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On Holders, Blades and Other Tie-In Sales

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Abstract

Tie-in sales have a bad image because of anti-competitive effects. Notably, tying contracts allow monopolists to carry over monopoly power into markets where they meet competition. Most of the literature assumes a firm being monopolist in one market and facing competition in another. In contrast, we analyze two firms which both are monopolists in one market and competitors in the other. Under such a symmetric structure tying has competitive effects. Tie-in sales may increase the consumers' expected utility. By tying their products, the firms insure consumers against uncertain future demand.

Keywords: Tie-in sales, leverage theory of tying, competition, expected utility.
JEL-Classification Numbers: $D21, D31, D43, L11$

1 Introduction

Tie-in sales force the buyer of a product (tying good) to buy one or more other goods (tied good) exclusively from the same supplier. A classic example of tie-in sales is the former IBM selling rule. IBM required its lessees of computing machines to purchase the associated punching cards from IBM too. Another example are blade holders and blades. By technical means blade holder manufacturers force their customers to also purchase their blades. Printers and ink cartridges, electric toothbrush and brushes, or hot glue gun and glue sticks are other examples of tie-in sales.

One of the most cited and contended rationale for tie-in sales is the leverage theory of tying. This theory holds that a firm may try to leverage its monopoly power from one market to another where it faces competition. Consider a multiproduct firm which is monopolist in a specific market and one of many competitors in the other markets. By tying, the monopolist links the purchase of the monopolized good to the purchase of the competitively sold good. The tying firm extends its monopoly power to competitive markets to strengthen its position there. Exponents of the leverage theory attribute anticompetitive effects to such a behavior. Whinston (1990) describes leverage as a mechanism to foreclose sales in the competitive market, thereby monopolizing it. Carbajo, De Meza and Seidman (1990) find that bundling, a special form of tying, causes rivals in the competitive market to act less aggressively.

In most leverage models firm j has a monopoly in one market and faces (imperfect) competition in another. The classic tie-in example fits this description very well: IBM has a monopoly position in the market for computing machines and faces competition in the market for punching cards. But is the assumption that only firm j has a monopoly in one market appropri-

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ate for all cases? Take for instance the shaving systems industry. This case describes a symmetric market structure. We understand symmetric as two firms j and i competing in one market and having some market power in the other market. Competition prevails in the market for blade holders. But as soon as customers buy a holder from firm j, they are forced to buy the blades from the same firm j. Customers do so unless it pays to buy blade holder and blades from firm i. Hence, firm j has a monopoly in the market for blades with respect to all customers who buy holders from firm j. The same is true for firm i. This symmetric structure holds for many real world situations.

In our model consumers do not know if they need to shave little or a lot. Therefore, expenditures for blades are uncertain. *Ex ante*, the consumers have an expected utility from shaving. We are interested in the effects of tying on buyers' expected utility. Therefore, we compare the situation under tie-in sales with independent pricing. Under independent pricing the firms do not tie their products. We show that consumers may enjoy a higher expected utility under tie-in sales than under independent pricing. The leverage of tying does not necessarily have anti-competitive effects. Especially, tie-in sales may insure buyers against uncertain future demand just because of leveraging. If the firms use tie-in sales, the consumers must buy the holder and blades from the same firm. Due to competition in the handle market, shaving system suppliers price the handle below marginal costs to attract consumers. Thereby, the firms set blade prices above marginal costs and earn positive profits in the blade market. The firms do so because they can compensate losses on the handles with profits on the blades. In our model such tie-in sales reduce the variance of consumers' expenditures compared to selling the goods independently. If consumers are risk averse they can be

better off under tie-ins than under independent pricing.

Like the leverage theory of tying, the literature about tie-in sales and risk reduction uses the approach with a monopoly in the tying good market. Burstein (1988) states that tying contracts combined with leases insure the risk averse lessee against uncertain future demand. Such a tying arrangement can achieve the effects of variable rental arrangements. Suppose a seller needs a machine and material to produce a good. How much material the seller processes depends on uncertain demand for her good. Leasing the machine and buying the material also from the lessor allows the lessee to use the machine at a variable renting price.

In line with the preceding rationale for tie-ins, Liebowitz (1983) describes various agreements between a provider and a purchaser of a machine with a belonging commodity. He finds that tie-in sales reduce a firm's performance risk. Especially, the risk reduction hypothesis can also explain the lowered price of the tying good and the raised price for the tied good.

Firms may use tie-in sales to speed early adoption of a new capital good. Lunn (1990) illustrates this variant of the risk reduction rationale for tying contracts. A firm markets a new capital-embodied innovation. Potential users are uncertain about the innovation's usefulness and profitability. To encourage the new product's early adoption the innovator offers a tying contract. Purchasers buy (lease) the innovation tied to the purchase of some other innovator's product. By lowering the (rental) price on the tying product and raising the tied good's price above the competitive level, the seller reduces the risk to the buyer (lessee).

A more formal approach by Kamecke (1998) develops the risk-reduction hypothesis further. A monopolist with private information about the quality of a basic product overcomes informational inefficiency by tying in a comple-

mentary commodity. Selling the basic product at a low price and the tied good at a supracompetitive level signals the basic product's high quality. Tie-in sales serve as an instrument to send price signals which overcome problems of asymmetric information in the introductory phase of a new durable product.

In section 2 we describe a model with consumers which have perfectly correlated demand. All consumers have either the same high or low demand for blades. To obtain a benchmark, section 3.1 solves the model under independent pricing. The following section determines the equilibrium if the firms use tie-in sales. Section 3.3 compares the outcomes from sections 3.1 and 3.2. Section 4 introduces a more fundamental utility function and consumers who are different *ex post*. In sections 4.1 and 4.2 we derive the firms' pricing behavior when consumers are heterogeneous and demand for blades is not completely inelastic. Next, we analyze the effects from tying on consumers' utility in the extended framework. Section 5 concludes.

2   The Model

Consider a set of identical consumers of measure 1. Consumers want to shave, say. To do so, they need a blade holder and one set of blades. Products are bundled as follows: A is a blade holder plus one set of blades. B is just a set of blades. Consumers' willingness to pay for A is r. If the consumer has a blade holder, her willingness to pay for B is also r. Because the consumer can shave when buying good A or buying good B while having a holder, the reservation price is the same for good A and B. In the following we stick to the willingness to pay as maximum amount of money buyers are willing to give up for the benefit from shaving. Hence, the reservation price represents

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the monetary benefit of being well shaved. With probability π consumers' demand for good B is low, q_l. Demand is high, q_h, with probability $1-\pi$. We assume that customers' demands are perfectly correlated. The realization of high or low demand is the same for all consumers. Demand is thus completely inelastic up to the reservation price. With probability π consumers demand the amount q_l of good B if the price is less or equal to r. Otherwise, demand for good B is zero. Analogously, demand is q_h with probability $1-\pi$ if good B's price does not exceed consumers' willingness to pay. The reservation price is common knowledge. Each consumer has initial income I. Income is such that spending for shaving is small relative to income. In sections 3.2 and 4.2 we show that the price for a set of blades does not exceed the price for package A, $p_A \geq p_B$. Thus, the consumers buy package B if they already have a handle. The consumers' utilities are

$$
U(r, p_A, p_B) = \begin{cases} U(I + r - p_A + q_l(r - p_B)) & \text{if demand is low,} \\ U(I + r - p_A + q_h(r - p_B)) & \text{if demand is high.} \end{cases}
$$

The utility function $U(\cdot)$ is strictly increasing and strictly concave, that is $U'(\cdot) > 0$ and $U''(\cdot) < 0$. Concavity of the expected utility function implies risk aversion. Buying good A and good B at prices p_A and p_B generates expected utility:

$$
E[U] = \pi U(I + r - p_A + q_l(r - p_B)) + (1 - \pi)U(I + r - p_A + q_h(r - p_B)).
$$

Two firms serve markets A and B. The costs are the same for both firms. Fixed costs are zero. Marginal costs are c_A for good A and c_B for good B. Unit costs for good A are higher than for good B. For simplicity marginal costs for blades are equal to or greater than one. Furthermore, marginal costs are smaller than the consumers' willingness to pay, $r > c_A > c_B \geq 1$. This assumption ensures that the consumer buys the goods if priced at marginal

costs. Firm j, $j = 1, 2$, offers the goods at prices p_{jA} and p_{jB}. The firms compete in standard Bertrand manner in both markets. Potential demand is one in market A and $q \in \{q_l, q_h\}$ with $q_l > 1$ in market B.

If the firms do not tie, good A is compatible with the other firm's good B. The consumers can use blades from firm 1 in conjunction with blade holders from firm 2 and *vice versa*. In contrast to independent pricing, tie-in sales restrict the consumers in using good A with the competitor's good B. If firms price independently, firm j faces demand $D_{jA}(p_{1A}, p_{2A})$ for good A, and $D_{jB}(p_{1B}, p_{2B})$ for B,

$$
D_{jA} = \begin{cases} 1 & \text{if } p_{jA} < p_{iA}, \\ 0.5 & \text{if } p_{jA} = p_{iA}, \quad \text{and} \quad D_{jB} = \begin{cases} q & \text{if } p_{jB} < p_{iB}, \\ 0.5q & \text{if } p_{jB} = p_{iB}, \\ 0 & \text{if } p_{jA} > p_{iA} \end{cases} \qquad 0 & \text{if } p_{jB} > p_{iB}. \end{cases}
$$

We analyze a two stage game for independent pricing and tying. Independent pricing allows buyers to combine the firms' products. Tying coerces consumers to buy good B associated with good A, i.e. both goods from the same firm.

At the first stage firms simultaneously choose prices p_{jA} for good A. By assumption this price holds for the first and second period[1]. Then, consumers buy good A and payoffs in market A are realized. Given first stage prices and demand, firms simultaneously choose prices p_{jB} in market B at the second stage. Similar to the first stage, consumers purchase good B in the second stage and firms earn profits. We assume that, at the beginning of stage 2, all players know if total demand q for good B is low or high. We solve the game by backwards induction. In short, the events take place as follows:

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[1]The assumption that package A's prices remain fixed prevents consumers from waiting until stage 2 to buy the holder. We exclude the sequential pricing problem arising from durable goods to focus on the tie-ins' effect on competition.

Stage 1: The firms simultaneously choose prices for package A. Consumers buy good A and payoffs are realized.

Stage 2: At first, all players learn if demand q is high or low. The firms simultaneously set the price for good B. Consumers purchase the blades from the original firm or switch seller. Firms and consumers realize their payoffs.

3 Homogeneous Consumers With Perfectly Correlated Demand

3.1 Independent Pricing

In this section we derive the firms' behavior without tying. We refer to these results as the independent pricing equilibrium, (IP).

Suppose we are at stage 2. Then, whatever happened in stage 1, both firms set prices equal marginal costs, $p_{jB} = c_B$ for $j = 1, 2$. Given that the firms price at marginal costs in stage 2, they will also price at marginal costs in stage 1. The standard Bertrand result that prices equal marginal costs is the subgame-perfect Nash equilibrium.

Proposition 1 *In the independent pricing equilibrium firms set prices equal to marginal costs in all markets.*

3.2 Tie-In Sales

At the second stage, firm j is a monopolist for customers who bought good A from j. But firm j can not extract consumers' surplus without restrictions. First, the price for B does not exceed the firm's own price for good A,

$p_B \leq p_A$. If this were the case, consumers would substitute B by A. Firm j's profits would drop because marginal costs for B are lower than for A. Secondly, consumers can change their supplier. Consumers switch as soon as their utility is higher by buying an access package A and $q - 1$ units of B from the initial seller's rival than by buying q units of B from the initial firm. In the second stage, demand corresponds to the demand for B. Package A renders the same utility as already having a holder and buying B. Therefore, consumers buy $q - 1$ packages B if they switch supplier. Formally, the no-switching condition is as follows:

$$U\left(I - p_{jA}^T + q(r - p_{jB})\right) \geq U\left(I - p_{jA}^T + r - p_{iA}^T + (q-1)(r - p_{iB})\right)$$

Note that the benefit from shaving in the first stage does not appear in the no-switching condition. The consumers used up the blades contained in A.

Firm j ensures that the consumers who bought its package A do not change firm. Then, firm j serves the fraction D_{jA}^* corresponding to the amount of sold packages A in stage 1. Total demand is qD_{jA}^*. In the second stage, firm j maximizes its profit subject to $p_{jB} \leq p_{jA}^T$ and the no-switching condition:

$$\max_{p_{jB}} \quad qD_{jA}^*(p_{jB} - c_B)$$

s.t.

$$p_{jB} \leq p_{jA}^T,$$
$$p_{jB} \leq \frac{p_{iA}^T + (q-1)p_{iB}}{q},$$

where p_{jA}^T denotes firm j's price for the first good set at stage 1. The no-switching condition also implies that firm j does not set a price for B above firm i's price for A:

$$p_{jB} \leq \frac{p_{iA}^T + (q-1)p_{iB}}{q} \leq p_{iA}^T.$$

Because profits are increasing in the price for B, firm j sets its price as high as possible. We now show that the firms set prices for B equal to their prices for A. Assume firm j charges a price $p_{jB} \leq (p_{iA}^T + (q-1)p_{iB})/q$ and firm i sets $p_{iB} = p_{iA}^T$. Then, firm j's price for B is $p_{jB} \leq p_{iA}^T$. This is equivalent to the first constraint, because firm j's price for A is smaller or equal to firm i's price. Otherwise, firm j did not sell any holders in stage 1. Now, assume the firms set prices

$$p_{1B} = \frac{(p_{2A}^T + (q-1)p_{2B})}{q},$$
$$p_{2B} = \frac{(p_{1A}^T + (q-1)p_{1B})}{q}.$$

The solution to this system of equations is $p_{jB}^T = p_{jA}^T$. Consequently, firm j sets the same price for B as for A.

With the pricing behavior at the second stage we now turn to the first stage. The firms are no longer monopolists, they compete in prices for good A demand. Consumers buy from the firm whose prices guarantee a higher expected utility. Anticipating second stage prices the consumers' expected utilities when buying from firm j is:

$$\pi U(I + r - p_{jA} + q_l(r - p_{jB}^T)) + (1 - \pi)U(I + r - p_{jA} + q_h(r - p_{jB}^T)).$$

Because the firms' pricing behavior is the same at the second stage, consumers buy from the firm with the lower good A price. Consequently, the standard Bertrand argument holds and the firms drive each other down to zero profits. But not only profits on product A are zero, expected overall profits Π_j are zero. So, the firms set the price p_{jA}^T such that overall profits are zero:

$$\Pi_j = \left[(p_{jA}^T - c_A) + \pi q_l(p_{jB}^T - c_B) + (1 - \pi)q_h(p_{jB}^T - c_B) \right] = 0.$$

Plugging in the result from stage 2, $p_{jB}^T = p_{jA}^T$, firm j's price for A is

$$p_{jA}^T = \frac{c_A + c_B(\pi q_l + (1-\pi)q_h)}{1 + \pi q_l + (1-\pi)q_h}.$$

Multiplying c_A and c_B with the denominator of equilibrium prices gives:

$$c_B(1 + \pi q_l + (1-\pi)q_h) < c_A + c_B(\pi q_l + (1-\pi)q_h) < c_A(1 + \pi q_l + (1-\pi)q_h).$$

Because $c_A > c_B$ equilibrium prices lie between marginal costs.

Proposition 2 *In the tie-in sales equilibrium the firms set the same price p_{jA}^T for both products. The price for good A is below marginal costs c_A and the price for good B is above marginal costs c_B. Each firm serves half the market.*

3.3   Comparison of Consumers' Utility

Because the firms' behavior is symmetric we suppress the index j in the following. According to proposition 1, prices equal marginal costs under independent pricing. If the firms use tie-in sales they set the same price for each good. Then, the consumers expect the utility $E[U_{IP}]$ under independent pricing and utility $E[U_T]$ under tying:

$$E[U_{IP}] = \pi U(I + r - c_A + q_l(r - c_B)) + (1-\pi)U(I + r - c_A + q_h(r - c_B)),$$
$$E[U_T] = \pi U(I + (1 + q_l)(r - p_A^T)) + (1-\pi)U(I + (1 + q_h)(r - p_A^T)).$$

Let the arguments in the utility function be consumers' wealth. If demand is low, consumers' wealth is higher under tying than under independent pricing. However, if consumers have high demand, tie-in sales lead to lower wealth than independent selling does. Hence, consumers enjoy higher expected utility with tie-in sales if demand is low and expect lower utility if

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demand is high. Figure 1 shows graphically that expected utility under tying lies above expected utility under independent pricing. Because expected wealth is the same for the two selling regimes consumers have higher expected utility under tying.
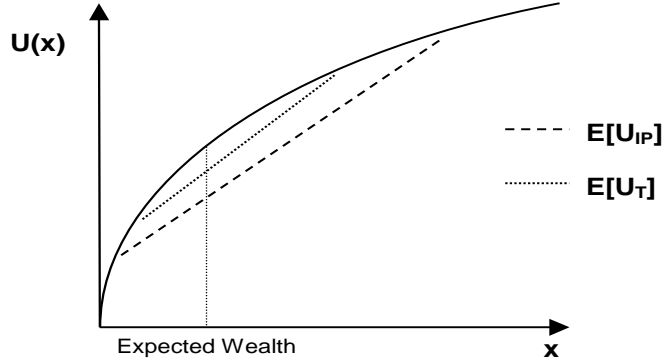

Figure 1: Expected Utility

To see the tie-ins' favorable effect for risk averse consumers let the consumers' wealth from a close and comfortable shave be lotteries. The lottery under independent pricing L_{IP} pays $r - c_A + q_l(r - c_B)$ with probability π and $r - c_A + q_h(r - c_B)$ with probability $1 - \pi$. Tying is the lottery L_T which gives $(1 + q_l)(r - p_A^T)$ if demand is low and $(1 + q_h)(r - p_A^T)$ if demand is high. The expected value is the same for both lotteries. But the lotteries' risks measured by their variance differ. Since the tie-in price is higher than B's marginal costs the tying lottery exhibits a smaller variance.

$$Var[L_{IP}] = (r - c_B)^2 \pi(1 - \pi)(q_h - q_l)^2$$
$$> (r - p_A^T)^2 \pi(1 - \pi)(q_h - q_l)^2 = Var[L_T]$$

Since consumers are risk averse they are better off under tying. If the firms tie their products, they set prices for the holder with blades below marginal costs. For blades only, the price is above marginal costs. The consumers

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benefit from the reduced good A price with certainty. Whereas the higher blade prices adversely affect the buyers only with some probability. Tie-in sales insure consumers against uncertain future demand.

4   Consumer Heterogeneity and a More Fundamental Utility Function

Hitherto, demand for package B is exogenous and the same for all consumers. A more fundamental utility function endogenizes demand for B. This extension also entails that the demand in the second stage is no longer completely inelastic. The strictly concave function $U(\cdot)$ defined on individuals' incomes and benefits from a clean shave exhibits risk aversion. However, the marginal benefit from shaving decreases as the number of shaves increase. Consumers' utility is

$$U(r, p_A, p_B) = U(I - p_A + r \ln(q+1) - p_B q),$$

where q is demand for blades only. Buying package A and q units of package B renders benefit $r \ln(q+1)$. The willingness to pay $r \in [\underline{r}, \overline{r}]$ is a random variable with uniform distribution. Consumers and firms both know the reservation price's density function $f(r) = 1/(\overline{r} - \underline{r})$. The reservation price's lower bound is greater than the marginal costs for A. This assumption ensures that consumers buy package A even if their demand for subsequent blades was zero.

The game's timing remains unchanged. *Ex ante*, the reservation price r is unknown. The consumers learn their reservation price at the second stage, after buying package A.

4.1 Independent Pricing

At the second stage consumers buy blades from the cheaper supplier because they can freely combine handle and blades. Therefore, firms price B at marginal costs. Given the prices for good B equal marginal costs, the firms also charge marginal costs for good A in the first stage. Proposition 1 still holds.

4.2 Tie-In Sales

After buying package A, consumers only demand blades at the second stage. Knowing their reservation prices, consumers demand q units of package B. Note that buyers used up package A's blades. Thus, only shaves in the second stage yield a benefit. A consumer with reservation price r demands the amount q which maximizes his utility. Let $q^*(r, p_B)$ denote the consumers' optimal demand for B at price p_B depending on their willingness to pay:

$$q^*(r, p_B) = \arg\max_q U(I - p_A + r\ln(q) - p_B q).$$

The first order condition for the consumers' maximization problem determines the optimal demand $q^*(r, p_B) = r/p_B$. Because any critical point of a concave function is a global maximizer, $q^*(r, p_B)$ is the optimal demand in stage two for a consumer who is willing to pay r.

If the firms tie their goods, they have monopoly power in the second stage. Like in section 3.2 two restrictions limit this monopoly power. First, the firms do not price package B higher than package A. Secondly, buyers still can change firms. Therefore, it is possible that firm j does not sell B to all consumers who initially bought A from j. Consumers with j's handle buy good B from firm j or buy A and subsequent goods B from i. If the consumers' utility is higher in the latter case, they switch firm. Denote the

individual who is indifferent between staying at firm j and switching to firm i by $\hat{r}_j$. Then, firm j supplies the set $\hat{R}_j$ of consumers. If firm j served the fraction α of consumers in the first stage it's maximization problem in the second stage is:

$$\max_{p_{jB}} \quad \alpha(p_{jB} - c_B) \int_{\hat{R}_j} q^* \left(r, p_{jB}\right) f(r) dr$$

s.t.

$$p_{jB} \leq p_{jA}^T,$$

$$r \ln(q^*(r, p_{jB})) - (q^*(r, p_{jB})) \, p_{jB}$$

$$\geq r \ln(q^*(r, p_{iB})) - (q^*(r, p_{iB}) - 1) \, p_{iB} - p_{iA}^T.$$

Again, we show that the price for B equals the price for A. Consider the firms' no switching constraints for the indifferent consumers:

$$\hat{r}_j \ln\left(\frac{\hat{r}_j}{p_{jB}}\right) - \hat{r}_j = \hat{r}_i \ln\left(\frac{\hat{r}_i}{p_{iB}}\right) - \hat{r}_i + p_{iB} - p_{iA}^T$$

$$\hat{r}_i \ln\left(\frac{\hat{r}_i}{p_{iB}}\right) - \hat{r}_i = \hat{r}_j \ln\left(\frac{\hat{r}_j}{p_{jB}}\right) - \hat{r}_j + p_{jB} - p_{jA}^T$$

The consumer with reservation price $\hat{r}_j$ initially bought A from firm j and is indifferent between buying q^* sets of B from j and buying A plus $q^* - 1$ sets of B from i. In the same way, the consumer with $\hat{r}_i$ who first purchased i's holder is indifferent between staying with i and switching to firm j. We next show that these indifferent consumers have the same willingness to pay, $\hat{r}_j = \hat{r}_i$.

Assume the situation $\hat{r}_i < \hat{r}_j$. Further, suppose that consumers with $r > \hat{r}_j$ stay with firm j while consumers with $r < \hat{r}_i$ stick to firm i as depicted in figure 2. However, it is impossible then that consumers with $r \in [\hat{r}_i, \hat{r}_j]$ change from j to i while consumers with the same reservation prices switch from i to j. Hence, $\hat{r}_i \geq \hat{r}_j$.
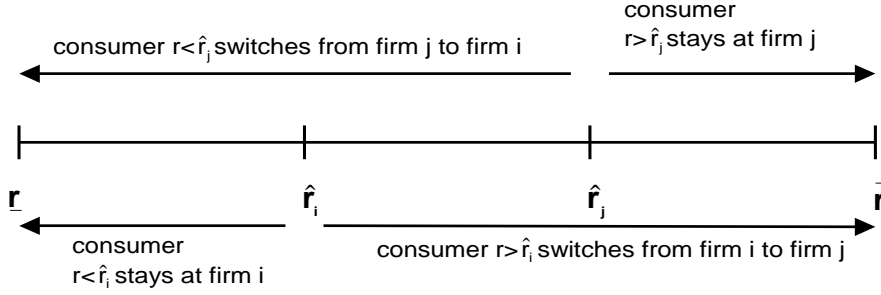
Figure 2: Firms' Indifferent Consumers with $\hat{r}_i < \hat{r}_j$

Now assume $\hat{r}_i > \hat{r}_j$ and consumers with $r > \hat{r}_j$ stay with firm j while consumers with $r < \hat{r}_i$ stick to firm i. Figure 3 illustrates this situation. Then, a consumer with reservation price $\hat{r}_i$ who initially bought from firm
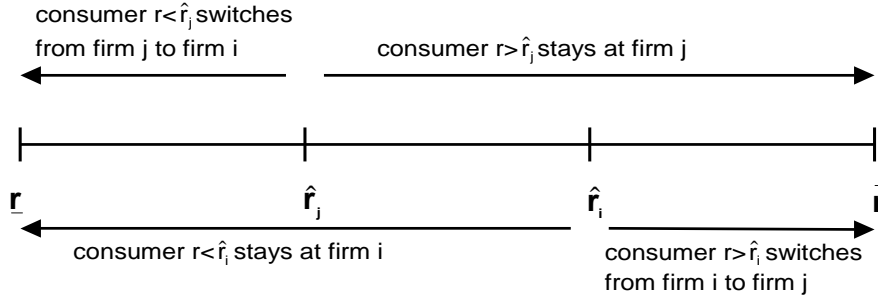

Figure 3: Firms' Indifferent Consumers with $\hat{r}_i > \hat{r}_j$

j has a higher utility than the indifferent consumer $\hat{r}_i$ who initially bought from firm i.

$$\hat{r}_i \ln\left(\frac{\hat{r}_i}{p_{jB}}\right) - \hat{r}_i > \hat{r}_i \ln\left(\frac{\hat{r}_i}{p_{iB}}\right) - \hat{r}_i + p_{iB} - p_{iA}.$$

Obviously, $r \ln(r/p_B) - r$ is decreasing in p_B for all r. The consequence is that firm i's price for package B is higher than firm j's price, $p_{jB} < p_{iB}$. Similarly, a consumer with willingness to pay $\hat{r}_j$ who initially bought from firm i must have a higher utility than the indifferent consumer $\hat{r}_j$. It follows that $p_{iB} <$

p_{jB} which contradicts the first consequence and hence the assumption $\hat{r}_i > \hat{r}_j$ is not valid.

It follows from the analysis above that the firms' indifferent consumers have the same willingness to pay $\hat{r}$. Substituting firm j's LHS of the no-switching condition into i's RHS gives $p_{iA}^T - p_{iB} = p_{jB} - p_{jA}^T$. Only identical prices for package A and B solve this equation, $p_{jA}^T = p_{jB}, j = 1, 2$.

Given the pricing behavior at the second stage, the firms set prices for good A at the first stage. Like in section 3.2, the firms use second stage profits to subsidize price reductions for A to compete for demand. The firms charge the price for handle with blades such that overall profits from selling package A and B are zero. Accordingly, the price p_{jA} solves the equation

$$\pi_j = p_{jA} - c_A + (p_{jB} - c_B)E[q^*(r, p_B)] = 0.$$

At the first stage, firm j expects demand $E[q^*(r, p_{jB})] = (\overline{r} + \underline{r})/(2p_B) = \mu_r/p_B$ for blades. Plugging consumers' expected demand in firm j's zero profit condition and replacing blade price by the price for the access package gives a cubic equation in p_{jA}. The solution for $p_{jA} \in \mathbb{R}^+$ is:

$$p_{jA}^T = p_{jB}^T = \frac{1}{2}c_A - \frac{1}{2}\mu_r + \frac{1}{2}\sqrt{(\mu_r - c_A)^2 + 4\mu_r c_B}.$$

Easy algebra shows that the price p_{jA}^T is always positive. Furthermore, the price is smaller than package A's marginal costs and greater than B's marginal costs. Finally, Proposition 2 still holds.

4.3 Comparison of Consumers' Utility

Because demand for package B is not completely inelastic the effects from tie-in sales are ambiguous. The utility function's general form precludes a direct comparison between expected utility under tying and independent pricing.

Therefore, we point out the combinations under which tie-ins provide the buyers a higher expected utility.

Let the argument in the utility function be consumers' wealth again. Then, the selling scheme with higher expected wealth and lower variation is superior. We first show that wealth's variance under tying is lower than under independent pricing. Then, we characterize the cost values for which tying leads to higher wealth compared to marginal pricing.

In the following analysis we neglect the income I. Income is deterministic and equal under the two selling schemes. The benefit from shaving in monetary terms is a random variable $W = r \ln(r/p_B + 1) - r - p_A$. The general form for a random variables' spread assigns wealth's variance as

$$\sigma_W^2 = E\left[\left(r \ln\left(\frac{r}{p_B} + 1\right) - r\right)^2\right] - \left(E\left[r \ln\left(\frac{r}{p_B} + 1\right) - r\right]\right)^2.$$

Note that the variance is independent of A's price. The consumers always buy package A and know its price with certainty. So, p_A has no effect on wealth's variance. Differentiating the variance with respect to the price p_B gives

$$\frac{\partial \sigma_W^2}{\partial p_B} = -2E\left[\left(r \ln\left(\frac{r}{p_B} + 1\right) - r\right)\frac{r^2}{rp_B + p_B^2}\right]$$
$$+ 2E\underbrace{\left[r \ln\left(\frac{r}{p_B} + 1\right) - r\right]}_{f(r)} E\underbrace{\left[\frac{r^2}{rp_B + p_B^2}\right]}_{g(r)}.$$

Denote the first term inside the expectation by $f(r)$ and the second term by $g(r)$. The function $g(r)$'s derivative with respect to r is always positive. Differentiating the other function $f(r)$ makes

$$\ln\left(\frac{r}{pB} + 1\right) + \frac{r}{r + pB} - 1.$$

If the price for package B equals the consumers' willingness to pay, the differential $\partial f(r)/\partial r$ is $\ln(2) + 0.5r - 1$. We remember that the reservation price

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is greater than one. The price for B is smaller than the willingness to pay. Therefore, $f(r)$ also increases in r. The Fortuin-Kasteleyn-Ginibre-inequality states, for two increasing functions f and g and any random variable R, that $E[f(R)g(R)] \geq E[f(R)]E[g(R)]$. Applying this fact on the variance's derivative shows that wealth's variation does not depend positively on the price for package B, $\partial \sigma_W^2 / \partial p_B \leq 0$. Under tying the price for blades is always higher than under marginal pricing. Thus, the variation in consumers' wealth decreases if firms tie their products.

To pin down the conditions under which tie-ins lead to higher utility consider the consumers' expected wealth

$$E[W] = E\left[r \ln\left(\frac{r}{p_B} + 1\right)\right] - \mu_r - p_A.$$

Bear in mind that prices satisfy the firms' zero profit condition. Then, the price for package A is a function of the price for package B, $p_A(p_B) = c_A - \mu_r + \mu_r c_B / p_B$. Plugging in $p_A(p_B)$ and differentiating expected wealth with respect to p_B is

$$\frac{1}{p_B^2}\left(-E\left[\frac{r^2 p_B}{r + pB}\right] + \mu_r c_B\right).$$

If $\partial E[W]/\partial p_B$ is positive, consumers expect higher wealth when the price for B increases. On one hand, a reduction in the price for A has a positive effect on wealth. On the other hand, the price reduction negatively effects the monetary benefit from shaving. Consumers expect higher wealth if the positive effect outweighs the negative,

$$\mu_r c_B \geq E\left[\frac{r^2 p_B}{r + p_B}\right].$$

The adverse effect due to a price increase is highest for the willingness to pay's upper bound. Let then $\overline{p}_B$ denote the price which solves the above

equation evaluated at $\overline{r}$. Tie-in sales yield a higher expected wealth than marginal pricing if the tying price p_A^T is smaller than this critical value $\overline{p}_B$:

$$p_A^T = \frac{1}{2}c_A - \frac{1}{2}\mu_r + \frac{1}{2}\sqrt{c_A^2 - 2c_A\mu_r + \mu_r^2 + 4\mu_r c_B} \leq \frac{\mu_r c_B \overline{r}}{\overline{r}^2 - \mu_r c_B} = \overline{p}_B.$$

For all values c_A which solve the above inequality $p_A^T \leq \overline{p}_B$ tie-in sales generate at least the same expected wealth as independent pricing does. The upper bound for package A's marginal costs is then

$$\overline{c}_A \leq \frac{3\overline{r}^2\mu_r c_B + \mu_r\overline{r}^3 - \mu_r^2 c_B\overline{r} - \mu_r^2 c_B^2 - \overline{r}^4}{\overline{r}(\overline{r} - \mu_r c_B)}.$$

Marginal costs for A are greater than the costs for B. The upper limit for package A's marginal costs must also be greater than B's marginal costs. Solving $c_B < \overline{c}_A$ bounds c_B above at $\overline{r}(\overline{r} - \mu_r)/\mu_r$. At last, consumers expect higher wealth under tying if marginal costs are small enough, that is

$$c_A \leq \frac{3\overline{r}^2\mu_r c_B + \mu_r\overline{r}^3 - \mu_r^2 c_B\overline{r} - \mu_r^2 c_B^2 - \overline{r}^4}{\overline{r}(\overline{r} - \mu_r c_B)},$$

$$c_B < \overline{r}(\overline{r} - \mu_r)/\mu_r.$$

Yet, tie-in sales insure consumers against uncertainty. Under tying the variation in wealth is smaller than under independent pricing. But expected wealth can be higher if the firms price at marginal costs. Tie-in sales increase the price for B. A higher price p_B has two negative effects. First, the spending for B increases and, second, the benefit from shaving decreases. The positive effect due to a price decrease in good A does not always compensate the negative effects in good B. Risk averse buyers account for expected wealth and its variance. If marginal pricing leads to higher wealth the utility function's general form makes a clear statement impossible. The increase in utility due to higher wealth can outweigh the utility decrease from a higher variance. However, marginal costs small enough allow an assertion. In this case, consumers prefer tie-in sales to independent pricing.

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5 Conclusions

This paper introduces tie-in sales in a symmetric setting with price competition. We analyze the case of two firms. Each firm sells two goods. A good example is the market for shaving systems. The razor basically consists of a razor holder and blades. We compare the outcome under tie-in sales with the benchmark independent pricing. If the firms do not tie, a firm's own goods are independent of each other. Under tying the firms design their products such that their parts are not compatible with other firms' components. Tie-in sales allow the firms to create monopolies in one of the two markets. The firms earn positive profits on the monopolized products. Because of competition in the market for the access good the firms drive each other down to zero overall profits. They use the profits from monopolized goods to subsidize losses in the competitive market. Although the firms tie their products, the symmetry in the firms' market power maintains price competition.

Beside the firms' behavior we examine the effects from tie-in sales on consumers' expected utility. The buyers are uncertain about their future spending. Before buying the holder, the consumers do not know their future demand. In a simple model, risk averse consumers who are homogeneous have completely inelastic demand. If demand is completely inelastic, tie-in sales insure the buyers against uncertainty in expenditures. The consumers prefer tie-in sales to independent pricing. We extend the simple model by relaxing the assumption of completely inelastic demand and allowing for consumer heterogeneity. In the extended version the tying's effect is ambiguous. Tie-in sales still have an insurance effect. But demand is no longer completely inelastic and thus the benefits from shaving decrease under tie-in sales. However, we characterize cost combinations which lead to higher expected utility under tie-in sales than under independent pricing.

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