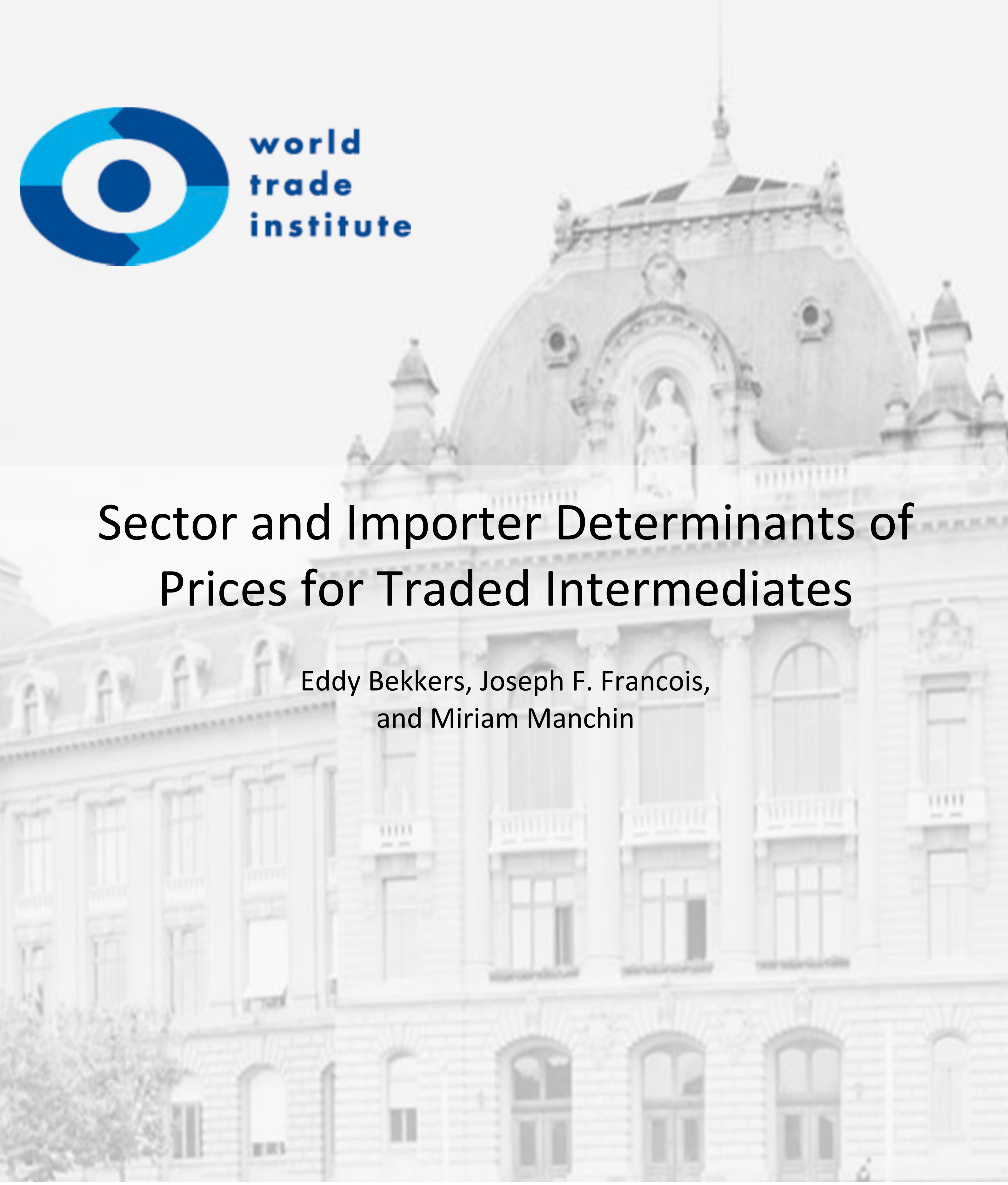


Sector and Importer Determinants of Prices for Traded Intermediates

Eddy Bekkers, Joseph F. Francois,
and Miriam Manchin

WTI Working Paper No. 04/2019

UNIVERSITÄT
BERN

Sector and Importer Determinants of Prices for Traded Intermediates*

Eddy Bekkers[†]
World Trade Organization
Joseph Francois[‡]
University of Bern and CEPR (London)
Miriam Manchin[§]
Politecnico di Milano

October 2019

Abstract

While the literature on traded goods prices emphasizes final goods prices and related consumer theory to explain variation in goods prices with importer characteristics, trade in intermediates actually constitutes about two-thirds of total trade. We propose a mechanism for explaining variations in the prices of intermediates as a function of importer characteristics, wherein production is vulnerable to failure and the probability of failure declines in the quality of intermediates. Higher wages mean a greater opportunity cost of failure, leading to a stronger demand for high-quality intermediates where firms face higher wages. We find empirical support for this mechanism in the case of intermediate goods using IV regressions. In addition, our findings indicate that while the cost of labor explains about one-fifth of variation in imported intermediate prices, it is a non-significant determinant of imported final good prices.

Keywords: Traded Goods Prices, Intermediates Trade
JEL codes: F12, F14

* This is the final working paper version of the text. Please cite as *Canadian Journal of Economics*, forthcoming.

*We want to thank Richard Baldwin, Gordon Hanson, Peter Egger, Miklos Koren, Beata Javorcik, participants at ETSG, various CEPR organized workshop presentations, and a seminar at Central European University for useful comments and discussions. All errors are our own.The opinions expressed in this article should be attributed to its authors and are not meant to represent the positions or opinions of the WTO and its members and are without prejudice to Members' rights and obligations under the WTO. The authors do not have conflicts of interest. Corresponding address: Eddy Bekkers, World Trade Organization, Rue de Lausanne 154, 1211 Geneve, Switzerland.

[†]eddy.bekkers@wto.org
[‡]joseph.francois@wti.org
[§]m.manchin@ucl.ac.uk

1 Introduction

Explaining the pattern of traded goods prices has been the subject of a rapidly expanding literature in international trade with studies using both firm level prices (Manova and Zhang, 2012; Verhoogen, 2008; Kugler and Verhoogen, 2012; Crozet, et al., 2012; Martin, 2012; Görg, et al., 2010; Bastos and Silva, 2010; Hallak and Sivadasan, 2010; Feenstra and Romalis , 2014; Dingel, 2017; Flach , 2016; Gervais, 2015) and aggregate prices measured by unit values at various product-aggregation levels (Schott, 2004; Hummels and Skiba, 2004; Hummels and Lugovskyy, 2009; Baldwin and Harrigan, 2011; Harrigan, et al., 2015; Bekkers, et al., 2012). To explain the pattern of traded goods based on importer characteristics, the theoretical literature has focused on final goods and consumer theory (Hummels and Lugovskyy (2009), Simonovska (2016), Alessandria and Kaboski (2011), Bekkers, et al. (2012)).[1]

The emphasis on final goods means factors linked to demand for intermediate goods have been largely neglected.[2] Yet, the majority of trade actually consists of trade in intermediates. Hummels et al. (2001) is one of the first papers highlighting the importance of intermediates trade. In this paper we aim to fill this gap, and offer an explanation for why more developed countries might import more expensive intermediate inputs. In doing so, we develop a model linking the pattern of traded goods prices of intermediates to importer characteristics, and apply the model to a panel dataset on traded intermediates goods prices and importer production costs. We show that sector-specific characteristics play an important role in explaining the pattern of prices for traded intermediates. In particular, we show that country-sector combinations with higher wages buy higher priced intermediates.

In principle, there are likely to be multiple mechanisms accounting for the correlation between imported intermediate goods prices and importer characteristics. In this paper we identify one such mechanism for which we also find empirical support. In the model developed here, firms produce goods using both intermediates and primary factors. The production process involves risk (it is vulnerable to occasional failure) and the probability of failure is declining in the quality of intermediates. Failures in the production process mean wasted resources (the inputs going into production). As these inputs include labor services, higher wages mean higher opportunity costs for failure and therefore lead to a stronger demand for high quality intermediates. This means, all other things being equal, that higher quality (and so higher priced) intermediates should be in greater demand in higher wage countries.

Our theoretical framework is related to three classes of models found in the literature. The first one is Kremer's (1993) O-ring model of production, where like the model here the quality of each of the inputs enters multiplicatively in the production function. When one of the inputs fails, the entire unit is rendered worthless, implying complementarity in the quality of inputs. A key difference is that Kremer (1993) assumes an unspecified distribution of the qualities of the labor inputs with assortative matching of the labor inputs as a result of complementarity, implying that higher quality labor inputs receive a higher wage. Our model instead contains an explicit relation between the quality of intermediates and the cost of the quality of those intermediates. This relation is motivated by the fact that higher quality goods are more expensive to produce or alternatively that higher quality goods require more factor inputs.

Like Kremer (1993), Verhoogen (2008) also works with a wage schedule in which wages rise (linearly) with the quality of labor inputs. However, he does not motivate this exclusively from the O-ring theory of Kremer (1993) but also with reference to efficiency wage and fair

[1] Hummels and Lugovskyy (2009) show in a Salop circle model that by adding an increased willingness to achieve the ideal variety when consumers are richer, firms can charge a higher markup in higher income markets. Simonovska (2016) uses a model of firm heterogeneity with non-homothetic Stone Geary preferences, where firms can charge a larger markup in higher income markets with consumers buying more of each good. Bekkers, et al. (2012) compare the two mentioned theories by addressing the impact of within country inequality in the importer country on traded goods prices, adding also a theory on the increased demand for quality in higher income markets. Their empirical results lend support to explanations emphasizing non-homothetic preferences. Francois and Kaplan (1996) provide early evidence linking inequality and non-homothetic preferences to import demand patterns, though they focus on the composition rather than the pricing of imports.

[2] Based on a recent classification of detailed trade data by use category (Bekkers, et al., 2012), about two-thirds of world trade is in intermediate goods.

wage theories. A wage schedule type approach assumes that there is some distribution of the qualities of workers (or intermediates), whereas in our approach it is costly (in terms of a larger amount of labor resources required) to produce higher quality goods. As a variation to our principal approach, we have also included a specification with a wage schedule for the quality of intermediates. With a wage schedule approach, labor is heterogeneous and higher quality intermediates are produced by more skilled workers, who receive a higher wage. This also gives rise to a positive relation between the quality of intermediates and the price of intermediates. In contrast with Verhoogen (2008), we assume that the wage schedule is non-linear, implying a non-linear relation between the quality and price of intermediates and the wage relative to productivity in the importing country. This heterogenous labor approach contrasts with our basic specification, where higher quality intermediates are instead costlier to produce because they require a larger amount of (homogeneous) labor resources.

A second model with a related mechanism on the probability of failure is proposed in Keller and Yeaple (2013) with respect to a gravity specification of multinational production. Affiliates producing intermediate inputs themselves, instead of sourcing them from headquarters, have to complete a sequence of tasks and communicate with the headquarters about each task to produce successfully. If one of the tasks fails, production is wasted. The model is employed to show that the forces of economic gravity are stronger for affiliates in technologically complex industries, where all other things being equal, more intermediates are sourced from headquarters.

A third set of related models are the ones by Kugler and Verhoogen (2012). The first model in Kugler and Verhoogen (2012) features complementarity between firm-level productivity and quality of inputs. Complementarity between productivity and quality of inputs provides an explanation for the empirical finding that larger firms display both higher output prices and higher input prices for intermediates. The model of Kugler and Verhoogen (2012) could also be employed to explain our empirical findings by extending it with productivity differences between countries.[3] With complementarity between productivity and the quality of inputs, more productive countries with higher wages would demand higher quality intermediates. Conceptually, our model adds an additional feature to the quality-productivity complementarity story of Kugler and Verhoogen (2012) with its emphasis on failure in the production process: more productive countries display higher wages and therefore a higher opportunity cost of failure generating a demand for higher-quality inputs. In Kugler and Verhoogen (2012) instead the complementarity between quality and productivity emerges by assumption.

Kugler and Verhoogen (2012) propose a second model to explain the firm-level stylized facts based on Sutton (1991) and Sutton (1998), with complementarity between the quality of inputs and fixed costs with fixed costs rising in the quality of output. As a result, more productive firms spend more on fixed costs, use higher-quality intermediates with higher input prices, and sell higher-quality output with higher output prices. This framework could also be used to explain our stylized facts: higher income countries demand higher quality goods, leading to more expenditures on fixed costs and by the complementarity between fixed costs and quality to higher priced, higher quality intermediates. We can differentiate between such a model and our model by evaluating whether wages in the importing country still matter once controlling for GDP per capita. In the Sutton-model of Kugler and Verhoogen (2012) higher priced intermediates are caused by a stronger demand for quality by consumers (proxied by GDP per capita) and wages at the sector level should not matter anymore. Our empirical results instead show that wages at the sector level stay significant once controlling for GDP per capita, providing support for the framework developed here. To summarize, our model contains an explicit relation between the quality and costs of intermediates in comparison to the framework employed in Kremer (1993) and adds an additional feature to the quality-complementarity mechanism of Kugler and Verhoogen (2012) (the role of the opportunity cost of failure).

In our empirical analysis, we test the theoretical hypothesis that the price of labor in the importing country leads to higher imported intermediate goods prices, controlling for

[3]Our modeling framework could also be extended with heterogeneous firms to explain the empirical findings of Kugler and Verhoogen (2012) based on the increased opportunity cost theory in more productive higher wage paying firms. Since we work with industry level data and are not interested in this paper in differences between firm-level prices we do not pursue this extension here.

labor productivity at sectoral level, market size and the level of development of the country. To measure import prices we use import unit value data (import prices) from the BACI database [4] which is constructed from COMTRADE and contains bilateral imports at the 6-digit HS level of classification (the greatest level of detail common across countries). Using the WIOD dataset, an extension of the original EU-KLEMS dataset (Dietzenbacher, et al., 2013; Koopman, Wang, Wei, 2014) we are able to identify imported intermediate goods. To measure labor costs and productivity, we employ the EU-KLEMS database that includes data on labor compensation and productivity at sectoral level. The WIOD database also contains world input-output tables. This enables us to map the imported inputs from partner countries into the importer sectors in the destination countries. As such we are able to map the labor costs in a given importing sector to the product-level price of imported inputs. This also means that in comparison to most of the literature we have substantially more degrees of freedom for employing sector specific explanatory variables. (Most of the literature on traded goods prices instead works with country-specific variables like GDP and GDP per capita.)

In order to establish causality, we work with instrumental variable regressions. To instrument the price of labor we use an instrument constructed by interacting the real exchange rate with the hours worked by high-skilled persons engaged (share in total hours) per sector. Using our product-level bilateral panel dataset, we find that higher labor costs relative to productivity in the importing country leads to higher imported prices of intermediate goods. In other words, countries where production is more expensive import higher quality, more expensive intermediate goods for further processing. This holds even when controlling for the level of development, the market size of the importing country, and the level of market power of input providers. Moreover, we find that while sector-country-specific wages explain about one-fifth of the variation in highly disaggregated intermediate goods prices, wages are only marginally important in explaining final goods prices, providing further evidence consistent with the O-ring theory. In addition, this indicates that not controlling for destination-sector specific wage effects is likely to generate a significant omitted variables bias.

In the next section we outline our theoretical model. In Section 3 we develop our estimating equation. In Section 4 we discuss data, while in Section 5 we discuss our results. Finally, in Section 6 we conclude.

2 Theory

2.1 Quality of Intermediates and Failure of Production

Consider a firm producing a final good X using both intermediate inputs Z and labor inputs V. The firm uses I different intermediate inputs Z_i with quality ϕ_i. The production process is vulnerable to failure. Every intermediate input has to be of sufficient quality. The production process is interrupted when there is a failure related to one of the intermediates used, and the corresponding unit is lost. So, if one of the intermediate inputs is of bad quality, the firm faces wasted inputs related to failed units of output.[5]

We define the probability that intermediate input i is of bad quality as p_i. The expected output $E(X)$ of the firm is equal to the probability that none of the intermediate inputs i is of bad quality times the output generated by the intermediate inputs Z and factor inputs V, $f(Z,V)$:

$$E(X) = \left[\prod_{i=1}^{I}(1-p_i)\right] f(Z,V) \tag{1}$$

[4] http://www.cepii.fr/anglaisgraph/bdd/baci/baciwp.pdf

[5] Our concept of quality is different from the usual interpretation of quality in consumer theory where it can be interpreted as product appeal or more generally a shifter in demand for given price. In our model quality is defined by its effect on production failure: a higher quality intermediate generates a lower probability of failure in production. So our definition of quality is similar to the definition of quality in Kremer (1993) where "a worker's skill (or quality) at a task, q, is defined by the expected percentage of maximum value the product retains if the worker performs the task (p. 553)."

The probability of intermediate input i being of sufficient quality to prevent failure, $q_i = 1 - p_i$, is a function of the quality of the intermediate input, ϕ_i. We specify the following function:

$$q_i(\phi_i) = \phi_i^{\xi}; \xi > 0 \tag{2}$$

The expected output is therefore equal to the following expression:

$$E(X) = \left[\prod_{i=1}^{I} \phi_i^{\xi}\right] f(Z, V) \tag{3}$$

We see that expected output is a function of intermediate inputs and labor inputs and of the quality of the intermediate inputs. Equation (3) could also be motivated directly from the fact that higher quality intermediates lead to higher output, but our exposition provides an intuitive motivation for the production function used in the next subsection based on failure in production.

The production function in equation (3) is similar to the production function in Kremer (1993). In our model there is a set of intermediate inputs combined with labor, while in Kremer (1993) there is only a set of labor inputs. Each of the factor inputs can fail, making the entire product worthless. This setup implies a complementarity between the level of quality of the inputs like in Kremer (1993).

2.2 Baseline Model with N Countries

We study an economy with N countries and J industries. Final goods in industry j in target country t are produced according to a CES production function with output X_{tj} a function of a bundle of intermediates Z_{tj} consisting of I_{tj} intermediates and labor inputs V_{tj}. Production is subject to failure as specified in the previous subsection, implying the following production function:

$$X_{tj} = \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}\right) \left(\alpha_{z_{tj}} Z_{tj}^{\frac{\eta-1}{\eta}} + \alpha_{v_{tj}} (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{4}$$

$$Z_{tj} = \left(\sum_{i=1}^{I_{tj}} z_{stij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{5}$$

φ_{tj} is a measure of labor productivity in industry j in country t. ξ measures the importance of quality for productivity, η is the substitution elasticity between intermediate inputs and labor inputs and σ is the substitution elasticity between different intermediate inputs. We assume that $\eta < 1$, reflecting that both intermediates and labor inputs are essential in the production process. The composite intermediate input bundle Z_{tj} consists of different intermediates z_{stij} bought from different source countries s and source sectors i. [6] $\alpha_{Z_{tj}}$ and $\alpha_{V_{tj}}$ are taste shifters indicating the importance of respectively intermediate inputs and labor in the production function.[7] Quality ϕ_{stij} is also $i-$ specific, but does not vary with the different units of variety i produced. So, quality is a property of a variety i and not of the different units of variety i.

Factor markets are perfectly competitive and ω_{tj} is the wage in country t in sector j. The market for intermediate goods is perfectly competitive and firms use labor inputs in production. The cost function of producing an intermediate product i in country s for industry j in country t is given by:

$$C(z_{stij}, \phi_{stij}) = \phi_{stij}^{\gamma_{si}} z_{stij} \frac{\omega_{si}}{\varphi_{si}} \tag{6}$$

[6]The source country s should in principle contain an additional subscript tij on its subscript as the source country is a function of the type of intermediates i bought and the target country t and industry j that buys them. We omit this sub-subscript to prevent burdensome notation.

[7]Since intermediate input shares vary across sectors and countries we include these parameters. We do not use variation in the shifters in the empirical part though, since they cannot be empirically observed, as shares are also driven by prices.

The parameter γ_{si} is country specific and is an inverse measure of the productivity to produce quality. Like above ω_{si} and φ_{si} are respectively the price and productivity of labor in country s in intermediate goods sector i the marginal cost of producing an intermediate rises in the quality of the intermediate, reflecting that higher quality goods are more expensive to produce or alternatively that higher quality goods require more labor inputs.[8]

There are iceberg trade costs τ_{stij} for shipping intermediate i from country s to final good sector j in country t. Given perfect competition, price is equal to marginal cost. Therefore, the landed price in the target country, $p_{Z_{stij}}$, is given by:

$$p_{Z_{stij}} = \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \tag{7}$$

Final goods producers minimize costs $\sum_{i=1}^{I_{tj}} p_{z_{stij}} z_{stij} + \omega_{tj} V_{tj}$ subject to the production function in equation (4), choosing the optimal quantities of z_{stij} and V_{tj} and the optimal level of quality ϕ_{stij}. In A.1 it is shown that cost minimization implies the following quality level and landed price for intermediate i produced in country s and used in sector j in country t:

$$\phi_{stij}^{\gamma_{si}} = \frac{\varphi_{si}}{\tau_{stij}\omega_{si}} \frac{\frac{1}{I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}}}{\left(\frac{\gamma_{si}\widetilde{\gamma_{stij}}^{\sigma-1}}{\xi I_{tj}} - 1\right)^{\frac{1}{1-\eta}}} \left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\frac{\eta}{1-\eta}} \left(\frac{\omega_{tj}}{\varphi_{tj}}\right) \tag{8}$$

$$p_{Z_{stij}} = \frac{\frac{1}{I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}}}{\left(\frac{\gamma_{si}\widetilde{\gamma_{stij}}^{\sigma-1}}{\xi I_{tj}} - 1\right)^{\frac{1}{1-\eta}}} \left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\frac{\eta}{1-\eta}} \left(\frac{\omega_{tj}}{\varphi_{tj}}\right) \tag{9}$$

$\widetilde{\gamma_{stij}}$ is a measure for the average productivity to produce quality for country t and industry j of intermediate i in country s relative to other intermediates k from other countries u, $\widetilde{\gamma_{stij}} = \left(\frac{1}{I_{tj}} \sum_{k=1}^{I_{tj}} \left(\frac{\gamma_{si}}{\gamma_{uk}}\right)\right)^{\frac{1}{1-\sigma}}$. Equations (8) and (9) imply that the quality of intermediates imported and also the price of intermediates imported rise proportionally in the wage ω_{tj} relative to labor productivity φ_{tj} in importing country t. The intuition is that the opportunity cost of failure rises when wages relative to productivity are higher. With either a higher wage or a lower productivity, value added is more expensive and the opportunity cost of failure is higher, giving a stronger incentive to source high-quality intermediates.[9]

2.3 Extension with Per Unit Trade Costs

The literature on traded goods prices has identified distance as an important determinant. Hummels and Skiba (2004) show that a larger distance between trading partners raises traded goods prices driven by the Alchian Allen "shipping the good apples out" effect. This effect appears due to per unit trade costs. In A.2 it is shown that our model also displays

[8] On the cost side our model deviates from Kremer (1993), with the costs of intermediate inputs being an explicit function of the quality of intermediate inputs. In Kremer (1993) there is an unspecified distribution of qualities of the labor inputs and because of assortative mating of workers following from the complementarity in the production function, firms pay a higher wage for higher quality workers. We work instead with an explicit relation between the cost and thus the price of intermediate inputs and the quality of the inputs without assuming a distribution of qualities of the inputs. The cost of the produced intermediates rises in the quality of the intermediates, since more inputs are required to produce higher quality goods. In A.3 we incorporate a (non-linear) wage schedule for labor used in the production of intermediates as in Verhoogen (2008). This leads to a similar expression for the equilibrium quality and price of intermediates as with the cost function in (6).

[9] The multiplicative functional form of the cost function in equation (7) in combination with endogenous quality implies that marginal costs $\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}$ do not affect the price in equilibrium. With a more general cost function, marginal costs in the exporting country would affect traded goods prices, but preclude us from finding an analytic solution. Since we focus in this paper on importer characteristics, we stick to our tractable cost formulation.

the Alchian Allen effect after adding per unit trade costs to the model. The effect of wages keeps the same sign with an elasticity deviating from 1 after the extension with per unit trade costs.[10]

3 Estimating Equations

We start from the expression for the price of intermediates product i from country s sold in sector j in country t in the baseline model in equation (9), omitting time subscripts. The price of intermediate products, or unit values, is defined as the value of exports divided by the volume of exports, $p_{z_{stij}} = \frac{p_{z_{stij}} z_{stij}}{z_{stij}}$. Ideally, we would have data for exporting product - importing sector pairs. However, we only have data on unit values for an exporting product i sold to all importing sectors j from country s to country t. In order to link the exporting product to the importing sector and to explain unit values with our theoretical expression for prices in equation (9), we use weights for adding up destination sectors j. The weights used are the volume shares of sector j in total sales of sector i in source country s sold to target country t, $w_{stij} = \frac{Z_{stij}}{\sum_{k=1}^{J} Z_{stik}}$, where J is the number of destination sectors. This allows us to have importer sector specific labor costs and productivity matched into the products imported from different countries. Taking logs of the theoretical expression in equation (9) and using these weights, we get the following expression for the unit values of product i shipped from country s to country t:

$$\begin{aligned}
\ln p_{z_{sti}} &= \ln \sum_{j=1}^{J} w_{stij} f\left(\gamma_{si} \widetilde{\gamma_{stij}}^{\sigma-1}\right)\left(\frac{\omega_{tj}}{\varphi_{tj}}\right) \\
&\approx \sum_{j=1}^{J} sh_{stij} \ln f\left(\gamma_{si}\right) + \sum_{j=1}^{J} sh_{stij} (\ln \omega_{tj} - \ln \varphi_{tj})
\end{aligned} \tag{10}$$

In the second line we applied a log linear approximation and moved from volume shares, w_{stij}, to value shares, sh_{stij}, by lack of data on volume shares. sh_{stij} is the value share of sector j in total sales of sector i in source country s to target country t defined as $sh_{stij} = \frac{p_{z_{stij}} z_{stij}}{\sum_{k=1}^{J} p_{z_{stik}} Z_{stik}}$. Equation (10) shows that wages ω_{tj} and productivity φ_{tj} in the importing country have the same effect on price $p_{Z_{sti}}$ (with opposite sign) and thus should be entered jointly in the regression. Therefore, we define the ratio of wages and productivity as $\theta_{tj} = \frac{\omega_{tj}}{\varphi_{tj}}$.

We add exporter-product-time fixed effects, ν_{siu}, to capture variation in exporter productivity, γ_{si}.[11] We include importer-exporter-product fixed effects, χ_{sti}, to account for the role of distance and also of other trade costs not related to distance appearing in the model with per unit trade costs. Moreover, we need this fixed effect to correct for differences in measurement of unit values across sectors which are specific to country-pairs.[12]

While our unit value data is HS6 aggregation, the world input-output data used for the mapping and calculation of weights is in a corresponding nace classification which is more aggregated. Hence our explanatory variables are at a more aggregate level.

This leads to the following estimating equation with u a subscript for time:

$$\ln p_{z_{stiu}} \approx \widetilde{\theta_{sti*u}} + \nu_{siu} + \chi_{sti} \tag{11}$$

[10]The specification with a non-linear wage schedule instead of the cost function in (6) implies a linear relation between the price of intermediates and the opportunity cost of failure, the wage relative to productivity in the importing country. The reason is that both quality and the wage (through the wage schedule) adjust endogenously to the opportunity cost of failure, leading to a linear relation. Hence, a non-linear wage schedule does not generate an elasticity different from 1. Further discussion is can be found in A.3.

[11]Exporter-product-time fixed effects are included to control for time varying exporter characteristics like income and market size (Schott (2004)) and other potential omitted variables which are time varying and exporter-product specific.

[12]Importer-product fixed effects are needed as well because our measure for the price of labor is an index with a base level differing per country-sector combination.

With $\widetilde{\theta_{sti*u}}$ defined as follows:

$$\widetilde{\theta_{sti^*u}} = \sum_{j=1}^{J} sh_{sti^*ju} \ln \theta_{tju} \tag{12}$$

Where i^* refers to the more aggregated nace product code corresponding to HS product level i. Thus we explain exporter sector i specific prices between country s and t by a weighted sum of wages and productivity in the different importer sectors j. With this approach our explanatory variables vary by exporter, importer, and sector and have thus dimension sti^*. The variation follows from variation in the underlying labor cost and productivity by importer t and sector j and the value weights varying by all four dimensions. Our approach with variation along more dimensions provides a clear advantage in comparison to most of the papers on traded goods prices using variables that vary only at the importer country level like GDP and GDP per capita.

Reflecting the baseline model, we estimate equation (11) first without further controls. To compare our results with the literature on traded goods prices discussed in the introduction and to test the extended model with per unit trade costs, we then estimate a specification with only time fixed effects where we also include importer GDP, importer GDP per capita, exporter GDP per capita, and distance in the regression besides importer wages relative to productivity.

4 Data

In our empirical analysis we proxy quality with import prices, more specifically with import unit values.[13] In line with the theory we work with prices inclusive of transportation costs, i.e. cif(cost insurance freight) prices. The data used for unit values come from the BACI database[14] which contains the quantity and the value of bilateral imports in 6-digit Harmonized System (HS) classification. We use data for the period between 2000-2007.[15] The BACI database is constructed from COMTRADE (Commodities Trade Statistics database) which provides very detailed trade data, accounting for more than 95% of the world trade. BACI takes advantage of the double information on each trade flow to fill out the matrix of bilateral world trade providing a "reconciled' value for each flow reported at least by one of the partners. Therefore the missing values in BACI are those concerning trade between non-reporting countries. The data is provided in 6-digit disaggregation level. At the international level, this is the most disaggregated product classification that one can obtain. [16] We winsorize unit values at 5 and 95% to address outliers which are most likely the result of measurement error in the unit value data [17]. As a robustness check we also trimmed unit values instead of winsorizing (for which results are presented in the Appendix) and we also used unit values without corrections, both leading to very similar results. [18] We also drop homogenous goods from the sample as identified by Rauch (1999) , since these products should not have significant differences in quality. [19]

[13]Unit values might not always be a good proxy for quality in some sectors as shown in Khandelwal (2010). While there have been new approaches aiming to better measure quality, including Khandelwal, due to data limitations we are not able to follow the approach in Khandelwal (2010). (We would lose roughly 80% of the sample.). Instead, we undertake robustness checks and construct and use a measure of market power in intermediate-input markets proposed by Kugler and Verhoogen (2012). Also see Bernini et al (2015), who follow the approach used by Khandelwal (2010) to estimate quality and find similar results using both Khandelwal's measure for quality and unit values, as we use here.

[14]http://www.cepii.fr/anglaisgraph/bdd/baci/baciwp.pdf

[15]This is the period for which we were able to obtain all the required data for control variables.

[16]It is important to keep in mind that what consumers or firms buying intermediate goods think of as a product is likely to be even more disaggregated than is implied by the 6-digit HS classification. For example, the product with HS 611220 code in the HS6 classification scheme is defined as ski suits, of textile materials, knit. A consumer is likely to consider a range of different products that fall within this same category as actually being distinct from each other. As such, our HS6 digit level unit values are average prices for those products within these categories.

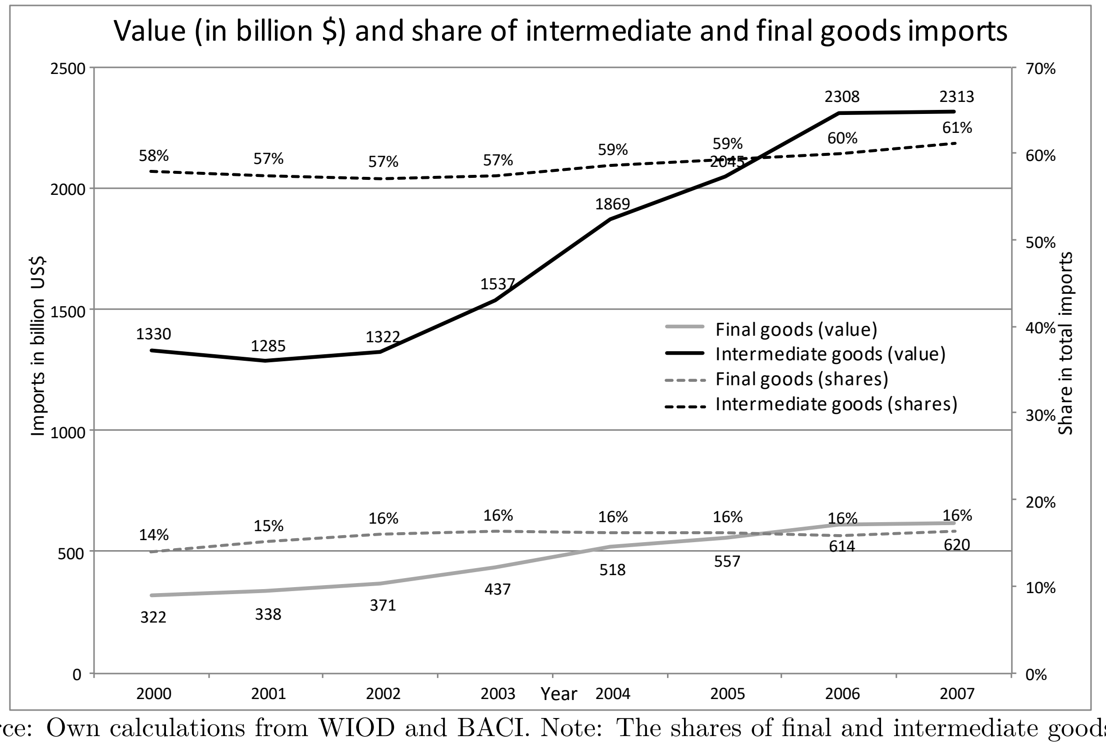
[17]A similar approach has been followed in Manova and Zhang (2012).

[18]These results using unit values without corrections are available upon request from the authors.

[19]As a robustness check we also run all regressions with keeping in the homogenous goods and our results hold.

We limit our analysis to intermediate products. The classification of products into intermediate and final consumption comes from the WIOD database, which is based on the BEC classification of the OECD (Dietzenbacher, et al., 2013; Bekkers, et al., 2012). The WIOD database provides an update to the EU-KLEMS database. Both are large scale, multi-year databases funded by the European Commission.[20] While the WIOD mapping starts with a reclassification of the HS6-BEC (UN) mapping at HS6 level, it is somewhat different from the original HS6-BEC scheme. This is because more emphasis is placed under the BEC scheme on whether goods are durables or not, with some products that clearly need processing before final consumption are classified under BEC as consumption goods. For example, televisions are classified as capital goods at HS6 level. Also, whole meat carcasses are classified as consumption goods at HS6 level under the BEC, though they are in fact bought by industry. There are further problems due to revisions to the HS classification scheme since the original mapping from BEC to HS was developed. Over 10 percent of current HS lines are orphans in the BEC-HS concordance. As such, at the end of the day, the mapping we work from WIOD better reflects both the most current HS-combined product lines, and our need for a breakdown of products by use. Using this classification, we end up with 2467 HS-6 product categories which are defined as intermediate products while we identify from the sample 949 as final products (and we exclude further 83 product categories which could be both final and intermediate goods). The Annex provides details on the number and share of observations classified as intermediate products in our sample belonging to each HS2 category (see Tables in A.7). Figure 1 depicts both the share of intermediates and final goods imports and their value for our sample. [21] The share of intermediates is around 58-61% for our sample, while the final goods' share is only about 15%. Thus the importance of intermediate goods is much more pronounced than that of the final goods, with the value of intermediate goods imported reaching 2313 billion dollars in 2007, while the final goods imports value amounting to 620 billion US dollars only.

Figure 1: Intermediate vs final goods imports



Source: Own calculations from WIOD and BACI. Note: The shares of final and intermediate goods do not add up to 100, the remainder being products that could be both final and intermediate goods.

[20]The WIOD consortium includes a number of European research centers and universities, as well as the OECD and UNCTAD.

[21]The shares do not add up to 100, the remainder being those products that could not be identified as final or intermediate goods.

The price of labor is constructed using labor compensation per person engaged (and as a robustness labor compensation per hour worked) obtained from the 2012 EU-KLEMS database.[22] The data are provided in local currency and so we convert them into US dollars using exchange rates from OECD. In addition we calculate indexes from the variables, using the same base year, 1995, as the rest of the database. Given that we are interested in the effect of the price of labor relative to productivity, we divide labor compensation per person with productivity of labor, which we proxy with sectoral total factor productivity from the 2012 EU-KLEMS database [23], again applying weights to map the importing sectors into exporting sectors.

We use the value weights sh_{stij} calculated from WIOD world input-output tables as defined in the previous section in order to convert importing country importing sector specific data into importing country exporter sector specific variables defined in equations (11)-(12). The world input-output table from WIOD is similar to national input-output tables except that the use of products is broken down according to their origin. Each product is produced either by a domestic industry or by a foreign industry. For a country A, flows of products both for intermediate and final use are split into domestically produced or imported. In addition, the dataset contains information on which foreign industry produced a product. The basic data for WIOD on import flows, similarly to our unit value data, originates from the UN COMTRADE database. The WIOD classification list has 59 products and 35 industries based on the CPA and NACE rev 1 (ISIC rev 2) classifications. The 35-industry list is identical to the list used in the EU-KLEMS database with additional breakdown of the transport sector. Both the price index of labor, and the weights from WIOD have the same industry aggregation. This is then mapped into HS6-level import price data. So, unit values are observed at the six-digit level, whereas our explanatory variables display a higher level of aggregation.

To construct our instrument for the price of labor relative to productivity, we use the interaction of the real exchange rate index and the share of high-skilled persons engaged. First, to calculate the real exchange rate index, we obtained a purchasing-power-parity based and current dollar price-based measure of gross domestic product from the International Monetary Fund's World Economic Outlook Database. We calculated the real exchange rate as a ratio of gross domestic product at current US dollars prices to GDP based on PPP valuation of the country GDP. From this we calculated the index of real exchange rate using the same base year (1995) as used for the rest of the dataset. For the second variable, data on the hours worked by high-skilled persons engaged (share in total hours) per sector were obtained from the WIOD database.

The Annex Table A.4 contains summary statistics for our variables. In addition, Annex Table A.5 and A.6 contain the list of countries included in our sample as exporters and importers. The countries included in the sample were driven by the availability of data from WIOD and EU-KLEMS. Since data needed to be available for importers from EU-KLEMS, and both for exporters and importers from WIOD, the number of importers is smaller due to less countries being available from EU-KLEMS than WIOD.

[22]The EU-KLEMS database contains industry-level measures of output, inputs and productivity and other derived variables for 25 European countries, Australia, Japan and the US for the period from 1970 onwards. The input variables include various categories of capital (K), labor (L), energy (E), material (M) and service inputs (S). The variables are organized around the growth accounting methodology with the exception of the basic input data-series which are derived independently from the assumptions underlying the growth-accounting method. The EU-KLEMS database has largely been constructed on the basis of data from national statistical institutes. Nominal and price series for output and total intermediate inputs at the industry level in the dataset are taken directly from the National Accounts. Further information about the database can be found at http://www.euklems.net/ and in O'Mahony and Timmer (2009).

[23]For the countries in our sample, the correlation between TFP and labor productivity (based on WIOD and EU-KLEMS) is close to 99%, meaning the results will be the same with either measure.

5 Empirical Results

5.1 Instrumental variable regressions

There could be a potential for reverse causality, whereby higher imported input prices cause higher prices of value added, or wages. Thus we estimate equation (10) over the period 2000-2007 employing an IV regression allowing for high-dimensional fixed effects. In our main specifications we include importer-exporter-product (HS 6 level) and exporter-time-product (HS 6 level) fixed effects. Since errors could be correlated across observations within exporter-importer-sector we clustered standard errors at exporter-importer-sector level, at the same dimension as our instrument and our main variable of interest.

We construct our instrument by interacting two variables. The first variable is the importer's real exchange rate calculated against the US dollar. The real exchange rate gives a proxy for the cost of goods and services in the importing country and is driven to a large extent by Balassa-Samuelson effects generating higher price levels in more productive countries. Employing the real exchange rate, we capture variation in the price of labor that is not driven by prices of intermediates but instead by the strength of the Balassa-Samuelson effect. For example, in a very rich country like Switzerland, there is a strong Balassa-Samuelson effect generating high wage levels and we capture this strong Balassa-Samuelson effect with the real exchange rate. The second variable used to construct our instrument is the hours worked by high-skilled persons engaged as a share of total hours worked per sector. This variable varies by time, importer and sector. To construct our instrument we interact the two variables.[24] The motivation for this sectoral variable is that more skilled workers have a stronger bargaining position vis-a-vis employers and will thus be able to negotiate a better ratio of wage relative to productivity. Hence, the Balassa-Samuelson effect will have a stronger effect in sectors with a larger share of high-skilled workers. Indeed our combined instrument has a higher correlation when the share of skilled workers is higher (the correlation is about 20% for the sample with the share of skilled workers being below the sample mean, and about 55% for the sample with above average skilled worker share).

Table 1 presents the results of our main specifications (first stage results are presented in Annex Table A.2). In the first column of the table, we only include the price of labor relative to productivity as explanatory variable (which is constructed as the (weighted average) price of labor divided by productivity). [25] As expected, based on our theoretical model, the cost of production in the importer country measured by the labor cost given productivity increases the import price of the intermediate goods. A 10 percent increase in the price of labor relative to productivity leads to about 6.3 percent increase in the price of imported goods. In other words, countries where workers (or the production process in general) are more expensive will import more expensive intermediate goods (which are also more likely to be of higher quality) for further processing. At the bottom of the table we also present test results for under-identification and weak instruments which indicate that we do not have a problem of weak instrument.

Results presented in column (2) include two additional control variables, GDP per capita, measuring the level of development of the country, and GDP, measuring market size. The coefficient on the price of labor relative to productivity becomes higher, implying that a one percent increase in the cost of labor (relative to productivity) increases the price of imported intermediates by about 2.5%. In addition, market size has a negative impact on imported good prices while GDP per capita has a positive impact. The result on market size provides support for models where the intensity of competition rises with market size (Hummels and Lugovskyy (2009)). Furthermore, the finding that labor compensation matters even when GDP per capita is controlled for provides further support for our model. In the Sutton-model of Kugler and Verhoogen (2012) higher priced intermediates are caused by a stronger demand for quality (or higher priced products) by consumers (proxied by GDP per capita) while wages at the sector level should not matter under the model's predictions. Our

[24] We find an about 36 percent correlation in our sample between our constructed instrument and the price of labor relative to productivity.

[25] As a robustness check we also run our regressions including labor price index and productivity separately (results are available upon request from the authors). Our main variable of interest, price of labor, remains positive and significant in all different specifications.

Table 1: IV Estimation Results, intermediate goods

	(1)	(2)	(3)	(4)
	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$
$\widetilde{\theta_{sti^*u}}$	0.633	2.477	2.475	1.431
	(5.40)***	(5.32)***	(5.32)***	(2.05)**
$LnGDP_t$		-4.844	-4.842	-3.244
		(6.74)***	(6.74)***	(3.32)***
$LnGDPpercapita_t$		4.479	4.478	3.049
		(7.00)***	(7.00)***	(3.55)***
$Lndistance_{st}$				0.179
				(17.40)***
$LnGDPpercapita_s$				0.174
				(8.84)***
HI_t			0.014	0.032
			(3.50)***	(3.94)***
Countrypair-HS6 fe.	YES	YES	YES	NO
Exporter-HS6-year fe.	YES	YES	YES	NO
Exporter fe.	NO	NO	NO	YES
Importer fe.	NO	NO	NO	YES
HS6-year fe.	NO	NO	NO	YES
LM test for underid	478.98	146.72	146.57	36.33
p-value	0.00	0.00	0.00	0.00
F-stat for weak id	555.95	134.86	134.72	41.66
R2	0.78	0.78	0.78	0.54
N	6,584,575	6,584,575	6,584,575	6,755,827

[a] Standard errors in parentheses, clustered by importer-exporter-sector.

[b] *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

[c] The sample is restricted to intermediate goods. The dependent variable, $\ln p_{z_{sti}}$, is the product-level unit value of imports from country s to country t. $\widetilde{\theta_{sti^*u}}$ is labor compensation per person engaged divided by productivity in importing country t, specific to sector i^* imported from country s, HI is the Herfindahl index in importing country t. Results for the first stage results are presented in the Annex table A.2.

empirical results instead show that wages at the sector level stay significant once controlling for GDP per capita, providing support for our model.

Kugler and Verhoogen (2012) show that imperfect competition in input markets can influence input prices. In particular, they find evidence on the importance of endogenous markups, costs differences, and imperfect competition in the input side to explain prices. Hence, we extend our specification in column (3) including a proxy for the level of market power in the intermediate-input side using a Herfindahl index for suppliers of each six-digit intermediate good, defined as the sum of squared market shares of exporters of the input. Controlling for the intermediate-input side market power does not alter the effect of labor cost (relative to productivity). In addition, our results indicate a significant and positive relationship between higher level of market power of intermediate suppliers/exporters and higher import prices of these goods.

Finally, in the last column, we also control for the distance between the exporter and importer and for the exporter's level of development (by including the log of exporter's country GDP per capita). This also implies that instead of including country-pair-product and exporter-product-year fixed effects, we include exporter, importer, and product-year fixed effects. This specification enables us to explore how import prices vary in a cross-section way with our main variable of interest. We find that our main variable of interest, the price of labor relative to productivity, still has a positive and significant impact on the price of imported inputs. In addition, both distance and GDP per capita of the exporter have the expected sign (in line with earlier findings in the literature (Hummels and Skiba, 2004; Baldwin and Harrigan, 2011; Martin, 2012; Schott, 2004)).[26]

5.2 Robustness checks

If the mechanism we propose in this paper is indeed the one behind the results presented in the previous section, we expect that the labor cost is not significant or much less important in explaining the variation in imported goods prices of final goods. In addition, when quality differences are not present, which is the case of homogenous goods, again we expect that the price of labor is not driving imported goods' prices. In order to verify this, in this section we present results using a final goods and homogeneous goods sample.

The first three columns of table 2 show results for our main specification using final goods while the last three present results for homogenous goods (first stage results are presented in the Annex in Table A.3). The cost of labor (given productivity) is insignificant in all specifications and the country-level variables are explaining the variation in imported goods prices both in the case of final and homogenous goods.

As further robustness we run our main specifications trimming the imported goods prices instead of winsorizing. Our results hold (results can be found in the annex Table A.1).

5.3 Economic significance

We also undertake a Shapley decomposition based on IV regressions separately for intermediate and final goods.[27] A summary of this decomposition can be found in Table 3. We find that while in the case of intermediate goods the price of labor explains about 20% of variation in the imported good prices (share being calculated excluding the fixed effects), in the case of final goods this is only 1.9%. In other words, while sector-country-specific wages explain about one-fifth of the variation in highly disaggregated intermediate goods prices, wages are only marginally important in explaining final goods prices, providing further evidence consistent with the O-ring theory. In addition, this indicates that not controlling for destination-sector specific wage effects is likely to generate a significant omitted variables bias.

[26] Hummels and Skiba (2004) find that free on board (fob) prices, so prices exclusive of transport costs, are rising in distance. This provides support for their Washington apples effect (terminology from Feensta and Romalis, 2014) that quality rises with distance. Since distance is not our main variable of interest we stick to the use of cif prices, so prices inclusive of transport costs. Since distance obviously raises transport costs and thus also cif-prices, our empirical results on distance could also be driven by this effect through transport costs. The results do, however, not contradict the Washington apples effect.

[27] Due to computational difficulties instead of HS6-year fixed effects, sector-year fixed effects.

Table 2: IV results with final and homogenous goods

	Final goods sample			Homogenous goods sample		
	(1)	(2)	(3)	(5)	(6)	(7)
	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$
$\widetilde{\theta_{sti^*u}}$	-0.056	0.112	-0.102	-0.400	1.533	-0.050
	(0.19)	(0.06)	(0.28)	(1.12)	(1.26)	(0.12)
$LnGDP_t$		-1.928	-1.536		-2.361	-0.553
		(1.42)	(3.62)***		(2.18)**	(1.13)
$LnGDPpercapita_t$		1.951	1.556		2.120	0.458
		(1.66)*	(3.92)***		(2.18)**	(1.00)
$Lndistance_{st}$			0.121			0.157
			(10.65)***			(13.20)***
$LnGDPpercapita_s$			0.267			0.173
			(9.33)***			(5.65)***
Countrypair-HS6 fe.	YES	YES	NO	YES	YES	NO
Exporter-HS6-year fe.	YES	YES	NO	YES	YES	NO
Exporter fe.	NO	NO	YES	NO	NO	YES
Importer fe.	NO	NO	YES	NO	NO	YES
HS6-year fe.	NO	NO	YES	NO	NO	YES
LM test for underid	199.45	12.02	74.00	102.88	13.87	55.63
p-value	0.00	0.00	0.00	0.00	0.00	0.00
F-stat for weak id	197.23	10.74	125.45	93.51	13.10	75.98
R2	0.84	0.84	0.67	0.82	0.82	0.59
N	1,977,513	1,977,513	2,018,782	567,499	567,499	603,775

Standard errors in parentheses, clustered by importer-exporter-sector.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$
The sample is restricted to intermediate goods. The dependent variable, $\ln p_{z_{sti}}$, is the product-level unit value of imports from country s to country t. $\widetilde{\theta_{sti^*u}}$ is labor compensation per person engaged divided by productivity in importing country t, specific to sector i^* imported from country s. Results for the first stage results are presented in the Annex Table A.3.

Table 3: Shapley decomposition, percentage of variation explained

	Intermediate sample	Final sample
$\widetilde{\theta_{sti^*u}}$	20.0%	1.9%
$LnGDPpercapita_t$	6.0%	12.2%
$LnGDP_t$, $Lndistance_{st}$, $LnGDPpercapita_s$	74.0%	85.9%

Shapley decomposition is based on IV regression results using sector-year instead of product-year fixed effects due to computational difficulties. The shares above are calculated excluding the variation explained by fixed-effects.

6 Summary and Concluding Remarks

Intermediate goods trade amounts to roughly two-thirds of world merchandise trade. Even so, the empirical and theoretical literature on traded goods prices has thus far concentrated on final goods and neglected intermediate goods. In this paper, we fill this gap in the literature, studying the relation between traded goods prices of intermediates and importer characteristics.

We first develop a simple model of production with final goods produced from intermediates and labor where the production process is vulnerable to failure and the probability of failure is declining in the quality of intermediates. Countries with a higher price of labor relative to productivity have a higher opportunity cost of failure. Therefore they demand higher quality intermediates. Including per unit trade costs, our model also contains a Washington apples effect with traded goods prices rising in distance.

In the empirical section we use import prices from a large disaggregated dataset containing bilateral imports at 6-digit HS classification. Intermediate goods are identified in our dataset using a classification scheme developed for an update of the EU-KLEMS database. Using WIOD input-output tables and the EU-KLEMS database we create an exporter-sector-importer specific labor price also varying over time. As such, our main explanatory variable is varying by time, sector and country-pair, an important advantage relative to most of the literature on traded goods prices using country level variables. We work with instrumental variable regressions to establish causality.

Our results show that labor costs relative to productivity in the importing country exert a positive, significant effect on traded goods prices for intermediate goods. Countries with more expensive production factors (higher priced workers) import higher quality, higher priced intermediate goods for further processing. These results also hold after controlling for market size and the level of development of the importing country. Basically, while we may observe systematic variation in import prices for both final and intermediate goods, the factors driving these patterns are different for intermediates than those for final goods.

A Supplementary Derivations, Data Summary, and Supplementary Regressions

A.1 Deriving Optimal Quality, Quantity and Price in Baseline Model

Final goods producers minimize costs $\sum_{i=1}^{I_{tj}} p_{Z_{stij}}(\phi_{stij}) z_{stij} + \omega_{tj} V_{tj}$ with $p_{Z_{stij}}(\phi_{stij})$ given in equation (7) subject to the production function in equations (4) and (5). The choice variables are the quantity of value added V_{tj}, the quantity of intermediates z_{stij}, and the quality of intermediates ϕ_{stij}. The respective first order conditions are given by:[28]

$$\omega_{tj} - \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \alpha_{v_{tj}} \varphi_{tj}^{\frac{\eta-1}{\eta}} V_{tj}^{-\frac{1}{\eta}} = 0 \tag{A.1}$$

$$p_{Z_{stij}} - \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \alpha_{z_{tj}} Z_{tj}^{-\frac{1}{\eta}} Z_{tj}^{\frac{1}{\sigma}} (z_{stij})^{-\frac{1}{\sigma}} = 0 \tag{A.2}$$

$$\gamma_{si} \phi_{stij}^{\gamma_{si}-1} \frac{\tau_{stij} \omega_{si}}{\varphi_{si}} z_{stij} - \lambda \xi \frac{X_{tj}}{\phi_{stij}} = 0 \tag{A.3}$$

Solving for z_{stij} from equation (A.2) gives:

$$z_{stij} = \frac{P_{Z_{tj}}^{\sigma}}{p_{Z_{stij}}^{\sigma}} Z_{tj} \tag{A.4}$$

With

$$p_{Z_{tj}} = \left(\sum_{i=1}^{I_{tj}} p_{Z_{stij}}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{A.5}$$

Substituting equation (A.4) back into the FOC for z_{stij}, equation (A.2), leads to:

$$P_{Z_{tj}} = \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \alpha_{z_{tj}} Z_{tj}^{-\frac{1}{\eta}} \tag{A.6}$$

Dividing the FOC for V_{tj} and Z_{tj} in equations (A.1) and (A.6) gives:

$$V_{tj} = \left(\frac{P_{Z_{tj}}}{\omega_{tj}} \frac{\alpha_{v_{tj}}}{\alpha_{z_{tj}}} \right)^{\eta} \varphi_{tj}^{\eta-1} Z_{tj} \tag{A.7}$$

Substituting back into the production function, equation (4), we find the following solutions for V_{tj} and Z_{tj}:[29]

$$V_{tj} = \alpha_{v_{tj}}^{\eta} \varphi_{tj}^{\eta-1} \omega_{tj}^{-\eta} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{\eta-1} P_{X_{tj}}^{\eta} X_{tj} \tag{A.8}$$

$$Z_{tj} = \alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} X_{tj} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{\eta-1} P_{X_{tj}}^{\eta} \tag{A.9}$$

With

$$P_{X_{tj}} = \frac{1}{\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}} \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \right)^{\frac{1}{1-\eta}} \tag{A.10}$$

[28] We also could have used two stage budgeting, but we decided to expose a straightforward derivation, as the lower nest seems non-homothetic with quality playing a role casting doubts about the appropriateness of two stage budgeting.

[29] Derivation of this equation and several equations below can be found in a separate appendix available upon request.

Substituting equation (A.8) into equation (A.1) generates the following expression for λ:

$$\lambda = P_{X_{tj}} \tag{A.11}$$

Dividing the FOC for ϕ_{stij}, equation (A.3), by a similar equation with different subscript and substituting equation (7), gives us:

$$\phi_{stij} = \left(\frac{\gamma_{uk}}{\gamma_{si}}\right)^{\frac{1}{\gamma_{si}(1-\sigma)}} \left(\frac{\tau_{utkj}\omega_{uk}}{\tau_{stij}\omega_{si}}\frac{\varphi_{si}}{\varphi_{uk}}\right)^{\frac{1}{\gamma_{si}}} \phi_{utkj}^{\frac{\gamma_{uk}}{\gamma_{si}}} \tag{A.12}$$

Substituting equation (A.12) into equation (A.5) implies for p_{Z_i}:

$$p_{Z_{tj}} = p_{z_{stij}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}; \ \ \widetilde{\gamma_{stij}} = \left(\frac{1}{I_{tj}} \sum_{k=1}^{I_{tj}} \left(\frac{\gamma_{si}}{\gamma_{uk}}\right)\right)^{\frac{1}{1-\sigma}} \tag{A.13}$$

Substituting the expression for Z_{tj}, equation (A.4), the expression for λ, equation (A.11), and the expression for p_{Z_i}, equation (A.13), into the FOC of ϕ_{stij}, equation (A.3) leads to:

$$\begin{aligned} \gamma_{si}\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \qquad & \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}\right)^{\sigma} \alpha_{Z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} \\ & - \left(\prod_{i=1}^{I_{tj}} \phi_{vtlj}^{\xi}\right)^{\xi(1-\eta)} \qquad P_{X_{tj}}^{1-\eta}\xi = 0 \end{aligned} \tag{A.14}$$

Using equation (7) and (A.13), we can rewrite as follows:

$$\begin{aligned} \gamma_{si}\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}\right)^{\sigma} &= \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{d_{si}} p_{stij} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \xi \\ & + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta} \left(\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{d_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}\right)^{\eta} \xi \end{aligned} \tag{A.15}$$

And solving for ϕ_{stij}:

$$\phi_{stij}^{\gamma_{si}} = \frac{\varphi_{si}}{\tau_{stij}\omega_{si}} \frac{\frac{1}{I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}}}{\left(\frac{\gamma_{si}\widetilde{\gamma_{stij}}^{\sigma-1}}{\xi I_{tj}} - 1\right)^{\frac{1}{1-\eta}}} \left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\frac{\eta}{1-\eta}} \left(\frac{\omega_{tj}}{\varphi_{tj}}\right) \tag{A.16}$$

A.2 Model with Per Unit Trade Costs

We add per unit trade costs a_{stij} to the baseline model and to keep the model tractable we work with a Cobb Douglas aggregator of intermediates instead of CES. The remainder of the model stays the same. Production is given by equation (4), but with Z_{tj} defined by:

$$Z_{tj} = \prod_{i=1}^{I_{tj}} Z_{stij}^{\beta}; \ \ \beta = \frac{1}{I_{tj}} \tag{A.17}$$

The cost function is as in equation (6), but the cif price of intermediate good i used in country t and sourced from country s is now:

$$p_{Z_{stij}} = \phi_{stij}^{\gamma} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} + a_{stij} \tag{A.18}$$

Minimizing costs $\sum_{i=1}^{I_{tj}} p_{Z_{stij}} z_{stij} + \omega_{tj} V_{tj}$ subject to the production function in equations (4) and (A.17) with $p_{Z_{stij}}$ defined in equation (A.18) leads to the following first order conditions:

$$\omega_{tj} - \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \alpha_{v_{tj}} \varphi_{tj}^{\frac{\eta-1}{\eta}} V_{tj}^{-\frac{1}{\eta}} = 0 \tag{A.19}$$

$$p_{Z_{stij}} - \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \alpha_{z_{tj}} Z_{tj}^{-\frac{1}{\eta}} \frac{Z_{tj}}{z_{stij}} = 0 \tag{A.20}$$

$$\gamma_{si} \phi_{stij}^{\gamma_{si}-1} \frac{\tau_{stij} \omega_{si}}{\varphi_{si}} z_{stij} - \lambda \xi \frac{X_{tj}}{\phi_{stij}} = 0 \tag{A.21}$$

The expressions for Z_{tj}, V_{tj} and λ remain the same and are still given by equations (A.9), (A.8) and (A.11). For z_{stij} we can easily find:

$$z_{stij} = \beta \frac{P_{Z_{tj}}}{p_{Z_{stij}}} Z_{tj} \tag{A.22}$$

With

$$p_{Z_{tj}} = \prod_{i=1}^{I_{tj}} \left(\frac{p_{Z_{stij}}}{\beta} \right)^{\beta} \tag{A.23}$$

Substituting equations (A.9) and (A.22) into (A.21) gives:

$$\gamma_{si} \phi_{stij}^{\gamma_{si}-1} \frac{\tau_{stij} \omega_{si}}{\varphi_{si}} \beta \frac{P_{Z_{tj}}}{p_{Z_{stij}}} \alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} X_{tj} \left(\prod_{i=1}^{I_{tj}} \phi_{vtlj}^{\xi} \right)^{\eta-1} P_{X_{tj}}^{\eta} - \lambda \xi \frac{X_{tj}}{\phi_{stij}} = 0 \tag{A.24}$$

Dividing equation (A.24) by a similar equation with different subscript and substituting equation (A.18), gives us:

$$\phi_{utkj}^{\gamma_{uk}} = \phi_{stij}^{\gamma_{si}} \frac{a_{utkj}}{a_{stij}} \frac{\tau_{stij}}{\tau_{utkj}} \frac{\omega_{si}}{\omega_{uk}} \frac{\varphi_{uk}}{\varphi_{si}} \tag{A.25}$$

Substituting equation (A.25) into equation (A.23) implies for $p_{Z_{tj}}$:

$$p_{Z_{tj}} = p_{Z_{stij}} \frac{\widetilde{a}_{utkj}}{\beta a_{stij}}; \widetilde{a}_{utkj} = \prod_{k=1}^{I_{tj}} a_{utkj}^{\beta} \tag{A.26}$$

Substituting equations (A.25) and (A.26) back into equation (A.24) leads after several steps to:

$$\phi_{stij}^{\gamma_{si}} \tau_{stij} \frac{\omega_{si}}{\varphi_{si}} \frac{\widetilde{a}_{utkj}}{a_{stij}} \left(\frac{\gamma_{si}}{\xi} - \frac{1}{\beta} \right) = \frac{\widetilde{a}_{utkj}}{\beta} + \left(\frac{\omega_{tj}}{\varphi_{tj}} \right)^{1-\eta} \alpha_{v_{tj}}^{\eta} \alpha_{z_{tj}}^{-\eta} \left(\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij} \omega_{si}}{\varphi_{si}} \frac{\widetilde{a}_{utkj}}{\beta a_{stij}} + \frac{\widetilde{a}_{utkj}}{\beta} \right)^{\eta} \tag{A.27}$$

We assume that $\gamma_{si} \beta > \xi$ to guarantee a positive solution for ϕ_{stij}. We cannot solve equation (A.27) for optimal quality. Still, we can derive comparative statics on the effect of changes in per unit trade costs a_{stij}, iceberg trade costs τ_{stij} and the price of labor given productivity in the importing country, $\frac{\omega_{tj}}{\varphi_{tj}}$:

$$\gamma_{si} \widehat{\phi_{stij}} = \widehat{a_{stij}} - \widehat{\tau_{stij}} + \frac{A_{stij}(1-\eta)}{(1 - A_{stij} B_{stij} \eta)} \widehat{\frac{\omega_{tj}}{\varphi_{tj}}} + \frac{A_{stij} \eta}{(1 - A_{stij} B_{stij} \eta)} \widehat{\frac{\alpha_{V_{tj}}}{\alpha_{Z_{tj}}}} \tag{A.28}$$

With

$$A_{stij} = \frac{\left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\eta}\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}+\frac{\widetilde{a}_{utkj}}{\beta}\right)^{\eta}}{\frac{\widetilde{a}_{utkj}}{\beta}+\left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\eta}\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}+\frac{\widetilde{a}_{utkj}}{\beta}\right)^{\eta}}$$

$$B_{stij} = \frac{\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}}{\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}+\frac{\widetilde{a}_{utkj}}{\beta}}$$

With the cif price positively dependent upon the quality level and $\gamma\beta > \xi$ and $\eta < 1$ as assumed, we can draw the following conclusions from expression (A.28). The quality and traded goods price of intermediate imports into country t from country s are rising in per unit trade costs a_{stij}, the wage relative to productivity in the importing country $\frac{\omega_{tj}}{\varphi_{tj}}$ and the labor taste shifter $\alpha_{V_{tj}}$ and declining in the iceberg trade costs τ_{stij} and the taste shifter of intermediate inputs $\alpha_{Z_{tj}}$. The effects of the different types of trade costs in the model are in line with the work of Hummels and Skiba (2004) on shipping the good apples out. The effects of the wage relative to productivity and the labor and intermediates taste shifters are in line with the baseline model above.

A.3 Model with Non-Linear Wage Schedule

The cost function of producing an intermediate of quality ϕ_{stij} is given by:

$$C\left(z_{stij},\phi_{stij}\right) = z_{stij}\frac{\omega_{si}\left(\phi_{stij}\right)}{\varphi_{si}} \tag{A.29}$$

The quality-wage schedule is given by the following non-linear function:

$$\omega_{si}\left(\phi_{stij}\right) = \nu_{si}\phi_{stij}^{\kappa_{si}} \tag{A.30}$$

Cost minimization of final goods producers leads to the same first order conditions for V_{tj} and z_{stij} as in A.1, equations (A.1)-(A.2). The first order condition for ϕ_{stij} becomes:

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}-1}}{\varphi_{si}}z_{stij} - \lambda\xi\frac{X_{tj}}{\phi_{stij}} = 0 \tag{A.31}$$

Marginal cost pricing of intermediate goods producers implies:

$$p_{Z_{stij}} = \frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}} \tag{A.32}$$

Dividing the FOC for ϕ_{stij} by a similar equation with different subscript and substituting the expression for the price of intermediates in equation (A.32) gives:

$$\phi_{stij} = \left(\frac{\kappa_{uk}\nu_{uk}}{\kappa_{si}\nu_{si}}\right)^{\frac{1}{\kappa_{si}(1-\sigma)}}\left(\frac{\tau_{utkj}}{\tau_{stij}}\frac{\varphi_{si}}{\varphi_{uk}}\right)^{\frac{1}{\kappa_{si}}}\phi_{utkj}^{\frac{\kappa_{uk}}{\kappa_{si}}} \tag{A.33}$$

Substituting equation (A.33) into equation (A.5) implies for p_{Z_i}:

$$p_{Z_{tj}} = p_{z_{stij}}I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}};\ \widetilde{\kappa_{stij}} = \left(\frac{1}{I_{tj}}\sum_{k=1}^{I_{tj}}\left(\frac{\kappa_{si}\nu_{si}}{\kappa_{uk}\nu_{uk}}\right)\right)^{\frac{1}{1-\sigma}} \tag{A.34}$$

Substituting the expression for Z_{tj}, equation (A.4), the expression for λ, equation (A.11), and the expression for p_{Z_i}, equation (A.34), into the FOC of ϕ_{stij}, equation (A.31) leads to:

$$\begin{aligned}\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}} \qquad & \left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma}\alpha_{Z_{tj}}^{\eta}P_{Z_{tj}}^{-\eta} \\ & -\left(\prod_{i=1}^{I_{tj}}\phi_{vtlj}^{\xi}\right)^{\xi(1-\eta)} \qquad P_{X_{tj}}^{1-\eta}\xi = 0\end{aligned} \tag{A.35}$$

Using equations (A.34) and (A.32), we can rewrite as follows:

$$\begin{aligned}\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma} &= \frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\xi \\ &\quad + \alpha_{v_{tj}}^{\eta}\omega_{tj}^{1-\eta}\varphi_{tj}^{\eta-1}\alpha_{Z_{tj}}^{-\eta}\left(\frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\eta}\xi \end{aligned} \tag{A.36}$$

And solving for ϕ_{stij}:

$$\phi_{stij}^{\kappa_{si}} = \frac{\varphi_{si}}{\tau_{stij}\nu_{si}}\frac{\frac{1}{I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\gamma_{stij}}}}{\left(\frac{\kappa_{si}}{\xi}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma-1}-1\right)^{\frac{1}{1-\eta}}}\left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\frac{\eta}{1-\eta}}\left(\frac{\omega_{tj}}{\varphi_{tj}}\right) \tag{A.37}$$

So from marginal cost pricing in equation (A.32), we get:

$$p_{Z_{stij}} = \frac{\frac{1}{I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\gamma_{stij}}}}{\left(\frac{\kappa_{si}}{\xi}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma-1}-1\right)^{\frac{1}{1-\eta}}}\left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\frac{\eta}{1-\eta}}\left(\frac{\omega_{tj}}{\varphi_{tj}}\right) \tag{A.38}$$

Hence, also with a non-linear wage schedule the price of intermediates rises linearly in opportunity cost of failure, the wage relative to productivity in the importing country. The reason is that both quality and the wage (through the wage schedule) adjust endogenously to the opportunity cost of failure, leading to a linear relation.

A.4 Derivations Suporting Sections A.1, A.2, and A.3

A.4.1 Deriving *Equation (A.7)*

Dividing the FOC for E_i and Z_i in equations (A.1) and (A.6) gives:

$$\begin{aligned}\frac{P_{Z_{tj}}}{\omega_{tj}} &= \frac{\lambda\left(\prod_{i=1}^{I_{tj}}\phi_{stij}^{\xi}\right)\left(Z_{tj}^{\frac{\eta-1}{\eta}}+(\varphi_{tj}V_{tj})^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}}\alpha_{z_{tj}}Z_{tj}^{-\frac{1}{\eta}}}{\lambda\left(\prod_{i=1}^{I_{tj}}\phi_{stij}^{\xi}\right)\left(Z_{tj}^{\frac{\eta-1}{\eta}}+(\varphi_{tj}V_{tj})^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}}\alpha_{v_{tj}}\varphi_{tj}^{\frac{\eta-1}{\eta}}V_{tj}^{-\frac{1}{\eta}}} \\ \frac{P_{Z_{tj}}}{\omega_{tj}}\frac{\alpha_{v_{tj}}}{\alpha_{z_{tj}}}\varphi_{tj}^{\frac{\eta-1}{\eta}} &= \frac{Z_{tj}^{-\frac{1}{\eta}}}{V_{tj}^{-\frac{1}{\eta}}} \\ Z_{tj} &= \left(\frac{\omega_{tj}}{P_{Z_{tj}}}\frac{\alpha_{z_{tj}}}{\alpha_{v_{tj}}}\right)^{\eta}\frac{1}{\varphi_{tj}^{\eta-1}}V_{tj}\end{aligned}$$

A.4.2 *Deriving Equation (A.8)*

Substituting equation (A.7) into equation (4) gives:

$$\begin{aligned}
X_{tj} &= \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}\right) \\
&\quad \times \left(\alpha_{z_{tj}} \left(Z_{tj}\right)^{\frac{\eta-1}{\eta}} + \alpha_{v_{tj}} \left(d_{tj} \left(\frac{P_{Z_{tj}}}{\omega_{tj}} \frac{\alpha_{v_{tj}}}{\alpha_{z_{tj}}}\right)^{\eta} \varphi_{tj}^{\eta-1} Z_{tj}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \\
&= \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}\right) \left(\alpha_{z_{tj}} \left(Z_{tj}\right)^{\frac{\eta-1}{\eta}} + \alpha_{v_{tj}} \left(\left(\frac{P_{Z_{tj}}}{\omega_{tj}} \frac{\alpha_{v_{tj}}}{\alpha_{z_{tj}}}\right)^{\eta} \varphi_{tj}^{\eta} Z_{tj}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \\
&= \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}\right) \left(\alpha_{z_{tj}} \left(Z_{tj}\right)^{\frac{\eta-1}{\eta}} + \alpha_{v_{tj}} \left(\frac{P_{Z_{tj}}}{\omega_{tj}} \frac{\alpha_{v_{tj}}}{\alpha_{z_{tj}}}\right)^{\eta-1} \varphi_{tj}^{\eta-1} Z_{tj}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \\
&= \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}\right) \left(\alpha_{z_{tj}} \left(Z_{tj}\right)^{\frac{\eta-1}{\eta}} + \alpha_{v_{tj}}^{\eta} \alpha_{z_{tj}}^{1-\eta} \left(\frac{P_{Z_{tj}}}{\omega_{tj}}\right)^{\eta-1} \varphi_{tj}^{\eta-1} Z_{tj}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \\
&= Z_{tj} \alpha_{z_{tj}}^{-\eta} P_{Z_{tj}}^{\eta} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}\right) \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1}\right)^{\frac{\eta}{\eta-1}}
\end{aligned}$$

Solving for Z_{tj}:

$$Z_{tj} = \alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} \frac{X_{tj}}{\left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}\right)^{\xi}} \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1}\right)^{\frac{\eta}{1-\eta}}$$

Total expenditure is therefore given by:

$$\begin{aligned}
P_{Z_{tj}} Z_{tj} \quad &+ \omega_{tj} V_{tj} \\
&= \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \varphi_{tj}^{\eta-1} \omega_{tj}^{1-\eta}\right) \frac{X_{tj}}{\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}} \\
&\quad \times \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1}\right)^{\frac{\eta}{1-\eta}} \\
&= \frac{1}{\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}} \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \varphi_{tj}^{\eta-1} \omega_{tj}^{1-\eta}\right)^{\frac{1}{1-\eta}} X_{tj}
\end{aligned}$$

We can therefore define the price index $P_{X_{tj}}$ as follows:

$$P_{X_{tj}} = \frac{1}{\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}} \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1}\right)^{\frac{1}{1-\eta}}$$

Z_{tj} and V_{tj} can now be written as follows:

$$\begin{aligned}
Z_{tj} &= \alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} \frac{X_{tj}}{\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}} \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \right)^{\frac{\eta}{1-\eta}} \\
&= \alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} X_{tj} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{(\eta-1)} \\
&\quad \times \left(\frac{1}{\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi}} \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \right)^{\frac{1}{1-\eta}} \right)^{\eta} \\
&= \alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} X_{tj} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{(\eta-1)} P_{X_{tj}}^{\eta} \\
V_{tj} &= \alpha_{v_{tj}}^{\eta} \varphi_{tj}^{\eta-1} \omega_{tj}^{-\eta} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{(\eta-1)} P_{X_{tj}}^{\eta} X_{tj}
\end{aligned}$$

A.4.3 Deriving *Equation (A.11)*

Substituting equation (A.8) into equation (A.1), followed by a bit of manipulation, generates an expression for λ:

$$\begin{aligned}
&\omega_{tj} - \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \alpha_{v_{tj}} \varphi_{tj}^{\frac{\eta-1}{\eta}} V_{tj}^{-\frac{1}{\eta}} = 0 \\
&\omega_{tj} - \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \alpha_{v_{tj}} d_{tj}^{\frac{\eta-1}{\eta}} \\
&\times \left(\alpha_{v_{tj}}^{\eta} \varphi_{tj}^{\eta-1} \omega_{tj}^{-\eta} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{(\eta-1)} P_{X_{tj}}^{\eta} X_{tj} \right)^{-\frac{1}{\eta}} = 0 \\
&P_{X_{tj}} - \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{\frac{1-\eta}{\eta}} X_{tj}^{-\frac{1}{\eta}} = 0 \\
&P_{X_{tj}} - \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{\frac{1-\eta}{\eta}} \\
&\times \left(\left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \right)^{-\frac{1}{\eta}} = 0 \\
&P_{X_{tj}} - \lambda \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right) \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{\frac{1-\eta}{\eta}} \\
&\times \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{-\frac{1}{\eta}} \left(Z_{tj}^{\frac{\eta-1}{\eta}} + (\varphi_{tj} V_{tj})^{\frac{\eta-1}{\eta}} \right)^{-\frac{1}{\eta-1}} = 0 \\
&P_{X_{tj}} = \lambda
\end{aligned}$$

A.4.4 Deriving *Equation (A.12)*

Substituting equation (A.4) into the FOC for ϕ_{stij}, equation (A.3), gives:

$$\gamma_{si}\phi_{stij}^{\gamma_{si}-1}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{P_{Z_{tj}}^{\sigma}}{p_{Z_{stij}}^{\sigma}}Z_{tj}-\lambda\xi\frac{X_{tj}}{\phi_{stij}}=0$$

Substituting equation (7) leads to:

$$\gamma_{si}\phi_{stij}^{\gamma_{si}-1}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{P_{Z_{tj}}^{\sigma}}{\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\right)^{\sigma}}Z_{tj}-\lambda\xi\frac{X_{tj}}{\phi_{stij}}=0$$

Rearranging and defining a similar equation for ϕ_{utkj} leads to:

$$\gamma_{si}\phi_{stij}^{\gamma_{si}(1-\sigma)}\left(\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\right)^{1-\sigma}P_{Z_{tj}}^{\sigma}Z_{tj}-\lambda\xi X_{tj}=0$$
$$\gamma_{uk}\phi_{utkj}^{\gamma_{uk}(1-\sigma)}\left(\frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}}\right)^{1-\sigma}P_{Z_{tj}}^{\sigma}Z_{tj}-\lambda\xi X_{tj}=0$$

Combining the two equations:

$$\gamma_{si}\phi_{stij}^{\gamma_{si}(1-\sigma)}\left(\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\right)^{1-\sigma}=\gamma_{uk}\phi_{utkj}^{\gamma_{uk}(1-\sigma)}\left(\frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}}\right)^{1-\sigma}$$
$$\phi_{stij}^{\gamma_{si}(1-\sigma)}=\frac{\gamma_{uk}}{\gamma_{si}}\left(\frac{\tau_{utkj}\omega_{uk}}{\tau_{stij}\omega_{si}}\frac{\varphi_{si}}{\varphi_{uk}}\right)^{1-\sigma}\phi_{utkj}^{\gamma_{uk}(1-\sigma)}$$
$$\phi_{stij}=\left(\frac{\gamma_{uk}}{\gamma_{si}}\right)^{\frac{1}{\gamma_{si}(1-\sigma)}}\left(\frac{\tau_{utkj}\omega_{uk}}{\tau_{stij}\omega_{si}}\frac{\varphi_{si}}{\varphi_{uk}}\right)^{\frac{1}{\gamma_{si}}}\phi_{utkj}^{\frac{\gamma_{uk}}{\gamma_{si}}}$$

A.4.5 Deriving *Equation (A.13)*

Substituting equation (A.12) into equation (A.5) leads to:

$$p_{Z_{tj}}=\left(\sum_{i=1}^{I_{tj}}\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$
$$=\left(\sum_{i=1}^{I_{tj}}\left(\left(\left(\frac{\gamma_{uk}}{\gamma_{si}}\right)^{\frac{1}{\gamma_{si}(1-\sigma)}}\left(\frac{\tau_{utkj}\omega_{uk}}{\tau_{stij}\omega_{si}}\frac{\varphi_{si}}{\varphi_{uk}}\right)^{\frac{1}{\gamma_{si}}}\phi_{utkj}^{\frac{\gamma_{uk}}{\gamma_{si}}}\right)^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$
$$=\left(\sum_{i=1}^{I_{tj}}\left(\left(\left(\frac{\gamma_{uk}}{\gamma_{si}}\right)^{\frac{1}{(1-\sigma)}}\left(\frac{\tau_{utkj}\omega_{uk}}{\tau_{stij}\omega_{si}}\frac{\varphi_{si}}{\varphi_{uk}}\right)\phi_{utkj}^{\gamma_{uk}}\right)\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$
$$=\left(\sum_{i=1}^{I_{tj}}\left(\left(\frac{\gamma_{uk}}{\gamma_{si}}\right)^{\frac{1}{(1-\sigma)}}\left(\frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}}\right)\phi_{utkj}^{\gamma_{uk}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$
$$=\left(\frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}}\right)\phi_{utkj}^{\gamma_{uk}}\left(\sum_{i=1}^{I_{tj}}\left(\frac{\gamma_{uk}}{\gamma_{si}}\right)\right)^{\frac{1}{1-\sigma}}$$
$$=p_{z_{utkj}}\left(\sum_{i=1}^{I_{tj}}\left(\frac{\gamma_{uk}}{\gamma_{s_{tij}i}}\right)\right)^{\frac{1}{1-\sigma}}$$
$$=p_{z_{stij}}\left(\sum_{k=1}^{I_{tj}}\left(\frac{\gamma_{si}}{\gamma_{u_{tkj}k}}\right)\right)^{\frac{1}{1-\sigma}}$$
$$=p_{z_{stij}}I_{tj}^{\frac{1}{1-\sigma}}\left(\frac{1}{I_{tj}}\sum_{k=1}^{I_{tj}}\left(\frac{\gamma_{si}}{\gamma_{u_{tkj}k}}\right)\right)^{\frac{1}{1-\sigma}}$$

We can rewrite this expression as follows:

$$p_{Z_{tj}} = p_{z_{stij}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}};\ \widetilde{\gamma_{stij}} = \left(\frac{1}{I_{tj}} \sum_{k=1}^{I_{tj}} \left(\frac{\gamma_{si}}{\gamma_{u_{tkj}k}} \right) \right)^{\frac{1}{1-\sigma}}$$

A.4.6 Deriving *Equation (A.14)*

First rewrite equation (A.3) as follows using equations (A.4) and (A.11):

$$\gamma_{si} \phi_{stij}^{\gamma_{si}-1} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} z_{stij} - \lambda \xi \frac{X_{tj}}{\phi_{stij}} = 0$$

$$\gamma_{si} \phi_{stij}^{\gamma_{si}-1} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \frac{P_{Z_{tj}}^{\sigma}}{p_{Z_{tsij}}^{\sigma}} Z_{tj} - \lambda \xi \frac{X_{tj}}{\phi_{stij}} = 0$$

$$\gamma_{si} \phi_{stij}^{\gamma_{si}-1} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \frac{P_{Z_{tj}}^{\sigma}}{p_{Z_{tsij}}^{\sigma}} \alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{(\eta-1)} P_{X_{tj}}^{\eta} X_{tj} - P_{X_{tj}} \xi \frac{X_{tj}}{\phi_{stij}} = 0$$

Substitute the expression for $\frac{P_{Z_{tj}}^{\sigma}}{p_{Z_{tsij}}^{\sigma}}$ from equation (A.13) and rearrange:

$$\gamma_{si} \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma} \alpha_{Z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{(\eta-1)} P_{X_{tj}}^{\eta} - P_{X_{tj}} \xi = 0$$

$$\gamma_{si} \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma} \alpha_{Z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} - \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{(1-\eta)} P_{X_{tj}}^{1-\eta} \xi = 0$$

A.4.7 Deriving *Equation (A.15)*

Substituting equation (A.10) and rearranging equation (A.14) gives:

$$\gamma_{si} \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma} \alpha_{Z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta}$$

$$- \left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{(1-\eta)} \frac{1}{\left(\prod_{i=1}^{I_{tj}} \phi_{stij}^{\xi} \right)^{(1-\eta)}} \times \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \right) \xi = 0$$

$$\gamma_{si} \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma} \alpha_{Z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta} - \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \right) \xi = 0$$

$$\gamma_{si} \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma} - \left(\alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \right) \alpha_{Z_{tj}}^{-\eta} P_{Z_{tj}}^{\eta} \xi = 0$$

$$\gamma_{si} \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma} - \left(P_{Z_{tj}} + \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta} P_{Z_{tj}}^{\eta} \right) \xi = 0$$

Further substituting equations (7) and (A.13) leads to equation (*A.15*).

$$\begin{aligned} \gamma_{si} \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma} &= p_{z_{stij}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \xi \\ &+ \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta} \left(p_{z_{stij}} \widetilde{\gamma_{tj}} \right)^{\eta} \xi \\ \gamma_{si} \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma} &= \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \xi \\ &+ \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta} \left(\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\eta} \xi \end{aligned}$$

A.4.8 Deriving *Equation (A.16)*

Start from equation (A.15) and solve for $\phi_{ij_{im}m}$:

$$\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \left(\frac{\gamma_{si}}{\xi} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma-1} - 1 \right)$$

$$= \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta} \left(\frac{\sigma}{\sigma-1} \phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\eta}$$

$$\left(\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{1-\eta} \left(\frac{\gamma_{si}}{\xi} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma-1} - 1 \right)$$

$$= \alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta}$$

$$\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}$$

$$= \frac{1}{\left(\frac{\gamma_{si}}{\xi} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma-1} - 1 \right)^{\frac{1}{1-\eta}}} \left(\alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta} \right)^{\frac{1}{1-\eta}}$$

$$\phi_{stij}^{\gamma_{si}} = \frac{\varphi_{si}}{\tau_{stij}\omega_{si}} \frac{\frac{1}{I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}}}{\left(\frac{\gamma_{si}}{\xi} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\sigma-1} - 1 \right)^{\frac{1}{1-\eta}}} \left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}} \right)^{\frac{\eta}{1-\eta}} \left(\frac{\omega_{tj}}{\varphi_{tj}} \right)$$

A.4.9 Deriving *Equation (A.25)*

We divide equation (A.24) by a similar equation with intermediate subscript l and substitute equation (A.18):

$$\begin{aligned}
\frac{\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}}}{pZ_{stij}} &= \frac{\phi_{utkj}^{\gamma_{uk}} \frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}}}{pZ_{utkj}} \\
\frac{\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}}}{\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} + a_{stij}} &= \frac{\phi_{utkj}^{\gamma_{uk}} \frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}}}{\phi_{utkj}^{\gamma_{uk}} \frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}} + a_{utkj}} \\
\frac{1}{1 + \frac{a_{stij}}{\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}}}} &= \frac{1}{1 + \frac{a_{utkj}}{\phi_{utkj}^{\gamma_{uk}} \frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}}}} \\
\frac{a_{stij}}{\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}}} &= \frac{a_{utkj}}{\phi_{utkj}^{\gamma_{uk}} \frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}}} \\
\phi_{stij}^{\gamma_{si}} \frac{\tau_{stij}\omega_{si}}{\varphi_{si}} &= \frac{a_{stij}}{a_{utkj}} \phi_{utkj}^{\gamma_{uk}} \frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}} \\
\phi_{utkj}^{\gamma_{uk}} &= \phi_{stij}^{\gamma_{si}} \frac{a_{utkj}}{a_{stij}} \frac{\tau_{stij}}{\tau_{utkj}} \frac{\omega_{si}}{\omega_{uk}} \frac{\varphi_{uk}}{\varphi_{si}}
\end{aligned}$$

A.4.10 Deriving *Equation (A.26)*

We substitute equation (A.25) into equation (A.23):

$$
\begin{aligned}
p_{Z_{tj}} &= \prod_{k=1}^{I_{tj}} \left(\frac{p_{Z_{utkj}}}{\beta} \right)^{\beta} \\
&= \prod_{k=1}^{I_{tj}} \left(\frac{\phi_{utkj}^{\gamma_{uk}} \frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}} + a_{utkj}}{\beta} \right)^{\beta} \\
&= \prod_{k=1}^{I_{tj}} \left(\frac{\phi_{stij}^{\gamma_{si}} \frac{a_{utkj}}{a_{stij}} \frac{\tau_{stij}}{\tau_{utkj}} \frac{\omega_{si}}{\omega_{uk}} \frac{\varphi_{uk}}{\varphi_{si}} \frac{\tau_{utkj}\omega_{uk}}{\varphi_{uk}} + a_{utkj}}{\beta} \right)^{\beta} \\
&= \prod_{k=1}^{I_{tj}} \left(\frac{\phi_{stij}^{\gamma_{si}} \frac{a_{utkj}}{a_{stij}} \tau_{stij} \frac{\omega_{si}}{\varphi_{si}} + a_{utkj}}{\beta} \right)^{\beta} \\
&= \prod_{k=1}^{I_{tj}} \left(\frac{\left(\frac{\phi_{stij}^{\gamma_{si}}}{a_{stij}} \tau_{stij} \frac{\omega_{si}}{\varphi_{si}} + 1 \right) a_{utkj}}{\beta} \right)^{\beta} \\
&= \prod_{k=1}^{I_{tj}} \left(\frac{\left(\phi_{stij}^{\gamma_{si}} \tau_{stij} \frac{\omega_{si}}{\varphi_{si}} + a_{stij} \right) \frac{a_{utkj}}{a_{stij}}}{\beta} \right)^{\beta} \\
&= \prod_{k=1}^{I_{tj}} \left(\frac{\left(\phi_{stij}^{\gamma_{si}} \tau_{stij} \frac{\omega_{si}}{\varphi_{si}} + a_{stij} \right) \frac{a_{utkj}}{a_{stij}}}{\beta} \right)^{\beta} \\
&= p_{Z_{stij}} \frac{\prod_{k=1}^{I_{tj}} a_{utkj}^{\beta}}{\beta a_{stij}} \\
&= p_{Z_{stij}} \frac{\widetilde{a}_{utkj}}{\beta a_{stij}}; \widetilde{a}_{utkj} = \prod_{k=1}^{I_{tj}} a_{utkj}^{\beta}
\end{aligned}
$$

A.4.11 Deriving *Equation (A.27)*

Substituting equations (A.25) and (A.26) back into equation (A.24), we get:

$$\gamma_{si}\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\beta\frac{P_{Z_{tj}}}{p_{Z_{stij}}}\alpha_{z_{tj}}^{\eta}P_{Z_{tj}}^{-\eta}\left(\prod_{i=1}^{I_{tj}}\phi_{vtlj}^{\xi}\right)^{\eta-1}P_{X_{tj}}^{\eta}-\lambda\xi=0$$

$$\gamma_{si}\frac{a_{stij}}{a_{utkj}}\frac{\tau_{utkj}}{\tau_{stij}}\frac{\omega_{uk}}{\omega_{si}}\frac{\varphi_{si}}{\varphi_{uk}}\phi_{utkj}^{\gamma_{uk}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\beta\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}\alpha_{z_{tj}}^{\eta}P_{Z_{tj}}^{-\eta}\left(\prod_{i=1}^{I_{tj}}\phi_{vtlj}^{\xi}\right)^{\eta-1}P_{X_{tj}}^{\eta}$$
$$-P_{X_{tj}}\xi=0$$

$$\gamma_{si}\phi_{utkj}^{\gamma_{uk}}\tau_{utkj}\frac{\omega_{uk}}{\varphi_{uk}}\frac{\widetilde{a}_{utkj}}{a_{utkj}}-\left(\prod_{i=1}^{I_{tj}}\phi_{vtlj}^{\xi}P_{X_{tj}}\right)^{1-\eta}\alpha_{z_{tj}}^{-\eta}P_{Z_{tj}}^{\eta}\xi=0$$

$$\gamma_{si}\phi_{utkj}^{\gamma_{uk}}\tau_{utkj}\frac{\omega_{uk}}{\varphi_{uk}}\frac{\widetilde{a}_{utkj}}{a_{utkj}}-\alpha_{z_{tj}}^{-\eta}P_{Z_{tj}}^{\eta}\xi\times$$
$$\left(\prod_{i=1}^{I_{tj}}\phi_{vtlj}^{\xi}\frac{1}{\prod_{i=1}^{I_{tj}}\phi_{stij}^{\xi}}\left(\alpha_{z_{tj}}^{\eta}P_{Z_{tj}}^{1-\eta}+\alpha_{v_{tj}}^{\eta}\omega_{tj}^{1-\eta}\varphi_{tj}^{\eta-1}\right)^{\frac{1}{1-\eta}}\right)^{1-\eta}=0$$

$$\gamma_{si}\phi_{utkj}^{\gamma_{uk}}\tau_{utkj}\frac{\omega_{uk}}{\varphi_{uk}}\frac{\widetilde{a}_{utkj}}{a_{utkj}}-\alpha_{z_{tj}}^{-\eta}P_{Z_{tj}}^{\eta}\xi\times$$
$$\left(\alpha_{z_{tj}}^{\eta}P_{Z_{tj}}^{1-\eta}+\alpha_{v_{tj}}^{\eta}\left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\right)=0$$

$$\gamma_{si}\phi_{utkj}^{\gamma_{uk}}\tau_{utkj}\frac{\omega_{uk}}{\varphi_{uk}}\frac{\widetilde{a}_{utkj}}{a_{utkj}}-\left(P_{Z_{tj}}+\alpha_{v_{tj}}^{\eta}\alpha_{z_{tj}}^{-\eta}P_{Z_{tj}}^{\eta}\left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\right)\xi=0$$

Substituting equations (A.18) and (A.26), changing the subscript index and rearranging we get:

$$\gamma_{si}\phi_{stij}^{\gamma_{si}}\tau_{stij}\frac{\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{a_{stij}}=\left(P_{Z_{tj}}+\alpha_{v_{tj}}^{\eta}\alpha_{z_{tj}}^{-\eta}P_{Z_{tj}}^{\eta}\left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\right)\xi$$

$$\frac{\gamma_{si}}{\xi}\phi_{stij}^{\gamma_{si}}\tau_{stij}\frac{\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{a_{stij}}=\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}+a_{stij}\right)\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}$$
$$+\left(\alpha_{v_{tj}}^{\eta}\alpha_{z_{tj}}^{-\eta}\left(\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}+a_{stij}\right)\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}\right)^{\eta}\left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\right)$$

$$\frac{\gamma_{si}}{\xi}\phi_{stij}^{\gamma_{si}}\tau_{stij}\frac{\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{a_{stij}}=\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}+a_{stij}\right)\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}$$
$$+\left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\alpha_{v_{tj}}^{\eta}\alpha_{z_{tj}}^{-\eta}\left(\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}+a_{stij}\right)\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}\right)^{\eta}$$

$$\phi_{stij}^{\gamma_{si}}\tau_{stij}\frac{\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{a_{stij}}\left(\frac{\gamma_{si}}{\xi}-\frac{1}{\beta}\right)$$
$$=\frac{\widetilde{a}_{utkj}}{\beta}+\left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\eta}\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}+\frac{\widetilde{a}_{utkj}}{\beta}\right)^{\eta}$$

A.4.12 Deriving *Equation (A.28)*

Log differentiating equation (A.27) with respect to a_{stij}, τ_{stij}, $\frac{\omega_{tj}}{\varphi_{tj}}$ and $\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}$ gives us:

$$\gamma_{si}\widehat{\phi_{stij}} + \widehat{\tau_{stij}} - \widehat{a_{stij}} = A_{stij}\left(1-\eta\right)\frac{\widehat{\omega_{tj}}}{\varphi_{tj}} + A_{stij}\eta\frac{\widehat{\alpha_{V_{tj}}}}{\alpha_{Z_{tj}}}$$
$$+A_{stij}B_{stij}\eta\left(\gamma_{si}\widehat{\phi_{stij}} + \widehat{\tau_{stij}} + \frac{\widehat{\omega_{si}}}{\varphi_{si}} - \widehat{a_{stij}}\right)$$

With

$$A_{stij} = \frac{\left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\eta}\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{\beta a_{stij}} + \frac{\widetilde{a}_{utkj}}{\beta}\right)^{\eta}}{\frac{\widetilde{a}_{utkj}}{\beta} + \left(\frac{\omega_{tj}}{\varphi_{tj}}\right)^{1-\eta}\left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}}\right)^{\eta}\left(\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{\beta a_{stij}} + \frac{\widetilde{a}_{utkj}}{\beta}\right)^{\eta}}$$

$$B_{stij} = \frac{\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{\beta a_{stij}}}{\phi_{stij}^{\gamma_{si}}\frac{\tau_{stij}\omega_{si}}{\varphi_{si}}\frac{\widetilde{a}_{utkj}}{\beta a_{stij}} + \frac{\widetilde{a}_{utkj}}{\beta}}$$

Rearranging generates equation (A.28):

$$\left(1-A_{stij}B_{stij}\eta\right)\gamma_{si}\widehat{\phi_{stij}} = -\left(1-A_{stij}B_{stij}\eta\right)\left(\widehat{\tau_{stij}} - \widehat{a_{stij}}\right) +$$
$$A_{stij}\left(1-\eta\right)\frac{\widehat{\omega_{tj}}}{\varphi_{tj}} + A_{stij}\eta\frac{\widehat{\alpha_{V_{tj}}}}{\alpha_{Z_{tj}}}$$
$$\gamma_{si}\widehat{\phi_{stij}} = \widehat{a_{stij}} - \widehat{\tau_{stij}} + \frac{A_{stij}\left(1-\eta\right)}{\left(1-A_{stij}B_{stij}\eta\right)}\frac{\widehat{\omega_{tj}}}{\varphi_{tj}}$$
$$+\frac{A_{stij}\eta}{\left(1-A_{stij}B_{stij}\eta\right)}\frac{\widehat{\alpha_{V_{tj}}}}{\alpha_{Z_{tj}}}$$

A.4.13 Deriving *Equation (A.33)*

Substituting equation (A.4) into the FOC for ϕ_{stij}, equation (A.31), gives:

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}-1}}{\varphi_{si}}\frac{P_{Z_{tj}}^{\sigma}}{p_{Z_{stij}}^{\sigma}}Z_{tj} - \lambda\xi\frac{X_{tj}}{\phi_{stij}} = 0$$

Substituting equation (A.32) leads to:

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}-1}}{\varphi_{si}}\frac{P_{Z_{tj}}^{\sigma}}{\left(\frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\right)^{\sigma}}Z_{tj} - \lambda\xi\frac{X_{tj}}{\phi_{stij}} = 0$$

Rearranging and defining a similar equation for ϕ_{utkj} leads to:

$$\begin{aligned}
\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}(1-\sigma)}\left(\frac{\tau_{stij}}{\varphi_{si}}\right)^{1-\sigma}P_{Z_{tj}}^{\sigma}Z_{tj} - \lambda\xi X_{tj} &= 0 \\
\kappa_{uk}\nu_{uk}\phi_{utkj}^{\kappa_{uk}(1-\sigma)}\left(\frac{\tau_{utkj}}{\varphi_{uk}}\right)^{1-\sigma}P_{Z_{tj}}^{\sigma}Z_{tj} - \lambda\xi X_{tj} &= 0
\end{aligned}$$

Combining the two equations:

$$\begin{aligned}
\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}(1-\sigma)}\left(\frac{\tau_{stij}}{\varphi_{si}}\right)^{1-\sigma} &= \kappa_{uk}\nu_{uk}\phi_{utkj}^{\kappa_{uk}(1-\sigma)}\left(\frac{\tau_{utkj}}{\varphi_{uk}}\right)^{1-\sigma} \\
\phi_{stij}^{\gamma_{si}(1-\sigma)} &= \frac{\kappa_{uk}\nu_{uk}}{\kappa_{si}\nu_{si}}\left(\frac{\tau_{utkj}}{\tau_{stij}}\frac{\varphi_{si}}{\varphi_{uk}}\right)^{1-\sigma}\phi_{utkj}^{\gamma_{uk}(1-\sigma)} \\
\phi_{stij} &= \left(\frac{\kappa_{uk}\nu_{uk}}{\kappa_{si}\nu_{si}}\right)^{\frac{1}{\kappa_{si}(1-\sigma)}}\left(\frac{\tau_{utkj}}{\tau_{stij}}\frac{\varphi_{si}}{\varphi_{uk}}\right)^{\frac{1}{\kappa_{si}}}\phi_{utkj}^{\frac{\kappa_{uk}}{\kappa_{si}}}
\end{aligned}$$

A.4.14 Deriving *Equation (A.35)*

First rewrite equation (A.31) as follows using equations (A.4) and (A.11):

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}-1}}{\varphi_{si}} z_{stij} - \lambda\xi\frac{X_{tj}}{\phi_{stij}} = 0$$

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}-1}}{\varphi_{si}} \frac{P_{Z_{tj}}^{\sigma}}{p_{Z_{tsij}}^{\sigma}} Z_{tj} - \lambda\xi\frac{X_{tj}}{\phi_{stij}} = 0$$

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}-1}}{\varphi_{si}} \frac{P_{Z_{tj}}^{\sigma}}{p_{Z_{tsij}}^{\sigma}} \alpha_{z_{tj}}^{\eta} P_{Z_{tj}}^{-\eta}\left(\prod_{i=1}^{I_{tj}}\phi_{stij}^{\xi}\right)^{(\eta-1)} P_{X_{tj}}^{\eta}X_{tj}$$

$$-P_{X_{tj}}\xi\frac{X_{tj}}{\phi_{stij}} = 0$$

Substitute the expression for $\frac{P_{Z_{tj}}^{\sigma}}{p_{Z_{tsij}}^{\sigma}}$ from equation (A.13) and rearrange:

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma}\alpha_{Z_{tj}}^{\eta}P_{Z_{tj}}^{-\eta}\left(\prod_{i=1}^{I_{tj}}\phi_{stij}^{\xi}\right)^{(\eta-1)} P_{X_{tj}}^{\eta}$$

$$-P_{X_{tj}}\xi = 0$$

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma}\alpha_{Z_{tj}}^{\eta}P_{Z_{tj}}^{-\eta}$$

$$-\left(\prod_{i=1}^{I_{tj}}\phi_{stij}^{\xi}\right)^{(1-\eta)} P_{X_{tj}}^{1-\eta}\xi = 0$$

A.4.15 Deriving *Equation (A.36)*

Substituting equation (A.10) and rearranging equation (A.35) gives:

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma}\alpha_{Z_{tj}}^{\eta}P_{Z_{tj}}^{-\eta} - \left(\prod_{i=1}^{I_{tj}}\phi_{stij}^{\xi}\right)^{(1-\eta)} \times$$

$$\frac{1}{\left(\prod_{i=1}^{I_{tj}}\phi_{stij}^{\xi}\right)^{(1-\eta)}}\left(\alpha_{z_{tj}}^{\eta}P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta}\omega_{tj}^{1-\eta}\varphi_{tj}^{\eta-1}\right)\xi = 0$$

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma}\alpha_{Z_{tj}}^{\eta}P_{Z_{tj}}^{-\eta}$$

$$-\left(\alpha_{z_{tj}}^{\eta}P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta}\omega_{tj}^{1-\eta}\varphi_{tj}^{\eta-1}\right)\xi = 0$$

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma} -$$

$$\left(\alpha_{z_{tj}}^{\eta}P_{Z_{tj}}^{1-\eta} + \alpha_{v_{tj}}^{\eta}\omega_{tj}^{1-\eta}\varphi_{tj}^{\eta-1}\right)\alpha_{Z_{tj}}^{-\eta}P_{Z_{tj}}^{\eta}\xi = 0$$

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma} - \left(P_{Z_{tj}} + \alpha_{v_{tj}}^{\eta}\omega_{tj}^{1-\eta}\varphi_{tj}^{\eta-1}\alpha_{Z_{tj}}^{-\eta}P_{Z_{tj}}^{\eta}\right)\xi = 0$$

Substituting equation (A.32) and (A.34) leads to equation (*A.31*):

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma} =$$

$$p_{z_{stij}}I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\xi + \alpha_{v_{tj}}^{\eta}\omega_{tj}^{1-\eta}\varphi_{tj}^{\eta-1}\alpha_{Z_{tj}}^{-\eta}\left(p_{z_{stij}}\widetilde{\kappa\nu_{stij}}\right)^{\eta}\xi$$

$$\frac{\tau_{stij}\kappa_{si}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}\left(I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\sigma} = \frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\xi$$

$$+\alpha_{v_{tj}}^{\eta}\omega_{tj}^{1-\eta}\varphi_{tj}^{\eta-1}\alpha_{Z_{tj}}^{-\eta}\left(\frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}}I_{tj}^{\frac{1}{1-\sigma}}\widetilde{\kappa\nu_{stij}}\right)^{\eta}\xi$$

A.4.16 Deriving *Equation (A.37)*

Start from equation (A.15) and solve for ϕ_{stij}:

$$\frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\kappa\nu_{stij}} \left(\frac{\kappa_{si}}{\xi} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\kappa\nu_{stij}} \right)^{\sigma-1} - 1 \right) =$$
$$\alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta} \left(\frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}} \right)^{\eta}$$

$$\left(\frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\kappa\nu_{stij}} \right)^{1-\eta} \left(\frac{\kappa_{si}}{\xi} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\kappa\nu_{stij}} \right)^{\sigma-1} - 1 \right) =$$
$$\alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta}$$

$$\frac{\tau_{stij}\nu_{si}\phi_{stij}^{\kappa_{si}}}{\varphi_{si}} I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\kappa\nu_{stij}} =$$
$$\frac{1}{\left(\frac{\kappa_{si}}{\xi} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\kappa\nu_{stij}} \right)^{\sigma-1} - 1 \right)^{\frac{1}{1-\eta}}} \left(\alpha_{v_{tj}}^{\eta} \omega_{tj}^{1-\eta} \varphi_{tj}^{\eta-1} \alpha_{Z_{tj}}^{-\eta} \right)^{\frac{1}{1-\eta}}$$

$$\phi_{stij}^{\kappa_{si}} = \frac{\varphi_{si}}{\tau_{stij}\nu_{si}} \frac{\frac{1}{I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\gamma_{stij}}}}{\left(\frac{\kappa_{si}}{\xi} \left(I_{tj}^{\frac{1}{1-\sigma}} \widetilde{\kappa\nu_{stij}} \right)^{\sigma-1} - 1 \right)^{\frac{1}{1-\eta}}} \left(\frac{\alpha_{v_{tj}}}{\alpha_{Z_{tj}}} \right)^{\frac{\eta}{1-\eta}} \left(\frac{\omega_{tj}}{\varphi_{tj}} \right)$$

A.5 Data Appendix and Additional Regression Tables

Table A.1: Results using a trimmed instead of winsorized sample

	(1)	(2)	(3)
	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$	$\ln p_{z_{sti}}$
$\widetilde{\theta_{sti^*u}}$	0.800	1.780	0.857
	(9.10)***	(6.44)***	(1.78)*
$LnGDP_t$		-3.457	-2.130
		(8.23)***	(3.26)***
$LnGDPpercapita_t$		3.278	2.109
		(8.72)***	(3.69)***
$Lndistance_{st}$			0.175
			(25.49)***
$LnGDPpercapita_s$			0.179
			(13.90)***
LM test for underid	463.15	160.09	37.23
p-value	0.00	0.00	0.00
F-stat for weak id	552.11	146.57	42.17
R2	0.78	0.78	0.52
First stage:			
	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$
IV	0.122	0.055	0.037
	(23.50)***	(12.11)***	(6.49)***
$LnGDP_t$		1.267	1.281
		(23.04)***	(23.83)***
$LnGDPpercapita_t$		-1.156	-1.145
		(19.96)***	(20.39)***
$Lndistance_{st}$			-0.003
			(1.43)
$LnGDPpercapita_s$			-0.004
			(0.99)
R2	0.94	0.95	0.78
N	5,849,899	5,849,899	6,052,284

Table A.2: First stage results, intermediate goods

	(1)	(2)	(3)	(4)
	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$
IV	0.121	0.053	0.053	0.036
	(23.58)***	(11.61)***	(11.61)***	(6.45)***
$LnGDP_t$		1.293	1.293	1.293
		(24.02)***	(24.02)***	(24.35)***
$LnGDPpercapita_t$		-1.179	-1.178	-1.156
		(20.78)***	(20.78)***	(20.85)***
$Lndistance_{st}$				-0.004
				(1.56)
$LnGDPpercapita_s$				-0.004
				(0.85)
HI			0.003	0.004
			(6.90)***	(2.77)***
R2	0.94	0.95	0.95	0.78
N	6,584,575	6,584,575	6,584,575	6,755,827

Table A.3: First stage results, final and homogenous goods

	Final goods sample			Homogenous goods sample		
	(1)	(2)	(3)	(4)	(5)	(6)
	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$	$\widetilde{\theta_{sti^*u}}$
IV	0.058	0.015	0.054	0.050	0.023	0.041
	(14.04)***	(3.28)***	(11.20)***	(9.67)***	(3.62)***	(8.72)***
$LnGDP_t$		0.667	0.643		0.798	0.829
		(10.37)***	(9.86)***		(9.23)***	(9.35)***
$LnGDPpercapita_t$		-0.587	-0.604		-0.736	-0.785
		(8.87)***	(9.00)***		(8.54)***	(8.58)***
$Lndistance_{st}$			0.004			0.003
			(1.64)			(0.92)
$LnGDPpercapita_s$			0.001			0.013
			(0.17)			(1.82)*
R2	0.93	0.94	0.69	0.94	0.94	0.71
N	1,977,513	1,977,513	2,018,782	567,499	567,499	603,775

Table A.4: Summary Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
$\ln p_{z_{sti}}$	6,755,827	2.122299	1.578961	-0.4879416	5.44458
$\widetilde{\theta_{sti^*u}}$	6,755,827	0.1655888	0.1688882	-0.6576125	1.163728
$LnGDP_t$	6,755,827	20.45467	1.474147	16.81022	23.35959
$LnGDPpercapita_t$	6,755,827	3.314647	0.4747392	1.513526	4.082383
$Lndistance_{st}$	6,755,827	7.747976	1.159812	4.087945	9.828204
$LnGDPpercapita_s$	6,755,827	2.55059	1.152499	-0.786483	4.082383
IV	6,755,827	9.443422	0.3529929	8.489432	11.63757

Table A.5: Importer countries in the sample (only intermediate goods)

Importers		Observations	Share (in %)
Australia	AUS	256,515	3.8
Austria	AUT	393,527	5.83
Czech Republic	CZE	348,202	5.15
Germany	DEU	636,375	9.42
Denmark	DNK	333,045	4.93
Spain	ESP	446,670	6.61
Finland	FIN	308,439	4.57
France	FRA	529,177	7.83
United Kingdom	GBR	536,828	7.95
Hungary	HUN	290,622	4.3
Ireland	IRL	223,013	3.3
Italy	ITA	563,504	8.34
Japan	JPN	333,050	4.93
Netherlands	NLD	475,261	7.03
Slovenia	SVN	182,275	2.7
Sweden	SWE	380,749	5.64
United States	USA	518,575	7.68

Table A.6: Exporter countries in the sample (only intermediate goods)

Exporters		Observations	Share (in %)
Australia	AUS	151,612	2.24
Austria	AUT	221,326	3.28
Bulgaria	BGR	132,068	1.95
Brazil	BRA	155,722	2.31
Canada	CAN	181,086	2.68
China	CHN	252,185	3.73
Cyprus	CYP	79,670	1.18
Czech Republic	CZE	196,986	2.92
Germany	DEU	307,450	4.55
Denmark	DNK	194,270	2.88
Spain	ESP	230,955	3.42
Estonia	EST	115,163	1.7
Finland	FIN	176,392	2.61
France	FRA	270,965	4.01
United Kingdom	GBR	277,403	4.11
Greece	GRC	138,547	2.05
Hungary	HUN	177,098	2.62
Indonesia	IDN	152,210	2.25
India	IND	203,138	3.01
Ireland	IRL	152,182	2.25
Italy	ITA	275,052	4.07
Japan	JPN	199,631	2.95
Korea, Rep.	KOR	185,095	2.74
Lithuania	LTU	120,348	1.78
Latvia	LVA	102,327	1.51
Mexico	MEX	156,150	2.31
Netherlands	NLD	251,879	3.73
Poland	POL	209,490	3.1
Portugal	PRT	168,412	2.49
Romania	ROM	167,489	2.48
Russian Federation	RUS	184,032	2.72
Slovak Republic	SVK	160,004	2.37
Slovenia	SVN	143,553	2.12
Sweden	SWE	210,437	3.11
Turkey	TUR	193,723	2.87
United States	USA	261,777	3.87

Table A.7: Observations belonging to different HS2 categories

only products identified as non-homogenous intermediate goods)

hs2	Freq.	Percent	hs2	Freq.	Percent
1	9,316	0.07	48	374,043	2.91
2	39,296	0.31	49	80,695	0.63
3	132,305	1.03	50	14,243	0.11
4	26,101	0.2	51	40,862	0.32
5	31,055	0.24	52	241,296	1.87
6	33,704	0.26	53	35,880	0.28
7	97,856	0.76	54	206,092	1.6
8	92,314	0.72	55	274,671	2.13
9	929	0.01	56	92,576	0.72
11	52,497	0.41	57	71,599	0.56
12	68,263	0.53	58	125,716	0.98
13	28,302	0.22	59	81,263	0.63
14	11,951	0.09	60	64,681	0.5
15	55,423	0.43	61	435,501	3.38
16	68,617	0.53	62	531,619	4.13
17	27,100	0.21	63	196,382	1.53
18	31,094	0.24	64	121,056	0.94
19	58,293	0.45	65	35,980	0.28
20	106,551	0.83	68	162,412	1.26
21	59,731	0.46	69	109,150	0.85
22	72,942	0.57	70	212,220	1.65
23	15,120	0.12	71	1,181	0.01
24	14,352	0.11	72	417,336	3.24
25	18,751	0.15	73	477,024	3.71
27	52,797	0.41	74	136,515	1.06
28	268,377	2.09	75	25,689	0.2
29	630,877	4.9	76	123,835	0.96
30	110,351	0.86	78	10,124	0.08
31	37,710	0.29	79	14,484	0.11
32	173,352	1.35	80	3,384	0.03
33	128,217	1	81	29,773	0.23
34	95,658	0.74	82	285,896	2.22
35	52,212	0.41	83	169,405	1.32
36	11,256	0.09	84	1,880,106	14.61
37	84,760	0.66	85	1,134,277	8.81
38	179,073	1.39	86	38,515	0.3
39	506,551	3.94	87	303,515	2.36
40	232,791	1.81	88	34,106	0.26
41	51,920	0.4	89	32,425	0.25
42	93,523	0.73	90	18,733	0.15
43	24,934	0.19	91	1,505	0.01
44	176,497	1.37	93	41,353	0.32
45	15,136	0.12	94	62,401	0.48
46	15,685	0.12	96	3,222	0.03
47	14,768	0.11	97	9,635	0.07
			Total	12,870,684	100

The final sample used for the regressions is somewhat smaller due to data restrictions on some of the explanatory variables.

References

Alcala, F. and A. Ciccone (2004) "Trade and productivity." *Quarterly journal of economics* 119(2): 613-646.

Alessandria, G. and J. P. Kaboski(2011) "Pricing-to-Market and the Failure of Absolute PPP." *American Economic Journal: Macroeconomics* 3(1): 91-127.

Baldwin, R. and J. Harrigan (2011) "Zeros, Quality and Space: Trade Theory and Trade Evidence." *American Economic Journal: Microeconomics* 3(2): 60-88.

Bastos, P. and J. Silva (2010) "The quality of a firm's exports: Where you export to matters." *Journal of International Economics* 82(2): 99 –111.

Bernini, M., and S. Guillou, and F. Bellone (2015). "Financial leverage and export quality: Evidence from France."" *Journal of Banking & Finance* 59: 280-296

Bekkers, E. (2016) "Firm Heterogeneity, Endogenous Quality, and Traded Goods Prices." *World Economy* 39(1): 72-96.

Bekkers, E., J. Francois and M. Manchin (2012) "Import Prices, Income, and Inequality." *European Economic Review* 56: 848–869.

Clair, G., G. Gaulier, T. Mayer, and S. Zignago (2004). "Notes on CEPII's distances measures." CEPII: Paris.

Crozet M., K. Head and T. Mayer (2012) "Quality sorting and trade: Firm-level evidence for French wine." *Review of Economic Studies* 79(2): 609-644.

Dietzenbacher, E., B. Los, R. Stehrer, M. Timmer and G. de Vries (2013) "The Construction of World Input-Output Tables in the WIOD Project." *Economic Systems Research* 25 (1): 71-98.

Dingel, J. (2017) "The Determinants of Quality Specialization." *The Review of Economic Studies* 84(4): 1551-1582.

Feenstra, R.C. and J. Romalis (2014) "International Prices and Endogenous Quality." *The Quarterly Journal of Economics* 129 (2): 477–527.

Flach, L. (2016) "Quality upgrading and price heterogeneity: Evidence from Brazilian exporters." *Journal of International Economics* 102: 282-290.

Francois, J. and S. Kaplan (1996) "Aggregate demand shifts, income distribution, and the Linder hypothesis." *The Review of Economics and Statistics* May(1): 244-250.

Francois, J. and Reinert, K.A. (ed.) (1998). *Applied Methods for Trade Policy Analysis.* Cambridge Books, Cambridge University Press.

Gervais, A. (2015) "Product quality and firm heterogeneity in international trade." *Canadian Journal of Economics* Volume 48 (3):1152–1174.

Görg, H., L. Halpern and B. Muraközy (2010) "Why Do Within Firm-Product Export Prices Differ Across Markets?" Kiel Working Paper No. 1596.

Guimaraes,P. and P. Portugal (2010) "A Simple Feasible Alternative Procedure to Estimate Models with High-Dimensional Fixed Effects." Stata Journal 10(4): 628-649, 2010.

Hertel, T.W. (ed.) (1997) *Global Trade Analysis: Modeling and Applications.* Cambridge: Cambridge University Press.

Hall, R. and C. Jones(1996) "The productivity of nations." NBER Working Paper No. 5812.

Hall,R. and C. Jones(1999) "Why do some countries produce so much more output per worker than others?" *Quarterly Journal of Economics* 114(1): 83-116

Hallak, J.C. and J. Sivadasan (2010) “Firms’ Exporting Behavior under Quality Constraints.” NBER Working Paper No. 14928.

Harrigan, J., X. Ma, and V. Shlychkov (2015) “Export Prices of US Firms.” *Journal of International Economics* 97 (1): 100-111.

Hummels, D., J. Ishii, and Kei-Mu Yi (2001) “The nature and growth of vertical specialization in world trade.” *Journal of International Economics* 54(1): 75-96

Hummels, D., V. Lugovskyy (2009) “International Pricing in a Generalized Model of Ideal Variety.” *Journal of Money, Credit and Banking* 41(1): 3-33.

Hummels, D. and A. Skiba (2004) “Shipping the Good Apples Out? An Empirical Confirmation of the Alchian Allen Conjecture.” *Journal of Political Economy* 112(6): 1384-1402.

Keller, W. and S.R. Yeaple (2013) “The Gravity of Knowledge.” *American Economic Review* 103(4): 1414-44.

Khandelwal, A. (2010) “The Long and Short (of) Quality Ladders.” The Review of Economic Studies 77(4): 1450-1476

Koopman,R., Z. Wang, S.-J. Wei. (2014). “Tracing Value-added and Double Counting in Gross Exports.” *American Economic Review* 104(2): 459-494.

Kremer, M. (1993) “The O-ring Theory of Economic Development.” *Quarterly Journal of Economics* 108: 551-575.

Verhoogen, E. and M. Kugler (2012) “Prices, Plant Size, and Product Quality.” *Review of Economic Studies* 79 (1): 307-339.

Manova, K. and Z. Zhang (2012) “Export Prices across Firms and Destinations.” *Quarterly Journal of Economics* 127: 379-436.

Martin, J. (2012) “Markups, Quality, and Transport Costs.” *European Economic Review* 56: 777-791.

Melitz, Marc J. (2003) “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity.” *Econometrica* 71 (6), pp. 1695-1725.

O’Mahony, M. and M.P. Timmer (2009) “Output, Input and Productivity Measures at the Industry Level: the EU KLEMS Database.” *Economic Journal* 119(538): 374-403.

Rauch, J. (1999), “Networks versus Markets in International Trade.” *Journal of International Economics* 48, pp. 7-35.

Rigobon, R., and D. Rodrik (2005) “Rule of law, democracy, openness, and income.” *Economics of transition* 13(3): 533-564.

Schott, P.K. (2004) “Across-Product Versus Within-Product Specialization in International Trade.” *Quarterly Journal of Economics* 119 (2): 647-678.

Simonovska, I. (2016) “Income Differences and Prices of Tradables: Insights from an Online Retailer.” The *Review of Economic Studies* 82 (4): 1612–1656

Sutton, J. (1991) *Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration.* MIT Press, Cambridge, Mass.

Sutton, J. (1998) *Technology and Market Structure: Theory and History.* MIT Press, Cambridge, Mass.

Verhoogen, E.A. (2008) “Trade, Quality Upgrading and Wage Inequality in the Mexican Manufacturing Sector.” *Quarterly Journal of Economics* 123(2): 489-530.

Verhoogen, E. and M. Kugler (2012) “Prices, Plant Size, and Product Quality.” *Review of Economic Studies* 79 (1): 307-339.