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# $|V_{cb}|$ and $\gamma$ from $B$ -mixing — Addendum to “ $B_s$ mixing observables and $|V_{td}/V_{ts}|$ from sum rules”

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ABSTRACT: In this addendum to “ $B_s$  mixing observables and  $|V_{td}/V_{ts}|$  from sum rules” [1] we study the impact of the recent improvements in the theoretical precision of  $B$  meson mixing onto CKM unitarity fits. Our key results are the most precise determination of the angle  $\gamma = (63.4 \pm 0.9)^\circ$  in the unitarity triangle and a new value for the CKM element  $|V_{cb}| = (41.6 \pm 0.7) \cdot 10^{-3}$ .

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In our recent works we have determined the hadronic matrix elements for  $B$ -mixing with HQET sum rules [1, 2] (cf. also [3]) and combined the results with lattice determinations [4–6] to obtain updated predictions [7] for the mass differences  $\Delta M_d$  and  $\Delta M_s$ . Here we use the weighted averages for the matrix elements presented in [7] to determine the following combinations of CKM elements

$$|V_{ts}V_{tb}| = (40.91_{-0.64}^{+0.67}) \cdot 10^{-3} \tag{1}$$

$$= \left( 40.91_{-0.62}^{+0.65} \Big|_{f_B^2} \pm 0.17 \Big|_{m_t} \pm 0.05 \Big|_{\alpha_s(M_Z)} \pm 0.02 \Big|_{\Delta M_s} \right) \cdot 10^{-3},$$

$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.2043_{-0.0011}^{+0.0010} \tag{2}$$

$$= 0.2043_{-0.0010}^{+0.0009} \Big|_{\xi} \pm 0.0003 \Big|_{\Delta M_d} \pm 0.0001 \Big|_{\Delta M_s},$$

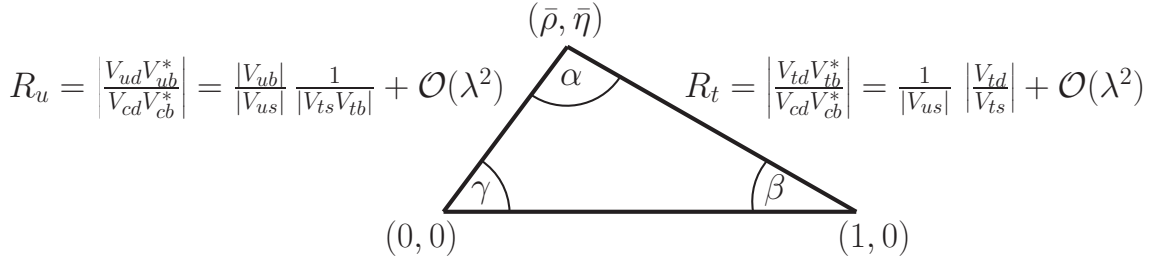
from the experimental measurements of the mass differences, updating the results in [1]. As discussed in [1, 7], the small theory uncertainty on  $|V_{td}/V_{ts}|$  is due to the combination of recent lattice results [8–10] for the ratio  $f_{B_s}/f_{B_d}$  and the precise sum rule results [1] for the ratio of the Bag parameters which yields the most precise result for the ratio  $\xi$  [7]. Motivated by the well-known discrepancy between the direct determination of the CKM elements  $V_{cb}$  and  $V_{ub}$  from semi-leptonic  $b$ -hadron decays (see [11] for some recent discussion) and the prospect of a measurement of the CKM angle  $\gamma$  with an uncertainty of  $1.5^\circ$  by 2023 from the LHCb collaboration [12] we study the impact of these values on CKM unitarity fits.

The effects of  $B$ -mixing on CKM unitarity fits can be illustrated with the unitarity triangle shown in figure 1. The combinations of CKM elements (1) and (2) we determined from  $\Delta M_s$  and  $\Delta M_d$  appear in the lengths of the two non-trivial sides of the triangle if we expand to leading order in the Wolfenstein parameter  $\lambda = |V_{us}|$ . Up to reflection with respect to the  $\bar{\rho}$  axis the apex of the triangle is exactly fixed with the addition of  $|V_{ub}|$  and the precisely measured  $|V_{us}|$ . Here, we use this information to determine the angle  $\gamma$ . Furthermore, we can extract  $|V_{cb}| = |V_{ts}V_{tb}| \times [1 + \mathcal{O}(\lambda^2)]$  with a precision that is competitive with direct measurements.

We perform a minimalistic CKM unitarity fit (cf. the appendix for a description of the fit procedure), first taking only the direct measurements of the CKM element  $|V_{us}| = 0.2243 \pm 0.0005$  [16] and the mass differences  $\Delta M_d$  and  $\Delta M_s$  into account. This strongly constrains the length of the side  $R_t$ . Figure 2 shows our results in the  $|V_{ub}| - \gamma$  and  $|V_{cb}| - \gamma$  planes where the shaded blue regions indicate the parameter space satisfying the inputs within one and two standard deviations. Obviously, the CKM fit with only three inputs is underconstrained which is reflected by the fact that the blue region traces a one-dimensional best-fit path in the 4-dimensional parameter space of the CKM matrix. Nevertheless, the underconstrained fit is sufficient to obtain an important constraint. Namely, for values of  $\gamma$  larger than about  $65^\circ$  the unitarity triangle does not close within the two-sigma region — independently of the value of the unconstrained degree of freedom which corresponds to the length of the side  $R_u$  in the unitarity triangle.<sup>1</sup> This behaviour

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<sup>1</sup>Similar observations were made in e.g. [15, 17].



**Figure 1.** Our conventions for the unitarity triangle in the  $\bar{\rho} - \bar{\eta}$  plane.

is illustrated in figure 3 and allows us to derive a stringent upper limit on  $\gamma$ . At the level of five standard deviations we obtain

$$\gamma \leq 66.9^\circ \quad [5\sigma], \quad (3)$$

which is indicated by the horizontal dashed line in figure 2 and quite a bit smaller than the direct measurements of  $\gamma$  [13, 14, 16, 18, 19] summarised there. We note that the indirect determinations of  $\gamma$  from the CKMfitter [18] and UTfit [19] collaborations also yield smaller values than direct measurements, albeit larger ones than our analysis. We used the CKMlive [20] tool to perform the standard CKMfitter analysis without direct measurements of  $\gamma$  or the mass differences and obtained the result

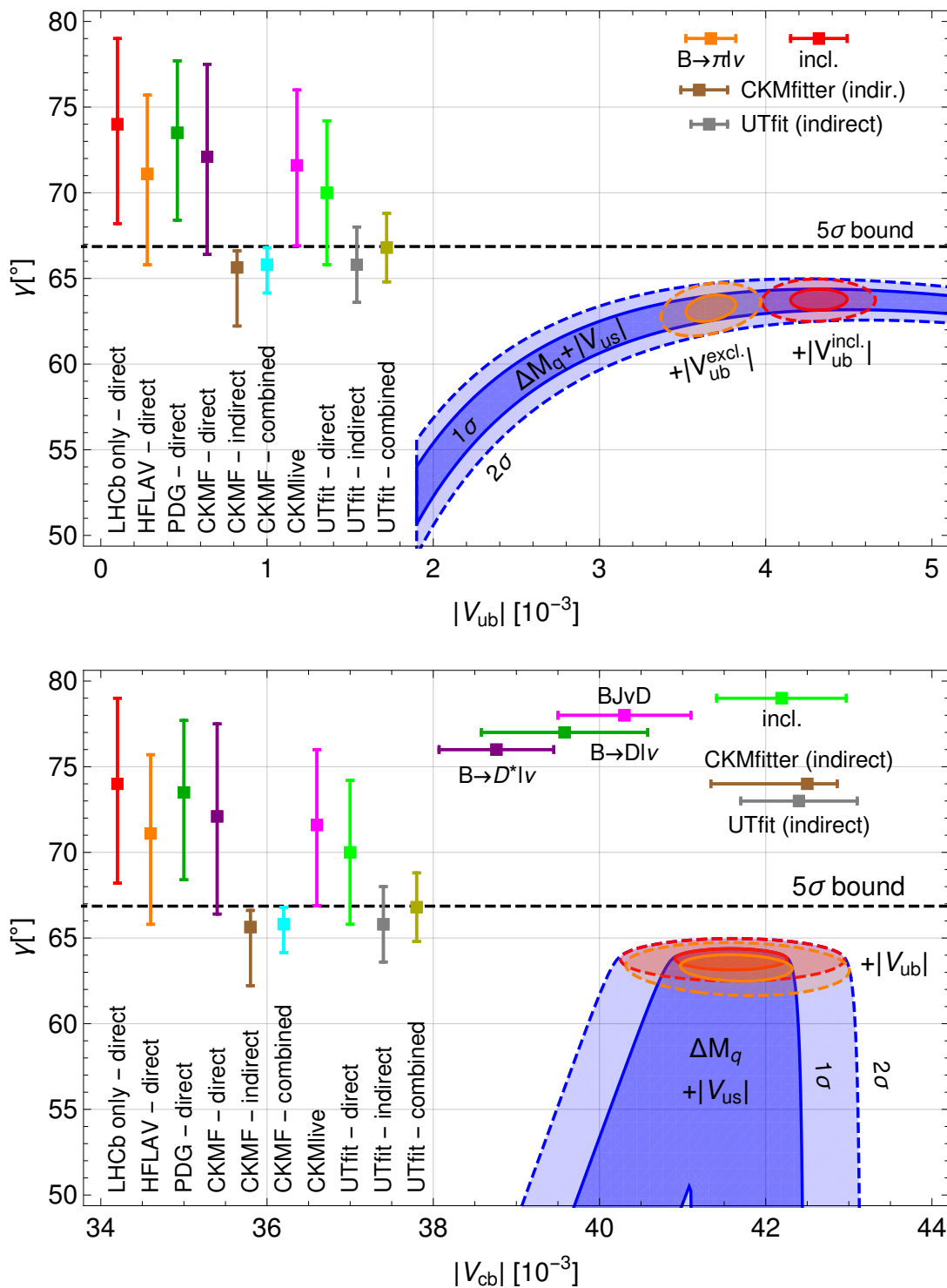
$$\gamma = (71.6_{-4.7}^{+4.4})^\circ, \quad \text{CKMlive — fit without } \gamma, \Delta M_s, \Delta M_d, \quad (4)$$

which is in good agreement with the direct measurements of  $\gamma$  and has a significantly larger uncertainty than the indirect fit results. This demonstrates that the smaller indirect values in the CKMfitter and UTfit studies are solely driven by  $\Delta M_s$  and  $\Delta M_d$  and implies that the confrontation of the planned improvements by LHCb and Belle II for the experimental determination of  $\gamma$  with constraints from the mass differences is a very promising indicator for BSM physics. Assuming the central value of the direct measurement remains the expected precision of  $\pm 1.5^\circ$  by 2023 will lead to a significant tension as indicated in figure 3.

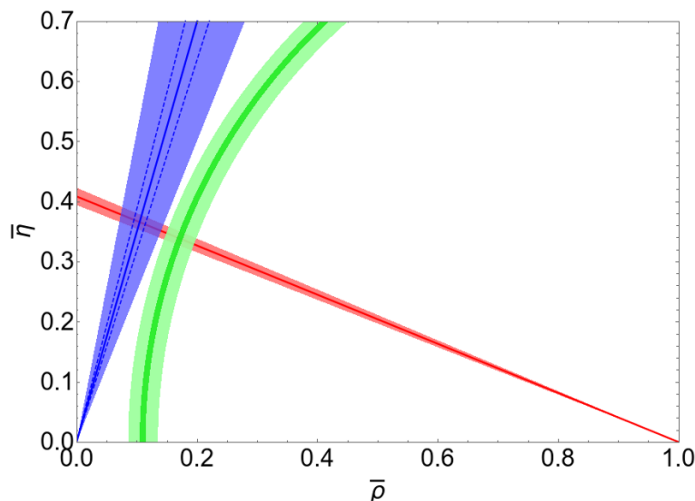
For smaller values of  $\gamma$  there are two intersections between the circle of length  $R_t$  around the point (1,0) and the line crossing the origin at angle  $\gamma$ , leading to two degenerate perfect-fit results for  $|V_{ub}|$  and  $|V_{cb}|$  at a fixed value of  $\gamma$ . This degeneracy can be broken by constraining the length of the side  $R_u$  by including the measurements of  $|V_{ub}|$  in the fit. Due to the well-known puzzle about different results in exclusive and inclusive measurements (shown by the orange and red horizontal error bars in figure 2, values from HFLAV [14]) this step would normally have to be taken with a grain of salt. However, due to a lucky numerical coincidence the values of  $|V_{ub}|$  are very close to the region where the intersection point of the circles of length  $R_t$  and  $R_u$  lies at the maximal value of  $\gamma$  allowed by  $R_t$  as shown by the orange and red ellipses in figure 2 which are the results of the fit when the exclusive or inclusive measurements of  $|V_{ub}|$  are included. Thus, the dependence of  $\gamma$  on the exact value of  $|V_{ub}|$  is rather small. Indeed we find

$$\gamma = (63.3_{-0.8}^{+0.7})^\circ, \quad \text{from } |V_{ub}^{\text{excl.}}|, \quad (5)$$

$$\gamma = (63.8_{-0.6}^{+0.6})^\circ, \quad \text{from } |V_{ub}^{\text{incl.}}|. \quad (6)$$



**Figure 2.** Our results for a minimalistic CKM unitarity fit based on direct measurements of  $|V_{us}|$  and the mass differences  $\Delta M_d$  and  $\Delta M_s$  are given as shaded blue regions. Including the exclusive or inclusive measurements of  $|V_{ub}|$  yields the orange and red regions, respectively. See text for details.



**Figure 3.** We show the constraints on the apex of the unitarity triangle from the direct measurement of  $\gamma$  from LHCb [13] (blue),  $B$ -mixing (green) and the value of  $\beta$ , taken from HFLAV [14] (red). The dark and light green regions indicate the  $1\sigma$  and  $5\sigma$  bounds, while the blue and red regions refer to the  $1\sigma$  constraints. The dashed blue lines illustrate the future precision of  $\pm 1.5^\circ$  on the measurement of  $\gamma$ . See [15] for a version of this plot prior to the recent improvements in the theory predictions for the mass differences.

We take the envelope of both values

$$\gamma = (63.4 \pm 0.9)^\circ, \tag{7}$$

as our final result to be sufficiently conservative about the uncertainty associated with the direct measurements of  $|V_{ub}|$ . Eq. (7) represents the most precise determination of  $\gamma$  to date. The result is fairly insensitive to the input value for  $|V_{us}|$ . If we inflate the error in  $|V_{us}|$  by a factor of three we obtain  $\gamma = (63.4 \pm 1.3)^\circ$  and the upper five-sigma bound (3) becomes  $68.9^\circ$ . A similar value of  $\gamma = (63.4 \pm 1.5)^\circ$  with an upper five-sigma bound of  $69.4^\circ$  results from doubling the theory uncertainty on the ratio  $\xi$ . Even in both of these more conservative scenarios, a future LHCb measurement of  $\gamma$  with an accuracy of  $1.5^\circ$  and an unchanged central value would still correspond to a tension at the level of five standard deviations. This demonstrates the robustness of refined direct measurements of  $\gamma$  as a probe of new physics effects.

The effect of the exclusive or inclusive  $|V_{ub}|$  measurements on the fit is also indicated in the  $|V_{cb}| - \gamma$  plane by the orange and red ellipses, respectively. The difference in the extracted values of  $|V_{cb}|$  is negligible and we again adopt the envelope as our final result

$$|V_{cb}| = (41.6 \pm 0.7) \cdot 10^{-3}. \tag{8}$$

We also show the exclusive and inclusive HFLAV averages [14] and the result of a recent reanalysis BJvD [21] of exclusive determinations in figure 2. Our result yields a competitive uncertainty and the one-sigma region overlaps with the inclusive and the BJvD results, while there is a 1.7 and 2.9  $\sigma$  tension with respect to the  $B \rightarrow D\ell\nu$  and  $B \rightarrow D^*\ell\nu$  values

quoted by HFLAV. The result (8) remains unaffected when we inflate the  $|V_{us}|$ -uncertainty by a factor of three or the theory uncertainty for  $\xi$  by a factor of two.

In summary, we have performed a minimal  $\chi^2$  fit of the CKM parameters based on the mass differences in the  $B$  system and direct measurements of  $|V_{us}|$  and  $|V_{ub}|$ . We found competitive results for  $|V_{cb}|$  which are in good agreement with the inclusive determinations and obtained the currently most precise value for the angle  $\gamma$  in the unitarity triangle. Our analysis clearly shows that more precise measurements of  $\gamma$  are a sensitive probe of new physics effects in the flavour sector. We are looking forward to updates of the complete CKM unitarity fits by the CKMfitter and UTfit collaborations where the latest theoretical developments [1–7] in  $B$ -mixing are taken into account.

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**Note added.** The latest LHCb measurement of  $|V_{cb}|$  from exclusive semileptonic  $B_s$  decays [22] is in excellent agreement with our indirect result (8).

## A Description of the fit procedure

We perform the fit in the standard parametrization of the CKM triangle with the three angles  $\theta_{12}$ ,  $\theta_{13}$  and  $\theta_{23}$  as well as the phase  $\delta$  which will be denoted by the four-dimensional vector  $\boldsymbol{\theta}$  below. No approximations related to the Wolfenstein parametrization are made in the numerical analysis. We define a  $\chi^2$  function as

$$\chi^2(\boldsymbol{\theta}) = \sum_{x \in X} \left( \frac{x(\boldsymbol{\theta}) - x_{\text{in}}}{\Delta x} \right)^2 \tag{A.1}$$

where the underconstrained fit, corresponding to the blue regions in figure 2, utilizes  $X = \{|V_{ts}V_{tb}|, |V_{td}/V_{ts}|, |V_{us}|\}$  and the orange and red regions are obtained by including  $|V_{ub}|$  in  $X$ . The  $x_{\text{in}}$  and  $\Delta x$  correspond to the central values and total uncertainties of these quantities which are given in eq. (1), eq. (2) and the text. The contours in figure 2 are then obtained by scanning over the parameter space, e.g. the one-sigma regions in the  $|V_{cb}| - \gamma$  plane corresponds to all points  $\vec{x}$  which satisfy

$$\text{Min}_{\boldsymbol{\theta} \text{ s.t. } (|V_{cb}|(\boldsymbol{\theta}), \gamma(\boldsymbol{\theta})) = \vec{x}} (\chi^2(\boldsymbol{\theta})) \leq 1. \tag{A.2}$$

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## References

- [1] D. King, A. Lenz and T. Rauh,  $B_s$  mixing observables and  $|V_{td}/V_{ts}|$  from sum rules, *JHEP* **05** (2019) 034 [[arXiv:1904.00940](#)] [[INSPIRE](#)].
- [2] M. Kirk, A. Lenz and T. Rauh, Dimension-six matrix elements for meson mixing and lifetimes from sum rules, *JHEP* **12** (2017) 068 [[arXiv:1711.02100](#)] [[INSPIRE](#)].
- [3] A.G. Grozin, R. Klein, T. Mannel and A.A. Pivovarov,  $B^0 - \bar{B}^0$  mixing at next-to-leading order, *Phys. Rev. D* **94** (2016) 034024 [[arXiv:1606.06054](#)] [[INSPIRE](#)].
- [4] FERMILAB LATTICE and MILC collaborations,  $B_{(s)}^0$ -mixing matrix elements from lattice QCD for the Standard Model and beyond, *Phys. Rev. D* **93** (2016) 113016 [[arXiv:1602.03560](#)] [[INSPIRE](#)].
- [5] RBC/UKQCD collaboration, SU(3)-breaking ratios for  $D_{(s)}$  and  $B_{(s)}$  mesons, [arXiv:1812.08791](#) [[INSPIRE](#)].
- [6] R.J. Dowdall et al., Neutral B-meson mixing from full lattice QCD at the physical point, *Phys. Rev. D* **100** (2019) 094508 [[arXiv:1907.01025](#)] [[INSPIRE](#)].
- [7] L. Di Luzio, M. Kirk, A. Lenz and T. Rauh,  $\Delta M_s$  theory precision confronts flavour anomalies, *JHEP* **12** (2019) 009 [[arXiv:1909.11087](#)] [[INSPIRE](#)].
- [8] FLAVOUR LATTICE AVERAGING GROUP collaboration, FLAG Review 2019, *Eur. Phys. J. C* **80** (2020) 113 [[arXiv:1902.08191](#)] [[INSPIRE](#)].
- [9] C. Hughes, C.T.H. Davies and C.J. Monahan, New methods for B meson decay constants and form factors from lattice NRQCD, *Phys. Rev. D* **97** (2018) 054509 [[arXiv:1711.09981](#)] [[INSPIRE](#)].
- [10] A. Bazavov et al., B- and D-meson leptonic decay constants from four-flavor lattice QCD, *Phys. Rev. D* **98** (2018) 074512 [[arXiv:1712.09262](#)] [[INSPIRE](#)].
- [11] P. Gambino, M. Jung and S. Schacht, The  $V_{cb}$  puzzle: An update, *Phys. Lett. B* **795** (2019) 386 [[arXiv:1905.08209](#)] [[INSPIRE](#)].
- [12] LHCb collaboration, Physics case for an LHCb Upgrade II — Opportunities in flavour physics and beyond, in the HL-LHC era, [arXiv:1808.08865](#) [[INSPIRE](#)].
- [13] LHCb collaboration, Update of the LHCb combination of the CKM angle  $\gamma$ , [LHCb-CONF-2018-002](#).
- [14] HFLAV collaboration, Averages of b-hadron, c-hadron and  $\tau$ -lepton properties as of 2018, [arXiv:1909.12524](#) [[INSPIRE](#)].
- [15] M. Blanke and A.J. Buras, Emerging  $\Delta M_d$ -anomaly from tree-level determinations of  $|V_{cb}|$  and the angle  $\gamma$ , *Eur. Phys. J. C* **79** (2019) 159 [[arXiv:1812.06963](#)] [[INSPIRE](#)].
- [16] PARTICLE DATA GROUP collaboration, Review of Particle Physics, *Phys. Rev. D* **98** (2018) 030001 [[INSPIRE](#)].
- [17] M. Blanke and A.J. Buras, Universal Unitarity Triangle 2016 and the tension between  $\Delta M_{s,d}$  and  $\varepsilon_K$  in CMFV models, *Eur. Phys. J. C* **76** (2016) 197 [[arXiv:1602.04020](#)] [[INSPIRE](#)].
- [18] CKMFITTER GROUP collaboration, CP violation and the CKM matrix: Assessing the impact of the asymmetric B factories, *Eur. Phys. J. C* **41** (2005) 1 [[hep-ph/0406184](#)] [[INSPIRE](#)].

- [19] UTFIT collaboration, *The Unitarity Triangle Fit in the Standard Model and Hadronic Parameters from Lattice QCD: A Reappraisal after the Measurements of  $\Delta M_s$  and  $BR(B \rightarrow \tau\nu_\tau)$* , *JHEP* **10** (2006) 081 [[hep-ph/0606167](#)] [[INSPIRE](#)].
- [20] CKMFITTER collaboration, *CKMlive*, <http://ckmlive.in2p3.fr>.
- [21] M. Bordone, M. Jung and D. van Dyk, *Theory determination of  $\bar{B} \rightarrow D^{(*)}\ell^-\bar{\nu}$  form factors at  $\mathcal{O}(1/m_c^2)$* , *Eur. Phys. J. C* **80** (2020) 74 [[arXiv:1908.09398](#)] [[INSPIRE](#)].
- [22] LHCb collaboration, *Measurement of  $|V_{cb}|$  with  $B_s^0 \rightarrow D_s^{(*)-}\mu^+\nu_\mu$  decays*, [arXiv:2001.03225](#) [[INSPIRE](#)].