Diskussionsschriften

**Subgame Perfect Punishment
for Repeat Offenders**

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Abstract

First we show that for wealth-constrained agents who may commit an act twice the optimal sanctions are the offender's entire wealth for the first and zero for the second crime. Then we ask the question whether this decreasing sanction scheme is subgame perfect (time consistent), i.e., does a rent-seeking government stick to this sanction scheme after the first crime has occurred. If the benefit and/or the harm from the crime are not too large, this is indeed the case; otherwise, equal sanctions for both crimes are optimal.
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1. Introduction

The literature on optimal law enforcement typically assumes that the government can commit to sanction schemes.[1] This means the government can use any set of threats to penalize wrongdoers. In particular, if a crime occurs, the government actually sanctions the wrongdoers even though, ex post, it may have no incentive to do so. Potential wrongdoers believe that the government will carry out the threat at any cost and, therefore, do not engage in the act in the first place.

In this paper we give up the assumption that the government can commit to whatever sanction scheme. We consider the analysis of optimal sanctions without the possibility to commit important because judges often have a lot of discretion as to the size of the penalty: they may, for example, adjust sanctions to the financial possibilities, the age, the education, etc. of the wrongdoer. Accordingly, we allow only for sanctions that the government actually wishes to implement should a crime have occurred.

Ruling out full commitment changes the optimal enforcement schemes. Suppose, for example, the government does not care about the sanction as is typically assumed in the literature. Then it will not enforce the penalty if a crime has happened given that there is, say, a small cost of doing so. The rational criminal will anticipate the ex post enforcement behavior of the government. Therefore, she will commit the crime because the threat of being sanctioned is not credible. Once we drop the commitment assumption, the typical deterrence equilibria of the law enforcement literature between potential wrongdoers and the government are based on empty threats. In the language of game theory, the equilibria are not subgame perfect or time consistent.

We study the problem of subgame perfect sanctions using the framework of Emons (2003). Agents may commit a crime twice. The act is inefficient; the agents are thus to be deterred. The agents are wealth constrained so that increasing the fine for the first offense means a reduction in the possible sanction for the second offense and vice versa. The agents may follow history dependent strategies, i.e., commit the crime a second time if and only if they were (were not) apprehended the first time. The government seeks to minimize the probability of apprehension.

Ignoring the government's commitment problem, it is optimal to set the sanction for the first offense equal to the entire wealth of the agents while

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[1]See, e.g., Garoupa (1997) or Polinsky and Shavell (2000) for surveys.

the sanction for the second offense equals zero. The intuition is as follows: A money penalty imposed for the second offense reduces the amount a person can pay for the first offense, since the wealth available to pay penalties is assumed to be fixed over the two periods. For that reason, a higher probability event – namely, a first offense that is detected – will be more effective use of the scarce money penalty resource than a lower probability event – namely, a second detected offense. Why is the probability of detection lower for the second rather than for the first crime? Simply, because an agent faces the *possibility* of being sanctioned for the second crime if and only if she has already been sanctioned for the first time. For our further results it is important to note that the optimal probability of apprehension increases with the benefit from the crimes.

This decreasing sanction scheme raises of course the issue of time consistency. Will the government really charge the agent the entire wealth when she was apprehended for the first crime, knowing that then she will commit the second act for sure? Isn't it better for the government to renege and charge little for the first act so that the agent still has sufficient wealth to pay a sanction that deters the second crime? Given that the first act has been committed anyway, that way the government can at least deter the second act.

To study this problem we consider a rent-seeking government. The sanctions paid by the criminals enter the government's welfare function. Our government, therefore, has an ex post incentive to collect fines. The government can commit to a probability of apprehension but not to sanctions. Our basic result is that if the agent's benefit and/or the harm from the crime are not too large, then the scheme where the sanction for the first crime is the entire wealth and the sanction for the second crime is zero is indeed subgame perfect.

To see this, consider the government after the agent has been apprehended for the first crime. If it implements our decreasing sanction scheme, it appropriates the entire wealth yet incurs the harm of the second crime. Thus, the lower the harm of the second crime, the more attractive is this option.

The alternative is to set the sanction for the second crime to a level that deters the act. With this option the government doesn't incur the harm of the second crime, yet forgoes the sanction for the second crime because it is deterred. If the benefit from the crime goes up, the optimal probability of apprehension increases, yet by more than the benefit; therefore, the actual

sanction necessary to deter the second crime falls. Since a low sanction for the second crime means a high amount the government can charge for the first crime, a high benefit of the second crime makes this option attractive. Accordingly, only for low benefits the government sticks to the decreasing sanction scheme.

If the benefit and/or the harm of the second crime are large, our decreasing sanction scheme is no longer time consistent. The government prefers to deter the second crime should the first crime have occurred. Accordingly, only sanction schemes where each sanction by itself deters the corresponding crime are time consistent. In this case the optimal subgame perfect sanction scheme entails equal sanctions in both periods. Enforcement costs are higher than with the decreasing sanction scheme.

The only paper we are aware of that deals with the problem of time consistent sanctions is Boadway and Keen (1998). They consider a government choosing a capital income tax rate and an enforcement policy. The government can commit to the enforcement policy but not to the tax rate. Ex ante the government wishes to announce a low tax rate to induce savings; ex post, when savings have been made, it will renege and apply a high tax rate. Boadway and Keen show that by committing to a lax enforcement policy the government can alleviate the welfare loss implied by its inability to commit to the tax rate.

In the next section we describe the model. In section 3 we derive the optimal sanctions for a government that can commit and in section 4 for a government that cannot commit. Section 5 concludes.

2. The Model

Consider a set of potential wrongdoers which has measure 1. Individuals live for two periods. In each period the agents can engage in an illegal activity, such as false parking, illegally raising prices, polluting the environment, or evading taxes. If an agent commits the act in either period, she receives a monetary benefit $b > 0$. We consider crimes without social gains. Using the language of Polinsky and Rubinfeld (1991), b is the illicit gain and the crime creates no acceptable gain.[2] The act causes a monetary harm h to society

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[2]See also Chu, Hu, and Huang (2000) for an analysis of crimes without social gains. They argue that the gains to the offender are not considered because the crime is not socially acceptable or because the gains of offenders (such as theft or other zero-sum crimes) offset with the victims' losses.

which is borne by the government. Since $h > 0$, the act is not socially desirable. The individuals are thus to be deterred from the activity.

To achieve deterrence the government chooses sanctions and a probability of apprehension. The government cannot tell whether an agent is in the first or second period of her life. The government only observes whether the crime is the first or the second one. Accordingly, the government uses fines $s_1, s_2 \geq 0$ where s_1 applies to first-time and s_2 to second-time observed offenders.

We assume that the government cannot commit to sanctions. This means that the government can choose a different sanction from the one announced at the outset once a crime occurred. Typically, a judge always finds good reasons to reduce or increase sanctions. In addition to sanctions, the government chooses a probability of apprehension p. This probability is the same for first- and second-time offenses.[3] It is irrevocably fixed before the agents take their actions. The government cannot easily change the amounts spent on, say, training the police. Accordingly, we assume that the government can commit to p while it cannot commit to sanctions.[4]

In the law enforcement literature the optimal policy is derived by maximizing the sum of the offenders' benefits minus the harm caused by the offenses minus law enforcement expenditures. Sanctions do not enter the benevolent government's objective function because they are a mere transfer of money.[5] Within this framework the literature derives the results on optimal fines and optimal probabilities of apprehension. See, e.g., Garoupa (1997) or Polinsky and Shavell (2000).

Nevertheless, these results hold true if and only if the government can fully commit to the probability of apprehension *and* to the announced sanction. To see this, suppose the government incurs a small cost $\varepsilon > 0$ of collecting the fine. Suppose the agent has been apprehended for the crime and then the government strategically decides whether or not to impose the sanction. With such a sequencing, the rational government will not impose the fine: it

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[3]We thus rule out the case where agents with a criminal record are more closely monitored than agents without a record. See Landsberger and Meilijson (1982) for an analysis of optimal detection probabilities.

[4]Boadway and Keen (1998) use the same commitment structure when studying the time consistency problem in the taxation of capital income.

[5]In the explicit formulation welfare is the criminal's utility (benefit minus expected sanction) plus the government's utility (expected sanction minus harm) minus enforcement costs.

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does not care about the fine anyway and it can safe the cost ε. Anticipating this ex post behavior of the government, the threat of being sanctioned is not credible and the agent will commit the act in the first place. To put it in the language of game theory: the equilibrium in the game between the offender and the government is not subgame perfect.

If we want to take the issue of subgame perfection (or time consistency) seriously, we must give the government an incentive to actually collect the fines. We do so by including the sanctions in the government's payoffs.[6] Our government maximizes revenues from sanctions minus the harms minus the enforcement expenditure and has thus an incentive to collect the fine should a crime have occurred. To save on notation we take the probability of apprehension p as an indicator of the enforcement expenditure.

This approach can be motivated in several ways. Garoupa and Klerman (2002) take the public choice perspective of a self-interested, rent-seeking government which maximizes revenues minus the harm borne by the government minus expenditure on law enforcement.[7] Polinsky and Shavell (2000) consider the standard benevolent welfare function and add a term reflecting individuals' fairness-related utility. If this fairness-related utility equals the actual sanction, their government maximizes the same welfare function as ours.[8]

Individuals are risk neutral and maximize expected income. They have initial wealth $W > 0$. Think of W as the value of the privately owned house or assets with a long maturity. The agents hold on to their wealth over both periods unless the government interferes with sanctions. Any additional income they receive in both periods, be it through legal or illegal activities, is consumed immediately. Accordingly, all the government can confiscate is W. If the fine exceeds the agent's wealth, she goes bankrupt and the government seizes the remaining assets. This implies that the fines s_1 and s_2 have to satisfy the "budget constraint" $s_1 + s_2 = W$.[9]

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[6]In terms of the explicit welfare function given in the preceding footnote, we simply exclude the criminal's utility (benefit minus expected sanction).

[7]Dittmann (2001) uses a similar approach.

[8]In Rubinstein (1979) the government's payoffs also depend on whether or not it punishes the offender. Unlike the other papers, Rubinstein's government is worse off if it punishes the offender, independently of whether the act was committed intentionally or not.

[9]This assumption distinguishes our approach from Polinsky and Shavell (1998) who work with a maximum per period sanction s_m. Accordingly, they may set $s_1 = s_2 = s_m$, which is typically the optimal enforcement scheme. In their framework s_m is like a per

To save on notation let the interest rate be zero. An agent can choose between the following strategies:

- She can choose not to commit the act at all. We call this strategy (0,0) which gives rise to utility $U(0,0) = W$. This is the strategy we wish to implement.

- She can choose to commit the act in period 1 and not in period 2. Call this strategy (1,0); here we have $U(1,0) = W + b - ps_1$. The act generates benefit b; with probability p the agent is apprehended and pays the sanction s_1.

- The agent can opt to commit the crime in period 2 but not in period 1. Call this strategy (0,1) generating utility $U(0,1) = W + b - ps_1$. With strategy (0,1) the agent has the same utility as with strategy (1,0) because the government observes only one offense.

- Moreover, the agent can commit the act in both periods which we denote by $(1,1)$ and $U(1,1) = W + b - ps_1 + b - p((1-p)s_1 + ps_2))$. The second crime is detected with probability p. With probability p the agent has a criminal record in the second period and thus is fined s_2; with probability $(1-p)$ she has no record and pays s_1 if apprehended.

- Finally, the agent can choose two history dependent strategies.[10] First, she commits the act in period 1. If she is not apprehended, she also commits the act in period 2; however, if she is apprehended in period 1, she does not commit the act in period 2. Call this strategy (1,(1|no record;0|otherwise)) with $U(1,(1|\text{no record};0|\text{otherwise})) = W + b - ps_1 + (1-p)(b - ps_1)$. Since the agent stops her criminal activities if she is apprehended once, she is never sanctioned with s_2.

- Second, she commits the act in period 1. If she is not apprehended, she does not commit the act in period 2; however, if she is apprehended in period 1, she commits the act in period 2. Call this strategy (1,(0|no record;1|otherwise)) with $U(1,(0|\text{no record};1|\text{otherwise})) = W + b -$

period income which cannot be transferred into the next period. Burnovski and Safra (1994) use the same budget constraint as we do.

[10]These history dependent strategies distinguish our paper from Burnovski and Safra (1994) where individuals decide ex ante simply on the number of crimes.

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$ps_1 + p(b - ps_2)$. It turns out that this strategy defines the agents' binding incentive constraint for the optimal sanctions.

Before we start deriving optimal sanctions, we have to ensure that the government indeed wants complete deterrence. We achieve this by assuming $W - 2h < -1$. If the government completely deters, there is neither harm nor revenue and the maximum possible expenditure for deterrence is 1 (recall that we take the probability of apprehension as a measure for enforcement cost). If the government doesn't deter at all, enforcement costs are zero, the government incurs the harm twice, and the maximal revenue it can obtain is the agents' wealth W. Accordingly, if the harm is sufficiently large, the rent-seeking government wants complete deterrence.

Let us now analyze sanctions that give the agents proper incentives not to engage in the activity in either period. We first derive the cost-minimizing sanction scheme that achieves perfect deterrence ignoring the government's commitment problem. The analysis follows Emons (2003). We will consider the government's incentives to implement this penalty scheme in section 4.

3. Optimal Sanctions if the Government can commit

We assume that agents have enough wealth so that deterrence is always possible, i.e., $2b < W$. The agent does not follow strategy (1,0), if $U(1,0) \leq U(0,0)$, she does not follow strategy (0,1), if $U(0,1) \leq U(0,0)$, etc. Straightforward computations confirm that the agent does not engage in strategies (1,0), (0,1), and (1,(1|no record;0|otherwise)), if

$$s_1 \geq b/p; \tag{1}$$

she does not pick strategy $(1, 1)$, if

$$s_2 \geq (2b/p^2) - s_1((2/p) - 1); \tag{2}$$

and she does not pick strategy (1,(0|no record;1|otherwise)), if

$$s_2 \geq (b(1 + p)/p^2) - s_1/p. \tag{3}$$

insert **Figures 1 and 2** around here

Accordingly, with all sanction schemes (s_1, s_2) to the right of the bold line in Figures 1 and 2, the agent has proper incentives and commits no crime. For example, the scheme $s_1 = s_2 = b/p$ induces no crimes.

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Let us next minimize the enforcement costs, as given by p, while providing incentives not to commit any crime.[11] We will minimize p taking the incentive constraint (3) into account. Then we show that the optimal $\hat{p}$ also satisfies the incentive constraints (1) and (2).

Obviously, Becker's (1968) maximum fine result applies here, meaning that in order to minimize p the government will use the agent's entire wealth for sanctions.[12] Accordingly, plugging the budget constraint $s_1 + s_2 = W$ into (3) and differentiating the equality yields

$$dp/ds_1 = (p - p^2)/(b - s_1 - 2p(W - s_1)) < 0$$

for $b < s_1 \leq W$. Consequently,

$$\hat{s}_1 = W, \ \hat{s}_2 = 0, \ \text{and} \ \hat{p} = b/(W - b).$$

Since $b/p < 2b/p(1 - p) < b(1 + p)/p \ \forall p \in (0, 1)$, the incentive constraints (1) and (2) are also satisfied.

We thus find that the optimal sanction scheme sets $\hat{s}_1 = W$ and $\hat{s}_2 = 0$. First time offenders are punished with the maximal possible sanction while second time offenders are not punished at all. The sanction s_1 is high enough that it not only deters first-time offenses but also second-time offenses even though they come for free.

The intuition for this result follows immediately from the incentive constraint (3). The agent pays the sanction s_1 with probability p and the sanction s_2 only with probability p^2. To put it differently: The agent is charged s_2 with probability p if and only if she has paid already s_1. Since paying the fine s_1 is more likely than paying s_2, shifting resources from s_2 to s_1 increases deterrence for given p. Consequently, p is minimized by putting all the scarce resources into s_1.

It is perhaps somewhat surprising that the strategy (1,(0|no record;1|otherwise)) and not the strategy (1,(1|no record;0|otherwise)) defines the binding incentive constraint in the optimal penalty structure. Given that the optimal penalties are declining, an agent who was not apprehended for the first crime has a strong incentive not to commit the act a second time: if she is apprehended she pays the high sanction s_1. If the agent was, however, apprehended for the first crime, the second crime comes for free. The sanction

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[11]Since in our setup the harm of the crime exceeds its acceptable benefit, maximizing social welfare boils down to minimizing enforcement costs.

[12]If $s_1 + s_2 < W$, sanctions can be raised and p lowered so as to keep deterrence constant.

s_1 has to be high enough so that she doesn't commit the first crime in the first place.

4. Optimal Sanctions if the Government cannot commit

Let us now check under which conditions the sanction scheme $\hat{s}_1 = W$, $\hat{s}_2 = 0$ together with the minimal enforcement probability $\hat{p} = b/(W-b)$ is subgame perfect. This means: Does the government really implement these sanctions once the agent has committed a crime? To do so, consider the subgame starting when the agent has been apprehended for the first crime.

If the government sticks to the penalty scheme $\hat{s}_1 = W$, $\hat{s}_2 = 0$, the agent will commit the second offense for sure because it comes for free. The government's payoff then is $W - 2h - \hat{p}$. It incurs the harm twice and seizes the agent's entire wealth with s_1.

The alternative is to lower s_1 and at the same time increase s_2 such that the agent doesn't commit the second act. Obviously, the rent-seeking government will set $s_2 = b/\hat{p}$, the minimal sanction achieving deterrence. The government goes for the minimal sanction guaranteeing deterrence because, by its very nature, the government will not get this money; that way, s_1 is as large as possible. Using $\hat{p} = b/(W - b)$, we find $s_2 = W - b$ and $s_1 = b$. If the government follows this strategy, its payoffs are $-h + b - \hat{p}$. It incurs the harm from the first crime, collects $s_1 = b$ and there is no more crime.

Comparing the two payoffs, obviously the government prefers to stick to $\hat{s}_1 = W$, $\hat{s}_2 = 0$ if $W - h \geq b$. The government gets the entire wealth less the harm by sticking to the optimal incentive scheme whereas it gets $s_1 = b$ if it chooses to deter the second offense. We may, therefore, conclude that $s_1^* = W$, $s_2^* = 0$ is subgame perfect if the agent's benefit b and/or the harm are not too large. See Figure 1.

Let us now determine the optimal subgame perfect sanction scheme together with the probability of detection p if $W - h < b$. Consider again the government deciding on sanctions after the wrongdoer has been apprehended for the first act. If the government wants to deter the second act, it will set $s_2 = b/p$. It chooses the minimal sanction ensuring deterrence because it will not get the money. This way it can collect the maximum amount $s_1 = W - b/p$ for the first act from the agent.

In contrast, the government may wish to induce the second crime. It does so by setting $s_2 < b/p$. The government collects s_2 only with probability p; it collects s_1 for sure because we are in the node where the government has just

apprehended the agent for the first crime. Since $W = s_1 + s_2$, the revenue maximizing government sets $s_1 = W$ and $s_2 = 0$ if it wants to induce the second crime. This generates a payoff of $W - 2h - p$ for the government.

The government prefers the strategy of inducing the second crime to optimally deterring the second crime if $W - 2h - p \geq W - h - b/p - p \Leftrightarrow b/p > h$. Deterring the second crime has the cost of the foregone revenue $s_2 = b/p$; encouraging the second crime has the cost of the harm h.

The left-hand side of the inequality $b/p > h$ is decreasing in p. Therefore, if it is not satisfied for the minimal probability of apprehension inducing no crimes $\hat{p} = b/(W - b)$, it does not hold for any p deterring both crimes. Thus, if $b/\hat{p} < h \Leftrightarrow W - h < b$, the government prefers to deter the second crime and does so optimally by setting $s_1^* = s_2^* = W/2$ and $p^* = 2b/W$. See Figure 2.

A low probability of apprehension increases b/p, the sanction which is necessary to deter the second crime. Deterring a second crime thus becomes unattractive. By choosing a low p, the government commits not to raise s_2 to a level which deters. This result is similar to Boadway and Keen (1998) where the government commits to a lax enforcement in order not to raise tax rates after savings decisions have been made.

We summarize the preceding observations with the following proposition.

Proposition: *If $W - h \geq b$, the optimal subgame perfect sanction scheme is given by $s_1^* = W$, $s_2^* = 0$ and $p^* = b/(W - b)$.*

If $W - h < b$, the optimal subgame perfect sanction scheme is given by $s_1^* = s_2^* = W/2$ and $p^* = 2b/W$.

Obviously, the government is better off in the first case where it uses the decreasing sanction scheme. In both cases crime is completely deterred. With the decreasing sanction scheme the probability of apprehension and hence enforcement cost is lower than in the second case of constant sanctions.

5. Conclusions

The purpose of this paper is to analyze subgame perfect sanction schemes, i.e., sanctions which the government indeed wants to implement should a crime have occurred. We consider the problem of time consistency important because judges tend to have a lot of discretion as to the size of the penalty. They anticipate that a high penalty now may reduce the potential for future

sanctions. Rational criminals will anticipate this and thus not be deterred by empty threats.

A rent-seeking government will stick to the optimal decreasing sanction scheme if it gets more money by allowing the second crime and cashing in the agent's entire wealth with the first sanction than by deterring the second crime. In the opposite case the government prefers to deter the second crime. It does so with equal sanctions for both crimes.

Accordingly, the constraint of time consistency has bite. If the government can commit, decreasing sanctions are always optimal; if the government cannot commit, decreasing sanctions may still be optimal but so may be equal sanctions. We haven't explained escalating sanctions based on offense history which are embedded in many penal codes and sentencing guidelines. Explaining escalating sanctions seems to be fairly difficult for the law enforcement literature.[13] Nevertheless, in our set-up the commitment issue ruled out decreasing sanction schemes in some cases. Perhaps the problem of time consistency is a fruitful track for future research to explain escalating sanction schemes.

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[13]See Emons (2003) for a discussion of the problems the law enforcement has in explaining escalating sanctions.

References

BECKER, G.: Crime and Punishment: An Economic Approach, *Journal of Political Economy*, 76 (1968), 169-217.

BOADWAY, R. AND M. KEEN: Evasion and Time Consistency in the Taxation of Capital Income, *International Economic Review*, 39 (1998), 461-476.

BURNOVSKI, M. AND Z. SAFRA: Deterrence Effects of Sequential Punishment Policies: Should Repeat Offenders be more Severely Punished, *International Review of Law and Economics*, 14 (1994), 341-350.

CHU, C. Y. CYRUS, SHENG-CHENG HU, AND TING-YUAN HUANG: Punishing Repeat Offenders more Severely, *International Review of Law and Economics*, 20 (2000), 127-140.

DITTMANN, I.: The Optimal Use of Fines and Imprisonment if Governments Don't Maximize Welfare, Discussion Paper, Humboldt-Universität Berlin, $http://papers.ssrn.com/sol3/papers.cfm?abstract\_id = 274449$

EMONS, W.: A Note on the Optimal Punishment for Repeat Offenders, *International Review of Law and Economics*, (2003), forthcoming, $http://www.vwi.unibe.ch/theory/papers/emons/pun\_0502.pdf$

GAROUPA, N.: The Theory of Optimal Law Enforcement, *Journal of Economic Surveys*, 11 (1997), 267-295.

GAROUPA, N. AND D. KLERMAN: Optimal Law Enforcement with a Rent-Seeking Government, *American Law and Economics Review*, 4 (2002), 116-140.

LANDSBERGER, M. AND I. MEILIJSON: Incentive Generating State Dependent Penalty System, The Case of Income Tax Evasion, *Journal of Public Economics* 19 (1982), 333-352.

POLINSKY, M. AND S. SHAVELL: The Economic Theory of Public Enforcement of Law, *Journal of Economic Literature*, 38 (2000), 45-76.

POLINSKY, M. AND S. SHAVELL: On Offense History and the Theory of Deterrence, *International Review of Law and Economics*, 18 (1998), 305-324.

POLINSKY, M. AND S. SHAVELL: The Fairness of Sanctions: Some Implications for optimal Enforcement Policy, *American Law and Economics Review*, 2 (2000), 223-237.

POLINSKY, M. AND D. RUBINFELD: A Model of Fines for Repeat Offenders, *Journal of Public Economics* 46 (1991), 291-306.

RUBINSTEIN, A.: An Optimal Conviction Policy for Offenses that May Have Been Committed by Accident, *in:* Applied Game Theory, S. Brams, A. Schotter, and G. Schwödiauer (Eds.) Würzburg, Physica-Verlag, 1979, 406-413.
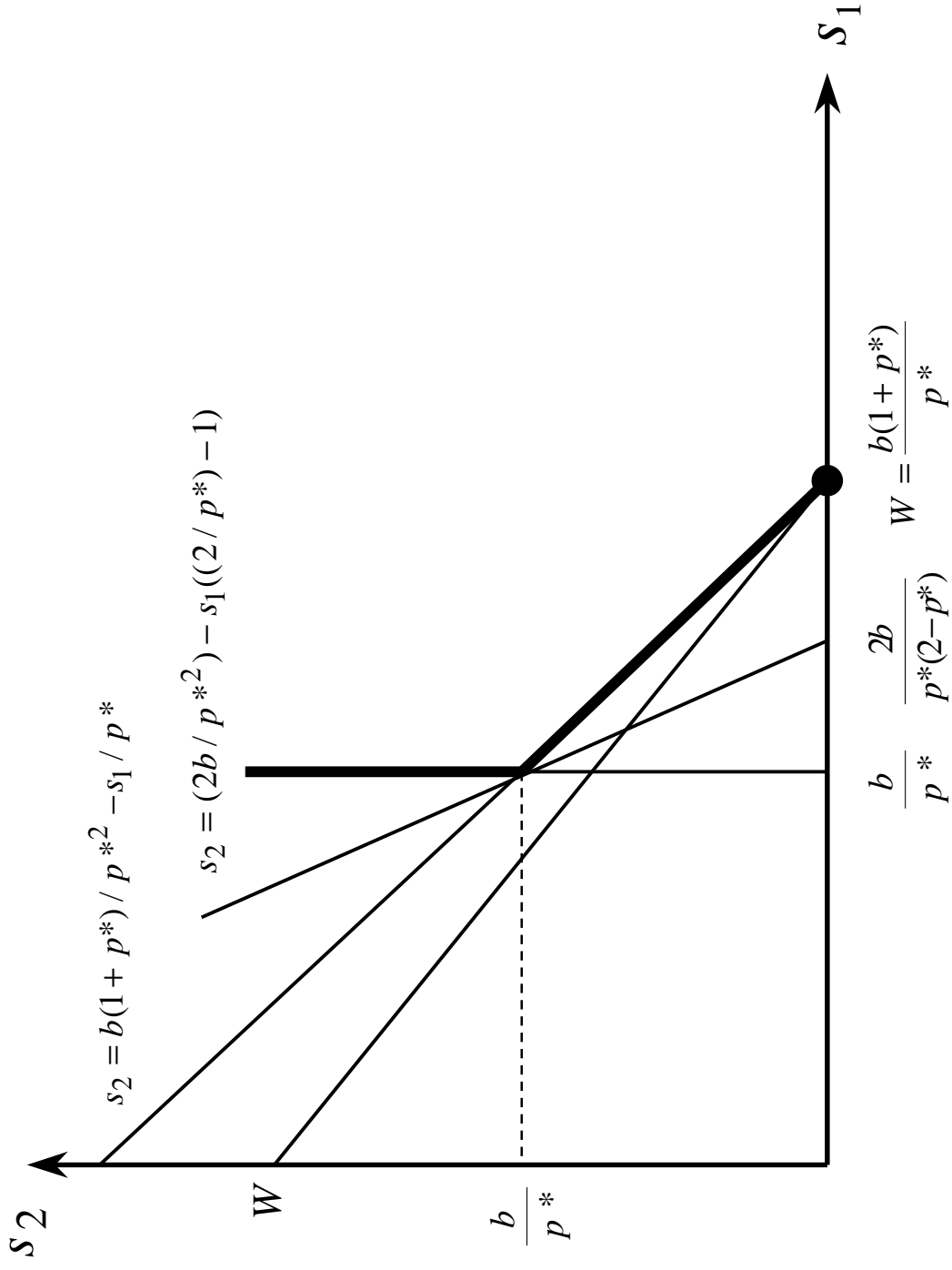
Figure 1: The Set of Incentive Compatible Sanctions and the Optimal Sanction Scheme $(s_1^*, s_2^*) = (W, 0)$ and $p^* = b/(W-b)$
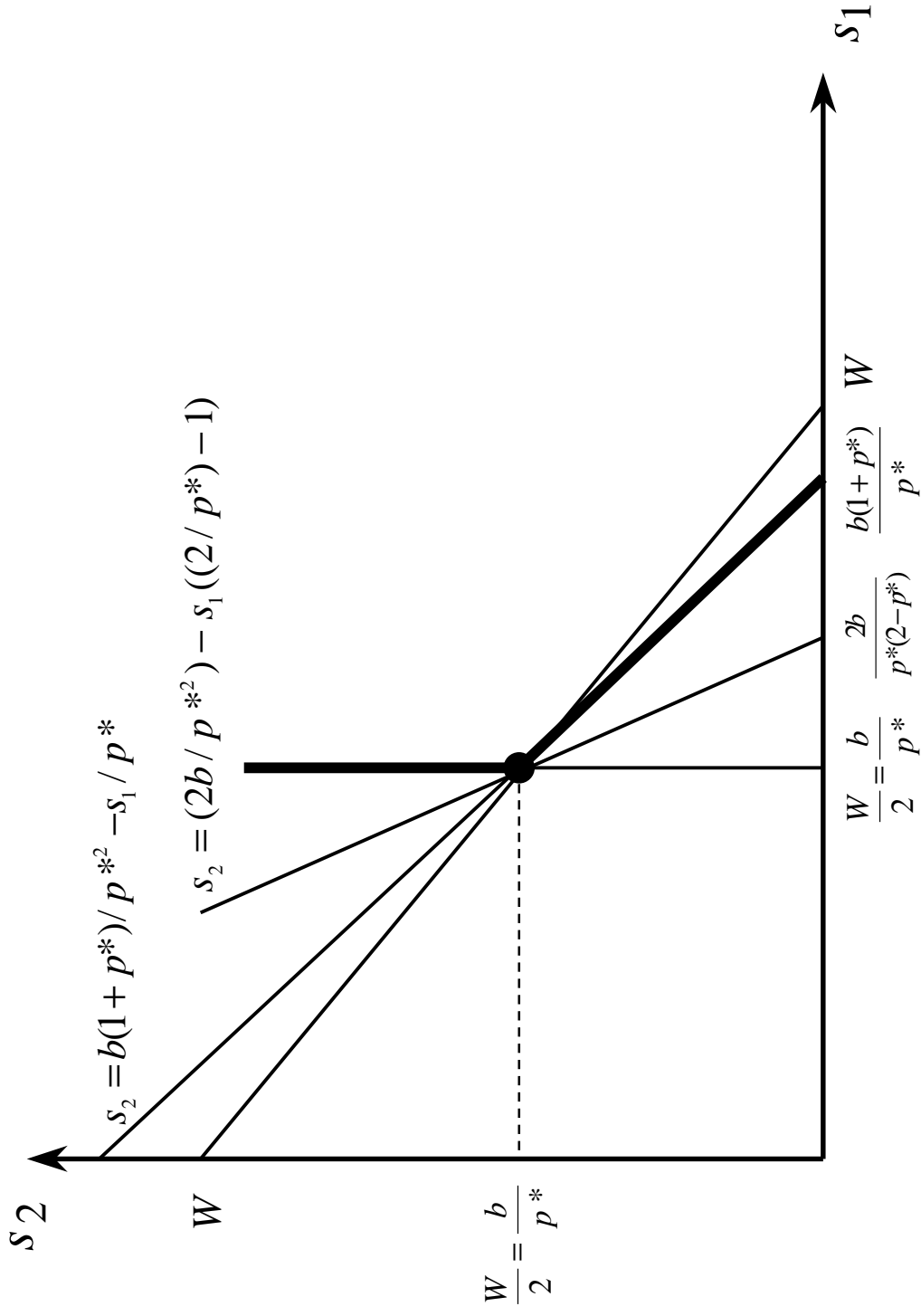
Figure 2: The Set of Incentive Compatible Sanctions and the Optimal Sanction Scheme $(s_1^*, s_2^*) = (W/2, W/2)$ and $p^* = 2b/W$

The axes are labeled s_2 (vertical) and s_1 (horizontal).

$s_2 = b(1+p^*)/p^{*2} - s_1/p^*$

$s_2 = (2b/p^{*2}) - s_1((2/p^*)-1)$

W

$\dfrac{W}{2} = \dfrac{b}{p^*}$

$\dfrac{W}{2} = \dfrac{b}{p^*}$

$\dfrac{2b}{p^*(2-p^*)}$

$\dfrac{b(1+p^*)}{p^*}$

W