Global and local players in a model of spatial competition

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DISCUSSION PAPERS
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Abstract

We consider Hotelling location games with global and local players. Global players are active in several markets, while local players act in a single market only. The decisive feature is that global players cannot tailor their product to each market but have to choose a location on the Hotelling line that is valid for all markets in which they are active. Obvious examples include the media industry and politics, where competitors typically compete in several markets with basically the same product. We determine equilibrium configurations for simple specifications of such games. We then show that the presence of global players tends to induce lower product diversity across markets. Finally, when the number of firms is endogenous, we show how global players may use their location choice as a preemptive device.

JEL-Code: D45, K21, K23, L11, L51

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1 Introduction

Motivation In this paper, we consider simple location games à la Hotelling with two sorts of players: Global players are active in several markets, while local players act in a single market only. The decisive feature is that global players can choose only one position for their product. That is, they cannot tailor their product to every individual market in which they are active. Since these markets will in general differ with respect to salient characteristics such as market size, the distribution of customers and the number and type of competitors, optimal location choices will change non-trivially compared to the standard case where all players are local. In particular, a global player’s gain by changing his position in one market is typically (partially) offset by a loss in another market as competitors will respond optimally to any such change.

Two obvious fields of application for our theory are media markets and politics: For example, in most countries, there are newspapers such as the New York Times in the U.S., which are sold in many different regions or local markets (e.g., on the East and the West Coast).\footnote{Although a geographical interpretation is given to the term “market” in this and subsequent examples, our arguments do not rely on it. One can equally plausibly think of other criteria such as gender, age, or education level. For a newspaper, say, the group of 35-40 year old females with high education levels might then be considered a different “market” than the group of 55-60 year old males with low education levels. Nevertheless, it might want to attract members of both groups with the same outlet, while it faces competitors which are only targeting one of these groups.} While characteristics such as size, the number of competitors, customer preferences are very likely to differ across these local markets, the daily issue of the New York Times is basically the same in New York as in Los Angeles. Thus, when the editors decide to publish more articles about New York politics to attract more readers in New York, this decision might result in a loss of customers on the Los Angeles market, provided that these customers do not care as much
about New York politics as New Yorkers do.\textsuperscript{2} On the other hand, there are outlets such as the New York Post or New York a.m., which are distributed in New York only. These competitors are typically much smaller in size and have a stronger focus on local issues, as their decisions about which articles to publish are driven by considerations for this market only. A similar argument holds for broadcasting markets, where large networks such as CNN and Fox basically provide the same program across the U.S. (and even to foreign countries), thereby competing against each other as well as against smaller channels, which serve one regional or local market only.\textsuperscript{3}

In politics, competition often takes place both on (possibly many different) local levels and on a federal level. Again, these “markets” tend to be highly heterogenous with respect to importance, voter preferences or voter characteristics, as voters in jurisdictions with high average income, say, prefer different tax policies than voters in jurisdictions with low average income. Also here, there often seem to be two groups of parties involved: On the one hand, there are big parties such as the Democrats or the Republicans in the U.S., which compete on all levels. These are also global players in our sense, as they cannot fully tailor their political views or decisions to each market separately. For example, suppose Democrats leaders announce that, should the party win the federal elections, taxes will be raised to increase redistribution. Then presumably, on a local election level, it would

\textsuperscript{2}Sometimes nationwide outlets have so-called “local windows” in which a limited number of pages contain articles which are tailored towards the relevant local market. We will discuss this issue in section 6 below.

\textsuperscript{3}In fact, it seems that program choices are truly interdependent as the following quotation suggests: “Well, Fox has had a profound impact on cable television not only because it is, I guess, the No. 1 watched program of cable television stations out there, even though they don’t have as many stations as CNN, but they’ve also forced the other cable television stations to put on more conservative hosts and pay attention to the conservative market out there. So the other cable television stations are more conservative today because of Fox.” R. Viguerie, co-author of “America’s right turn: how conservatives used new and alternative media to take power” (quotation taken from an interview at http://www.booknotes.org/transcript/?ProgramID=1796).
be hardly convincing to argue in favor of tax cuts to gain votes in a high average income jurisdiction. On the other hand, there are often small parties or political groups which compete only in a subset of jurisdictions.

**Framework and main results** In this paper we set up a simple model which reflects many common features of the above-mentioned examples: Global players compete in several markets against different local competitors, and markets differ with respect to size and distribution of customers. We characterize equilibrium configurations and compare them to the standard model where all players are local, such that markets are not “connected” through the location choices of global players.

Apart from characterizing equilibrium configurations of such location games, we are concerned with the resulting degree of diversity *within* and *across* markets. As for media markets in Germany and Switzerland, say, many people argue that there seems to exist a (too) great degree of product homogeneity not only within but even across regional markets, despite differences in taste of consumers and an increasing number of suppliers. While it is a well-known feature of many Hotelling location games that firms cluster at the median position of the distribution of customers, thereby inducing full product homogeneity (or zero diversity) *within* a market, we argue that the presence of global players can help to explain why product diversity is low also *across* markets. The basic mechanism at work is that the concern of a global player to cater customers preferences in all markets induces him to move towards a “weighted average” of all relevant markets. In turn, the best response of a local competitor is to locate adjacently to the global player. This brings the equilibrium configuration closer together compared to the case where all players are local and where markets are separated. As

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4See, e.g., NZZ (2003), Spiegel online (2003), or Weltwoche (2005).
a result, diversity across markets decreases.

So as to compare different equilibrium outcomes, we develop a criterion based on the preference costs of consumers. With respect to diversity within markets (“local diversity”), we show that clustering is suboptimal in terms of minimization of preferences costs. Moreover, within the class of configurations with clustering, we show that doing so at the median position would lead to the lowest preference costs. This criterion allows us the compare welfare in equilibrium outcomes where clustering occurs at different positions.

We then analyze three extensions to the basic model: First, we allow for entry, where each firm has to incur a cost when entering a market. We show how preemption may become an important additional aspect for an incumbent global player when deciding on where to locate. Second, we extend the basic model to a case with two global players where each of them competes with a local competitor on his “home market”, but where they additionally compete directly against each other on a third market. Finally, while for the above-mentioned examples, price competition seems to play only a minor role, we illustrate how our approach could be set in relation to a model à la Gabszewicz, Laussel, and Sonnac (2001), where media firms compete in prices for readers and for revenue from selling advertisement space. We give a sufficient condition such that equilibrium prices are zero for all locations because of the intensity of the competition for advertisements. In these situations, our approach could be viewed as a reduced form of such a model, thereby justifying our abstracting from price competition in the basic model.

**Relation to the Literature**  To the best of our knowledge, the issue of global players has not yet been addressed in the literature so far. However,
our modelling approach is clearly in the tradition of the large literature on horizontal differentiation starting with Hotelling (1929) and Downs (1957). In this literature, as in our paper, a major topic has also been the degree of product diversity resulting in equilibrium. Starting from Hotelling’s famous “Principle of Minimum Differentiation” derived from a simple location model without price competition as ours, it has been enquired under which circumstance the claim is correct (see, e.g., d’Aspremont, Gabszewicz, and Thisse (1979), Osborne and Pitchik (1987), and Irmen and Thisse (1998) and, in the context of political competition, the survey by Osborne (1995)).

In the context of media markets, Gabszewicz, Laussel, and Sonnac (2001) analyze a model where apart from location choices and prices, firms also compete for receipts from selling advertisement space in their outlets. Contrary to our basic model, since firms also compete in prices and customers face quadratic preference costs, their benchmark model without advertisement receipts involves maximum differentiation (as was shown by d’Aspremont, Gabszewicz, and Thisse (1979)). Including advertisement receipts, they show that the new equilibrium may now induce clustering, thereby reducing the degree of diversity to zero. Along these lines, our contribution is that we identify an additional channel through which clustering may occur, namely the interaction of global and local players.

Finally, the relationship between the socially optimal level of diversity and the equilibrium level is discussed by Calvo-Armengol and Zenou (2002) in a “circular city” - model à la Salop (1979).

The remainder of the paper is organized as follows. In section 2 the basic model is introduced, and in section 3 we derive the equilibrium configurations for our basic model and compare it to a benchmark where all players are local. The issue of diversity is discussed in section 4. In Section
we consider the three extensions of the basic model, while in section 6 we discuss our results and conclude. All proofs are delegated to the appendix.

2 The basic model

We analyze a Hotelling location model in which competition takes place in two markets \( k = A, B \). There are two types of players: local players are active in one market only, while global players are active in both markets. As outlined above, the decisive feature is that the location choice of a global player is necessarily the same for all markets in which he operates.

There is a unit mass of consumers in each market \( k \) which are distributed on the unit interval according to a continuous distribution function \( F_k \) with density \( F_k' \) and with full support. The median of each \( F_k \) is denoted \( m_k \) and implicitly given by \( F_k(m_k) = \frac{1}{2} \). All distributions are common knowledge.

Each individual consumes one unit of the good from the closest provider. There is no price competition.

As for firms’ payoff, different markets will in general be of different size and importance to firms. Let \( s_k \) measure the market size or importance of market \( k \). Thus, \( s_A > s_B \) means that capturing a customer in market \( A \) is more valuable for a firm than in market \( B \). In media markets, for example, \( s_k \) might depend on the number of people living in one market place and their average income since these factors constitute the advertisement value of a firm’s product. In politics, \( s_k \) would measure the number of seats of a district or state. For the moment, we assume that the number of competitors is fixed. Therefore since consumers have unit demand, in each market \( k \) each firm’s revenue is equal to the number of customers it attracts times the

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5 See section 5.3.
6 Getting a one percent market share in a large metropolitan area might be more valuable than getting half the market in a small rural area.
7 This assumption is relaxed in section 5.1
market potential $s_k$.

Concerning the timing of the game, we follow Prescott and Visscher (1977), Neven (1987) and Anderson and Engers (2001) and assume that location choices occur sequentially. The order of moves is exogenously given and global players are assumed to move first. We believe that thinking of global players choosing their location before local players is reasonable, as in most examples we have in mind, they have been active long before the advent of their local competitors. In addition, from a technical point of view, this assumption ensures the existence of pure strategy equilibria. Furthermore, we also assume that, once a location choice has been made, it is prohibitively costly to move to another location. As Prescott and Visscher (1977) have argued, such costs might for example be pure physical re-location costs, or advertisement costs to change the perception of the firm in the mind of its customers when it is at a new location of the spectrum. This implies that there is no commitment problem for global players as they never find it profitable to change their location \textit{ex post}, \textit{after} local players have chosen their location.

3 Equilibrium Analysis

3.1 Benchmark: Separated Hotelling markets

As a natural benchmark we first consider the case with local players only in which case the two markets are separated from each other. In particular, suppose that in each market $k$ there are two local players and that, respectively, player 1, say, moves first, and then player 2. Prescott and Visscher (1977, pp. 381) have shown that in such a framework, whether players move sequentially or simultaneously does not affect the equilibrium outcome. Therefore, this benchmark is trivial to analyze as it is equivalent to
the simple standard model of spatial competition.

Denote by \( l_{ki} \) the location choice of player \( i \) in market \( k \), where \( i = 1, 2 \). It then follows that in each market the best response of player 2 as a function of the location choice of player 1, denoted by \( l_{k2}^*(l_{k1}) \) is given by

\[
l_{k2}^*(l_{k1}) = \begin{cases} 
  l_{k1}^- & \text{if } l_{k1} > m_k \\
  l_{k1} & \text{if } l_{k1} = m_k \\
  l_{k1}^+ & \text{if } l_{k1} < m_k
\end{cases}
\]

where \( l_{k1}^- := l_{k1} - \varepsilon \) and \( l_{k1}^+ := l_{k1} + \varepsilon \) and where \( \varepsilon \) is an arbitrarily small positive number. In words, if \( l_{k1} \) is to the left of the median of \( F_k \), then it is optimal for player 1 to position just to the right of player 2 and vice versa.

Finding the optimal choice for player 1 and denoting equilibrium outcomes with a “*”, this leads immediately to the well-known result that in each market, firms cluster at the median position:

**Proposition 1 (Hotelling, Prescott-Visscher).** On each market \( k = A, B \), in the unique subgame perfect equilibrium outcome both players locate at the median position, i.e., \( l_{k1}^* = l_{k2}^* = m_k \).

Intuitively, as in the standard Hotelling game with simultaneous moves, player 1 anticipates that wherever he locates, player 2 will always locate adjacent to that side of the \([0,1]\)-interval which has the larger number of consumers (the “long side”). It follows that player 1’s optimal location is where the remaining “short side” is maximized which is at the median position (see Figure 1).

Note that unless \( m_A = m_B \), the equilibrium locations will be different across markets. As will become clear below, the introduction of global players is only of interest when the medians of the respective distributions are different. Otherwise, the best response for a player in market A would be the same as in market in B and this would also hold if he is a global player who is active in these two markets.
3.2 From local to global competition

In our analysis of games with global players, we consider the simplest case where one global player competes in both markets against one local player, respectively. The location choice of the local player in market \( k = A, B \) and the global player are denoted by \( l_k \) and \( g \), respectively.

We solve this game backwards by first determining the best response for a local player in market \( k = A, B \) as a function of the location choice \( g \) of the global player which is given by:

\[
l^*_k(g) = \begin{cases} 
g^- & \text{if } g > m_k 
g & \text{if } g = m_k 
g^+ & \text{if } g < m_k 
\end{cases}
\]

where \( g^- := g - \varepsilon \) and \( g^+ := g + \varepsilon \).

Given these best responses, we can then state the following preliminary result concerning the optimal location choice \( g^* \) of the global player:

**Lemma 1.** On each market \( k = A, B \), \( g^* < m_k \) only if \( m_{-k} < m_k \). It then follows that in any equilibrium \( m_B \leq g^* \leq m_A \) if \( m_A > m_B \) and \( m_A \leq g^* \leq m_B \) if \( m_A < m_B \).

Thus, the global player will only depart from his preferred position in market A, say, when thereby gaining customers in market B. Given the best
response of his local competitor there, this is only possible when moving
towards $m_B$. Throughout the remainder of the paper, we make the following
assumption:

**Assumption 1.**

i) $m_A > m_B$

ii) $F''_A < 0$ and $F''_B > 0$

iii) $s_A \cdot F'_A(m_B) - s_B \cdot F'_B(m_B) > 0$ and $s_A \cdot F'_A(m_A) - s_B \cdot F'_B(m_A) < 0$

The first part of the assumption is without loss of generality and simply
limits the number of cases we have to consider, while the analysis of the
omitted cases is completely analogous. Parts ii) and iii) ensure an interior
solution to the maximization problem of the global player as moving away
from a median becomes increasing costly. For market A (B), the relevant
direction of deviation is to the left (right) and thus the concavity (convexity)
of $F_A$ ($F_B$) ensures that this is the case. Part iii) simply says that the
marginal benefit from moving to the right at $g = m_B \ (g = m_A)$ is positive
(negative).

Using Lemma 1, it follows that the local players’ relevant best responses
are $l^*_A = g^{*+} > g^*$ and $l^*_B = g^{*−} < g^*$ so that the global player’s profit as a
function of $g$ can be written as:

$$\Pi(g) = s_A \cdot F_A(g) + s_B \cdot (1 - F_B(g)) \quad \forall g \in [m_B, m_A] \quad (3)$$

That is, the global player captures all customers to the left of his com-
petitor in market A and those to the right in market B. Assumption 1 ensures
that this function achieves its unique maximum at $g^*$, where the following
first order condition is satisfied:

$$s_A \cdot F'_A(g^*) = s_B \cdot F'_B(g^*) \quad (4)$$

This condition simply says that the marginal gain from moving closer
towards $m_A$ and getting more customers there, weighted by $s_A$ (measuring
the importance of market A) must equal the marginal loss from moving further away from $m_B$, thereby losing customers there. This leads to the following result:

**Proposition 2.** With one global player competing with one local player on each market A and B, the subgame perfect equilibrium outcome is \( \{ g = g^*, l_A^* = g^{*+}, l_B^* = g^{*-} \} \), where \( g^* \) is implicitly given by Eqn. (4).

As can be seen from Figure 2, there is still clustering in equilibrium, but it does no longer occur at the median positions in each market. Instead, both markets are “tied” together through the presence of the global player. In the following section, we will discuss in more detail what this means in terms of diversity. But it is already clear that as a consequence of the introduction of a global player, all firms are located at the same position in both markets despite the fact that $m_A \neq m_B$.

Furthermore, it is also intuitively clear how the relative importance of the different markets affects the equilibrium location choice of the global player. Applying the implicit function theorem it can easily be shown that $g^*$ is more (less) distorted from $m_A$ when market B (A) is more important, i.e., we have $\frac{dg^*}{ds_B} < 0$ and $\frac{dg^*}{ds_A} > 0$. 

Figure 2: Equilibrium with a global player
For instance, with respect to the case of media markets, our analysis might suggest that the currently observed trend towards more product homogeneity does not necessarily mean that these strategies are targeted towards the median consumer in a certain market, but rather towards a “weighted average” of consumers of all markets in which firms are active.

4 Diversity

We now analyze the impact of global players on the degree of diversity and on total preference costs resulting in equilibrium and compare them to the benchmark where all players are local. Let $y_k$ and $y_k$ denote the respective maximum and minimum location choices in market $k$. We introduce the following two measures:

**Definition 1.** We refer to
i) “local diversity” as the spread of location choices within each market $k = A, B$, i.e., $LD_k := y_k - y_k$.
ii) “global diversity” as the maximum spread of location choices across markets, i.e., $GD := \max_k \{y_k\} - \min_k \{y_k\}$.

Instead of simply claiming that “more diversity is good”, we derive the desirability of diversity endogenously from the minimization of preference (or transportation) costs consumers incur when buying from a provider whose location is different from their own. For simplicity, we normalize these costs to 1 per distance unit. That is, the preference costs for a consumer at $x \in [0, 1]$ when consuming the product from a provider at $y \in [0, 1]$ is simply $|x - y|$. Let $LPC_k(y_{k1}, y_{k2})$ denote the aggregate preference costs in market $k$ when firms locate at positions $y_{k1}, y_{k2} \in [0, 1]$, respectively. Moreover, define $GPC := \sum_k LPC_k$ as total preference costs across both markets.
With two players on each market, the first best location choices are denoted \( \{y_{k1}^f, y_{k2}^f\} \) and minimize \( LPC_k \). Assuming \( y_{k1}^f \geq y_{k2}^f \) without loss of generality, we therefore have

\[
\{y_{k1}^f, y_{k2}^f\} \in \arg \min_{y_{k1}, y_{k2}} LPC_k(y_{k1}, y_{k2})
\]

\[
= \int_0^{y_{k1} + y_{k2}} |y_{k2} - x|F_k'(x)dx + \int_0^{y_{k1} - y_{k2}} |y_{k1} - x|F_k'(x)dx
\]  

where the first term of the expression gives the preference costs for all customers buying from firm 1 (i.e., those for which firm 1 is the nearest firm). Analogously, the second expression refers to the preference costs of customers buying from firm 2. The level of local diversity minimizing preference cost is then given by \( LD_{k}^f = y_{k1}^f - y_{k2}^f \) and the following result holds:

**Lemma 2.** i) Minimization of preference costs in market \( k \) involves a positive amount of local diversity, i.e., \( LD_{k}^f > 0 \) and thus clustering is not optimal.

ii) Given that clustering occurs in equilibrium in market \( k \), i.e., \( y_{1k}^* = y_{2k}^* = y \), so that \( LD_{k}^* = 0 \), preference costs are lowest if firms cluster at \( y = m_k \).

This result says that, given that clustering occurs in equilibrium, from a preference cost minimizing point of view, firms should cluster at the median \( m_k \). However, preference costs would be further decreased by generating a positive degree of local diversity around \( m_k \) so that at the first best location choices there will be some local diversity. Note however, that it does not follow from Lemma 2 that any degree of local diversity is better in terms of preference costs than clustering. The result is useful because it allows us to compare equilibrium outcomes in which clustering occurs in both markets, but at different locations:

**Corollary 1.** While \( LD_k = 0 \) for all \( k = A, B \) in both models, with and without global players, global diversity is strictly lower and total preference
costs are strictly higher in the model with a global player compared to the benchmark.

Intuitively, for the benchmark case, we know from Proposition 1 that clustering occurs at the median of each market, and so in terms of preference costs, the "second best" outcome is achieved. As for global diversity, given Assumption 1, we have $GD = m_A - m_B > 0$.

For the case with the global player, from Proposition 2 we know that there is still clustering in each market, but it does no longer occur at the medians $m_A$ and $m_B$. Therefore, it immediately follows that global diversity goes down. Moreover, Lemma 2 nicely allows to compare different clustering equilibria with respect to the preference costs they generate: Not only is clustering socially inefficient, but if it does not occur at the median location, then it is even worse. Note also that in this case, the reduction in global diversity comes together with an increase in total preference costs, because preference costs in each individual market are higher than in the benchmark case. However, as will become clear in section 5.2, it cannot generally be concluded that the introduction of global players leads to higher total preference costs compared to the benchmark case where all players are local.

5 Extensions

In this section we extend the basic model and consider i) the case of entry, ii) a setup with more than one global player, and iii) competition in prices. We address each issue in turn.
5.1 Entry and preemption

We have so far not addressed the case when a global player faces competition from both sides in each market, i.e., if there is potentially more than one local competitor on each market.

To study such a situation with entry, we extend the basic model of section 3.2 and consider first the case where markets are small such that, in equilibrium, at most one local firm enters in each market. We then derive the conditions under which the global player is able to simultaneously deter entry in both markets or in one market only. Finally, we discuss the case of larger markets, where more than one local player will enter in equilibrium.

5.1.1 Preliminaries

We assume now that the global player faces potential competition from sequentially entering local firms in each of its relevant markets, $A$ and $B$, and as before, the global player moves first.

Let $T$ denote the fixed cost of operation for a local player, which is assumed to be the same in each market. Then, $\alpha_k := T/s_k$ is the market share a local player needs to capture in market $k$ in order to break even. Neglecting integer constraints, the maximal number of local players entering market $k$ in equilibrium will be $1/\alpha_k$. In contrast to local players, the global player does not necessarily have to obtain a share of $\alpha_k$ in each market in order to break even, since the fixed cost when operating in both markets will, in general, be smaller than $2T$. For simplicity, we set it to zero so that he is always willing to enter both markets.\footnote{Of course in reality, global players are also likely to face a cost when entering a certain market such as opening a sales office. All we need, however, is that these costs are lower than for a local competitor who does not already have a ready-made product.}

When distributions are not uniform, one has to distinguish between lo-
cations and market shares. For this purpose, we generalize the method proposed by Prescott and Visscher (1977) and define the function $\lambda_k(x, \alpha_k)$ which is the location that leaves a market share of $2\alpha_k$ between itself and the next competitor to the left located at some $x > 0$. That is, $\lambda_k$ is such that $F_k(\lambda_k) := 2\alpha_k + F_k(x)$. Since in equilibrium, no firm will ever locate at $x = 0$, 

$$\lambda_k(0) := F_k^{-1}(\alpha_k)$$

(6) gives the largest position on the Hotelling line a firm can take such that there is no subsequent entry to its left. Analogously, let $\rho_k(x, \alpha_k)$ be the location that generates a market share of $2\alpha_k$ when the closest competitor is located at $x < 1$ to its right. That is, $\rho_k(x, \alpha_k)$ is such that $F_k(x) - F_k(\rho_k) := 2\alpha_k$. Also, as for the right corner, the smallest position a local player can take without inducing subsequent entry to its right is then given by 

$$\rho_k(1) := F_k^{-1}(1 - \alpha_k).$$

(7)

5.1.2 Small markets

Let us assume first that $\frac{1}{2} < \alpha_k < 1$ for $k = A, B$, i.e., that markets are small so that in any equilibrium, at most one local player will enter market $k$. Note that $\alpha_k > \frac{1}{2}$ implies $\rho_k(1) < \lambda_k(0)$ so that for the global player, any choice $g \in [\rho_k(1), \lambda_k(0)]$ would deter entry in $k$, while any other choice would not, since $g < \rho_k(1)$ ($g > \lambda_k(0)$) would induce entry to his right (left).

Consider first the case depicted in Figure 3, in which there is an overlap of the two intervals for which a local firm would enter in each market, should the global player locate outside this interval. Then, the global player can forestall entry in both markets by choosing any location in the overlapping interval $[\rho_A(1), \lambda_B(0)]$, because no potential entrant would then be able to get a market share of $\alpha_k$. Clearly, this is what the global player will do in
Figure 3: Global player can deter entry in both markets.

Figure 4: Global player can deter entry only in one market.

Next, assume that there is no overlap, which is, e.g., the case if \( \lambda_B(0) < \rho_A(1) \), as shown in Figure 4. In this case, whenever the global firm chooses to forestall entry in market \( k \), which it can do only by locating in \([\rho_k(1), \lambda_k(0)]\), then it necessarily opens the way to entry in market \(-k\). Finally, by locating in one of the intervals \([0, \rho_B(1)]\), \([\lambda_B(0), \rho_A(1)]\), or \([\lambda_A(0), 1]\), the global player invites entry in both markets. We analyze each of these three cases in turn:

**Preempting entry in market** \( A \)  
Note first, that for any choice of the global player which prevents entry in market \( A \) (i.e., \( g \in [\rho_A(1), \lambda_B(0)] \)), the best response of the first local player in market \( B \) is to choose \( l_B^* = \lambda_B(0) \), i.e., to move as closely to the global player’s position as possible without leaving too big a gap to its left, which would induce a second local firm to
enter in market $B$. For the global player this means that he strictly prefers to choose $g^* = \rho_A(1)$, i.e., the smallest location that prevents entry in market $A$ and maximizes his share in market $B$. It follows that the equilibrium payoff of the global player is

$$\Pi(g^*) = s_A + s_B \cdot \left(1 - F_B\left(\frac{\lambda_B(0) + \rho_A(1)}{2}\right)\right)$$

for $g^* = \rho_A(1)$. Obviously, $\Pi(g^*) \geq s_A$.

**Preempting entry in market $B$** This case is completely analogous: Given any $g \in [\rho_B(1), \lambda_B(0)]$ to deter entry in market $B$, the first local player in market $A$ will locate at $\rho_A(1)$ thereby preventing further entry. In turn, the best response for the global player is to locate at $g^* = \lambda_B(0)$, which yields an equilibrium payoff of

$$\Pi(g^*) = s_A \cdot F_A\left(\frac{\lambda_B(0) + \rho_A(1)}{2}\right) + s_B.$$  

Clearly, for $g^* = \lambda_B(0)$, $\Pi(g^*) \geq s_B$.

Thus, when deterring entry in one market in an optimal way, the payoff to the global player is at least $\max[s_A, s_B]$.

**Accommodating entry in both markets** We focus on the case $g \in (\lambda_B(0), \rho_A(1))$. The first local player will choose his location to maximize his respective market share, while making sure that no further entry occurs. Thus, we have $l_A^* = \rho_A(1)$ and $l_B^* = \lambda_B(0)$. It follows that the payoff for the global player is

$$\Pi(g) = s_A \cdot F_A\left(\frac{g + \rho_A(1)}{2}\right) + s_B \cdot \left(1 - F_B\left(\frac{\lambda_B(0) + g}{2}\right)\right).$$

Note that $F_A\left(\frac{g + \rho_A(1)}{2}\right) \leq 1 - \alpha_A < \frac{1}{2}$ and $1 - F_B\left(\frac{\lambda_B(0) + g}{2}\right) \leq 1 - \alpha_B < \frac{1}{2}$.

Hence, the maximal payoff of accommodating entry in both markets is no

\[9\text{All arguments for the cases } g < \rho_B(1) \text{ and } g > \lambda_B(0) \text{ are analogous.}\]
larger than \( s_A(1 - \alpha_A) + s_B(1 - \alpha_B) < \frac{1}{2}(s_A + s_B) \), which is strictly less than the minimal profit of optimally deterring entry in one market, \( \max[s_A, s_B] \).

This leads immediately to the following result:

**Proposition 3.** When the global player cannot preempt entry in both markets, then

i) it is never optimal for him to allow entry in both markets.

Assume \( \lambda_B(0) < \rho_A(1) \). Then

ii) the global player prefers to prevent entry in market \( A \) (\( B \)), thereby accommodating entry in market \( B \) (\( A \)) if

\[
 s_A \cdot (1 - F_A(\frac{\lambda_B(0) + \rho_A(1)}{2})) \geq s_B \cdot F_B(\frac{\lambda_B(0) + \rho_A(1)}{2}).
\]

(11)

The second part of the proposition follows by comparing (9) and (10).

Intuitively, the global player has always the option to monopolize at least one market. The result shows that he strictly prefers to do so to locating “in the middle”, thereby allowing entry in both markets. Whether it is optimal to allow entry in market \( A \) or \( B \) depends on the relative importance of both markets and the distributions of customers.

### 5.1.3 Large markets

It is straightforward to extend the analysis to the case of larger markets in which more than one local player will enter in equilibrium. Consider for example the case \( \frac{1}{3} < \alpha_k < \frac{1}{2} \) so that \( \lambda_k(0) < m_k < \rho_k(1) \). When, say, locating to the left of \( \lambda_k(0) \), the global player invites one subsequent entrant (at \( t_k^* = \rho_k(1) \)), while when locating inside the interval (\( \lambda_k(0), \rho_k(1) \)), two local players enter, one at \( \lambda_k(0) \) and one at \( \rho_k(1) \). Hence, the global player gets “hit” from both sides. Although his optimal choice will depend on the characteristics of market \(-k\) as well, it follows that there is a strong incentive for the global player to avoid locating inside the interval (\( \lambda_k(0), \rho_k(1) \)).
Thus, since the global player needs a smaller market share in any local market than local firms in order to break even, the global player can behave more aggressively than local players can.

Finally, with respect to diversity, we observe that the tendency towards clustering (or product homogeneity) is weakened when there is entry. Speaking somewhat loosely, the reason is that to preempt subsequent entry, firms have to leave gaps no larger than $\alpha_k$ to the closest end of the unit interval and no larger than $2\alpha_k$ between themselves. On the other hand, these gaps may not be too small either because otherwise the firms would not cover the fix cost $T > 0$. Thus, one would expect that the issue of diversity is less problematic when the threat of entry is virulent than when it is not. This seems consistent with the observation that excessive product homogeneity seems to be particularly prevalent in radio and TV markets, where the threat of entry is rather weak whenever there exists only a limited number of available licenses.

5.2 Two global players in three markets

As a further extension, we now consider the case of two global players, labelled $G_A$ and $G_B$, where each of them is competing against one local player in a “home market” ($A$ and $B$ for $G_A$ and $G_B$, respectively). In addition, both global players are competing directly against each other on a third market $C$. Denote the location choices by the global players by $g_A$ and $g_B$, and those of the local players by $l_A$ and $l_B$, respectively.

Note first that the best responses $L_A$ and $L_B$ are still given by

$$l_A^*(g_A) = \begin{cases} g_A^- & \text{if } g_A > m_A \\ g_A & \text{if } g_A = m_A \\ g_A^+ & \text{if } g_A < m_A \end{cases}$$  \hspace{1cm} (12)
and
\[
l_B^*(g_B) = \begin{cases} 
  \bar{g}_B & \text{if } g_B > m_B \\
  \bar{g}_B & \text{if } g_B = m_B \\
  \bar{g}_B^+ & \text{if } g_B < m_B
\end{cases}
\] (13)

To determine the equilibrium configuration, the following preliminary result is crucial:

**Lemma 3.** Given \( m_B < m_A \) by Assumption 1, \( \min(m_B, m_C) \leq g_B^* \leq \max(m_A, m_C) \) holds in any pure strategy equilibrium.

Thus, the mere fact that \( m_A > m_B \) implies that, in equilibrium, \( G_A \) (whose home market is \( A \)) will always locate to the right of \( G_B \) (whose home market is \( B \)), independent of the characteristics of market \( C \).

Although we know that in equilibrium, \( G_A \) will always locate to the right of \( G_B \), the equilibrium payoffs and the exact equilibrium configuration will depend on the relative position of the medians. As in the basic model, to limit the number of cases to consider, we make the following assumption in addition to Assumption 1:

**Assumption 2.** i) \( F_C \) is uniform so that \( m_C = \frac{1}{2} \) and \( F_C'' = 0 \)

ii) \( m_B < m_C = \frac{1}{2} < m_A \)

Given this Assumption it then follows from Lemma 3 that \( g_A^* < m_A \) and \( g_B^* > m_B \) must hold so that the payoff for the global players are given by

\[
\Pi_{G_A}(g_A, g_B) = s_A \cdot F_A(g_A) + s_C \cdot \left(1 - F_C\left(\frac{g_A + g_B}{2}\right)\right) \] (14)

and

\[
\Pi_{G_B}(g_A, g_B) = s_B \cdot \left(1 - F_B(g_B)\right) + s_C \cdot F_C\left(\frac{g_A + g_B}{2}\right) \] (15)

Thus, \( G_A \) serves all customers to the left of \( g_A \) in market \( A \) (and \( L_A \) chooses \( l_A^* = g_A^+ \) thereby serving all customers to the right of \( g_A \)). In market \( C \), \( G_A \) serves all customers to the right of \( g_A \) plus all customers to the left
between the middle of \( g_A \) and \( g_B \) (which is at \( \frac{g_A + g_B}{2} \)) and \( g_A \), because \( G_A \) is the closest firm for these. An analogous interpretation holds for \( G_B \).

Again, our assumptions ensure interior solutions for the equilibrium location choices \( g_A^* \) and \( g_B^* \), which, when taking into account that \( F_C \) is uniform, are then implicitly given by the following first order conditions:

\[
\begin{align*}
  s_A \cdot F'_A(g_A) - \frac{s_C}{2} &= 0 \quad (16) \\
  -s_B \cdot F'_B(g_B) + \frac{s_C}{2} &= 0 \quad (17)
\end{align*}
\]

Summarizing, we have the following result:

**Proposition 4.** In the model with two global players and three markets, the subgame perfect equilibrium location choices are \( \{ g_A = g_A^*, g_B = g_B^*, l_A^* = g_A^+, l_B^* = g_B^- \} \), where \( g_A^* \) and \( g_B^* \) are implicitly given by Eqn. (16) and (17).

The result is illustrated in Figure 5. Invoking again the implicit function theorem, it is straightforward to show that \( g_A^* \) is increasing in \( s_A \) and decreasing in \( s_C \). Intuitively, a higher \( s_C \), say, makes market C more attractive for \( G_A \), so that he becomes more willing to sacrifice customers in market A to attract more customers in market C. Analogously, \( g_B^* \) is decreasing in \( s_B \).
and increasing in $s_C$. Finally, note that since $g^*_A$ is decreasing in $s_C$ and $g^*_B$ is increasing in $s_C$, the difference $g^*_A - g^*_B$ is decreasing in $s_C$. That is, the less important is market $C$, the more do $G_A$ and $G_B$ prefer to locate near the median of their respective “home” market.

In terms of diversity, note from Figure 5 that we have clustering in markets $A$ and $B$, but not in market $C$. This means that preference costs in markets $A$ and $B$ are strictly higher compared to the benchmark (this follows from Lemma 2). However in market $C$, since there is no clustering, preference costs can be lower compared to the benchmark. If this effect is sufficiently strong, then it can be easily shown that both global diversity and preference costs may decrease compared to the benchmark case.

5.3 Advertising and price competition in a two-sided market

So far, we have assumed that firms compete for customers via locations but not in prices. Arguing along the lines of Gabszewicz, Laussel, and Sonnac (2001), we now show that a model where firms (like, e.g., newspapers, broadcasting companies, internet platforms or political parties) compete for customers by location choice only can be viewed as a reduced form of a two-sided market model in which firms first compete for customers and then for advertisers. The basic idea is that if customers are sensitive to price differences and if advertising revenue is sufficiently important, then price competition for customers is degenerate, so that the equilibrium prices charged to consumers are zero and competition takes place via locations only. Because we want to allow for more general distributions of customers, we slightly extend Gabszewicz, Laussel, and Sonnac (2001) to general distribution functions.

In their model, two newspapers $i = 1, 2$ compete on a single market,\(^\text{10}\)

\(^{10}\)Once the case with a single market has been laid out, the extension to the multi-market case is immediate.
where customers are distributed between $[0,1]$ according to a distribution function $F(\cdot)$. Note that it is immaterial whether firms are local and global players. For our analysis to be correct, it is only required that global firms can in general set different prices in different markets. Firms first choose a location $x_i \in [0,1]$, then set a price $p_i \geq 0$ when competing for readers, and finally set a fee $\sigma_i \geq 0$ for advertisers. Since all else equal, advertisers prefer newspapers with more readers, this is a two-sided market with externalities, as, e.g., analyzed by Armstrong (2004).

An advertiser’s net valuations when placing an add in newspaper $i$ with $d_i$ readers and fee $\sigma_i$ is $\theta d_i - \sigma_i$, where $\theta$ measures an advertiser’s gross valuation and is uniformly distributed on $[0,1]$ with density $4\gamma$. If an advertiser $\theta$ places an add in both newspapers, his net utility is $\theta d_1 - \sigma_2 + \theta d_2 - \sigma_2$. It follows that type $\tilde{\theta}_i := \sigma_i/d_i$ is indifferent between advertising and not advertising in newspaper $i$, while all $\theta > \tilde{\theta}_i$ strictly prefer to do so. Thus, the demand for advertisement which $i$ faces is $4\gamma(1 - \frac{\sigma_i}{d_i})$, so that the fee which maximizes advertising revenue is $\sigma_i^* = d_i/2$ and maximal revenue is $\gamma d_i$.

Next, assume for simplicity that readers do not care about the number of adds in a newspaper and, without loss of generality, assume $x_1 \leq x_2$. Moreover, let the transportation cost be quadratic in the distance travelled, so that reader $\tilde{x} \in [x_1, x_2]$ is indifferent between buying from 1 and 2 if and only if $-t(\tilde{x} - x_1)^2 - p_1 = -t(\tilde{x} - x_2)^2 - p_2$, where $t \geq 0$ measures the ease with which readers switch from one newspaper to another. Thus, if $\tilde{x} \in [x_1, x_2]$ exists, $\tilde{x} = \frac{x_1 + x_2}{2} + \frac{p_2 - p_1}{2t(x_2 - x_1)}$ is indifferent, so that newspapers 1 and 2 face demands $d_1(p_1, p_2) = F(\tilde{x})$ and $d_2(p_1, p_2) = 1 - F(\tilde{x})$, respectively. Otherwise, demand for one firm is zero and one for the other one. Given
locations $x_1, x_2$, $i$’s profit when setting prices thus is

$$\Pi_i(p_1, p_2) = (p_i + \gamma)d_i(p_1, p_2).$$

(18)

Rather than characterizing the price equilibria for all conceivable distribution functions, parameter configurations and locations, we now derive a simple sufficient condition such that the equilibrium prices are zero for any location choices. From the corresponding first order condition for Eqn. (18), such a condition can be derived as

$$\frac{\gamma}{2t} \geq \frac{1}{\min_x f(x)}.$$  

(19)

Under standard regularity assumptions on $F(\cdot)$ such as full support, the right hand side is finite, and the condition is always satisfied for $\gamma$ sufficiently large or $t$ sufficiently small. Thus, our modelling approach can be viewed as a reduced form of a game in which the price paid by customers is not the primary source of revenue. Indeed, this seems to be a realistic feature of many media markets.

6 Conclusion

To the best of our knowledge, this paper is the first to analyze location games in which a subset of players compete on several markets with the same product, a feature frequently observed in reality. We have shown how the equilibria of such games differ from the standard Hotelling game and that global players tend to induce lower degrees of diversity across markets. If one accepts the common view that one aspect of globalization is that it has lead to more market integration (e.g., through international expansion strategies or mergers), our results seem much in line with the frequently observed (and lamented) trend towards more product homogeneity in markets such as the media industry, for which our model seems most appropriate.
We would now like to discuss some of our assumptions and suggest directions in which the analysis could be further extended. As mentioned in the introduction, the products of global players are sometimes not fully identical throughout markets, but some product differentiation is achieved through so-called “local windows”. For example, newspapers often have local sections where mainly local topics are covered. Such local windows can easily be incorporated in our model without affecting qualitatively our results as long as at least part of the product remains “global”.

This brings us to the question why there are global players in the first place. That is, why does the New York Times not increase the size of its local windows so that in fact, it would publish completely different newspapers in New York and Los Angeles? One might think of the basic model with one global player (see section 3.2) as arising from the benchmark case through a merger of two local firms. It can then be easily seen that the global player would be less profitable than two separate local firms: Local firms earn $\frac{1}{2}s_k$ in each market, so that an owner who runs both firms separately in each market would earn $\frac{1}{2}s_A + \frac{1}{2}s_B$, while when running them as a global player, he would only earn $s_A \cdot F_A(g^*) + s_B \cdot (1 - F_B(g^*))$ which is strictly less as local competitors now capture more than half of each market. Thus, the emergence of global players cannot be explained endogenously within our model and other reasons for this phenomenon such as returns to scale might be the key factors (see Anderson and de Palma (2000)).

An interesting question to be addressed in future work arises in areas where a government has the power to define markets, which is, e.g., the case when issuing radio or TV licences for different regions such as metropolitan areas. Assume that metropolitan areas differ with respect to consumer tastes and market size and that, as seems reasonable, the objective of the
government is to minimize consumer preference costs. Then, should markets be defined narrowly or broadly, i.e., should a license be given for each metropolitan area alone or should several areas be bundled into a single license? And if bundling is optimal, should one bundle similar or rather different markets into one license? The answer obviously depends on the number and the scope of applicants, which in turn depend on the scope and number of licenses, giving rise to a new game. In this game, government and applicants for a license anticipate that players with a license will choose locations along the lines of the present paper, but also the number and scope of applicants for a license depends on the number and scope of licenses chosen by government.

A further extension aims at providing empirical support for the predictions derived in the model. For example, we would expect the content of the New York Times to respond to significant changes in its competitive environment such as its own expanding into a new market or the advent of new, strong competitors in one or several of its relevant markets. Depending on the political orientation of its new competitors, one reasonable measure would be the number of articles published on more liberal topics such as health or family issues as compared to more conservative topics such as defense or crime. For that purpose, we have access to an article database containing more than 36,000 articles published in the New York Times since 1946 with detailed information, for example, on the topic of the article and whether its content is of local or broader interest.
Appendix

A Proof of Lemma 1

If there were no other markets to consider for the global player then we know from Proposition 1 that he would locate at \( m_k \) in equilibrium. Thus, he is willing to depart from this location only if this increases his profit in market \(-k\). However, given the best responses of his local competitors, he can increase his profit in market \(-k\) only if he moves towards the median of \( F_{-k} \).

B Proof of Lemma 2

It is instructive to begin with part ii): Given that both firms cluster at some point \( y \in [0, 1] \), preference costs in market \( k \) are given by

\[
LPC_k(y) = \int_0^y (y - x)F'_k(x)dx + \int_y^1 (x - y)F'_k(x)dx
\]

which has to be minimized with respect to \( y \). The first order condition is given by

\[
\int_0^y F'_k(x)dx + (y - y)F'_k(y) = \int_y^1 F'_k(x)dx + (y - y)F'_k(y) \iff \\
\int_0^y F'_k(x)dx = \int_y^1 F'_k(x)dx
\]

and clearly, the solution is at \( y = m_k \) where both integrals are equal to \( \frac{1}{2} \).

Moreover, the second order condition is satisfied as

\[
\frac{\partial^2}{\partial y^2} PC_k(\cdot) = F''_k(y) + F''_k(y) > 0
\]

so that the objective function is strictly convex for all \( y \in (0, 1) \) and the solution is unique.

Part i): Assume to the contrary that total preference costs are minimized if both locate at the median position, i.e., if \( y_1 = y_2 = m_k \). But because the
distribution is non-degenerate, this cannot be a minimum. To see this, let only firm 1 change his position to either \( y^+ \) or \( y^- \). Because all who previously went to \( y \) can still do so, all customers who want to can still do so. But because either the interval \([0, m_k]\) or the interval \((m_k, 1]\) contains a positive measure of customers, these will prefer to go to firm 1 if \( y_1 = y^- (y^+) \) because this firm is closer. Therefore, total preference costs cannot be minimized with clustering.

C Proof of Lemma 3

We first prove \( \min(m_B, m_C) \leq g^*_B, g^*_A \leq \max(m_A, m_C) \). Since the proof for \( \min(m_B, m_C) \leq g^*_B, g^*_A \) is analogous, it suffices to show that \( g^*_B, g^*_A \leq \max(m_A, m_C) \). For that purpose, assume first that \( g_B > \max(m_A, m_C) \). In this case, the best response of \( G_A \) is to locate to the left of \( g_B \) since by doing so his profits increase on both markets \( A \) and \( C \). If \( G_A \)'s best response is such that \( g_A > \max(m_A, m_C) \), \( G_B \)'s best response will be to locate at \( g_B < g_A \) since by doing so his profits increase both in market \( B \) and \( C \). Either \( g_B > \max(m_A, m_C) \), in which case the same chain of arguments begins anew, or \( g_B < \max(m_A, m_C) \). But if \( g_B \leq \max(m_A, m_C) \), the best response of \( G_A \) will be \( g_A \leq \max(m_A, m_C) \) because doing so increases his profits in both markets. Thus, we have shown that in any pure strategy equilibrium \( g^*_B, g^*_A \leq \max(m_A, m_C) \) and by analogy \( \min(m_B, m_C) \leq g^*_B, g^*_A \), and we are left to show \( g^*_B \leq g^*_A \), which we do by contradiction. There are three cases to be distinguished, given that \( m_B < m_A \) by Assumption 1.

**Case 1:** \( m_B < m_C < m_A \). If \( m_B < g_A < g_B < m_C < m_A \), \( G_A \) can increase profits in both markets relevant to him by moving to the right of \( g_B \). Similarly, when \( m_B < m_C < g_A < g_B < m_A \), \( G_B \) can increase his profits in both markets \( B \) and \( C \) by moving to the left of \( g_A \).
If \( m_B < g_A < m_C < g_B < m_A \), then \( G_A \) could increase his payoff in both markets by moving to the right, (weakly) beyond \( m_C \) and as closely as possible near \( G_B \). In the neighborhood of \( g_B \) at \( g_B^- \), he would not prefer to surpass \( G_B \) because this would give a discontinuous loss in market C and only a marginal gain in market A. But for any \( m_C \leq g_A < g_B < m_A \), we have already shown that \( G_B \) would strictly prefer to locate at \( g_B \leq g_A \) as he would gain in both relevant markets. Thus, \( g_B^* \leq g_A^* \) has been shown for case 1.

**Case 2:** \( m_C < m_B < m_A \). When \( m_C < g_A < g_B < m_B < m_A \), then \( G_A \) has an incentive to move as closely to the left of \( G_B \) as he gains in both relevant markets. But when \( g_A = g_B^- \), then \( G_B \) has an incentive to locate to the left of \( G_A \) at \( g_A^- \) since this would give him a discontinuous gain in market C and only a marginal loss in market B. When \( m_C < g_A < m_B < g_B < m_A \), then, again, \( G_A \) prefers to locate at \( g_A = g_B^- \). But then, \( G_B \) can gain in both markets by choosing \( g_B = g_A^- \). When \( m_C < m_B < g_A < g_B < m_A \), then \( G_B \) can gain in both markets by moving to the left of \( G_A \). Therefore, we have shown \( g_B^* \leq g_A^* \) for case 2.

**Case 3:** \( m_B < m_A < m_C \). Cases 2 and 3 being symmetric, the proof for case 3 is analogous to the one for case 2.
References


