Inequality and Aggregate Savings in the Neoclassical Growth Model

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DISCUSSION PAPERS

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Abstract

Within the context of the neoclassical growth model I investigate the implications of (initial) endowment inequality when the rich have a higher marginal savings rate than the poor. More unequal societies grow faster in the transition process, and therefore exhibit a higher speed of convergence. Furthermore, there is divergence in consumption and lifetime wealth if the rich exhibit a higher intertemporal elasticity of substitution.

Unlike the Solow-Stiglitz model, the steady state is always unique although the consumption function is concave.

JEL classification: O40, D30, O10

Keywords: Marginal propensity to consume, income distribution, growth, concave consumption function.

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1 Introduction

Rich people save more. Will inequality necessarily continue to increase over time? Furthermore, how does this savings behavior affect the growth path? Tackling these questions is a priori a complex task: Inequality affects capital accumulation when the marginal propensities to consume (MPC) differ. However, inequality changes through the accumulation process because savings rates differ and factor prices change. It is the purpose of this paper to analyze this relationship between inequality and this savings behavior within the context of the neoclassical growth model with perfect and complete markets.

Theoretical arguments that consumption propensities decrease with wealth date back at least to Fisher (1930) and Keynes (1936).[1] Carroll and Kimball (1996) show that when agents are subject to uninsurable risks or liquidity constraints the consumption function is concave except in special cases. The empirical relevance of decreasing MPC is unquestioned. Looking at household data, it is a well-established fact that rich people save more - not only on average but also at the margin - out of wealth or permanent income; see the paper by Dynan, Skinner, and Zeldes (2004) and their references.[2]

What are the macroeconomic effects of decreasing MPC? To make my point as simple as possible, I perform my analysis under full certainty but where the different

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[1] Fisher and Keynes stated their argument in terms of the saving function. Note that a convex saving function is equivalent to a concave consumption function. I will state the relevant properties in terms of the consumption function throughout the paper.

[2] Perhaps unsurprisingly, the empirical picture is less clear on the aggregate level. The differences between studies are due to different data sets and different approaches to the endogeneity problems. Although Schmidt-Hebbel and Servén (2000) and Li and Zou (2004) could not find a robust effect of inequality on saving or consumption, Cook (1995) and Smith (2001) found a positive effect of inequality on (private) saving rates. The well-known studies of Barro (2000) and Forbes (2000) obtain a positive inequality growth relationship, at least for rich countries. This is also consistent with the view that inequality increases savings.

MPC arise due to non-homothetic preferences. To the best of my knowledge there is so far no study of inequality and growth under the conditions that consumers behave optimally and the resulting consumption function is concave. Stiglitz (1969) studies the dynamics of distribution when consumption is an exogenously given linear function of wealth. Chatterjee (1994), Caselli and Ventura (2001) and Bertola, Foellmi, and Zweimueller (2006, Chap. 3) study the same question in a Ramsey model where the linear consumption rule is a result of dynamic optimization. The impact of concave consumption functions on the evolution of inequality and growth was previously studied by Bourguignon (1981) and Schlicht (1975) in the context of the Solow-Stiglitz model with exogenous consumption propensities. Bourguignon (1981) shows that multiple steady states may emerge that can be Pareto-ranked. In this paper, a concave consumption function is the result of a dynamic optimization with intertemporally separable preferences. Surprisingly, the analysis is greatly simplified: the steady state equilibrium is unique and independent of the initial distribution. This result implies that more unequal societies must exhibit a higher speed of convergence because they grow faster in the transition process.

A related important strand of the literature includes papers by Becker (1980), Lucas and Stokey (1984) and, more recently, Sorger (2002). They study conditions when the long run distribution of wealth is non-degenerate in steady state. Furthermore, Bliss (2004) analyzes a general class of preferences to determine whether convergence occurs in the accumulation process. However, the focus of these papers is not to analyze the impact of inequality on growth.

The paper is structured as follows. Section 2 presents the model. Both the competitive equilibrium and the social planner's solution are analyzed. Section 3 then presents a numerical simulation. In the final section, Section 4, the differences from Bourguignon's model are discussed.

2 The model

2.1 Set-up

Preferences All consumers have the same intertemporal additive preferences and the same discount rate. The time horizon is infinite. Hence, the intertemporal utility function is given by

$$U_i = \int_0^\infty e^{-\rho t} u(c_i(t)) dt \tag{1}$$

where $c_i(t)$ denotes consumption of individual i at date t. We assume that (i) $u(\cdot)$ is twice continuously differentiable above some (subsistence) level $\bar{c} \geq 0$. (ii) We take the usual assumption that $u' > 0 > u''$, i.e. marginal utility is declining but the individual is non-satiated (at least over the relevant range). (iii) Further we assume $\lim_{c \to \bar{c}} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. Assumptions (i) and (ii) imply that the elasticity of substitution is positive for all $c > \bar{c}$:

$$-\frac{u'(c)}{u''(c)c} > 0 \text{ for } c > \bar{c} \geq 0.$$

Individual factor endowments We assume that - at date 0 - household i is endowed with l_i units of labor, which is assumed to be constant over time, and $k_i(0)$ units of capital. We restrict the inequality in the way that all households are viable, i.e., each household can afford to consume more than $\bar{c}$ in every period of time. We will come back to this assumption below. The number of households is constant. Hence total amount of labor L is also constant and we normalize it to one. Hence, the total amount of labor and capital in the economy is given by

$$K \equiv \int_{\mathcal{N}} k_i(t) dP_i$$

$$1 \equiv \int_{\mathcal{N}} l_i dP_i$$

where $\mathcal{N}$ denotes the set of families and dP_i the size of family i.

Technology and competitive factor rewards The inputs labor and capital are used to produce a homogenous output good Y which can be both used for consumption and investment. Production takes place with a standard neoclassical production function $F(\cdot, \cdot)$ with constant returns to scale and diminishing marginal products. The production function shall be twice continuously differentiable in its arguments. There is no technological progress, i.e., we focus on transitional dynamics only.[3]

$$Y(t) = F(K(t), 1) \equiv f(K(t))$$

The factors are rewarded their marginal products, hence the interest rate and the wage rate are given by

$$
\begin{aligned}
r(t) &= f'(K(t)) \\
w(t) &= f(K(t)) - K(t)f'(K(t))
\end{aligned}
\tag{2}
$$

and are uniquely determined by the current capital stock $K(t)$.

2.2 The social planner's problem

Before turning to the market equilibrium it is useful to consider the social planner's problem. The planner assigns welfare weights ω_i to the individuals which are pinned down by the (initial) distribution of k_i and l_i in the decentralized optimum analyzed in the next section.[4] Setting up the current value Hamiltonian with $\{c_i(t)\}$ as control and $K(t)$ as state variable

$$H = \int_{\mathcal{N}} \omega_i u(c_i(t)) dP_i + \lambda(t)\dot{K}(t)$$

---

[3] As is well known, with positive growth we get steady states only if the intertemporal elasticity of substitution is constant, i.e., utility is CRRA.

[4] In a decentralized equilibrium, consumption depends monotonically on lifetime resources - which in turn are determined by the initial distribution of k_i and l_i . Hence for each distribution of lifetime resources there is a distribution of welfare weights ω_i such as to mimic the decentralized solution.

subject to the capital accumulation constraint (the output good can be used both for consumption and investment)

$$\dot{K}(t) = f(K(t)) - \int_{\mathcal{N}} c_i(t) dP_i \tag{3}$$

leads to the first order conditions

$$\omega_i u'(c_i(t)) - \lambda(t) = 0 \tag{4}$$

and

$$\rho \lambda(t) - \dot{\lambda}(t) = \lambda(t) f'(K(t)). \tag{5}$$

We may disregard the Kuhn-Tucker conditions because of the Inada conditions and the distributional assumptions. The first order conditions (4) and (5) and the capital accumulation equation (3) give the standard pair of differential equations, we omit time indices,

$$
\begin{aligned}
\frac{\dot{\lambda}}{\lambda} &= \rho - f'(K) \\
\dot{K} &= f(K) - \int_{\mathcal{N}} c(\omega_i, \lambda) dP_i
\end{aligned}
\tag{6}
$$

where $c(\omega_i, \lambda)$ is implicitly defined by $\omega_i u'(c_i) = \lambda$. Figure 1 depicts equations (6) with K on the horizontal and λ on the vertical axis. The $\dot{\lambda} = 0$ locus is vertical at $f'(K) = \rho$, and the $\dot{K} = 0$ locus is monotonically decreasing as $c(\omega_i, \lambda)$ is decreasing in λ. The system has a unique saddle path with negative slope. Hence the policy function $\lambda(K)$ is uniquely determined.

Figure 1

2.3   The decentralized equilibrium

Markets are perfect and complete. We assume that each household is able to consume more than $\bar{c}$. All individuals face the same factor prices, thus the household's income

is given by $w(t)l_i + r(t)k_i(t)$. The evolution of individual wealth then reads $\dot{k}_i(t) = w(t)l_i + r(t)k_i(t) - c_i(t)$. Imposing the transversality condition we get the intertemporal budget constraint. The utility maximization problem of the consumer reads

$$\max_{\{c_i(t)\}} \int_0^\infty e^{-\rho t} u(c_i(t)) dt \quad \text{s.t.} \quad \int_0^\infty e^{-R(t)} c_i(t) dt \leq k_i(0) + \int_0^\infty e^{-R(t)} w(t) l_i dt$$

where $R(t) = \int_0^t r(s) ds$. The first order condition reads

$$e^{-\rho t} u'(c_i(t)) - \mu_i e^{-R(t)} = 0 \tag{7}$$

where μ_i denotes the marginal utility of wealth. Our assumptions on the production function imply that $R(t)$ is differentiable. Hence, we may differentiate (7) with respect to time and get the familiar Euler equation

$$\dot{c}_i(t) = -\frac{u'(c_i(t))}{u''(c_i(t))} \left(r(t) - \rho \right). \tag{8}$$

It is easy to see that the FOC of the decentralized equilibrium are equivalent to those of the social planner's problem. Differentiating (4) with respect to time, we get $\dot{c}_i = \dot{\lambda} u'(c_i)/(\lambda u''(c_i))$. Using (6) to replace $\dot{\lambda}/\lambda$, immediately leaves us with the Euler equation (8). The resource constraint is clearly the same in both cases. Hence, the decentralized equilibrium is unique and Pareto-efficient.

Aggregating (8) we obtain the equation of motion for aggregate consumption C

$$\dot{C}(t) = (r(t) - \rho) \int_{\mathcal{N}} -\frac{u'(c_i(t))}{u''(c_i(t))} dP_i. \tag{9}$$

2.3.1 Steady State

Because there is no technical progress, the economy will be in steady state when C, Y, and K are constant. Setting $\dot{\lambda} = 0$ and $\dot{K} = 0$ in (6) yields us the steady state value of the interest rate and the consumption level

$$r^* = f'(K^*) = \rho$$
$$C^* = f(K^*).$$

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Hence, the steady state capital stock is unique and independent of the distribution.[5] This shows that the macroeconomic analysis of decreasing MPC on the individual level is much simpler in a model with optimizing agents. Because individual consumption increases - although the growth rate may differ due to the varying intertemporal rate of substitution - if the interest rate exceeds the rate of time preference, there must be a unique stationary steady state. This is a sharp difference to Bourguignon's (1981) result. In a model with optimizing agents the macroeconomic analysis of decreasing MPC on individual level turns out to be much simpler. Since individual consumption increases - although the growth rate may differ because of the varying intertemporal rate of substitution - if the interest rate exceeds the rate of time preference, there must be a unique stationary steady state. For any (separable) utility function (1), it is optimal to choose a constant consumption flow only if $r = \rho$.

2.3.2 Transitional dynamics

Although distribution does not affect the steady state, the transitional dynamics are affected when the consumption function is non-linear. The following Lemmas 1-4 describe the properties of the consumption $c_i(t)$ as a function of wealth $a_i(t)$, for a given path of interest rates.

Lemma 1 *Iff $c_i(0) > c_j(0)$, then $c_i(t) \geq c_j(t) \ \forall t$.*

Proof. The first order condition (7) may be rewritten $u'(c_i(t)) = e^{-R(t)+\rho t}u'(c_i(0))$. This implies that $c_i(t)$ is monotonic in $c_i(0)$. ∎

Lemma 2 *Individual consumption is monotonically increasing in wealth $k_i(0) + \int_0^\infty e^{-R(t)}w(t)l_i dt$.*

Proof. Assume to the contrary that a poorer agent's consumption today is higher than that of a richer agent. Lemma 1 implies that the poor's consumption will not be

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[5]Although the aggregate values of K and C are unique, the individual c_i- and k_i-distribution is indeterminate and is governed by the initial distribution (see Sorger, 2002).

lower than of the rich in the future. As the rich agent's intertemporal budget constraint is satisfied with equality, the poor would violate his budget constraint. ∎

To proceed, it is useful to define $\phi(x) \equiv -u'(x)/u''(x)$.

Lemma 3 *If $\phi(.)$ is convex and $c_i(t) > c_i(0)$, then $c_i(t)$ is a convex function of $c_i(0)$.*

Proof. We use $u'(c_i(t)) = e^{-R(t)+\rho t}u'(c_i(0))$. Differentiate this equation with respect to $c_i(t)$ and $c_i(0)$ to get

$$\frac{\partial c_i(t)}{\partial c_i(0)} = \frac{\phi(c_i(t))}{\phi(c_i(0))}.$$

where we replaced $e^{-R(t)+\rho t}$ by $u'(c_i(t))/u'(c_i(0))$. To determine $d^2c(t)/dc_i(0)^2$, we take the derivative with respect to $c_i(0)$

$$\begin{aligned}
\frac{\partial^2 c_i(t)}{\partial c_i(0)^2} &= \frac{\phi'(c_i(t))\frac{\phi(c_i(t))}{\phi(c_i(0))}\phi(c_i(0)) - \phi(c_i(t))\phi'(c_i(0))}{[\phi(c_i(0))]^2} \\
&= \frac{\phi(c_i(t))}{[\phi(c_i(0))]^2}\left[\phi'(c_i(t)) - \phi'(c_i(0))\right].
\end{aligned}$$

Hence, $\partial^2 c_i(t)/\partial c_i(0)^2 > 0$ iff $\phi'(c_i(t)) > \phi'(c_i(0))$. This holds true if $\phi(.)$ is convex and $c_i(t) > c_i(0)$. ∎

Lemma 4 *If $\phi(.)$ is convex and $c_i(t) > c_i(0)$, consumption is a concave function of wealth.*

Proof. Define $a_i(0) \equiv k_i(0) + \int_0^\infty e^{-R(t)}w(t)l_i dt$. We differentiate the intertemporal budget with respect to wealth and get

$$\begin{aligned}
\frac{\partial c_i(0)}{\partial a_i(0)} &= \left[\int_0^\infty e^{-R(t)}\frac{\partial c_i(t)}{\partial c_i(0)}dt\right]^{-1} \\
\frac{\partial^2 c_i(0)}{\partial a_i(0)^2} &= -\left[\int_0^\infty e^{-R(t)}\frac{\partial c_i(t)}{\partial c_i(0)}dt\right]^{-2}\int_0^\infty e^{-R(t)}\frac{\partial^2 c_i(t)}{\partial c_i(0)^2}\frac{\partial c_i(0)}{\partial a_i(0)}dt \\
&= -\left(\frac{\partial c_i(0)}{\partial a_i(0)}\right)^3\int_0^\infty e^{-R(t)}\frac{\partial^2 c_i(t)}{\partial c_i(0)^2}dt.
\end{aligned}$$

By Lemma 3, $\partial^2 c_i(t)/\partial c_i(0)^2 > 0$, this implies $\partial^2 c_i(0)/\partial a_i(0)^2 < 0$. ∎

For a given path of interests and wages, Lemma 4 gives the condition such that the rich have a lower marginal consumption propensity than the poor. If $\phi(.)$ is convex and the economy is growing (such that $c_i(t) > c_i(0)$), aggregate savings are higher in a more unequal society when the conditions in Lemma 4 hold.

Let us now consider two economies that are identical except for the fact that the second economy's wealth distribution is generated from the first economy's wealth distribution via a mean-preserving spread. The following Proposition 1 states that the second economy exhibits a higher saving rate and grows faster.

Proposition 1 *If $-u'(c)/u''(c)$ is convex and the economy is growing, more unequal societies (aggregate wealth held constant) have a higher savings rate and a higher rate of output growth.*

Proof. According to Lemma 4, a regressive transfer in wealth decreases aggregate consumption for a *given* path interests and wages because the consumption function is concave with $-u'(c)/u''(c)$ is convex. However, to determine the impact on aggregate consumption, we must take into account the change in the path of factor prices.

To tackle this problem, we formulate the private consumption allocations in terms of the social planner's solution. Equation (4) implies that the ratio of marginal utilities of two agents i and j must remain constant over time

$$\frac{\omega_j}{\omega_i} = \frac{u'(c_i(t))}{u'(c_j(t))} = \frac{u'(c_i^*)}{u'(c_j^*)}$$

where $t \geq 0$ and c_i^* denotes the steady state consumption of agent i. Assume w.l.o.g that $c_i(0) > c_j(0)$. Consider now a regressive transfer from j to i at $t = 0$. According to Lemma 1 and 2, steady state consumption of i must be higher and steady state consumption of j must be lower than before, since consumption is monotone in wealth and aggregate consumption is constant in steady state independent of the distribution. Hence, $d\left[c_i^* + c_j^*\right] = 0$ (assume for ease of notation i and j have the same weight in the population). Using the implicit function theorem, we may calculate the implied

change in welfare weights

$$\frac{\partial \omega_i}{\partial c_i^*} = -\omega_i \frac{u''(c_i^*)}{u'(c_i^*)} > 0 \text{ and } \frac{\partial \omega_j}{\partial c_i^*} = \omega_j \frac{u''(c_j^*)}{u'(c_j^*)} < 0.$$

We determine the change in consumption at date $t \geq 0$. Applying the chain rule yields

$$\frac{\partial c_i(t)}{\partial c_i^*} = \frac{\partial c_i(t)}{\partial \omega_i} \frac{\partial \omega_i}{\partial c_i^*} = \frac{u'(c_i(t))}{u''(c_i(t))} \frac{u''(c_i^*)}{u'(c_i^*)},$$

analoguous for $\partial c_j(t)/\partial c_i^*$. This allows us to determine the change in total consumption

$$\frac{\partial \left[c_i(t) + c_j(t) \right]}{\partial c_i^*} = \frac{u'(c_i(t))}{u''(c_i(t))} \frac{u''(c_i^*)}{u'(c_i^*)} - \frac{u'(c_j(t))}{u''(c_j(t))} \frac{u''(c_j^*)}{u'(c_j^*)}.$$

This expression is negative iff

$$\frac{u'(c_i(t))}{u''(c_i(t))} \frac{u''(c_i^*)}{u'(c_i^*)} < \frac{u'(c_j(t))}{u''(c_j(t))} \frac{u''(c_j^*)}{u'(c_j^*)}$$

or

$$\frac{\phi(c_i(t))}{\phi(c_i^*)} < \frac{\phi(c_j(t))}{\phi(c_j^*)}.$$

From the proof of Lemma 3 we know that this is equivalent to $\partial c_i^*/\partial c_i(t) > \partial c_j^*/\partial c_j(t)$, which holds true if $c_i(t) > c_j(t)$ and ϕ is convex. ∎

As a corollary note that $-u'(c)/u''(c)$ being concave would imply that more unequal societies save less. In addition, note that the results reverse if we consider a shrinking economy where $r(t) < \rho$ and hence $c(t) < c(0)$, see the proof of Lemma 3. Finally, savings are independent of distribution when $-u'(c)/u''(c)$ is linear. Income distribution has no effect on accumulation when preferences take the HARA (hyperbolic risk aversion) form (see the discussion in Bertola et al., 2006, chap. 3).

Furthermore, we are able to draw conclusions on the evolution of the consumption and the wealth distribution.

Proposition 2 *Consumption and wealth inequality increases (decreases) in a growing economy if the elasticity of substitution $-u'(c)/u''(c)c$ increases (decreases) in c.*

Proof. From (8) we see that the growth rate of individual consumption $\dot{c}_i/c_i$ increases in c_i when $-u'(c)/u''(c)c$ increases (decreases) in c. Wealth inequality moves pari passu with consumption inequality since consumption is monotone in wealth. ∎

Comparing Propositions 1 and 2, the conditions on the evolution of inequality are not the same as for the concavity of the consumption function. The reason is the following: The concavity of the consumption function follows from different *marginal* propensities to consume. Instead, the evolution of the wealth and consumption inequality is governed by differences in saving or consumption rates, i.e. by different *average* propensities to consume.

The difference between the two conditions can be best seen by considering an example: Assume that the consumption function is linear but exhibits a positive axis intercept due to subsistence consumption.[6] In that case, the marginal propensities to consume are the same, but the rich exhibit lower average propensity to consume. As a result, inequality will widen over time.

2.3.3 Speed of convergence

We saw that all economies converge to the same steady state but unequal economies grow faster in the transitional process. To bring these two results together we must follow that more unequal societies exhibit a higher speed of convergence towards the steady state. To calculate the speed of convergence $\dot{K}(t)/(K(t) - K^*)$ we linearize the economy around its steady state

$$\frac{\dot{C}(t)}{C(t) - C^*} \cong \frac{\dot{K}(t)}{K(t) - K^*} \cong \mu \equiv \frac{1}{2}\left[\rho + \sqrt{\rho^2 + 4f''(K^*)\int_{\mathcal{N}}\frac{u'(c_i^*)}{u''(c_i^*)}dP_i}\right]. \qquad (10)$$

---

[6]An example for a utility function giving rise to an affine linear consumption function is the Stone-Geary utility $u(c) = \ln(c - \bar{c})$. Intuitively, the subsistence consumption level $\bar{c} > 0$ forces a poor individual to start off with a high level of consumption in a growing economy which precludes them from capital accumulation. Consequently, the subsequent growth rate of wealth and consumption is lower for the poor.

The derivation of equation (10) is shown in the appendix. The following proposition proves our intuition.

Proposition 3 *More unequal societies exhibit a higher speed of convergence.*

Proof. As (10) is evaluated at the steady state, an increase in wealth dispersion increases consumption dispersion. Hence, when $-u'(c)/u''(c)$ is convex, $\int_{\mathcal{N}} -\frac{u'(c_i^*)}{u''(c_i^*)} dP_i$ is larger, this increases the absolute value of μ. ∎

Along the same lines, we get the expressions for the evolution of aggregate consumption and capital stock around the steady state

$$
\begin{aligned}
\frac{C(t) - C^*}{C^*} &\cong \int_{\mathcal{N}} -\frac{u'(c_i^*)}{u''(c_i^*)c_i^*} \frac{c_i^*}{C^*} dP_i \frac{f''(K^*)}{\mu} e^{\mu t} \frac{K(0) - K^*}{K^*} \\
\frac{K(t) - K^*}{K^*} &\cong e^{\mu t} \frac{K(0) - K^*}{K^*}.
\end{aligned}
$$

These results highlight a further difference from the Bourguignon-Solow model. Bourguignon's (1981) analysis of the Solow model with concave consumption (or convex savings) suggests that the poor might indirectly gain from redistribution. He showed that inegalitarian steady states may occur where the consumption of the rich and the poor is higher than in an egalitarian steady state. More inequality raises savings and investment and therefore wages as the economy produces more capital intensive. This mechanism is the reason why the consumption levels of the poor and the rich are higher in the inegalitarian steady state than in the egalitarian one. Hence, the inegalitarian steady state is Pareto-dominant. (Of course such a comparison is not possible because there are no utility functions in the Solow model and the transitional process would have to be taken into account).

In the Ramsey model with perfect and complete markets, the equilibrium allocation is always Pareto optimal. Bourguignon's result, however, appears when the utility level of a single agent is examined. Consider a growing economy that undergoes a mean-preserving spread in its wealth distribution. This raises the welfare of an agent i, who is unaffected by the mean-preserving spread, because the more unequal economy

grows faster in the transition process. Importantly, the Ramsey model does not have a Pareto improvement because at least some of the agents whose wealth is taken in the regressive transfer must be worse off.

The analysis on convergence was restricted to a neighborhood of the steady state. In particular, the consumption inequality is evaluated at its steady state level. Hence, the linearization does not allow for "feedback" effects of income distribution on growth and vice versa. To study the dynamics outside of steady state we therefore have to refer to numerical simulations; this is done in the next section 3.

3   Numerical exercise

To study the quantitative effects involved, we perform a simple quantitative exercise. Let marginal utility be given by $u'(c) = (c^\gamma - 1)^{-\sigma}$ where a consumption of unity may be interpreted as the subsistence level and $\gamma < 1$. It is easy to show that the resulting consumption function is concave in wealth when the interest rate exceeds the rate of time preference. Furthermore, the elasticity of substitution $-u'(c)/u''(c)c$ is increasing in consumption. The preference parameters are chosen as $\rho = 0.02$, $\sigma = 2$, and $\gamma = 0.01$. The new parameter γ determines the concavity of the consumption function. The MPC will react more strongly to changes in wealth as γ increases. The aggregate production function takes a Cobb-Douglas form, $Y = K^\alpha$. The capital share is given by $\alpha = 0.33$. Hence the steady states values of capital and consumption are given by $K^* = (\alpha/\rho)^{1/(1-\alpha)} = 65.6$ and $C^* = (K^*)^\alpha = 3.94$.

To simplify further we assume that there are only two groups in the population: β poor and $1-\beta$ rich agents. According to Wolff (1998), the top 20% of the US population owns about 80% of financial wealth. To match the (financial) wealth distribution, let $\beta = 0.8$ be the group size of the poor. I choose the following individual wealth levels at date 0: $k_P(0) = 10$ and $k_R(0) = 110$. Therefore, with this specification, the richest 20% own 73% of aggregate wealth. The aggregate capital stock is $K(0) = 30$, or around

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45% of its steady state value. The only free parameter left is the distribution of wage incomes (labor endowments). In the low inequality simulation I choose $l_P = 0.8$ (a poor individual earns 80% of the average wage income) whereas in the high inequality case I set $l_P = 0.5$.

<div align="center">Table 1, Figure 2</div>

How well can this simple model with perfect markets account for differences in savings rates? The difficulties in estimating cross country relationships between inequality and savings rates notwithstanding, Smith (2001) estimated that an increase in the Gini coefficient by one standard deviation or 10 percentage points is associated with a 1.5% increase in the country's savings rate. In Table 1 we see that, with the chosen values of the parameters, an increase in the consumption Gini of 10 percentage points result in a savings rate increase of 0.6 - 1 percentage points,[7] with higher marginal effects for higher levels of inequality. Although no elements of uncertainty are present, the model is able to generate reasonable quantitative effects. Furthermore, the simulation shows the evolution of inequality and, in particular, the influence of the higher savings rates of the rich. The positive subsistence consumption $\bar{c} = 1$ forces the poor to choose a flat consumption path (see Figure 2) which results in a slow accumulation of assets. For the high inequality specification, the poor's assets in steady state are even lower than at the starting date (see Table 1, last column).

4 Conclusion

I analyzed the macroeconomic implications of decreasing marginal consumption propensities. With optimal savings and *infinite* horizons, the equilibrium sequences of interest

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[7]Note that we evaluate the savings rates at the starting point of the transition process. Obviously, as the economy moves closer to the steady state, the savings rates decline and equal zero in steady state.

rates and wages are unique and Pareto-efficient. If consumption is a concave function of wealth, more inequality leads to a higher speed of convergence. This holds true although inequality affects accumulation in the transition path with a general utility function $u(c)$. These results stand in a sharp contrast to Bourguignon's findings when assuming exogenous concave consumption behavior. Intuitively, the extreme differences in the outcomes are analogous to those resulting from a comparison of the Ramsey and the OLG models. The Solow-Stiglitz model with exogenous savings can be rationalized by an OLG economy with (warm glow) bequests. In the OLG models, multiple steady states may emerge as new generations enter the economy and agents have finite horizons. In this paper, horizons are infinite, which precludes multiple equilibriums. However, this conjecture needs to be explored further.

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5   Appendix

To derive the speed of convergence, we take a first-order Taylor approximation around the steady state (where $f'(K) = \rho$). For the evolution of individual consumption (8) we get

$$
\begin{aligned}
\dot{c}_i &\cong \frac{\partial \dot{c}_i}{\partial c_i}\left[c_i - c_i^*\right] + \frac{\partial \dot{c}_i}{\partial K}\left[K - K^*\right] \\
&= -f''(K)\frac{u'(c_i)}{u''(c_i)}\left[K - K^*\right].
\end{aligned}
$$

By aggregation we get the evolution of aggregate consumption (note that $\dot{C} = \dot{C} - \dot{C}^*$)

$$
\dot{C} - \dot{C}^* \cong f''(K)\int_{\mathcal{N}} -\frac{u'(c_i)}{u''(c_i)}dP_i\left[K - K^*\right]. \tag{A1}
$$

In the same way we approximate the capital accumulation equation $\dot{K} = f(K) - C$,

$$
\dot{K} - \dot{K}^* \cong \rho\left[K - K^*\right] - \left[C - C^*\right]. \tag{A2}
$$

As (A1) and (A2) are linear in C and K, the growth rates of $[C - C^*]$ and $[K - K^*]$ coincide. The solution of this log linearized system is (10).

Table 1: Calibration

	High inequality	Low inequality	Representative Agent
Initial values			
Labor endowment l_P	0.5	0.8	1
Asset endowment $k_P(0)$	10	10	30
Asset endowment $k_R(0)$	110	110	30
Consumption of the poor $c_P(0)$	1.367	1.912	2.652
Consumption of the rich $c_R(0)$	7.320	5.430	2.652
Aggregate consumption $C(0)$	2.557	2.615	2.652
Consumption GINI	37.2	21.5	0
Savings rate	16.8%	14.9%	13.7%
Steady State			
Consumption of the poor $c_P{}^*$	1.512	2.421	3.940
Consumption of the rich $c_R{}^*$	13.799	10.018	3.940
Aggregate consumption C^*	3.940	3.940	3.940
Assets of the poor $k_P{}^*$	9.70	16.16	64.74
Assets of the rich $k_R{}^*$	289.38	263.44	64.74

Figure 1: Phase Diagram
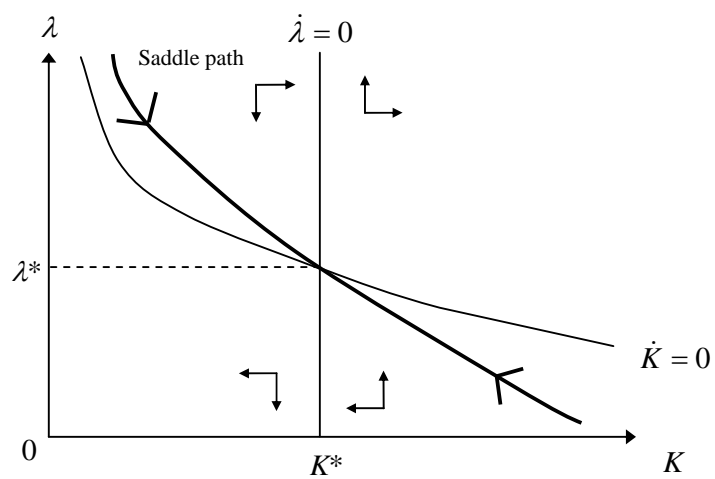
**Figure 2: Dynamics of individual variables
for low initial inequality**

Evolution of consumption

Evolution of financial wealth