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Coordination Failure and Financial Contagion

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Abstract

This paper explores a unique equilibrium model of "informational" financial contagion. Extending the global game model of Morris and Shin (1999), I show that the failure of a single firm can trigger a chain of failures merely by affecting the behavior of investors. In contrast to the existing multiple equilibria models of financial and banking panics, there is no indeterminacy in the present model. Thus, it provides a clear framework to assess the consequences of contagion and yields some important and hitherto unnoticed insights. Most importantly, if contagion is compared to an appropriate benchmark, its impact can be both positive or negative, which contrasts sharply with the traditional view of contagion. Moreover, contagion increases the correlation between firms, but the effect on the unconditional probability of failure is exactly zero.

Keywords: financial contagion, systemic risk, financial crises, global games, unique equilibrium.

JEL-Classifications: G15, G21, C72

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1 Introduction

As is well known in the growing literature on financial contagion and systemic risk, there are at least two potential channels through which a single firm can affect other firms. The first channel rests upon on direct capital connections between the firms, which make it possible that the failure of a debtor leads to the failure of its creditors simply because the latter have to write off their claims. While it is difficult to get an accurate picture of these credit linkages in practice, the mechanism of this transmission channel seems to be well understood. Recent theoretical models which deal with such contagion in a banking framework include Rochet and Tirole (1996), Allen and Gale (2000) and Dasgupta (2001). A second channel, which I will refer to as "informational contagion", hinges on the beliefs of financial market participants. In a banking context, depositors (or also other banks) may decide to withdraw their claims when observing the collapse of another bank because they lack precise information on how the failure is related to their own bank. Likewise, international investors might withdraw their investments from a country when they observe trouble in a different country, and due to such reactions of investors, a crisis can spread from one firm or country to others. Since the present paper focuses on this second channel, I will refer to informational contagion whenever I speak of (financial) contagion hereafter.[1,2]

Although informational contagion is widely believed in[3] and is a major concern behind regulatory measures such as introducing a deposit insurance, providing implicit government guarantees or imposing capital controls, there are few formal models which deal with the informational propagation channel. Recent exceptions include Chen (1999) and Aghion, Bolton and Dewatripont (2000). Obviously, a major impediment to a sound theoretical foundation of informational contagion is the presence of multiple equilibria in the usual

[1] Typically, contagion is either discussed in a banking or in an international crises context, so I will also have these examples in mind, although my model is rather general. Since the literature is far from converging to a common definition of the widely used terms contagion and systemic risk, I renounce to give a precise definition until section 4.

[2] For a discussion of different bank contagion channels, see Kaufman (1994). Dornbusch et al. (2000) survey international contagion channels.

[3] See e.g. Park (1991), who argues that the lack of bankspecific information is the main source of bank contagion and offers evidence in favour of this view. Calomiris and Mason (1997), and Gorton (1988) also find evidence of informational bank contagion. For evidence on informational contagion across countries, see Ahluwalia (2000), Park and Song (2000), and also Eichengreen, Rose and Wyplosz (1996).

coordination failure and bank run models as in Diamond and Dybvig (1983). Since the events which determine the beliefs of the depositors and may trigger a run are not part of these models, it remains open which equilibrium will occur. Hence, there is an apparent indeterminacy in any sunspot model. If theory offers little guidance on the circumstances which trigger a single bank run, it seems even less suited to predict in which situations contagion will occur. Put differently, if a model "predicts" that depending on the agents' beliefs, any outcome of firm A can be an equilibrium, but remains completely silent about the beliefs, it is hardly able to predict if and how the outcome of firm A could affect a different firm B. The same problems arise in models of international financial crises with self-fulfilling features in the manner of Obstfeld (1996).

There is, however, a recent strand of literature, initiated by Carlsson and van Damme (1993) and further advanced by Morris and Shin (1998) in a speculative attack framework, which has developed a technique that selects a unique equilibrium in many coordination failure models.[4] The common feature of this "global games" approach is that the payoffs of the players depend on some random state of the world, which is however not common knowledge. Instead, each player privately observes a noisy signal of the true state. Interestingly, this assumption is not only a step towards a more realistic information structure, but also introduces a useful structure into the beliefs of agents which leads a unique equilibrium. In another contribution, Morris and Shin (1999) provide a model of a coordination failure among numerous small creditors financing a common project. They show that in the presence of noisy but sufficiently precise private information of the creditors, there is a unique threshold of the realization of some fundamental variable below which the borrower is "run" or denied sufficient credit such that the project fails. Although systemic risk is neither an issue in Morris and Shin (1999) nor in any other model with only one firm, there is little doubt that Carlsson and van Damme's (1993) unique equilibrium selection technique provides a promising framework to analyze contagion.

In this paper, I therefore extend a modified version of the Morris and Shin (1999) model to include two (or more) firms and two (kinds of) fundamental

[4]A precursor of this literature is Rubinstein (1989). Morris and Shin (2001) provide a good survey of this "global games" approach. Goldstein and Pauzner (2000) apply the technique to the Diamond and Dybvig (1983) model. Rochet and Vives (2000) also explore a banking model. Finally, it shall be mentioned that an earlier bank model with a unique Bayesian equilibrium is found in Postlewaite and Vives (1987).

variables. The first fundamental represents states which are specific for each firm, while the second fundamental is a common state influencing all firms within an industry or region. Further, I introduce a time structure such that the investors of the second firm can observe what happens to the first project before deciding whether to roll over their loans. In this model, there is still a unique equilibrium in which each firm succeeds if and only if the firm specific fundamental exceeds some threshold, which depends on the state of the common fundamental. In addition, it can be shown that the equilibrium threshold of the second firm is larger when the first firm fails, which implies that a failure of the first firm increases the probability that the second firm fails. Hence, there is a contagious link between both projects which stems from a purely informational channel. If the creditors of the second firm observe that the first firm fails, they adjust their prior beliefs on the state of the common fundamental accordingly. As a consequence, they become more reluctant to roll over their loans, which in turn may cause the collapse of the second firm. On the other hand, by the same mechanism, a good result of the first firm can rescue the other, which is a positive aspect of contagion which - as far as I know - has received no attention in the literature so far. Finally, the outcome of the model can be compared to a natural benchmark in which there is no contagion. As an important result, if we are only concerned about the unconditional likelihood of failure of the firms, the net effect of contagion is exactly zero. This result emerges despite the fact that a failure of the first firm can trigger the failure of the second firm (and possibly more firms). Therefore, the model provides not only a rigorous theoretical foundation of an informational channel of contagion, but also reveals that the implications of such contagion are quite different from those which the traditional view of contagion suggests.

The present work is probably most closely related to the recent contributions of Dasgupta (2001) and Goldstein and Pauzner (2001), which both analyze the issue of contagion in a global game framework. However, the propagation mechanisms in their models are different. Dasgupta (2001) develops a banking model in which contagion stems from the existence of capital links between financial institutions. Goldstein and Pauzner (2001) explore a model with two countries subject to self-fulfilling financial crises, in which contagion emerges due to wealth effects. In their framework, a crisis in one country reduces investors' wealth, which makes them more risk averse and more inclined to withdraw their investments from the second country. This in turn raises the likelihood of a crisis in the second country. The model presented in this paper, by contrast, presumes neither capital links nor wealth effects. To my knowledge, it is the first model of informational contagion in a global game framework. Since the considered firms need not be banks, the model is rather general, though banks seem to be typical examples of borrowers facing numerous small creditors. If the different firms are interpreted as representative firms of different countries, the model can also be applied in an international setting to explore why and how different countries are affected when one country experiences a financial crisis.

The remainder of the paper is structured as follows. In section 2, I present the basic structure of the model and derive the unique equilibrium of the game. For presentation purposes, the model is introduced with only one firm, while the second firm is introduced in the extended model presented in section 3. This part of the paper also contains the important result that there is a contagious link between the firms. In section 4, I discuss the meaning and the consequences of financial contagion in some more detail and define the concepts of "negative" and "positive" contagion, while section 5 concludes.

2 The Basic Model

2.1 Structure of the Model with one Firm

To begin with and for ease of exposition, suppose there is only one firm, which runs a project that is financed by a continuum of creditors normalized to one.[5] In section 3, I will extend the model to include two or more firms, which is of course a crucial step for the analysis of contagion. The basic structure of the model and many features of the equilibrium, however, can well be discussed in the framework with one firm. The model focuses on the behavior of the individual creditors, who, ad an ad interim stage of the project, receive the opportunity to rethink their investment and to decide on whether to roll over their loans. The initial lending decision is not part of the model, yet assume that the loan conditions are such that it is ex ante rational to lend. Together with the realizations of two economic fundamental variables, the fraction l of creditors who decide to withdraw their loans in the intermediate period determines whether the firm succeeds or fails. At a final stage, the project is terminated and yields a gross return of $R \geq 1$ if it

[5] The project represents the whole and only activity of the firm, hence I will use the terms firm and project interchangeably.

succeeds, whereas the return is 0 if it fails.[6]

The return $r(\theta, u, l)$ of the project is specified as

$$r(\theta, u, l) = \begin{cases} R \text{ if } \theta > u + zl \\ 0 \text{ if } \theta \leq u + zl \end{cases}, \tag{1}$$

where θ and u index the states of two independent economic fundamentals. More specifically, assume that the state θ is drawn according to a uniform distribution on the unit interval, while the state u is either u_1 or u_2 with probabilities $P(u = u_1) = w^a$ and $P(u = u_2) = 1 - w^a$, where $u_1 > u_2$. Notice from (1) that strong economic fundamentals are associated with a high θ and a low u. Of course, as long as only one project is considered, it seems unnecessarily complicated to introduce two fundamentals. However, the role of the second fundamental u will become clear in the model with two firms in section 3, where I will assume that the state u equally affects both firms, while θ is related to a specific firm. That is, θ summarizes factors that affect only one firm and could for example represent the ex ante unknown quality of the hired chief manager, whereas u could be a proxy variable for the demand facing the products of both firms.

The parameter $z > 0$ in equation (1) captures the disruption caused by early loan withdrawals, which can be due to the firm having to sell illiquid long-term assets at a fire-sale loss in order to pay out the lenders. The higher z, the more harm is caused by withdrawing funds before the project matures. Further, assume that

$$u_2 > 0, \; u_1 + z < 1, \tag{2}$$

which implies that the firm specific fundamental θ is in some sense dominant, because a sufficiently high (low) state θ can always save (ruin) the project. Technically, assumption (2) guarantees the existence of a lower and an upper "dominance region" of θ for which agents have dominant actions. There is now a tripartite classification of the fundamental variables. If the state of fundamentals turns out to be very poor, i.e. if $\theta \leq u$, the project fails even under the best financing conditions where nobody withdraws funds at the intermediate state. Conversely, if $\theta > u + z$, the project succeeds irrespective

[6] The date of termination may well depend on whether the project succeeds. In many cases, including all kinds of runs, it seems plausible to assume that the project must be abandoned immediately after the intermediate period if it is a failure, while otherwise, it can be orderly terminated at its time of maturity.

of the actions of the creditors, whereas if $u < \theta \leq u + z$, the outcome of the project depends in a crucial way on the actions of the creditors.

Creditors have Von Neumann - Morgenstern preferences and are risk neutral. Hence, they maximize expected payoffs. If rolling over the loan, they receive the face value of debt normalized to 1 if the projects succeeds and nothing otherwise. If they withdraw early, they obtain some amount λ with certainty, where $0 < \lambda < 1$. One interpretation of this payoff structure is that lenders can liquidate a collateral which yields λ in the intermediate period but becomes worthless if the project fails. Alternatively, the difference $1 - \lambda$ may represent an interest rate which is paid to patient lenders, provided the project succeeds. The payoffs of the creditors can be summarized as follows:

	$\theta \leq u + lz$	$\theta > u + lz$
Roll over	0	1
Withdraw	λ	λ

It is important to see that if the states of θ and u were common knowledge, the model would result in multiple equilibria whenever $u \leq \theta < u + z$. In this case, if everyone rolls over the loan, no single agent has an incentive to deviate given the others' strategies, since the project succeeds anyway. On the other hand, if nobody rolls over, nobody has an incentive to deviate either, because the project certainly fails. Therefore, even if one neglects mixed strategies, there are at least two obvious Nash equilibria. However, these conclusions change considerably if we put the game in a "global game" framework with incomplete information.

Assume, therefore, that the distributions of the fundamental variables are common knowledge, but the true state of the fundamentals is not observable. Instead, before deciding whether to roll over the loan, each creditor i privately observes a noisy signal

$$s_i = \theta + \varepsilon_i \tag{3}$$

on the state θ, where ε_i is uniformly distributed on $[-\varepsilon, \varepsilon]$ and independent of θ and $\varepsilon_k \; \forall k \neq i$. Assume further that

$$2\varepsilon \leq \min[u_2, 1 - u_1 - z], \tag{4}$$

which means that the signal is at least of some minimal precision. One natural interpretation of this framework with noisy signals is that investors have access to different sources of information. Alternatively, they might slightly differ by their interpretation of publicly available information.

Finally, an important assumption of the model is that the creditors cannot coordinate their behavior, i.e. they cannot meet to share their private information - in which case they could learn θ by a law of large numbers - or to coordinate their action before deciding on their credit. This assumption is certainly plausible for depositors of a bank or other small and dispersed investors. Thus, the decision whether to withdraw funds or not is the result of a noncooperative game among the creditors. Moreover, since the type of each player, which is characterized by the signal s_i, is private knowledge, it is a game of incomplete information. In the next section, I show that this game has a unique Bayesian Nash equilibrium.

2.2 Unique Equilibrium

Creditors maximize expected payoffs. Hence, they roll over their loan if and only if the expected payoff from doing so exceeds the opportunity costs λ.[7] The only available information upon which this decision can be conditioned is the signal s_i, so a strategy of a creditor is a plan which maps each signal s_i into one of the two actions "withdraw" respectively "roll over". In an equilibrium profile of strategies, each creditor's strategy maximizes her expected payment given that the other creditors follow the strategies in the profile.

Before turning to the main results of this section, consider some preliminaries. Since the error term ε_i of the signal is drawn from a uniform distribution, the posterior belief $\theta \mid s_i$ of creditor i is also uniformly distributed, that is:

$$\theta \mid s_i \overset{u}{\sim} [s_i - \varepsilon, s_i + \varepsilon]. \tag{5}$$

More precisely, this is only true if $\varepsilon \leq s_i \leq 1-\varepsilon$, but given the assumption in (4), the optimal response to any signal below ε or above $1-\varepsilon$ is trivial anyway. To see this, note that any creditor who observes a signal $s_i < \underline{s} = u_2 - \varepsilon$ knows that $\theta < u_2$, which implies that the project fails even in the best case where $u = u_2$ and $l = 0$. Hence, the payoff from rolling over the loan is zero with certainty, and the lender will withdraw her loan. Conversely, any signal $s_i > \overline{s} = u_1 + z + \varepsilon$ implies $\theta > u_1 + z$, such that the project succeeds even in the worst case where $u = u_1$ and $l = 1$, and so the loan is certainly not withdrawn. To summarize, any rational creditor withdraws if observing a signal $s_i < \underline{s}$ and rolls over whenever $s_i > \overline{s}$. Hence, from here on, I will

[7] Where it is implicitly assumed that they terminate the loan if the expected payoff from both actions is exactly equal. This assumption is not crucial for any results.

restrict attention to signals in the interval $[\underline{s}, \overline{s}]$. Since the assumption in (4) guarantees that $\underline{s} \geq \varepsilon$ and $\overline{s} \leq 1 - \varepsilon$, the distribution given in (5) is correct for all relevant $s_i \in [\underline{s}, \overline{s}]$.

What is now the optimal strategy of a creditor who receives a signal between $\underline{s}$ and $\overline{s}$? Notice that the role of the signal is twofold. First, the signal contains (possibly very precise) information on the state θ. Second and perhaps more importantly, the signal is known to be correlated with the private signals of others, which allows an inference regarding their beliefs and actions. Unlike in a setup where θ is common knowledge, it is no longer the case that *any* beliefs about the beliefs of others are equally permissible. The signal structure, therefore, serves as a device to coordinate the (higher order) beliefs of agents. It is exactly this role of the signal, together with the existence of dominance regions ensured by assumption (2) and the strategic complementarity proved in appendix 1, which leads to a unique equilibrium of the game. Intuitively, observing a high signal is good news for each creditor, because it means that the state θ is good. Since observing a high signal also implies that the others have observed a high signal (and that all believe that all have observed a high signal, and so on), it makes each creditor believe that the other lenders are more inclined to roll over their loans. This is again good news and increases the incentives to roll over the loan even further. Hence, one might conjecture that there is some threshold s of the signal above which the loan is rolled over. In the remainder of this section, I will show that this guess turns out to be exactly true and that the following result holds:

Proposition 1 *The game among creditors has a unique equilibrium in which there is a threshold $\theta^*(u)$ of θ, depending on the state u of the other fundamental, such that the firm succeeds if and only if $\theta > \theta^*(u)$.*

The proposition can be proved in two main steps which are stated in lemma 1 and 2. To begin with, suppose that all lenders follow a switching or monotone strategy around some threshold parameter s. Thus, depending on the observed signal s_i, each creditor i takes either action 0 (withdraw) or 1 (roll over). The switching strategy around s can be formalized as:

$$I_s(s_i) = \begin{cases} 0 \text{ if } s_i \leq s \\ 1 \text{ if } s_i > s \end{cases}.$$

Further, denote by $Q(s, I_s)$ the expected payoff from rolling over the loan of a creditor who observes the signal $s_i = s$ and believes that all other creditors follow strategy I_s. We can then derive the following result:

Lemma 1 *If all creditors follow a switching strategy around some threshold s, there is a unique value of s which solves $Q(s, I_s) = \lambda$.*

In order to prove lemma 1, it is sufficient to establish that $Q(s, I_s)$ raises from 0 to 1 and is strictly increasing in s over the whole range $[\underline{s}, \overline{s}]$. This is what I now show. If every lender applies the switching strategy I_s, the proportion of creditors terminating the loan in the intermediate period conditional on s and θ is

$$l(s,\theta) = P(s_i \leq s \mid \theta) = \int_{\min[\theta-\varepsilon,s]}^{\min[s,\theta+\varepsilon,]} (2\varepsilon)^{-1} ds_i$$

respectively

$$l(s,\theta) = \begin{cases} 0 & \text{if } s < \theta - \varepsilon \\ \frac{1}{2\varepsilon}[s - (\theta-\varepsilon)] & \text{if } \theta - \varepsilon \leq s \leq \theta + \varepsilon \\ 1 & \text{if } s > \theta + \varepsilon \end{cases} \tag{6}$$

For both realizations $u \in \{u_1, u_2\}$, there is a unique critical value of θ, solving

$$\theta = u + zl(s,\theta),$$

above which the project succeeds. Given equation (6) and conditional on $u \in \{u_1, u_2\}$, this threshold of θ solves

$$\theta = \begin{cases} u & \text{if } s < \theta - \varepsilon \\ u + \frac{z}{2\varepsilon}[s - (\theta-\varepsilon)] & \text{if } \theta - \varepsilon \leq s \leq \theta + \varepsilon \\ u + z & \text{if } s > \theta + \varepsilon. \end{cases} \tag{7}$$

Let $\theta^{crit}(s,u)$ denote this critical threshold of θ, conditional on s and on $u \in \{u_1, u_2\}$. Hence, solving (7) for θ, $\theta^{crit}(s,u)$ is determined as

$$\theta^{crit}(s,u) = \begin{cases} u & \text{if } s < u - \varepsilon \\ \frac{2\varepsilon u + z(s+\varepsilon)}{2\varepsilon + z} & \text{if } u - \varepsilon \leq s \leq u + z + \varepsilon \\ u + z & \text{if } s > u + z + \varepsilon \end{cases} \tag{8}$$

Of course, if the state u were known, no s below $u - \varepsilon$ or above $u + z + \varepsilon$ could ever be an optimal switching threshold, but the creditors must find their optimal strategy without knowing u. Conditional on $u \in \{u_1, u_2\}$, and given the posterior distribution of θ defined in (5), the expected payoff $\widetilde{Q}(s, I_s, u)$ from rolling over the loan is

$$\widetilde{Q}(s, I_s, u) = P(\theta \geq \theta^{crit}(s, u) \mid s_i = s) = \int_{\max[s-\varepsilon, \theta^{crit}(s,u)]}^{\max[s+\varepsilon, \theta^{crit}(s,u)]} (2\varepsilon)^{-1} d\theta.$$

From equation (8), $Q(s, I_s, u)$ can then be computed as

$$\widetilde{Q}(s, I_s, u) = \begin{cases} 0 & \text{if } s < u - \varepsilon \\ \frac{1}{2\varepsilon + z}[s + \varepsilon - u] & \text{if } u - \varepsilon \leq s \leq u + z + \varepsilon \\ 1 & \text{if } s > u + z + \varepsilon \end{cases} \tag{9}$$

Given $\widetilde{Q}(s, I_s, u)$ for both states $u \in \{u_1, u_2\}$, it is straightforward to compute $Q(s, I_s)$. To keep the notation general enough for later purposes, I denote in the following by w the probability that u is u_1, conditional on all available information. Of course, in the model with one firm, w is simply the unconditional probability w^a, but in section 3, w will depend on some additional information of the creditors. Since the best guess of $P(u = u_1)$ is given by w, the unconditional expected payoff $Q(s, I_s)$ of the marginal creditor is then

$$Q(s, I_s) = w\widetilde{Q}(s, I_s, u_1) + (1 - w)\widetilde{Q}(s, I_s, u_2).$$

Together with equation (9), this gives us an expression for $Q(s, I_s)$ in terms of the model parameters. Notice that $Q(s, I_s) = 0$ for all $s < \underline{s} = u_2 - \varepsilon$ and that $Q(s, I_s) = 1$ for all $s > \overline{s} = u_1 + z + \varepsilon$. Furthermore, as long as $s < u_1 - \varepsilon$, $\widetilde{Q}(s, I_s, u_1)$ is zero, and for all $s > u_2 + z + \varepsilon$, $\widetilde{Q}(s, I_s, u_2)$ is one. Therefore, assuming $u_1 - \varepsilon \leq u_2 + z + \varepsilon$,[8] the range $[\underline{s}, \overline{s}]$ can be split into three parts, and the expected payoff from rolling over the loan is:

$$Q(s, I_s) = \begin{cases} \frac{(1-w)[s+\varepsilon-u_2]}{2\varepsilon+z} & \text{if } \underline{s} \leq s < u_1 - \varepsilon \\ \frac{s+\varepsilon-[wu_1+(1-w)u_2]}{2\varepsilon+z} & \text{if } u_1 - \varepsilon \leq s \leq u_2 + z + \varepsilon \\ \frac{w[s+\varepsilon-u_1]}{2\varepsilon+z} + (1-w) & \text{if } u_2 + z + \varepsilon \leq s \leq \overline{s}. \end{cases} \tag{10}$$

Figure (1) plots the graph of $Q(s, I_s)$, which is characterized by three linear intercepts with different slopes, for an arbitrary parameter setting. Although

[8] This assumption ensures that the intermediate range is at least of measure zero. Otherwise, if $u_1 - \varepsilon > u_2 + z + \varepsilon$, there would be a range in which $Q(s, I_s)$ is $1 - w$ and we would have to assume λ does not take the particular value $1 - w$ to guarantee the uniqueness of the equilibrium.

the slope changes twice, $Q(s, I_s)$ is continuous in s over the whole range $[\underline{s}, \overline{s}]$ and the derivative with respect to s is

$$\frac{\partial Q(s, I_s)}{\partial s} = \begin{cases} \frac{1-w}{2\varepsilon+z} & \text{if } \underline{s} < s < u_1 - \varepsilon \\ \frac{1}{2\varepsilon+z} & \text{if } u_1 - \varepsilon < s < u_2 + z + \varepsilon \\ \frac{w}{2\varepsilon+z} & \text{if } u_2 + z + \varepsilon < s < \overline{s} \end{cases}$$

Clearly, this derivative is strictly positive.[9] Thus $Q(s, I_s)$ runs from 0 to 1 and is monotonically increasing in s $\forall$ $s \in [\underline{s}, \overline{s}]$, which implies that for each λ, there is exactly one value of s which solves $Q(s, I_s) = \lambda$, as is illustrated in figure (1). This proves lemma 1.

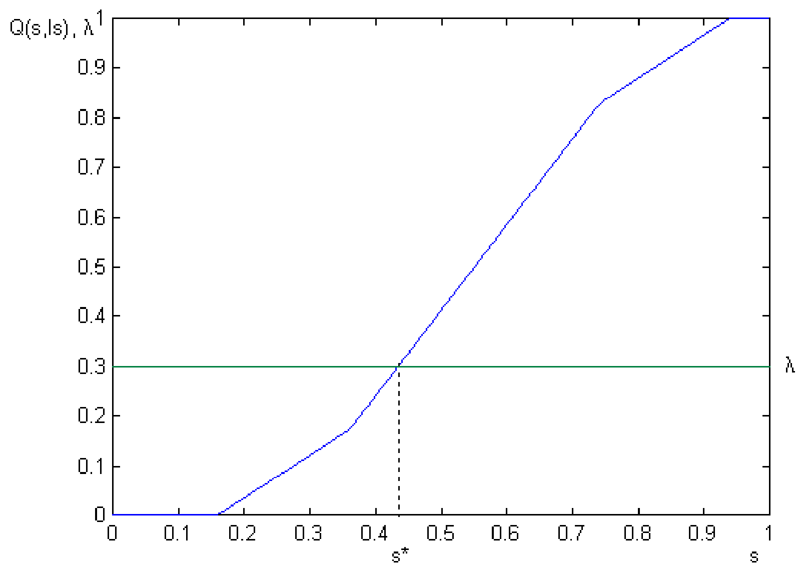

Figure 1: *Threshold* s^* *for* $(u_1, u_2, z, \varepsilon, \lambda, w^a)$=*(0.4,0.2,0.5,0.05,0.3,0.5)*

So far, I have restricted attention to pure switching strategies in which creditors roll over their loan if and only if the signal exceeds some threshold, though in principal they might also follow mixed strategies, presumably giving more weight to the action "roll over" if the signal s_i increases.[10] However,

[9]The fact that the derivative is not defined if s is either $\underline{s}$, $u_1 - \varepsilon$, $u_2 + z + \varepsilon$ or $\overline{s}$ does not change the conclusion that $Q(s, I_s)$ is strictly increasing in s in the range $[\underline{s}, \overline{s}]$.

[10]Some authors restrict attention to such switching strategies from the beginning. See for instance Dasgupta (2001) and Rochet and Vives (2000).

the following lemma states that the unique equilibrium strategy of the game is indeed a pure switching strategy around the threshold s^*.

Lemma 2 *If there is a unique value s^* of s solving $Q(s, I_s) = \lambda$, there is a unique equilibrium of the game in which every creditor rolls over the loan if and only if the signal exceeds the threshold s^*.*

There are several ways to prove this threshold structure of the unique equilibrium, which is now familiar in the global games literature. The proof presented in appendix A follows Morris and Shin (1998). The crucial step of this (or any other) proof is to show that the decisions to roll over the loan are strategic complements, which, together with lemma 1, implies the result of lemma 2. Hence, lemma 1 and 2 together establish that there is a unique switching threshold s^* and that the unique equilibrium strategy is $I_{s^*}(s_i)$. This in turn implies, together with equation (8), that there is a unique equilibrium threshold $\theta^*(u) \equiv \theta^{crit}(s^*, u)$ for each state $u \in \{u_1, u_2\}$ such that the project succeeds if and only if $\theta > \theta^*(u)$. This proves proposition 1. Since the discrete threshold function $\theta^*(u)$ takes only two different values, I will also use the notation $\theta^*(u_1) = \theta_1^*$ and $\theta^*(u_2) = \theta_2^*$. Note that while the threshold $\theta^*(u)$ depends on u, the strategy parameter s^* does not, for u is unknown at the time when the decision whether to roll over the loan is taken.

2.3 Equilibrium Thresholds

Given the unique equilibrium profile of strategies, we can solve the model for the equilibrium thresholds s^*, θ_1^* and θ_2^*. First, $Q(s^*, I_{s^*}) = \lambda$ can be solved to obtain the equilibrium switching threshold s^*. Notice that in order to solve $Q(s^*, I_{s^*}) = \lambda$ for s^*, either the first, second or third line of equation (10) has to be used, depending on the particular value of λ. In a second step, s^* and equation (8) determine the two equilibrium thresholds θ_1^* and θ_2^*, where one has to bear in mind that θ_1^* is simply u_1 as soon as $s^* < u_1 - \varepsilon$ and that $\theta_2^* = u_2 + z$ if $s^* > u_2 + z + \varepsilon$. Summarizing the results, the model yields the following equilibrium thresholds

$$s^* = \begin{cases} u_2 + \lambda \frac{2\varepsilon + z}{1-w} - \varepsilon & \text{if } 0 < \lambda < \underline{\lambda} \\ wu_1 + (1-w)u_2 + \lambda(2\varepsilon + z) - \varepsilon & \text{if } \underline{\lambda} \leq \lambda \leq \overline{\lambda} \\ u_1 + \frac{(\lambda + w - 1)(2\varepsilon + z)}{w} - \varepsilon & \text{if } \overline{\lambda} < \lambda < 1 \end{cases}$$

$$\theta_1^* = \begin{cases} u_1 & \text{if } 0 < \lambda < \underline{\lambda} \\ \frac{2\varepsilon u_1 + z u_2 + z w(u_1 - u_2)}{2\varepsilon + z} + \lambda z & \text{if } \underline{\lambda} \leq \lambda \leq \overline{\lambda} \\ u_1 + z\left(1 - \frac{1-\lambda}{w}\right) & \text{if } \overline{\lambda} < \lambda < 1 \end{cases} \tag{11}$$

$$\theta_2^* = \begin{cases} u_2 + \frac{\lambda z}{1-w} & \text{if } 0 < \lambda < \underline{\lambda} \\ u_2 + \frac{z w(u_1 - u_2)}{2\varepsilon + z} + \lambda z & \text{if } \underline{\lambda} \leq \lambda \leq \overline{\lambda} \\ u_2 + z & \text{if } \overline{\lambda} < \lambda < 1 \end{cases},$$

where the different ranges of λ are separated by

$$\underline{\lambda} = \frac{(1-w)\left[u_1 - u_2\right]}{2\varepsilon + z} \text{ and } \overline{\lambda} = 1 - \frac{w(u_1 - u_2)}{2\varepsilon + z}. \tag{12}$$

It is important to stress that although the fundamentals uniquely determine whether the project succeeds, the outcome is still driven by the expectations of the agents. In many cases, creditors withdraw their loans only because they believe that others are going to do so, and since the other creditors share the same fear, a collapse of the firm can be the result of self-fulfilling beliefs. If for example $u_1 + 2\varepsilon < \theta < \theta^*(u)$, which may well be the case if λ or z is large enough, the firm fails although everyone knows that this is not justified by the fundamentals.[11] Only for extreme values of θ that lead to signals below $\underline{s}$ or above $\overline{s}$ for which agents have dominant actions, there is no need to care about other players' beliefs. Therefore, the point I want to make is that the crucial role of the fundamental is not to exclude self-fulfilling failures. Rather, given the assumed information structure, it uniquely determines the beliefs of the agents and thereby their behavior.

Another lesson of the preceding analysis is that the outcome of the model is not efficient, for if the creditors could coordinate their actions, they would agree to roll over their loans for more states of the economic fundamentals, thereby lowering the thresholds and the likelihood of a failure. In the first best outcome, everyone would roll over her loan if and only if $\theta > u$, which would require both coordination and the knowledge of u. As a weaker conclusion, even if u is unknown, there is no doubt that any fundamental threshold above u_1 is inefficient.

[11] At the risk of raising confusion, one may even emphasize the subtle but essential role of higher order beliefs. A player may be reluctant to roll over even if she knows that $\theta > u_1$ and that everyone knows that $\theta > u_1$, for she may not know that everyone knows that everyone knows that $\theta > u_1$, which is in fact never common knowledge in this game. See Morris and Shin (1998), (2001) for more on higher order beliefs in global games.

In the remainder of this section, I discuss some properties of the thresholds θ_1^* and θ_2^*, which in turn determine the likelihood of failure of the project. A first important result which follows easily from the thresholds in (11) and from $u_1 > u_2$ is that

$$\theta_1^* > \theta_2^*, \tag{13}$$

which simply means that the threshold above which the projects succeeds is higher in the worse case $u = u_1$. Second, differentiating the thresholds with respect to the model parameters z and λ yields

$$\frac{\partial \theta^*(u)}{\partial z} \geq 0 \text{ and } \frac{\partial \theta^*(u)}{\partial \lambda} \geq 0. \tag{14}$$

In economic terms, an increasing disruption z caused by early loan terminations raises ceteris paribus the critical threshold above which the firm survives. Also consistent with intuition, a growing λ, implying higher opportunity costs of lending, increases the thresholds, since it makes the lenders more inclined to withdraw their funds before the project matures, which in term lowers the likelihood of success. The dispersion ε of the signal s_i, on the other hand, has no clear impact on the thresholds. More precisely,

$$\frac{\partial \theta_1^*}{\partial \varepsilon} \geq 0 \text{ and } \frac{\partial \theta_2^*}{\partial \varepsilon} \leq 0. \tag{15}$$

Thus, an increasing ε drives the two thresholds apart, if it has any influence at all. In order to analyze how u_1 and u_2 affect the thresholds, it seems useful to define the numbers $\overline{u}$ and k such that $u_1 = \overline{u} + k$ and $u_2 = \overline{u} - k$. Differentiating the thresholds yields then

$$\frac{\partial \theta^*(u)}{\partial \overline{u}} = 1, \tag{16}$$

whereas the sign of the derivative with respect to k is ambiguous. There is again an intuitive interpretation of the result in (16). Raising the average $\overline{u}$ of u_1 and u_2 corresponds to an adverse shift in the distribution of u, which is equivalent to a downward shift of the distribution of the firm specific variable θ. Due to the assumed uniform distribution of θ, such a shift raises the thresholds by the same amount.

Finally, it is important to see that both thresholds $\theta_1^*$and θ_2^* satisfy

$$\frac{\partial \theta^*(u)}{\partial w} \geq 0. \tag{17}$$

Note that although $\theta_1^* = u_1$ or $\theta_2^* = u_2 + z$ for some values of λ, at least one of the two thresholds θ_1^* and θ_2^* is always strictly increasing in w. In other words, if the creditors attach a higher probability to the undesirable state $u = u_1$, the project is more likely to fail.

With these results in mind, we are now ready to incorporate a second firm into the model and to explore the main implications of this extension.

3 The Model with 2 Firms

3.1 The Time Structure of the Model

In this part of the paper, I extend the model to include a second firm. Thus, suppose there are two basically identical firms A and B both of which run a project financed by two separate continua of creditors. Assume further that the state u is the same for both firms, while there are two independent firm specific states θ_A and θ_B. The latter can be interpreted either as two independent draws from one uniformly distributed fundamental variable or as the joint realization of two independent uniform random variables. The return of each firm is still given by equation (1), except that for firm A (firm B), θ is replaced by θ_A (θ_B).

In addition, I assume the following time structure, which is summarized in figure (2). In a first stage, both firms start their projects, and nature chooses the states u, θ_A and θ_B according to the underlying probability distributions. In a second stage, each creditor of firm A receives a signal on θ_A as specified in (3) and individually decides whether to withdraw her loan. Subsequently, the project of firm A matures and everyone, including the creditors of firm B, observes whether firm A fails or not. However, neither u nor θ_A is observed by anyone at this time. The agents may learn these realizations after the termination of project B or even never, where the latter seems more plausible. Having observed the outcome of A, the creditors of firm B receive a signal on θ_B and decide whether to withdraw their loans.[12] Finally, the project of firm B is completed.

Since the creditors of A have no additional information beyond the signal s_i when deciding on their loan, they behave like the creditors in the model

[12]They may also observe the signal earlier, e.g. together with the creditors of firm A. The crucial point is that the creditors of firm B can wait longer until they have to decide whether they roll over their loans.

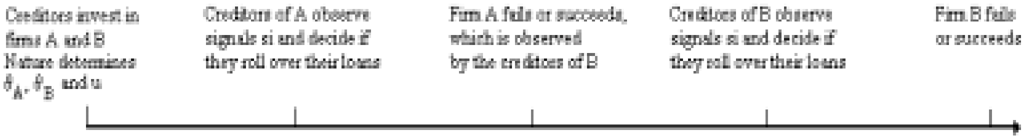

Figure 2: *Timing of Events*

with one firm, and the equilibrium results of the previous sections go through unchanged. The creditors of the second firm, however, can observe the outcome of the first firm. Does this observation provide useful information which affects the behavior of the creditors? Clearly, the answer is yes, as is shown in the next subsection.

3.2 Updating the Beliefs

To see what information can be gained from observing the outcome of A, consider first what happens to firm A. Since the first firm is equivalent to the single firm in the model of section 2, except that the fundamental θ is replaced by θ_A, we know that there is a unique equilibrium in which the creditors of A roll over their loans if and only if their signal exceeds some critical level s_A^*, so that firm A succeeds if and only if θ_A exceeds some threshold $\theta_A^*(u)$. The thresholds of the signal and of θ_A are still given by the terms in (11), where w is the unconditional probability $w = P(u = u_1) = w^a$. In the following, I define by $\theta_{A,1}^*$ and $\theta_{A,2}^*$ the two thresholds θ_1^* and θ_2^* computed with $w = w^a$, which can be summarized in the discrete threshold function $\theta_A^*(u)$ given by $\theta^*(u)$ with $w = w^a$. Further, let F_A denote the event where firm A fails and $S_A \equiv \overline{F_A}$ be the complementary event where A survives. In equilibrium, firm A fails whenever $\theta_A \leq \theta_A^*(u)$. Thus, the formal definitions of the events "failure of A" and "success of A" are:

$$\begin{aligned} F_A &= \{\{\theta_A, u\} \mid \theta_A \leq \theta_A^*(u)\} \\ S_A &= \{\{\theta_A, u\} \mid \theta_A > \theta_A^*(u)\}. \end{aligned}$$

Since θ is uniformly distributed over the unit interval, the probability that the realization of θ does not exceed a specific threshold is equivalent to the threshold itself, so it is straightforward to compute the following conditional probabilities:

$$\begin{aligned} P(F_A \mid u = u_1) &= \theta^*_{A,1} \\ P(F_A \mid u = u_2) &= \theta^*_{A,2} \\ P(S_A \mid u = u_1) &= 1 - \theta^*_{A,1} \\ P(S_A \mid u = u_2) &= 1 - \theta^*_{A,2} \ . \end{aligned} \tag{18}$$

In a next step, let $w^f \equiv P(u = u_1 \mid F_A)$ denote the probability that u is u_1, conditional on the failure of A, while $w^s \equiv P(u = u_1 \mid S_A)$ shall denote the corresponding probability conditional on a success of A. It can then be shown that $w^s < w^a < w^f$, which is an important result for what follows later. To prove this claim, notice that by Bayes rule, the probability of u being u_1 conditional on F_A can be computed as:

$$w^f = \frac{P(F_A \mid u = u_1)P(u = u_1)}{P(F_A \mid u = u_1)P(u = u_1) + P(F_A \mid u = u_2)P(u = u_2)}.$$

Together with $P(u = u_1) = w^a$ and $P(u = u_2) = 1 - w^a$, equation (18) provides all probabilities that are needed to compute w^f. Proceeding the same way, we can also compute the probability w^s. The results in terms of the fundamental thresholds are:

$$w^f = \frac{w^a\theta^*_{A,1}}{w^a\theta^*_{A,1} + (1 - w^a)\theta^*_{A,2}} \tag{19}$$

$$w^s = \frac{w^a(1 - \theta^*_{A,1})}{1 - w^a\theta^*_{A,1} - (1 - w^a)\theta^*_{A,2}}. \tag{20}$$

Finally, notice from inequality (13) in section 2.3 that $\theta^*_{A,1} > \theta^*_{A,2}$, which implies

$$\theta^*_{A,1} > w^a\theta^*_{A,1} + (1 - w^a)\theta^*_{A,2}$$

and, after multiplying both sides by $\frac{w^a}{w^a\theta^*_{A,1} + (1-w^a)\theta^*_{A,2}}$,

$$w^f = \frac{w^a\theta^*_{A,1}}{w^a\theta^*_{A,1} + (1 - w^a)\theta^*_{A,2}} > w^a. \tag{21}$$

A symmetric argument establishes that

$$w^s = \frac{w^a(1 - \theta^*_{A,1})}{1 - w^a\theta^*_{A,1} - (1 - w^a)\theta^*_{A,2}} < w^a. \tag{22}$$

Together, (21) and (22) prove that $w^s < w^a < w^f$ as claimed. Hence, it is rational (and quite intuitive) that the creditors of firm B consider the worse state $u = u_1$ to be more likely when they observe that A fails. Though such failure can be due to a low θ_A and not due to a high realization of u, it provides additional information (i.e. bad news) on the true state of u. On the other hand, conditional on a success of project A, the event $u = u_1$ is less likely than unconditionally.

3.3 Failure Probabilities and Contagion

What is now going to happen to the second firm? From the discussion in the previous section, we know that the only difference between the creditors of the two firms lies in their different assessment of the state u. The outcome of the first project serves as a signal which leads to an updated information on the probability distribution of u. Since this signal is commonly observed, all creditors of B share the same updated beliefs over u, and the only consequence for their decision making is that the probability w is either w^s or w^f. Therefore, the unique equilibrium of the game among the creditors of B is still obtained as in section 2.2, with the important difference that the equilibrium depends on the outcome of firm A. If the first firm fails, the two fundamental thresholds of θ_B can be calculated as in (11) with $w = w^f$, whereas if A succeeds, the thresholds of θ_B are computed with $w = w^s$.

In the following, let $\theta^f_{B,j}$ (respectively $\theta^s_{B,j}$) denote the thresholds of θ_B if A fails (respectively succeeds), where $j \in \{1, 2\}$ captures the state of u. The thresholds of θ_B can also be written as a function $\theta^*_B(u, \theta_A)$, where $\theta^*_B(u, \theta_A)$ is $\theta^*(u)$ computed with $w = w^f$ if $\theta_A \leq \theta^*_A(u)$ and $\theta^*_B(u, \theta_A) = \theta^*(u)$ with $w = w^s$ if $\theta_A > \theta^*_A(u)$. Further, define by F_B and $S_B \equiv \overline{F_B}$ the events where firm B fails respectively survives. Failure of B occurs whenever $\theta_B \leq \theta^*_B(u, \theta_A)$, i.e. if A fails and $\theta_B \leq \theta^f_{B,j}$ or if A succeeds and $\theta_B \leq \theta^s_{B,j}$ for $j \in \{1, 2\}$. Formally, the two mutually exclusive and exhaustive sets F_B and S_B are defined as:

$$
\begin{aligned}
F_B &= \{\{\theta_A, \theta_B, u\} \mid \theta_B \leq \theta^*_B(u, \theta_A)\} \\
S_B &= \{\{\theta_A, \theta_B, u\} \mid \theta_B > \theta^*_B(u, \theta_A)\}
\end{aligned}
$$

The important point to see is that the threshold of θ_B depends not only on u, but also on θ_A, although θ_A is independent of any economic fundamental which affects firm B.

Since θ_B is uniformly distributed over the unit interval, the thresholds of θ_B represent at the same time conditional failure probabilities, which are summarized in the following equation:[13]

$$\begin{aligned} P\left(F_B \mid (u = u_1) \cap F_A\right) &= \theta^f_{B,1} \\ P\left(F_B \mid (u = u_1) \cap S_A\right) &= \theta^s_{B,1} \\ P\left(F_B \mid (u = u_2) \cap F_A\right) &= \theta^f_{B,2} \\ P\left(F_B \mid (u = u_2) \cap S_A\right) &= \theta^s_{B,2}. \end{aligned} \tag{23}$$

Furthermore, the failure probability of B conditional on failure of A can be written as

$$\begin{aligned} P(F_B \mid F_A) = \; & P\left(F_B \mid (u = u_1) \cap F_A\right) P\left(u = u_1 \mid F_A\right) \\ & + P\left(F_B \mid (u = u_2) \cap F_A\right) P\left(u = u_2 \mid F_A\right), \end{aligned}$$

which, after plugging in the results of (19) and (23), yields the probability

$$P(F_B \mid F_A) = \frac{w^a \theta^*_{A,1} \theta^f_{B,1} + (1 - w^a)\theta^*_{A,2}\theta^f_{B,2}}{w^a \theta^*_{A,1} + (1 - w^a)\theta^*_{A,2}}. \tag{24}$$

Accordingly, if the creditors of the second firm observe that A is successful, firm B fails with probability

$$P(F_B \mid S_A) = \frac{w^a (1 - \theta^*_{A,1})\theta^s_{B,1} + (1 - w^a)(1 - \theta^*_{A,2})\theta^s_{B,2}}{1 - w^a \theta^*_{A,1} - (1 - w^a)\theta^*_{A,2}}. \tag{25}$$

The probabilities $P(S_B \mid F_A)$ and $P(S_B \mid S_A)$ of a success of firm B, conditional on either the failure or survival of A, are calculated analogously, or simply as $1 - P(F_B \mid F_A)$ and $1 - P(F_B \mid S_A)$.

Finally, we can also calculate the *unconditional* failure probability of each firm. First, note that the likelihood of failure of A can be written as

$$P(F_A) = P(F_A \mid u = u_1)P(u = u_1) + P(F_A \mid u = u_2)P(u = u_2)$$

Therefore, plugging in $P(u = u_1) = w^a$, $P(u = u_2) = 1 - w^a$ and the relevant probabilities from (18), firm A fails with probability

$$P(F_A) = w^a \theta^*_{A,1} + (1 - w^a)\theta^*_{A,2} \tag{26}$$

[13] Naturally, the corresponding survival probabilities of firm B are easily found by substracting the relevant threshold from one.

and survives with probability $P(S_A) = 1 - P(F_A)$. The unconditional probability of failure of the second firm can then be computed as

$$P(F_B) = P(F_B \mid F_A)P(F_A) + P(F_B \mid S_A)P(S_A).$$

which, using equations (26), (24) and (25), results in

$$\begin{aligned} P(F_B) &= w^a \left[\theta^*_{A,1}\theta^f_{B,1} + (1 - \theta^*_{A,1})\theta^s_{B,1}\right] \\ &\quad + (1 - w^a) \left[\theta^*_{A,2}\theta^f_{B,2} + (1 - \theta^*_{A,2})\theta^s_{B,2}\right]. \end{aligned} \tag{27}$$

The unconditional probability that B survives, of course, is then given by $P(S_B) = 1 - P(F_B)$.

To complete the discussion of this section, it is worthwhile having a closer look at the thresholds of firm B conditional on the outcome of the first firm. From the crucial result (17) in section 2.3, we know that the threshold $\theta^*(u)$ is increasing in w for at least one state of u. Together with $w^f > w^s$, this implies that

$$\theta^s_{B,1} \leq \theta^f_{B,1} \tag{28}$$

and

$$\theta^s_{B,2} \leq \theta^f_{B,2}, \tag{29}$$

where at least one of the inequalities (28) and (29) holds strictly. This in turn proves that the failure of A affects the behavior of the creditors of B in a way which increases the thresholds and thereby the likelihood of failure of B. By the same mechanism, however, a success of A has a positive impact on B. The following proposition summarizes these important findings.

Proposition 2 *(Contagion) If the creditors of firm B observe the outcome of firm A before deciding on their loan, there exists (for at least one state u) a range of θ_B for which B fails if A fails and succeeds if A survives. Hence, a failure of A increases the likelihood of failure of firm B, whereas a success of firm A makes a failure of firm B less likely.*

Proposition 2 is a crucial result, establishing that there is a contagious link between the two firms such that the probability that B fails is higher if A fails than conditional on a success of A. While the first part of the proposition follows directly from the fact that at least one of the inequalities (28) and (29) holds strictly, the second statement which concerns the likelihood of failure may require some additional explanations, for which I refer to the next subsection.

3.4 Contagion and Correlation

According to proposition 2, the failure of A increases the probability that B fails. However, this is a potentially misleading statement which deserves closer attention. Of course, because of their joint dependence on the common state u, the outcomes of the two firms are not uncorrelated even without contagion, but the point I want to make is that contagion leads to an additional increase of this "natural" correlation. To see this, let me sketch the following (and hopefully not confusing) reasoning. Define by $\widetilde{P}(F_B \mid F_A)$ the probability that B fails conditional on the failure of A, but under the assumption that there is no contagion. The latter assumption implies that the thresholds of B are left unchanged and correspond to those of A. Accordingly, let $\widetilde{P}(F_B \mid S_A)$ be the likelihood that B fails provided that A succeeds and that there is no contagion. In other words, we might think of the probabilities from the perspective of an outsider who can observe the outcome of A, while the creditors of B cannot. Given these definitions, I show in appendix $B.1$ that the following chain of inequalities holds:

$$P(F_B \mid F_A) > \widetilde{P}(F_B \mid F_A) > \widetilde{P}(F_B \mid S_A) > P(F_B \mid S_A). \qquad (30)$$

Thus, compared to the unconditional likelihood of failure, the conditional probability that B fails is higher if A fails than if A succeeds even without contagion. But if there is contagion, inequality (30) shows that the conditional failure probability is further increased if A fails and lowered if A succeeds. The difference between $P(F_B \mid F_A)$ and $\widetilde{P}(F_B \mid F_A)$ or between $\widetilde{P}(F_B \mid S_A)$ and $P(F_B \mid S_A)$ captures the effect on the failure probability which stems from the change in the behavior of investors.

The first part of the correlation which is caused by common (macroeconomic) fundamentals should not be mixed up with contagion. Taking the banking industry as an example, it may happen that several banks crash simultaneously because they jointly face extremely bad economic conditions, which need not mean that they have contagiously affected each other. In my view, a reasonable understanding of financial contagion can only include forces which originate in financial markets (e.g. changes in the behavior of investors) and which remain effective even if one controls for any common third influencing factors. In the present model, contagion implies that even conditional on the state u, the likelihood that B fails is higher (lower) if A fails (succeeds). In this sense, it is correct to conclude that by altering the behavior of creditors, a collapse of A can *cause* the failure of B.

3.5 The Model with N firms

So far, there are only two firms in the model. Before turning to a rigorous discussion of the consequences of contagion in section 4, I would like to mention that in principle, the model is easily generalized to include $N \geq 2$ firms. In a first step, we can introduce a third firm C which is equivalent to A or B, except that its investors can observe the outcome of both A and B before having to decide on their loans. Proceeding analogously, we can include arbitrarily many firms. The imposed time structure implies that every project has a contagious impact on all projects which follow later in time but on none which matures earlier. This framework raises the possibility that the failure of one or a few firms can trigger a chain of failures and may even ruin all other firms, which is however an extremely unlikely event. Moreover, this is again only one side of contagion, because a single successful firm might as well rescue the other firms. Since the basic mechanisms and insights of the model with two firms remain unchanged, whereas the Bayesian updating process becomes already involved with three firms, a thorough discussion of this generalization is beyond the scope of this paper.

4 Contagion Revisited

4.1 Positive versus Negative Contagion

A major result of the model with two (or more) firms is that there exists financial contagion, such that the failure of firm A can trigger the collapse of firm B. Thus, the outcome of project B depends on θ_A even though the firm specific fundamental variables θ_A and θ_B are independent. There is also no direct connection in the sense that B is a creditor, a supplier or a customer of A. The contagious link between the two firms stems from a purely informational mechanism, which arises because the creditors of firm B can observe whether A survives or not. Based on this signal, they (reasonably) adjust their beliefs on the state of the fundamental u, which in turn affects the thresholds of B in such a way that project B is more likely to share the same fate as project A. In this section, I discuss the basic mechanism and the consequences of contagion in some more detail. The following discussion is meant to provide a better understanding of contagion and on the conclusions which can be drawn, since these may prove less obvious than suggested in the existing literature.

Given the result in proposition 2, one might take the difference between the thresholds $\theta^f_{B,j}$ and $\theta^s_{B.j}$ or the probabilities $P(F_B \mid F_A)$ and $P(F_B \mid S_A)$ as a measure to quantify the negative impact of contagion and jump to the conclusion that this would be the gain from eliminating contagion. However, $\theta^s_{B,j}$ and $P(F_B \mid S_A)$ are not necessarily the appropriate benchmarks with which $\theta^f_{B,j}$ and $P(F_B \mid F_A)$ should be compared when discussing the impact of contagion. Rather, the outcome of the model should be compared to a scenario in which there is no financial contagion. One approach to get rid of the contagious link is to suppose that the creditors of B cannot observe the outcome of project A before they have to take action. Equivalently, we could assume that two different states u_a and u_b are relevant in the return function (1) of firm A and B, where u_a and u_b are independent. A third benchmark scenario is simply to assume that firm B is the only present firm. Each of these three scenarios provides a reasonable benchmark case in which the relevant thresholds of firm B are the same as those of A.

Taking this hypothetical scenario of an "autark" firm as a starting point, we can apply the following definition of contagion: Contagion occurs whenever the outcome of project B deviates from the outcome which would prevail in the benchmark case. If the second firm fails only because of the adverse signal sent out by the collapse of A, the owner and the lenders of firm B are apparently worse off because of A. Therefore, I define this event as "negative contagion". On the other hand, there is also an event where firm B is successful but would not be so in the benchmark case. At the risk of depriving the literature on contagion of its only agreement, I define this event as "-positive contagion". Since contagion is typically perceived as an undesirable phenomenon, these definitions may prove to be controversial. Those readers who insist on contagion being a negative event per definition may regard the term "negative contagion" as a pleonasm, while others may find the notion of "positive contagion" hard to accept. Of course, this is a matter of definition, but it cannot be overemphasized that whatever name we choose for the two phenomena, they occur by the same mechanism.

This does by no means contradict the traditional argument of the informational channel that the failure of one firm reveals adverse information on other firms. To be sure, there is nothing wrong with this explanation. However, if we accept this argument, then the success of the very same firm must also reveal some information, which must lead to an adjustment of the players' priors in the opposite direction. To see this, consider the following counter-example. Suppose the investors are rational and have some common prior beliefs on an economic fundamental (here the variable u), and that they adjust their priors in some direction conditional on bankruptcy of A. If they adjust their priors in the same direction if A survives, the priors are systematically adjusted in the same direction for all possible states of the world, that is with probability one. Clearly, this is a contradiction, for no rational player could have formed such prior beliefs. Hence, the model cannot produce the result of negative contagion without creating the possibility of positive contagion. This should also make clear that the traditional discussion, which takes only one of two directions of the propagation channel into account, leads at best to partial insights on informational contagion.

Formally, the events C_n and C_p of positive and negative contagion can be defined as:

$$\begin{aligned} C_n &= \{\{\theta_A, \theta_B, u\} \mid \theta_A^*(u) < \theta_B \le \theta_B^*(u, \theta_A)\} \\ C_p &= \{\{\theta_A, \theta_B, u\} \mid \theta_B^*(u, \theta_A) < \theta_B \le \theta_A^*(u)\}\,. \end{aligned}$$

In order to get a better understanding of the notion of negative and positive contagion, it may be illuminating to consider figure 3, which plots the critical equilibrium thresholds of θ_A and θ_B for both cases $u = u_1$ and $u = u_2$ and an arbitrary parameter constellation.[14] While the benchmark threshold is simply a constant $\theta_{A,j}^*$ for $j = 1, 2$, the threshold $\theta_B^*(u, \theta_A)$ of firm B is a step function with a sharp drop at $\theta_A = \theta_{A,j}^*$ for both cases $u = u_1$ and $u = u_2$. Given the equilibrium strategies of the creditors, firm B collapses if and only if $\theta_B \le \theta_B^*(u, \theta_A)$, that is, if the joint realization of θ_A and θ_B falls into the area under the solid step function in the relevant field of figure 3. In the benchmark case without contagion, by contrast, it would fail whenever θ_B happened to lie below one of the dashed horizontal lines in figure 3.

Since negative contagion can only occur if the first firm fails, suppose now that A fails. Hence, firm B collapses when θ_B does not exceed $\theta_{B,1}^f$ or $\theta_{B,2}^f$. As long as θ_B does not even exceed $\theta_{A,1}^*$ or $\theta_{A,2}^*$, however, B would also fail in the hypothetical benchmark case. In this case, it makes no sense to speak of contagion, because B would even fail if there were no firm A. Negative contagion, therefore, is effective if and only if $u = u_1$ and $\theta_{A,1}^* < \theta_B \le \theta_{B,1}^f$ or if $u = u_2$ and $\theta_{A,2}^* < \theta_B \le \theta_{B,2}^f$. In figure 3, negative contagion occurs in the areas labelled I and III. On the other hand, positive contagion will be crucial

[14] In this example: u_1=0.53, u_2=0.2, z=0.2, ε=0.09, w^{a}=0.5 and λ=0.5.

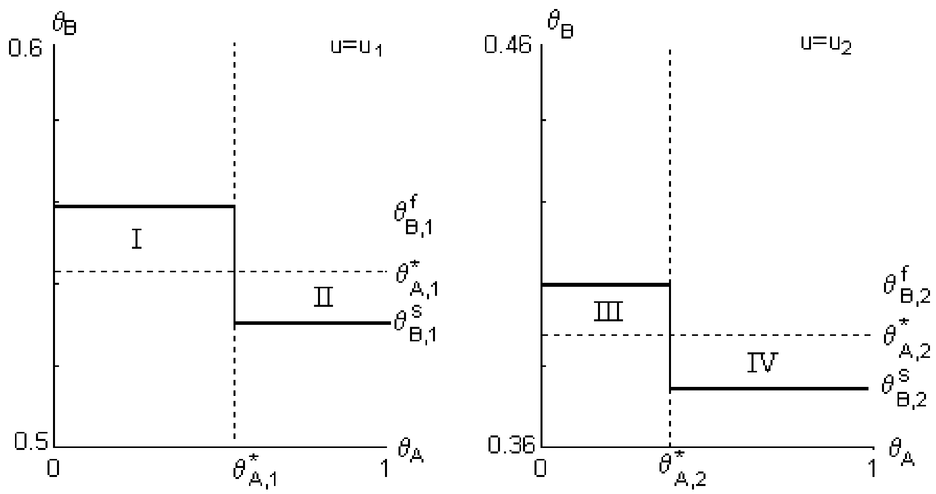

Figure 3: *Thresholds* $\theta_B^*(u, \theta_A)$, $\theta_A^*(u)$ *and Contagion*

for the success of firm B if and only if A survives and θ_B falls between $\theta_{B,1}^s$ and $\theta_{A,1}^*$ or between $\theta_{B,2}^s$ and $\theta_{A,2}^*$, which is associated with regions II and IV in figure 3. In this case, the fact that the investors have observed a success of the first firm saves the second.

Given these considerations, it is straightforward to compute the probabilities associated with positive or negative contagion in terms of the fundamental thresholds. The results are:[15]

$$P(C_n) = w^a \theta_{A,1}^* \left(\theta_{B,1}^f - \theta_{A,1}^*\right) + (1 - w^a)\theta_{A,2}^* \left(\theta_{B,2}^f - \theta_{A,2}^*\right) \quad (31)$$

$$P(C_p) = w^a (1 - \theta_{A,1}^*) \left(\theta_{A,1}^* - \theta_{B,1}^s\right) + (1 - w^a)(1 - \theta_{A,2}^*) \left(\theta_{A,2}^* - \theta_{B,2}^s\right) \quad (32)$$

The natural question which arises next is whether positive or negative contagion is more likely to occur. As will be shown below, the answer to this question is surprisingly (or may be not so surprisingly) simple.

4.2 The Net Effect of Contagion

In order to capture the "net effect" of contagion, we have to define a reasonable measure of this impact. Since the model allows to calculate the

[15] The probabilities are derived in appendix B.2.

likelihood of positive or negative contagion, an obvious approach to define a net measure of contagion is to compare the probabilities of positive and negative contagion. More specifically, I define the net effect of contagion (NEC) by

$$NEC = P(C_p) - P(C_n). \tag{33}$$

This measure is restricted to the intervall $[-1, 1]$ and is positive if and only if positive contagion is more frequent. Alternatively, by comparing the probability of failure in the presence of contagion to the benchmark case, the above defined net effect of contagion can be computed as

$$NEC = P(F_A) - P(F_B)$$

or as $P(S_B) - P(S_A)$. Naturally, the two definitions of the net effect must yield the same result. Given the first definition above and equations (31) and (32), the net effect in terms of the fundamental thresholds turns out to be :

$$\begin{aligned} NEC &= w^a \left[\theta^*_{A,1}(1-\theta^f_{B,1}) - (1-\theta^*_{A,1})\theta^s_{B,1}\right] \\ &\quad + (1-w^a)\left[\theta^*_{A,2}(1-\theta^f_{B,2}) - (1-\theta^*_{A,2})\theta^s_{B,2}\right]. \end{aligned} \tag{34}$$

This, however, is not the end of the story, for the magnitude of the net effect is much simpler than the above term may suggest. If we plug in the appropriate thresholds, the result simplifies to

$$NEC = 0.$$

Hence, whatever the likelihood of positive or negative contagion is, the net effect of contagion defined in (33) is exactly zero, as stated in the following proposition:

Proposition 3 *In the uniform model, the events of negative and positive contagion are equally likely. Therefore, the net effect of contagion is zero.*

In order to understand the meaning of the proposition, we may also consider an investor who can choose whether to buy equity claims of firm A or B which yield a constant dividend in case of success and nothing otherwise. Both firms are identical except that B is subject to contagion while A is not. Proposition 3 then tells us that before the game starts, this investor is indifferent between both choices. In other words, the present framework suggests that if we are only interested in whether the firm fails or not, there is no need to care about contagion.[16] As a matter of course, this finding

[16] Notice that there are indeed no other states of the firm in this model: either the return is R or 0.

contrasts sharply with the widespread concern about contagion both among academics and politicians.

The proposition can be proved by simply computing the term in (34) with the thresholds given by (11), where w has to be replaced appropriately by either w^a, w^f or w^s. Notice that although this is in principle straightforward, the thresholds are computed differently for different ranges of λ as discussed in section 2.3, and in combination with a second firm there are already quite many cases to look at.[17]

4.3 Some further Remarks on Contagion

4.3.1 Incidence of Contagion

Besides identifying the net effect of contagion, one might also be interested in a measure of the frequency or incidence of contagion. Such a measure, denoted IC, can be obtained by summing up the probabilities of positive and negative contagion, where due to proposition 3 $IC = 2P(C_n) = 2P(C_p)$. Plugging in the appropriate thresholds from (11) into equation (31) or (32) and simplifying the resulting expression yields the following result in terms of the model parameters:[18]

$$IC = \begin{cases} \frac{2\lambda z w^a((1-w^a)(u_1-u_2)-\lambda z)}{(1-w^a)} & \text{if } 0 < \lambda < \underline{\lambda} \\ \frac{4\varepsilon z w^a(1-w^a)(u_1-u_2)^2}{(2\varepsilon+z)^2} & \text{if } \underline{\lambda} \le \lambda \le \overline{\lambda} \\ \frac{2z(1-w^a)(1-\lambda)(w^a(u_1-u_2)-z(1-\lambda))}{w^a} & \text{if } \lambda > \overline{\lambda} \end{cases} \tag{35}$$

This measure shows how frequently the outcome of the first firm is decisive for the outcome of the second firm. Notice that while the effect of λ, w^a and z is ambiguous, the incidence of contagion is increasing in u_1 and decreasing in u_2. Clearly, the larger the difference between the two possible realizations of u is, the more important is the updated information which the creditors of B can gain from observing the outcome of A. This finding leads us to a more general discussion of the role of the fundamental state u.

[17] The computations of the net effect are available from the author on request.

[18] $\underline{\lambda}$ and $\overline{\lambda}$ are defined in equation (12) in section 2.3.

4.3.2 The Role of the Common Fundamental and of Rationality

An evident but nevertheless important insight of the preceding analysis is that with rational agents, informational contagion can only occur if there exist some common factors influencing both considered firms or countries. If none of the fundamentals affecting the firms are correlated, the failure of the first firm does not reveal any information that is relevant for the creditors of the second firm. Hence, they have no reason to condition their strategies on the outcome of the first firm and there can be no contagion. Moreover, to understand the role of the common state u, we could also consider the case where the common fundamental is negatively correlated, although this case is certainly less realistic in practice. Instead of assuming that the state u is the same for both firms, suppose now that the success of firm A (B) depends on the state u_A (u_B), where $u_B = u_1$ if $u_A = u_2$ and $u_B = u_2$ if $u_A = u_1$. In this case, leaving all other assumptions unchanged, the model would result in a peculiar kind of contagion which one might call "anti-contagion", because the failure of A would *lower* the chances of a failure of B.

Thus, the sign of the correlation of the common fundamental u determines the sign of contagion. But while the dependence on some common economic fundamentals is an important prerequisite, it is not sufficient for observing contagious events, because firms with sufficiently strong firm-specific fundamentals θ will withstand negative contagion. Very weak firms, on the other hand, cannot be rescued by positive contagion.

Finally, notice that a second prerequisite for having an informational channel of contagion is the rationality of agents. In contrast to the popular view that contagion must be driven by some weird and irrational "animal spirits", the present model underlines that it is exactly the rational reasoning of the creditors which leads to a systematic contagious link between the firms.

4.3.3 Empirical Evidence

The preceding subsection has established some features of the model which are different from pure sunspot theories and which have empirically testable implications. First, firms can only suffer (or benefit!) from informational contagion if they are in some sense perceived to be similar, that is, if agents think they are equally affected by some common fundamental(s). This issue has been explored by Aharony and Swary (1996) in the context of bank failures. In their study on 33 US banks in the mid-1980s, they find that the extent of the negative impact of contagion is greater for banks that are similar to the failed bank.[19] Likewise, Ahluwalia (2000) shows for a sample of 19 emerging countries and three episodes of crises that a country's vulnerability to contagious currency crises depends on the *visible similarities* between that country and other countries experiencing a crisis.[20] This is, again, exactly what the model predicts: After a crisis in one country, investors tend to withdraw their funds from different countries if the latter seem to have something in common with the first country. Second, in contrast to pure sunspot models, informational contagion cannot randomly ruin (or save) any firm or country. Even in the presence of contagion, the firms which survive are invariably those with the stronger firm specific fundamentals. Again, most empirical studies on contagion clearly support this implication.[21]

4.3.4 Transparency and Contagion

An important issue in the current discussion about strengthening the international financial system is the call for increased transparency. Morris and Shin (1999), however, show that in a global game framework, increasing transparency, interpreted as raising the precision of the private signals, need not mitigate the inefficiency of the coordination failure. To see why this is true in the present model, notice that by equation (15) in section 2.3, a decreasing ε need not lower the failure thresholds. What are the consequences for contagion if $\varepsilon \to 0$? From equation (35), it is easily seen that the likelihood of contagion does not depend on ε whenever $\lambda < \underline{\lambda}$ or $\lambda > \overline{\lambda}$. Therefore, if $\varepsilon \to 0$, the incidence of contagion converges to zero if and only if $\underline{\lambda} \leq \lambda \leq \overline{\lambda}$, and before becoming too enthusiastic, one should bear in mind that the inefficiency of the coordination failure has nothing to do with contagion and is not eliminated even in this case.

[19] See also Gorton (1988), who finds that bank failures are related to the (of course, common) business cycle.

[20] See also Park and Song (2000), who argue in their empirical study that foreign bank creditors refused to roll over short term debt in Korea after the crisis in Thailand of 1997, apparently because of a perceived similarity. Eichengreen, Rose and Wyplosz (1996) also find a role of macroeconomic similarity, although in their study, it seems to be dominated by trade links.

[21] According to Saunders and Wilson (1996), for example, poor banks faced more withdrawals than sound banks during the US banking panics between 1929-1933. Calomiris and Mason (1997) find that despite some confusion, it were the weaker banks that finally failed in the Chicago Bank Panic of 1932.

4.3.5 Diversification and Contagion

Another implication of the analysis is that contagion limits the benefits of portfolio diversification, since it increases the correlation between the returns of the two firms. From the perspective of our creditors in the model, this is not a problem. Being risk neutral, they would not be eager to diversify even if they had the opportunity to do so. If, on the other hand, we think of investors who hold equity claims that yield a constant dividend if the firm survives and nothing otherwise, then these investors would prefer a world without financial contagion. Contagion implies that the undiversfiable risk is higher than it appears if only takes the impact of the fundamentals into account. This is an implication which is also emphasized by Goldstein and Pauzner (2001). In their model, there would be no correlation between returns in different countries without contagion. Their endogenous correlation arises because the two countries share the same investors, while in the present model, the firms face different investors, but the behavior of the creditors of the second firm increases the "natural" correlation between the firms.

4.3.6 When Should we Care about Systemic Risk?

The presentation thus far suggests that there are little if any adverse consequences of contagion, apart from the implications on diversification. Before turning to the concluding summary of the paper, let me therefore take up the important question if there is any reason to be concerned about contagion or systemic risk. While the present model cannot not come up with a definitive answer to that question, it helps to clarify where the problems of systemic risk are to be looked for and where not. It may be helpful, for this purpose, to highlight again the difference between a world with and without contagion. If one neglects the uncountable possible outcomes for each single creditor, there are only three distinguishable states of the world in the model with 2 firms: either no firm fails, one of the firms fails, or both fail. The consequence of contagion is that the states in which no or both firms fails become more likely, whereas the failure of a single firm is less probable. But due to the symmetry result of proposition 3, failures are not more frequent if there is contagion. Hence, the problem of systemic risk is not that a failure of a single member of the system can trigger the failure of other members, because this is compensated for. One should also bear in mind that the inefficiency of the equilibrium thresholds, which has been discussed in section 2.3, is a consequence of the coordination failure and has nothing to do with systemic risk. This makes clear that one should be concerned about systemic risk if and only if there exist some additional costs of a joint failure of both (respectively many) firms, that is, if the joint failure of two firms is considered to be worse than twice the failure of a single firm. One might argue, for example, that the functioning of some system is endangered or that some know-how is forever lost if too many firms fail together. While I leave it open for future research whether there really exist such additional systemic effects, there are no such costs in the present model.

5 Conclusions

Extending a global game model in the manner of Morris and Shin (1999) to include two fundamentals and two or more firms has produced four main results. First, the unique equilibrium result of the basic model with only one fundamental and one firm still holds, and second, the first firm has a contagious impact on the performance of the second or any further firm merely by affecting the beliefs of the creditors. Unlike any multiple equilibria model in the tradition of Diamond and Dybvig (1983), the present model is able to determine exactly in which states of economic fundamentals contagion occurs. Beyond the theoretical satisfaction of obtaining a unique equilibrium and more importantly, such a model provides a rigorous framework to analyze contagion and reveals some hitherto unnoticed insights. In particular, from a neutral perspective, there exists also positive contagion in the sense that a good performance of one firm can rescue other firms. This is a third crucial result, or rather an important insight, which stems from comparing contagion with a well defined and natural benchmark in which there is no contagion. While the present model can also explain that a collapsing firm can trigger a chain of failures, it makes clear that this is only one side of contagion. The occurrence of positive contagion is not a marginal peculiarity of the model, it is as natural as negative contagion and happens exactly by the same mechanism. To my knowledge, this finding is new to the literature, which centers around negative contagion so far. Finally, a fourth major result of the paper shows that in the model with uniformly distributed fundamentals, negative and positive contagion are equally likely to occur, implying that the net effect of contagion on the firms is exactly zero.

In contrast to the contributions of Dasgupta (2001) and Goldstein and

Pauzner (2001), who also use the unique equilibrium selection technique of Carlsson and van Damme (1993) to explore contagion, the present model focuses on purely informational contagion. There are neither capital links nor wealth effects at work. As should be intuitively evident, a closer analysis of the transmission channel makes clear that there can be no informational contagion if none of the relevant economic fundamentals affecting the firms are correlated. This is also consistent with the empirical literature on banking and international financial crises, which finds ample evidence that contagion does not arbitrarily spread to *any* firm or country. Rather, similarities between firms or countries help to predict contagion. Another implication which distinguishes the present informational model from pure sunspot theories is that even with contagion, the failing firms are invariably those with weaker fundamentals. This finding is again confirmed by the bulk of empirical studies on contagion, which find that investors can at least to some extent discriminate between weak and strong firms (or countries). Further, the model implies that contagion does not necessarily vanish if the investors' signals become arbitrarily precise. Therefore, creating transparency need not eliminate contagion. This is, on the other hand, again no reason for tremendous concern since the consequences of contagion are not as weird as usually suggested.

Is there any reason for policy intervention in an industry in which there is informational contagion? From the discussion above, it is clear that any benevolent regulator should try to eliminate negative contagion while preserving the possibility of positive contagion, which is a tricky task, to say the least, for both possibilities are caused by the same mechanism. Since the net effect of contagion is zero, the model suggests that informational contagion provides no sound background to justify any intervention. Nevertheless, the equilibrium outcome of the model is inefficient, for both projects fail in some states where they would survive if the creditors were able to coordinate. Thus, there is a potential for improvement, provided some regulator is able to eliminate the coordination failure. This inefficiency, however, does not result from contagion, but is a consequence of the coordination failure itself. Informational financial contagion neither adds nor removes any inefficiency.

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Appendix

1 Proof of Lemma 2

The proof of lemma 2 presented here is taken from Morris and Shin (1998) except that some intermediate steps work just other way round. Let $\pi(s_i)$ denote the fraction of creditors who roll over the loan when receiving signal s_i and let $Q(s_i, \pi)$ be the expected payoff conditional on strategy π and on observing the signal s_i. Then the proof will proceed in the following two steps:

1. $\pi(s_i) \geq \pi'(s_i)\ \forall s_i$ implies $Q(s_i, \pi) \geq Q(s_i, \pi')\ \forall s_i$.
2. Together with lemma 1, this implies that $I_{s^*}(s_i)$ is the unique equilibrium strategy.

For the first result, note that conditional on θ and π, the fraction of creditors foreclosing on the loan is

$$l(\theta, \pi) = \int_{\theta-\varepsilon}^{\theta+\varepsilon} [1 - \pi(s_i)]\, ds_i. \tag{36}$$

Next, denote by

$$S(\pi) = \{\{u, \theta\} \mid \theta > u + zl(\theta, \pi)\}$$

the event in which the project is a success given aggregate strategy π. Because of the discrete nature of u, the set $S(\pi)$ of all $\{u, \theta\}$ combinations for which the firm survives can be split up in to the two mutually exclusive events

$$S(\pi) = \{u = u_1, \theta \in S_1(\pi)\}\ \cup \{u = u_2, \theta \in S_2(\pi)\} \tag{37}$$

where

$$S_1(\pi) = \{\theta \mid \theta > u_1 + zl(\theta, \pi)\} \tag{38}$$

and

$$S_2(\pi) = \{\theta \mid \theta > u_2 + zl(\theta, \pi)\}\,. \tag{39}$$

The expected payment $Q(s_i, \pi)$ is then simply the probability of a success conditional on π and s_i, i.e. $P(S(\pi) \mid s_i)$. Given (37) and the conditional distribution $\theta \mid s_i$ in (5), and noticing that the posterior probability of u being u_1 is w (and that θ and u are independent), the expected payoff is

$$Q(s_i, \pi) = w \int_{S_1(\pi)\cap[s_i-\varepsilon, s_i+\varepsilon]} (2\varepsilon)^{-1} d\theta + [1-w] \int_{S(\pi)\cap[s_i-\varepsilon, s_i+\varepsilon]} (2\varepsilon)^{-1} d\theta. \quad (40)$$

Now the argument can be stated as follows. If $\pi(s_i) \geq \pi'(s_i)\ \forall s_i$, then by (36) $l(\theta, \pi) \leq l(\theta, \pi')$ for all θ and by (38) and (39) $S_1(\pi) \supseteq S_1(\pi')$ and $S_2(\pi) \supseteq S_2(\pi')$. Thus, from (40) follows $Q(s_i, \pi) \geq Q(s_i, \pi')$ as claimed. Hence, the decisions to roll over the loan are strategic complements, that is, the decision of some players to prolong credit makes it more attractive for other players to do so for any state of the fundamental.[22]

Turning to the second step of the proof, consider the strategy $\pi(s_i)$ in any equilibrium of the game among creditors and define $\underset{\sim}{s}$ and $\tilde{s}$ as

$$\begin{aligned} \underset{\sim}{s} &= \inf\{s_i \mid \pi(s_i) > 0\} \\ \tilde{s} &= \sup\{s_i \mid \pi(s_i) < 1\}. \end{aligned}$$

Then, $\underset{\sim}{s} \leq \inf\{s \mid 0 < \pi(s_i) < 1\} \leq \sup\{s \mid 0 < \pi(s_i) < 1\} \leq \tilde{s}$ implies

$$\underset{\sim}{s} \leq \tilde{s}. \quad (41)$$

Note that whenever $\pi(s_i) < 1$, some creditors end up foreclosing on the loan, which is only consistent with equilibrium behavior when the expected payoff $Q(s_i, \pi)$ does not exceed λ. By continuity, this must also hold for $s_i = \tilde{s}$:

$$Q(\tilde{s}, \pi) \leq \lambda. \quad (42)$$

Since $I_{\tilde{s}}(\tilde{s}) \leq \pi(\tilde{s})$ and because of the result derived in the first step above, (42) implies

$$Q(\tilde{s}, I_{\tilde{s}}) \leq \lambda. \quad (43)$$

[22] Games which exhibit strategic complementarities are known as supermodular games. In *global games with strategic complementarities*, there exist a smallest and largest strategy that survive iterated deletion of dominated strategies and which coincide in the limit as the signal error vanishes. In the present model with uniform distributions, there is even no need to let the signal error shrink to zero in order to obtain a unique equilibrium. See Frankel, Morris and Pauzner (2002) for a general theoretical discussion of global games with strategic complementarities.

Next, we know from lemma 1 that $\widetilde{s} = s^*$ is the unique solution to $Q(\widetilde{s}, I_{\widetilde{s}}) = \lambda$, which implies

$$Q(\widetilde{s}, I_{\widetilde{s}}) \leq Q(s^*, I_{s^*}), \tag{44}$$

Further, in the proof of lemma 1 it has been shown that $Q(\widetilde{s}, I_{\widetilde{s}})$ is increasing in $\widetilde{s}$, which, together with (44), establishes

$$\widetilde{s} \leq s^*. \tag{45}$$

A symmetric argument shows that

$$\underset{\sim}{s} \geq s^*. \tag{46}$$

Now, together with (41), the above inequalities (45) and (46) imply $s^* \leq \underset{\sim}{s} \leq \widetilde{s} \leq s^*$ and therefore

$$\underset{\sim}{s} = \widetilde{s} = s^*. \tag{47}$$

In an equilibrium strategy π, the greatest lower bound of the set of s_i in which at least one creditor rolls over coincides with the least upper bound of the set in which at least one creditor terminates the loan. This means if at least one lender rolls over her loan, everyone does so, or put differently, $\pi(s_i)$ is either 0 or 1, and the switching point is $s_i = s^*$. Hence, the unique strategy which is consistent with equilibrium is switching around s^*. This proves lemma 2.

2 Probabilities of Contagion

2.1 Proof of Inequality (30)

The first inequality

$$P(F_B \mid F_A) = w^f \theta^f_{B,1} + (1 - w^f)\theta^f_{B,2} > w^f \theta^*_{A,1} + (1 - w^f)\theta^*_{A,2} = \widetilde{P}(F_B \mid F_A)$$

in (30) follows from $\theta^f_{B,j} \geq \theta^*_{A,j}$, which holds strictly at least for one $j = 1, 2$. Then

$$\widetilde{P}(F_B \mid F_A) = w^f \theta^*_{A,1} + (1 - w^f)\theta^*_{A,2} > w^a \theta^*_{A,1} + (1 - w^a)\theta^*_{A,2} = P(F_A)$$

follows because $w^f > w^a$ and $\theta^*_{A,1} > \theta^*_{A,2}$. Next, $w^s < w^a$ and $\theta^*_{A,1} > \theta^*_{A,2}$ together imply

$$P(F_A) = w^a\theta^*_{A,1} + (1 - w^a)\theta^*_{A,2} > w^s\theta^*_{A,1} + (1 - w^s)\theta^*_{A,2} = \widetilde{P}(F_B \mid S_A),$$

establishing $\widetilde{P}(F_B \mid F_A) > \widetilde{P}(F_B \mid S_A)$. Finally,

$$\widetilde{P}(F_B \mid S_A) = w^s\theta^*_{A,1} + (1 - w^s)\theta^*_{A,2} > w^s\theta^s_{B,1} + (1 - w^s)\theta^s_{B,2} = P(F_B \mid S_A)$$

results from $\theta^s_j \leq \theta^a_j$, holding again strictly for a least one $j = 1, 2$. This proves the claim (30).

2.2 Likelihood of negative and positive Contagion

The probability of negative contagion is

$$\begin{aligned} P(C_n) &= P\left((u = u_1) \cap F_A \cap (\theta^*_{A,1} < \theta_B \leq \theta^f_{B,1})\right) \\ &\quad + P\left((u = u_2) \cap F_A \cap (\theta^*_{A,2} < \theta_B \leq \theta^f_{B,2})\right). \end{aligned}$$

Noticing that θ_B is independent of u and does not affect the outcome of A, the above probability can be expressed as

$$\begin{aligned} P(C_n) &= P\left(\theta^*_{A,1} < \theta_B \leq \theta^f_{B,1}\right) P(F_A \mid u = u_1)P(u = u_1) \\ &\quad + P\left(\theta^*_{A,2} < \theta_B \leq \theta^f_{B,2}\right) P\left(F_A \mid u = u_2\right) P\left(u = u_2\right). \end{aligned}$$

Accordingly, the likelihood of positive contagion is given by

$$\begin{aligned} P(C_p) &= P\left(\theta^s_{B,1} < \theta_B \leq \theta^*_{A,1}\right) P(S_A \mid u = u_1)P(u = u_1) \\ &\quad + P\left(\theta^s_{B,2} < \theta_B \leq \theta^*_{A,2}\right) P\left(S_A \mid u = u_2\right) P\left(u = u_2\right). \end{aligned}$$

Plugging in (18), (26) and $P(u = u_1) = w^a$ yields, after some algebra, the probabilities given in equations (31) and (32).