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Equilibrium and Segregation with Local Income Taxation When Households Differ in Both Preferences and Incomes*

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Abstract

This paper presents a model of an urban area with local income taxes used to finance a local public good. Households differ in both incomes and their taste for housing. The existence of an asymetric equilibrium is shown in a calibrated two-community model assuming single-peaked distributions for income and housing taste. The equilibrium features segregation of households by both incomes and tastes. The high-tax community shows lower housing prices and lower public good provision than the low-tax community. The model is able to explain the substantial differences in local income tax level and average income across communities as observed in, e.g., Switzerland. The numerical investigation suggests that taste heterogeneity reduces the distributional effects of local tax differences. The numerical investigation also suggests that the ability of the rich community to set low taxes is higher when this community is physically small. However, a tax haven need not be small.

Key Words: Segregation, Fiscal Federalism, Income Taxation, Local Public Goods

JEL-classification: H71, H73, R13

*The term 'stratification' has been replaced by 'segregation' in the course of revision.

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1 Introduction

Decentralized decision making at appropriate levels of government is in many countries viewed as an essential factor for good government. In some countries the federal system involves a decentralized fiscal structure. In the United States, for example, the central government raises progressive income taxes while the individual states collect retail sales taxes and many communities are financed by property taxes. Fiscal Federalism has recently been intensively debated in the European Union. On the one hand there are attempts to coordinate fiscal policies across EU member states. On the other, increased regional self-government, as implied by the subsidiarity principle, calls for some regional fiscal autonomy. Oates (1972) argues that local units deciding upon public programs are more likely to trade off costs against benefits if these programs are financed by local taxes.

The formal framework for the study of local provision of local public goods originates in Tiebout's (1956) seminal work. Tiebout showed that fiscal decentralization leads to an efficient provision of local public goods because people with similar preferences would settle in a particular location and vote for their desired level of public goods provision. Tiebout's result rests heavily on the assumption that households have equal incomes. When households differ in incomes, this result may no longer hold. In this situation it is advantageous for high-income households to locate with other high-income households and hence reduce their tax burden. Furthermore, low-income households may also prefer to locate with high-income households to benefit from their large tax base. So, while the rich try to gather in a rich community, the poor may seek to follow and hence leave an ever smaller and poorer community. However, as the literature outlined in the next paragraph has shown, this 'race to the bottom' need not take place. The paper extends this finding to local income taxation.

Following Tiebout, there is a long tradition of modelling fiscal decentralization at community level. The consideration of heterogeneous household income by Ellickson (1971) and Westhoff (1977) moved the focus away from seeking optimal community size to the study of urban areas with given community borders. While this strand of literature was followed by a large number of studies investigating local *property taxation* (surveyed in Ross and Yinger 1999), there have been few contributions on local taxation of *income* (e.g. Hansen and Kessler 2001a).

Multi-community models with heterogeneous agents predict a segregation of the population by income, i.e. households of the same income group live in the same community. While differences of average income across local jurisdictions are typically observed, perfect segregation of income groups is not an empirical phenomenon. This incomplete income segregation of the population may be attributed to heterogeneous tastes for public goods and housing or to a certain preference for a particular place. Epple and Platt (1998) study a model with property taxation and show that the introduction of heterogeneous taste for housing indeed predicts a more realistic incomplete segregation of the population. Epple and Sieg (1999) test the predicted income segregation and show that it is able to explain the differences of income distributions across communities in the Boston metropolitan area.

This paper follows Epple and Platt (1998) but introduces heterogeneous tastes in a multi-community model with local *income taxation* and a partly substitutable public good. The income taxation model has been investigated by Goodspeed (1989). This study generalizes Goodspeed's analysis both by introducing heterogeneous tastes and by using a realistic single-peaked distribution of the population. Not only does this single-peakedness capture a realistic feature of urban economies, but it also challenges the existence of equilibria in multi-community models with income taxation. The possible non-existence of segregated equilibria in a model with local income taxation is shown by Hansen and Kessler (2001b).

Switzerland is an exemplary case of a federal fiscal system. Switzerland is a federation of 26 states, the so-called cantons. The cantons are divided into individual communities of varying size and population. The roughly 3000 communities form individual jurisdictions with great autonomy in terms of providing local public goods such as school services or infrastructure. The unique situation in Switzerland is that the communities finance their expenditures mainly by local income taxes. While cantons autonomously organize the whole tax system, e.g. the degree of tax progression or the split between income and corporate taxes, the communities can generally only set a tax shifter in a given cantonal tax scheme. There is considerable variation in income taxes across Swiss communities. For example, for a two-child family with a gross income of 100,000 Swiss francs (CHF) the sum of cantonal and community income tax ranged from

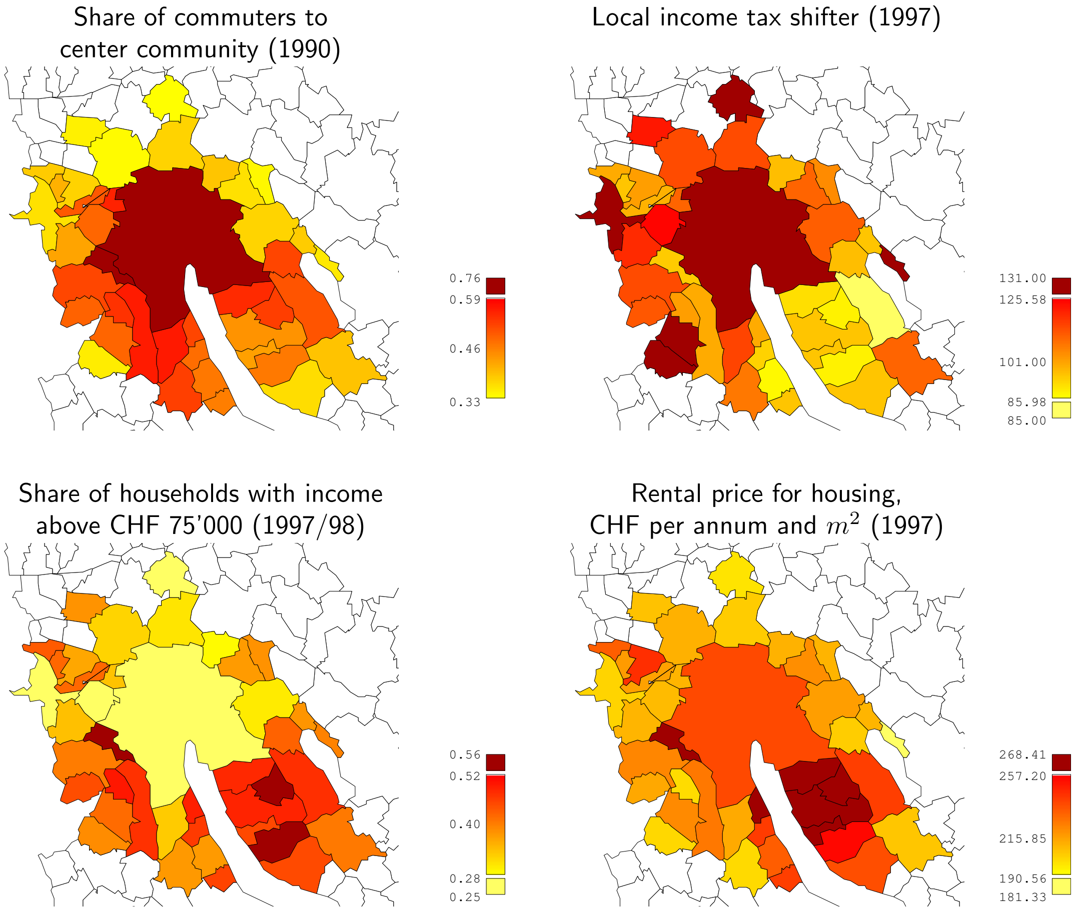

Figure 1: Community characteristics in the metropolitan area of Zurich.[2]

CHF 3,500 (city of Zug) to CHF 14,500 (city of Neuchâtel) in the year 2001.[1] Within metropolitan areas the (community) tax differences are smaller but may still differ by a factor of up to 1.5 in the Zurich area for example. Figure 1 shows the substantial differences in local tax levels, income and housing prices across this community system.[2] The bottom-left map visualizes the considerable segregation by incomes in the Zurich area. The top-right and the bottom-left maps demonstrate a striking relationship between income taxation and spatial income

[1] Taken into consideration are the tax rates of the cantonal capitals. Some smaller communities show even higher respectively lower tax rates.

[2] Data from the following sources: Commuter: Swiss Federal Statistical Office, Census 1990. Tax rates: Statistisches Amt des Kantons Zürich, Steuerfüsse 1997. Income distribution: Swiss Federal Tax Administration. Housing prices: Wüest und Partner, Zurich. Considered are all communities where more than 1/3 of the working population is commuting to the center community.

distribution: the local share of rich households is almost an inverted picture of the local tax levels. It is particularly interesting to see whether multi-community models are able to explain the observed tax differentials.

The paper is organized as follows: Section 2 introduces the formal model and derives the properties of the household utility function that induce segregation of the population. In the first part of Section 3 the calibration used for the numerical investigation of the equilibrium is described. In the second part of Section 3 the numerical equilibrium is presented and the welfare implications of the decentralized decision making are discussed. Section 4 draws conclusions.

2 The Model

The model economy is divided into J distinct communities. The area is populated by a continuum of heterogeneous households, which differ in both income $y \in [\underline{y}, \overline{y}]$, $0 < \underline{y}, \overline{y} < \infty$, and a parameter $\alpha \in [0, 1]$ describing their taste for housing. Income and taste are jointly distributed according to the density function $f(y, \alpha) > 0$. There are three goods in the economy: private consumption b, housing h and a local publicly provided good g.[3] The latter is local in the sense that it is only consumed by the residents of a community.

A household can move costlessly and chooses the community in which its utility is maximized as place of residence. Each community indexed by j can individually set the amount of the local public good g_j and the local income tax rate $t_j \in [0, 1]$. These decisions are made in a majority rule vote by the residents respecting budget balance in the community. Each community has a fixed amount of land L_j from which housing stock is produced. All households are renters and the housing stock is owned by an absentee landlord. The price for housing p_j in community j is determined in a competitive housing market. The private good is considered as numeraire. A community j is fully characterised by the triple (t_j, p_j, g_j). The set of all possible community characteristics is given by $\Gamma = [0, 1] \times \mathbb{R}^{++} \times \mathbb{R}^{+}$.

Location choice and voting are examined in a two-stage game. In the first stage, households choose their place of residence. In the second stage the inhabitants of a community vote for the level of public good provision and consequently

[3] See Section 2.4 for a discussion of the nature of the public good.

for the community tax rate. The model is solved using backward induction.

2.1 Households

The preferences of the households are described by a Stone-Geary utility function

$$U(h, b, g; \alpha) := \alpha\, ln(h - \beta_h) + (1 - \alpha)\, ln(b - \beta_b) + \gamma\, ln(g - \beta_g)\,,$$

where h is the consumption of housing, b the consumption of the private good and g the consumption of the publicly provided good. $\beta_h > 0$, $\beta_b > 0$ and $\beta_g > 0$ are sometimes referred to as existential needs for housing, private good and public good, respectively. The parameter $\alpha \in [0, 1]$ describes the households' taste for housing, as will become apparent below.

Households face a budget constraint:

$$ph + b \leq y(1 - t)\,,$$

where p is the price of housing and t the local income tax. Note that the price of the private good is set to unity. Maximisation of the utility function with respect to h and b subject to the budget constraint yields the housing demand function

$$h^* := h(t, p, g; y, \alpha) = \frac{\alpha[y(1 - t) - p\beta_h - \beta_b]}{p} + \beta_h$$

and the demand for the the private good $b^* = y(1 - t) - ph^*$. Both demand functions are linear functions of after-tax income $y(1 - t)$, reflecting the fact that a linear demand system implies a Stone-Geary utility function and vice-versa (see Deaton and Muellbauer 1980). Housing demand is increasing in α as long as the household can satisfy its existential needs, i.e. $\partial(h)/\partial(\alpha) > 0$ iff $y(1 - t) > p\beta_h + \beta_b > 0$. $\alpha = 0$ implies that the housing demand is equal to the existential needs and hence does not change with household income. $\alpha = 1$ denotes a household which spends all extra income on housing after paying his existential need. The income elasticity of housing

$$\varepsilon := \frac{\partial h^*}{\partial y(1 - t)} \frac{y(1 - t)}{h^*} \lesseqgtr 1 \quad \text{iff} \quad \alpha \lesseqgtr \frac{p\beta_h}{p\beta_h + \beta_b}$$

depends on the housing price and is smaller or bigger than 1 depending on the household's tastes α (see the Appendix).

The indirect utility

$$V(t,p,g;y,\alpha) := U(h^*,b^*,g;\alpha)$$

gives the utility of a household with income y and preference parameter α in a community with income tax t, housing prices p and a public good provision g.

The following properties of the indirect utility function determine the distribution of households across communities. Properties 1 to 3 are directly derived from this indirect utility function assuming that existential needs are strictly satisfied, i.e. $y(1-t) > p\beta_h + \beta_b > 0$ and $g > \beta_g$. The calculations are provided in the Appendix.

Property 1 (Relative preferences)
For all $(t,p,g,y,\alpha) \in \Gamma \times \mathbb{R}^+ \times [0,1]$

$$M_{g,t}(t,p,g,y,\alpha) := \left.\frac{dg}{dt}\right|_{dV=0,dp=0} > 0\,,$$

$$M_{g,p}(t,p,g,y,\alpha) := \left.\frac{dg}{dp}\right|_{dV=0,dt=0} > 0\,,$$

$$M_{t,p}(t,p,g,y,\alpha) := \left.\frac{dt}{dp}\right|_{dV=0,dg=0} < 0\,.$$

Property 1 defines and signs the marginal rate of substitution $M_{\cdot,\cdot}$ between two community characteristics. Property 1 states that a household can be compensated for a tax increase either by more public good provision or by lower housing prices. Westhoff (1977) calls this trade-off the *relative preference* for the public good. Property 1 also states that a household can be made indifferent to higher housing prices if it is compensated by more public good provision. Property 1 holds under the standard assumption about the influence of prices, taxes and public goods on the household's well-being and is not specific to the assumed utility function.

Property 2 (Monotonicity of relative preferences)

(a) *For all* $(t,p,g,y) \in \Gamma \times \mathbb{R}^+$ *and any* $\alpha \in [0,1]$,

$$\frac{\partial M_{g,t}}{\partial y} < 0 \quad \text{and} \quad \frac{\partial M_{g,p}}{\partial y} < 0\,.$$

(b) *For all* $(t,p,g,\alpha) \in \Gamma \times [0,1]$ *and any* $y \in \mathbb{R}^+$,

$$\frac{\partial M_{g,p}}{\partial \alpha} > 0 \quad \text{and} \quad \frac{\partial M_{t,p}}{\partial \alpha} < 0\,.$$

Property 2 states that the relative preference for community characteristics changes *monotonically* with both income and taste. This property is equivalent to the Spence-Mirrless condition in information economics. It implies that a rich household can be compensated for a tax increase by strictly less public goods than a poor household. The compensation for higher housing prices by public goods decreases with income and increases with housing taste. The marginal rate of substitution between tax and housing price falls with housing taste.[4] Property 2 is a consequence of the changing weight of housing in the household's budget with income and taste. Note that Property 2 therefore does not hold for homothetic preferences.[5]

Property 3 (Proportional shift of relative preferences)

(a) *For all* $(t, p, g, y) \in \Gamma \times \mathbb{R}^+$ *and any given* $\alpha \in [0, 1]$*, both*

$$\frac{\partial M_{g,p}}{\partial y} / \frac{\partial M_{g,t}}{\partial y} \quad \text{and} \quad \frac{\partial M_{t,p}}{\partial y} / \frac{\partial M_{t,g}}{\partial y}$$

are independent of y*, where* $M_{t,g} = 1/M_{g,t}$.

(b) *For all* $(t, p, g, \alpha) \in \Gamma \times [0, 1]$ *and any given* $y \in \mathbb{R}^+$*,*

$$\frac{\partial M_{g,t}}{\partial \alpha} = 0 \,.$$

Property 3 results from the linear demand system in combination with the additive separability between g and (h, b). Although very specific, Property 3 is an indispensable condition to get segregation of the population.[6]

[4] The marginal rate of substitution between tax and housing price decreases with income if $\varepsilon > 1$ and increases if $\varepsilon < 1$.

[5] The property $\partial M_{g,t}/\partial y < 0$ is shared with Goodspeed (1986, 1989), who shows that it is equivalent to $\varepsilon_{g,y}/\varepsilon_{g,p} > 1$, where $\varepsilon_{g,y}$ is the income elasticity and ε_{g,p_g} is the (shadow) price elasticity of demand for the public good. $\partial M_{g,p}/\partial y < 0$ is shared with Ellikson (1971), Westhoff (1977), Epple, Filimon and Romer (1984, 1993) and Goodspeed (1986, 1989). Goodspeed reformulates this assumption as $\varepsilon_{g,y}/\varepsilon_{g,p} > \varepsilon_{h,y}$, where $\varepsilon_{h,y}$ is the income elasticity of demand for housing. He also points to empirical evidence that shows that both assumptions are reasonable. Property 2b shows preference heterogeneity in the same spirit as Epple and Platt (1998).

[6] It seems difficult to justify either Property 3a or Property 3b empirically. Schmidheiny (2002) shows that Property 3 is a necessairy condition for perfect income segregation. Goodspeed seems to derive income segregation without Property 3 in the same setting. However, the graphical proof he provides in Goodspeed (1986) is incomplete. Goodspeed (1989) uses the Stone-Geary utility function for the numerical simulation.

2.2 Location Choice

A household chooses to locate in the community in which its utility is maximal. Defining $V_j(y,\alpha) := V(t_j, p_j, g_j; y, \alpha)$ as the household's utility in j, a household chooses j if and only if

$$V_j(y,\alpha) \geq V_i(y,\alpha) \quad \text{for all } i\,. \tag{1}$$

The distribution of the households across communities implied by Properties 2 and 3 is described in the following paragraphs. A first observation is that all households are indifferent between all communities when the communities have identical community characteristics, i.e. $(t_i, p_i) = (t_j, p_j)$ for all j, i. In this case the households settle such that all communities show the same income distribution. In addition, it is always possible to think of equilibria in which subsets of communities have identical characteristics, i.e. $(t_i, p_i) = (t_j, p_j)$ for some j, i. However, these equilibria may not be stable.[7] The focus of this paper is on the empirically interesting case of equilibria where all communities exhibit distinct characteristics.

The following paragraphs describe how the utility maximizing households will be allocated across communities.

Lemma 1 (Boundary indifference)
Consider the subpopulation with taste α. If a household with income y' prefers to live in community j and another household with income $y'' > y'$ prefers to live in community i, then there is a 'border' household with income $\hat{y}_{ji}(\alpha)$, $y' \leq \hat{y}_{ji}(\alpha) \leq y''$, which is indifferent between the two communities.

Proof: The household with income y' prefers j to i, hence $V_j(y') - V_i(y') \geq 0$. The opposite is true for a household with income y'' thus $V_j(y'') - V_i(y'') \leq 0$. $V_j(y) - V_i(y)$ is continuous in y as V is continuous in y. The intermediate value theorem implies that there is at least one $\hat{y}_{ji}$ between y' and y'' s.t. $V_j(\hat{y}_{ji}) - V_i(\hat{y}_{ji}) = 0$. The existence of $\hat{y}_{ji}$ follows from $f(y,\alpha) > 0$. $\square$

[7] The notion of 'stability' in an intrinsically static model is rather peculiar. Nevertheless, equilibria in static multi-community models are often judged by their 'dynamic' behavior. In this ad-hoc interpretation, an equilibrium is called 'stable' when the change of community characteristics induced by the migration of 'few' households gives these households an incentive to move back.

The set of 'border' households is described by the function $\hat{y}_{ji}(\alpha)$. Equivalently, the set of border households is given by the inverse function $\hat{\alpha}_{ji}(y)$, implicitly defined by $V_j(\hat{\alpha}_{ji}(y)) = V_i(y, \hat{\alpha}_{ji}(y))$.

Definition 1 (Conditional income segregation)
An allocation of households is called conditionally segregated by incomes if the J sets $I_j = \{y : \text{household with income } y \text{ and taste } \alpha \text{ prefers community } j\}$ satisfy

- *I_j is an interval for all j,*
- *$I_j \cap I_i = \emptyset$ for all $i \neq j$,*
- *$I_1 \cup ... \cup I_J = [\underline{y}, \overline{y}]$*

for any α and for any j: $I_j \neq \emptyset$ for at least one α.

Definition 1 means that in a subpopulation with equal tastes any community is populated by a single and distinct income class.

Proposition 1 (Conditional income segregation)
When the household preferences are described by a Stone-Geary utility function and all J communities exhibit distinct characteristics, $(t_j, p_j, g_j) \neq (t_i, p_i, g_i)$ for all $i \neq j$, then the allocation of households is conditionally segregated by incomes.

Proof: The proof uses the fact that the utility difference $V_j - V_i = V(t_j, p_j, g_j; y, \alpha) - V(t_i, p_i, g_i; y, \alpha)$ between community j and i is strictly monotonic in y (see the Appendix):

$$sign \frac{\partial V_j - V_i}{\partial y} = sign(\frac{p_j \beta_h + \beta_b}{1 - t_j} - \frac{p_i \beta_h + \beta_b}{1 - t_i}).$$

Consider three households with income $y' < y'' < y'''$ respectively and suppose that the communities are not formed of non-overlapping intervals: y' as well as y''' prefer community j, but y'' strictly prefers community i. Given the opposed preference of y' and y''' it follows from Lemma 1 that there is an indifferent household $\hat{y}$, $y' \leq \hat{y} < y''$. The above sign condition implies that all households richer than $\hat{y}$, e.g. y''' , also prefer i, which is a contradiction. $\square$

Schmidheiny (2002) shows that the Properties 1, 2a to 3a are sufficient conditions for income segregation.

Definition 2 (Conditional taste segregation)
An allocation of households is called conditionally segregated by tastes if the J sets $I_j = \{\alpha$: household with income y and taste α prefers community $j\}$ satisfy

- *I_j is an interval for all j,*
- *$I_j \cap I_j = \emptyset$, for all $i \neq j$,*
- *$I_1 \cup ... \cup I_J = [0, 1]$*

for any y and for any j: $I_j \neq \emptyset$ for at least one y.

Definition 2 means that in a subpopulation with equal incomes any community is populated by a single and distinct interval of tastes.

Proposition 2 (Conditional taste segregation)
When the household preferences are described by a Stone-Geary utility function and all J communities exhibit distinct characteristics, $(t_j, p_j, g_j) \neq (t_i, p_i, g_i)$ for all $i \neq j$, then the allocation of households is conditionally segregated by tastes. Households in communities with lower housing prices have stronger tastes for housing than households in communities with higher prices.

Proof: The proof of the first sentence is analogous to Proposition 1 using the sign condition (derived in the appendix) $sign(\partial(dV_j - dV_i)/\partial\alpha) = sign(p_i - p_j)$. Second sentence: Consider $p_i < p_j$ and a household $(\hat{y}, \hat{\alpha})$ which is indifferent between the two communities j and i, hence $V_j(\hat{y}, \hat{\alpha}) = V_i(\hat{y}, \hat{\alpha})$. Then any household with the same income y and taste parameter $\alpha > \hat{\alpha}$ prefers community i, i.e. $V_j(\hat{y}, \hat{\alpha}) < V_i(\hat{y}, \hat{\alpha})$, since $\partial(dV_j - dV_i)/\partial\alpha < 0$ if $p_i < p_j$. $\square$

Schmidheiny (2002) shows that the Properties 1, 2b to 3b are sufficient conditions for segregation by tastes.

Propositions 1 and 2 offer two ways of calculating a community's population:

$$n_j = \int_0^1 \int_{\underline{y}_j(\alpha)}^{\overline{y}_j(\alpha)} f(y, \alpha)\, dy\, d\alpha = \int_{\underline{y}}^{\overline{y}} \int_{\underline{\alpha}_j(y)}^{\overline{\alpha}_j(y)} f(y, \alpha)\, d\alpha\, dy,$$

where $\underline{y}_j(\alpha)$ and $\overline{y}_j(\alpha)$ are the lowest and highest income in community j given the subpopulation with taste α. $\underline{y}_j(\alpha)$ is given by the locus of indifferent households $\hat{y}_{ji}$ between community j and its 'adjacent' community i with lower income households. The other boundaries $\overline{y}_j(\alpha)$, $\underline{\alpha}_j(y)$ and $\overline{y}_j(\alpha)$ are given analogously. Note that the adjacent community might not be the same for all subpopulations. This is demonstrated in Figure 2 showing four examples of possible segregation patterns in the case of three populated communities.

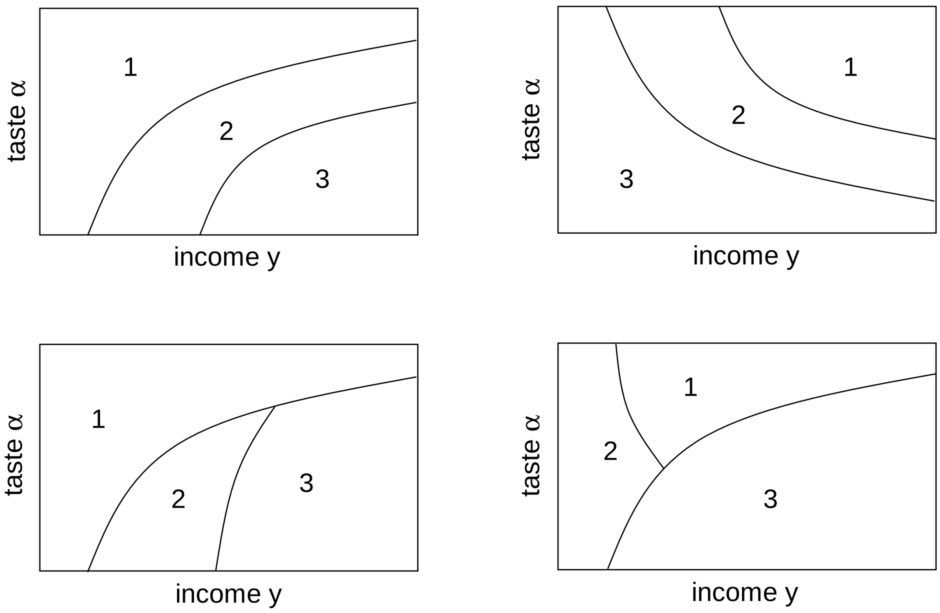

Figure 2: Examples of segregation patterns in the three-community case. The areas denoted by '1', '2' and '3' show the attributes of the households that prefer community 1, 2 or 3 respectively.

2.3 Housing market

Within each community housing is produced from land and non-land factors. The housing supply in each community j is an increasing function of the housing price p_j and the land dedicated to housing L_j. The housing supply function

$$HS_j = L_j \cdot p_j^{\theta}$$

is adopted from Epple and Romer (1991), who derive the supply function from an explicit production function, where θ is the ratio of non-land to land input.

The aggregate housing demand in community j is

$$HD_j = \int_0^1 \int_{\underline{y}_j(\alpha)}^{\overline{y}_j(\alpha)} h(t_j, p_j, g_j; y, \alpha) f(y, \alpha) \, dy \, d\alpha.$$

In equilibrium, the price for housing in community j clears the housing market

$$HD_j = HS_j. \tag{2}$$

Definition 3 (Housing market tightness)
The housing market in community j with given population is called not too tight if

$$\frac{dp_j}{d(1-t_j)} \frac{1-t_j}{p_j}\Big|_{HD_j=HS_j} < 1 \, .$$

Definition 3 defines the housing market as *not too tight* if the reaction of housing prices to changes in the tax rate and hence in the disposable income of the population is moderate. Note that the reaction of the housing price depends not only on the housing supply function but also on the characteristics, i.e. tastes and incomes, of the local population.

2.4 Public Choice

A community j provides a certain amount of a local public good to all its residents. The cost of providing this good is an increasing function of the amount provided g_j and the number of inhabitants n_j in the community. For simplicity, a linear function is assumed:

$$C(g_j, n_j) = c_0 + c_1 \, g_j \, n_j \, ,$$

where $c_0 \geq 0$ and $c_1 > 0$. Note that there are no spillovers in the production of the good across communities. The increasing cost in the number of beneficiaries means that the good is not a pure public good since there is rivalry in consumption. It is public in the sense that it is publicly provided and that all residents consume the same amount of the good. One can think of e.g. schools, street construction and maintenance, city planning activities, etc. A positive constant c_0 implies increasing returns to scale in the production of the public good.

The community finances the publicly provided good by a proportional income tax. The tax revenue is

$$T_j = \int_0^1 \int_{\underline{y}_j(\alpha)}^{\overline{y}_j(\alpha)} t_j \, y \, f(y,\alpha) \, dy \, d\alpha = n_j \, t_j \, Ey_j \, ,$$

where Ey_j is the mean income in community j. In equilibrium, the community's budget is balanced:

$$C(g_j, n_j) = T_j \, . \tag{3}$$

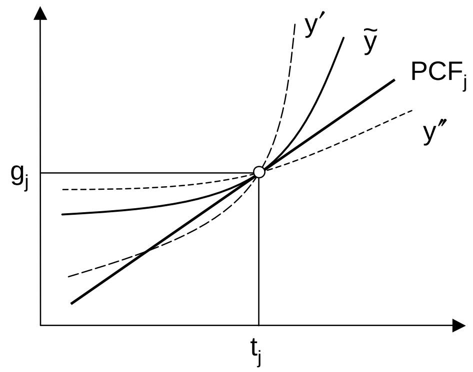

Figure 3: Voters' indifference curves in the (t, g) space.

The tax rate and the amount of public goods are determined in a majority rule vote by the residents of the community. At this stage, households take the population of the community as given.

Definition 4 (Public choice frontier PCF)
The public choice frontier PCF_j in community j is the set of (p_j, g_j, t_j) triples, where the pair (g_j, t_j) satisfies budget balance and p_j clears the housing market given the housing demand with tax rate t_j.

Proposition 3 (Segregation of voters)
Consider the subpopulation of households with taste α in community j and assume that the housing market is not too tight for all (p_j, t_j) on the PCF_j. If a household $\tilde{y}_j(\alpha)$ prefers the triple (p_j, g_j, t_j) on the PCF_j to all other triples on the PCF_j, then any richer (poorer) household opposes a reduction (increase) in taxes.

Proof: The proof refers to Figure 3. Consider the indifference curves of three voters with household income $y' < \tilde{y} < y''$ respectively, given the same taste parameter α. These indifference curves take into account the reaction of the housing prices to a change in the income tax rates. The straight line is the PCF in community j. One can verify in the figure that the pivotal voter $\tilde{y}$ prefers the pair (g_j, t_j) to all other combinations on the PCF. It is shown in the Appendix that the indifference curve is monotonically increasing in t and that its derivative w.r.t. t is decreasing in y. Therefore, all richer voters, e.g. y'', dislike all $(g, t) \in PCF_j$ combinations with taxes lower than t_j, while all poorer voters, e.g. y', dislike higher taxes. $\square$

$\tilde{y}_j(\alpha)$ is called the locus of pivotal voters. It is a decreasing function in α, as the price reduction induced by higher taxes is more appreciated by households with a stronger taste for housing. Note that from the perspective of a naïve voter who ignores the housing market, Proposition 3 holds without the additional assumption of the housing market tightness.

Definition 5 (Majority rule voting equilibrium)
A triple (p_j, g_j, t_j) on community j's PCF is called a majority rule voting equilibrium when no other triple on the PCF is strictly preferred by a majority of the community's residents.

As an implication of Proposition 3, a majority rule voting equilibrium in community j is established when

$$\int_0^1 \int_{\underline{y}_j(\alpha)}^{Min(\tilde{y}_j(\alpha),\overline{y}_j(\alpha))} f(y,\alpha)\, dy\, d\alpha = \frac{1}{2} \int_0^1 \int_{\underline{y}_j(\alpha)}^{\overline{y}_j(\alpha)} f(y,\alpha)\, dy\, d\alpha \tag{4}$$

and the housing market is not too tight.

2.5 Equilibrium

The equilibrium as defined below is a subgame-perfect Nash equilibrium in the two-stage game of residential choice and voting.

Definition 6 (Equilibrium)
A set of community characteristics (p_j, g_j, t_j), $j = 1, ..., J$, and an allocation of individual households across communities is an equilibrium if and only if

- *all households choose their community to maximise their utility,*
- *the housing market clears in all communities,*
- *there is a majority rule voting equilibrium in all communities.*

Existence of the equilibrium is proofed by Goodspeed (1986) in a model with income taxes, taste homogeneity, naïve voters and a uniform income distribution. Epple, Filimon and Romer (1993) show existence in a model with property taxes and homogeneous tastes. Unfortunately, as in other models with taste heterogeneity (Epple and Platt, 1998), a proof of existence and uniqueness of this equilibrium could not be established. However, equations (1) to (4) provide the basis for a computational strategy to numerically find an equilibrium.

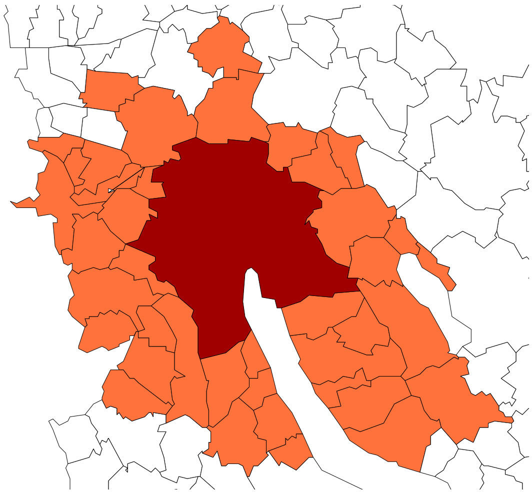

Figure 4: The Zurich metropolitan area around the lake of Zurich.

3 Numerical Equilibrium

In this section the qualitative and quantitative properties of the model are investigated in a fully specified model calibrated to the biggest Swiss metropolitan area.

3.1 Calibration

The area around the city of Zurich forms the biggest Swiss metropolitan area. The city of Zurich has about 330 thousand inhabitants and is the capital of the canton (state) of Zurich. The canton of Zurich counts 1.2 Million inhabitants in 171 individual communities. As described in the introduction, any of these communities can set an individual level of income taxes.

The analysis is restricted to the city of Zurich and a ring of the most integrated communities around the center. This ring is formed by all communities in the canton of Zurich with more than 1/3 of the working population commuting to the center.[8] Figure 4 shows a map with the city of Zurich and the thus defined ring of 40 communities. The community characteristics of this area are also discussed in the introduction (see Figure 1).

The whole area has a physical size of $349km^2$, of which $88km^2$ (25%) form

[8]The number of commuters to the city of Zurich and the size of the working population in the communities is based on the 1990 Census.

the city of Zurich. $140km^2$ are dedicated to development, $53km^2$ (38%) in the inner city and $87km^2$ in the fringe communities. In 1998, the whole area was populated by around 628'000 inhabitants, of whom 334'000 lived in the city and 294'000 in the fringe communities.[9] This agglomeration is modelled as two distinct jurisdictions with land size $L_1 = 0.4$ and $L_2 = 0.6$ respectively.

In the year 1997, the communal tax level in the fringe communities was on average 19% (minimum -35%, maximum +1%) lower than in the inner city.[10] The rental price for housing in the periphery was on average 6% lower than in the center (minimum -24%, maximum +13%).[11] The lowest-tax communities south-east of the city center exhibit substantially higher housing prices than the center. Figure 1 visualizes the spatial distribution of tax rates, incomes and housing prices in the area.

The income distribution is calibrated with data from the Swiss labor force survey.[12] The 1995 cross-section contains detailed information on 1124 households in the above defined region. These households had an average income of CHF 92,000 (median CHF 66,700) after state and federal taxes.[13] A log-normal distribution truncated at a minimum and a maximum income level is used to approximate the income distribution. A mean of $E(ln\, y) = 11.1$ and a standard deviation of $SD(ln\, y) = 0.55$ are close to the observed median and quartile distance. The minimum income is assumed to be $y_{min} = 23,000$ and the maximum income $y_{max} = 500,000$.[14] The median income in the city of Zurich is CHF 58,700 opposed to CHF 75,200 in the fringe communities.

The distribution of the taste parameter is described by a beta distribution. The Swiss labor force survey also contains the monthly housing expenditure of renters which allows to calibrate the distribution of tastes.[15] If the housing de-

[9]Source: Statistisches Amt des Kantons Zürich, Gemeindedaten per 31.12.1998.

[10]Source: Statistisches Amt des Kantons Zürich, Steuerfüsse 1997.

[11]Source: Wüest und Partner, Zurich. Offer prices for flats in newspapers and in the internet in 1997.

[12]Swiss Federal Statistical Office, Schweizerische Arbeitskräfterhebung (SAKE) 1995.

[13]State and Federal taxes were deducted from net household income (after social security contribution) assuming a two-child family.

[14]The minimum income is subsistence level for a one-person-household as defined by the Schweizerische Konferenz für Sozialhilfe (SKOS) and adjusted for inflation. The maximum income is chosen arbitrarily, but has no influence on the numerical simulation due to the low weight on high incomes.

[15]Of course, there is a selection bias by only considering renters. Because the proportion of renters is very high in Switzerland (65% in the data set used), this is justified.

mand function is correctly specified, the taste parameter α of a household with disposable income y_d can be calculated as $(ph - ph_{min})/(y_d - y_{d,min})$, where ph is expenditure on housing and ph_{min} is the housing expenditure of the household with minimal disposable income $y_{d,min}$. The disposable income of a household y_d is calculated as reported household income minus federal, state and communal taxes. The average yearly housing expenditure of households around subsistence level is taken to approximate ph_{min}. This enables to approximate each household's taste parameter α. A beta distribution with mean $E(\alpha) = 0.17$ and standard deviation $SD(\alpha) = 0.11$ describes the distribution of the so calculated taste parameter fairly well. Taste and income are assumed to be uncorrelated.

The price elasticity of housing supply is $\theta = 3$ as in Epple and Romer (1991) and Goodspeed (1989). The production of the public good exhibits constant returns to scale, i.e. $c_0 = 0$ and $c_1 = 1$. $\beta_h = 700$, $\beta_b = 13000$ are chosen such that the consumption bundle of the minimal income household in equilibrium corresponds to the empirical findings. It is assumed that the existential needs for the public good are fairly high and that the benefit from additional units is limited: $\beta_g = 4000$ and $\gamma = 0.02$.[16] The assumed parameters result in equilibrium tax rates close to the observed ones. The parameters are summarized at the bottom of Table 1.

3.2 Simulation Results

The equilibrium values p_j, g_j and t_j, $i = 1, 2$, must satisfy equations (2), (3) and (4) and guarantee that the households reside in the community they prefer as expressed in equation (1). Unfortunately, there is no closed form solution to this nonlinear system of 6 equations and 6 unknowns. The equation system is therefore numerically solved for the equilibrium values of the model.[17]

[16]The properties of the equilibrium depends decisively on the preference parameters of the public good. The assumption that the existential need is small but the benefit from additional units is important can lead to numerical equilibria in which the high-tax communities exhibit higher housing prices but *lower* public goods provision. The rich households will then prefer the high-tax communities. This situation is in clear contradiction to the observed pattern in Swiss metropolitan areas.

[17]Numerically solving the equation system is tedious and time-consuming. The aggregation of individual demand and voting behavior requires double integrals over the community population. These integrals cannot be calculated analytically. Gauss-Legendre Quadrature with 40 nodes in each dimension is used to approximate the various double integrals. Numerically minimizing the sum of squared deviations from the equilibrium conditions with the Gauss-Newton method solves for the equilibrium values. Appropriate scaling of the arguments and

Table 1: Equilibrium values of the simulation.

	unified	heterogen. tastes center	heterogen. tastes periphery	homogen. tastes center	homogen. tastes periphery
L: land size	1	0.40	0.60	0.40	0.60
p: housing price	11.40	10.48	11.92	9.19	12.37
t: income tax rate	0.064	0.085	0.059	0.110	0.056
g: public good prov.	5032	4488	5225	4335	5390
n: inhabitants	1	0.284	0.716	0.314	0.686
Ey: average income	78,547	52,995	88,687	39,368	96,460
Average CV		50.9	84.4	41.8	455.9
n (CV>0)		0.171	0.506	0.180	0.686

The model parameters are: $\beta_h = 700$, $\beta_b = 13000$, $\beta_g = 4000$, $\gamma = 0.02$, $E(\alpha) = 0.17$, $SD(\alpha) = 0.11$ (heterogeneous tastes), $SD(\alpha) = 0$ (homogeneous tastes), $E(ln\, y) = 11.1$, $SD(ln\, y) = 0.55$, $y_{min} = 23'000$, $y_{max} = 500'000$, $\theta = 3$, $c_0 = 0$ and $c_1 = 1$.

Table 1 shows the equilibrium values for the calibrated model in columns 2 and 3. The equilibrium values for the case of a unified jurisdiction and for the case of taste homogeneity are given for comparison. As can be seen, the equilibrium values of the two communities differ substantially. I will refer to the high-tax community 1 as the 'center' and the low-tax community 2 as the 'periphery'.[18] The tax rate t_1 in the center is 44% higher than in the suburbs, whereas the housing price is 12% and the public provision 14% lower. The average household income in the center is CHF 53 thousand a year compared to CHF 89 thousand in the suburbs. Thus, the simulated model is able to explain tax and income differences of the magnitude observed in the Zurich area. Not surprisingly, the model is very poor at explaining the high housing prices in the center as any immanent center advantages such as closeness to the main business and cultural activities are neglected.

The segregation of the population in the two communities is shown in the left picture in Figure 5. The locus of indifferent households, $\hat{y}_{12}$, turns out to

of the equilibrium conditions is important for the accuracy of the result. Convergence is only achieved with 'good' starting values. Starting values are obtained from a grid search over the six-dimensional space of possible values. Different starting values lead to the same equilibrium values.

[18]These labels are arbitrary. There is always a second equilibrium with lower taxes in community 1. If the two communities have the same physical size, these two equilibria are identical except for the community index.

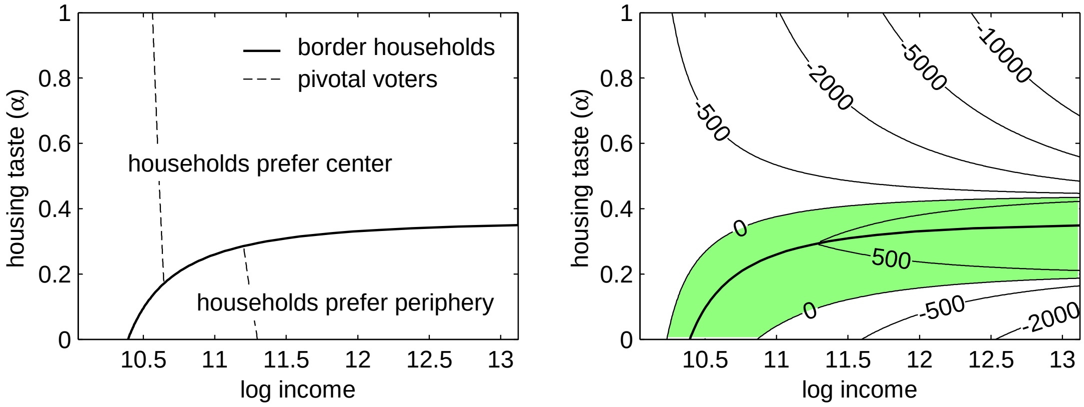

Figure 5: Income and taste segregation in equilibrium. The left figure includes the loci of pivotal voters. The right figure shows contour lines of the compensating variation (CV).

be an increasing function of income in the present equilibrium. This implies that, given a subpopulation with equal tastes, richer households prefer the low-tax-high-price community.[19] However, this does not lead to a perfect income segregation between the two communities since the households have different preferences. Although the average income in the center is much lower than in the periphery, households from almost all income groups can be found in both communities. Figure 6 presents the resulting income distributions in the two communities. Figure 5 also shows the loci of pivotal voters which split the communities' populations into half. Households in the 'rich' suburbs vote for more public goods than households in the 'poor' center, yet this generous public good provision can be financed by a lower tax rate, due to the higher average income of the residents.

The above segregated equilibrium is now compared to the equilibrium when the two jurisdictions were unified. The latter is equivalent to the equilibrium that would emerge if the two distinct jurisdictions harmonized their income tax levels and households located randomly. The equilibrium values of the unified community are presented in column 1 in Table 1. One can immediately see that the housing price, tax level and public good provision lie between the corresponding

[19]Note that all households with a very high taste for housing prefer to live in the low-price community. This, however, applies to only 5% of the population, as the weight on taste parameters above 0.38 is low.

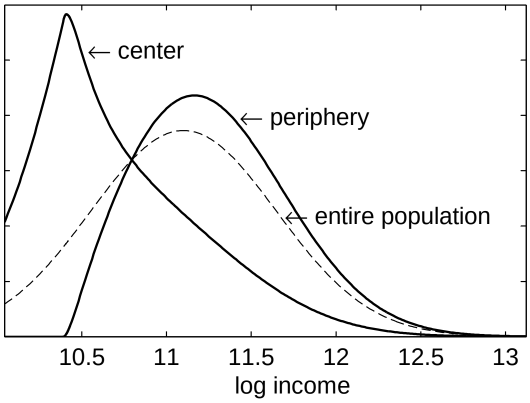

Figure 6: Income distribution in the center and the periphery.

values in the two-community model. The competition of the two communities for households does thus not lead to an overall reduction of taxes, but to relatively lower taxes in the 'rich' community and higher taxes in the 'poor' community. The welfare effects associated with the segregated equilibrium depend on both the households' incomes and tastes.

The welfare effects are revealed by inspecting the compensating variation (CV) for the different types of households. The compensating variation is the additional gross income that a households needs in order to be compensated for a shift from the symmetric to the segregated equilibrium.[20] The right picture in Figure 5 shows contour lines of the CV. Households in the shaded band between the two zero contour lines exhibit positive values of the CV and thus prefer the one-community situation. Households further away from the border household prefer competing jurisdictions. The average CV's are reported in Table 1. Households in the 'poor' community have to be compensated by an average income allowance of CHF 51 compared to CHF 84 in the 'rich' community. Note that this amount is only one-tenth of a percent of average gross income. The number of households that prefer tax harmonization is also given in Table 1. 60% of the population in the 'poor' community and 71% of the population in the 'rich' community prefer tax harmonization.

How does the taste heterogeneity affects the properties of the equilibrium?

[20]The CV defined above is not a comprehensive measure of the welfare implications since it ignores the welfare implications for the (absentee) landlord. The analytical solution for the CV given household characteristics is given in the Appendix.

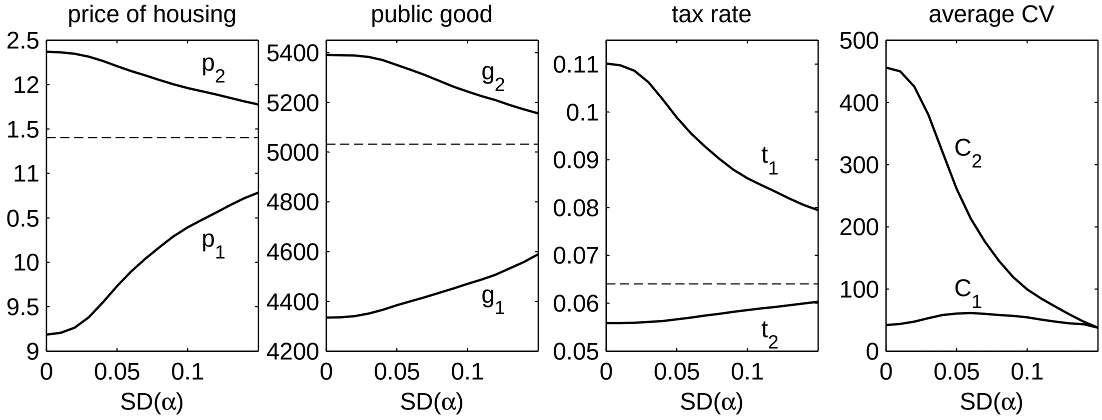

Figure 7: Equilibria with changing standard deviation of taste parameter.

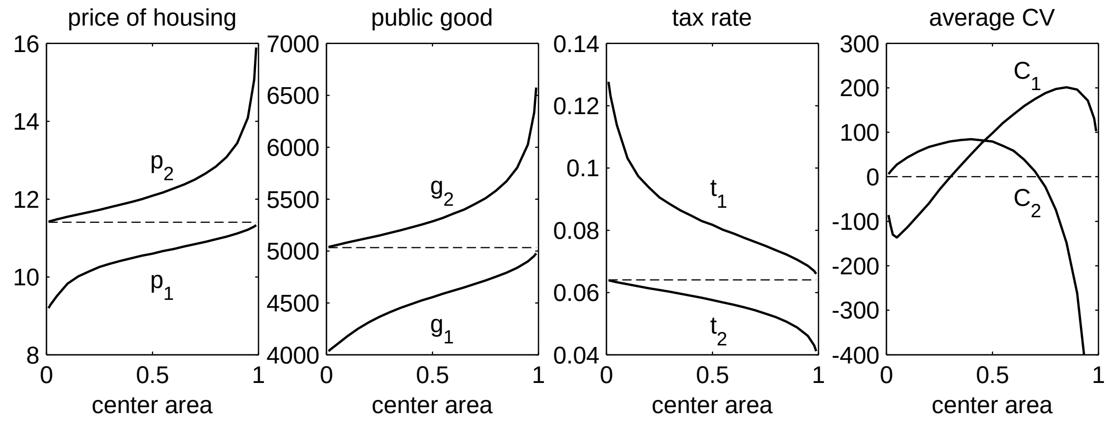

Figure 8: Equilibria with changing relative size of the communities.

The last two columns in Table 1 give the equilibrium values in the case of constant tastes that equal the average heterogeneous tastes. This equilibrium features perfect income segregation. Consequently, the income difference between the two communities is much larger than with heterogeneous tastes. Also, the price, tax and public good provision differences across the communities are stronger. The equilibrium values in a unified community equal the ones with taste homogeneity.[21] The welfare effects under the assumption of taste homogeneity are substantially greater than under heterogeneity.

[21] The symmetric equilibrium in the case of taste homogeneity is theoretically different from the one in the case of taste heterogeneity as the pivotal voter varies with the taste parameter. However, this difference is numerically negligible if the constant taste parameter equals the mean of the heterogeneous tastes.

Figure 7 shows the equilibrium values for different levels of taste heterogeneity measured by the standard deviation $SD(\alpha)$, leaving the mean of tastes constant. The horizontal axes cover a range from $SD(\alpha) = 0$ (taste homogeneity) to a maximum $SD(\alpha) = 0.143$[22] including the calibrated case $SD(\alpha) = 0.11$. The picture reveals that the equilibrium converges towards the values of the symmetric case, indicated by the dotted lines. This is explained by the fact that with increasing taste variance, the population is more and more segregated by taste rather than income. This result suggests that taste heterogeneity is able to lower the negative distributional effects of decentralized tax authority. Figure 7 (far right) shows the corresponding change of the average compensating variation. While the average CV in the poor center community is almost unaffected by the amount of taste heterogeneity in the population, it falls sharply in the rich periphery. The fraction of households in the periphery which would prefer harmonized taxes falls accordingly from 100% ($SD(\alpha) = 0$) to 65% ($SD(\alpha) = 0.143$).

Up to now, the physical land size was given by the calibration. However, the relative size of the two jurisdictions is likely to influence the equilibrium values and is therefore investigated in detail. The influence of the relative land size is reported in Figure 8. The housing price and the public good provision in both communities increase with the physical size of the center community (community 1). Recall that the high-tax community is called the center by convention. The low-tax community shows lower public good provision and lower housing prices than the low-tax community throughout all possible partitions of land between the two communities. Furthermore, the average income in the low-price community is always lower than in the high-price community. The order of community characteristics is hence not affected by the relative land size. Not surprisingly, the equilibrium values of a community that virtually covers the whole area ($L_1 = 1$ or $L_1 = 0$, hence $L_2 = 1$) equal the values of one unified community, marked by the dotted lines. The equilibrium values in the remaining very small community differ maximally from the values in the case of unified communities. The tax rate in the rich community 2 declines with increasing relative land size in community 1. This shows that the rich community's ability to set low taxes is higher when it is physically small.

[22]Given the mean $E\alpha = 0.17$, $SD(\alpha) = 0.143$ is the maximal standard deviation that preserves the bell-shaped form of the beta distribution. Higher values lead to a u-shaped distribution.

The influence of the relative land size on welfare is particularly interesting. Recall that in the calibrated situation ($L_1 = 0.4$), the average household prefers harmonized taxes: average CV is CHF 51 in the center and CHF 84 in the periphery. This result does strongly depend on the relative community size as can be seen in Figure 8 (far right). The average CV in the poor center community is negative if this community is small ($L_1 < 0.3$), meaning that the population in the center does on average prefer (higher) local taxes to (lower) harmonized taxes, as they are associated with lower housing prices.[23] Note that it is the poorer part of the population in the poor community that profits most from the local differences. The rich periphery shows a similar picture. Its increased ability to set low taxes when it is small ($L_2 < 0.27$) leads to a negative average CV, i.e. an average preference for local taxation.[24] Note that it is the richer part of the population that profits most from the decentralized tax setting.

4 Conclusions

This paper presents a model of an urban area with local income taxes used to finance a local public good. The main assumptions of the model are the following: Households differ in income and taste for housing. The demand for housing and private goods is a linear demand system. The share of housing in the budget of the households is on average declining. The private good can only partly substitute for the publicly provided good.

The existence of a segregated equilibrium is shown in a calibrated two-community model assuming realistic single-peaked distributions for income and taste in housing. The high-tax community shows both lower housing prices and lower public good provision than the low-tax community. The equilibrium features segregation of households by both income and tastes. The emerging segregation pattern is such that rich households prefer the low-tax-high-price community given a subpopulation with equal tastes. As tastes differ across households, this does not lead to a perfect income segregation of the population but to an income distribution in the 'rich' low-tax community that stochastically dominates the income distribution in the 'poor' high-tax community. The model is able to

[23] The majority of households in the center prefer decentralized tax setting when $L_1 < 0.26$.

[24] When the rich community is even smaller ($L_2 < 0.11$), the supporters of local taxation is on the majority.

explain the substantial differences of the local income tax level and of average income across communities as observed in Switzerland.

The numerical investigation suggests that taste heterogeneity reduces the distributional effects of local tax differences. The differences of characteristics across communities are maximal when tastes are equal for all households and the population is accordingly perfectly segregated by incomes. These differences decrease with increasing taste heterogeneity as the income segregation of the population becomes more and more diffuse.

The numerical investigation also suggests that the relative size of the individual jurisdictions has great impact on the equilibrium situation. The characteristics of a relatively large community are close to the equilibrium characteristics of a single jurisdiction that covers the whole area. Conversely, the relatively small community differs substantially from the single jurisdiction. The ability of the rich community to set low taxes, for example, is higher when this community is physically small. However, contrary to the findings by Hansen and Kessler (2001a), a tax haven need not be small.

Multi-community models are especially well-suited to study metropolitan areas as it is assumed that the residence choice of a household is made after and independent of the decision of where its members work. This assumption does not seem justifiable on the level of federal states or even countries. Therefore, the results presented in this paper may only be indicative for the analysis of fiscal federalism on the level of states or countries.

From an empirical perspective, a weakness of the model is its poor explanatory power for the typically observed high housing prices in the highly taxed center of Swiss metropolitan areas. This is due to neglect of the advantages of the central business (and cultural) district for households. The incorporation of distance between residence and business or other activities – hence the incorporation of geography – is an interesting task for future research.

Appendix

The household problem is

$$\max_{h,b} \; U(h,b,g,\alpha) \;=\; \alpha\, ln(h-\beta_h) + (1-\alpha)\, ln(b-\beta_b) + \gamma\, ln(g-\beta_g)$$
$$s.t. \qquad ph + b \le y(1-t)\,.$$

This leads to the housing demand

$$h^* = h(t,p,y,\alpha) = \frac{\alpha[y(1-t) - p\beta_h - \beta_b]}{p} + \beta_h\,,$$

the income elasticity of housing

$$\varepsilon = \frac{\partial h^*}{\partial y(1-t)}\frac{y(1-t)}{h^*} = \frac{\alpha y(1-t)}{\alpha[y(1-t) - p\beta_h - \beta_b] + p\beta_h}$$

and the indirect utility function

$$V = \alpha\, ln(\alpha) + (1-\alpha)\, ln(1-\alpha) - \alpha\, ln(p) + ln[y(1-t) - p\beta_h - \beta_b] + \gamma\, ln(g-\beta_g)\,.$$

The marginal rates of substitution in Property 1 are derived by totally differentiating the indirect utility function:

$$\begin{aligned}
M_{g,t} &= -\frac{\partial V/\partial t}{\partial V/\partial g} = \frac{y(g-\beta_g)}{\gamma[y(1-t) - p\beta_h - \beta_b]}\,,\\
M_{g,p} &= -\frac{\partial V/\partial p}{\partial V/\partial g} = \frac{h^*(g-\beta_g)}{\gamma[y(1-t) - p\beta_h - \beta_b]}\,,\\
M_{t,p} &= -\frac{\partial V/\partial p}{\partial V/\partial t} = -\frac{h^*}{y}\,.
\end{aligned}$$

Differentiation of the MRS w.r.t. income and taste yields Property 2:

$$\frac{\partial M_{g,t}}{\partial y} = -\frac{(g-\beta_g)(p\beta_h + \beta_b)}{\gamma[y(1-t) - p\beta_h - \beta_b]^2}\,, \qquad \frac{\partial M_{g,t}}{\partial \alpha} = 0\,,$$
$$\frac{\partial M_{g,p}}{\partial y} = -\frac{(1-t)(g-\beta_g)\beta_h}{\gamma[y(1-t) - p\beta_h - \beta_b]^2}\,, \qquad \frac{\partial M_{g,p}}{\partial \alpha} = \frac{g-\beta_g}{p\gamma}\,,$$
$$\frac{\partial M_{t,p}}{\partial y} = \frac{(1-\alpha)p\beta_h - \alpha\beta_b}{py^2}\,, \qquad \frac{\partial M_{t,p}}{\partial \alpha} = -\frac{y(1-t) - p\beta_h - \beta_b}{py}\,.$$

The independence of the MRS ratio in Property 3 follows directly:

$$\frac{\partial M_{g,p}}{\partial y}\Big/\frac{\partial M_{g,t}}{\partial y} = \frac{(1-t)\beta_h}{p\beta_h + \beta_b}\,,$$

$$\frac{\partial M_{t,p}}{\partial y} / \frac{\partial M_{t,g}}{\partial y} = \frac{[(1-\alpha)p\beta_h - \alpha\beta_b](g-\beta_g)}{\gamma p(p\beta_h+\beta_b)},$$

where $M_{t,g} = 1/M_{g,t}$.

The locus of indifferent households between community j and i

$$\hat{y}_{ji}(\alpha) = \frac{(p_j\,\beta_h+\beta_b)\,{p_i}^{\alpha}\,(g_j-\beta_g)^{\gamma} - (\beta_b+p_i\,\beta_h)\,{p_j}^{\alpha}\,(g_i-\beta_g)^{\gamma}}{(1-t_j)\,{p_i}^{\alpha}\,(g_j-\beta_g)^{\gamma} - (1-t_i)\,{p_j}^{\alpha}\,(g_i-\beta_g)^{\gamma}}.$$

solves $V(t_j,p_j,g_j,y,\alpha) = V(t_i,p_i,g_i,y,\alpha)$ for y. Alternatively, the locus solves for α:

$$\hat{\alpha}_{ji}(y) = \frac{ln[\frac{y(1-t_j)-p_j\beta_h-\beta_b}{y(1-t_i)-p_i\beta_h-\beta_b}] + \gamma\,ln[\frac{g_j-\beta_g}{g_i-\beta_g}]}{ln(p_j/p_i)}.$$

The locus $\hat{\alpha}_{ji}(y)$ is either strictly increasing and concave in y or strictly decreasing and convex, as can easily be verified by inspecting the first and second derivative

$$\frac{\partial\hat{\alpha}_{ji}}{\partial y} = -\frac{(1-t_j)[p_i\beta_h-\beta_b]-(1-t_i)[p_j\beta_h-\beta_b]}{[y(1-t_j)-p_j\beta_h-\beta_b][y(1-t_i)-p_i\beta_h-\beta_b]\,ln(p_j/p_i)}$$

$$\frac{\partial^2\hat{\alpha}_{ji}}{\partial y^2} = -\frac{\partial\hat{\alpha}_{ji}}{\partial y}\cdot\frac{(1-t_j)[y(1-t_i)-p_i\beta_h-\beta_b]+(1-t_i)[y(1-t_j)-p_j\beta_h-\beta_b]}{[y(1-t_j)-p_j\beta_h-\beta_b][y(1-t_i)-p_i\beta_h-\beta_b]}$$

and provided that all household reach the subsistence level, i.e. $y(1-t) > p\beta_h+\beta_b > 0$, in both communities.

The utility difference between community j and i is

$$V_j(y,\alpha) - V_i(y,\alpha) = -\alpha\,ln(\frac{p_j}{p_i}) + ln[\frac{y(1-t_j)-p_j\beta_h-\beta_b}{y(1-t_i)-p_i\beta_h-\beta_b}] + \gamma\,ln(\frac{g_j-\beta_b}{g_i-\beta_b}).$$

Differentiation of the above expression w.r.t. y and α is used in the proof of Propositions 1 and 2:

$$\frac{\partial(V_j-V_i)}{\partial y} = \frac{1}{y-\frac{p_j\beta_h+\beta_b}{1-t_j}} - \frac{1}{y-\frac{p_i\beta_h+\beta_b}{1-t_i}}, \quad \frac{\partial(V_j-V_i)}{\partial\alpha} = ln(p_i) - ln(p_j).$$

The rate of substitution between tax rate and public good provision a voter faces is derived from totally differentiating the indirect utility function considering the housing market reaction, $dp/dt|_{HD=HS}$ (community subscripts ommited):

$$\begin{aligned}\left.\frac{dg}{dt}\right|_{dV=0,HD=HS} &= \frac{-\frac{\partial V}{\partial t} - \frac{\partial V}{\partial p}\cdot\left.\frac{dp}{dt}\right|_{HD=HS}}{\frac{\partial V}{\partial g}} = M_{gt} + M_{gp}\cdot\left.\frac{dp}{dt}\right|_{HD=HS}\\ &= \frac{g-\beta_g}{\gamma[y(1-t)-p\beta_h-\beta_b]}\left[y + h^*\left.\frac{dp}{dt}\right|_{HD=HS}\right].\end{aligned}$$

The voter's rate of substitution is positive when the price effect on the housing market is not too large:

$$\left.\frac{dg}{dt}\right|_{dV=0,HD=HS} > 0 \quad \text{iff} \quad \left.\frac{dp/p}{d(1-t)/(1-t)}\right|_{HD=HS} < \frac{y(1-t)}{ph^*} .$$

The voter's rate of substitution decreases with income

$$\begin{aligned}\frac{\partial \left.\frac{dg}{dt}\right|_{dV=0,HD=HS}}{\partial y} &= \frac{\partial M_{g,t}}{\partial y} + \frac{\partial M_{g,p}}{\partial y} \left.\frac{dp}{dt}\right|_{HD=HS} . \\ &= -\frac{g-\beta_g}{\gamma[y(1-t)-p\beta_h-\beta_b]^2}\left[p\beta_h+\beta_b+(1-t)\beta_h \left.\frac{dp}{dt}\right|_{HD=HS}\right]\end{aligned}$$

if the price effect on the housing market is not too large:

$$\frac{\partial \left.\frac{dg}{dt}\right|_{dV=0,HD=HS}}{\partial y} < 0 \quad \text{iff} \quad \left.\frac{dp/p}{d(1-t)/(1-t)}\right|_{HD=HS} < \frac{p\beta_h+\beta_b}{p\beta_h} .$$

Both the condition on the sign of the voter's marginal rate of substitution and the sign of its derivative w.r.t. y are fulfilled if $\left.\frac{dp/p}{d(1-t)/(1-t)}\right|_{HD=HS} < 1$ (see Definition 3) and all households reach the subsistence level.

The compensating variation cv_j is the additional gross income that a household in e.g. community j needs in order to be compensated for a shift from the symmetric (unified) equilibrium, (t_u, p_u, g_u), to the segregated equilibrium, (t_j, p_j, g_j). Solving

$$V(t_u, p_u, g_u; y, \alpha) = V(t_j, p_j, g_j; y + cv, \alpha)$$

for cv yields the compensating variation for a household with income y and taste α in community j:

$$cv_j(y,\alpha) = \frac{[y(1-t_u)-\beta_b-p_u\beta_h](\frac{p_u}{p_j})^{-\alpha}(\frac{g_u-\beta_g}{g_j-\beta_g})^{\gamma} - [y(1-t_j)-\beta_b-p_j\beta_h]}{1-t_j} .$$

The average compensating variation in community j is then computed as

$$CV_j = \frac{1}{n_j}\int_0^1 \int_{\underline{y}_j(\alpha)}^{\overline{y}_j(\alpha)} cv_j(y,\alpha)\, f(y,\alpha)\, dy\, d\alpha .$$

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