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Abstract

We modify the basic Gehrig (1993) model. In this model, individual agents are either buyers or sellers. They can choose between joining the search market, joining the monopolistic intermediary or remaining inactive. In the search market, agents are randomly matched and the price at which exchange takes place is set bilaterally. If agents join the intermediary, buyers have to pay an ask price set in advance by the intermediary. Likewise, if sellers decide to deal through the intermediary, they get the bid price set by the intermediary. As Gehrig shows, this model has an equilibrium in which the search market and the market of the monopolistic intermediary are simultaneously open. The intermediary makes positive profits because he trades at a positive ask-bid spread, and the set of individual agents is tripartite: High valuation buyers and low cost sellers deal through the intermediary, buyers and sellers with average valuations and average costs are active in the search market, and low valuation buyers and high cost sellers remain inactive. We modify this basic model by imposing a sequential structure. We assume that the monopolistic intermediary first has to buy the good from sellers on the input market before he can sell it to buyers on the output market. As a consequence of the sequential structure, the subgame following capacity setting has a unique subgame perfect equilibrium with an active search market. On the equilibrium path, the equilibrium analyzed by Gehrig is replicated.

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1 Introduction

A question models of perfect competition leave unanswered is which institution coordinates the decisions of the great number of agents necessary for perfect competition to work. While attributing the role of the coordination mechanism to prices, these models typically remain silent about the origin of these prices. A second issue that remains open is how exchange of the goods takes place. If a thousand sellers and thousand buyers trade some good at a given price, these models do not say how and where the agents exchange the good. Since questions relating to the microstructure of markets are not treated in these models, the microeconomic model of perfect competition can be said to lack a microeconomic foundation.

In this (and a forthcoming companion) paper, we deal with some of these questions. Building on the work of Gehrig (1993), we study a model in which a monopolistic intermediary coordinates the decisions of buyers and sellers willing to trade with him at the ask and bid prices he quotes. Another subset of buyers and sellers is active in a search market where goods are exchanged without the intermediary's services. In this model, (ask and bid) prices originate from a profit seeking intermediary. The intermediary establishes the exchange of the good for those traders who are willing to trade through him, while prices in the search market are determined through a bargaining process and the good is exchanged from an individual seller to an individual buyer. Intermediation is a profitable business because search market participants are matched at random and therefore those buyers and sellers who could exchange the good with the greatest mutual benefit will, in general, not find each other. As a consequence, the search market will not exhaust all potential gains from trade. The *dispersed* rather than the *asymmetric* nature of information gives thus rise to profitable intermediation. Therefore, the model departs from the strand of literature focussing on asymmetries of information that give rise to - financial - intermediaries (e.g. Diamond and Dybvig, 1983; Diamond, 1984; Freixas and Rochet, 1997; Dixit, 2001). The intermediary in our type of model does not reduce or eliminate inefficiencies due to informational advantages of one party involved in trade and its strategic use thereof. What intermediation in this type of model achieves is that those buyers and sellers who have the most to gain can trade with greater certainty and at a price which leaves them a greater gain than they can expect from search market participation.

We think there is a fair justification to not a priori distinguish between financial intermediaries like banks or insurance companies on the one hand and non-financial intermediaries like retailers on the other hand. After all, why should financial intermediaries by their very nature be characterized as mitigating inefficiencies due to asymmetries of information while non-financial intermediaries arise for some other reasons, like, say, increasing returns to scale? Of course, it is beyond question that asymmetric information is important for the businesses financial intermediaries are engaged in. The requirement of a collateral in credit contracts is hard to understand without referring to asymmetric information. But granting this does not imply that such asymmetries are of no or only of minor importance in other industries like, say, retailing. A simple transaction like buying food can involve considerable uncertainties and risks because quality differences can be hard to detect for customers before consuming the good. Therefore, reducing these uncertainties may just be one of the services provided by retailers. This is very similar to the services provided by a financial intermediary, whose job consists among other things of making sure that the credit-takers are worth the credits given. In retailing, intermediaries make sure that the producers are worth the credit the consumers give them when buying their products. Accordingly, the fees retailers charge to producers are just the analogue to the collateral banks require when they give credit. Thus, the non-financial intermediary may mitigate the same sort of problems arising from asymmetric information as financial intermediaries.[1] On the other hand, retailing is certainly a profitable business for other reasons, too. It allows customers to find at less cost what they are looking for. This sort of service is provided by banks as well, of course. Though a person might find a more profitable opportunity for a credit contract outside a bank, searching for such an opportunity typically involves the costs of time spent searching for (and the risk of not finding) this opportunity. Hence, financial and non-financial intermediaries might provide basically the same sort of services with respect to the dispersed nature of information, too, namely that of mitigating or eliminating search cost.

[1] See also Friedman (1962, p.146) for a suggestion to view retailers as providing, among other things, these kinds of services.

The immediate goal of the present research is to incorporate Cournot (or Cournot-like) competition between intermediaries into the model set out by Gehrig (1993). In order to do so, we have to modify the model in two main ways. First, we introduce a dynamic (or sequential) structure, so that the intermediary can only start selling after he has finished buying. As a consequence of this sequential structure, the number of (subgame perfect) equilibria is reduced considerably. This is what we do in this present paper. In a companion paper, we then introduce competition between capacity constrained intermediaries à la Kreps and Scheinkman (1983).

This paper is also related to Spulber (1996), Rust and Hall (2003) and Neeman and Vulkan (2002). What distinguishes this paper from Spulber's and Rust and Hall's work is that though we introduce a dynamic structure, our model is basically a static one-shot game. Its structure is the same as that of a partial equilibrium model, with the exception that a "central market place" is not taken as given. This is also what makes the difference to Neeman and Vulkan (2002), who study whether agents will choose to trade in a decentralized or in a centralized market, but do not explain how the centralized market operates. An important contrast to Spulber (1996) is that the prices the intermediary sets are publicly observable. Dixit (2001) finally studies profit maximizing intermediation in a model where the basic informational friction is the trust-worthiness of individual agents. Intermediaries in his model provide information about past behavior of agents and enforce contracts. However, extending our model to asymmetric information is beyond the scope of the present and the companion paper.

The paper is structured as follows. Section 2 describes, the basic model. Section 3 derives the solution of the dynamic intermediation game, and Section 4 concludes. Discussions of issues concerning rationing and the structure of the Gehrig model are relegated to the Appendix.

2 The Basic Model

Our model world consists of a large number of individuals who can engage in trade in a decentralized (search) market. More specifically, there is a continuum of buyers willing to buy one unit of an indivisible good of homogenous quality (which is known to every one). Their preferences are described by reservation prices r

which are uniformly distributed over the unit interval, $r \sim U[0,1]$. If a buyer with reservation price r buys the product at price p (where the volunteer nature of exchange and individual rationality require $p \leq r$), his utility gain is $r-p$. This generates an aggregate demand schedule $D(p) = 1-p$, $p \in [0,1]$, which can be interpreted as a (Walrasian) market demand. Analogously, sellers' preferences are described by reservation prices or unit costs of production s which are uniformly distributed on the unit interval $[0,1]$. If a seller with reservation price s sells the product at price p (where again volunteer exchange under individual rationality requires $p \geq s$), his utility gain is $p-s$, so that aggregate supply is $S(p) = p$, $p \in [0,1]$. A buyer with reservation price r owns another good that he can exchange for the good in question. This good is called money. We assume that buyers have money and that sellers accept money in exchange for the good. Given the demand function $D(p) = 1-p$ and the supply function $S(p) = p$, the Walrasian market outcome is characterized by price $p^W = \frac{1}{2}$ and quantity exchanged $Q^W = \frac{1}{2}$.

At the core of the model is the assumption that there is no benevolent auctioneer quoting market clearing prices and coordinating trading activities at zero costs. Therefore, the agents are forced to establish the allocation by their own actions. The dispersed nature of information makes search for a trading partner costly insofar as search is time consuming and involves uncertainty. Following Gehrig, we assume that only individuals with expected utility gain from search market participation greater than zero enter the search market. When buyers and sellers enter the search market, they are matched at random by some matching technology. As Spulber (1999, p. 561) observes, the search market is static in the sense that search market participants are randomly and pairwise matched at most once. Gehrig (1993, p.102/3, emphasis added) describes the matching technology as follows:

> The technology is such that each market participant on the *short side* of the market is matched with some probability $\lambda \in [0,1]$ with an agent of the opposite type. The matching probabilities of agents on the *long side* consequently are adjusted by the relative numbers and, therefore, less than λ.

When a seller and a buyer are matched, they bargain over the price. We briefly review the most prominent bargaining procedures and then explain why we choose an even sharing rule.

Brief Review of Bargaining Procedures In Gehrig (1993), search market offers are take-it-or-leave-it offers and the trader who can make the offer is determined by nature. Accordingly, agents' reservation prices are always private information, so that the trader who makes the offer does not know the reservation of the other trader. Consequently, the optimal offer depends on (the distribution of) traders active in the search market. As an alternative, Spulber (1999) suggests to introduce alternating offers à la Rubinstein (1982) so as to get rid of the asymmetry of information inherent in the assumption that reservation prices are private information in the bargaining process. In a Rubinstein alternating offers game, both reservation prices and subjective discount factors are assumed to be knowledge common to both parties engaged in the process. Finally, Freixas and Rochet (1997, exercise 2.1) assume that reservation prices are known when a buyer and a seller have been successfully matched and that the two parties share the gains from trade $r - s$ evenly, provided $r - s > 0$. (If $r - s \leq 0$ there is no trade.) We refer to this as the solution under an even sharing rule. Interestingly, this corresponds to the Nash bargaining solution (Mas-Collel et al., 1995, p. 842).[2]

Even Sharing Lacking a generally accepted theory and/or robust empirical evidence about people's bargaining behavior, one procedure is as good as any other. However, it should be noted that the even sharing rule coincides with the expected payoff of the Rubinstein alternating offers game if both players have the same discount factor and if both players have the same chance of making the first offer. (This is shown in Appendix B.) Since for the game as a whole, it is this expected payoff that matters only, the even sharing rule can therefore be seen as a combination of the Gehrig and the Spulber-Rubinstein procedure if the person who makes the first offer is determined by nature. Moreover, because adding a Rubinstein bargaining game to the last stage of the game (with nature determining the first mover) yields the same results as simply assuming that the gains from trade are shared evenly, we assume that buyers and sellers who are successfully matched learn each other's reservation price and then share the

[2]The Nash bargaining solution is the partition p which maximizes $\prod_{i \in N} U_i(p)$, where $U_i(p)$ is individual i's utility gain under p. Denote the aggregate utility gain from cooperation by U. In our case, $U = r - s := x > 0$, $U_\sigma = p$, $U_\beta = x - p$, so that $\prod_{i \in N} U_i = U_\sigma U_\beta = p(x - p)$, which is maximized at $p = \frac{x}{2}$.

gains from trade evenly. Not doing so would only make the whole game more cumbersome and necessitate a lot of notation not needed otherwise.[3]

The Dynamic Intermediation Game

Next we describe the dynamic intermediation game with a monopolistic intermediary. The main differences to the original model is the time structure of the game and as a consequence of this, a reduction of the number of (subgame perfect) equilibria with an active search market (see Gehrig, 1993; Freixas and Rochet, 1997; Spulber, 1999). In Appendix A.2 we also clarify what happens with agents who get rationed by the intermediary.

In the presence of intermediation, buyers and sellers face three decisions. They can either join the intermediary, enter the search market or choose to remain inactive. Using Gehrig's notation, we denote by I_σ (I_β) the set of all sellers (buyers) who join the intermediary. The set of sellers (buyers) active in the search market is denoted by S_σ (S_β), and the set of sellers (buyers) who decide not to be active is denoted by Z_σ (Z_β). Finally, we denote by Ω_σ (Ω_β) the set of all sellers (buyers), so that by definition $Z_\sigma \equiv \Omega_\sigma \backslash (I_\sigma \cup S_\sigma)$ and $Z_\beta \equiv \Omega_\beta \backslash (I_\beta \cup S_\beta)$. The (Lebesgue) measure of these sets is denoted by $v(.)$.

In the first stage, the intermediary sets a maximum quantity he is willing to buy $\overline{q}$, to which we refer as a capacity constraint, and he quotes the bid price b at which he is willing to buy. Sellers decide whether they want to sell to the intermediary. In the second stage, which begins after buying is finished, the intermediary sets the ask price a at which he is willing to sell. Buyers decide whether they want to join the intermediary. It is assumed that the intermediary's prices and the sets of individuals joining the intermediary are observed by all agents without costs. In the presence of an intermediary, the market where buyers interact with the intermediary will be called *output market* and the place where sellers interact with the intermediary will be called *input market*. The intermediary is also assumed to accept money in exchange for the good. Because the intermediary first buys and then sells, he is assumed to have enough money to buy from the sellers the quantity he wants to buy.

The intermediary must have two technologies individual search market traders do not have. The first one is an information or communication technology, i.e.

[3]This is the same motivation as in Rubinstein and Wolinksy (1987, p. 283).

the capacity to communicate to all traders the prices at which he is willing to trade. The second one is the capacity to trade the volume the intermediary wants to trade (counters, transportation facilities etc.), which will be referred to as capacity constraint. If more sellers join the intermediary than he can serve given his capacity constraint, some sellers get rationed. Likewise, if more buyers join the intermediary than he can serve, some buyers will get rationed. Throughout, we assume that sellers and buyers who get rationed by the intermediary cannot go back to the search market.

Let us summarize. The intermediation game with a monopolistic intermediary has three stages.

1. **Input Market**: The intermediary determines a maximum quantity - a capacity constraint $\overline{q}$ - he is willing to buy and sets a bid price b. After observing b, sellers decide simultaneously whether to join the intermediary. Up to $\overline{q}$, the intermediary is obliged to buy any quantity sellers want to sell to him. For all those sellers who join the intermediary, the game is over, regardless of whether they can actually sell or not.[4] When a seller with cost s has joined the intermediary, this information becomes public. Accordingly, the aggregate quantity bought by the intermediary, denoted as q^b, becomes public information, too. These assumptions imply that $q^b = \min[\overline{q}, \upsilon(I_\sigma)]$.

2. **Output Market**: On the output market, the intermediary sets an ask price a at which he has to sell any quantity buyers want to buy up to his whole stock q^b. If $\upsilon(I_\beta) < q^b$, the intermediary can dispose of the extra units for free. For buyers who decide to join the intermediary the game is over, regardless of whether they can buy or get rationed. The set of buyers who have joined the intermediary is observed by all players remaining in the game.

3. **Search Market**: Sellers and buyers who have not joined the intermediary may join the search market. Those who participate in the search market meet randomly. The matching technology is such that all traders in the search market are matched with probability $\lambda \in [0, 1]$ if the set of sellers and

[4]See Appendix A.2.

buyers active in the search market have the same measure. Otherwise, the traders on the long side of the search market are matched with probability $\gamma_i\lambda$, where $\gamma_i = \frac{\upsilon(I_j)}{\upsilon(I_i)} < 1$ with $i = \sigma, \beta, j \neq i$. There is no further possibility to trade after a match has been established. For those who are not matched, the game is over. A buyer r and a seller s who are successfully matched share the gains from trade evenly by agreeing on the price $\frac{r-s}{2}$ if $r - s > 0$. If $r - s \leq 0$, they do not exchange the good. After that, the game is over.

Strategies There are three types of agents, sellers s, buyers r and the monopolistic intermediary I. A strategy for seller s is

$$\tau_s = (I_s(b, \overline{q}); S_s(a, b, \overline{q}, I_\sigma, I_\beta)) \,. \tag{1}$$

Similarly, for a buyer a strategy is

$$\rho_r = (I_r(a, b, \overline{q}, I_\sigma,), S_r(a, b, \overline{q}, I_\sigma, I_\beta)) \,, \tag{2}$$

where the functions $I_i(.)$ and $S_i(.)$ specify the conditions under which agent i joins the intermediary or the search market, respectively, $i = s, r$. Note that both for sellers and buyers, we do not have to specify the decision to be inactive, because it is contained in the case where an agent decides to join neither the intermediary nor the search market. Finally, for the intermediary, a strategy is

$$\varphi = (\overline{q}, b; a(I_\sigma)) \,, \tag{3}$$

where $\overline{q}$ and b are real positive numbers and $a(I_\sigma)$ is a real valued function. In general, a strategy for this game is a complicated expression, because there are many states of the world for which each agent must have a complete contingent plan. For example, every small change in the set of sellers deciding to join the intermediary will require a different optimal response by all other players in subsequent periods. Since there is an infinity of such contingencies, it will not be possible to write down these strategies in closed forms in general.

3 Results

We first show that a slight modification of Proposition 1 of Gehrig (1993) applies for the dynamic intermediation game with even sharing, so that the space over which strategies must be defined can be reduced considerably.

Proposition 1 (Gehrig (1993), Proposition 1) *In any equilibrium with an active search market,*[5] *there are critical reservation values* $\underline{r}$ *and* $\overline{r}$*, such that the set of buyers can be partitioned into three subsets . If* $r \in [0, \underline{r})$*, then* $r \in Z_\beta$*; if* $r \in [\underline{r}, \overline{r}]$*, then* $r \in S_\beta$ *and if* $r(\overline{r}, 1]$*, then* $r \in I_\beta$*. There are critical unit costs* $\underline{s}$ *and* $\overline{s}$*, such that the set of sellers can be partitioned into three subsets (in any equilibrium with an active search market). If* $s \in [0, \underline{s})$*, then* $s \in I_\sigma$*; if* $s \in [\underline{s}, \overline{s}]$*, then* $s \in S_\sigma$ *and if* $s \in (\overline{s}, 1]$*, then* $s \in Z_\sigma$*.*

The Proposition is proved with the help of the following three Lemmas.

Lemma 1 (Gehrig (1993), Lemma 1) *For any positive ask bid spread* $a - b > 0$*, some traders will be active in the search market.*

Proof: Buyers with $r < a$ and sellers with $s > b$ can expect positive utility gains from search market participation.■

Lemma 2 (Gehrig (1993), Lemma 2) *In equilibrium, the sets of inactive buyers and sellers,* Z_β *and* Z_σ*, are closed and convex sets such that* $0 \in Z_\beta$ *and* $1 \in Z_\sigma$*.*

Proof: Let buyer r be inactive and suppose $\tilde{r} < r$ is active. Then r could imitate $\tilde{r}$ and get at least his payoff, whereas his payoff when inactive is zero. Completely symmetric reasoning applies for sellers. Finally, buyer 0 and seller 1 remain inactive because they never expect a positive gain from trade.■

Lemma 3 (Modification of Lemma 3, Gehrig (1993)) *In any equilibrium with an active search market (i.e.* $S_\sigma \neq \emptyset, S_\beta \neq \emptyset$*),*
(i) $r_0 \in S_\beta \Rightarrow r \notin I_\beta$ *for* $r < r_0$ *and*
(ii) $s_0 \in S_\sigma \Rightarrow s \notin I_\sigma$ *for* $s > s_0$*.*

Proof: Parts of the proof very closely mimic the one by Gehrig (1993). We denote by $\alpha_i, i = \sigma, \beta$ a seller's and a buyer's probability of being rationed when joining the intermediary. This probability is the same regardless of the seller's

[5] We have added this phrase because there is also an equilibrium where no one joins the search market. If no one goes to the search market, unilateral deviation to join the search market does obviously not pay. However, as long as there is no fix cost of joining the search market, in this equilibrium, two continua of agents play weakly dominated strategies.

and buyer's valuations s and r. That is, we consider the case with proportional rationing.[6] Also, we denote by $\gamma_i, i = \sigma, \beta$ a seller's and a buyer's probability of being successfully matched in the search market with probability λ. Thus, for example a seller is matched with probability $\lambda\gamma_\sigma = \lambda \min[\frac{\upsilon(S_\beta)}{\upsilon(S_\sigma)}, 1]$. Since each agent has measure zero, α_i and $\gamma_i, i = \sigma, \beta$ can be taken as given by every individual agent.

We first consider (ii) of Lemma 3 of Gehrig. Two cases can be distinguished, the case where there is rationing at the intermediary's, i.e. $\alpha_\sigma < 1$, and the case where there is no rationing, i.e. $\alpha_\sigma = 1$. We consider the latter case first. Note that $s_0 \in S_\sigma \Leftrightarrow \gamma_\sigma U_\sigma(s_0) \geq \alpha_\sigma(b - s_0)$, where $U_\sigma(s_0)$ is the expected utility gain of seller s_0 of search market participation for $\upsilon(S_\beta) = \upsilon(S_\sigma)$. (For the case under consideration now, we have $\alpha_\sigma = 1$.) Let $F(r)$ be the cumulative distribution function of buyers active in the search market. Then, we have

$$U_\sigma(s_0) = \lambda \int_{s_0 \leq r} \frac{r - s_0}{2} dF(r), \text{and} \tag{4}$$

$$U_\sigma(s) = \lambda \int_{s \leq r} \frac{r - s}{2} dF(r). \tag{5}$$

Because $s > s_0$, $U_\sigma(s_0) > U_\sigma(s)$. Subtracting (5) from (4) we get

$$\begin{aligned} U_\sigma(s_0) - U_\sigma(s) &= \lambda \int_{s_0 \leq r} \frac{s - s_0}{2} dF(r) - \lambda \int_{s_0 \leq r \leq s} s dF(r), \text{or} \\ U_\sigma(s) &= U_\sigma(s_0) - \lambda \int_{s_0 \leq r} \frac{s - s_0}{2} dF(r) + \lambda \int_{s_0 \leq r \leq s} s dF(r). \end{aligned}$$

Since $s > 0, \lambda \int_{s_0 \leq r \leq s} s dF(r) > 0$, so that

$$U_\sigma(s) > U_\sigma(s_0) - \lambda \int_{s_0 \leq r} \frac{s - s_0}{2} dF(r).$$

Because $\lambda \int_{s_0 \leq r} \frac{s - s_0}{2} dF(r) < s - s_0$,

$$U_\sigma(s) > U_\sigma(s_0) - (s - s_0).$$

Multiplying both sides by γ_σ, $0 < \gamma_\sigma \leq 1$, we get $\gamma_\sigma U_\sigma(s_0) > \gamma_\sigma U_\sigma(s) - \gamma_\sigma(s - s_0)$, so that

$$\gamma_\sigma U_\sigma(s) > \gamma_\sigma U_\sigma(s_0) - (s - s_0).$$

[6] If rationing is efficient, then, for example, the $\overline{q}$ sellers with the lowest costs joining the intermediary will be able to sell with probability one, and all others with probability 0. Therefore, the proof for the case with efficient rationing is much simpler than with proportional rationing.

But $s_0 \in S_\sigma \Leftrightarrow \gamma_\sigma U_\sigma(s_0) \geq \alpha_\sigma(b - s_0)$ (where for the case we are considering $\alpha_\sigma = 1$). Therefore,

$$\gamma_\sigma U_\sigma(s) > (b - s_0) - (s - s_0) = b - s,$$

where $b - s$ is the utility gain for s of joining the intermediary. Thus, $s > s_0$ will not join the intermediary if s_0 joins the search market. For buyers, the case (i) under the assumption $\alpha_\beta = 1$ is completely analogous and can be found in Gehrig (1993, p.114/5).

Next, let us consider (ii) with $\alpha_\sigma < 1$. The utility gain of seller s from joining the search market $\gamma_\sigma U_\sigma(s)$ is certainly as great as the utility he gets when making a deal in exactly the same matches as seller s_0 does (in a sense, this is the value s attributes to the expected deals s_0 accepts).[7] That is,

$$\begin{aligned} \gamma_\sigma U_\sigma(s) &\geq \gamma_\sigma \lambda \int_{s_0 \leq r} \frac{r - s}{2} dF(r) \\ &= \gamma_\sigma U_\sigma(s_0) - (s - s_0)\gamma_\sigma \lambda \int_{s_0 \leq r} \frac{1}{2} dF(r). \end{aligned}$$

Now two cases have to be distinguished: (1) $\alpha_\sigma > \gamma_\sigma \lambda \int_{s_0 \leq r} \frac{1}{2} dF(r)$ and (2) $\alpha_\sigma \leq \gamma_\sigma \lambda \int_{s_0 \leq r} \frac{1}{2} dF(r)$. It will be shown first that in case (1) s will not join the intermediary for $s > s_0$ and second that in case (2), $S_\sigma = S_\beta = \emptyset$.

For case (1), it is true that

$$\gamma_\sigma U_\sigma(s) > \gamma_\sigma U_\sigma(s_0) - \alpha_\sigma(s - s_0).$$

Add and subtract $\alpha_\sigma b$ on the right-hand side to get

$$\gamma_\sigma U_\sigma(s) > \gamma_\sigma U_\sigma(s_0) - \alpha_\sigma(b - s_0) + \alpha_\sigma(b - s).$$

But because s_0 joins the search market, $U_\sigma(s_0) - \alpha_\sigma(b - s_0) \geq 0$. Therefore, $\gamma_\sigma U_\sigma(s) > \alpha_\sigma(b - s)$, where $\alpha_\sigma(b - s)$ is s's expected utility gain from joining the intermediary. Thus, for $s > s_0$, s will not join the intermediary if $s_0 \in S_\sigma$.

The case for buyers being completely symmetric, it will not be added here.

In case (2), for any seller $s < s_0$ we would have

$$\begin{aligned} \gamma_\sigma U_\sigma(s) &\geq \gamma_\sigma U_\sigma(s_0) + \alpha_\sigma(s_0 - s) \\ &= \gamma_\sigma U_\sigma(s_0) - \alpha_\sigma(b - s_0) + \alpha_\sigma(b - s). \end{aligned}$$

[7]Note that s is not required to be greater than s_0.

Because $s_0 \in S_\sigma$, $\gamma_\sigma U_\sigma(s_0) - \alpha_\sigma(b - s_0) \geq 0$. Thus, $\gamma_\sigma U_\sigma(s) > \alpha_\sigma(b - s)$ and therefore, $s \in S_\sigma$ for $s < s_0$. Because this holds for any $s < s_0$, then if $s_0 \in S_\sigma$ for one s_0, then $I_\sigma = \emptyset$. But this contradicts $\alpha_\sigma = \frac{\overline{q}}{\upsilon(I_\sigma)} < 1$. Therefore, it must be that $S_\sigma = \emptyset$. Completely analogous reasoning applies for buyers, establishing that $S_\beta = \emptyset$. This completes the proof of Lemma 3.■

Note that $S_\sigma = S_\beta = \emptyset$ and $\alpha_i < 1, i = \sigma, \beta$ can happen only if $a = b$ and if for all $s \leq b$, $s \in I_\sigma$ and for all $r \geq a$, $r \in I_\beta$. Now all sellers $s > b$ and buyers $r < a$ will be inactive, so that indeed $r_0 \in Z_\beta \Rightarrow r \notin I_\beta$ for $r < r_0$ and $s_0 \in Z_\sigma \Rightarrow s \notin I_\sigma$ for $s > s_0$, which is just Gehrig's Lemma 2.

Proof of Proposition 1: These three Lemmas state that the sets of inactive buyers and sellers and the sets of buyers and sellers active in the search market are convex and directed sets. Therefore, only buyers with high reservation prices and sellers with low costs can potentially gain by trading with the intermediary. ■

Rationing in the Dynamic Game As a consequence of the dynamic structure, rationing occurs in a way slightly different from the one in Gehrig's paper. Sellers are rationed if and only if the "number" of sellers willing to sell at bid price b exceeds the capacity $\overline{q}$ the intermediary has set (i.e. iff $\upsilon(I_\sigma) > \overline{q}$), while buyers are rationed if and only if the number of buyers willing to buy at ask price a exceeds the quantity the intermediary has in stock, which is $\min[\upsilon(I_\sigma), \overline{q}]$. This is in contrast to the game in Gehrig's setting, where rationing of sellers (buyers) occurs if and only if $\upsilon(I_\sigma) > (<) \upsilon(I_\beta)$. Note that this is so independent of the rationing rule that applies in case rationing occurs.

3.1 Input Supply and Output Demand Functions

For $a > b$, Gehrig's Lemma 1 implies that all buyers with $r \in [\underline{s}, \overline{r}]$ and all sellers with $s \in [\underline{s}, \overline{r}]$ are active in the search market so that $S_\beta = S_\sigma = [\underline{s}, \overline{r}]$. Therefore, in any equilibrium with $a > b$, $\gamma_\beta = \gamma_\sigma = 1$. Moreover, because reservation prices of all agents are uniformly distributed on the unit interval, we know that for $r \in S_\beta, r \sim U[\underline{s}, \overline{r}]$ and for $s \in S_\sigma, s \sim U[\underline{s}, \overline{r}]$. Therefore, $dF(r) = \frac{1}{\overline{r} - \underline{s}} dr$ and $dG(s) = \frac{1}{\overline{r} - \underline{s}} ds$, where $F(r)$ and $G(s)$ are the cumulative distribution functions of buyers and sellers active in the search market. Since all previous actions are assumed to be observable, $\underline{s}$ and $\overline{r}$ will be known when agents decide whether to join the search market. Therefore, it suffices to condition this decision on $\underline{s}$ and $\overline{r}$, so that a strategy for seller s can be written as $\tau_s = (I_s(b, \overline{q}); S_s(a, b, \overline{q}, \underline{s}, \overline{r}))$. Similarly, for a buyer a strategy can be written as $\rho_r = (I_r(a, b, \overline{q}, \underline{s}); S_r(a, b, \overline{q}, \underline{s}, \overline{r}))$, and for the intermediary, a strategy simplifies to $\varphi = (\overline{q}, b; a(\underline{s}))$. This allows us to compute explicitly the expected utility gains from search market participation and to characterize completely agents' equilibrium strategies in the game. This is what we do next.

We begin by briefly describing the equilibrium of the bargaining subgame. With even sharing, a buyer r and a seller s who are matched in the search market share the gains from trade $r - s$ equally, provided $r - s > 0$. We will refer to seller $\underline{s}$ and buyer $\overline{r}$ as the critical seller and buyer. The expected utility gain for seller s with $s \in [\underline{s}, \overline{r}]$ from search market participation is then

$$\begin{aligned} U_\sigma(s) &= \lambda \int_s^{\overline{r}} \frac{(r-s)}{2} dF(r) = \frac{\lambda}{2} \frac{1}{\overline{r} - \underline{s}} \int_s^{\overline{r}} (r-s) dr \\ &= \frac{\lambda}{2} \frac{\left[\frac{r^2}{2} - rs\right]_s^{\overline{r}}}{\overline{r} - \underline{s}} = \frac{\lambda}{4} \frac{(\overline{r} - s)^2}{\overline{r} - \underline{s}}, \end{aligned}$$

which is the same as that derived by Gehrig under the alternative bargaining schedule with take-it-or-leave-it offers. Thus, for the critical seller $\underline{s}$ we have

$$U_\sigma(\underline{s}) = \frac{\lambda}{4} (\overline{r} - \underline{s}) . \tag{6}$$

Likewise, for a buyer with reservation price $r \in [\underline{s}, \overline{r}]$ the expected utility gain from being active in the search market is

$$\begin{aligned} U_\beta(r) &= \lambda \int_{\underline{s}}^{r} \frac{(r-s)}{2} dG(s) = \frac{\lambda}{2} \frac{1}{\overline{r} - \underline{s}} \int_{\underline{s}}^{r} (r-s) ds \\ &= \frac{\lambda}{4} \frac{(r - \underline{s})^2}{\overline{r} - \underline{s}}, \end{aligned}$$

so that for the critical buyer

$$U_\beta(\overline{r}) = \frac{\lambda}{4} (\overline{r} - \underline{s}) = U_\sigma(\underline{s}) . \tag{7}$$

Now, the utilities of critical buyers and sellers participating in the search market in equation (7) can be used to derive the reservation prices of these agents for joining the intermediary. If buyers and sellers joining the intermediary are rationed with probability $\alpha_\beta \leq 1$ and $\alpha_\sigma \leq 1$, buyer $\overline{r}$ is indifferent[8] between joining the intermediary and joining the search market if and only if

$$\alpha_\beta(\overline{r} - a) = \frac{\lambda}{4}(\overline{r} - \underline{s}) \tag{8}$$

and seller $\underline{s}$ is indifferent if and only if

$$\alpha_\sigma(b - \underline{s}) = \frac{\lambda}{4}(\overline{r} - \underline{s}). \tag{9}$$

For $\alpha_\beta = \alpha_\sigma = 1$, solving equation (8) and (9) yields

$$a\left(\overline{r}, \underline{s}\right) = \frac{4-\lambda}{4}\overline{r} + \frac{\lambda}{4}\underline{s} \quad \text{and} \quad b\left(\underline{s}, \overline{r}\right) = \frac{4-\lambda}{4}\underline{s} + \frac{\lambda}{4}\overline{r}. \tag{10}$$

Thus, $a\left(\overline{r}, \underline{s}\right)$ and $b\left(\underline{s}, \overline{r}\right)$ are reservation prices of buyer $\overline{r}$ and seller $\underline{s}$ for joining the intermediary, given all $s < \underline{s}$ and all $r > \overline{r}$ have joined the intermediary and provided there is no rationing.

Recall that we assume that buyers and sellers who are indifferent between joining the intermediary and joining the search market join the search market. If the intermediary quotes ask price $a = a\left(\overline{r}, \underline{s}\right)$ and bid price $b = b\left(\underline{s}, \overline{r}\right)$ and if there is no rationing, buyers with $r > \overline{r}$ and sellers with $s < \underline{s}$ will then join the intermediary. Note that if there is to be no rationing on the input market, the capacity constraint $\overline{q}$ needs to be at least as great as $\underline{s}$. On the other hand, the quantity the intermediary sells on the output market cannot exceed the quantity bought on the input market, which is denoted as q^b. Note that this is the quantity bought *by* the intermediary. Clearly, $q^b = \min[\underline{s}, \overline{q}]$. Thus, without rationing, $\overline{r} \geq 1 - q^b$.

There being $1 - \overline{r}$ buyers whose reservation prices are greater than or equal to $\overline{r}$, quantity demanded at ask price $a\left(\overline{r}, \underline{s}\right)$ is $1 - \overline{r}$. Let q^d denote this quantity. Note that this is quantity demanded *at* the intermediary's. For the same reasons as for buyers, there are $\underline{s}$ sellers who are willing to sell at bid price $b\left(\overline{r}, \underline{s}\right)$, provided the buyer with the highest reservation price in the search market is buyer $\overline{r}$. Therefore, $\underline{s}$ is equal to the quantity the intermediary can buy at the bid price $b\left(\overline{r}, \underline{s}\right)$, which is q^b. If we replace $\overline{r}$ by $1 - q^d$ and $\underline{s}$ by q^b in equation (10), we get the inverse output demand and (inverse) input supply functions

$$a\left(q^d, q^b\right) = \frac{4-\lambda}{4} - \frac{4-\lambda}{4}q^d + \frac{\lambda}{4}q^b \tag{11}$$

[8] Throughout, we assume that all agents - buyers, sellers and the intermediary - are risk neutral.

and

$$b\left(q^d, q^b\right) = \frac{\lambda}{4} - \frac{\lambda}{4}q^d + \frac{4-\lambda}{4}q^b, \tag{12}$$

while the output demand and the input supply functions are

$$q^d\left(a, q^b\right) = 1 - \frac{4}{4-\lambda}a + \frac{\lambda}{4-\lambda}q^b, \tag{13}$$

and

$$q^b\left(b, q^d\right) = -\frac{\lambda}{4-\lambda} + \frac{4}{4-\lambda}b + \frac{\lambda}{4-\lambda}q^d. \tag{14}$$

Note that the ask price elasticity of output demand, given q^b, is

$$\varepsilon\left(a, q^b\right) := -\frac{4a}{4-\lambda-4a+\lambda q^b}. \tag{15}$$

Note also that these functions are valid only under the provision that there is an active search market from which some agents can expect positive utility gains. This requires that $\overline{r} > \underline{s}$. If $\overline{r} \leq \underline{s}$, agents lose the outside option of search market participation. In this case, seller s would join the intermediary whenever $b > s$ and a buyer r will buy from the intermediary whenever $a < r$. Graphically, therefore, beyond the point of intersection of the (inverse) output demand function $a\left(q^d, q^b\right)$ with the (inverse) Walrasian demand function $1 - q^d$, the willingness to pay for intermediated trade is given by the (inverse) Walrasian demand function. Therefore, the reservation prices of buyers an intermediary faces are actually given by the maximum of these two functions

$$\min\left[a\left(q^d, q^b\right), 1 - q^d\right]. \tag{16}$$

It is easy to verify that the intersection of $a\left(q^d, q^b\right)$ with $1 - q^d$ is at the point where $1 - q^d = q^b$.

Analogously, the (inverse) input supply function $b\left(q^d, q^b\right)$ in equation (12) is valid only to the left of the intersection with q^b. Beyond that point, expected utility gain from search market participation in not positive, and the reservation prices for trading through the intermediary are given by the (inverse) Walrasian supply function. Hence, the sellers' reservation prices the intermediary faces are given by the maximum of these two functions

$$\max\left[b\left(q^d, q^b\right), q^b\right]. \tag{17}$$

Again, the point of intersection is where $1 - q^d = q^b$.

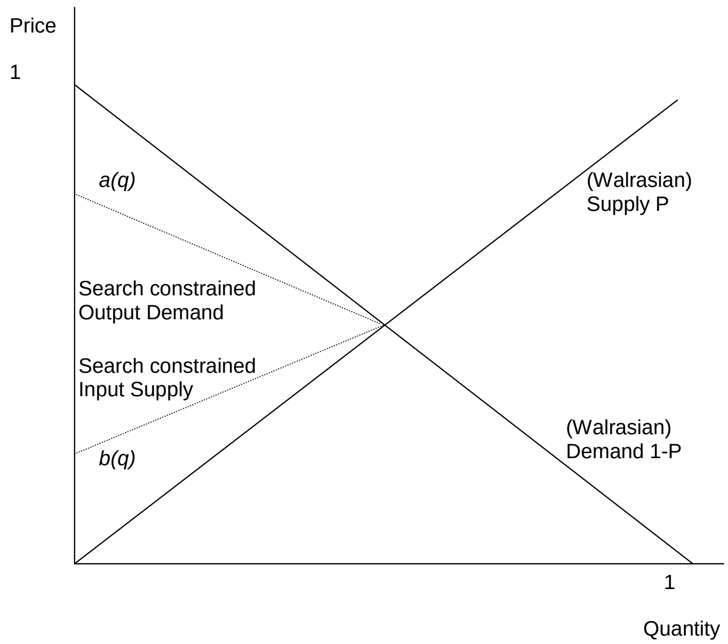

Figure 1: Walrasian and search market constrained demand and supply functions.

If quantity bought equals quantity sold, i.e. $q^d = q^b = q$, trade by the intermediary is said to be *balanced*. In this case, the inverse demand and supply functions are

$$a(q) := \frac{4-\lambda}{4} - \frac{2-\lambda}{2} q \tag{18}$$

$$b(q) := \frac{\lambda}{4} + \frac{2-\lambda}{2} q. \tag{19}$$

Under balanced trade, the input supply function is

$$q^b(b) := \frac{4b-\lambda}{2(2-\lambda)}, \tag{20}$$

so that under balanced trade the inverse output demand function can be written as a function of b only

$$a\left(q^b(b)\right) = 1 - b. \tag{21}$$

Figure 1 depicts the Walrasian demand and supply functions and the search constrained output demand and input supply functions for the intermediary, under the assumption that intermediated trade is balanced.

3.2 Subgame Perfect Equilibria

Next, we analyze the subgame perfect equilibria of the dynamic intermediation game. These are described in Proposition 2 below. We call the subgame that begins after capacity has been built "capacity constrained subgame" and show that the capacity constraint subgame has a unique subgame perfect equilibrium with an active search market. Before we turn to Proposition 2, we state and prove the following Lemmas.

Because whether or not there is rationing on the input side may affect output demand, in principle we have to distinguish these two cases when analyzing the output market subgame. However, as we show in Lemmas 4 and 5, whether or not there has been rationing on the input side, it will not be in the interest of the intermediary to ration buyers on the output market. That is, if the intermediary has bought quantity q and if all $s \in [0, \underline{s})$ have joined the intermediary (where $q < \underline{s}$), there is an ask price a such that a buyer wants to join the intermediary if and only if $r > 1 - q$. This result is summarized in Lemma 4.

Lemma 4 *For $q \leq \underline{s} \leq \frac{1}{2}$ and for $a = \frac{4-\lambda}{4}(1-q) + \frac{\lambda}{4}\underline{s}$, in any equilibrium, $r \in I_\beta$ if and only if $r \in (1-q, 1]$.*

Proof: Buyer $r_0 := 1 - q$ is indifferent between joining the search market and joining the intermediary, since $r = r_0$ is the solution to

$$r - \frac{4-\lambda}{4}(1-q) - \frac{\lambda}{4}\underline{s} = \frac{\lambda}{4}(r - \underline{s}).$$

Since we have assumed that indifferent buyers join the search market, $r_0 \in S_\beta$. However, for any $r > r_0$, the utility gain from joining the intermediary is greater than the utility gain from search market participation. To see this, consider buyer with $r = r_0^+$, where r_0^+ denotes a reservation price marginally greater than r_0. His utility gain from buying from the intermediary is greater than his expected utility gain from joining the search market under the hypothesis that he is the critical buyer. Therefore, $r = r_0^+$ will join the intermediary. From Lemmas 2 and 3 it then follows that $r \in I_\beta$ for any $r \geq r_0^+$. ■

Lemma 5 *For a given $\overline{r}$ and a given $\underline{s}$, buyer $\overline{r}$'s reservation price for joining the intermediary, $a(\overline{r}, \underline{s})$, is greatest if $\alpha_\beta = 1$.*

Proof: Solving $\alpha_\beta(\overline{r} - a) = \frac{\lambda}{4}(\overline{r} - \underline{s})$ (see equation (8)) for a yields $a = \frac{4\alpha_\beta - \lambda}{4}\overline{r} + \frac{\lambda}{4}\underline{s}$. Clearly, this is greatest for $\alpha_\beta = 1$. ■

Together, Lemmas 4 and 5 state that (1) there is an ask price such that all that has been bought (with or without rationing on the input side) can be sold without rationing on the output market and that (2) rationing of buyers is not in the intermediary's interest because it only decreases the reservation price of buyer $\overline{r}$ for joining the intermediary.[9] Thus, the intermediary will not set an ask price below the one at which he can sell everything. What has not be shown, however, is whether it is in the intermediary's interest to sell everything he has bought (i.e. to set an ask price such that $q^d = q^b$). Lemma 6 states the condition under which the intermediary wants to sell everything.

Lemma 6 *The unique subgame perfect ask price function for the intermediary is*

$$a^*\left(q^b\right) = \max\left[a\left(q\right), \frac{1}{2}\right],$$

for any q^b.

Proof:

By assumption, there are no costs involved with disposing any quantity the intermediary cannot sell. It is also assumed that there are no costs associated with selling. Therefore, if the quantity bought allows him to do so (that is, if q^b is large enough), the monopolistic intermediary will sell exactly the quantity for which elasticity of output demand is -1. (Otherwise, he will set the market clearing price, which is greater than the price at which elasticity is -1.) The intersection between the (inverse) output demand function and the (inverse) Walrasian demand function in (16) being given by

$$q^d = 1 - q^b,$$

this intersection occurs at $q^d < \frac{1}{2}$ for $q^b > \frac{1}{2}$. Thus, for $q^b > \frac{1}{2}$,

$$\min\left[a\left(q^d, q^b\right), 1 - q^d\right] = 1 - q^d.$$

[9] Note that this does not involve any quantity effects, yet; $a(\overline{r})$ decreases not because more buyers have to be attracted by the intermediary in order to have rationing, which in turn requires a to go down. Merely because he is less likely to get served does the reservation price of $\overline{r}$ for joining the intermediary decrease.

That is, the relevant (inverse) demand function is the (inverse) Walrasian demand function, the elasticity of which is -1 for $a = \frac{1}{2}$. Hence, for $a\left(q^d, q^b\right) > 1 - q^d$, $a^*\left(q^b\right) = \frac{1}{2}$.

For $q^b < \frac{1}{2}$, the relevant inverse demand function is $a(q)$. The elasticity of output demand is -1 (see equation (15) above) for

$$a = \left.\frac{4 - \lambda + \lambda q^b}{8}\right|_{q^b \in [0,1]} \leq \frac{1}{2},$$

but for $q^b < \frac{1}{2}$, $a \leq \frac{1}{2}$ will not be market clearing. Without selling less, therefore, the intermediary can increase a up to the price for which $a = a(q)$, where $a(q)$ is the ask price function for balanced trade as defined in (18). But because in this range, demand is elastic, the intermediary has no incentive to increase a beyond this point and to sell less than q^b, so that for $a\left(q^d, q^b\right) < 1 - q^d$, $a^*\left(q^b\right) = a(q)$. ■

What is not yet clear is under which conditions a seller will decide to join the intermediary. Inspection of the inverse input supply function $b(\underline{s}, \overline{r})$ in (10) shows immediately that this decision depends among other things on the reservation price of the critical buyer active in the search market, $\overline{r}$. But since this price depends on the quantity the intermediary sells (which depends on the quantity he buys), this reservation price depends in turn on the decision of all sellers to join or not to join the intermediary, which in turn depends on their expectations what the intermediary and buyers will do in stage 2 of the game, and so on. This is a potential source of indeterminacy: If all other sellers with $s < \underline{s}$ sell, then selling might be optimal for an individual seller, while if all others do not sell, then not selling will be optimal for him as well. However, based on the insights provided by Lemmas 4 and 6, the following Lemma shows that this indeterminacy disappears.

Lemma 7 *For $q \leq \overline{q} \leq \frac{1}{2}$ and $b = b(q)$, $s \in I_\sigma$ if and only if $s \in [0, q)$, where $b(q)$ is as defined in (19).*

Proof: The proof consists of a iterating the same argument. The argument consists of two parts.

(1) There exists a set of sellers with positive measure whose dominant strategy is to sell at bid price $b(q)$, even if all buyers are active in the search market (i.e.

even if the intermediary does not sell anything). Formally, $\exists\ s_1 > 0$ such that $\frac{\lambda}{4}(1-s) \leq b(q) - s$ for all $s \leq s_1$. To see this, solve $\frac{\lambda}{4}(1-s_1) = b(q) - s_1$ for s_1 to get $s_1 = \frac{2(2-\lambda)}{4-\lambda}q > 0$. Note that unless $\lambda = 0, s_1 < q$.

(2) By virtue of Lemmas 4 and 5 the intermediary will want to sell everything he has bought and the buyers with $r \in (1-s_1, 1]$ will join the intermediary. Therefore, the buyer with the highest reservation price active in the search market will be $r = 1 - s_1$. Given this, $\exists\ s_2 > s_1$ such that $\frac{\lambda}{4}(1-s_1-s) \leq b(q) - s$ for all $s \in (s_1, s_2]$. To see this, solve $\frac{\lambda}{4}(1-s_1-s_2) = b(q) - s_2$ to get $s_2 = \left[1 + \frac{\lambda}{(4-\lambda)}\right]\frac{2(2-\lambda)}{(4-\lambda)}q > s_1$ for $\lambda > 0$.

(3) Iterating step (1) and (2) n times, we get

$$\begin{aligned} s_{n+1} &= \left[1 + \frac{\lambda}{4-\lambda} + \left(\frac{\lambda}{(4-\lambda)}\right)^2 + ... + \left(\frac{\lambda}{(4-\lambda)}\right)^n\right]\frac{2(2-\lambda)}{(4-\lambda)}q \\ &= \left[\sum_{i=0}^{n}\left(\frac{\lambda}{(4-\lambda)}\right)^i\right]\frac{2(2-\lambda)}{(4-\lambda)}q. \end{aligned}$$

Let n go to infinity. Since $0 < \frac{\lambda}{(4-\lambda)} < 1$ for $\lambda > 0$, $\lim_{n\to\infty}\sum_{i=0}^{n}\left(\frac{\lambda}{(4-\lambda)}\right)^i = \frac{4-\lambda}{2(2-\lambda)}$ so that $\lim_{n\to\infty} s_{n+1} = q$. ■

Lemma 7 eliminates the multiplicity of equilibria present in Gehrig's original model (in the capacity constrained subgame).[10] Note that in determining $s_k, k = 1, ..., n$ the fact that the intermediary will sell what the sellers with $s \in [s_{k-1}, s_k)$ sell has not been used.

There is a resemblance between this mechanism and a suggestion made by Spulber (1999, p.125), who says that

> ... inventory holdings by intermediaries could reduce buyer and seller concerns about being rationed, which could also alter the equilibrium outcome.

Due to the sequential structure of the game, the intermediary can be seen as holding inventories at the beginning of the output market subgame. At least some buyers then have a dominant strategy to buy from the intermediary. Sellers in turn take this into account when making their decisions to sell to the intermediary. In a sense, by selling to the intermediary and through the intermediary's inventory

[10] Without variable costs of building capacity, there is a continuum of capacity constraints the intermediary can set in equilibrium.

holding, sellers can induce the buyers to buy from the intermediary and thereby to leave the search market.

Next we show that rationing will not occur on the input market for $\overline{q} \leq \frac{1}{2}$.

Lemma 8 *Bid prices $b > b(\overline{q})$ cannot be part of an equilibrium strategy.*

Proof: Because we know from the previous Lemma that with $b = b(\overline{q})$ the intermediary buys $\overline{q}$ when setting $b(\overline{q})$, $b > b(\overline{q})$ has no quantity effect. The only positive effect it has for the intermediary is that it increases $\underline{s}$ in case sellers with $s > \underline{s}$ are attracted and thereby increases $a(\overline{r}, \underline{s})$. The negative effect is, of course, that it is a higher bid price, which by itself decreases the intermediary's profits. It can be shown that the negative effect outweighs the positive effect: For $\underline{s}$ to be indifferent between joining the intermediary and the search market, this requires the following equality to hold

$$\alpha_\sigma(b - \underline{s}) = \frac{\lambda}{4}(1 - \overline{q} - \underline{s}),$$

where we have used the fact that the intermediary will sell on the output market what he has bought on the input market (see Lemmas 4, 5 and 6). Note that $\alpha_\sigma = \frac{\overline{q}}{\underline{s}}$. Plugging this expression in we get after some re-arranging $b = \underline{s}\left(\frac{\lambda}{4}\frac{1}{\overline{q}} + \frac{4-\lambda}{4} - \frac{\lambda}{4}\frac{\underline{s}}{\overline{q}}\right)$. Define this function as $b(\underline{s}, \overline{q})$. Bid price $b > b(\overline{q})$ will not pay if

$$a(\overline{q}, \underline{s}) - b(\underline{s}, \overline{q}) \quad < \quad a(\overline{q}) - b(\overline{q}),$$

where $a(\overline{q}, \underline{s}) := \frac{4-\lambda}{4}(1-q) + \frac{\lambda}{4}\underline{s}$ like in Lemma 4.[11] This inequality implies

$$\begin{aligned} \frac{4-\lambda}{4} - \frac{4-\lambda}{4}\overline{q} - \frac{2-\lambda}{2}\underline{s} - \frac{\lambda}{4}\frac{\underline{s}}{\overline{q}}(1-\underline{s}) \quad &< \quad \frac{2-\lambda}{2} - (2-\lambda)\overline{q} \\ &\Leftrightarrow \\ -\frac{2-\lambda}{2}\underline{s} - \frac{\lambda}{4}\frac{\underline{s}}{\overline{q}}(1-\underline{s}) \quad &< \quad \frac{4-\lambda}{4}(1-\overline{q}) - \frac{2-\lambda}{2}\overline{q}. \end{aligned}$$

But because $\underline{s} > \overline{q}$, $-\frac{2-\lambda}{2}\underline{s} < -\frac{2-\lambda}{2}\overline{q}$. Therefore, if the inequality

$$-\frac{\lambda}{4}\frac{\underline{s}}{\overline{q}}(1-\underline{s}) < \frac{4-\lambda}{4}(1-\overline{q})$$

[11]Strictly speaking, for $b > b(\overline{q})$ not to pay, $a(1-\overline{q}, \underline{s}) - \max[b(\underline{s}, \overline{q}), \underline{s}] < a(\overline{q}) - b(\overline{q})$ suffices, which certainly holds if the above inequality holds; see (17) above.

holds, then $b > b(\overline{q})$ does not pay. Since the right-hand side is positive for any $\overline{q} < 1$, while the left-hand side is negative for any $\underline{s} \in (0,1)$, the inequality holds always. Therefore, $b > b(\overline{q})$ does not pay.■

What remains to be determined is the optimal bid price b^* and the capacity constraint $\overline{q}$ the intermediary sets in stage 1. We first consider the optimal bid price and then the optimal capacity constraint.

Lemma 9 *The optimal bid price is $b^* = \frac{1}{4} + \frac{\lambda}{8}$. If $b = b^*$, all sellers with reservation prices $s < \frac{1}{4}$ will sell to the intermediary.*

Proof: We can neglect the constraint $q \leq \overline{q}^*$, which can still be chosen accordingly. (We only have to assume that $\overline{q}$ is large enough so that quantity q^b can be bought.) We observe first that bid prices b inducing $q^b\left(b\right) > \frac{1}{2}$ cannot be optimal. For if the bid price b is such that $q^b\left(b\right) > \frac{1}{2}$, the intermediary's profits are

$$\pi\left(b\right) \quad = \quad \left[a-b\right] q^b\left(b\right) = \left[\frac{1}{2} - b\right] q^b\left(b\right),$$

where $a^*\left(q^b\right) = \left.\frac{1}{2}\right|_{q^b > \frac{1}{2}}$ has been used. But for $q^b\left(b\right) > \frac{1}{2}$, the search market shuts down (because $\overline{r} < \underline{s}$) and sellers' reservation prices for intermediated trade are given by $b = q^b$ (see (17) above). Therefore, profits are

$$\pi\left(b\right) = \left.\left[\frac{1}{2} - q^b\right] q^b\right|_{q^b > \frac{1}{2}} < 0,$$

which cannot be an optimum given the intermediary's outside option of $\pi = 0$.

Second, by Lemma 8 bid prices $b > b\left(\overline{q}\right)$ can be ruled out. Given $a^*\left(q^b\right) = \max\left[\left. a\left(q^d, q^b\right)\right|_{q^d = q^b}, \frac{1}{2}\right]$ in the second stage of the game (see Lemma 6) and given that for any bid price $\left. b\left(q\right)\right|_{q \leq \min\left(\frac{1}{2}, \overline{q}\right)}$, q sellers are willing to sell to him (see Lemma 7), the intermediary's first stage maximization problem is

$$\max_b \pi\left(b\right) = \left[a\left(q\left(b\right)\right) - b\right] q\left(b\right),$$

where $q\left(b\right)$ is given by (20) and $a\left(q\left(b\right)\right)$ is given by (21). Thus,

$$\pi\left(b\right) = \left[1 - 2b\right] \frac{4b - \lambda}{2\left(2 - \lambda\right)}.$$

The first order condition is

$$0 \quad = \quad \frac{2 - 8b + \lambda}{2 - \lambda},$$

and the second order condition $\left(-\frac{8}{2-\lambda}<0\right)$ is satisfied as well, so that $b^*=\frac{1}{4}+\frac{\lambda}{8}$ and $q\left(b^*\right)=\frac{1}{4}$. ■

Note that Lemma 9 implies that equilibrium profits are $\pi^*\left(b^*\right)=\frac{2-\lambda}{16}$ and equilibrium quantity traded is $\frac{1}{4}$, provided the capacity constraint is greater than or equal to $\frac{1}{4}$.

What therefore remains to be determined is the optimal capacity constraint $\overline{q}^*$. Since we have assumed no costs of building capacity, any capacity constraint greater than or equal to $\frac{1}{4}$ will allow the intermediary to earn his the equilibrium profits π^*. Therefore, in any equilibrium, $\overline{q}^*\geq\frac{1}{4}$.

Together with the modified Proposition 1 of Gehrig, Lemmas 4 through 9 imply that the capacity constrained subgame of the dynamic intermediation game has a unique subgame perfect equilibrium with an active search market. This equilibrium is summarized in the following Proposition. Let $\varphi\mid\overline{q}$ be a strategy of the intermediary in the capacity constrained subgame and assume $\overline{q}\geq\frac{1}{4}$.

Proposition 2 *For $\overline{q}\geq\frac{1}{4}$, the capacity constrained subgame of the dynamic intermediation game has a unique subgame perfect equilibrium with an active search market, in which*

$$\begin{aligned}
\tau_s^* &= \left(I \quad \textit{iff} \quad b\geq\max\left[b\left(q\right)|_{\overline{q}\geq q>s},s\right],\ S \quad \textit{iff} \quad \overline{r}>s\right) \quad \forall s\in[0,1] \\
\rho_r^* &= \left(I \quad \textit{iff} \quad a\leq\min\left[a\left(q\right)|_{r\geq 1-q^b},1-r\right],\ S \quad \textit{iff} \quad r>\underline{s}\right) \quad \forall r\in[0,1] \textit{ and} \\
\varphi^*\mid\overline{q} &= \left(\ b=1/4+\lambda/8,\ a^*(q^b)=a(q)\right),
\end{aligned}$$

where the functions $a\left(q\right)$ and $b\left(q\right)$ are the inverse demand and inverse supply functions as defined in equation (18) and (19) above.

Because capacity building is costless, there is a continuum of otherwise identical subgame perfect equilibria in the full game. In all of these equilibria, the intermediary sets $\overline{q}^*\geq\frac{1}{4}$.

Proof: From Lemma 4 we know that if the $\underline{s}\geq q^b=q$ sellers with the lowest reservation prices have joined the intermediary and thereby cannot be on the search market, there is an ask price $a(q,\underline{s})$ such that buyer r will join the intermediary if and only if $r>1-q$. Lemmas 5 and 6 state that rationing on the output market (which would occur only if $a<a(q)$) and selling less than the quantity he has bought (which would occur only if $a>a(q)$) is not in the

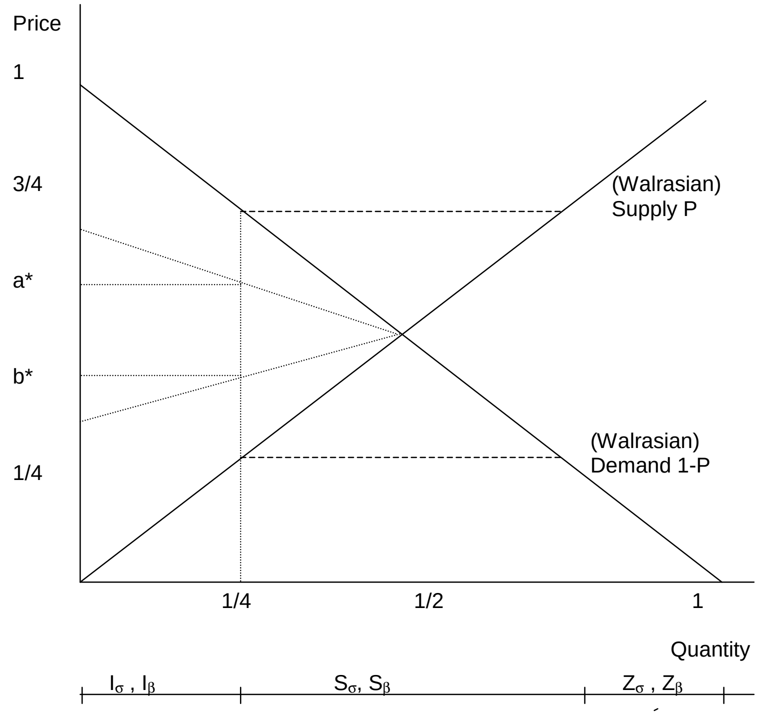

Figure 2: The equilibrium outcome.

intermediary's interest, provided $q \leq \frac{1}{2}$. This establishes that $a^*(q^b) = a(q)$ for $q \leq \overline{q} \leq \frac{1}{2}$. Lemma 8 says that rationing on the input market is not in the intermediary's interest, so that $b \leq b(\overline{q})$ in equilibrium. Lemma 9 determines the optimal bid price and the quantity traded by the intermediary, while Lemma 7 establishes uniqueness in the capacity constrained subgame. Finally, from the modified Proposition 1 of Gehrig we know that all low cost sellers and all buyers with high reservation prices deal with the intermediary and that those sellers and buyers with average reservation prices are active in the search market, while the remaining traders are not active. ■

In this equilibrium, quantity traded by the intermediary is $\frac{1}{4}$. All sellers with reservation prices $s < \frac{1}{4}$ and buyers with reservation prices $r > \frac{3}{4}$ trade with the intermediary. The ask price set by the intermediary is $\frac{3}{4} - \frac{\lambda}{8}$, and the bid price $\frac{1}{4} + \frac{\lambda}{8}$. Sellers and buyers with reservation prices $s, r \in \left[\frac{1}{4}, \frac{3}{4}\right]$ are active in the search market and sellers with valuations $s > \frac{3}{4}$ and buyers with reservation prices $r < \frac{1}{4}$ remain inactive. The intermediary's equilibrium profits are $\frac{2-\lambda}{16}$, as illustrated in Figure 2.

The intermediary trades with the subsets of buyers I_β and I_σ. The sets of buyers and sellers active in the search market are S_β and S_σ, while the sets of sellers and buyers Z_β and Z_σ remain inactive. Note that there are other payoff equivalent equilibria, which are, however, not subgame perfect. For example, setting $a(q^b) = \frac{3}{4} - \frac{\lambda}{8}$ would induce the same equilibrium payoffs. But if e.g. $q^b < \frac{1}{4} - z$, where z is a positive number smaller than $\frac{1}{4}$, $a(q^b) = \frac{3}{4} - \frac{\lambda}{8}$ is not optimal, so that $a(q^b) = \frac{3}{4} - \frac{\lambda}{8}$ is not a subgame perfect strategy.

4 Conclusions

In this paper, we have modified the Gehrig (1993) model by imposing a sequential structure. According to this modification, a monopolistic intermediary first sets a capacity constraint and a bid price. Up to the capacity constraint, the intermediary is committed to buy any quantity sellers are willing to sell to him at the bid price he sets. The intermediary starts selling only after having finished the acquisition of the inputs. As a consequence of this structure, the capacity constrained subgame has a unique subgame perfect equilibrium with an active search market. This is in contrast to the original model, which exhibits a multiplicity of equilibria (which are not payoff equivalent). In addition, we have addressed a problem concerning rationing and the structure of the model (see Appendix A.2). We consider the following extensions. In a companion paper, we introduce competition between capacity constrained intermediaries. As another extension, one could introduce a fix cost of search market participation. Further extensions could introduce asymmetric information or analyze the model for (more) general distributions of buyers' and sellers' reservation prices.

Appendix

A Intermediation and Rationing

A.1 Excursion on Rationing

Rationing occurs whenever the price is such that the market does not clear. Though either side of the market can get rationed, rationing is more often discussed as occurring on the demand side.

A.1.1 Rationing on the Output market

Typically, economists describe rationing by a rule which states what parts of demand get served at the (non-market clearing) price. Such a rule is called a rationing rule. In the literature, basically two rationing rules are used (see e.g. Tirole, 1988; Vives, 1999). One rule is often called *random* or *proportional rationing* rule (PR for short), and the other one *efficient rationing* rule (ER). The difference between the two rules is easiest to see by investigating the differences between the residual demand functions. These functions depict what part of the market demand remains unserved after rationing has occurred. Assume that there are two sellers selling given stocks q_1 and q_2 at prices p_1 and p_2. Let market demand be $D(p)$ and assume $p_1 < p_2$. If we are to have rationing at price p_1, it must be that $D(p_1) > q_1$. In this case and under PR, seller 2 faces the residual demand function

$$D^R_{PR}(p_1, p_2, q_1) = D(p_2) \frac{D(p_1) - q_1}{D(p_1)}, \tag{22}$$

while under ER, seller 2 faces the residual demand function

$$D^R_{ER}(p_2, q_1) = D(p_2) - q_1. \tag{23}$$

That is, under ER the buyers with the highest reservation prices get served at the non-market clearing price p_1, while under PR a proportional fraction of all buyers with willingness to pay greater than p_1 get served, where proportional means that $\frac{q_1}{D(p_1)}$ of all buyers willing to pay p_1 get served, and $1 - \frac{q_1}{D(p_1)}$ get rationed.

A.1.2 Rationing on the Input market

Rationing on the input market is completely analogous. Let $S(p)$ be the aggregate supply function for an input factor and let q_1 and q_2 be capacity constraints such that firm i cannot buy more than q_i. Then, assuming $p_1 > p_2$ and $S(p_1) > q_1$ (for otherwise there is no rationing) the residual supply function firm 2 faces is under PR and

$$S_{PR}^{R}\left(p_1, p_2, q_1\right) = S\left(p_2\right) \frac{S\left(p_1\right) - q_1}{S\left(p_1\right)}, \tag{24}$$

and under ER it is

$$S_{ER}^{R}\left(p_2, q_1\right) = S\left(p_2\right) - q_1. \tag{25}$$

A.2 Rationing and the structure of the basic model

As mentioned above (see footnote 4), the assumption that rationed traders cannot go back to the search market is crucial. This point deserves some emphasis because it may help prevent misunderstandings. Note that Gehrig (1993, p.106) writes:

> ... intermediaries may ration the long side and send unsuccessful traders back to the search market

and Spulber (1999, p.121) says:

> Those rationed by the intermediary can move on to the matching market.

We now show that (1) that the assumption that "rationed traders are sent back to the search market" would not fit with the remainder of Gehrig's analysis and (2) that the assumption that rationed traders cannot go back to the search market is crucial for the model. We begin with (1).

If some agents, e.g. sellers, are rationed at the intermediary's with probability $(1 - \alpha_\sigma)$, the expected utility gain for seller s from joining the intermediary is, according to Gehrig (1993, equations (3.1), (3.2) and (A.2))

$$W_\sigma(s) := \alpha_\sigma(b - s). \tag{26}$$

But if s can join the search market if he gets rationed, then his utility gain from joining the intermediary is rather

$$W_\sigma(s) = \alpha_\sigma(b - s) + (1 - \alpha_\sigma)\gamma_\sigma U_\sigma(s),$$

because with the probability that he gets rationed he joins the search market, where he grasps expected utility gain $\gamma_\sigma U_\sigma(s)$.[12]

But even more importantly, if agents can join the search market after being (proportionally) rationed by the intermediary, this affects the expected utility gains from search market participation. Assume again that sellers are rationed,[13] so that the fraction $(1-\alpha_\sigma)$ of sellers with reservation prices $s < \underline{s}$ will subsequently be active in the search market. The consequences of this will be first that all buyers with $r \in (0, \underline{s}]$ will join the search market, so that the set of inactive buyers is reduced to the buyer with $r = 0$. All other buyers can now expect positive utility gains from search market participation. Thus, Gehrig's Proposition 1 would not hold (or only hold for a case with measure zero), since $\underline{r}$ were almost 0. Second, there would be more buyers than sellers active in the search market, which by itself is not a problem. However, the reservation prices of sellers active in the search market were no longer uniformly distributed. Rather, this distribution would have a kink at $\underline{s}$. In Gehrig's setting with take-it-or-leave-it offers, this has a third consequence, namely that the optimal offers for buyers active in the search market are hard to compute if rationing occurs (and if a proportional rationing rule applies). Thus, it will not be trivial to compute the expected utility from search market participation for the critical buyer $\overline{r}$, which will certainly not be $U_\beta(\overline{r}) = \frac{\lambda}{4}(\overline{r} - \underline{s})$.

This gets us to the claim (2), namely that the assumption that rationed traders cannot go back to the search market is crucial for the model. Suppose they could. Then, rationing on the input side would have two effects for the expected utility gain from search for buyers. The positive effect for buyers is that the set of sellers active in the search market increases, so that all else equal, their utility increases. The negative effect is that the probability of being successfully matched, $\gamma_\beta \equiv \min[\frac{v(S_\sigma)}{v(S_\beta)}, 1]$, decreases, since now there are more buyers active in

[12]Interestingly, rationing would so far not affect the reservation price to deal with the intermediary, provided $\gamma_\sigma = 1$ with or without rationing. To see this, consider the critical buyer $\underline{s}$ for whom

$$\alpha_\sigma(b - \underline{s}) + (1 - \alpha_\sigma)U_\sigma(\underline{s}) = U_\sigma(\underline{s})$$

holds, so that as before $b - \underline{s} = U_\sigma(\underline{s})$.

[13]In Gehrig's and Spulber's setting, the game is played in simultaneous moves, so that it is immaterial on which side rationing occurs. But in the dynamic game of this paper, rationing of buyers is not a credible threat by the intermediary because it is not time consistent. This is why we concentrate here on rationing of sellers.

the search market than sellers, implying $\gamma_\beta < 1$.

For a seller s, the expected utility gain from search decreases because there are more buyers with $r \leq s$ active in the search market, so that the probability of being matched to a buyer with whom he cannot engage in mutually beneficial trade increases.

Since the reservation prices to trade with the intermediary increase as the utility gain from search market participation decreases, it is in the intermediary's interest to make this utility gain small. The cost of inducing rationing on the input side is, of course, that the intermediary has to pay a bid price above the one at which the market (or capacity) clears. However, with proportional rationing and without fix cost of search market participation, it suffices to set b only marginally above the market clearing price. Then, $\alpha_\sigma < 1$, which induces all buyers with $r \leq \underline{s}$ to join the search market. In the limit as $\lim \alpha_\sigma \to 1$, therefore, $v(S_\sigma) = \overline{r} - \underline{s}$ and $v(S_\beta) = \overline{r}$, so that $\gamma_\beta < 1$, which unambiguously decreases buyers' utility gain from search market participation and thus increases their reservation prices for buying from the intermediary. Therefore, if rationing is proportional and if rationed traders can join the search market, it would be in the interest of the intermediary to (marginally) ration the input side. Thus, the assumption that rationed traders cannot join the search market is crucial for the model.

B Rubinstein Alternating Offers Bargaining

Following Freixas and Rochet (1997), we have assumed that buyers and sellers who are matched share the gains from trade evenly, if there are any such gains (i.e. if $r - s > 0$). Alternatively, we could assume that when two traders are successfully matched, the reservation prices r and s are common knowledge to both parties. Provided $r > s$, the buyer and the seller engage in a Rubinstein alternating offers bargaining game, where they have the same discount factor δ. The player who can make the first move is determined by flipping a fair coin. The reason for assuming that the player to make the first offer is determined by chance is merely that it allows us to get rid of the discount factor δ outside the bargaining subgame.

When a seller s and a buyer r meet, they have a cake of size $(r - s)$ to share, if $r > s$. Otherwise, they cannot engage in mutually beneficial trade, and the search market ends without utility gain for both of them. Let the common discount rate be δ. Then, if the buyer is given the chance to make the first offer, he offers himself $\frac{1}{1+\delta}(r-s)$, leaving $\frac{\delta}{1+\delta}(r-s)$ to the seller. That is, the price p_r the buyer sets solves

$$r - p_r = \frac{1}{1+\delta}(r-s),$$

so that

$$p_r = \frac{\delta r + s}{1+\delta},$$

which the seller accepts. On the other hand, if the seller is given the chance to make the first offer, he gives himself the fraction $\frac{1}{1+\delta}$ of the cake and leaves $\frac{\delta}{1+\delta}$ to the buyer. That is, the seller sets a price p_s that solves

$$p_s - s = \frac{1}{1+\delta}(r-s),$$

so that

$$p_s = \frac{r + \delta s}{1+\delta}.$$

Let the chance that a buyer or a seller can make the first offer be equal. Then, the price a buyer or a seller can expect on average if a match is successful (i.e. if $r > s$), is

$$\begin{aligned} \frac{1}{2}(p_r + p_s) &= \frac{1}{2}\left(\frac{\delta r + s}{1+\delta} + \frac{r + \delta s}{1+\delta}\right) \\ &= \frac{r+s}{2}. \end{aligned}$$

See also Rubinstein (1982), Shaked and Sutton (1984), Mas-Collel et al. (1995, p.298) and Gibbons (1992); Spulber (1999) introduces Rubinstein bargaining into a model in this spirit.

B.1 Expected utility gain from Search

The expected utility gain for a seller s with $s \in [\underline{s}, \overline{r}]$ from search market participation is then

$$\begin{aligned}
U_\sigma(s) &= \frac{\lambda}{2}\int_s^{\overline{r}} (p_s - s)\frac{1}{\overline{r}-\underline{s}}dr + \frac{\lambda}{2}\int_s^{\overline{r}} (p_r - s)\frac{1}{\overline{r}-\underline{s}}dr \\
&= \frac{\lambda}{2}\int_s^{\overline{r}} (p_s + p_r - 2s)\frac{1}{\overline{r}-\underline{s}}dr = \frac{\lambda}{2}\int_s^{\overline{r}} (r - s)\frac{1}{\overline{r}-\underline{s}}dr \\
&= \frac{\lambda}{2}\frac{\left[\frac{r^2}{2} - rs\right]_s^{\overline{r}}}{\overline{r}-\underline{s}} = \frac{\lambda}{4}\frac{(\overline{r}-s)^2}{\overline{r}-\underline{s}},
\end{aligned}$$

which is the same as in the Gehrig model.

Thus for the critical seller $\underline{s}$

$$U_\sigma(\underline{s}) = \frac{\lambda}{4}(\overline{r}-\underline{s}).$$

Likewise, for a buyer with reservation price $r \in [\underline{s}, \overline{r}]$ the expected utility gain from being active in the search market is

$$\begin{aligned}
U_\beta(r) &= \frac{\lambda}{2}\int_{\underline{s}}^{r} (r - p_s)\frac{1}{\overline{r}-\underline{s}}ds + \frac{\lambda}{2}\int_{\underline{s}}^{r} (r - p_r)\frac{1}{\overline{r}-\underline{s}}ds \\
&= \frac{\lambda}{2}\int_{\underline{s}}^{r} (2r - p_r - p_s)\frac{1}{\overline{r}-\underline{s}}dr \\
&= \frac{\lambda}{2}\int_{\underline{s}}^{r} (r - s)\frac{1}{\overline{r}-\underline{s}}ds \\
&= \frac{\lambda}{4}\frac{(r-\underline{s})^2}{\overline{r}-\underline{s}},
\end{aligned}$$

so that for the critical buyer

$$U_\beta(\overline{r}) = \frac{\lambda}{4}(\overline{r}-\underline{s}) = U_\sigma(\underline{s}).$$

References

Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy 91*(3), 401–419.

Diamond, D. W. (1984). Financial intermediation and delegated monitoring. *Review of Economic Studies 51*(3), 393–414.

Dixit, A. (2001). On modes of economic governance. *Working Paper*.

Freixas, X. and J.-C. Rochet (1997). *Microeconomics of banking*. MIT Press, Cambridge, Massachusetts.

Friedman, M. (1980 [1962]). *Capitalism and Freedom*. University of Chicago Press.

Gehrig, T. (1993). Intermediation in search markets. *Journal of Economics & Management Strategy 2*, 97–120.

Gibbons, R. (1992). *A Primer in Game Theory*. Prentice Hall, New York.

Kreps, D. M. and J. A. Scheinkman (1983). Quantity precommitment and bertrand competition yield cournot outcomes. *Bell Journal of Economics 14*(2), 326–337.

Mas-Collel, A., M. D. Whinston, and J. R. Green (1995). *Microeconomic Theory*. Oxford University Press.

Neeman, Z. and N. Vulkan (2002). Markets versus negotitations: the predominance of centralized markets. *Working Paper*.

Rubinstein, A. and A. Wolinksy (1987). Middlemen. *Quarterly Journal of Economics 102*(3), 581–593.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica 50*(1), 97–109.

Rust, J. and G. Hall (2003). Middlemen versus market makers: A theory of competitive exchange. *Journal of Political Economoy 111*(2), 353–403.

Shaked, A. and J. Sutton (1984). Involuntary unemployment as a perfect equilibrium in a bargaining model. *Econometrica 52*(6), 1351–1364.

Spulber, D. F. (1996). Market making by price-setting firms. *Review of Economic Studies 63*, 559–580.

Spulber, D. F. (1999). *Market Microstructure: Intermediaries and the Theory of the Firm*. Cambridge University Press, Cambridge.

Tirole, J. (1988). *The theory of industrial organization*. MIT Press, Cambridge, Massachusetts.

Vives, X. (1999). *Oligopoly Pricing*. MIT Press, Cambridge, Massachusetts.