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Market making oligopoly

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Abstract

This paper analyzes price competition between market makers who set costly capacity constraints before they intermediate between producers and consumers. The key finding is that the unique perfect equilibrium outcome is Cournot if capacity is costly and rationing efficient. This result is interesting for two main reasons: It generalizes Kreps and Scheinkman (1983) to an arbitrary number of market makers, and it contrasts with Stahl (1988) and the broader literature on market making, such as Gehrig (1993), Fingleton (1997) and Rust and Hall (2003), where due to the absence of capacity constraints on the input market the Bertrand paradox typically prevails.

Keywords: Market making, capacity constraints, price competition.

JEL-Classification: C72, D41, D43, L13.

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1 Introduction

In many industries, firms act as price setters both on the input and on the output market. For example, commercial banks set both deposit rates on the input market and loan and mortgage rates on the output market. Similarly, retailers like Wal-mart take neither input nor output prices as given, as witnessed by the much publicized complaints of farmers and Wal-mart's less efficient competitors alike.

Acting as arbitrageurs who buy and sell a good, these firms bring together supply and demand much in the same way as a Walrasian auctioneer does. This is why we call call them market makers, following the recent literature (see, e.g., Stahl, 1988; Gehrig, 1993; Spulber, 1996; Fingleton, 1997; Rust and Hall, 2003). Quite naturally, it is to be expected that a monopolistic market maker will set a lower bid price on the input market and a higher ask price on the output market than a Walrasian auctioneer would, and that it will net a positive profit. As the number of market makers increases, one would expect that these bid and ask prices come closer and closer to the Walrasian price, so that in the limit perfect competition amongst market makers coincides with perfect competition à la Walras.

However, as first observed by Stahl (1988), the transition from monopolistic to perfectly competitive market making is quite discontinuous. It is easy to understand why if one assumes that two competing firms first buy and then sell a homogenous good, observing in the interim stage the quantity bought by the other firm. Thus, in this setting market makers first bid for the capacity they face in the second stage when selling the good on the output market. In this case, price competition on the input market is a winner-takes-all competition for the monopoly profit accruing on the output market: Even if in equilibrium the two firms would share revenue on the output market, either firm fares strictly better by slightly overbidding the other firm's bid price on the input market, thereby taking over the whole market. This remains true as long as the opponent's bid price is below the zero-profit price, which in many settings coincides with the Walrasian price. As zero profits become an equilibrium condition, two market makers will often be enough to have perfect competition as the equilibrium outcome just like in Bertrand product market competition.

Obviously, this motivates to see whether devices to solve the Bertrand paradox in product market competition can be applied for market makers as well. Basically, this is what the present paper is about. The paper's focus is on capacity constraints. Borrowing from the seminal work of Kreps and Scheinkman

(1983), we address the question what happens if market makers have to set capacities prior to competing in prices on either the input or the output market. That is, in contrast to Stahl's paper, which analyzes market making when capacities are set in an *interim* stage, we analyze competition between market makers when capacities are set *ex ante*. There are two motivations for taking this approach. First, as a matter of fact, market makers need to have the capacity to trade so as to be able to compete with one another. Absent the capacities to trade, the Bertrand-Stahl threat to take over the whole input market by slightly overbidding the competitor's price is simply empty. Second, since most models of market making assume a homogenous good,[1] it seems a good advice to pursue the approach with capacity constraints, which naturally allows to maintain the homogenous good assumption.

Our main finding is that for a wide range of alternative settings, the unique equilibrium outcome is Cournot rather than Bertrand if capacities are costly. Thus, we generalize the key results of Kreps and Scheinkman (1983) to an arbitrary number of market makers. The intuition for this result is first that capacity constraints substantially soften price competition, as first observed by Edgeworth (1897): If all firms face sufficiently small capacity constraints, none of them can take over the whole market. Consequently, price competition will be less aggressive. Second, due to the assumption of efficient rationing, the residual demand and supply functions market makers face are the same as under Cournot competition. Consequently, on the equilibrium path Cournot behavior ensues.

These findings are interesting, and surprising, for two reasons. First, the paper shows that it makes a big difference for models of market making whether capacity constraints are set in an interim stage as in Stahl (1988) or ex ante. Second, the fact that the findings of Kreps and Scheinkman generalize to an arbitrary number of market makers is interesting news in itself. As pointed out, e.g., by Stahl (1988) and Yanelle (1989, 1996), models of market making may behave quite differently from the underlying oligopoly model. For example, if demand is inelastic at the Walrasian price in Stahl's model, then the equilibrium will be non-Walrasian. Therefore, the robustness we find is by no means a foregone conclusion.

Apart from the extensive literature on capacity constrained product market competition, the paper is closest related to Stahl (1988), Gehrig (1993), Fingleton (1997) and Rust and Hall (2003). The paper by Neeman and Vulkan

[1] The only exception we are aware of is Shevchenko (2004).

(2003), which analyzes how a given centralized market drives out trade through direct negotiations, is largely complementary to ours as we investigate how an intermediated market operates and under what conditions it approaches the ideal or centralized market they take as given. In a very recent paper, Ju et al. (2004) study capacity constrained price competition between market makers. However, they do not consider mixed strategies, so that our paper complements theirs. The main difference between our model as well as the models of Stahl, Gehrig, Fingleton and Rust and Hall and the model of Spulber (1996) is that in the former models, market makers set publicly observable prices, whereas the prices in the latter model are private information. Shevchenko (2004) analyzes competition between middlemen in a setting with heterogenous goods and preferences. Apart from that, the main difference is that we model price competition between market makers, whereas in his model terms of trade are determined through Nash bargaining. Similarly, in Rubinstein and Wolinksy (1987) all trade occurs at terms that result from bargaining. Moreover, in their setting a middleman's capacity is exogenously given, while in our model, the capacity of market makers is determined endogenously.[2]

The remainder of the paper is structured as follows. Section 2 introduces the basic model, and section 3 derives the equilibrium for this model. Section 4 extends the basic model and deals in turn with forward contracts, inelastic demand and simultaneous ask and bid price setting. Section 5 concludes.

2 The model

In this section, we develop the basic model. Except for the requirement that market makers have to set capacities prior to setting prices on either side of the market, the model is very similar to the one in Stahl (1988, section 3).[3] The assumptions are as follows.

There are n market makers, which are indexed as $i = 1, .., n$ and occasionally also called firms. A typical market maker is indexed as i, j or k. We take the number of market makers as exogenously given, though we argue at the end of

[2] As market making is by its very nature a two-sided activity, the paper relates also loosely to the recent literature on two-sided markets or platforms like, e.g., Caillaud and Jullien (2001), Rochet and Tirole (2002, 2004), Armstrong (2004), or McCabe and Snyder (2004). However, for platforms it is typically assumed that customers of one type, say, sellers exert an externality on the utility enjoyed by customers of the other type. In contrast, in this paper conditional on being served at a given price buyers and sellers do not care about the number of other sellers or buyers served by a given market maker.

[3] The time structure of this section also corresponds to the one analyzed by Yanelle (1996) in her Game 2. The time structure with forward contracts we analyze in section 4.1 is analogous to her Game 1.

section 3 that the equilibrium number of market makers can easily be derived as a function of the fix cost of entry in a game with an additional entry stage preceding this game. Each market maker maximizes its own profit. In stage 1, market makers simultaneously set physical capacity constraints, which are denoted as $\overline{q}_i$. The cost of capacity $\overline{q}_i$ is denoted as $C(\overline{q}_i)$, where $C' > 0$ and $C'' \geq 0$ is assumed.[4] A capacity constraint is such that trading quantity up to the constraint involves no direct costs, while beyond capacity trade is prohibitively costly.[5] Throughout we denote by $\overline{q}_i$ the capacity of market maker i and by $\overline{q}_{-i}$ the aggregate capacity of all others than i, and aggregate capacity is denoted as $\overline{Q}$, so that by definition $\overline{Q} \equiv \overline{q}_i + \overline{q}_{-i}$. In stage 2, market makers simultaneously set bid prices b_i on the so called input market, and in stage 3, they simultaneously set ask prices a_i on the output market. All previous actions are assumed to be observed, and in case rationing occurs, the efficient rationing rule applies. Quantity of i and aggregate quantity of all others than i are denoted as q_i and q_{-i}, respectively, and aggregate quantity is denoted as $Q \equiv q_i + q_{-i}$. We will make clear where necessary what quantity (stock, quantity sold or quantity demanded) is meant by q_i, q_{-i} or Q.

Let $A(Q)$ denote the inverse demand function, which depicts the market clearing ask price $A(.)$ as a function of aggregate quantity demanded Q, and consider Figure 1 for an illustration of the basic assumptions. The inverse supply function is denoted as $B(Q)$, where $B(Q)$ is the market clearing bid price for aggregate quantity supplied Q. Let $D(a) \equiv A^{-1}(Q)$ and $S(b) \equiv B^{-1}(Q)$, respectively, denote the demand and supply function. Both functions represent the behavior of perfectly competitive agents. As usual, we assume $A' < 0$. Moreover, we assume $0 \leq B(0) < A(0) < \infty$, $B' > 0$, $A(0) - B(0) > C'(0)$ and that the Walrasian quantity Q^W, given by $A(Q^W) = B(Q^W)$, is less than infinity. Furthermore, it is assumed that the ask price elasticity of demand, denoted as $\varepsilon_a(Q)$, does not exceed minus one, i.e., $\varepsilon_a(Q) \leq -1$ for any $Q \leq Q^W$.[6] We say that demand is (price) elastic whenever $\varepsilon_a \leq -1$. Also, we assume

[4] To be precise, there are different types of capacity used by market makers. On the one hand, they need to have the capacity to store, transport and sell the good in order to be able to compete on the output market. On the other hand, they must also have the capacity to buy the good, residing, e.g., in the number of clerks or salesmen employed. In general, these different kinds of capacities may involve different costs. For our analysis to be exactly correct, it is required that market makers do not set different capacities at different levels, i.e., they do not set, say, a capacity to sell that exceeds their capacity to buy.

[5] The assumption of prohibitive production cost beyond capacity is quite standard in the literature on capacity constrained product market competition. An exception is Boccard and Wauthy (2000) who consider the possibility that the cost of production beyond capacity may be less than prohibitively large, though it is still larger than below capacity.

[6] A sufficient condition for this is $D''a + D' - \frac{(D')^2}{D} \leq 0$.

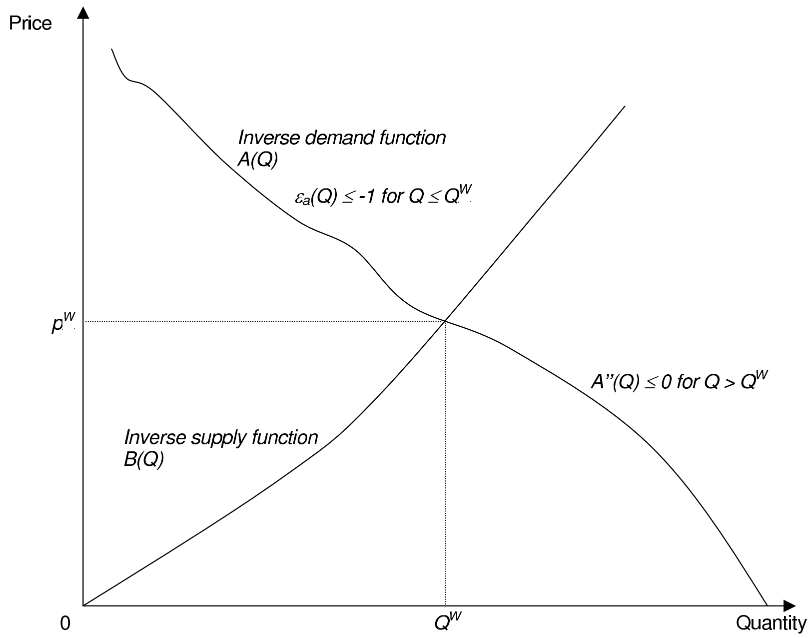

Figure 1: The basic setting.

$B'' \geq A''$ for any $Q \leq Q^W$. This last assumption makes sure that the spread function $Z(Q)$, defined as $Z(Q) \equiv A(Q) - B(Q)$, is weakly concave. Because $A' < 0$ and $B' \geq 0$, we also have $Z' < 0$. For simplicity, we assume $A'' \leq 0$ for $Q > Q^W$, which will allow us to directly apply results of Kreps and Scheinkman (1983). Note that the above assumptions imply that there is a quantity $\tilde{Q}$ such that $A(\tilde{Q}) = 0$. For $Q \geq \tilde{Q}$, we let $A(Q) = 0$. So as to distinguish the market clearing prices given by the functions $A(.)$ and $B(.)$ from prices set by market makers, the latter are denoted by small letters and a subscript, like a_i or b_i, and we will occasionally denote the prices of all firms other than i as a_{-i} and b_{-i}.

The rationales for these assumptions are as follows. Concavity of $Z(.)$ turns out be very helpful. It is less restrictive than assuming that $A(.)$ is concave, which is often assumed in models of product market competition. The assumption that demand is price elastic for any quantity not exceeding the Walrasian one makes sure that setting market clearing prices is a subgame perfect strategy in any equilibrium. Though it is satisfied in many applications (e.g., Gehrig, 1993; Fingleton, 1997; Rust and Hall, 2003) and maintained in large parts of Stahl (1988), relaxing this assumption seems very desirable. As we argue in section 4.2, this does not seem impossible, but for the time structure outlined above, it involves some technicalities that have not been solved yet.

With respect to capacity constraints, a key simplifying assumption is of course that trade beyond capacity is possible only at prohibitive costs. At first glance, this assumption may seem very restrictive. After all, a firm whose capacity constraint is binding might rent idle capacity from another firm. However, this raises the question whether a firm, say i, can rent additional capacity from a competitor when its own capacity is binding. For simplicity, consider the case of product market competition, where i's capacity constraint is binding if, e.g., all firms set the market clearing price. In this case, there clearly is no possibility to rent idle capacity from another firm. Alternatively, i's constraint can be binding if it sets a lower price than one of its competitors with idle capacity. Under efficient rationing, selling one unit of its idle capacity to the low priced firm will reduce the residual demand for this firm by one unit.[7] The maximal willingness to pay of i will be given by its price, which is lower than that of the firm with idle capacity. Therefore, there are no gains from trade for the two firms. What therefore is required is merely that aggregate capacity is given, which is far less restrictive.[8]

The assumption of efficient rationing follows the approach taken in the largest part of the literature.[9] As noted by Davidson and Deneckere (1986), the assumption is not without consequences in the sense that for alternative rationing schemes like, say, proportional rationing, equilibrium behavior is likely to be more aggressive than Cournot. However, the fact that the equilibrium behavior in models of capacity constrained price competition à la Kreps and Scheinkman (1983) or Levitan and Shubik (1972) is *less* competitive than Bertrand is nowhere put into question. To be sure, the main motivation for assuming efficient rationing is analytical ease. But there is also fair justification for it, namely that it is, at least qualitatively, innocuous.

As to timing, the crucial assumption is that capacity can be observed. In particular, it cannot be increased before price competition starts without having the competitors take notice. Whether this assumption is realistic depends of course on the application. It is arguably a good approximation if capacity takes the form of sale space or number and size of branches as in retail trade. It is certainly less accurate if the binding constraint is given by computer capacity

[7] The same holds for proportional rationing whenever aggregate capacity exceeds the monopoly quantity. If aggregate capacity is smaller, firms set the market clearing price in equilibrium.

[8] Nevertheless, we have to assume that capacity cannot be resold among firms (or market makers) because otherwise firms could act as a cartel.

[9] See, e.g., Levitan and Shubik (1972), Kreps and Scheinkman (1983), Osborne and Pitchik (1986), Deneckere and Kovenock (1992, 1996) and Boccard and Wauthy (2000).

like, e.g., for providers of internet platforms.

The structure of the basic model is most appropriate when market makers are retailers or shops. These are capacity constrained and do not sell forward contracts but rather must have the goods in stock if they want to be able to sell. Hence, the acquisition of stocks precedes selling. In other instances, such as wholesale trade, forward contracts are frequently used. In section 4 we extend the model to forward contracts, which corresponds to a reversion of the input and the output market stages. The informational assumption that all previous actions are observed is partly made for convenience and can be relaxed. For example, if the prices in stage 2 are observed, then the quantities competitors have in stock (or in the presence of forward contracts, the quantities they are obliged to buy) can be inferred from the observation of capacities and prices.

3 Equilibrium analysis

We proceed as follows. The game outlined in section 2 is a dynamic game with complete information. Hence, it can be solved using backward induction. Because concepts from Cournot competition are crucial for the analysis that follows, we first define Cournot capacities and derive the Cournot outcome for our game. Then we solve for the equilibrium of each stage in turn, beginning with stage 3.

3.1 Preliminary: Cournot competition

As Cournot competition typically refers to competition on a product market organized by a Walrasian auctioneer, whereas we study competition between market makers, we have to make clear what we mean by Cournot competition and Cournot outcome in our setting. If we counterfactually assume that both on the input and on the output market a Walrasian auctioneer quotes market clearing ask and bid prices and every market maker names the quantity it wants to trade, taking as given the inverse supply and demand functions $B(.)$ and $A(.)$ and the quantities its competitors name, then the quantities traded in equilibrium are called Cournot equilibrium quantities. That is, as a Cournot competitor i maximizes its profit by choosing its optimal quantity q_i^* given the quantities of all others q_{-i} and the inverse supply and demand functions $B(Q)$ and $A(Q)$ and its cost function $C(q_i)$ with $C' > 0$ and $C'' \geq 0$. Let $\Pi_i(q_i, q_{-i})$ denote firm i's profit when setting quantity q_i. Then, the maximization problem for i is

$$\max_{q_i} \Pi_i(q_i, q_{-i}) = (A(q_i + q_{-i}) - B(q_i + q_{-i}))\, q_i - C(q_i)$$
$$= Z(q_i + q_{-i})q_i - C(q_i), \tag{1}$$

which yields the following first order condition

$$0 = Z'(q_i^* + q_{-i})q_i^* + Z(q_i^* + q_{-i}) - C'(q_i^*). \tag{2}$$

The solution is called i's best response or reaction function $r_c(q_{-i})$. It is implicitly defined as

$$r_c(q_{-i}) = \frac{Z(r_c(q_{-i}) + q_{-i}) - C'(r_c(q_{-i}))}{-Z'(r_c(q_{-i}) + q_{-i})}. \tag{3}$$

Because $Z(Q)$ has a negative slope and is weakly concave, the maximization problem (1) is a concave problem, so that the solution in (3) is the unique interior maximum.

Since the concept is repeatedly used, let us also define the Cournot best response function with zero costs of production or trade. Let $r(q_{-i})$ denote the Cournot best response function when marginal costs are zero. Then, i's profit is $Z(Q)q_i$ and its best response to its competitors supply of q_{-i} is implicitly given as

$$r(q_{-i}) = \frac{Z(r(q_{-i}) + q_{-i})}{-Z'(r(q_{-i}) + q_{-i})}. \tag{4}$$

The solution in (4) is the unique interior maximum. The corner solution with $r(q_{-i}) = 0$ arises only if q_{-i} is so large that $Z(q_{-i}) \leq 0$, i.e., if $q_{-i} \geq Q^W$. If we assume $C' = 0$, differentiate (2) with respect to q_{-i} and set the result equal to zero, we can solve for $r'(q_{-i})$ to get

$$r'(q_{-i}) = \frac{ZZ'' - (Z')^2}{-ZZ'' + 2(Z')^2}, \tag{5}$$

where we have dropped the argument of $Z(.)$. The property $r' < 0$ is readily established for any concave function Z, because the nominator is negative and the denominator is positive. Moreover, $r' > -1$. To see this, note that $-(-ZZ'' + 2(Z')^2) < ZZ'' - (Z')^2$. This implies also that $r(q_{-i}) + q_{-i}$ increases in q_{-i}, i.e., $\frac{d(r(q_{-i})+q_{-i})}{dq_{-i}} > 0$. Moreover, the fact that for $q_{-i} < Q^W, r_c(q_{-i}) < r(q_{-i})$ is also readily established, using $Z' < 0, Z'' \leq 0$ and $C' > 0$ to get a contradiction for $r_c(q_{-i}) \geq r(q_{-i})$.

Equilibrium quantities Individual firms' Cournot equilibrium quantities when trade is costly are given by the unique fix point of the equation $q^C = r_c((n-1)q^C)$. Aggregate Cournot quantity is denoted as $Q^C \equiv nq^C$, and we refer to the ask price $A(Q^C)$ and the bid price $B(Q^C)$ as Cournot (ask and bid) prices. For the case with zero marginal costs, equilibrium quantity q_Z is given by the fix point of the equation $q_Z = r((n-1)q_Z)$. Because $r_c(q) < r(q)$ for $q < Q^W$, $q^C < q_Z$ follows.

3.2 The output market subgame

Let Q denote the aggregate stock of market makers and assume that Q is observed. Recall that for $Q \leq Q^W$, the demand function is price elastic. In this case $a_i = A(Q)$ for all i is the unique Nash equilibrium. To see this, note first that prices $a_i' < A(Q)$ are strictly dominated by the market clearing ask price $a_i = A(Q)$ since by setting a_i', i sells the same quantity as it would by setting a_i but at a lower price. Therefore, the only equilibrium candidates are prices $a_i \geq A(Q)$. Suppose first that all firms other than i set $a_{-i} = A(Q)$ and let i contemplate deviation to some $a_i > A(Q)$. Because demand is elastic, increasing price by one percent will result in a reduction of demand by more than one percent. Therefore, the deviation will not pay, and hence, given $a_{-i} = A(Q)$, $a_i = A(Q)$ is a best response for all i.[10] Uniqueness follows once it is noted that for any other combination of ask prices with $a_i \geq A(Q)$, at least one player could strictly increase his profit by changing his price.

For $Q > Q^W$, there are two possibilities, the exact conditions for either one to materialize will be derived shortly. The first possibility is that individual quantities bought are such that the equilibrium is in pure strategies. In this case, $a_i = A(Q) \geq 0$ for all i. The second one is that the equilibrium is in mixed strategies. Let $s(x) \equiv \frac{A(s(x)+x)}{-A'(s(x)+x)}$ denote the Cournot best response of a firm with zero marginal costs when its competitors sell x and let k be one of the firms (perhaps the only one) with the largest quantity in stock. As Kreps and Scheinkman (1983) show, in the mixed strategy equilibrium of this game, firms randomize over prices no larger than $A(s(q_{-k}) + q_{-k})$. We will return to this result below, but for now we take it as granted. Note that for $q_{-k} > 0$,

[10] Note that this result holds both for proportional and efficient rationing. Under proportional rationing, the residual demand function for i when all others set a non-market clearing price a_{-i} is $D(a)\frac{D(a_{-i})-\overline{q}_{-i}}{D(a_{-i})}$ for $a > a_{-i}$. Obviously, the price elasticity of the residual demand function equals the elasticity of the demand function $D(a)$. Under the same conditions the residual demand function under efficient rationing is $D(a) - \overline{q}_{-i}$, the ask price elasticity of which is $D'(a)\frac{a}{D(a)-\overline{q}_{-i}}$, which is strictly smaller (i.e., greater in absolute terms) than the elasticity of the demand function $D(a)$.

$A(s(q_{-k}) + q_{-k}) < A(s(0)) \equiv a^M$, where a^M is the price a monopoly without costs would charge. That is, at a^M the price elasticity of demand equals minus one. Note also that $a^M \leq A(Q^W)$.

Given our observations of the behavior on the output market, we can now prove:

Lemma 1 *In any equilibrium, (i) aggregate quantity bought Q does not exceed Q^W and (ii) ask prices are market clearing, i.e., $a_i = A(Q)$ for all i.*

Proof: For aggregate stock $Q \leq Q^W$, the unique equilibrium outcome has just been shown to be $a_i = A(Q)$ for all i. Therefore, (ii) follows as soon as (i) is shown.

Part (i): Similar to the price setting behavior on the output market, on the input market bid prices exceeding the capacity clearing price $B(\overline{Q})$ are dominated by $b_i = B(\overline{Q})$ for all i. Therefore, in any equilibrium, for the aggregate quantity to exceed Q^W, the market maker who sets the lowest price on the input market while still buying a positive amount pays a bid price greater than $B(Q^W)$. However, for any aggregate stock $Q > Q^W$, the price any seller gets in the output market equilibrium will be less than $A(Q^W) \equiv B(Q^W)$. Either there is a pure strategy equilibrium with $a_i = A(Q) < B(Q^W)$ or the equilibrium will be in mixed strategies where the range of prices over which firms randomize will not exceed $a^M \leq B(Q^W)$. Thus, each market maker who trades a positive amount will make negative profit, which cannot be an equilibrium given the possibility to make zero profit (e.g., by setting $b_i = 0$).■

Equilibrium in Stahl's (1988) model: The case of elastic demand It is now straightforward to derive the equilibrium when capacity on the input market is not binding, which is the case analyzed by Stahl (1988). Assume that there are two firms. For any aggregate quantity not exceeding the Walrasian quantity, the equilibrium output price will be market clearing. Therefore, if both firms set the same price on the input market they would share revenue on the output market. However, either firm has an incentive to slightly overbid the competitor's bid price since this discontinuously increases its profits. As in Bertrand product market competition, the unique equilibrium with two firms has thus both firms quote the Walrasian price on the input and on the output market and net zero profits.

3.3 The input market subgame

We now turn to the analysis of the bid price setting or input market subgame. We first show that there is a unique Nash equilibrium in the bid price setting subgame if each market maker has a capacity no greater than the quantity given by its Cournot best response function for zero costs. In this equilibrium each i plays the pure strategy $b_i = B(\overline{Q})$. This establishes that given Cournot capacities, $b_i = B(\overline{Q})$ is a Nash equilibrium. Second, we show that there is another region of pure strategy equilibria in which capacity constraints are by and large irrelevant, and we characterize this region. Third, we determine the expected equilibrium revenue for the largest firm for those capacity combinations for which the equilibrium of the bid price setting subgame is in mixed strategies. In particular, we show that the crucial result of Kreps and Scheinkman (1983) and Boccard and Wauthy (2000, 2004), according to which the largest firm earns the Stackelberg follower profit in the mixed strategy region, carries over to the present model.[11]

3.3.1 Region I of pure strategy equilibria

For $Q \leq Q^W$, an obvious candidate for a pure strategy equilibrium is the market clearing bid price $B(\overline{Q})$. To see whether $b_i = B(\overline{Q})$ for all i is indeed an equilibrium, suppose that all firms other than i set $b_{-i} = B(\overline{Q})$, and consider whether or when deviation from $b_i = B(\overline{Q})$ pays for i. Bid prices above $B(\overline{Q})$ being strictly dominated, we only have to consider downward deviation. As i sets $b_i < B(\overline{Q})$, it faces a residual supply of $\max[S(b_i) - \overline{q}_{-i}, 0]$. Since all other market makers set a bid price not larger than $B(\overline{Q})$, it will be the case that $S(b_i) - \overline{q}_{-i} < \overline{q}_i$. Therefore, $\max[S(b_i) - \overline{q}_{-i}, 0]$ will be the quantity bought by i when underbidding its competitors. Also note that for $S(b_i) - \overline{q}_{-i} > 0$ and $b_{-i} \leq B(\overline{Q})$, aggregate quantity bought will just be $S(b_i)$. Since the unique equilibrium of the ask price setting game is to set $a_i = A(Q)$, the equilibrium price on the output market is a direct function of the smallest bid price for which residual supply is positive. If all others set $b_{-i} = B(\overline{Q})$, it is a function only of i's bid price. If $b_i > B(\overline{q}_{-i}) \Leftrightarrow S(b_i) > \overline{q}_{-i}$, then $A(Q) = A(S(b_i))$. Otherwise, $A(Q) = A(\overline{q}_{-j})$, but then the profit of i is zero independently of $A(.)$. Given that its profit is positive when setting $B(\overline{Q})$ whenever $\overline{Q} < Q^W$, the deviation $b_i \leq B(\overline{q}_{-i})$ will not pay. Therefore, we can concentrate on $b_i > B(\overline{q}_{-i})$. In this

[11] See also De Francesco (2003).

case, i's profit when deviating from $B(\overline{Q})$ is

$$\Pi_i(b_i, b_{-i}, \overline{q}_{-i}) = (A(S(b_i)) - b_i)\left(S(b_i) - \overline{q}_{-i}\right), \tag{6}$$

which is what i maximizes over b_i when optimally deviating. If we define $x \equiv S(b_i) - \overline{q}_{-i}$, we have $A(S(b_i)) = A(x + \overline{q}_{-i})$ and $b_i = B(x + \overline{q}_{-i})$. Therefore, maximizing $\Pi_i(b_i, b_{-i}, \overline{q}_{-i})$ over b_i is equivalent to maximizing

$$\Pi_i(x, \overline{q}_{-i}) = \left(A(x + \overline{q}_{-i}) - B(x + \overline{q}_{-i})\right) x = Z(x + \overline{q}_{-i})x \tag{7}$$

over x, whence it becomes clear that the optimal deviation over b_i is equivalent to choosing the optimal quantity under Cournot competition with zero costs. In other words, the optimal x will be such that $x = r(\overline{q}_{-i})$ implying that the optimal bid price b_i is equal to $B(r(\overline{q}_{-i}) + \overline{q}_{-i})$, the Cournot best response bid price. Note also that this price is the optimal price for a firm who is certain to be the lowest price bidder on the input market, which is a property that will be used below.

Having thus established that the optimal deviation of i is to set $b_i = B(r(\overline{q}_{-i}) + \overline{q}_{-i})$ when all others set higher prices, it is now straightforward to see when such deviation does not pay. Since prices above $B(\overline{Q})$ are dominated, it follows that whenever $B(r(\overline{q}_{-i}) + \overline{q}_{-i}) \geq B(\overline{Q})$, deviation does not pay. Because $B' > 0$, this implies that whenever $\overline{q}_i \leq r(\overline{q}_{-i})$, setting a price below the market clearing bid price does not pay for i. The intuition for this result is pretty clear. If firm i could, it would buy $r(\overline{q}_{-i})$, but because this is more than $\overline{q}_i$, it cannot buy that much. Therefore, it does not pay for i to set a price higher than $B(\overline{Q})$. Clearly, we therefore have an equilibrium where all firms set $B(\overline{Q})$ if for all firms i, $\overline{q}_i \leq r(\overline{q}_{-i})$.[12]

The argument establishing uniqueness is analogous to the one of the output market subgame. Bid prices above $B(\overline{Q})$ being strictly dominated, the only alternative candidates for an equilibrium are bid prices smaller than $B(\overline{Q})$. However, whenever a firm i sets a bid price $b_i < B(\overline{Q})$, at least one other firm, say j, will optimally set a price below $B(\overline{Q})$ and above b_i, so that firm i's profit would discontinuously increase by setting a slightly higher price than j does. Thus, there is no other equilibrium. These findings are summarized as follows:

Lemma 2 *For capacities $\overline{q}_i \leq r(\overline{q}_{-i})$ for all $i = 1, .., n$, there is a unique Nash equilibrium in the input market subgame, in which all market makers set the market clearing bid price $B(\overline{Q})$.*

[12] Note that this condition is exactly the same that has to hold in Kreps and Scheinkman (1983) for a pure strategy equilibrium (in their region I).

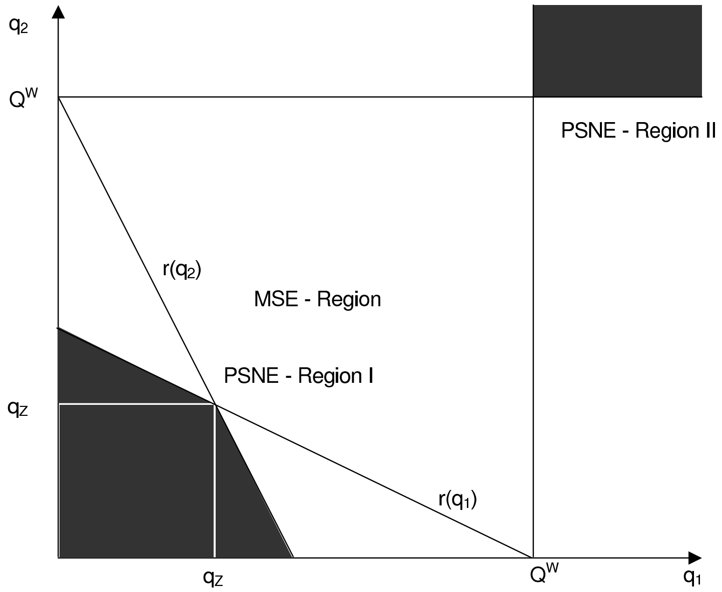

Figure 2: Cournot reaction functions and Cournot equilibrium.

Region I in Figure 2 depicts the region of pure strategy equilibria for the case of two firms and a linear spread function, where q_Z denotes the Cournot equilibrium quantity with zero marginal costs.[13]

A final note concerns the question for which market maker the constraint $\overline{q}_i \leq r(\overline{q}_{-i})$ becomes binding first. To answer this question, define $m \equiv \overline{Q} - \overline{q}_i - \overline{q}_j$. Then, the constraints for i and j are $\overline{q}_i \leq r(m + \overline{q}_j)$ and $\overline{q}_j \leq r(m + \overline{q}_i)$. Assume that initially $\overline{q}_i = \overline{q}_j < r(m + \overline{q}_j) = r(m + \overline{q}_i)$ and then let $\overline{q}_i$ increase, while $\overline{q}_j$ is kept fix. Since $\overline{q}_i$ increases by one while $r(m + \overline{q}_i)$ decreases by less than one in $\overline{q}_i$, it follows that the constraint becomes first binding for i, who is now the larger firm. Applying the argument for any two firms, it follows that if the constraint $\overline{q}_i \leq r(\overline{q}_{-i})$ is not violated for the largest firm, then it is satisfied for all other firms.

3.3.2 Region II of pure strategy equilibria

There is another region of capacity constraints for which equilibria are in pure strategies. The intuition is easily grasped if we assume that for two or more firms $\overline{q}_i \geq Q^W$. Since for these firms capacity constraints are not binding for any quantity $Q \leq Q^W$, we are back in the world of unconstrained Bertrand competition. If $n > 2$, there are multiple, but payoff equivalent equilibria: At

[13] In this figure and those that follow, linearity merely serves the purpose of simplification.

least two of the market makers with capacities greater than Q^W set their bid equal to $B(Q^W)$. The other firms can set any bid price not exceeding $B(Q^W)$, and any market maker buying a positive amount of quantity sets an ask price equal to $A(Q^W)$ in the final stage of the game, and all market makers earn zero profits. But it is easy to to see that $\overline{q}_i \geq Q^W$ for at least two firms is only a sufficient condition. The complete region II of pure strategy equilibria is illustrated in Figure 2, where the shaded areas are regions I and II of pure strategy Nash equilibria (PSNE) and the white area is the region of mixed strategy equilibria (MSE) of the bid price setting subgame. Formally, region II of pure strategy equilibria is given as follows:

Lemma 3 *If $\overline{q}_{-i} \geq Q^W$ for all i, there is always an equilibrium in which all firms play pure strategies. In this equilibrium, all firms that buy positive quantity set $B(Q^W)$. All firms make zero profits in any equilibrium.*

Proof: Given $\overline{q}_{-i} \geq Q^W$ for all i, if all firms other than i set $b_{-i} = B(Q^W)$, there is no way i can increase its profit by setting a price other than $B(Q^W)$. For $b_i \leq B(Q^W)$, i's profit is zero, while for $b_i > B(Q^W)$, i's profit is negative. Thus, $b_i = B(Q^W)$ for all i is an equilibrium.

Next, we show that the unique equilibrium outcome is that all firms make zero profits. Let b_k be the lowest bid price set by any of the $-i$ firms for which residual supply is positive, absent i's bidding. If $b_k < B(Q^W)$, the best response of i will be to set a price lower than $B(Q^W)$ but higher than b_k. (How much higher this price will be depends on the capacities and prices set by the other firms among i's competitors, but is not material.) Given i's best response, k's profit will discontinuously decrease. Since i's price is below $B(Q^W)$, k can increase its profit by slightly overbidding i's price. This race to the top does, obviously, stop only as the lowest bid price for which the residual supply absent i's price setting, b_k, equals $B(Q^W)$. Thus all firms that buy positive quantity must set $B(Q^W)$. Moreover, all of the firms that buy positive quantity make zero profit since the equilibrium ask price will be $A(Q^W) = B(Q^W)$. Trivially, firms that do not buy any quantity make zero profits. ■

Note that there may be multiple equilibria. A necessary condition for multiple equilibria to arise is that in addition to $\overline{q}_{-i} \geq Q^W$ for all i, $\overline{q}_{-i} - \overline{q}_j \geq Q^W$ holds for some j and i. In this case, j can set or randomize over any bid price $b \leq B(Q^W)$, provided the aggregate capacity of all firms other than j who set $B(Q^W)$ is at least Q^W. Because $\overline{q}_{-i} - \overline{q}_j \geq Q^W$, firm i cannot gain by deviating from $B(Q^W)$ if all other firms but j set $B(Q^W)$.

3.3.3 Region of mixed strategy equilibria

If $\overline{q}_{-j} < Q^W$ for at least one j and $\overline{q}_k > r(\overline{q}_{-k})$ for at least one k, there is no pure strategy equilibrium.[14] If $B(\overline{Q}) < B(Q^W)$, downward deviation from the market clearing bid price pays for k. If $B(\overline{Q}) \geq B(Q^W)$, the only candidate price at which quantity is traded in a pure strategy equilibrium is $B(Q^W)$. However, if all others set $B(Q^W)$, downward deviation pays for j since $\overline{q}_{-j} < Q^W$ implies that j's residual supply is positive for some $b_j < B(Q^W)$ and $b_{-j} = B(Q^W)$. When setting $b_j < B(Q^W)$, j buys therefore a positive quantity on which it earns a positive spread, while with $b_j = b_{-j} = B(Q^W)$, its profit is zero.

Determining the expected equilibrium payoffs in the mixed strategy region The existence of an equilibrium for our game is guaranteed by Dasgupta and Maskin (1986). The equilibrium involves non-degenerate mixed strategies. Though these mixed equilibrium strategies are hard to compute, it is possible to derive the expected equilibrium profit or revenue for the largest firm without completely characterizing these strategies. The expected equilibrium revenue is given in Lemma 4, which replicates the key finding of Kreps and Scheinkman (1983) for market makers facing a concave spread function. It states that in the mixed strategy equilibrium, the largest firm earns in expectation no more than it would have earned had it determined its capacity according to the Cournot best response function with zero costs.

Lemma 4 *Let i be one of the largest firms. In the mixed strategy equilibrium, the expected profit of any of the largest firms is equal to* $r(\overline{q}_{-i})Z(r(\overline{q}_{-i}) + \overline{q}_{-i})$.

Proof: The proof has three steps. First, it is shown that in equilibrium at most one firm sets the lowest bid price in the support over which firms randomize with positive probability. Therefore, there is a firm who is overbid with probability one when setting this price. In the second step, this fact is used to determine the expected equilibrium revenue of any such firm. Based on the indifference property of mixed strategy equilibria, the expected equilibrium revenue of such a firm equals the revenue it gets when setting the lowest price. Third, having determined this revenue, the firm that nets this revenue is determined.

Step 1: Let $\Phi_h(b)$ be the equilibrium distribution function of firm h, $h = 1, .., n$. Denote by $\underline{b}$ the lowest price in the support of any firm. That is,

[14] Note also that if $\overline{q}_{-j} < Q^W$ for at least one j and $\overline{q}_k > r(\overline{q}_{-k})$ for at least one k holds, then $\overline{q}_{-i} < Q^W$ and $\overline{q}_i > r(\overline{q}_{-i})$ holds also for i, where i is (one of) the largest firms(s).

$\underline{b} \equiv \sup_b\{b \mid \max_h \Phi_h(b) = 0\}$. Let j be one of the firms whose support includes $\underline{b}$. At most one firm will set this price with positive probability. To see this, note first that $\underline{b} < \min[B(\overline{Q}), B(Q^W)]$. Otherwise, we would have $\underline{b} \geq \min[B(\overline{Q}), B(Q^W)]$ implying that we are in a pure strategy Nash equilibrium since prices above $\min[B(\overline{Q}), B(Q^W)]$ are strictly dominated. But downward deviation from $\min[B(\overline{Q}), B(Q^W)]$ has been shown to pay for the largest firm, say k, because $\overline{q}_k > r(\overline{q}_{-k})$, implying that k must net more than $Z(\min[\overline{Q}, Q^W])\overline{q}_k$ in equilibrium. This implies then that j buys less than $\overline{q}_j$ when setting $\underline{b}$. Assume j sets $\underline{b}$ with positive probability. Then, if another firm set $\underline{b}$ with positive probability, j could strictly increase its expected profit by setting a slightly higher price. This leaves both aggregate quantity bought and thus the spread j gets almost unaffected, but it discontinuously increases the quantity traded by j and thus increases its expected profit.

Step 2: There is a firm who is overbid with probability one when setting $\underline{b}$. Either it sets $\underline{b}$ with positive probability. Then, no other firm sets $\underline{b}$ with positive probability, which implies that all other firms set higher prices with probability one. Or no firm sets $\underline{b}$ with positive probability. Then, obviously any firm whose support includes $\underline{b}$ will be overbid with probability one when setting $\underline{b}$. Therefore, $\underline{b}$ must maximize the profit for any such firm under the condition that this firm is overbid with probability one when setting $\underline{b}$. That is, for any firm, say j, who is overbid with probability one when setting $\underline{b}$, $\underline{b} = \arg\max_b (A(S(b)) - b)(S(b) - \overline{q}_{-j})$. Otherwise, j could not be indifferent between setting $\underline{b}$ and setting $\arg\max_b (A(S(b)) - b)(S(b) - \overline{q}_{-j})$, but would prefer the latter. As we saw above in Lemma 2, maximizing $(A(S(b)) - b)(S(b) - \overline{q}_{-j})$ over b is equivalent to maximizing $Z(r + \overline{q}_{-j})r$ over r, which yields the Cournot reaction function $r(\overline{q}_{-j})$, implying $\underline{b} = B(r(\overline{q}_{-j}) + \overline{q}_{-j})$. Moreover, by the indifference property of mixed strategy equilibria, firm j's expected equilibrium profit will be $R(\overline{q}_{-j}) \equiv Z(r(\overline{q}_{-j}) + \overline{q}_{-j})r(\overline{q}_{-j})$, which is the Stackelberg follower profit. Exactly like in Kreps and Scheinkman (1983) and Boccard and Wauthy (2000, 2004), there is thus a firm j earning $R(\overline{q}_{-j})$. The final thing to be shown is that it is (one of) the largest firm(s).

Step 3: Note that the problem is more complicated than in Kreps and Scheinkman because a firm's expected profit when setting a higher price than $\underline{b}$ will also depend on other firms' expected bid prices, since these influence aggregate quantity bought and thus the equilibrium ask price the firm gets. Therefore, no direct equivalent to their calculations in Lemma 5 (d) and (e) can be applied.

So as to see that j is one of the largest firms, note first that for any j to be a candidate for setting $\underline{b}$, $\overline{q}_j > r(\overline{q}_{-j})$ is required for otherwise $B(r(\overline{q}_{-j})+\overline{q}_{-j}) \geq B(\min[\overline{Q}, Q^W])$. Second, for any firm j who is among the candidates for earning revenue $R(\overline{q}_{-j})$, there is a bid price $\overline{b}_j$, $\underline{b} < \overline{b}_j < \min[B(\overline{Q}), B(Q^W)]$ such that j would never overbid $\overline{b}_j$ if all other firms set $\overline{b}_j$ with certainty. That is, for every firm j there is a "security level bid price" $\overline{b}_j$ implicitly defined by

$$Z(S(\overline{b}_j)) \equiv \frac{R(\overline{q}_{-j})}{\min[S(\overline{b}_j), \overline{q}_j]}. \tag{8}$$

Note that because in the mixed strategy region $R(\overline{q}_{-j}) > Z(\min[\overline{Q}, Q^W])\overline{q}_j$, it follows that $Z(S(\overline{b}_j)) > Z(\min[\overline{Q}, Q^W]) \Leftrightarrow \overline{b}_j < \min[B(\overline{Q}), B(Q^W)]$. Next, define $\overline{z}_i \equiv Z(S(\overline{b}_i))$ and bear in mind that $\overline{z}_i > \overline{z}_j \Leftrightarrow \overline{b}_i < \overline{b}_j$.

The crucial argument to be made is the following. So as to simplify the illustration, assume that $\overline{b}_i < \min_{k\neq i}\{\overline{b}_k\}$, so that i is the single firm with the lowest security level bid price. Then, i is the least aggressive firm and will earn $R(\overline{q}_{-i})$ and all other firms will earn more than $R(\overline{q}_{-k})$, $k \neq i$. The reason for this is that all $-i$ can set $\overline{b}_i$ (or a slightly higher price) and be sure not to be overbid by i. But since $\overline{b}_i < \overline{b}_k$, any $k \neq i$ earns more than $R(\overline{q}_{-k})$. So as to find out which firm i has the lowest $\overline{b}_i$, we have to determine the dependence of $\overline{z}_i$ on $\overline{q}_i$ and on $\overline{q}_{-i}$. Obviously, for $\overline{q}_i > S(\overline{b}_i)$, $\frac{\partial \overline{z}_i}{\partial \overline{q}_i} = 0$. Otherwise, we have

$$\frac{\partial \overline{z}_i}{\partial \overline{q}_i} = -\frac{R(\overline{q}_{-i})}{\overline{q}_i^2} = -\frac{r(\overline{q}_{-i})Z(r(\overline{q}_{-i})+\overline{q}_{-i})}{\overline{q}_i^2} < 0 \tag{9}$$

$$\frac{\partial \overline{z}_i}{\partial \overline{q}_{-i}} = \frac{R'(\overline{q}_{-i})}{\overline{q}_i} = \frac{r(\overline{q}_{-i})Z'(r(\overline{q}_{-i})+\overline{q}_{-i})}{\overline{q}_i} < 0. \tag{10}$$

The inequality in (9) follows immediately for $R(\overline{q}_{-i}) > 0$. As to the inequality in (10), drop arguments and note that

$$R'(\overline{q}_{-i}) = r'Z + rZ'(r'+1) = r'(Z + rZ') + rZ' = rZ' < 0, \tag{11}$$

where the last equality is due to the fact that by definition of a reaction function the term in parentheses is zero.

For any two firms with $\overline{q}_i = \overline{q}_h$, we have $\overline{z}_i = \overline{z}_h$. The question is thus whether $\overline{z}_i$ decreases more than $\overline{z}_h$ when $\overline{q}_i$ increases while $\overline{q}_h$ is kept constant.[15] Put differently, the crucial question is whether $\overline{z}_i$ decreases more in $\overline{q}_i$ than in $\overline{q}_{-i}$. But for $\overline{q}_i > r(\overline{q}_{-i})$ (which is a condition that must hold for one i for firms to be in the mixed strategy region in the first place), the inequality $\frac{\partial \overline{z}_i}{\partial \overline{q}_i} \geq \frac{\partial \overline{z}_i}{\partial \overline{q}_{-i}}$ holds. To see that this is true, note first that quite trivially for $r = 0$,

[15] For any $k \neq i$, $\frac{\partial \overline{q}_{-i}}{\partial \overline{q}_k} = 1$ implying $\frac{\partial \overline{z}_i}{\partial \overline{q}_k} = \frac{\partial \overline{z}_i}{\partial \overline{q}_{-i}}$.

$\frac{\partial \bar{z}_i}{\partial \bar{q}_i} \geq \frac{\partial \bar{z}_i}{\partial \bar{q}_{-i}}$ holds with equality. Second, note that $\bar{q}_i > r(\bar{q}_{-i})$ is equivalent to $0 > \bar{q}_i Z'\left(r(\bar{q}_{-i}) + \bar{q}_{-i}\right) + Z\left(r(\bar{q}_{-i}) + \bar{q}_{-i}\right) \Leftrightarrow \frac{Z'}{\bar{q}_i} < -\frac{Z}{\bar{q}_i^2}$. For $r > 0$ (which holds at least for one i because $\bar{q}_{-i} < Q^W$ for one i) this implies $-\frac{rZ}{\bar{q}_i^2} \equiv \frac{\partial \bar{z}_i}{\partial \bar{q}_i} > \frac{\partial \bar{z}_i}{\partial \bar{q}_{-i}} \equiv \frac{rZ'}{\bar{q}_i}$. Since $\frac{\partial \bar{z}_i}{\partial \bar{q}_i} > \frac{\partial \bar{z}_i}{\partial \bar{q}_{-i}}$ implies $\frac{\partial \bar{b}_i}{\partial \bar{q}_i} < \frac{\partial \bar{b}_i}{\partial \bar{q}_{-i}}$, it follows that $\bar{b}_i < \bar{b}_j$ if and only if $\bar{q}_i > \bar{q}_j$. Applying the argument for any i and j, it follows that $\bar{z}_i \in \max_k\{\bar{z}_k\}$ if and only if $\bar{q}_i \in \max_k\{\bar{q}_k\}$. Thus, i earns $R(\bar{q}_{-i})$ if and only if $\bar{q}_i \in \max_k\{\bar{q}_k\}$. ■

There is a fairly clear intuition for this result. Smaller firms are more aggressive in the bid price subgame because they have more to lose from low bid prices and, by the same token, more to win from high bid prices. To see this, note that when a small firm is overbid by a large firm, the small firm incurs the risk of not buying anything because, ultimately, the large firm may take the whole market. Note that this risk does not exist for the largest firm, say i, because its profit is positive even if it is overbid by small firms, which cannot take the entire market: Even at the lowest bid price $\underline{b} \equiv B(r(\bar{q}_{-i}) + \bar{q}_{-i})$, i buys a positive quantity, whereas there is no guarantee that any of its competitors gets to buy anything when being overbid while setting prices close to $\underline{b}$ since $\bar{q}_i$ might be larger than $S(\underline{b}) = r(\bar{q}_{-i}) + \bar{q}_{-i}$. As a consequence of this vulnerability from low prices, smaller firms are more aggressive, i.e., are willing to overbid higher prices than larger firms. As large firms incur the cost of high bid and low ask prices on larger quantities, they have a greater dislike for high bid prices. Thus, they are not willing to engage in high bid price wars. Consequently, all small firms can earn more than their Stackelberg follower profit. In the terminology of Fudenberg and Tirole (1984), large firms are thus fat cats while small firms are lean and hungry.

It should also be noted that the reasoning to determine expected equilibrium revenue for capacity constrained product market competition is completely analogous. Deneckere and Kovenock (1992, 1996) were the first to use ideas along these lines to derive expected equilibrium revenue for product market competition. There, of course, it is the firm with the *higher* "security level price" $\underline{p}_i$ that nets the Stackelberg follower profit. Moreover, in their setting, it is also quite easy to see that it is the lower bound of prices to be set because under efficient rationing on a product market a firm's profit depends on its competitor's expected price only because it determines the probability of being the lower or higher price bidder, but otherwise it is independent of the competitor's expected price.

3.4 Equilibrium of the full game

Lemma 2 establishes that a necessary condition for Cournot behavior to be replicated on the equilibrium path is met: Given Cournot capacities, market makers set Cournot prices. Lemma 4 says that the profit of the largest market maker, say i, in the mixed strategy region is equal to its Stackelberg follower profit, given the aggregate capacity of all other market makers. If all other market makers have set Cournot capacities, the Stackelberg follower profit is equal to the Cournot profit. Hence, if all others set Cournot capacities, then unilateral deviation does not pay even if capacity is costless, which proves the first part of

Proposition 1 *There exists a subgame perfect equilibrium outcome with Cournot actions. If capacity is costly, the Cournot outcome is the unique subgame perfect equilibrium outcome.*

Uniqueness If capacities are costless, then for $n \geq 3$ there is always a subgame perfect equilibrium in which the aggregate capacities of any firm's competitors are larger than the Walrasian quantity. On the equilibrium path, prices for which quantities are traded are equal to the Walrasian price. Equilibrium profits are zero for all firms. With $n = 2$, this is not an outcome of a subgame perfect equilibrium because any of the two firms could make positive profits by unilaterally deviating and setting a smaller capacity.[16] In addition, as pointed out by Kreps and Scheinkman (1983), there may also be subgame perfect equilibria in which one firm sets a large capacity and subsequently a mixed strategy equilibrium ensues in the price setting subgame.[17] However, if capacity is costly, then both types of these equilibria disappear. Setting aggregate capacity equal to the Walrasian quantity will no longer be an equilibrium outcome because revenue is zero while capacity is costly. The equilibrium with the mixed strategies on the equilibrium path breaks down because a firm is no longer indifferent between setting $r_c(.)$ and a larger capacity, where $r_c(.)$ is the Cournot best response function associated with the cost function $C(.)$ defined in (3). For any firm, its best response being uniquely given by $r_c(.)$, there are

[16] As noted by Kreps and Scheinkman (1983, p. 337) for product market competition.

[17] The basic reason for this is that with costless capacity there are multiple best responses for a firm whose competitor has a capacity $x < r(x)$. Either it sets capacity $r(x)$ or it sets a large capacity, e.g., larger than Q^W. The firm will be indifferent because capacity is costless and therefore it expects in both cases to earn the Stackelberg follower profit. If the firm responds with $r(x)$, then the Cournot equilibrium emerges, but if it sets a sufficiently large capacity, then the best response by the other firm will be to set $l^* \equiv \arg\max_l Z(r(l) + l)$. Thus, e.g., the capacities $(\overline{q}_1 = l^*, \overline{q}_2 = 2Q^W)$ will be part of a perfect equilibrium strategy profile.

no other subgame perfect equilibrium outcomes if capacities are costly.

Example Consider the following example. Assume that the inverse demand is $A(Q) = 1 - Q$ and the inverse supply is $B(Q) = Q$, which implies that the spread is $Z(Q) = 1 - 2Q$ and the Walrasian quantity is $Q^W = \frac{1}{2}$. Note that $\varepsilon_a(Q^W) = -1$ and $A'' = Z'' \leq 0$. Thus the above assumptions are satisfied. Assume also that the constant unit cost of capacity is $c \in (0, 1)$ and that the number of market makers is $n \geq 1$. Then it follows from Proposition 1 that the unique subgame perfect equilibrium outcome is $\overline{q}_i = \frac{1-c}{2}\frac{1}{n+1}$, $b_i = \frac{1-c}{2}\frac{n}{n+1}$ and $a_i = 1 - \frac{1-c}{2}\frac{n}{n+1}$ for all i.

3.5 Fix cost and entry

So far, the number n of active market makers has been taken as exogenous. However, assuming that market entry is possible at a positive fix cost f and that entry takes place prior to capacity setting, the number of active market makers can be determined endogenously. Since there is a negative relationship between the number of active market makers and profits, the relationship between the equilibrium number of active market makers, n^*, and fix cost f will also be negative.

Two comments are in order. First, if physical capacity is a necessary condition for firms to be able to engage in price competition, legal and other barriers to build capacity will have exactly the same effect as barriers to entry. To the extent that government policy affects the ease with which capacity is built, e.g., by shortening or lengthening the required legal procedures, policy has an effect on the size of fix costs of entry since there is little doubt that these legal measures will be used by incumbents to fight or at least delay entry by a competitor.[18]

Second, the result that lowering the fix cost of entry and thereby increasing the equilibrium number of market makers unambiguously increases welfare is due to the fact that there is no cost of switching from one market maker to another and that customers of market makers have measure zero. This is in contrast to Ellison and Fudenberg (2003), where equilibria with two competing market places can be inefficient because both buyers and sellers prefer markets where there are more agents of the opposite type. Specifically, in stage 1 of the game Ellison and Fudenberg analyze a finite number of buyers and sellers

[18] A recent example comes from Switzerland, where most legal objections to the new stores Aldi, a German retailer entering the Swiss market, wants to build are made by local competitors.

simultaneously decide to join one of two market places, and in stage 2 a market game ensues at the place they have joined, where the equilibrium price within a market arises, e.g., from competing market makers. The market makers in our model thus correspond to the competing market makers at a given market place in their model. Note that as the number of market makers becomes large, the zero measure assumption for individual producers or consumers may become harder to justify in our model since a consumer or a producer becomes relatively large compared to the size of a market maker. Consequently, fragmentation among market makers may give rise to a coordination problem (at least in the game with simultaneous moves analyzed below) similar to the one analyzed by Ellison and Fudenberg with two market places and a finite number of buyers and sellers.

4 Extensions

In this section, we deal with three extensions of the model, all of which show that the above results are fairly robust. We first introduce forward contracts and show that the basic results from the previous section carry over. Then we analyze the basic model of section 2 with the modification that demand is inelastic for some quantities smaller than the Walrasian one. Though a definite answer is beyond the scope of the present paper, we are able to show that a small deviation from the Cournot capacity does not pay if all others set the Cournot capacity. This suggests (but of course it does not prove) that Cournot is an equilibrium outcome even if demand is inelastic. Finally, we show that for the case where market makers simultaneously set ask and bid prices the key results from above are still valid. In all three cases, the key observation is that in the region with mixed strategy equilibria the largest firm nets the Stackelberg follower profit.

4.1 Forward contracts

As in Stahl (1988), introducing forward contracts amounts to a reversion of stages 2 and 3. The modified time structure with forward contracts is as follows. As before in stage 1, market makers simultaneously set capacities. In stage 2, they sell forward contracts to consumers, entitling each consumer with the right to get one unit of the good at the specified ask price. In stage 3, after forward contracts are sold, market makers buy the amount of the good on the input market necessary to fulfill the obligation arising from the volume of forward contracts sold. Following Stahl (1988, pp. 196-7), we assume that the penalty for default is sufficiently severe to deter any default in equilibrium. In addition, the legal proceedings are assumed to be lengthy (and time consuming) enough so that consumers involved in it cannot buy from another market maker while the proceeding lasts. This rules out that a market maker can gain additional consumers by forcing another one into default. Note that these "no default in equilibrium" assumptions imply that whatever volume of forward contracts is sold, market makers will set the market clearing bid price on the input market. As before, we assume that rationing on the output market is efficient.

Equilibrium in Stahl's (1988) model: The case of forward contracts Absent binding capacity constraints that are set ex ante, the equilibrium outcome in Stahl's model with forward is Walrasian regardless of whether demand is elastic or inelastic at the Walrasian price. Whenever a firm tries to sell forward contracts at a price above the Walrasian price, it will be undercut by a competitor. Consequently, the equilibrium condition that both (or all) firms net zero profits requires that all firms that trade a positive volume sell at the Walrasian price. Due the the severe default penalty, all firms fulfill their obligations to sell, and therefore buy at the Walrasian price.

Let us now return to our game, in which capacity constraints are set first. Since in stage 3 all market makers set market clearing bid prices, we can begin the analysis directly in stage 2. Given capacities $\overline{Q} < Q^W$, the profit for market maker i when setting $a_i \geq A(\overline{Q})$ while all other market makers set the market clearing ask price $a_{-i} = A(\overline{Q})$ is

$$\Pi_i(a_i, a_{-i}) = (a_i - B(D(a_i)))(D(a_i) - \overline{q}_{-i}). \tag{12}$$

When setting $a_i \geq A(\overline{Q})$, aggregate quantity demanded is $Q = D(a_i) \leq \overline{Q}$. Therefore, the market clearing bid price on the input market will be $B(Q) = B(D(a_i))$ implying that the spread i earns on its quantity traded is $a_i - B(D(a_i))$. Finally, since $a_i \geq a_{-i} = A(\overline{Q})$, quantity traded by i will be $r \equiv D(a_i) - \overline{q}_{-i} \leq \overline{q}_i$. Therefore, the maximization problem of i is equivalent to that of maximizing the profit of a market maker i who faces market clearing prices on both sides and trades quantity r when all others trade aggregate quantity $\overline{q}_{-i}$. That is,

$$\max_r \Pi_i(r, a_{-i}) = (A(r + \overline{q}_{-i}) - B(r + \overline{q}_{-i}))r \equiv Z(r + \overline{q}_{-i})r, \tag{13}$$

from where it becomes apparent that this problem is identical to the one previously studied. In particular, the region of pure strategy equilibria (i.e., the region where all market makers set the capacity clearing ask price $A(\overline{Q}))$ will be given by capacities $\overline{q}_i$ such that $\overline{q}_i \leq r(\overline{q}_{-i})$ for all i. Moreover, it follows that in the region of capacities with mixed strategies, the largest firm, say k, will have an expected profit of $Z(r(\overline{q}_{-k}) + \overline{q}_{-k})r(\overline{q}_{-k})$. Thus, we have established the following corollary of Proposition 1:

Corollary 1 *When market makers sell forward contracts, Cournot actions are a subgame perfect equilibrium outcome. If capacities are costly, the Cournot outcome is the unique subgame perfect equilibrium outcome.*

Note that it is not necessary to assume that demand is price elastic for any $Q \leq Q^W$ for this result to go through. Recall that in the model of the previous section, this assumption was used to ensure market clearing prices in the output market in the last stage. But with forward contracts and sufficiently severe punishment for default, bid prices will always be market clearing, be the equilibrium on the output market (i.e., in stage 2) in pure or in mixed strategies. Therefore, the results carry over even if demand becomes inelastic for some quantities traded smaller than the Walrasian one. This is in contrast to Stahl's findings, according to which with two market makers, zero costs and elastic demand the equilibrium outcome is Walrasian in the absence of capacity constraints with and without forward contracts. If demand is inelastic in his model, the equilibrium outcome with forward contracts is Walrasian for two market makers with zero costs. The contrast is therefore particularly stark with forward contracts: Without capacity constraints, price competition is Bertrand-like and yields the Walrasian equilibrium outcome, with costly capacity constraints, the equilibrium outcome is Cournot, regardless of whether demand is elastic or inelastic.

4.2 Inelastic demand

Another natural question is whether the above results without forward contracts carry over to a model where the demand function is inelastic for some quantity traded smaller than the Walrasian one. This question is also motivated by the observation of Stahl (1988) that without capacity constraints the unique subgame perfect equilibrium outcome is non-Walrasian and involves waste because the winner on the input market throws away some of the quantity bought, as will be shown shortly.

However, our model with inelastic demand gives rise to a region of capacity constraints where firms will randomize on the input market taking into account that they will, eventually, also randomize on the output market. As the equilibrium in such a model has not yet been derived, a definite and conclusive answer to the question cannot be given. Nevertheless, based on the previous analysis it is easy to show that small deviations from Cournot capacity do not pay if all other firms set Cournot capacities. This result strongly suggests that the previous results would carry over to a game where demand is inelastic.

We make the following assumptions. There are no forward contracts, and the timing is as in the basic model outlined in section 2. All previous actions are observed, and in case rationing occurs, the efficient rationing rule applies. As before, we assume $A' < 0$, $B(0) \geq 0$, $A(0) - B(0) > C'(0)$ and $B' > 0$. In addition, we now assume that $A''(Q) \leq 0$ for $Q \leq Q^W$ as well. These assumptions imply that the spread function $Z(Q) \equiv A(Q) - B(Q)$ has the properties $Z(Q) < A(Q), Z'(Q) < A'(Q)$ and $Z''(Q) \leq 0$. The Walrasian quantity Q^W is such that $A(Q^W) = B(Q^W)$. The main modification is that the ask price elasticity of demand at Q^W is now assumed to be greater than minus one, i.e., $\varepsilon_a(Q^W) \equiv \frac{A(Q^W)}{A'(Q^W)Q^W} > -1$.

Equilibrium in Stahl's (1988) model: The case of inelastic demand Assume that the winner on the input market in case of tied bids is determined by flipping a fair coin and that the winner takes over the whole supply.[19] As either bidder can gain by slightly overbidding its competitor as long as they expect positive profits, an equilibrium condition is that both make zero profits. Since the monopoly revenue on the output market exceeds the revenue at the Walrasian price, it follows that both must set a bid price above the Walrasian price on the input market. Consequently, quantity bought by the winner will be larger than Q^W, whereas the quantity sold on the output market will be the monopoly quantity, which is smaller than Q^W. Hence, there is waste in equilibrium,[20] as illustrated in Figure 3. Quantity bought by the winner on the input market is Q', which is such that $B(Q')Q' = \Pi^M \equiv a^M Q^M$, where Π^M is the profit of a monopoly seller with zero costs who sells Q^M at the price a^M. Since $\varepsilon_a(Q^W) > -1 \Leftrightarrow \Pi^M > A(Q^W)Q^W$, we have $B(Q') > B(Q^W)$ and $Q' > Q^W > Q^M$, so that there is waste in equilibrium of the size $Q' - Q^M > 0$.

[19] Without this somewhat peculiar tie-breaking rule, the game has no equilibrium; see Stahl (1988, p.195).

[20] Were a rigorous environmental regulation in place that forbid and effectively deterred waste, then the equilibrium outcome would be Walrasian even with inelastic demand. Such a strict environmental regulation would have exactly the same effect as the severe default penalty in the presence of forward contracts.

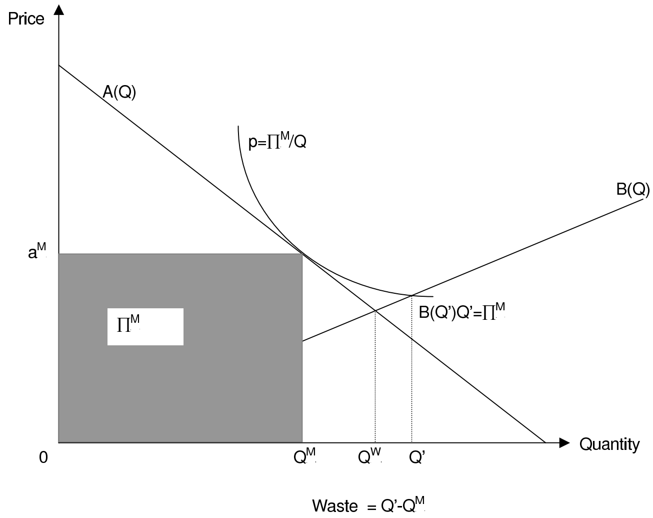

Figure 3: Stahl's (1988) model with inelastic demand and equilibrium waste.

Let us now return to our model and recall that the Cournot best response function for a firm facing the spread $Z(.)$ with zero marginal cost is $r(x) \equiv \frac{Z(r(x)+x)}{-Z'(r(x)+x)}$, where x is the quantity traded by all other firms and that $0 > r' > -1$. Note also that $r(0)$ is the quantity traded by a monopolistic market maker. Furthermore, for $x \geq Q^W$, $r(x) = 0$. As above, we let $s(x) \equiv \frac{A(s(x)+x)}{-A'(s(x)+x)}$ be the Cournot reaction function of a firm with zero marginal cost facing the demand $A(.)$ when all other firms supply quantity x. Since $A(.)$ is weakly concave, $0 > s' > -1$ follows. Note that $s(0) < Q^W$ is the quantity sold by a monopoly facing zero costs. We know that $s(x) > r(x)$ for any $x \leq Q^W$. To see this, assume to the contrary $s(x) \leq r(x)$, implying $\frac{A(s(x)+x)}{-A'(s(x)+x)} \leq \frac{Z(r(x)+x)}{-Z'(r(x)+x)}$. But since $Z(Q) < A(Q)$, $Z'(Q) < A'(Q) < 0$ and $Z'' \leq 0$, this yields a contradiction. Moreover, the Cournot equilibrium quantities for firms facing the spread function $Z(.)$ and the ask price function $A(.)$ with zero marginal costs, respectively, are given by $q_Z \equiv r((n-1)q_Z)$ and $q_A \equiv s((n-1)q_A)$. That these quantities are unique follows by noting that $x < r((n-1)x)$ and $x < s((n-1)x)$ for $x = 0$. Since both right-hand sides decrease in x while both left-hand sides increase in x, there is a unique fix point. Finally, $q_Z < q_A$. To see this, plug q_Z into $s(.)$. Since $s(x) > r(x)$ holds, $s((n-1)q_Z) > q_Z$ follows. Figure 4 provides an illustration. The functions $r(.)$ and $s(.)$ are the Cournot reaction functions and the quantities q_Z and q_A denote the corresponding equilibrium quantities for the spread function Z and the ask price function A.[21] Given $q_1 = q_Z$, equilibrium

[21] As before, linearity is assumed to simplify the illustration.

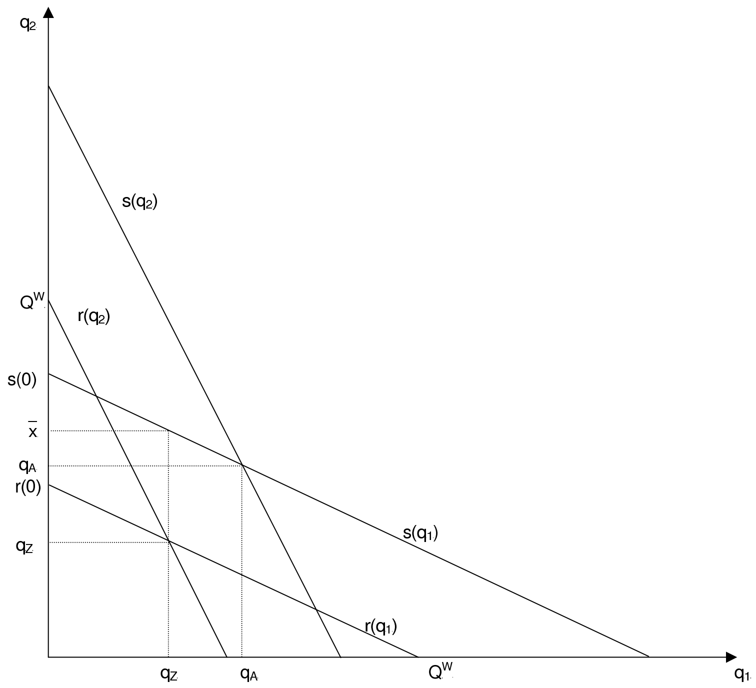

Figure 4: The model with inelastic demand.

ask prices will be market clearing for any $q_2 \leq \overline{x}$. Note also that $s(0) < Q^W$, which implies that demand is inelastic for some $Q < Q^W$.

These preliminaries are almost all we need to establish that small deviations from Cournot equilibrium capacities q_Z do not pay. Let every firm i set the Cournot capacity $\overline{q}_i = q_Z$, $i = 1, .., n$. Since $q_Z < q_A$, it follows that setting market clearing prices is an equilibrium outcome of the subsequent subgames. Next let firm j consider deviating when all $-j$ set q_Z. As usual, we only need to consider upward deviations, since for smaller capacities firms will set market clearing prices in the following subgames. A sufficient condition for the output market equilibrium to be in pure strategies is that $\overline{q}_i \leq s(\overline{q}_{-i})$ for all i. Recall that if the constraint is satisfied for the largest firm, then it is satisfied for any other. Define $\overline{x} \equiv s((n-1)q_Z + \overline{x})$. That is, $\overline{x}$ is such that if the largest firm has stocks not exceeding $\overline{x}$, while all others have stocks no greater than q_Z, then in the output market equilibrium all firms will set the market clearing price. Note that because $s' < 0$ and $q_Z < q_A$, $\overline{x} > q_A$. Therefore, if j deviates and sets a capacity $\overline{q}_j \in [q_Z, \overline{x}]$, the deviation will not pay. That is:

Proposition 2 *Let* $\overline{x} \equiv s((n-1)q_Z + \overline{x})$ *and assume* $\overline{q}_j \in [q_Z, \overline{x}]$ *while all* $-j$ *set* q_Z*. Then the expected equilibrium profit of* j *if capacity is costless is* $Z(r((n-1)q_Z) + (n-1)q_Z)r((n-1)q_Z) \leq Z(nq_Z)q_Z$*. If capacity is costly, then* j*'s expected profit when deviating to* $\overline{q}_j \leq \overline{x}$ *is strictly smaller.*

Proof: Given market clearing prices on the output market, it has been shown that deviation does not pay, since in the (possibly mixed strategy) equilibrium of the input market, the deviating firm will earn the Stackelberg follower profit. But for $\overline{q}_j \leq \overline{x}$ and $\overline{q}_k = q_Z$ for all $k \neq j$, $\overline{q}_i \leq s(\overline{q}_{-i})$ for all i. Therefore, the equilibrium ask price will be market clearing and deviation will not pay. If capacity is costly, costs of the excess capacity have to be borne, making expected equilibrium profits strictly smaller. ■

An example may be illustrative. Assume zero costs and let $A(Q) = 1 - Q$ and $B(Q) = \frac{Q}{2}$, so that $Z(Q) = 1 - \frac{3}{2}Q$, $Q^W = \frac{2}{3}$, $s(x) = \frac{1}{2} - \frac{1}{2}x$, $r(x) = \frac{1}{3} - \frac{1}{2}x$, $q_A = \frac{1}{n+1}$ and $q_Z = \frac{2}{3}\frac{1}{n+1}$. In this case, $\overline{x} = \frac{1}{3} - \frac{n-1}{n+1}\frac{2}{9}$. Among other things, this means that for $n \geq 7$, $(n-1)q_Z + \overline{x} \geq Q^W$. That is, for more than seven firms a deviation so large that aggregate capacity exceeds the Walrasian quantity does not pay. It thus seems very likely that any larger deviation would not pay either.

4.3 Simultaneous ask and bid price setting

To complete, we address the question what happens if market makers simultaneously set ask and bid prices. For that purpose, we assume again that the demand function is price elastic for any $Q \leq Q^W$, and the time structure is as follows. In stage 1, market makers simultaneously set capacity constraints $\overline{q}_i$, $i = 1, .., n$. In stage 2, having observed the capacity of all competitors, they simultaneously set a pair of bid and ask prices (a_i, b_i), $i = 1, .., n$. These prices are such that a market maker is obliged to buy up to its capacity and to pay b_i per unit supplied. The demand for a market maker can exceed its stock. In this case, the market maker is obliged to sell its stock at the ask price set. On both sides of the market, the efficient rationing rule applies.

We are now going to show that under these conditions, Cournot actions are a subgame perfect equilibrium outcome of the game. We first prove that the regions of pure strategy equilibria are identical with the game with sequential price setting.

Lemma 5 *If* $\overline{q}_i \leq r(\overline{q}_{-i})$ *for all* i*,* $b_i = B(\overline{Q})$ *and* $a_i = A(\overline{Q})$ *is the unique pure strategy equilibrium of the simultaneous price setting subgame. If* $\overline{q}_{-i} \geq Q^W$ *for* *all i, all market makers earn zero revenue. Those who trade positive quantities set $a_i = b_i = A(Q^W) = B(Q^W)$.*

Proof: If all $-i$ set market clearing prices, then the optimal deviation of i is $a_i = A(r(\overline{q}_{-i}) + \overline{q}_{-i})$ and $b_i = B(r(\overline{q}_{-i}) + \overline{q}_{-i})$. For $\overline{q}_i < r(\overline{q}_{-i})$, though, $A(r(\overline{q}_{-i})+\overline{q}_{-i}) < A(\overline{Q})$ and $B(r(\overline{q}_{-i})+\overline{q}_{-i}) > B(\overline{Q})$. These prices being dominated by setting the market clearing prices, deviation does not pay. Uniqueness of this equilibrium follows along the previous lines. If $\overline{q}_{-i} \geq Q^W$ for all i, capacity constraints are not binding for any market maker and the Bertrand argument applies. ■

However, if $\overline{q}_{-i} < Q^W$ for at least one i and $\overline{q}_j > r(\overline{q}_{-j})$ for at least one j, then there is no pure strategy equilibrium. If $B(\overline{Q}) < B(Q^W)$, deviation from setting $(A(\overline{Q}), B(\overline{Q}))$ pays at least for j. If $B(\overline{Q}) \geq B(Q^W)$, then deviation from setting $(A(Q^W), B(Q^W))$ pays at least for i. Since no other pair of prices can be a pure strategy equilibrium, it follows that there is no pure strategy equilibrium. Nevertheless, arguments by Dasgupta and Maskin (1986) can be used to show that an equilibrium exists. We now establish the property of this mixed strategy equilibrium that is key for our purpose:

Lemma 6 *Let i be (one of the largest) firms when capacities are such that there is no pure strategy equilibrium. Then i's expected revenue in the mixed strategy equilibrium is $R(\overline{q}_{-i}) \equiv r(\overline{q}_{-i})Z(r(\overline{q}_{-i}) + \overline{q}_{-i})$.*

Proof: The proof is based on a series of claims. Let $\overline{a}$ and $\underline{b}$ denote the upper and lower bound of the support of ask and bid prices in the mixed strategy equilibrium.

Claim 1: $\underline{b}$ and $\overline{a}$ can be set at most by one firm with positive probability in equilibrium. Proof: Suppose not. Then, either of these firms could strictly increase its expected profit by setting a slightly higher bid or slightly lower ask price with positive probability instead. This would increase its expected quantity traded while leaving the spread it earns largely unaffected, thereby increasing its expected profit.

Claim 2: $\overline{a} = A(S(\underline{b}))$. Proof: Since bid prices $b > B(\overline{Q})$ are dominated, aggregate quantity bought will be weakly larger than $S(\underline{b})$ implying that the market clearing ask price will be weakly smaller than $A(S(\underline{b}))$. Since by claim 1 there is a firm that is underbid with probability one when setting $\overline{a}$, $\overline{a}$ must maximize this firm's expected profit, conditional on being underbid with probability one on the output market. Since aggregate stock is weakly larger than $S(\underline{b})$ and because demand is price elastic, $\overline{a} \leq A(S(\underline{b}))$. Now consider a firm who is overbid with probability one when setting $\underline{b}$. In this case, its quantity bought can be zero in case the capacities of all other firms are larger than $S(\underline{b})$. In this case, though, the ask price it sets is immaterial. In the other case, its quantity bought is positive, and $A(S(\underline{b}))$ is the market clearing and profit maximizing for this firm. Thus, $\overline{a} = A(S(\underline{b}))$.

Claim 3: Let i be one of the firms who is overbid with probability one when setting $\underline{b}$. Then, $\underline{b} = B(r(\overline{q}_{-i}) + \overline{q}_{-i})$ and $\overline{a} = A(r(\overline{q}_{-i}) + \overline{q}_{-i})$. Proof: As soon as $\underline{b} = B(r(\overline{q}_{-i}) + \overline{q}_{-i})$ is shown, $\overline{a} = A(r(\overline{q}_{-i}) + \overline{q}_{-i})$ follows from claim 2. But $\underline{b} = B(r(\overline{q}_{-i}) + \overline{q}_{-i})$ is the profit maximizing bid price of firm i when it faces a market clearing ask price. Therefore, the claim is proved.

Claim 4: Firm i is (one of the) largest firm(s). Proof: Any smaller firm, say j, could set $\overline{b}_i$ (as defined in the proof of Lemma 4) and $\overline{a}_i \equiv A(S(\overline{b}_i))$, thereby making more profit than $R(\overline{q}_{-j})$, while i would make less profit by slightly overbidding $b_j = \overline{b}_i$ on the input and slightly underbidding $a_j = \overline{a}_i$ on the output market. ■

From Lemma 6 follows:

Proposition 3 *When ask and bid prices are set simultaneously, Cournot actions are a subgame perfect equilibrium outcome. If capacities are costly, the Cournot outcome is the unique subgame perfect equilibrium outcome.*

Proof: Given that all other market makers have set Cournot capacities, unilateral deviation does not pay. If a firm sets a smaller capacity, equilibrium prices will be market clearing, and given market clearing prices, the best response is setting the Cournot capacity. When increasing capacity, the price setting equilibrium will eventually be in mixed strategies, in which case the largest firm earns the Stackelberg follower profit. Uniqueness for costly capacity follows from the fact that for any firm, conditional on being the largest firm, its best response given capacity x of other firms is uniquely given by $r_c(x)$. ■

5 Conclusions

The question how and under what conditions a (neoclassical) market equilibrium emerges in a self-organized market is an important issue that has only recently become the focus of models with endogenous market making. A theme that prevails in these models is that two market makers are often enough for perfect competition if prices are public signals. The reason for this is that price competition is of the Bertrand-like winner-takes-all kind and thus yields the Walrasian outcome. The downside to this seemingly competitive result is, of course, that there will never be competition between market makers if there is a positive fix cost of entry for market making.

Given that the outcome is either that market making is perfectly competitive or monopolistic, it is natural to ask whether it is possible and plausible that there be a more gradual transition from monopoly to perfect competition. This is a question that bears importance beyond theory since policy prescriptions based on models of market making will clearly depend on whether or not this is the case. The present paper has addressed this question by emphasizing the role of capacity constraints market makers face. Apart from the fact that capacity constraints are obviously important for many market makers, our approach is motivated by the literature on capacity constraints in product market competition. This literature has shown that if capacity constraints are set and observed prior to price setting, price competition is substantially weakened. Though the literature dates back to Edgeworth, probably the most prominent and influential contribution has been made by Kreps and Scheinkman (1983). They showed that under fairly general assumptions on the demand function the equilibrium outcome is Cournot if rationing is efficient. Boccard and Wauthy (2000, 2004) have recently extended this model and shown that Cournot is the unique subgame perfect equilibrium for any number of firms. We have shown that these results carry over to market making under quite a wide range of settings. In particular, if capacities are costly and rationing is efficient the unique subgame perfect equilibrium outcome is Cournot if demand is price elastic, independent of whether bid prices are set first or ask prices are set first or bid and ask prices are set simultaneously.

To the best of our knowledge, this paper is the first to combine capacity constrained price competition and market making when capacities are set ex ante.[22] As the key results of Kreps and Scheinkman (1983) carry over to market making and to any number of firms, one conclusion from the present study is that their results are fairly robust in these respects. A more important lesson, though, is that capacity constraints matter for models of market making. Consequently, the Bertrand-Walras outcome should not be taken for granted for market making, but rather be seen as an exception that occurs only if costly capacity constraints are either not present or naturally larger than the Walrasian quantity.

Finally, capacity constraints and market making may be helpful in under-

[22] A qualification concerns the very recent paper by Ju et al. (2004), who introduce oligopolistic competition between market makers in a model à la Rust and Hall (2003). However, Ju et al. do not consider mixed strategies.

standing observed asymmetries in price adjustments. For instance, Peltzman (2000) finds that consumer prices typically increase after positive cost shocks for intermediate products, but do not decrease after negative cost shocks. As Peltzman notes, this finding is poorly explained by available theories. However, physical capacity constraints that are set prior to the realizations of the cost shocks provide a natural explanation: As a positive cost shock corresponds to an upward shift in the supply function market makers face, their capacity constraints eventually cease to be binding after such a cost increase. Therefore, in equilibrium aggregate quantity traded will be smaller than aggregate capacity, and thus the equilibrium output price will increase. On the other hand, a negative cost shock shifts the supply function downward. Consequently, the capacity constraints will become binding, so that aggregate quantity traded will not increase and, by the same logic, output price will not decrease. Hence, output prices adjust in an asymmetric way.

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