Diskussionsschriften

Information and Barometric Prices: An Explanation for Price Stickiness

Simon Lörtscher
Michael Manz

02-06

June 2002

Universität Bern
Volkswirtschaftliches Institut
Gesellschaftstrasse 49
3012 Bern, Switzerland
Tel: 41 (0)31 631 45 06
Web: www-vwi.unibe.ch

Information and Barometric Prices: An Explanation for Price Stickiness

Simon Lörtscher and Michael Manz*

June 2002

Abstract

Price stickiness plays a decisive role in many macroeconomic models, yet why prices are sticky remains a puzzle. We develop a microeconomic model in which two competing firms are free to set prices, but face uncertainty about the state of demand. With some probability, there is a positive demand shock, which is observed but by one firm. In equilibrium, only the informed firm adjusts its price after the shock, while the uninformed firm raises its price only with a delay, after observing the price of its competitor. Hence, prices are sticky in the sense that one firm's price does not adjust immediately. Further, if getting information is costly, the model implies that the larger firm tends to be better informed and to adjust its price first.

JEL-Classification: L16, D43, D82.

KEYWORDS: Price Setting, Sticky Prices, Asymmetric Information, Barometric Price Leadership.

---

*University of Bern, Vereinsweg 23, 3012 Bern, Switzerland.
E-mail: simon.loertscher@vwi.unibe.ch and michael.manz@vwi.unibe.ch.
Preliminary version. Comments are welcome. For helpful comments and discussions, we would like to thank Esther Brügger, Alain Egli, Winand Emons, Roland Hodler and Manuel Wälti.

1 Introduction

Both casual and systematic evidence suggest firms do not change prices very frequently.[1] While there exist various theories trying to explain sluggish price adjustment (e.g. by assuming menu costs), economists generally agree that the question remains a puzzle. Of course, the observation of prices remaining the same over time does not yet prove that prices are sticky. After all, as long as no change occurs in the economy, even completely flexible prices will not change. Thus, the idea of price stickiness must refer to the way in which prices adjust to a change in the economy. In particular, stickiness invokes the idea that adjustment is "slow" compared to some hypothetical benchmark in which all firms adjust prices immediately. Price stickiness, therefore, means that adjustment of prices is incomplete in the sense that not all firms adjust prices immediately after a change has come. Some firms set a new price, while others somehow wait and see and adjust their price only after a while.

Without recurring to irrationality, price stickiness thus understood can be explained in two different ways. Due to costs associated directly with price adjustment, firms may prefer not to alter prices after a change even if they are perfectly aware of the change. Alternatively, firms may not adjust prices because they face uncertainty the reduction of which is costly. While New Keynesian models typically stress direct adjustment costs, our focus in this paper is on the issue of uncertainty and costs of information acquisition.[2]

Of course, some prices are perfectly flexible, since they are determined in organized exchange markets that come close to the Walrasian model. But it is also an empirical fact that many prices are set by firms. Typically, consumer prices belong to the latter category. Obviously, when discussing price stickiness, one has to focus on prices that are set. Understanding why firms do not adjust prices more frequently therefore requires a theory of how firms set prices, which requires abandoning the idea of a Walrasian auctioneer. Unfortunately, price setting in markets with homogeneous goods is susceptible to anomalies such as the Bertrand paradox. In our view, the difficulties in modelling price setting constitute an important impediment to a better understanding of sticky prices.

In this paper, we develop a simple microeconomic model in which two

[1] See e.g. Kashyap (1995) or Blinder et al. (1998).

[2] See e.g. Ball and Romer (1991) or Ball and Mankiw (1994) for a Keynesian perspective. Meltzer (1995) and Brunner and Meltzer (1993) emphasize the importance of uncertainty and of costs of acquiring information.

competing firms are free to set prices, but face uncertainty about the state of demand. This uncertainty takes the following form. With some probability, there is a positive demand shock, which is observed but by one firm. In this setting, we derive three main results. First, we show that only the informed firm adjusts its price after a positive demand shock, while the price of the uninformed firm remains the same as without the shock. The uninformed firm's price is, then, sticky in the sense that it does not immediately adjust to a change in demand. Our framework thus allows both for a definition what price rigidity means and for an explanation why it occurs. Second, we assume that demand remains the same in a second period in which firms can set prices anew. By observing the competitor's price of the first period, the previously uninformed firm will then learn the state of demand and adjust its price in the case of a positive demand shock. Hence, the informed firm is what is called a barometric price leader in the literature of industrial organization.[3] Third, we extend the model by assuming that, before setting prices in the first period, each firm is given the opportunity to invest into a technology which reveals the true state of demand. Our model then implies that information is more valuable for the firm with the larger quantity. Hence, larger firms are more inclined to invest into information acquisition, and smaller firms are more likely to be uninformed and to exhibit sluggish price adjustment.

In order to evade the difficulties with price competition that are well known since Bertrand (1883), we separate firms' production and selling activities in time. Firms have to produce first. Then they carry the quantities produced to the market place, where they observe the quantity produced by the competitor. Then, after one firm has learnt the true state of demand, they simultaneously set prices. This sequential structure ensures that there is a lower (and non-zero) bound for the prices firms consider setting: No firm, be it informed or not, sets a price below the market clearing price on the low demand (i.e. the demand in the absence of a positive shock). Still, if demand is not sufficiently elastic, the price setting game need not have an equilibrium. After all, it might pay a firm to set a higher price and to risk throwing away quantity it cannot sell, in which case the Bertrand logic would apply again. However, if quantities have to be removed from the market place at some costs, this strategy does not pay, and the price setting game has a Perfect Bayesian Equilibrium which is unique as far as prices set in equilibrium and payoffs are concerned.

[3] See Cooper (1997).

On the one hand, our paper is related to the industrial organization literature dealing with price setting and imperfect competition. In particular, we share with Kreps and Scheinkman (1983) and with Stahl (1988) the idea of sequential structuring. On the other hand, prices in our model reveal information as in Walrasian models. Our paper can be seen as an attempt to incorporate informational costs in the spirit of Meltzer (1995) into a model in which firms set prices.

The paper is structured as follows: In section 2, we introduce the price setting game and derive our main results. Section 3 sketches an extension of the basic model by incorporating investment into market research technology. Section 4 concludes.

2 The Price Setting Game

2.1 Structure of the Basic Model

In the basic model, we consider the following set up. There are two firms $j \in \{1, 2\}$ which procude a homogeneous good. We assume that production and market places are separated through time and space, so that firms have to produce first, then carry the goods to the market place where they can sell their products. This separation of production and market makes sure that firms cannot further increase the quantity they wish to sell, once they are selling on the market. Further, we assume that there are two periods $t \in \{1, 2\}$ on the market place and that the state of demand is the same for both periods. We consider only the sellers' problem and treat production as exogenous. Hence, we explore the price setting game for given quantities q_1 and q_2 of the two firms, where we assume that these quantities are the same for both periods. We also assume that the good is perishable such that quantities cannot be shifted across time.

The two firms face a stochastic demand, which is the same in both periods. With probability α, there is no shock, and demand is low for both periods. With probability $(1 - \alpha)$, there is a positive demand shock at the beginning of the first period, in which case demand is high for both periods. Let $Q^D(p)$ denote the aggregate demand function, which is either

$$Q_H^D(p) = Q^D(p)$$

in case of high demand or

$$Q_L^D(p) = \max\left\{Q^D\left(p + \varepsilon\right), 0\right\}$$

in case of low demand, where $Q^D(.)$ is a twice differentiable, monotonically decreasing and weakly concave function and ε is a parameter which captures the magnitude of the demand shock.

On the market place, the two firms observe the quantities produced and engage in price competition. In each period $t \in \{1, 2\}$, they set prices simultaneously. While they are free to set any price they like, they remain committed to that price for the remainder of the period. Thus, prices are legal offers that oblige a firm to sell its whole stock if customers want to buy it at that price. If they like, firms may change the price for the second period. However, they cannot leave their unsold goods on the market. If firms do not sell the whole quantity they have brought with them, they have to carry the excess quantity from the market at constant marginal removal costs γ. We assume that this is the case for both periods.

Since firms need not set the same prices, we have to make an assumption on rationing. In the case where prices are different and the low price offer cannot satiate the entire demand, we have to specify which consumers are attracted by the seller with the lower and by the seller with the higher price. That is, we need a *rationing rule* which gives us the residual demand function, i.e. the part of the demand remaining for the firm with the higher price. Throughout this paper, we assume that an *efficient rationing* rule applies. Therefore, supposing that aggregate demand is $Q^D(.)$ and that firm $j \in \{1, 2\}$ sets the higher price $p_j > p_{-j}$, the residual demand function $Q^{DR}(p_j, p_{-j}, q_{-j})$ of firm j is given by $\max\left\{Q^D(p_j) - q_{-j}, 0\right\}$.[4] Figure 1 plots demand and residual demand of firm 1 for linear demand functions, where p_L and p_H denote the prices for which the low respectively high demand clears for a given supply $Q^S \equiv q_1 + q_2$. These market clearing prices are implicitly determined by $Q_L^D(p_L) = Q_H^D(p_H) = Q^S$.

Finally, we assume the following structure of information. Before the beginning of the first period, the firms do not know the state of demand, but the probabilities of the two states are common knowledge. At the beginning of the first period, one firm learns the true state of demand before setting the price for that period. For the time being, we assume that this firm is randomly chosen by nature. We will discuss how the emergence of an

[4] See Tirole (2000) for a discussion of these rationing rules.

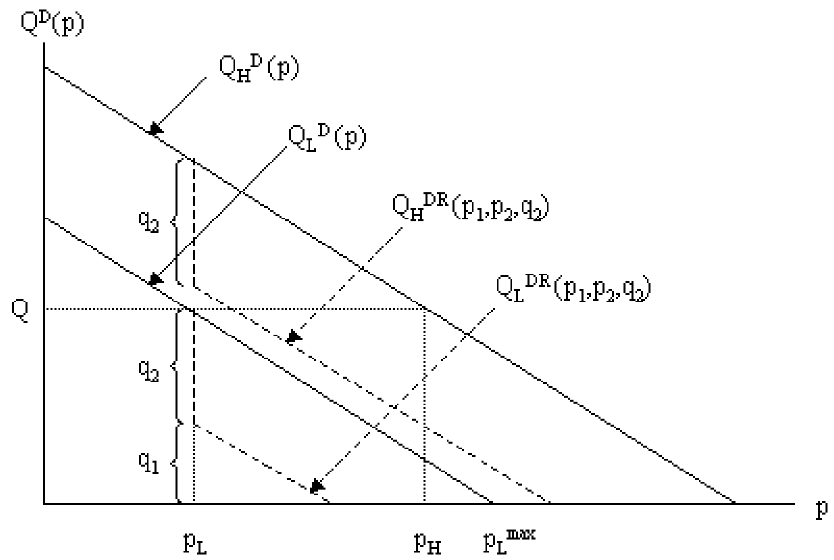

Figure 1: Demand and residual demand of firm 1

informed and an uninformed firm can be explained endogenously in section 3. When they set their prices, both firms are aware of which one knows the state of demand. When the market closes at the end of period 1, the firms can in addition observe the price and any unsold quantity of the other firm.

The two firms are risk neutral and maximize the total expected profit from both periods. The available information of the firms is asymmetric: while the informed firm can condition the price on the state of demand, the uninformed firm cannot. Hence, the firms face a two period game of incomplete information, in which the two states low and high demand can be interpreted as the two possible types of the informed firm. Since by observing its competitor's first period price, the uninformed firm receives a signal on the competitor's type, the game shares the essential characteristics of a signaling game. There are, however, two differences between our game and standard signaling games. First, both the sender (the informed firm) and the receiver (the uninformed firm) act in both periods. That is, the uninformed (informed) firm also sets a price in the first (second) period. Second, the receiver gets additional information by observing any unsold quantities of the informed firm.

A strategy of the *uninformed* firm (firm u henceforth) consists of choosing a price for each period, where the latter price can be conditioned on the price of the *informed* firm (firm i henceforth) in the first period. On the other hand, a strategy of firm i is given by a pair of prices (one for each state of demand) for period 1 and a pair of prices for period 2.[5] Let $\sigma_I = \{p_{IL,1}, p_{IH,1}, p_{IL,2}, p_{IH,2}\}$ and $\sigma_U = \{p_{U,1}, p_{U,2}(p_{I,1}, I)\}$ be the strategies of firm i and firm u. By $p_{IH,t}$ and $p_{IL,t}$, we denote the prices set by firm i if demand is high and low, respectively, while $p_{U,t}$ is the price set by firm u, $t \in \{1, 2\}$ indicates the time period, $p_{I,1}$ is the observed first period price of firm i, and I is an indicator function which deserves a few comments. Recall that prices are assumed to be legal offers that oblige the offering firm to sell the whole quantity if customers are willing to buy it at that price, so that the quantity a firm can sell is not a separate choice variable for that firm. Also, we have assumed that unsold quantities are observables. In particular, firm u can observe any first period quantity firm i cannot sell. Therefore, if firm i sets $p_{I,1} > p_L$, firm u can infer the true state of demand from observing prices and removed quantities. An indicator function I can hence be defined with the following properties: $I = 0$ if demand can be inferred to be high, and $I = 1$ if demand can be inferred to be low or if demand cannot be inferred.

To capture the beliefs of firm u, we further denote by $\mu(p_{I,1}, I)$ the probability with which the uninformed firm believes demand is low conditional on having observed $p_{I,1}$ and eventual removed quantities. Quite clearly, then, for $p_{I,1} > p_L$ the only beliefs consistent with the observations $p_{I,1}$ are $\mu^*(p_{I,1}, I) = I$. However, if $p_{I,1} \leq p_L$ and if $p_{U,1} \leq p_L$, no such certainty can be reached, since no quantities are removed for either state of demand, and thus any beliefs are consistent with the observations. In the next subsection, we show that this game has a simple Perfect Bayesian Equilibrium (PBE) with a unique equilibrium path.[6]

2.2 Equilibrium

Essentially, a PBE consists of a profile of strategies and a set of beliefs, such that at every information set of the game, the strategy of each firm maximizes

[5] In principle, the informed firm might also condition its second period prices on the first period price of the uninformed firm. But since the uninformed has only one type and therefore nothing to reveal, there is no reason to do so.

[6] In our game, the set of perfect Bayesian equilibria coincides with the set of sequential equilibria. See Fudenberg and Tirole (1991, Theorem 8.2).

its expected profit given the set of beliefs and the other player's subsequent strategy. In addition, beliefs are derived from equilibrium strategies and are updated according to Bayes' rule where possible. Defining

$$\overline{\gamma} = \max\{\overline{\gamma}_1, \overline{\gamma}_2\},$$

where

$$\overline{\gamma}_1 = -\left(\left.\frac{\partial Q_L^D(p)}{\partial p}\right|_{p=p_L}\right)^{-1} \frac{\max\{q_1, q_2\}}{\alpha} - p_L \tag{1}$$

$$\overline{\gamma}_2 = \frac{(1-\alpha)\varepsilon}{\alpha} - p_L \tag{2}$$

and focusing on pure strategies, we can prove the following result: If removal costs γ exceed $\overline{\gamma}$, then in any PBE, firm i sets market clearing prices p_L (p_H) in both periods if demand is low (high), and firm u sets its first period price to p_L, while its second period price mimics the first period price of the informed firm. The only potential difference between equilibria is off the equilibrium path: Firm u may have various beliefs, giving rise to various second period prices, if the firm i set a first period price below p_L, which, however, in equilibrium never happens.

Formally, this is stated in the following proposition:

Proposition 1 *If $\gamma \geq \overline{\gamma}$, the only pure strategy profiles and beliefs that constitute a PBE are*

$$\sigma_I^* = \{p_L, p_H, p_L, p_H\}, \sigma_U^* = \left\{p_L, p_{U,2}^*(p_{I,1}, I)\right\} \text{ and } \mu^*(p_{I,1}, I)$$

where $\forall\ p_{I,1} \geq p_L$,

$$\mu^*(p_{I,1}, I) = \text{ and } p_{U,2}^*(p_{I,1}, I) = p_L I + p_H (1-I),$$

and $\forall\ p_{I,1} < p_L$, any beliefs $\mu^*(p_{I,1}, I) \geq \alpha$, implying $p_{U,2}^*(p_{I,1}, I) = p_L I + p_H (1-I)$, can occur in equilibrium, while beliefs $\mu^*(p_{I,1}, I) < \alpha$ are only admissible if they imply second period prices $p_{U,2}^*(p_{I,1}, I) < 2p_L - p_{I,1}$.

To prove the proposition, we first consider a hypothetical reduced game in which there is only one period, such that any signaling interactions can be neglected. In this reduced game, the strategy of firm i consists of choosing two prices p_{IL} and p_{IH}, whereas firm u simply has to set a price p_U. For this reduced game, the following lemmata can be established:

Lemma 1 *Suppose $\gamma > \overline{\gamma}_1$. Then if demand is low, firm* i *strictly prefers to lower its price p_{IL} for any $p_{IL} \in \{p_{IL} \mid p_{IL} > p_L, p_{IL} \geq p_U\}$, and if demand is high, firm* i *strictly prefers to lower its price p_{IH} for any $p_{IH} \in \{p_{IH} \mid p_{IH} > p_H, p_{IH} \geq p_U\}$.*

Lemma 2 *If $\gamma > \overline{\gamma}$ and for any beliefs $\mu \geq \alpha$, firm* u *strictly prefers to lower its price p_U for any $p_U \in \{p_U \mid p_U > p_L, p_U \geq p_{IL}, p_U \leq p_H\}$ or any $p_U \in \{p_U \mid p_U \geq \max(p_H, p_{IL}, p_{IH})\}$.*

Though the proofs are rather tedious and therefore relegated to Appendices A and B, the mechanism at work is quite intuitive. Basically, the same mechanism applies throughout the price setting game. We begin with a brief comment on the role of removal costs. Consider, for example, Lemma 1. First, it should be noted that due to efficient rationing, firm i cannot sell its whole quantity q_I if demand is low and whenever $p_{IL} > p_L$ and $p_{IL} \geq p_U$. Therefore, under prices such as these, firm i will have to throw part of its quantity away, which will obviously not pay if removing quantity from the market is sufficiently costly. The basic role of removal costs is thus to make deviation from a market clearing price unattractive. It should be noted, however, that these costs are not necessarily needed (i.e. need not be positive): If low demand is elastic at p_L, there is no incentive for firm i to set a price p_{IL} higher than p_L, regardless of firm u's strategy, since revenue decreases with a higher price. What is behind Lemma 1, then, is a version of the mechanism uncovered by Bertrand: Whenever firm i considers setting a price above the one of its competitor and if these prices are such that firm i cannot sell everything (that is, if prices are above p_L or p_H), it pays firm i to decrease its price. A similar argument applies for firm u in Lemma 2.

Based on Lemma 1 and 2, we can then prove our main result stated in Proposition 1. The proof, which is basically an iterated application of Lemma 1 and 2, is relegated to Appendix C. We present the proof in six steps.

First, since a Bertrand like mechanism implies that no prices above p_H or between p_L and p_H can prevail in equilibrium, it can be shown that there exists no pooling equilibrium. Second, as a consequence of this, firm u learns the state of demand by observing first period prices, such that in equilibrium, both firms are informed in the second period. By virtue of Lemma 1, second period equilibrium prices must be p_L (p_H) if demand is low (high) in any PBE and for any first period equilibrium price of firm i. Third, speaking somewhat loosely, firm i will therefore set first period prices which were optimal in a one period game. Lemma 1 and 2 then imply that in the first period, firm u sets p_L , while firm i sets p_L (p_H) if demand is low (high) in *any* equilibrium. What remains to be specified is the equilibrium reaction function $p^*_{U,2}(\ p_{I,1}, I)$ for off equilibrium prices $p_{I,1}$. Note that for off equilibrium prices $p_{I,1} > p_L$, firm u will become informed about the state of demand after the first period, such that in the second period, firm u would set p_L (p_H) if demand is low (high). This is shown in step 4. Fifth, for off equilibrium prices $p_{I,1} < p_L$, firm u cannot infer the state of demand. Therefore, there are some degrees of freedom for the equilibrium reaction function $p^*_{U,2}(\ p_{I,1}, I)$ for off equilibrium prices $p_{I,1} < p_L$. However, even for such prices, only beliefs implying $p^*_{U,2}(\ p_{I,1}, I) \leq 2p_L - p_{I,1}$ are consistent with equilibrium. Otherwise, firm i would be tempted to set such a price $p_{I,1} < p_L$, which cannot be part of an equilibrium. Step 1 to 5 exclude all equilibrium candidates other than those stated in Proposition 1. What remains to be shown, therefore, is that the strategies and beliefs constitute a PBE. This is done in our final step. The only difference of equilibria lying in off equilibrium beliefs and behavior, payoffs and observable prices are identical across all equilibria. Hence, the game has a unique equilibrium path.

We would like to emphasize that despite being complicated when dealt with in detail, the equilibrium outcome of the game is fairly simple. In the first period, the uninformed firm always sets p_L, while the informed firm sets p_L if demand is low and p_H if demand is high. In the second period, both firms set p_L if demand is low and p_H if demand is high. Thus, our model exhibits price stickiness in the sense that one firm sets a new price after a positive demand shock, while the other one follows a price policy of "wait and see" and adjusts its price only after a while.

3 Information Acquisition

A natural extension of the model is to endogenize the appropriation of information at the beginning of the first period. We now make the following assumptions: Either firm can invest an amount F in market research, in which case the firm learns the true state of demand before setting the price for the first period. If no firm invests, we assume again that one firm learns demand by chance, where the a priori probability of becoming informed is $\frac{1}{2}$. Thus, investment provides information with certainty.

For any given quantities and if only one firm invests, we know from Section 2 that the informed firm sets p_L (p_H) in both periods if demand is low (high), whereas the uninformed firm sets p_L in the first period and $p_L I + p_H(1-I)$ in the second period. If both firms invest, it follows immediately that both firms set p_L (p_H) in both periods if demand is low (high). Finally, if no firm invests, one firm learns demand by chance, which leads to the same result as the game where only one firm invests.

The expected value for firm $j \in \{1, 2\}$ of being informed with certainty is then simply $\frac{1-\alpha}{2}(p_H - p_L) q_j = \frac{1-\alpha}{2} \varepsilon q_j$. A risk-neutral profit maximizing firm, therefore, would invest in market research whenever the gain from doing so exceeds the costs F. Moreover, for a given F and for $q_1 \neq q_2$, the firm with the greater quantity is more inclined to invest in market research because the value of being informed is increasing in q_j. While the firm with the smaller quantity invests if $F < \underline{F} = \frac{1-\alpha}{2} \varepsilon \min\{q_1, q_2\}$, the seller with the larger quantity invests whenever $F < \overline{F} = \frac{1-\alpha}{2} \varepsilon \max\{q_1, q_2\}$. Hence, if quantities are not identical and $F \in \left[\underline{F}, \overline{F}\right]$, the firm with the larger quantity invests and the other one does not. Under the assumption $F \in \left[\underline{F}, \overline{F}\right]$, our extended model predicts that large firms are better informed about demand and act as a price leader.

4 Discussion

We have presented a Non-Walrasian model in which two firms set prices under asymmetric information about demand. If there is a positive demand shock, the informed firm raises its price, whereas the other firm learns about the shock only by observing its competitor's price and thus adjusts its own price in the next period. Thus, the uninformed firm's price is sticky insofar as it does not immediately react to a change in demand. If firms can choose whether to acquire information at some fix cost, the firm with the larger quantity has stronger incentives to become informed. Therefore, prices of larger firms tend to move first if demand increases, which is an empirically testable hypothesis. Due to this prediction, the model escapes the criticism of Blinder et al. (1998) who argue that existing theories of sticky prices are epistemologically empty. In addition, the popular term market leader is given a natural meaning. A market leader is not only a large firm, but also, and maybe more importantly, the firm that adjusts its price first, because it is better informed.

Throughout the paper, we have treated quantities as exogenous data for the firms (sellers). This may be a reasonable assumption in the short run once the products are on the market, but in longer terms firms typically are engaged in production. A natural extension, therefore, would seem to introduce production in a stage previous to the price setting game. While this is possible, we do not think that it is a particularly useful endeavour, since we do not gain any additional insight into why prices seem to be sticky. Price stickiness is reflected solely by the fact that not all firms set the market clearing price on the high demand after a positive demand shock, whatever quantities firms have produced. Further, the model could be extended to include more than two firms. Again, we conjecture that the mechanics would remain the same and the gain from this extension would thus not be substantial. Informed firms would adjust immediately, while uninformed firms would wait and see.

In our view, uncertainty and costs of information are crucial to understand why prices are sticky. Menu costs, taken literally as direct costs of setting a new price, would have to be implausibly large to account for the price stickiness of our model. Uncertainty and costs of information, on the other hand, seem to be more fundamental frictions. Depending on firm specific variables such as firm size, the decision to remain uninformed may be completely rational. In our model, this is what makes prices sticky: firms do not adjust prices because they lack information.

If costs of information play a crucial role, they deserve explicit treatment. This paper is an attempt to model the role of information in the context of price competition. Of course, the fact that in our model prices are only adjusted in one direction may be seen as a shortcoming of the model. It is, however, a consequence of the difficulties that arise as one leaves the fiction of a Walrasian auctioneer. Further research on the microstructure of markets will hopefully help to reduce problems such as these.

References

[1] Ball, Laurence and David Romer (1991), Sticky Prices as Coordination Failure, American Economic Review: 539-52.

[2] Ball, Laurence and N. Gregory Mankiw (1994), A sticky price manifesto, Carnegie-Rochester Conference Series on Public Policy 41:127-51.

[3] Bertrand, Joseph (1883), Théorie Mathémathique de la Richesse Sociale. Journal des Savants: 499-508.

[4] Blinder, Alan S.,Canetti, Elie R. D., Lebow, David E., and Jeremy B. Rudd (1998), Asking about prices: A new approach to understanding price stickiness, New York, Russel Sage Foundation.

[5] Brunner, Karl and Allan H. Meltzer (1993), Money and the economy: Issues in monetary analysis, Raffaele Mattioli Lectures. New York, Cambridge University Press.

[6] Cooper, David J. (1997), Barometric Price Leadership, International Journal of Industrial Organization 15(3): 301-25.

[7] Fudenberg, Drew, and Jean Tirole (1991), Game Theory, Cambridge (Mass.): The MIT Press.

[8] Kashyap, Anil K., Sticky Prices (1995), New Evidence from Retail Catalogs, Quarterly Journal of Economics: 245-74.

[9] Kreps, David M. and José A. Scheinkman (1983), Quantity Precommitment and Bertrand competition yield Cournot outcomes. Bell Journal of Economics: 326-37.

[10] Meltzer, Allan H. (1995), Information, Sticky Prices and Macroeconomic Foundations, Federal Reserve Bank of St. Louis Review 77(3) : 101-18.

[11] Tirole, Jean (2000), The theory of industrial organization, Cambridge (Mass.): The MIT Press.

Appendix

A Proof of Lemma 1

To prove Lemma 1, we treat the price p_U of firm u as given and show that removal costs of $\overline{\gamma}_1$ are sufficient to ensure that the revenue of firm i is strictly decreasing in p_{IL} for any $p_{IL} \in \{p_{IL} \mid p_{IL} > p_L, p_{IL} \geq p_U\}$ if demand is low and strictly decreasing in p_{IH} for any $p_{IH} \in \{p_{IH} \mid p_{IH} > p_H, p_{IH} \geq p_U\}$ if demand is high. Together, this proves the lemma. We now consider the two cases of low and high demand separately. Throughout the proofs, we denote by $p_L^{\max}$ ($p_H^{\max}$) the prices for which the demanded quantity becomes zero in case of low (high) demand.

A.1 Low Demand

First, note that raising the price p_{IL} above $p_L^{\max}$ is never preferable (being strictly dominated by $p_{IL} = p_L$). Now, since the residual demand is binding whenever demand is low and the informed firm sets $p_{IL} \geq \max(p_L, p_U)$, the revenue of firm i is

$$\left(Q_L^D(p_{IL}) - q_U\right) p_{IL} - \gamma \left(q_I - Q_L^D(p_{IL}) + q_U\right).$$

Differentiating revenue with respect to p_{IL} yields[7]

$$\frac{\partial Q_L^D(p_{IL})}{\partial p_{IL}} p_{IL} + \left(Q_L^D(p_{IL}) - q_U\right) + \gamma \frac{\partial Q_L^D(p_{IL})}{\partial p_{IL}}, \tag{3}$$

for any $p_{IL} < p_L^{\max}$, and the second derivative

$$\frac{\partial^2 Q_L^D(p_{IL})}{\partial p_{IL}^2} p_U + 2 \frac{\partial Q_L^D(p_{IL})}{\partial p_{IL}} + \gamma \frac{\partial^2 Q_L^D(p_{IL})}{\partial p_{IL}^2}$$

is negative because $\frac{\partial^2 Q_L^D(p)}{\partial^2 p} \leq 0$ by assumption. This implies that if (3) is negative at $p_{IL} = p_L$, then it is negative for any p_{IL} up to $p_L^{\max}$. Next, the derivative (3) evaluated at $p_{IL} = p_L$ is negative if and only if

$$\gamma > -\left(\left. \frac{\partial Q_L^D(p)}{\partial p} \right|_{p=p_L} \right)^{-1} q_I - p_L, \tag{4}$$

[7] More precisely, if $p_{IL} = p_L$, this is only the right hand side derivative, but since it does not affect any proofs, we will neglect that remark in the remaining discussion.

(note that if $p_{IL} = p_L$, we can replace $Q_L^D(p_{IL}) - q_U$ by q_I). Since $\alpha < 1$ and $\max(q_1, q_2) \geq q_I$, it follows from (1) that condition (4) is certainly met if $\gamma > \overline{\gamma}_1$. This proves the first part of Lemma 1.

A.2 High Demand

Again, any $p_{IH} \geq p_H^{\max}$ is strictly dominated by $p_{IH} = p_H$, so we restrict attention to $p_{IH} < p_H^{\max}$. If demand is high and $p_{IH} \geq \max(p_H, p_U)$, the profit of firm i is

$$\left(Q_H^D(p_{IH}) - q_U\right) p_{IH} - \gamma \left(q_I - Q_H^D(p_{IH}) + q_U\right).$$

Proceeding the same way as in the previous subsection, it can be shown that if the derivative

$$\frac{\partial Q_H^D(p_{IH})}{\partial p_{IH}} p_{IH} + \left(Q_H^D(p_{IH}) - q_U\right) + \gamma \frac{\partial Q_H^D(p_{IH})}{\partial p_{IH}} \tag{5}$$

is negative for $p_{IH} = p_H$, which is the case if and only if

$$\gamma > -\left(\left.\frac{\partial Q_H^D(p)}{\partial p}\right|_{p=p_H} \right)^{-1} q_I - p_H, \tag{6}$$

then it is negative for any p_{IH} up to $p_H^{\max}$. Due to $p_L < p_H$ and $\left.\frac{\partial Q_H^D(p)}{\partial p}\right|_{p=p_H} = \left.\frac{\partial Q_L^D(p)}{\partial p}\right|_{p=p_L}$, restriction (6) is weaker than (4), hence it is also satisfied if $\gamma > \overline{\gamma}_1$. Therefore, firm i strictly prefers to lower its price for any $p_{IH} \in \{p_{IH} \mid p_{IH} > p_H, p_{IH} \geq p_U\}$ if $\gamma > \overline{\gamma}_1$. Q.E.D.

B Proof of Lemma 2

For firm u, the reasoning is slightly more complicated, since its expected revenue depends on the "belief" μ.[8] We proof the lemma in three steps presented below. First, we show that the derivative of firm u's revenu with respect to p_U is strictly negative for any $p_U \geq \max(p_H, p_{IL}, p_{IH})$. Then we show that firm u prefers to lower p_U for any $p_U \in \{p_U \mid p_U > p_L, p_U \geq p_{IL}, p_U \leq p_H\}$, where we distinguish between the two cases $p_H < p_L^{\max}$ and $p_H < p_L^{\max}$. Together, this proves the lemma.

[8] Of course, in a reduced one period game or in period 1, the only reasonable belief μ is the objective probability α, but we keep the presentation more general for later purposes.

B.1 Prices above p_H

For this part of the proof, we draw on the proof of Lemma 1 in Appendix A. If the uninformed firm sets a price p_U that exceeds p_H and is at least p_{IL} and at least p_{IH}, the derivative of its expected revenue with respect to p_U consists of two components: The first (and "low demand part") component is μ times the derivative expressed in (3) or zero, while the second ("high demand") part can be computed as $(1-\mu)$ multiplied by the derivative in equation (5).[9] Since we have shown above that the assumption $\gamma > \overline{\gamma}_1$ is sufficient to ensure that both derivatives (3) and (5) are strictly negative for any price above p_H (unless for dominated prices which imply a zero revenue), it follows that the derivative of firm u, which is some average of the two, is also strictly negative. Thus, firm u prefers to lower p_U whenever $p_U \geq \max(p_H, p_{IL}, p_{IH})$.

B.2 Prices below p_H if $p_H < p_L^{\max}$

Next, assume $p_H < p_L^{\max}$, as will be the case for a sufficiently large quantity Q. If firm u sets some $p_U \in \{p_U \mid p_U > p_L, p_U \geq p_{IL}, p_U \leq p_H\}$, the quantity sold is given by the residual demand $Q_L^{DR}(p_U, q_I) = Q_L^D(p_U) - q_I$ if demand is low and by q_U if demand is high. Hence, the expected revenue of firm u is

$$\mu \left(Q_L^D(p_U) - q_I\right) p_U - \mu\gamma \left(q_U - Q_L^D(p_U) + q_I\right) + (1-\mu) q_U p_U \tag{7}$$

Differentiating with respect to p_U yields

$$\mu \left[\frac{\partial Q_L^D(p_U)}{\partial p_U} p_U + Q_L^D(p_U) - q_I + \gamma \frac{\partial Q_L^D(p_U)}{\partial p_U}\right] + (1-\mu) q_U, \tag{8}$$

and the second derivative

$$\mu \left[\frac{\partial^2 Q_L^D(p_U)}{\partial p_U^2} p_U + 2 \frac{\partial Q_L^D(p_U)}{\partial p_U} + \gamma \frac{\partial^2 Q_L^D(p_U)}{\partial p_U^2}\right]$$

is again strictly negative because of $\frac{\partial^2 Q_L^D(p)}{\partial^2 p} \leq 0$. Therefore, if the first derivative (8) is negative at $p_U = p_L$, then it must be so for any higher p_U up to p_H. Noticing that $p_U = p_L$ implies by definition $Q_L^D(p_U) - q_I = q_U$, (8) evaluated at $p_U = p_L$ is

$$\mu \left[\left.\frac{\partial Q_L^D(p)}{\partial p}\right|_{p=p_L} p_L + \gamma \left.\frac{\partial Q_L^D(p)}{\partial p}\right|_{p=p_L}\right] + q_U,$$

[9] Except that, of course, p_{IL} or p_{IH} have to be replaced by p_U.

which is negative if γ exceeds the critical threshold

$$\gamma > -\left(\left.\frac{\partial Q_L^D(p)}{\partial p}\right|_{p=p_L}\right)^{-1}\frac{q_U}{\mu} - p_L. \tag{9}$$

From the definition in (1) it is straightforward to recognize that the restriction (9) is satisfied for any $\mu \geq \alpha$ if $\gamma > \overline{\gamma}_1$, implying that firm U clearly prefers to lower p_U.

B.3 Prices below p_H if $p_H > p_L^{\max}$

Now, consider the case $p_H > p_L^{\max}$. If $p_U \in \{p_U \mid p_U > p_L, p_U \geq p_{IL}, p_U \leq p_H\}$ and in the range $p_U \leq p_L^{\max}$, firm U's revenue is still given by (7). As we have shown in the previous section, revenue is therefore strictly decreasing in p_U up to $p_L^{\max}$ if $\gamma > \overline{\gamma}_1$.

Turning to prices $p_U \in \{p_U \mid p_U > p_L, p_U \geq p_{IL}, p_U \leq p_H\}$ but above $p_L^{\max}$, it is never optimal to set a price higher than $p_L^{\max}$, unless the price is p_H, since by raising p_U, firm U benefits in case of high demand without losing anything in case of low demand. Hence, in this case $p_U = p_H$ is the only alternative to setting p_L that we could not exclude so far. In a next step, we compare revenues from setting p_H and p_L. Given belief μ, firm U strictly prefers $p_U = p_L$ to any $p_U \in \{p_U \mid p_U > p_L, p_U \geq p_{IL}, p_U \leq p_H\}$ if

$$p_L q_U > (1-\mu)p_H q_U - \mu\gamma q_U.$$

Since $p_H = p_L + \varepsilon$, the above condition transforms to

$$\gamma > \frac{(1-\mu)}{\mu}\varepsilon - p_L.$$

If γ exceeds $\overline{\gamma}_2$ defined in (2), the above condition is met for any $\mu \geq \alpha$. Thus, $\gamma > \overline{\gamma}$ ensures that also if $p_H > p_L^{\max}$, firm U strictly prefers to lower p_U whenever $p_U \in \{p_U \mid p_U > p_L, p_U \geq p_{IL}, p_U \leq p_H\}$. Together with the results of the previous subsections, this proves Lemma 2. Q.E.D.

C Proof of Proposition 1

We prove Proposition 1 in six steps. To economize on notation, we drop the argument I in the functions $p_{U,2}^*(p_{I,1}, I)$ and $\mu(p_{I,1}, I)$.

Step 1: There exists no pooling equilibrium, i.e. no PBE with $p^*_{IL,1} = p^*_{IH,1}$.

Proof: To prove this claim, suppose to the contrary that there exists an equilibrium in which firm i sets $p^*_{IL,1} = p^*_{IH,1} = p^*_{I,1}$. In this case, the beliefs of firm u are $\mu(p^*_{I,1}) = \alpha$ if the equilibrium price $p^*_{I,1}$ is observed. We then show that for any $p^*_{I,1}$ and any $p^*_{U,1}$, the only second period prices $p^*_{IL,2}$, $p^*_{IH,2}$ and $p^*_{U,2}(p^*_{I,1})$ that could prevail in equilibrium are $p^*_{IL,2} = p^*_{U,2}(p^*_{I,1}) = p_L$ and $p^*_{IH,2} = p_H$. But from this follows that $p^*_{I,1} = p^*_{IL,1} = p^*_{IH,1}$ cannot be a PBE, which proves the above claim.

We first turn to the case of low demand and show that in any pooling equilibrium, only $p^*_{IL,2} = p_L$ can be part of an equilibrium. To see why, note that since prices in the last period bear no consequences on the future, any $p^*_{IL,2} < p_L$ is strictly dominated and can therefore be excluded. Suppose, instead, that $p^*_{IL,2} > p_L$ is chosen in equilibrium together with some $p^*_{U,2}(p^*_{I,1})$. If $p^*_{IL,2} \geq p^*_{U,2}(p^*_{I,1})$, this cannot be an equilibrium, because from Lemma 1 we know that firm i would like to lower its price. However, for any $p^*_{IH,2}$, $p_L < p^*_{IL,2} < p^*_{U,2}(p^*_{I,1})$ cannot occur in equilibrium either: If $p^*_{IH,2} \leq p_H$ and given beliefs $\mu(p^*_{I,1}) = \alpha$, Lemma 2 implies that firm u should lower its price. If, on the other hand, $p^*_{IH,2} > p_H$, then by virtue of Lemma 1, firm i would prefer to lower $p^*_{IH,2}$ if $p^*_{IH,2} > p^*_{U,2}.)$, and in the opposite case where $p_H < p^*_{IH,2} \leq p^*_{U,2}(p^*_{I,1})$, Lemma 2 implies that firm u would prefer to lower its price. Hence, setting $p^*_{IL,2} > p_L$ is inconsistent with equilibrium behavior, leaving $p^*_{IL,2} = p_L$ as the only equilibrium candidate.

An equivalent reasoning establishes that only $p^*_{U,2}(p^*_{I,1}) = p_L$ can prevail in equilibrium: Any strategy σ^*_U in which $p^*_{U,2}(p^*_{I,1}) < p_L$ is strictly dominated, whereas $p^*_{U,2}(p^*_{I,1}) > p_L$ can be excluded for the following reasons: Either, if $p^*_{IH,2} \leq p_H$ and $p^*_{U,2}(p^*_{I,1}) \geq p^*_{IL,2}$ or if $p^*_{IH,2} > p_H$ and $p^*_{IH,2} \leq p^*_{U,2}(p^*_{I,1})$, firm u should lower $p^*_{U,2}(p^*_{I,1})$ (see Lemma 2). Or, firm i would prefer to decrease $p^*_{IL,2}$ if $p^*_{IH,2} \leq p_H$ and $p^*_{U,2}(p^*_{I,1}) < p^*_{IL,2}$ or to decrease $p^*_{IH,2}$ if $p^*_{IH,2} > p_H$ and $p^*_{IH,2} > p^*_{U,2}(p^*_{I,1})$ (see Lemma 1).

Next, any $p^*_{IH,2} \neq p_H$ can also be excluded in equilibrium: Strict dominance reasons exclude all $p^*_{IH,2}$ below p_H, while the same reasoning as above implies that there can be no equilibrium with $p^*_{IH,2} > p_H$, since in this case at least one firm would like to lower its second period price.

So far, we have shown that a pooling equilibrium is only compatible with $p^*_{IL,2} = p^*_{U,2}(p^*_{I,1}) = p_L$ and $p^*_{IH,2} = p^*_{U,2}(p^*_{I,1}) = p_H$. From now on, we restrict attention to these strategies and consider whether within this restricted strategy space, there exists any $p^*_I = p^*_{IL,1} = p^*_{IH,1}$ that could prevail in equilibrium for any $p^*_{U,1}$. The answer is no, as we show next.

Note first that $p^*_{I,1} < p_L$ cannot occur in equilibrium, since in combination with $p^*_{IL,2} = p_L$ and $p^*_{IH,2} = p_H$, this is strictly dominated by $p^*_{I,1} = p_L$. Second, suppose $p^*_{I,1} \geq p_L$ but $p^*_{I,1} < p_H$. Again, given $p^*_{IL,2} = p_L$ and $p^*_{IH,2} = p_H$, this is not compatible with equilibrium behavior, because if demand is high, firm i can do better by setting $p^*_{IH,1} = p_H$. Third, suppose $p^*_{I,1} \geq p_H$ and $p^*_{U,1} \geq p^*_{I,1}$. In combination with $p^*_{IL,2} = p_L$ and $p^*_{IH,2} = p_H$, this cannot be part of a PBE, because from Lemma 2, we know that in this case $p^*_{U,1}$ cannot be an equilibrium response of firm u. Fourth, assume $p^*_{I,1} \geq p_H$ and $p^*_{U,1} < p^*_{I,1}$. This cannot be a PBE with $p^*_{IL,2} = p_L$ and $p^*_{IH,2} = p_H$ either, since Lemma 1 implies that firm i should lower $p^*_{I,1}$.

Summing up, we have established that there can be no equilibrium in which $p^*_{IL,1} = p^*_{IH,1}$. Hence, any equilibrium must be of a separating nature, in which case the informed firm reveals its type, i.e. the state of demand.

Step 2: In any PBE, i) $\mu(p^*_{IL,1}) = 1$ and $\mu(p^*_{IH,1}) = 0$, ii) $p^*_{U,2}(p^*_{IL,1}) = p_L$ and $p^*_{U,2}(p^*_{IH,1}) = p_H$, iii) $p^*_{IL,2} = p_L$ and $p^*_{IH,2} = p_H$.

Proof: Given the separating nature of any equilibrium derived in step 1, beliefs $\mu(p^*_{IL,1}) = 1$ and $\mu(p^*_{IH,1}) = 0$ follow immediately. So in equilibrium, both firms know the state of demand in the second period. In this last period, no firm sets a price below p_L (p_H) if demand is low (high), since these prices are strictly dominated by p_L (p_H). But no prices above p_L (p_H) if demand is low (high) can be part of an equilibrium either, because in this case, the price of one firm must be at least as high as the other firm's price. From Lemma 1 we know that in this case, the firm with the weakly higher price wants to decrease its price. Thus, only prices as specified above in ii) and iii) can occur in any PBE.

Step 3: In any PBE, i) $p^*_{U,1} = p_L$, ii) $p^*_{IL,1} = p_L$ and $p^*_{IH,1} = p_H$.

Proof: Steps 1 and 2 allow us to restrict attention to potential equilibrium strategies in which $p^*_{U,2}(p^*_{IL,1}) = p_L$, $p^*_{U,2}(p^*_{IH,1}) = p_H$, $p^*_{IL,2} = p_L$ and $p^*_{IH,2} = p_H$. We now turn to first period prices. Again, any $p^*_{U,1} < p_L$ is strictly dominated. But in equilibrium, no $p^*_{U,1} > p_L$ can occur either. To see this, assume $p^*_{U,1} > p_L$ together with some $p^*_{IL,1}$ and $p^*_{IH,1}$. If $p^*_{IL,1} > p^*_{U,1}$ (and $p^*_{U,1} > p_L$), Lemma 1 implies that firm i strictly prefers a lower price $p^*_{IL,1}$. However, $p^*_{IL,1} \leq p^*_{U,1}$ (and $p^*_{U,1} > p_L$) can also be excluded from equilibrium behavior for any $p^*_{IH,1}$. Either $p^*_{IH,1} \leq p_H$, in which case firm u wants to lower its price (see Lemma 2). Or $p^*_{IH,1} > p_H$, in which case $p^*_{IH,1}$ should be reduced if $p^*_{IH,1} > p^*_{U,1}$ (see Lemma 1) or firm u should decrease $p^*_{U,1}$ if $p^*_{IH,1} \leq p^*_{U,1}$ (see Lemma 2). Thus, only $p^*_{U,1} = p_L$ can occur in equilibrium, as claimed in i).

Imposing the additional restriction $p^*_{U,1} = p_L$ on the space of equilibrium strategy profiles, it follows from Lemma 1 that $p^*_{IL,1} \leq p_L$ and $p^*_{IH,1} \leq p_H$ in any equilibrium. Strict dominance implies $p^*_{IL,1} \geq p_L$ and $p^*_{IH,1} \geq p_H$, leaving only $p^*_{IL,1} = p_L$ and $p^*_{IH,1} = p_H$ as part of an equilibrium strategy of firm i, as claimed in ii) above.

Step 4: In any PBE and $\forall\ p_{I,1} > p_L$, $\mu^*(p_{I,1}) = I$ and $p^*_{U,2}(p_{I,1}) = p_L I + p_H\,(1 - I)$.

Proof: Since firm u learns the state of demand whenever $p_{I,1} > p_L$, $\mu^*(p_{I,1}) = I$ follows immediately, while $p^*_{U,2}(p_{I,1}) = p_L I + p_H\ (1 - I)$follows then by the same reasoning as in step 2.

Step 5: In any PBE and $\forall\ p_{I,1} < p_L$, either $\mu^*(p_{I,1}) \geq \alpha$ and $p^*_{U,2}(p_{I,1}) = p_L$ or $\mu^*(p_{I,1}) < \alpha$ and $p^*_{U,2}(p_{I,1}) \leq 2p_L -\ p_{I,1}$.

Proof: Since $p_{I,1} < p_L$ is off the equilibrium path and because there are no removed quantities to observe if firm u sets $p^*_{U,1} = p_L$ (which is u's only equilibrium choice), any beliefs $\mu(p_{I,1})$ are consistent with the observation $p_{I,1} < p_L$. However, for any beliefs $\mu(p_{I,1}) \geq \alpha$, $p^*_{U,2}(p_{I,1}) = p_L$ is the only equilibrium response, as we have shown in the fourth paragraph of step 1 (note that Lemma 2 holds for any beliefs $\mu \geq \alpha$).

Only if $\mu(p_{I,1}) < \alpha$, $p^*_{U,2}(p_{I,1}) > p_L$ is a priori not excluded in equilibrium, as becomes obvious if e.g. $\mu(p_{I,1}) = 0$. In this case, prices below p_H being dominated, firm u would set at least $p^*_{U,2}(p_{I,1}) = p_H$. However, $\mu(p_{I,1}) = 0$ and $p^*_{U,2}(p_{I,1}) = p_H$ are not compatible with equilibrium behavior for all $p_{I,1} < p_L$. To see why, suppose $p^*_{U,2}(p_{I,1}) = p_H$ and $\mu(p_{I,1}) = 0$ for some $p_{I,1} = p_L - z$, where z is an abritrarily small positive number. Given this reaction function, firm i will profitably set $p_{IL,1} = p_L - z$ and $p_{IL,2} > p_L + z$, thereby contradicting the beliefs $\mu(p_{I,1}) = 0$. Clearly, $p^*_{U,2}(p_{I,1}) = p_H$ cannot have been optimal for firm u in this case. Hence, even for $p_{I,1} < p_L$, not any combination of $\mu(p_{I,1})$ and $p_{U,2}(p_{I,1})$ can prevail in equilibrium. Only beliefs $\mu^*(p_{I,1})$ implying reaction functions $p^*_{U,2}(p_{I,1}) \leq 2p_L -\ p_{I,1}$ are feasible in any PBE, since $p^*_{U,2}(p_{I,1}) \leq 2p_L -\ p_{I,1}$ makes $p^*_{IL,1} < p_L$ (and $p^*_{IH,1} < p_L$) unprofitable for firm i. This proves step five.

Step 6: The strategies $\sigma_I^* = \{p_L, p_H, p_L, p_H\}, \sigma_U^* = \{p_L, p_L I + p_H (1 - I)\}$ and beliefs $\mu^*(p_{I,1}) = I$ constitute a PBE.

Proof: So far we have established that there can be no other PBE than those stated in Proposition 1. This step makes sure that the strategy profiles and beliefs stated in Proposition 1 *are* indeed an equilibrium. While we prove this for a certain set of beliefs $\mu^*(p_{I,1}) = I$, $\forall p_{I,1}$, it can be shown in an equivalent way for all other feasible set of beliefs.

Consider first firm u. In period 1 and given σ_I^*, firm u faces with probability α a competitor which plays $p_{IL,1}^* = p_L$ and with probability $(1 - \alpha)$ a competitor playing $p_{IH,1}^* = p_H$. Because firm i can never learn anything from observing prices of firm u, the latter need not take any second period consequences into account when making its first period decisions. For period 1, we can therefore refer to Lemma 2, which implies that firm u will have no incentive to set a price higher than p_L. But since setting a price $p_{U,1}^* < p_L$ is strictly dominated in any case, $p_{U,1}^* = p_L$ is best answer to σ_I^*.

In period 2, firm u can infer demand for any $p_{I,1} > p_L$, such that the only rational belief is $\mu^*(p_{I,1}) = I$. If $I = 0$, that is if demand is known to be high, prices $p_{U,2}^*(p_{I,1}) < p_H$ are strictly dominated, and since Lemma 1 implies that raising the price above p_H is not optimal given $p_{IH,2}^* = p_H$, it follows that $p_{U,2}^*(p_{I,1}) = p_H$ is best answer to σ_I^* for $I = 0$. For analogous reasons, firm u has no incentives to deviate from $p_{U,2}^*(p_{I,1}) = p_L$ if demand is known to be low ($I = 0$) and given σ_I^*. This implies that given σ_I^*, $p_{U,2}^*(p_{I,1}) = p_L I + p_H (1 - I)$ is optimal for any $p_{I,1} > p_L$. Next, if $p_{I,1} = p_L$, $\mu^*(p_{I,1}) = 1$ is the only belief consistent with σ_I^*, and for any $p_{I,1} < p_L$, $\mu^*(p_{I,1}) = 1$ is also consistent with σ_I^* and Bayesian updating (if $p_{I,1} < p_L$, any beliefs are feasible). Given $\mu^*(p_{I,1}) = 1$ and by virtue of Lemma 1, $p_{U,2}^*(p_{I,1}) = p_L$ for $p_{I,1} \leq p_L$ is best answer to σ_I^*. This establishes that $p_{U,2}^*(p_{I,1}) = p_L I + p_H (1 - I)$ for any $p_{I,1}$ is firm u's optimal response to σ_I^*.

We now turn to the informed firm. Given the equilibrium reaction function $p_{U,2}^*(p_{I,1})$ of firm u, firm i would give up something in the first period without gaining anything in the second period if it chose a price $p_{IL,1} < p_L$ or $p_{IH,1} < p_H$. By virtue of Lemma 1, $p_{IL,1} > p_L$ or $p_{IH,1} > p_H$ is also incompatible with equilibrium behavior given σ_U^*. This implies $p_{IL,1}^* = p_L$ and $p_{IH,1}^* = p_H$. In the second period, Lemma 1 (and again, the fact that lower prices are strictly dominated) imply $p_{IL,2}^* = p_L$ and $p_{IH,2}^* = p_H$, so that σ_I^* is strict best answer to σ_U^*. This proves our final step and thus completes the proof of Proposition 1. Q.E.D