

# Liquidity, the Mundell-Tobin Effect, and the Friedman Rule\*

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## Abstract

We investigate whether the Mundell-Tobin effect affects the optimal monetary policy prescription in a framework that is a combination of overlapping generations and new monetarist models. As is standard in the overlapping generations literature, we find that a constant money stock is welfare-maximizing regardless of other parameters if monetary policy is implemented by taxing the young. In that case, the Friedman rule becomes relatively more costly if the Mundell-Tobin effect is stronger. If monetary policy is implemented by taxing the old, the Friedman rule is optimal in the absence of the Mundell-Tobin effect. With the Mundell-Tobin effect present, the optimal money growth rate is an increasing function of the liquidity of capital, approaching a constant money stock for perfectly liquid capital.

Keywords: *New monetarism, overlapping generations, optimal monetary policy*

JEL codes: E4, E5

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# 1 Introduction

When it comes to optimal monetary policy, there is a stark contrast between practitioners and theorists. Most central banks in developed countries follow an inflation target of around 2% annually, and there is a general agreement among practitioners that deflation has to be avoided at any cost. Meanwhile, most theoretical models find that the Friedman rule, i.e. setting the inflation rate such that the opportunity cost of holding money balances is zero, is the optimal monetary policy. Since zero opportunity costs for holding money implies deflation in standard models, this prediction clearly differs from what practitioners believe to be optimal. The Friedman rule has been found to be optimal by Friedman himself in a model with money in the utility (Friedman, 1969), but also in a variety of other monetary models such as cash-in-advance (Grandmont and Younes, 1973; Lucas and Stokey, 1987), spatial separation (Townsend, 1980), and New Monetarism (Lagos and Wright, 2005). While there have been alterations of these models that render the Friedman rule suboptimal<sup>1</sup>, these have often been somewhat ad-hoc.

One mechanism that could make deviations from the Friedman rule optimal is the Mundell-Tobin effect (Mundell (1963) and Tobin (1965)). The Mundell-Tobin effect predicts that an increase in the return on nominal assets such as bonds or fiat money crowds out capital investment. Therefore, lower inflation rates reduce capital investment. However, inflation above the Friedman rule reduces people's willingness to hold liquid assets. If certain trades can only be settled with liquid assets, higher inflation rates thus reduce quantities traded. This implies that there is a trade-off between the benefits of a high return on liquid assets and the costs associated with reduced capital investment due to the Mundell-Tobin effect. We investigate this effect in a model that combines the overlapping generations (OLG) framework a la Wallace (1980) with a New Monetarist model a la Lagos and Wright (2005). This approach allows us to find novel results regarding both of these literatures, and settle some debates, as we explain in the literature review below.

In our model, each period is divided into two subperiods, called CM and DM. Agents are born at the beginning of the CM and live until the end of the CM of the following period; i.e., they are alive for three subperiods. There are two assets in the economy, productive capital and fiat money. With some probability, agents are relocated during the DM. If they are relocated, they can only use fiat money to settle trades, while they can use money and capital if they are not relocated. This relocation shock follows Townsend (1987). During the final CM of their lives, agents return to their original location and have access to all their remaining assets. Monetary policy is implemented

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<sup>1</sup>E.g. for the New Monetarist literature: theft as in Sanches and Williamson (2010), taxes as in Aruoba and Chugh (2010), or socially undesirable activities financed by cash as in Williamson (2012).

either by paying transfers to / raising taxes from the young or the old agents.

We first study a benchmark case where all agents are relocated, meaning that capital is perfectly illiquid. In this version of the model, there is no tradeoff between money and capital, as capital can never be used to provide DM consumption, but since it (weakly) dominates in terms of rate of return, it is more useful to provide CM consumption. If monetary policy is implemented over the young agents, a constant money stock is optimal in this version of the model, as is common in many OLG models. If the old are taxed instead, the Friedman rule is the optimal policy if the buyer's elasticity of DM consumption is sufficiently low, while otherwise a higher, but still deflationary money growth rate becomes optimal. We can also show that welfare is strictly higher if the old agents are taxed - the reason for this is that old agents can pay taxes by working when young and then earning the return on capital before paying the tax, making the tax payment strictly lower from a social perspective.

We then analyze the full model with partially liquid capital, which creates a tradeoff between money and capital: Acquiring more money means agents are better insured against the relocation shock, but away from the Friedman rule, they forego the higher return earned on capital. Lowering the inflation rate thus makes holding money relatively more attractive and thereby crowds out capital investment - the Mundell-Tobin effect is at play. In this version of the model, the optimal monetary policy still consists of taxing the old, and the optimal inflation rate now becomes a function of the liquidity of capital (and the elasticity of DM consumption if it is above 1). If capital is relatively illiquid, the optimal inflation rate is close to the Friedman rule. If capital is relatively illiquid however, the optimal inflation rate is close to a constant money growth rate, as in this case providing insurance against the relocation shock becomes really costly, as it crowds out a lot of capital investment.

**Existing literature.** While the Mundell-Tobin effect exists in many models<sup>2</sup>, the Friedman rule usually yields the optimal outcome in both dimensions - i.e., optimal capital investment and first-best allocations in trades that require liquid assets. While higher inflation rates would increase capital investment in these models, this investment is inefficient due to decreasing returns to scale. There have been a few papers that find deviations from the Friedman rule to be optimal due to the Mundell-Tobin effect - e.g. Venkateswaran and Wright (2013), Geromichalos and Herrenbrueck (2017), Wright et al. (2018), or Altermatt (2017). However, in these papers there is usually an additional friction that leads to underinvestment at the Friedman rule, e.g., limited pledgeability, taxes, or wage bargaining. If these frictions are shut down in the papers mentioned, the Mundell-

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<sup>2</sup>Lagos and Rocheteau (2008) is a prime example.

Tobin effect still exists, but the Friedman rule is optimal. In this paper, we are able to show that the Mundell-Tobin effect itself can make deviations from the Friedman rule optimal, even if there are no other frictions in the economy besides the one that causes the Mundell-Tobin effect.

In the OLG literature following Wallace (1980), the Mundell-Tobin effect has also been studied, and papers like Smith (2002, 2003) and Schreft and Smith (2002) have claimed to show that the Friedman rule is suboptimal in their models because of the Mundell-Tobin effect. However, OLG models typically find deviations from the Friedman rule to be optimal even without the Mundell-Tobin effect<sup>3</sup>, as in Weiss (1980), Abel (1987), or Freeman (1993). Bhattacharya et al. (2005) and Haslag and Martin (2007) build on these results to show that the results in Smith (2002) and the other papers mentioned are not driven by the Mundell-Tobin effect, but by the standard properties of the OLG models. The debate whether the Mundell-Tobin effect itself can render deviations from the Friedman rule to be optimal thus remained unsettled.

We are able to contribute to this debate by incorporating the setting of Smith (2002) into a New Monetarist model. The resulting model combines features of OLG models with those from models along the lines of Lagos and Wright (2005) (LW), and thus resembles other combinations of OLG and LW such as Zhu (2008) or Altermatt (2019). As explained above, this approach allows us to implement monetary policy in two different ways, either by taxing the young or by taxing the old.

We think that our paper contributes to the literature in two important ways. First, it is able to reconcile Smith (2002) with Bhattacharya et al. (2005) and Haslag and Martin (2007). If the young are taxed in our model, we are able to replicate the results of the latter two. But we are also able to show that it is strictly better to tax the old, and that in this case, the Friedman rule becomes optimal (for sufficiently low elasticity of DM consumption) in the absence of the Mundell-Tobin effect, while deviations from the Friedman rule become optimal if the Mundell-Tobin effect is at play, which confirms Smith's intuition that the Mundell-Tobin effect itself is enough to justify deviations from the Friedman rule. The reason we are able to do so is that in our framework, all agents have access to their capital during the final stage of their life (i.e., the second CM) independent of the relocation shock. This allows us to implement the Friedman rule in a way that is less costly than it is usually the case in OLG models. Second, our paper shows that the Mundell-Tobin effect itself can make deviations from the Friedman rule optimal in the New Monetarist literature, which

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<sup>3</sup>There is a further complication in the welfare analysis of OLG models due to the absence of a representative agent. Freeman (1993) shows that the Friedman rule is typically Pareto optimal, but not maximizing steady state utility in OLG models. In this paper, we are going to focus on steady-state optimality when analyzing optimal policies in OLG models.

differs from the results found by Lagos and Rocheteau (2008) and many others.

**Outline.** The rest of this paper is organized as follows. In Section 2, the environment is explained, and in Section 3, we present the market outcome for perfectly liquid capital. In Section 4, we discuss the market outcome for perfectly illiquid capital and monetary policy implementation. Section 5 presents the results of the full model, and finally, Section 6 concludes.

## 2 The model

Our model is a combination of the environment in Lagos and Wright (2005), and the overlapping generations model (OLG) with relocation shocks from Townsend (1987), as used by Smith (2002). Time is discrete and continues forever. Each period is divided into two subperiods, called the decentralized market (DM) and the centralized market (CM). There are two distinct locations, which we will sometimes call islands. At the beginning of a period, the CM takes place, and after it closes, the DM opens and remains open until the period ends. At the beginning of each period, a new generation of agents is born, consisting of one unit mass per island each of buyers and sellers. An agent born in period  $t$  lives until the end of the CM in period  $t + 1$ . Each generation is named after the period it is born in. Figure 1 gives an overview of the sequence of subperiods and the lifespans of generations. There is also a monetary authority.

Both buyers and sellers are able to produce a general good  $x$  during the first CM of their life at linear disutility  $h$ , whereas incurring the disutility  $h$  yields exactly  $h$  units of general goods; buyers and sellers also both receive utility from consuming the general good during the second CM of their life. During the DM, sellers are able to produce special goods  $q$  at linear disutility; buyers receive utility from consuming these special goods.

The preferences of buyers are given by

$$\mathbb{E}_t\{-h_t^b + u(q_t^b) + \beta U(x_{t+1}^b)\}. \quad (1)$$

Equation (1) states that buyers discount the second period of their life by a factor  $\beta \in (0, 1)$ , gain utility  $u(q)$  from consuming the special good in the DM and  $U(x)$  from consuming the general good in the CM, with  $u(0) = 0$ ,  $u'(q) > 0$ ,  $u''(q) < 0$ ,  $u'(0) = \infty$ ,  $U(0) = 0$ ,  $U'(x) > 0$ ,  $U''(x) < 0$ ,  $U'(0) = \infty$ , and linear disutility  $h$  from producing the general good during their first CM. The preferences of the sellers are

$$\mathbb{E}_t\{-h_t^s - q_t^s + \beta U(x_{t+1}^s)\}. \quad (2)$$

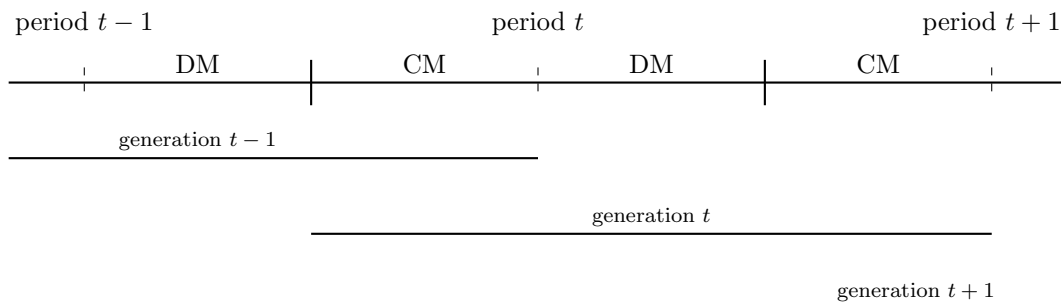


Figure 1: Timeline with lifespans of generations.

Sellers also discount the second period of their life by a factor  $\beta$ , gain utility  $U(x)$  from consuming in the CM and disutility  $q$  from producing in the DM, with  $\bar{q} = u(\bar{q})$  for some  $\bar{q} > 0$ . Furthermore, we define  $q^*$  as  $u'(q^*) = 1$ ; i.e., the socially efficient quantity.

During the CM, general goods can be sold or purchased in a centralized market. During the DM, special goods are sold in a centralized market. A fraction  $\pi$  of buyers are relocated during the DM, meaning that they are transferred to the other island without the ability to communicate with their previous location. Sellers are not relocated, and during the CM, no relocation occurs for both types of agents. Relocated buyers are transferred back to their original location for the final CM of their life<sup>4</sup>. Relocation occurs randomly, so for an individual agent, the probability of being relocated is  $\pi$ . Buyers learn at the beginning of the DM whether they are relocated or not.

The monetary authority issues fiat money  $M_t$ , which it can produce without cost. The monetary authority always implements its policies at the beginning of the CM. The amount of general goods that one unit of fiat money can buy in the CM of period  $t$  is denoted by  $\phi_t$ . The inflation rate is defined as  $\phi_t/\phi_{t+1} - 1 = \pi_{t+1}$ , and the growth rate of fiat money from period  $t - 1$  to  $t$  is  $\frac{M_t}{M_{t-1}} = \gamma_t$ . Monetary policy is implemented by issuing newly printed fiat money either to young or to old buyers via lump-sum transfers<sup>5</sup> (or lump-sum taxes in the case of a decreasing money stock). We denote transfers to young buyers as  $\tau^y$ , and transfers to old buyers as  $\tau^o$ . Furthermore, we will use an indicator variable  $\mathcal{I}$  to denote the regime, i.e., which generation is taxed. If  $\mathcal{I} = 1$  ( $\mathcal{I} = 0$ ), young buyers (old buyers) are taxed, which means  $\tau^y$  ( $\tau^o$ ) is set such that the money growth rate  $\gamma_t$  chosen by the monetary authority can be implemented, while  $\tau^o = 0$  ( $\tau^y = 0$ ).

<sup>4</sup>In Smith (2002), each agent lives only for two periods. Relocation occurs during the last period of an agent's life, meaning that all assets that he cannot spend during that period are wasted from his point of view. Our model crucially differs from Smith (2002) in that regard, as our agents have access to all their assets during the final period of their life.

<sup>5</sup>As we will show in this paper, the exact timing of the lump-sum taxes is irrelevant for consumption allocations, but not for welfare in OLG models with capital.

Besides fiat money, there also exists capital  $k$  in this economy. During the CM, agents can transform general goods into capital. One unit of capital delivers  $R > 1$  units of real goods in the CM of the following period. We will assume throughout the paper that

$$R\beta > 1. \quad (3)$$

Capital is immobile, meaning that it is impossible to move capital to other locations during the DM. It is also impossible to create claims on capital that can be verified by other agents.

## Decentralized market

In the DM, special goods are sold in competitive manner<sup>6</sup>. Due to anonymity and a lack of commitment, all trades have to be settled immediately. Therefore, buyers have to transfer assets to sellers in order to purchase special goods. Because capital cannot be transported to other locations and claims on capital are not verifiable, relocated buyers can only use fiat money to settle trades. Nonrelocated buyers can use both fiat money and capital to purchase special goods. We will use  $p_t$  to denote the price of special goods in terms of fiat money. All buyers face the same price, regardless of their means of payment. As sellers are not relocated during the DM, all of them accept both fiat money and capital of nonrelocated buyers as payment. Because the problem is symmetric, we will only focus on one location for the remainder of the analysis.

## Buyer's lifetime problem

A buyer's value function at the beginning of his life is given by

$$\begin{aligned} V^b = & \max_{h_t, q_t^m, q_t^b, x_{t+1}^m, x_{t+1}^b} - h_t + \pi(u(q_t^m) + \beta U(x_{t+1}^m)) + (1 - \pi)(u(q_t^b) + \beta U(x_{t+1}^b)) \\ \text{s.t. } & h_t + \mathcal{I}\tau_t^y = \phi m_t + k_t^b \\ & p_t q_t^m \leq m_t \\ & p_t q_t^b \leq m_t + \frac{Rk_t^b}{\phi_{t+1}} \\ & x_{t+1}^m = \phi_{t+1} m_t + Rk_t^b - \phi_{t+1} p_t q_t^m + (1 - \mathcal{I})\tau_t^o \\ & x_{t+1}^b = \phi_{t+1} m_t + Rk_t^b - \phi_{t+1} p_t q_t^b + (1 - \mathcal{I})\tau_t^o. \end{aligned}$$

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<sup>6</sup>The case of bilateral meetings in the DM is highly interesting, potentially more realistic, and leads to a number of additional frictions in this economy. For the points we want to make with this paper, however, we want to isolate the friction stemming from the Mundell-Tobin effect and the need for liquidity, while keeping the rest of the environment as frictionless as possible.

All variables with a superscript  $m$  indicate decisions of relocated buyers (movers). Variables with superscript  $b$  indicate decisions of buyers prior to learning about relocation, or those of buyers that aren't relocated, depending on the context. The first constraint is the standard budget constraint for the portfolio choice when young. The second constraint denotes that relocated buyers cannot spend more than their money holdings during the DM, and the third constraint denotes that nonrelocated buyers cannot spend more than their total wealth for consumption during the DM<sup>7</sup>. The fourth and fifth constraint denote that buyers use all remaining resources for consumption when old.

We can simplify the problem by substituting some variables. Additionally, we assume that  $\frac{\phi_t}{\phi_{t+1}} \geq \frac{1}{R}$ . In this case, the second constraint always holds at equality, as there is no reason for buyers to save money for the CM if capital pays a higher return. We also know that the third constraint never binds, because spending all wealth during the DM would imply  $x_{t+1} = 0$ , but this violates the Inada conditions. After simplification, the buyer's problem is

$$\begin{aligned}
V^b = \max_{m_t, k_t^b, q_t^b} & \quad \mathcal{I}\tau^y - \phi_t m_t - k_t^b + \pi \left( u \left( \frac{m_t}{p_t} \right) + \beta U(Rk_t^b + (1 - \mathcal{I})\tau^o) \right) \\
& \quad + (1 - \pi)(u(q_t^b) + \beta U(\phi_{t+1}m_t + Rk_t^b - \phi_{t+1}p_t q_t^b) + (1 - \mathcal{I})\tau^o).
\end{aligned} \tag{4}$$

## Seller's lifetime problem

A seller's value function at the beginning of his life is given by

$$\begin{aligned}
V^s = \max_{h_t, q_t^s, x_{t+1}^s} & \quad -h_t^s - q_t^s + \beta U(x_{t+1}^s) \\
s.t. & \quad h_t^s = k_t^s \\
& \quad x_{t+1}^s = Rk_t^s + \phi_{t+1}p_t q_t^s.
\end{aligned}$$

Here, we already assumed that sellers do not accumulate money in the first CM, which is true in equilibrium for  $\gamma \geq \frac{1}{R}$ . Thus, the first constraint denotes that sellers work only to accumulate capital, and the second constraint denotes that a seller's CM consumption is equal to the return on capital plus his revenue from selling the special good in the DM. Again, we can simplify the problem by substituting in the constraints. After simplification, the seller's problem is

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<sup>7</sup>The purchasing power of capital is scaled by  $\frac{R}{\phi_{t+1}}$  to ensure that buyers give up the same amount of CM consumption by paying with capital and money.



$$V^s = \max_{q_t^s, k_t^s} -k_t^s - q_t^s + \beta U(Rk_t^s + \phi_{t+1} p_t q_t^s). \quad (5)$$

## Planner's problem

Before we turn to market outcomes in the next section, we want to establish the solution to the planner's problem in order to derive a benchmark in terms of welfare. We will focus on stationary equilibria; due to the assumption stated in equation (3), a planner would actually prefer a nonstationary equilibrium with a capital stock that is growing over time. However, such an equilibrium cannot be implemented in a market economy due to the overlapping generations structure<sup>8</sup>, and is thus not of particular interest as a benchmark case. Note that we also focus on future generations here, while ignoring the initial old. By doing so, we follow papers like Smith (2002) and Haslag and Martin (2007), as we want to compare our results to theirs.

The planner maximizes the utility of a representative generation, which is given by

$$\begin{aligned} V^g &= -h^b - h^s + u(q^b) - q^s + \beta(U(x^b) + U(x^s)) \\ \text{s.t.} \quad & q^b = q^s \\ & x^b + x^s = Rk + h^b + h^s - k \\ & k \leq h^b + h^s, \end{aligned}$$

whereas the first constraint ensures market clearing for special goods, and the second and third constraints ensure market clearing for general goods. General goods can either be directly transferred from young agents to old agents, or invested in capital after production. Since  $R > 1$ , it is quite clear that a planner wants to set  $k = h^b + h^s$ , which then implies

$$h^s + h^b = \frac{x^s + x^b}{R}. \quad (6)$$

Thus, the planner's maximization problem can be written as

$$V^g = \max_{q, x^b, x^s} -\frac{x^s + x^b}{R} + u(q^b) - q^b + \beta(U(x^b) + U(x^s)). \quad (7)$$

Solving this problem yields the following first-order conditions:

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<sup>8</sup>More specifically, the reason is that agents only care about their own welfare and have finite lives. It would still be possible to implement an increasing capital stock by implementing a nonstationary tax/transfer scheme, but we are focusing on stationary taxes in this paper.

$$q^b : \quad u'(q^b) = 1 \tag{8}$$

$$x^b : \quad U'(x^b) = \frac{1}{\beta R} \tag{9}$$

$$x^s : \quad U'(x^s) = \frac{1}{\beta R}. \tag{10}$$

Together with equation (6), these first-order conditions determine the optimal amounts of consumption and labor in this economy.

Having laid out the environment and established the steady-state first-best equilibrium, we now want to turn to market equilibria. But before solving the full model, we study the corner cases with  $\pi = 0$  and  $\pi = 1$  in the next two sections, respectively. We do this for two reasons: First, understanding these simpler cases will make it easier to analyze what is going on in the full model. Second, it is easier to establish results about monetary policy implementation schemes (i.e., setting  $\mathcal{I} = 0$  or 1) in the simplified model with  $\pi = 1$ , but these results are also going to be relevant for the full model.

### 3 Equilibrium with perfectly liquid capital

In this section, we solve for the market equilibrium for the special case of  $\pi = 0$ , which represents perfectly liquid capital and abstracts from any uncertainty for all agents in the model. As there is no tradeoff between money and capital in this case (both are equally liquid and safe), only the rate of return of the assets matter, and agents will only hold the asset with the higher rate of return. For  $\gamma > \frac{1}{R}$ , capital is the asset with the higher rate of return, and as this is the case we are most interested in, we abstract from money (and transfers) in this section. As we used  $p_t$  to denote the price of the DM good in terms of fiat money, we have to alter the problem slightly, as this price is undefined if money is not held in equilibrium. In this section, we therefore introduce  $\rho_t$ , which is the price of the DM good in terms capital.

Given these alterations of the model, the buyer's problem from equation (4) becomes

$$V^b = \max_{k_t^b, q_t^b} -k_t^b + u(q_t^b) + \beta U(Rk_t^b - \rho_t q_t^b),$$

and yields the following first-order conditions:

$$q^b : \quad u'(q^m) = \rho_t \beta U'(Rk_t^b - \rho_t q_t^b) \quad (11)$$

$$k^b : \quad 1 = \beta R U'(Rk_t^b - \rho_t q_t^b). \quad (12)$$

The seller's problem is only affected by the change in notation. Solving equation (5) yields

$$q^s : \quad 1 = \beta R \rho_t U'(Rk_t^s + \rho_t q_t^s) \quad (13)$$

$$k^b : \quad 1 = \beta R U'(Rk_t^s + \rho_t q_t^s). \quad (14)$$

Combining equations (13) and (14) gives  $\rho_t = 1$ , which means that DM prices are such that the seller is indifferent between working in the CM or the DM. Then, combining this with equations (11) and (12) yields

$$u'(q^b) = 1.$$

Furthermore, it is easily confirmed that  $h^s + h^b = \frac{x^s + x^m}{R}$ . Finally, equations (12) and (14) show that CM consumption is equal to the first-best level in this equilibrium. Thus, we can conclude that perfectly liquid capital allows to implement the planner's solution<sup>9</sup>.

## 4 Equilibrium with perfectly illiquid capital

Having shown that there are no inefficiencies with perfectly liquid capital, we now want to investigate the other extreme case, which is perfectly illiquid capital. In the model, this is captured by  $\pi = 1$ , which means that all buyers are relocated during the DM. In this case, fiat money is the only way to acquire consumption during the DM. Thus, for  $\gamma \geq \frac{1}{R}$ , there is no tradeoff between holding fiat money and capital, as only fiat money provides DM consumption, while capital (weakly) dominates in terms of providing CM consumption. This means that the Mundell-Tobin

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<sup>9</sup>We need to add one qualifier to this statement. To implement the planner's solution with perfectly liquid capital, utility functions have to be such that sellers want to consume at least as much in the CM as they receive from selling the efficient amount of special goods at  $\rho = 1$  in the DM. If that is not the case, sellers only work in the DM and prices would increase. The way we derived the equilibrium is incorrect in that case, as we haven't formally included a non-negativity constraint on the seller's capital accumulation. Formally, this requires  $\frac{1}{\beta R} \leq U'(u'^{-1}(1))$ . We are assuming that this holds for the remainder of the paper. An alternative assumption we could make to prevent this issue is that the measure of sellers is sufficiently larger than the measure of buyers, such that individual sellers don't sell too many special goods in the DM. This friction might be interesting to study in other contexts, but it is not relevant for the points we want to make in this paper.

effect is not at play in this version of our model.

With  $\pi = 1$ , the buyer's lifetime value function then simplifies to

$$V^b = \max_{m_t, k_t^b} \mathcal{I}\tau^y - \phi_t m_t - k_t^b + u\left(\frac{m_t}{p_t}\right) + \beta U(Rk_t^b + (1 - \mathcal{I})\tau^o).$$

Solving this problem yields two first-order conditions:

$$m_t : \quad p_t \phi_t = u'\left(\frac{m_t}{p_t}\right) \quad (15)$$

$$k_t : \quad \frac{1}{\beta R} = U'(Rk_t^b + (1 - \mathcal{I})\tau^o), \quad (16)$$

while solving the seller's problem yields the following first-order conditions:

$$q^s : \quad 1 = \phi_{t+1} p_t \beta_t U'(Rk_t^s + \rho_t q_t^s) \quad (17)$$

$$k^b : \quad 1 = \beta R U'(Rk_t^s + \rho_t q_t^s). \quad (18)$$

Combining equations (17) and (18) yields  $p_t = \frac{R}{\phi_{t+1}}$ . Plugging this into equation (15) gives

$$u'(q^m) = \frac{\phi_t}{\phi_{t+1}} R. \quad (19)$$

There are a couple of things to observe here. First, equations (16) and (18) demonstrate that the CM consumption is always at the first-best level, independent of monetary policy. This shows that the Mundell-Tobin effect is not at play with perfectly illiquid capital. Second, equation (19) shows that the DM consumption is equal to its optimal level for  $\gamma = \frac{1}{R}$ , which can be interpreted as the Friedman rule. However, the total amount of work  $h^b + h^s$  that agents undertake in this equilibrium is strictly higher than in the planner's solution, thus reducing welfare compared to the first-best.

#### 4.1 Definition of the Friedman rule

Before continuing the analysis, it is worth spending a moment on the definition of the Friedman rule. There exists a variety of definitions, both in terms of explaining it or in the context of models. However, the one that - in our opinion at least - is best at capturing the economic intuition of Friedman's original statement, is the following: The rate of return on money has to be such that the opportunity cost of holding money is zero. In our model, this definition corresponds to  $\gamma = \frac{1}{R}$ , which is the definition we will be using for the remainder of this paper. In models based on Lagos

and Wright (2005), the Friedman rule typically corresponds to  $\gamma = \beta$ . This can be interpreted as setting the inflation rate such that carrying money across time is costless. This interpretation still holds in our model, but it does not correspond to no opportunity cost of holding money, as there is another asset (capital) in our model that offers an even higher rate of return. A third popular interpretation of the Friedman rule is to set the nominal interest rate to zero. However, models that take liquidity serious have shown that this is a bad definition, as there is a variety of different nominal interest rates in the economy. A refined definition that resulted from these models is to set the interest rate of a (hypothetical) perfectly *illiquid* bond to zero. In the context of our model, this definition corresponds to  $\gamma = \frac{1}{R}$  for  $\pi = 1$ , and  $\gamma < \frac{1}{R}$  for  $\pi < 1$ , with the exact level of  $\gamma$  depending on parameters and the supply of that illiquid bond.

This analysis shows that defining the Friedman rule is not straightforward. In models based on Lagos and Wright (2005), the quasilinear structure ensures that no asset can exist in the economy with a return higher than  $\frac{1}{\beta}$ , which means that all three definitions of the Friedman rule discussed here nicely coincide. In models that abandon the quasilinear utility structure, this is typically not the case, so the right interpretation of the Friedman rule becomes crucial for policy analysis.

## 4.2 Monetary policy implementation

Whether the Friedman rule is the welfare-maximizing policy in the economy with perfectly illiquid capital also depends on the cost of implementing it, which in turn depends on how monetary policy is implemented. In this section, we discuss how welfare is affected by the two tax regimes.

We first derive the stationary equilibrium when monetary policy is implemented over young buyers ( $\mathcal{I} = 1$ ). In a stationary equilibrium we must have:  $q^b = q^s = q$  (DM market clearing),  $m = M$  (money market clearing) and  $\phi/\phi_{+1} = \gamma$  i.e. the inflation rate must equal the growth rate of the money supply since the real value of money is constant over time or  $\phi M = \phi_{+1} M_{+1}$ . Furthermore the real value of taxes/transfers paid/received by young buyers is given by:  $\tau^y = \phi(M - M_{-1}) = \frac{\gamma-1}{\gamma}\phi M$ . Using this and the definitions and first-order conditions derived above for  $\pi = 1$ , we can then define a stationary equilibrium with perfectly illiquid capital as a list of eight variables  $\{h^b, h^s, k^b, k^s, \phi_{+1}M, q, x^b, x^s\}$  solving:

$$u'(q) = \gamma R \quad (20)$$

$$\phi_{+1}M = qR \quad (21)$$

$$x^b = x^s = x \quad \text{solving} \quad U'(x) = \frac{1}{\beta R} \quad (22)$$

$$h^b = \phi_{+1}M + k^b \quad (23)$$

$$h^s = k^s \quad (24)$$

$$x^b = Rk^b \quad (25)$$

$$x^s = R(k^s + q) \quad (26)$$

As already pointed out above, equation (25) shows that  $x^b$  is independent of the inflation rate and thus the Mundell-Tobin effect does not affect the buyer's capital accumulation. However, equation (26) shows that total capital accumulation is still indirectly affected by inflation, as sellers accumulate less capital if they expect to sell more goods in the DM - and as we know from equation (20), DM consumption and thus also real balances are decreasing in the inflation rate. This can be shown formally by differentiating both sides of (20) with respect to  $\gamma$  to get:

$$\frac{\partial q}{\partial \gamma} = \frac{R}{u''(q)} < 0 \quad (27)$$

which is negative from the concavity of  $u(q)$ .

Next we derive the stationary equilibrium when monetary policy is implemented over old buyers ( $\mathcal{I} = 0$ ). The only changes in the stationary equilibrium allocation affect the labor supply of the buyer (equation (23)) and the consumption of the buyer in the second CM (equation (25)). These now read:

$$h^b = \gamma \phi_{+1}M + k^b \quad (28)$$

$$x^b = Rk^b + \tau^o = Rk^b + \phi_{+1}M(\gamma - 1) = x \quad (29)$$

Combining these two equations yields

$$h^{b,o} = \gamma \phi_{+1}M + \frac{x - \tau^o}{R} \quad (30)$$

$$k^{b,o} = \frac{x - \tau^o}{R} \quad (31)$$

while in the model where monetary policy is implemented over the young buyers we had

$$h^{b,y} = \gamma\phi_{+1}M + \frac{x}{R} - \tau^y \quad (32)$$

$$k^{b,y} = \frac{x}{R} \quad (33)$$

For  $\gamma > 1$  and thus  $\tau^o = \tau^y > 0$  ( $\gamma < 1$  and  $\tau^o = \tau^y < 0$ ), we have  $k^{b,o} < k^{b,y}$  and  $h^{b,o} > h^{b,y}$  ( $k^{b,o} > k^{b,y}$  and  $h^{b,o} < h^{b,y}$ ), i.e., capital accumulation is lower (higher) when the old are receiving the transfers, while the hours worked when young are higher (lower). For  $\gamma = 1$ , the two equilibria coincide. To put this in other words, if monetary policy consists of making transfers to buyers ( $\gamma > 1$ ), it is better to implement monetary policy over young buyers, whereas if monetary policy consists of raising taxes ( $\gamma < 1$ ), it is better to raise these taxes from old buyers. The reason for this is that capital has a return  $R > 1$ ; thus, if agents receive a transfer, it is better to receive it when young and invest it in capital, whereas if agents have to pay a tax, it is better to use the return on capital to pay it when old instead of paying it directly from labor income when young.

Next we investigate the effects of inflation on total labor supply  $H = h^b + h^s$  and total capital accumulation  $K = k^b + k^s$  for both policy implementation schemes. As we will show, these effects can depend on the elasticity of DM consumption with respect to inflation, which we denote as  $\epsilon_q$ .

**Proposition 1.** *With  $\mathcal{I} = 0$  inflation affects total labor supply and total capital accumulation in the following way:*

1. If  $|\epsilon_q| < 1$ :  $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} > 0$  and  $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} < 0$ : *Reverse Mundell-Tobin effect.*
2. If  $|\epsilon_q| > 1$ :  $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} < 0$  and  $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$ : *Mundell-Tobin effect.*
3. If  $|\epsilon_q| = 1$ :  $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = 0$  and  $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} = 0$ : *No Mundell-Tobin effect.*

With  $\mathcal{I} = 1$ ,  $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} < 0$  and  $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$  and thus the Mundell-Tobin effect holds for all values of  $|\epsilon_q|$ .

The proof to this Proposition can be found in the appendix. With  $\mathcal{I} = 0$ , in the first case the elasticity of DM-consumption to inflation is low. The wealth effect is stronger than the substitution effect and labor supply increases in inflation. Total capital decreases, so we actually have a reverse Mundell-Tobin effect. In the second case, the elasticity of DM-consumption to inflation is high. The substitution effect is stronger than the wealth effect and total work decreases in inflation, while total capital increases, so we have a Mundell-Tobin effect. In the third case wealth and substitution effects exactly offset each other and total labor supply and capital are independent from inflation. With  $\mathcal{I} = 1$ , the wealth effect is zero and thus the effect of inflation on total labor

supply is always negative.

After having established the effects of inflation on capital and labor supply, we can derive the optimal monetary policy.

**Proposition 2.** *With  $\pi = 1$ , the optimal monetary policy is to set  $\mathcal{I} = 0$ , and  $\gamma = 1/R$  for  $|\epsilon_q| \leq 1$ , and  $\gamma = \frac{|\epsilon_q|}{|\epsilon_q| + R - 1} \in (1/R, 1)$  for  $|\epsilon_q| > 1$ .*

The proof to this Proposition can be found in the appendix. The intuition behind it is as follows: For  $\mathcal{I} = 1$ ,  $\gamma = 1$  is optimal, because if the young are taxed, it is too costly to run a deflationary policy in order to increase DM consumption. With  $\mathcal{I} = 0$ ,  $\gamma = 1/R$  is optimal if the elasticity of DM consumption is sufficiently low, whereas  $\gamma \in [1/R, 1)$  is optimal otherwise, because if the old are taxed instead, implementing the Friedman rule is less costly. Since the equilibrium allocation coincides between the two regimes for  $\gamma = 1$ , and the social planner would never choose  $\gamma = 1$  with  $\mathcal{I} = 0$ , we can conclude that the optimal monetary policy always consists of taxing the old.<sup>10</sup>

These findings link our results to Haslag and Martin (2007). They have shown that a constant money stock is typically optimal in an OLG model, even without the Mundell-Tobin effect. We can confirm these results for our model for the case of  $\mathcal{I} = 1$ . However, our model allows to implement the Friedman rule in a less costly way by taxing old agents instead<sup>11</sup>, and we can show that in this case,  $\gamma^* < 1$ , with the Friedman rule being the optimal policy if the elasticity of DM consumption is relatively low.

The analysis so far shows that the first-best allocation is achieved automatically if capital is perfectly liquid, while the first-best consumption levels can be implemented at the Friedman rule in the case of perfectly illiquid capital, but at the cost of having agents work more in the first CM of their lives. Whether agents still prefer the Friedman rule over higher money growth rates in the case of perfectly illiquid capital depends on the way the Friedman rule is implemented and on

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<sup>10</sup>It is not true that taxing the old strictly dominates taxing the young, however: If for some reason the policymaker is restricted to running inflationary policies ( $\gamma > 1$ ), taxing - respectively paying transfers to - the young agents is more efficient.

<sup>11</sup>While this works nicely in our model, it would not do the trick in pure OLG models. The difference is that relocation occurs during the final stage of an agent's life in models such as Smith (2002) or Haslag and Martin (2007). The reason that taxing the old is strictly cheaper in our model is that all agents know they have access to their capital when they have to pay the tax, and can thus fully pay the tax via capital investment. In pure OLG models, only non-relocated agents have access to their capital during the final stage of their life.



the utility functions of agents. But even if the Friedman rule is welfare maximizing in the case of perfectly illiquid capital, welfare is still strictly lower than in the case of perfectly liquid capital.

## 5 Equilibrium with partially liquid capital

After having analyzed both corner solutions, we can finally turn to the case of partially liquid capital, or partial relocation in terms of the model. This introduces uncertainty for buyers. There is now a clear tradeoff between acquiring money or capital: Money provides insurance against the relocation shock, while capital offers a higher rate of return for  $\gamma > \frac{1}{R}$ . The tradeoff just described is related to the Mundell-Tobin effect: At low inflation rates, acquiring money for insurance is connected with only a small loss of return, thus making money relatively more attractive and depressing capital accumulation. At high inflation rates, acquiring money for insurance is really costly in terms of rate of return foregone, and thus capital accumulation becomes relatively more attractive.

For sellers, there is no uncertainty even with partially liquid capital. Thus, the results we found in equations (17) and (18) still hold. This implies two things: First, the seller's CM consumption is still unaffected by monetary policy and always at the first-best level; second,  $p_t = \frac{R}{\phi_{t+1}}$  still holds.

Solving the buyers' lifetime problem while making use of this result yields the following first-order conditions:

$$m_t : \quad \frac{\phi_t}{\phi_{t+1}} = \pi \frac{1}{R} u' \left( \frac{\phi_{t+1} m_t}{R} \right) + (1 - \pi) \beta U'(\phi_{t+1} m_t + R(k_t^b - q_t^b) + (1 - \mathcal{I})\tau^o) \quad (34)$$

$$k_t^b : \quad \frac{1}{\beta R} = \pi U'(Rk_t^b) + (1 - \pi) U'(\phi_{t+1} m_t + R(k_t^b - q_t^b) + (1 - \mathcal{I})\tau^o) \quad (35)$$

$$q_t^b : \quad u'(q_t^b) = \beta R U'(\phi_{t+1} m_t + R(k_t^b - q_t^b) + (1 - \mathcal{I})\tau^o). \quad (36)$$

We are now ready to define an equilibrium in the full model:

**Definition 1.** *An equilibrium is a sequence of variables  $m_t, k_t^b, q_t^b \forall t$  that solve equations (34)-(36).*

Next, we are interpreting the equilibrium with a number of propositions. The proofs to all of them can be found in the appendix.

**Proposition 3.** *Capital accumulation is increasing with  $\gamma$ , while real balances and total wealth are decreasing in  $\gamma$ . Furthermore, capital accumulation increases more strongly with  $\gamma$  for lower  $\pi$ .*

Basically, this proposition describes the Mundell-Tobin effect - which exists in our model with partially liquid capital, while it is shut down with perfectly illiquid capital. The proposition also

shows that the Mundell-Tobin effect is stronger for more liquid capital. This makes intuitive sense, as insurance becomes less important at low risks of relocation, so agents react more strongly to differences in return between the two assets. The results that both total wealth (which corresponds to hours worked) and real balances go down with higher inflation rates also hold in the model with perfectly illiquid capital.

**Proposition 4.** *At  $\gamma = \frac{1}{R}$ , consumption of relocated and nonrelocated buyers is equal in both markets and perfectly smoothed across DM and CM. For  $\gamma > \frac{1}{R}$ , nonrelocated buyers still perfectly smooth consumption levels across DM and CM, but their consumption in both markets is decreasing with increases in  $\gamma$ . The DM consumption of relocated buyers is decreasing with  $\gamma$ , while their CM consumption is increasing with  $\gamma$ .*

This proposition shows that perfect insurance against the relocation shock is always achievable at the Friedman rule. Deviations from the Friedman rule lead to decreases in total consumption for all agents. For relocated agents, DM consumption decreases stronger than for nonrelocated agents, as they can only use money to purchase special goods. Therefore, their CM consumption has to increase with inflation, as they still have the capital they accumulated hoping to be able to spend it during the DM.

**Proposition 5.** *For  $\mathcal{I} = 1$ ,  $\gamma = 1$  is the optimal money growth rate, independent of all other parameters. Furthermore, the Friedman rule is relatively more costly in this regime for low  $\pi$ .*

This proposition once again confirms Haslag and Martin (2007), and shouldn't be surprising after the results we stated in Proposition 2. There, we showed that a constant money stock is optimal even in the absence of the Friedman rule if taxes are levied on young agents, as the implementation of the Friedman rule is just too costly in this regime. However, the second result in this proposition shows that there is something to the intuition in Smith (2002): While the Friedman rule is suboptimal with or without the Mundell-Tobin effect, a stronger Mundell-Tobin effect makes the Friedman rule even less desirable in this case. The intuition behind this is as follows: For high  $\pi$ , capital cannot be used to trade in most DM trades anyway. Thus, the insurance motive is relatively more important, and providing a high return on money does not crowd out a lot of capital. At low  $\pi$  however, a majority of DM trades could be made with capital, but at low inflation rates, most of them will be made with money - at the Friedman rule, all of them will be made with money actually. Thus, providing this insurance against the relocation shock becomes really costly, as it crowds out a lot of highly productive capital. While an individual agent does not care directly, because fiat money offers the same return as capital at the Friedman rule, this is costly for society, as lump-sum taxes have to be raised to implement this high return on fiat money, while capital delivers this return naturally.

**Proposition 6.** *For  $\mathcal{I} = 0$  and utility functions without wealth effects, the optimal money growth rate  $\gamma^*$  is a function of the liquidity of capital  $\pi$ , with  $\gamma^* \rightarrow \frac{1}{R}$  for  $\pi \rightarrow 1$ , and  $\gamma^* \rightarrow 1$  for  $\pi \rightarrow 0$ .*

This proposition shows that the premise behind Smith (2002) was correct, and the Mundell-Tobin effect is enough to make deviations from the Friedman rule optimal. In Proposition 2, we have shown that the Friedman rule can be optimal when capital is fully illiquid in this model for some utility functions and when monetary policy is implemented by taxing the old<sup>12</sup>. This proposition now shows that deviations from the Friedman rule become optimal once capital is partially liquid, and the optimal money growth rate is increasing in the liquidity of capital, approaching a constant money stock for fully liquid capital. The intuition is again the policy tradeoff between full insurance against the liquidity shock (which the Friedman rule delivers), and optimal capital accumulation (which a constant money stock delivers). The monetary authority has to weigh these two effects. For highly liquid capital, insurance becomes relatively less important, while the losses from capital accumulation can be quite large, and vice versa for relatively illiquid capital.

## 6 Conclusion

We have added a market which requires liquid assets to trade to an OLG model with relocation shocks, in order to study whether the Mundell-Tobin effect can make deviations from the Friedman rule optimal. We found that the implementation of monetary policy matters in OLG models with capital: If changes in the money stock are implemented by taxing young agents, a constant money stock is welfare-maximizing, independent of other model parameters. Even in this case, however, we can show that the Friedman rule is relatively more costly if the Mundell-Tobin effect is stronger. If instead old agents are taxed (or more generally, taxes can be paid by accumulating capital), the Friedman rule is optimal for some utility functions in the absence of the Mundell-Tobin effect. Once capital becomes partially liquid and thus the Mundell-Tobin effect is present in the model, we can show that deviations from the Friedman rule are optimal. In this case, the optimal level of money growth is an increasing function of the liquidity of capital. Furthermore, we have also shown that some definitions of the Friedman rule that are used interchangeably in the New Monetarist literature are not applicable in more general model structures, with the definition of the Friedman

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<sup>12</sup>This might sound like a special case without too much relevance. However, we follow a positive approach: We wanted to show that in an economy where the Friedman rule is optimal in the absence of the Mundell-Tobin effect, the Mundell-Tobin effect can make deviations from the Friedman rule optimal. We achieved to show this here. The reason that we need to make assumptions about the structure of the utility functions and the way monetary policy is implemented is stemming from the OLG structure we used in our model, which makes it notoriously hard to have the Friedman rule as the optimal policy prescription, even though it is commonly found in other models.

rule as setting the opportunity cost of holding money equal to zero proving to be the most reliable one.

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# Appendix A

## A.1 Proof of Proposition 1

*Proof.* We begin with the model where monetary policy is implemented over taxes/transfers to old buyers ( $\mathcal{I} = 0$ ). We can write total labor supply in the CM in equilibrium as:

$$H^{\mathcal{I}=0} = \gamma q R + \frac{x}{R} - q(\gamma - 1) + \frac{x}{R} - q \quad (37)$$

The first term are the new real money holdings of buyers, the second term and the third term are the new capital holdings (consumption in second CM minus transfers). The last two terms are the labor supply of the seller. Differentiating this w.r.t. inflation yields:

$$\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = R\left(q + \frac{\partial q}{\partial \gamma} \gamma\right) - \left(q + \frac{\partial q}{\partial \gamma} (\gamma - 1)\right) - \frac{\partial q}{\partial \gamma} \quad (38)$$

Looking at (37) and ignoring the indirect effect of inflation over  $q$  the first effect has two opposing components: First, inflation marginally increases the labor supply by  $qR$ . Inflation makes acquiring real balances more expensive, so buyers have to work more to acquire the same amount of real balances. Second, inflation marginally decreases the labor supply by  $q$ . Inflation gives buyers higher transfers in the second CM and thus makes real balances more valuable, which decreases the labor supply. Since  $R > 1$  the total effect is positive, i.e. the first effect dominates the second. Since these effects make a given level of real balances more or less valuable we interpret these effects as *wealth effects*.

The remainder of 38 are then effects which influence the labor supply over real balances or  $q$ . They are given by

$$\frac{\partial q}{\partial \gamma} \gamma (R - 1) < 0 \quad (39)$$

which must be negative from 27. A lower consumption level in the DM induced by higher inflation also lowers the labor supply in the CM. We can interpret this as a *substitution effect*. Agents substitute from consumption in the DM to less work (leisure) in the first CM if inflation rises. Summarizing we can rewrite the effect of inflation on total labor supply in the CM as:

$$\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = \underbrace{q(R - 1)}_{\text{wealth effect}} + \underbrace{\frac{\partial q}{\partial \gamma} \gamma (R - 1)}_{\text{substitution effect}} \quad (40)$$

As usual with opposing wealth and substitution effects there are three possible scenarios: Either the wealth effect dominates the substitution effect, the substitution effect dominates the wealth effect or they exactly offset each other. Using 40 this depends on whether

$$\begin{aligned}
q(R-1) &\underset{\geq}{\leq} -\frac{\partial q}{\partial \gamma} \gamma (R-1) \\
&\Leftrightarrow \\
-\frac{qu''(q)}{u'(q)} &\underset{\geq}{\leq} 1
\end{aligned} \tag{41}$$

The lefthand-side of this relationship denotes the coefficient of relative risk aversion, or the inverse of the elasticity of DM-consumption with respect to inflation,  $\epsilon_q$ . To see this, derive the elasticity as

$$\epsilon_q = \frac{dq/q}{d\gamma/\gamma} = \frac{\gamma}{q} \frac{\partial q}{\partial \gamma} = \frac{u'(q)}{qu''(q)} < 0 \tag{42}$$

Thus we can interpret the lefthand-side of 41 as the absolute value of the inverse of the elasticity of DM-consumption to inflation:

$$-\frac{qu''(q)}{u'(q)} = \frac{1}{|\epsilon_q|} \tag{43}$$

If  $\epsilon_q$  is below 1 in absolute value (the elasticity is low) the wealth effect dominates the substitution effect. If  $\epsilon_q$  is above 1 in absolute value (the elasticity is high) the substitution effect dominates the wealth effect.

We now turn to the Mundell-Tobin effect. Denoting total capital investment as  $K = k^b + k^s$  we can write

$$K^{\mathcal{I}=0} = \frac{2x}{R} - q(\gamma - 1) - q = \frac{2x}{R} - q\gamma \tag{44}$$

and the first derivative is

$$\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} = -(\gamma \frac{\partial q}{\partial \gamma} + q) = \frac{u'(q)}{-u''(q)} - q \tag{45}$$

Comparing the effects of inflation on labor supply from equation (41) and on capital investment from equation (45), we see that they are closely related. Plugging in the respective values for  $\epsilon_q$  yields the three cases from the proposition.

If monetary policy is instead implemented over the young buyers ( $\mathcal{I} = 1$ ), we can write total labor supply in the CM in equilibrium as:

$$H^{\mathcal{I}=1} = \gamma qR + \frac{x}{R} - qR(\gamma - 1) + \frac{x}{R} - q \tag{46}$$

So we see the only difference in the total labor supply is on the part with the taxes/transfers. We already commented on this above. Performing a similar analysis to before in terms of wealth and substitution effects yields the following:

$$\frac{\partial H^{\mathcal{I}=1}}{\partial \gamma} = \underbrace{q(R - R)}_{\text{wealth effect}} + \underbrace{\frac{\partial q}{\partial \gamma}(R - 1)}_{\text{substitution effect}} \quad (47)$$

The basic difference compared to equation (40) is that here the wealth effect is zero and thus the effect of inflation on total work is always negative since there is only a substitution effect. Total capital with  $\mathcal{I} = 1$  is given by:

$$K^{\mathcal{I}=1} = \frac{2x}{R} - q \quad (48)$$

since in this case only the capital holdings of sellers respond to inflation and  $q$ . The first derivative is thus just  $-\frac{\partial q}{\partial \gamma}$  which is positive. Thus we have a Mundell-Tobin effect (capital increases with inflation). ■

## A.2 Proof of Proposition 2

*Proof.* To prove this proposition, we can derive the optimal policy and the respective welfare achieved in each case from proposition 1 and compare them. The welfare of a representative generation in this model can be written as

$$V^g = -H + u(q) - q + 2\beta U(x) \quad (49)$$

We start with case 3, where  $|\epsilon_q| = 1$  and there is no Tobin-effect (this case only exists in the model where  $\mathcal{I} = 0$ ). We know that in this case inflation has no effect on total work either, i.e.  $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = 0$ . In this case the derivative of 49 is just

$$\frac{\partial V^g}{\partial \gamma} = \frac{\partial q}{\partial \gamma}(u'(q) - 1) = \frac{\partial q}{\partial \gamma}(\gamma R - 1) \quad (50)$$

Since  $\frac{\partial q}{\partial \gamma}$  is negative welfare clearly decreases for inflation rates above the Friedman rule. Therefore  $\gamma^* = 1/R$  must be the optimal inflation rate.<sup>13</sup>

Next we consider case 1 where  $|\epsilon_q| < 1$  and there is a reverse Tobin effect (this case also exists only in the model where  $\mathcal{I} = 0$ ). In this case, total work increases with inflation,  $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} > 0$  and thus the derivative of 49 is

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<sup>13</sup>We don't consider inflation rates below the FR because in this case no capital would be used also for the CM and this allocation would certainly be worse than the Friedman-rule allocation with capital.



$$\frac{\partial V^g}{\partial \gamma} = -\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} + \frac{\partial q}{\partial \gamma}(\gamma R - 1) \quad (51)$$

At the FR this derivative is negative and it gets more negative if the inflation rate goes above the FR. Thus welfare decreases for inflation rates above the FR and also here  $\gamma^* = 1/R$  is the optimal inflation rate.<sup>14</sup> Inflation is clearly bad for welfare because it decreases consumption and makes agents work more. So inflation should be as low as possible, i.e., at the Friedman rule.

Finally, we consider case 2 where there is a Mundell-Tobin effect. This case exists in both models but in the first only when  $|\epsilon_q| > 1$ . We know in both models that the derivative of total work in the CM to inflation is negative in this case, i.e.  $\frac{\partial H}{\partial \gamma} < 0$  and the total derivative is again

$$\frac{\partial V^g}{\partial \gamma} = -\frac{\partial H}{\partial \gamma} + \frac{\partial q}{\partial \gamma}(\gamma R - 1) \quad (52)$$

In contrast to cases 1 and 3, the derivative at the FR is now positive. So increasing inflation above the FR increases welfare. This is because in this case inflation actually presents a tradeoff for agents. Higher inflation means less consumption in the DM but at the same time less work in the first CM. The positive sign of the derivative at the FR indicates that agents prefer to consume less than first best in the DM and work less in the first CM.

What are the concrete optimal inflation rates in both models? We first look at the model where monetary policy is implemented over young buyers ( $\mathcal{I} = 1$ ). In this model we know total hours only change due to a substitution effect and the derivative of total hours to inflation is  $\frac{\partial q}{\partial \gamma}(R - 1)$  according to equation (47). Thus equation (52) becomes

$$\frac{\partial V^g}{\partial \gamma} = -\frac{\partial q}{\partial \gamma}(R - 1) + \frac{\partial q}{\partial \gamma}(\gamma R - 1) = \frac{\partial q}{\partial \gamma}R(\gamma - 1) \quad (53)$$

We know that  $\frac{\partial q}{\partial \gamma}$  is negative. Thus 53 is positive as long as  $\gamma < 1$  and negative if  $\gamma > 1$ . Thus  $\gamma^* = 1$  maximizes  $V^g$ . For  $\gamma < 1$  welfare increases in inflation and for  $\gamma > 1$  welfare decreases in inflation. This result holds independent of the curvature of the utility function.

If monetary policy is implemented over old buyers ( $\mathcal{I} = 0$ ) we know we need a high elasticity of DM-consumption to inflation ( $|\epsilon_q| > 1$ ) to be in case 2. Then from equation (40) we get the derivative of total work towards inflation as

$$\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = q(R - 1) + \frac{\partial q}{\partial \gamma}\gamma(R - 1) \quad (54)$$

Thus equation (52) becomes

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<sup>14</sup>Also here it might seem optimal to decrease inflation below the FR but this would again imply a jump into an economy without capital where welfare would be certainly lower than at the FR with capital. So again we don't consider inflation rates below the FR.

$$\frac{\partial V^g}{\partial \gamma} = -q(R-1) - \frac{\partial q}{\partial \gamma} \gamma(R-1) + \frac{\partial q}{\partial \gamma} (\gamma R - 1) = -q(R-1) + \frac{\partial q}{\partial \gamma} (\gamma - 1) \quad (55)$$

We know for  $|\epsilon_q| > 1$  equation (55) is positive at the FR and negative at  $\gamma = 1$ . The interior solution solves

$$\gamma^* = \frac{|\epsilon_q|}{|\epsilon_q| + R - 1} \in (1/R, 1) \quad (56)$$

The welfare results also imply an ordering of the two ways of implementing monetary policy. We know if  $\gamma = 1$  both regimes are equivalent in terms of allocation and we also know that if monetary policy is implemented over young buyers  $\gamma^* = 1$  is the optimal inflation rate. But this allocation is always feasible, but not optimal if monetary policy is implemented over old buyers (Remember that equilibrium allocations coincide for  $\gamma = 1$ ). Thus, we can conclude that the optimal monetary policy is to set  $\mathcal{I} = 0$  and  $\gamma = 1/R$  for  $|\epsilon_q| \leq 1$ , and to set  $\mathcal{I} = 0$  and  $\gamma = \in [1/R, 1)$  for  $|\epsilon_q| > 1$ . ■

### A.3 Proof of Proposition 3

*Proof. To be done. Show  $\frac{\partial k^b}{\partial \gamma} > 0$ ,  $\frac{\partial \phi^m}{\partial \gamma} < 0$ ,  $\frac{\partial h^b}{\partial \gamma} < 0$  (maybe this one depends on utility functions?),  $\frac{\partial \frac{\partial k^b}{\partial \gamma}}{\partial \pi} < 0$ .* ■

### A.4 Proof of Proposition 4

*Proof. To be done. Show  $q^m = q^b$  and  $x^m = x^b$  at  $\gamma = \frac{1}{R}$ ,  $\frac{\partial q^b}{\partial \gamma} < 0$ ,  $\frac{\partial x^b}{\partial \gamma} < 0$ ,  $\frac{\partial q^m}{\partial \gamma} < 0$ ,  $\frac{\partial x^m}{\partial \gamma} > 0$ . Perfect consumption smoothing for nonrelocated buyers can be seen directly in equation (36).* ■

### A.5 Proof of Proposition 5

*Proof. To be done.* ■

### A.6 Proof of Proposition 6

*Proof. To be done.* ■