not{D} + \Gamma^0 - m\Big)\Big[z_0^{-1/2} G_{\Delta_+}(z, w)\Big] w_0^{\Delta_3}\; K_{\Delta_4}(w, \vec{y}_4)\mathcal{P}_-\,, \end{aligned} \tag{C.10}$$

where ([C.10]) we used the inversion relation ([C.7]) for $S(\hat{z}, \hat{w})$ as well as the projection properties $\mathcal{P}_-^2 = \mathcal{P}_-$, $\mathcal{P}_+^2 = \mathcal{P}_+$, $\mathcal{P}_+\mathcal{P}_- = \mathcal{P}_-\mathcal{P}_+ = 0$.

Next, we may use integration by parts relations like[25]

$$\begin{aligned} \int \frac{d^{d+1}z}{z_0^{d+1}} f(z) z_0 \Gamma^a \partial_a g(z) &= \int \frac{d^{d+1}z}{z_0^{d+1}} \Big(- \partial_a f(z) z_0 \Gamma^a + d\, f \Gamma^0 \Big) g(z)\,, \\ \int d^{d+1}z\sqrt{g(z)}\; f \not{\partial} g &= \int d^{d+1}z\sqrt{g(z)}\; f\big(- \overleftarrow{\not{\partial}} + d\Gamma^0\big) g\,, \\ \int d^{d+1}z\sqrt{g(z)}\; f \not{D} g &= \int d^{d+1}z\sqrt{g(z)}\; f\big(- \overleftarrow{\not{D}}\big) g\;. \end{aligned} \tag{C.11}$$

Here $\overleftarrow{\not{D}}$, $\overleftarrow{\not{\partial}}$ do not act on the measure. One can also show that

$$\begin{aligned} &\frac{1}{\sqrt{z_0}} \mathcal{P}_- \Big(K_{\Delta'}(z, \vec{y}) K_\Delta(z, \vec{x}) U(z, \vec{x}) \Big) \overleftarrow{\not{D}} \mathcal{P}_- = - \Big(\Delta - \Delta' - \frac{d+1}{2} \Big) \mathcal{P}_- K_{\Delta'}(z, \vec{y}) K_\Delta(z, \vec{x}) \\ &\quad - 2\Big(\Delta' - \frac{d}{2} \Big) \mathcal{P}_- U(z, \vec{x}) U(z, \vec{y}) \mathcal{P}_- K_{\Delta'+1}(z, \vec{y}) K_\Delta(z, \vec{x}) \mathcal{P}_-\;. \end{aligned} \tag{C.12}$$

Using ([C.11]) and the following projections,

$$\frac{1}{\sqrt{z_0}} \mathcal{P}_- U(z, \vec{x}) \mathcal{P}_- = -\mathcal{P}_-, \quad \mathcal{P}_- \Gamma^0 = -\mathcal{P}_-, \quad \mathcal{P}_- U(z, \vec{y}) \Big(- \Gamma^0 + m \Big) \mathcal{P}_- = -(1+m)\mathcal{P}_-, \tag{C.13}$$

one can write B in ([C.10]) as

$$B(\vec{y}_1, \vec{y}_2, \vec{y}_4) = (2\Delta_2 - d) J(\vec{y}_1, \vec{y}_2, \vec{y}_3) + \Big(m + \Delta_1 - \Delta_2 - \frac{d}{2} + \frac{1}{2} \Big) I(\vec{y}_1, \vec{y}_2, \vec{y}_3), \tag{C.14}$$

where

$$\begin{aligned} I(\vec{y}_1, \vec{y}_2, \vec{y}_4) &= \int d^{d+1}z\sqrt{g(z)}d^{d+1}w\sqrt{g(w)}\; K_{\Delta_2}(z, \vec{y}_2) K_{\Delta_1}(z, \vec{y}_1) \\ &\quad \times G_{\Delta_+}(z, w) w_0^{\Delta_3} K_{\Delta_4}(w, \vec{y}_4) \mathcal{P}_-, \end{aligned} \tag{C.15}$$

$$\begin{aligned} J(\vec{y}_1, \vec{y}_2, \vec{y}_4) &= \int d^{d+1}z\sqrt{g(z)}d^{d+1}w\sqrt{g(w)}\; K_{\Delta_2+1}(z, \vec{y}_2) K_{\Delta_1}(z, \vec{y}_1) G_{\Delta_+}(z, w) w_0^{\Delta_3} \\ &\quad \times K_{\Delta_4}(w, \vec{y}_4) \mathcal{P}_- U(z, \vec{y}_1) U(z, \vec{y}_2) \mathcal{P}_-\;. \end{aligned} \tag{C.16}$$

[25] Here f and g are arbitrary matrix functions that decay rapidly enough at infinity, so no surface terms are needed.

Since $U(z,\vec{y}) = U(z,\vec{x}) - \frac{\vec{\not{y}}-\vec{\not{x}}}{\sqrt{z_0}}$, one can further show that

$$\mathcal{P}_- U(z,\vec{y})U(z,\vec{x})\mathcal{P}_- = \left[\frac{(z-x)^2}{z_0} - \frac{(\vec{\not{y}}-\vec{\not{x}})(\not{z}-\vec{\not{x}})}{z_0}\right]\mathcal{P}_- \; . \tag{C.17}$$

We also have

$$\frac{(\not{z}-\vec{\not{x}})}{z_0}K_{\Delta+1}(z,\vec{x}) = \frac{1}{2\Delta-d}\vec{\not{\partial}}_{\vec{x}}K_\Delta(z,\vec{x}), \quad \frac{(z-x)^2}{z_0}K_{\Delta+1}(z,\vec{x}) = \frac{2\Delta}{2\Delta-d}K_\Delta(z,\vec{x}) \, . \tag{C.18}$$

These formulae enable one to express J in (C.16) as

$$J(\vec{y}_1,\vec{y}_2,\vec{y}_4) = \frac{1}{2\Delta_2-d}\Big(2\Delta_2 - \vec{\not{y}}_{12}\vec{\not{\partial}}_{\vec{y}_2}\Big)I(\vec{y}_1,\vec{y}_2,\vec{y}_4)\,, \tag{C.19}$$

so that we can finally write B as

$$\begin{aligned} B(\vec{y}_1,\vec{y}_2,\vec{y}_4) &= \Big(-\vec{\not{y}}_{12}\vec{\not{\partial}}_{\vec{y}_2} + \Delta_1+\Delta_2+m+\frac{1}{2}-\frac{d}{2}\Big)I(\vec{y}_1,\vec{y}_2,\vec{y}_4) \\ &= \Big(-\vec{\not{y}}_{12}\vec{\not{\partial}}_{\vec{y}_2} + \Delta_1+\Delta_2+m+\frac{1}{2}-\frac{d}{2}\Big)\mathcal{I}(\vec{y}_1,\vec{y}_2,\vec{y}_4)\mathcal{P}_-, \end{aligned} \tag{C.20}$$

where

$$\mathcal{I}(\vec{y}_1,\vec{y}_2,\vec{y}_4) = \int d^{d+1}z\sqrt{g(z)}d^{d+1}w\sqrt{g(w)}\; K_{\Delta_2}(z,\vec{y}_2)K_{\Delta_1}(z,\vec{y}_1)G_{\Delta_+}(z,w)w_0^{\Delta_3}K_{\Delta_4}(w,\vec{y}_4)\,. \tag{C.21}$$

This expression can be simplified further by using again the inversion transformation. From the relations (C.5) for the scalar propagators, we can write

$$\mathcal{I}(\vec{y}_1,\vec{y}_2,\vec{y}_4) = \frac{1}{c_{\Delta_3}}|\hat{\vec{y}}_1|^{2\Delta_1}|\hat{\vec{y}}_2|^{2\Delta_2}|\hat{\vec{y}}_4|^{2\Delta_4}\; W^s_{\Delta_1\Delta_2\Delta_3\Delta_4,\Delta_+}(\hat{\vec{y}}_1,\hat{\vec{y}}_2,\hat{\vec{y}}_3=0,\hat{\vec{y}}_4)\,, \tag{C.22}$$

$$\begin{aligned} &W^s_{\Delta_1\Delta_2\Delta_3\Delta_4,\Delta_+}(\vec{y}_1,\vec{y}_2,\vec{y}_3,\vec{y}_4) \\ &\quad\equiv \int d^{d+1}z\sqrt{g(z)}d^{d+1}w\sqrt{g(w)}\; K_{\Delta_1}(z,\vec{y}_1)K_{\Delta_2}(z,\vec{y}_2)G_{\Delta_+}(z,w)K_{\Delta_3}(w,0)K_{\Delta_4}(w,\vec{y}_4)\,, \end{aligned} \tag{C.23}$$

where (C.24) is the contribution of the exchange diagram involving only scalar fields. Let us also note that the derivative of $\mathcal{I}$ may be written as

$$\begin{aligned} &\frac{\partial}{\partial y_2^j}\mathcal{I}(\vec{y}_1,\vec{y}_2,\vec{y}_4) = -|\hat{\vec{y}}_2|^2\frac{\partial}{\partial \hat{y}_2^j}\mathcal{I}(\vec{y}_1,\hat{\vec{y}}_2,\vec{y}_4) \\ &= -\frac{1}{c_{\Delta_3}}|\hat{\vec{y}}_1|^{2\Delta_1}|\hat{\vec{y}}_2|^{2\Delta_2}|\hat{\vec{y}}_4|^{2\Delta_4}\left(|\hat{\vec{y}}_2|^2\frac{\partial}{\partial\hat{y}_2^j}+2\Delta_2\hat{y}_2^j\right)W^s_{\Delta_1\Delta_2\Delta_3\Delta_4,\Delta_+}(\hat{\vec{y}}_1,\hat{\vec{y}}_2,\hat{\vec{y}}_3,\hat{\vec{y}}_4) \end{aligned} \tag{C.24}$$

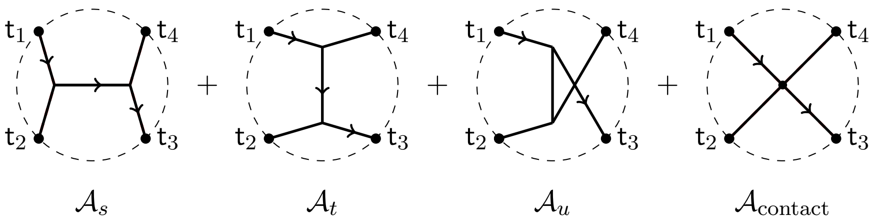
Thus finally we obtain for (C.2) (note that $\vec{y}_i = \hat{\vec{x}}_{i3} = \frac{\vec{x}_{i3}}{|\vec{x}_{i3}|^2}$, and thus $\hat{\vec{y}}_i = \vec{x}_{i3}$):

$$\begin{aligned}
A(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4) &= c_{\Delta_3}\frac{\not{\vec{x}}_{13}}{|\vec{x}_{13}|^{2\Delta_1}|\vec{x}_{23}|^{2\Delta_2}|\vec{x}_{43}|^{2\Delta_4}} \\
&\qquad \times\left(-\not{\vec{y}}_{12}\not{\partial}_{\vec{y}_2}+\Delta_1+\Delta_2+m+\frac{1}{2}-\frac{d}{2}\right)\mathcal{I}(\vec{y}_1,\vec{y}_2,\vec{y}_4)\mathcal{P}_- \\
&= \not{\vec{x}}_{13}\mathcal{P}_-\Bigg[\left(\hat{\not{\vec{x}}}_{13}-\hat{\not{\vec{x}}}_{23}\right)\left(|\not{\vec{x}}_{23}|^2\not{\partial}_{\vec{x}_{23}}+2\Delta_2\not{\vec{x}}_{23}\right) \\
&\qquad +\Delta_1+\Delta_2+m+\frac{1}{2}-\frac{d}{2}\Bigg]W^s_{\Delta_1\Delta_2\Delta_3\Delta_4,\Delta_+}(\vec{x}_{13},\vec{x}_{23},0,\vec{x}_{43}) \\
&= \not{\vec{x}}_{13}\mathcal{P}_-\Bigg[\left(\frac{\not{\vec{x}}_{13}}{|\vec{x}_{13}|^2}-\frac{\not{\vec{x}}_{23}}{|\vec{x}_{23}|^2}\right)\left(|\not{\vec{x}}_{23}|^2\not{\partial}_{\vec{x}_2}+2\Delta_2\not{\vec{x}}_{23}\right) \\
&\qquad +\Delta_1+\Delta_2+m+\frac{1}{2}-\frac{d}{2}\Bigg]W^s_{\Delta_1\Delta_2\Delta_3\Delta_4,\Delta_+}(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4)\ . \qquad \text{(C.25)}
\end{aligned}$$

As anticipated, this allows us to rewrite the fermion exchange diagram in terms of a suitable differential operator acting on the purely scalar exchange diagram which is much easier to compute using known general results (see, e.g., [46]).

C.2 $\langle\Phi^2\Psi^2\rangle$ correlator in super Liouville theory in AdS$_2$

The correlator $\langle\Phi^2\Psi^2\rangle$ involving two fermions and two scalars receives contributions from the following four diagrams



$$. \qquad \text{(C.26)}$$

A non-trivial calculation is required for the first diagram $\mathcal{A}_s$ only. Indeed, the second diagram $\mathcal{A}_t$ can be obtained by crossing, while the other two diagrams $\mathcal{A}_u$ and $\mathcal{A}_{\text{contact}}$ do not involve a bulk fermion propagator and can be computed by using the relation (5.18).

The contribution of the first diagram $\mathcal{A}_s$ is obtained from A defined in (C.2) by specializing the general result (C.25) to the particular $d = 1$ case with $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_+ = 2$, and taking the (12) component of the spinor matrix to project to the $\Psi\Psi$ channel. Finally, we have to insert the coupling and normalization factors to match our conventions for the fermionic propagators used in the main text as compared to appendix B. All steps are straightforward: the specialisation of (C.25) gives

$$\begin{aligned}
&A(\mathtt{t}_1,\mathtt{t}_2,\mathtt{t}_3,\mathtt{t}_4) \qquad\qquad \text{(C.27)} \\
&= \mathtt{t}_{13}\,\Gamma^{\mathtt{t}}\mathcal{P}_-\left[\left(\frac{1}{\mathtt{t}_{13}}-\frac{1}{\mathtt{t}_{23}}\right)\left(2\Delta_2\mathtt{t}_{23}+\mathtt{t}_{23}^2\partial_{\mathtt{t}_2}\right)+\Delta_1+\Delta_2+m+\frac{1}{2}-\frac{d}{2}\right]W^s_{2222,2}(\mathtt{t}_1,\mathtt{t}_2,\mathtt{t}_3,\mathtt{t}_4),
\end{aligned}$$

where

$$
\begin{aligned}
W^s_{2222,2}(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4) &= \int \frac{d^2z}{z_0^2}\frac{d^2w}{w_0^2}\, K_2(z,\mathsf{t}_1)K_2(z,\mathsf{t}_2)G_2(z,w)K_2(w,\mathsf{t}_3)K_2(w,\mathsf{t}_4)\\
&= c_2^4\,\frac{1}{4}|\mathsf{t}_3-\mathsf{t}_4|^{-2}D_{2211}(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4)\\
&= -\frac{\left(\chi^3-3\chi+2\right)\log|1-\chi|+\chi\left(-\chi^2\log|\chi|+\chi-1\right)}{2\pi^3(\chi-1)^2\chi^3\mathsf{t}_{13}^4\mathsf{t}_{24}^4}\,. \qquad \text{(C.28)}
\end{aligned}
$$

Here $\chi=\frac{\mathsf{t}_{12}\mathsf{t}_{34}}{\mathsf{t}_{13}\mathsf{t}_{24}}$ is the 1d cross-ratio, c_2 is given in (B.6), and the (standard) AdS integral is computed as in [46] (D-functions are discussed in [50, 51]). Evaluating (C.27) gives

$$
A(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4)=\mathsf{t}_{13}\,\Gamma^{\mathsf{t}}\mathcal{P}_-\frac{(2\chi^4-5\chi^3+7\chi-4)\log|1-\chi|-\chi(2\chi-5)\left(\chi^2\log|\chi|-\chi+1\right)}{2\pi^3(\chi-1)^2\chi^3\mathsf{t}_{13}^4\mathsf{t}_{24}^4}. \qquad \text{(C.29)}
$$

Adding the coupling and normalization factors,[26] we get the final expression for the contributions of the s-channel fermion exchange

$$
\begin{aligned}
\mathcal{A}_s(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4) &= (2b)^2\,(2\pi)^3\left(\frac{2\pi}{3}\right)^2\frac{1}{(4\pi)^2}\Big[A(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4)\Big]_{12} \qquad \text{(C.30)}\\
&= \mathsf{t}_{13}\frac{4\pi^3b^2}{9}\frac{(2\chi^4-5\chi^3+7\chi-4)\log|1-\chi|-\chi(2\chi-5)\left(\chi^2\log|\chi|-\chi+1\right)}{2\pi^3(\chi-1)^2\chi^3\mathsf{t}_{13}^4\mathsf{t}_{24}^4}\,.
\end{aligned}
$$

The t-channel contribution in (C.26) is obtained by exchanging the 2,4 legs

$$
\mathcal{A}_t(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4)=\mathcal{A}_s(\mathsf{t}_1,\mathsf{t}_4,\mathsf{t}_3,\mathsf{t}_2)\,. \qquad \text{(C.31)}
$$

The third (u-channel scalar exchange) diagram in (C.26) as well as the last contact diagram can be computed using (5.17), (5.18):

$$
\begin{aligned}
\mathcal{A}_u &= 2b\,(-8b)\left[\int \mathsf{d}^2w\,\mathsf{d}^2w'\;\mathsf{S}_\partial(\mathsf{t}_1,w)\mathsf{S}_\partial(w,\mathsf{t}_3)g(w,w')g_\partial(\mathsf{t}_2,w')g_\partial(\mathsf{t}_4,w')\right]_{12}\\
&= 2b\,(-8b)\left(-\frac{9}{2}\mathsf{t}_{13}\right)\int \mathsf{d}^2w\,\mathsf{d}^2w'\;g_\partial(\mathsf{t}_1,w)g_\partial(w,\mathsf{t}_3)g(w,w')g_\partial(\mathsf{t}_2,w')g_\partial(\mathsf{t}_4,w')\\
&= 72b^2\mathsf{t}_{13}\,2\pi\left(\frac{4}{3}\right)^4\frac{1}{(4\pi)^2}\,\frac{1}{4}|\mathsf{t}_2-\mathsf{t}_4|^{-2}D_{2121}(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4)\\
&= \frac{64b^2}{9\pi}\frac{\mathsf{t}_{13}}{\mathsf{t}_{24}^2}D_{2121}(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4)\,, \qquad \text{(C.32)}\\
\mathcal{A}_{\text{contact}} &= 2b^2\left(-\frac{9}{2}\mathsf{t}_{13}\right)\int \mathsf{d}^2w\;g_\partial(\mathsf{t}_1,w)g_\partial(w,\mathsf{t}_3)g_\partial(\mathsf{t}_2,w)g_\partial(\mathsf{t}_4,w)\\
&= -9b^2\mathsf{t}_{13}\left(\frac{4}{3}\right)^4\frac{1}{(4\pi)}\;D_{2222}(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4)=-\frac{64b^2}{9\pi}\frac{\mathsf{t}_{13}}{\mathsf{t}_{24}^2}D_{2222}(\mathsf{t}_1,\mathsf{t}_2,\mathsf{t}_3,\mathsf{t}_4)\,. \qquad \text{(C.33)}
\end{aligned}
$$

[26]Explicitly, $(\frac{2\pi}{3})^2$ is the rescaling of two scalar bulk-to-boundary propagator $g_\partial(\mathsf{t};w')=\frac{4}{3c_2}K(\mathsf{t},w')=\frac{2\pi}{3}K(\mathsf{t},w')$; $(2\pi)^3$ comes from different normalizations of the 3 fermionic propagators, $\mathsf{S}_\partial(w,\mathsf{t}')=2\pi\Sigma(w,\mathsf{t}')$ and $\mathsf{S}_\partial(\mathsf{t},w')=-2\pi\bar{\Sigma}(w',\mathsf{t})$; $\frac{1}{(4\pi)^2}$ arises from the rescaling of the integration measure; $(2b)^2$ is the coupling factor of the two vertices; the projection to Ψ gives $(\Gamma^{\mathsf{t}}\mathcal{P}_-)_{12}=\frac{1}{2}$.

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Summing up all the contributions (C.30), (C.31), (C.32), (C.33) to (C.26), we arrive at the final result

$$\langle \Psi(\mathsf{t}_1)\Phi(\mathsf{t}_2)\Psi(\mathsf{t}_3)\Phi(\mathsf{t}_4)\rangle_{0,\text{conn}} = \mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u + \mathcal{A}_{\text{contact}} = \frac{2b^2}{3}\frac{1}{\mathsf{t}_{13}^3\mathsf{t}_{24}^4}\frac{4\chi^2-4\chi+3}{\chi^2(\chi-1)^2}\,, \quad \text{(C.34)}$$

which is in full agreement with the prediction from (4.46) and (4.49).

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