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Driving Forces of the Swiss Output Gap

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Abstract

Contrary to standard agnostic statistical approaches an output gap estimate based on a New Keynesian Small Open Economy model provides the possibility to analyze the driving forces of the variation in GDP caused by nominal rigidities. This paper makes use of this and provides an estimate of a model based output gap that corresponds well with conventional measures. The results confirm conventional wisdom that most of the variation is due to foreign shocks. But the risk premium shock in the uncovered interest rate parity equation also plays an important role. It has a procyclical effect on the output gap except for the last recession.

JEL classification: C11, C51, E32, F41
Keywords: DSGE models; output gap; natural level of output; small open economy; business cycle; recessions

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1 Introduction

Taking a pure Keynesian view, economic fluctuations call for stabilization policies as they represent periods in which the economy does not operate at its efficient level. In contrast, pure real business cycle models propose that fluctuations represent the efficient response of the economy to disturbances. Regardless of their distinct interpretation of fluctuations, both views necessitate a measure to quantify these fluctuations. Most of the literature relies on agnostic approaches to estimate output gaps while this paper uses an estimate based on economic theory. In contrast to agnostic approaches, the model based approach permits analyzing and comparing periods of recession and their respective driving forces. The objective of this paper is to provide a theoretically consistent model based output gap and to perform a shock decomposition as the identification of the driving forces of a recession provides important information for policy makers.

The literature typically relies on statistical procedures to extract the business cycle. Hodrick-Prescott (HP) filter, unobserved components methods and the like are often used to extract a trend from the observed gross domestic product (GDP) series. The problem with these approaches is the underlying agnostic identification assumption. Potential output does not follow a smooth trend as implicitly imposed by these identification assumption because it is affected by real shocks. The production function approach, which is also used in the literature, estimates potential output, but is difficult to implement. Neiss and Nelson (2005) argue that this approach recognizes that demographic and labor-market developments affect the full-employment labor force and induce changes in potential output, but the approach neglects cyclical variations of potential output. However, such cyclical variations can be modeled in the New-Keynesian framework and are given by, for example, terms of trade shocks. These shocks affect the household's decision regarding timing and magnitude of consumption and therefore induce variation in labor supply and so in efficient and natural output (Neiss and Nelson, 2005). Hence, a model based approach is advisable.

The New Keynesian framework is used in academia as well as at central banks to study economic fluctuations and provides the researcher as well as policy makers with a tool for understanding the economy. An important property of these models is that they are derived from explicit microeconomic principles. Hence, they allow us to model the market imperfections and price rigidities explicitly. Turning off the nominal rigidities leads to a theoretically based estimate of natural output. The microfoundation helps to escape the

Lucas critique and a counterfactual analysis, like the estimation of output under flexible prices, can be performed.

Recently, some researches relied on a model based approach and used New Keynesian DSGE models to get estimates of the output gap. The approach chosen here relies on a New Keynesian Small Open Economy model. This approach is in spirit of McCallum and Nelson (1999) and McCallum (2001). They identify the level of natural output under flexible prices as the relevant measure of the output gap for monetary policy. However, these models apply to closed economies.[1] The estimation of small open economy models (Lubik and Schorfheide (2007), Del Negro and Schorfheide (2009), Justiniano and Preston (2010b), among others) is an active research area. However, only a few (Bäurle and Menz (2008), Beltran and Draper (2008)) are applied to Swiss data or focus on the estimation of the output gap and analysis of recessions. First approaches to measure the output gap for a small open economy were given by Adolfson et al. (2008), Beltran and Draper (2008), Caraiani (2009) and Leist and Neusser (2010).

The model used in this paper is based on Monacelli (2005) and incorporates external habit persistence as well as indexation and allows for deviations from the law of one price in addition to standard small open economy DSGE model properties like monopolistic competition and sticky prices. The log marginal densities indicate that including nominal exchange rate changes in the Taylor rule and allowing for indexation in the Phillips curve dominates other model specification. The validity of the model is examined by comparing the model implied correlations to the correlations of the data. Furthermore, impulse response functions are consistent with previous findings. Different recession dating approaches are used to assess the usefulness of the natural output gap as a measure of recessions. Finally, the output gap series is decomposed into its historical shocks.

The resulting estimate of the natural output gap corresponds well with conventional wisdom about Swiss business cycles and is consistent with alternative recession dating approaches. Moreover, in contrast to Adolfson et al. (2008), who use deviation from trend as the output gap measure, a model based output gap is used as the marginal likelihood favors this specification. The shock decomposition enables to identify the driving forces of the Swiss business cycle which is not possible when relying on pure agnostic approaches. The uncovered interest rate parity shock and foreign output shock are the main driving forces of Swiss recessions. The latter finding

[1] See also see also Justiniano and Primiceri (2008), Andres et al. (2005), Neiss and Nelson (2005), Edge et al. (2008)

contrasts Justiniano and Preston (2010a) who find that US shocks cannot explain economic fluctuations in Canada. Above all, the approach presented in this paper provides evidence that the last recession is different from previous ones as the effect of the risk premium shock on output and the output gap changed in the last recession. Thus, the model is able to capture the different characteristic of the last recession which originated in the financial sector.

Section 2 discusses the model with a focus on the treatment of the foreign economy in an otherwise standard small open economy model with incomplete pass through. Section 3 presents the estimation strategy including a thorough discussion of the data and prior distribution. Different model specifications are compared at the beginning of section 4, while the following discussion of the model validity and main results are provided only for the model that fits the data best. Section 5 concludes.

2 The Model

This section introduces the small open economy New Keynesian DSGE model used in the following analysis. The presentation of this model closely follows Beltran and Draper (2008), who extended the Monacelli (2005) model. The main elements of the model are as follows. Utility maximizing households choose between consumption and labor. Monopolistic competitive firms produce domestic goods and are subject to Calvo-style price setting. Retail firms importing foreign differentiated goods charge a mark-up over their costs as they are monopolistic competitive. Therefore, a wedge between the world market price and the price of foreign goods in domestic currency, which is labeled as the "law of one price gap" by Monacelli (2005), is introduced. Furthermore, retail firms are also subject to Calvo-style price setting. Beltran and Draper (2008) extend the basic model of Monacelli (2005) by introducing habit persistence.

The small open economy is considered to be negligible compared to the rest of the world. Therefore, we can treat the foreign economy as a closed economy. For our purposes we will approximate the foreign economy with a VAR process using data on foreign output (y_t^*), foreign inflation (π_t^*) and foreign interest rates (i_t^*). The derivation of the model can be found in the appendix. The log-linearized equations which describe this model and are used for estimation are given below. The small letter variables denote percentage deviations of the variables from their respective steady state.[2]

The cost minimization of firms gives the expression for the real marginal costs $mc_t = w_t - p_{H,t} - z_t$, where w_t are wages, z_t is productivity and $p_{H,t}$ is the log of the domestic price index. Then, the log linearized household first order condition with respect to labor input (N_t) as well as the definition of the terms of trade $X_t \equiv \frac{P_{F,t}}{P_{H,t}}$, where $P_{F,t}$ is the foreign price index , are used to derive

$$mc_t - \gamma x_t = \varphi y_t - (1+\varphi) z_t + \sigma(1-h)^{-1}(c_t - hc_{t-1}) \tag{2.1}$$

where c_t denotes consumption, y_t denotes output, h is the degree of habit persistence, γ denotes the share of foreign goods in the consumption bundle, σ is the coefficient of relative risk aversion and φ denotes the inverse of the labor supply elasticity. As Monacelli (2005) notes, the real marginal costs are affected by world output y_t^* via risk sharing through its effect on labor supply as well as by a "relative price effect" captured by terms of trade and the law of one price (LOP) gap (see equation (2.8)).

The New Keynesian Phillips Curve (NKPC) for domestic goods prices is derived from the optimization of domestic producers. They produce the domestic good and sell it in domestic and foreign markets. It is assumed that they face monopolistic competition and Calvo-style price setting, where θ_H is the probability for a firm not to be able to adjust prices. Hence, domestic inflation ($\pi_{H,t} = p_{H,t} - p_{H,t-1}$) is given by

$$\pi_{H,t} = \delta_H \pi_{H,t-1} + \beta \mathbb{E}_t \left(\pi_{H,t+1} - \delta_H \pi_{H,t}\right) + \kappa_H mc_t + \varepsilon_{\pi_H,t} \tag{2.2}$$

$$\text{where} \qquad \kappa_H \equiv \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}$$

with $0 < \beta < 1$ being the discount factor, δ_H the indexation parameter and $\varepsilon_{\pi_H,t}$ denoting an exogenous i.i.d. shock. In contrast to the standard SOE model by Galí and Monacelli (2005) there are also retailers in the domestic economy. They import foreign differentiated goods at world market prices and the law of one price holds at the docks. However, they are also facing monopolistic competition and staggered price setting. This leads to short run deviations from the law of one price. From the optimization of the retailers, the NKPC for imported inflation ($\pi_{F,t} = p_{F,t} - p_{F,t-1}$)

$$\pi_{F,t} = \delta_F \pi_{F,t-1} + \beta \mathbb{E}_t \left(\pi_{F,t+1} - \delta_F \pi_{F,t}\right) + \kappa_F \psi_{F,t} + \varepsilon_{\pi_F,t} \tag{2.3}$$

$$\text{where} \qquad \kappa_F \equiv \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F}$$

[2]In contrast to the appendix the hat notation $\hat{a}_t$ to denote percentage deviations of a variable A_t from its steady state is neglected to enhance readability.

can be derived. $\varepsilon_{\pi_F,t}$ denotes an exogenous i.i.d. shock and $\psi_{F,t}$ denotes the deviation from the law of one price from its steady state. The probability that a retailer is not able to adjust its prices is given by θ_F and δ_F is the indexation parameter of retailers.

Using the definition of the terms of trade, the log-linearized consumer price index (π_t) under the assumption that the elasticity of substitution between domestic and foreign goods (η) equals one is given by:

$$\pi_t = \pi_{H,t} + \gamma \Delta x_t \tag{2.4}$$

The real exchange rate is given by

$$q_t = (1-\gamma)x_t + \psi_{F,t} \tag{2.5}$$

while the change of the LOP gap is given by

$$\Delta \Psi_{F,t} = \Delta s_t + \pi_t^* - \pi_{F,t} \tag{2.6}$$

where s_t is the nominal exchange rate and π_t^* is foreign consumer price index inflation. The change in terms of trade can be denoted by

$$\Delta x_t = \pi_{F,t} - \pi_{H,t} \tag{2.7}$$

The following equation is derived using the assumption of internationally complete asset markets. This assumption implies that the domestic household Euler equation can be equated to the foreign Euler equation (in the spirit of Chari et al. (2002) and Galí (2008)). Therefore, home consumption depends on foreign output (y_t^*) as well as on the terms of trade and on the law of one price gap.

$$(c_t - hc_{t-1}) = (y_t^* - hy_{t-1}^*) + \frac{1}{\sigma}(1-h)\left[(1-\gamma)x_t + \psi_{F,t}\right] \tag{2.8}$$

From the completeness of asset markets assumption we also get the uncovered interest parity condition, which, after some manipulations can be written as

$$(i_t - E_t\pi_{t+1}) = (i_t^* - \mathbb{E}_t\pi_{t+1}^*) + \mathbb{E}_t\left[\Delta q_{t+1}\right] + \varepsilon_{q,t} \tag{2.9}$$

where $\varepsilon_{q,t}$ denotes an exogenous i.i.d. shock.

The market clearing equation is derived from imposing that domestic output must equal consumption of domestic goods and exports of domestic goods. Plugging in the demand functions for these goods yields the following market clearing condition:

$$(1-\gamma)c_t = y_t - \gamma\eta(2-\gamma)x_t - \gamma\eta\psi_{F,t} - \gamma y_t^* \tag{2.10}$$

Regarding the monetary policy (MP) several standard MP rules are assumed. They are all of the form

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(\psi_\pi \pi_t + \psi_a a_t + \psi_s \Delta s_t) + \varepsilon_{i,t} \tag{2.11}$$

where a_t is either detrended output (y_t), output growth ($\Delta y_t + z_t$) or the natural output gap ($\tilde{y}_t$). To derive the output gap[3] we use the fact that deviations of marginal costs from steady state (the desired markup) is zero under flexible prices. Furthermore, the LOP holds under flexible prices. Defining the output gap as the difference between output and output under flexible prices a model based measure of the output gap can be derived. This pure model based approach is given in the appendix. The DSGE model for the foreign closed economy is approximated by a VAR(2). Hence, there will be a slight modification compared to the complete structural model in the appendix when it comes to the implementation for estimation. The approximation by a VAR(2) implies that the foreign technology z_t^* and the foreign flexible price output y_t^{*flex} is not implemented into the equilibrium conditions. Hence, y_t^* is used as a proxy for y_t^{*flex} and z_t^* in the equation of the domestic flexible price output and in the equation of domestic terms of trade under flexible prices. Therefore, domestic output gap is given by

$$\tilde{y}_t = y_t - y_t^{flex} \tag{2.12}$$

where

$$y_t^{flex} = \frac{1+\varphi}{\varphi} z_t - \frac{x_t^{flex}}{\varphi} + \phi_{yy^*} y_t^* \tag{2.13}$$

$$x_t^{flex} = A x_{t-1}^{flex} + B\left[z_t - h z_{t-1}\right] - \phi_{xz^*} y_t^* \tag{2.14}$$

$$A = \frac{h\sigma\left[\varphi\gamma\eta(2-\gamma)+1\right]}{\sigma\left[\varphi\gamma\eta(2-\gamma)+1\right] + (1-h)(1-\gamma)^2\varphi}$$

$$B = \frac{\sigma(1+\varphi)}{\sigma\left[\varphi\gamma\eta(2-\gamma)+1\right] + (1-h)(1-\gamma)^2\varphi}$$

[3]The literature makes a distinction between the potential (or efficient) output and natural (or flexible price) output. The former is the level of output under flexible prices and perfect competition. The latter is the level of output under imperfect competition, but with flexible prices. The term output gap in this paper always refers to the difference of output and natural level of output. McCallum and Nelson (1999), McCallum (2001) and Neiss and Nelson (2005) identify the natural level of output as most appropriate theoretically to inform central banks about disequilibria caused by nominal rigidities.

For the productivity shock we assume an AR(1) process

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t} \tag{2.15}$$

while the foreign economy is approximated by a VAR(2).

$$\begin{pmatrix} y_t^* \\ \pi_t^* \\ i_t^* \end{pmatrix} = \mathbf{A_1} \begin{pmatrix} y_{t-1}^* \\ \pi_{t-1}^* \\ i_{t-1}^* \end{pmatrix} + \mathbf{A_2} \begin{pmatrix} y_{t-2}^* \\ \pi_{t-2}^* \\ i_{t-2}^* \end{pmatrix} + \mathbf{B}\boldsymbol{\varepsilon}_\mathbf{t} \tag{2.16}$$

with

$$\mathbf{A_1} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad \mathbf{A_2} = \begin{pmatrix} a_{14} & a_{15} & a_{16} \\ a_{24} & a_{25} & a_{26} \\ a_{34} & a_{35} & a_{36} \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

The vector $\boldsymbol{\varepsilon}_\mathbf{t}$ contains the exogenous i.i.d. shocks to foreign output, foreign inflation and foreign interest rates and is given by $[\varepsilon_{y^*,t}, \varepsilon_{\pi^*,t}, \varepsilon_{i^*,t}]$.

3 Estimation

Firstly, this section provides a short overview of the chosen estimation strategy. Secondly, the data is discussed and finally, the prior specification is described.

3.1 Estimation Strategy

Bayesian estimation strategy as exposed in An and Schorfheide (2007) and Fernández-Villaverde (2009) is applied. A full-information likelihood approach is chosen in order to use all information implied by the model. The Kalman filter provides the possibility to evaluate the implied likelihood function derived from the DSGE model. However, the likelihood function is flat in many regions which makes pure maximum-likelihood estimation a difficult task. Therefore, the log-likelihood is augmented with priors about parameter distributions. This approach allows prior information about parameters to be implemented and adds curvature to the likelihood. Therefore, knowledge about the economy derived from past observations can be incorporated in a straightforward manner. Furthermore, the validity of pure maximum-likelihood estimation procedure crucially hinges on a correctly specified model such that it can appropriately represent the data generating process. This criterion is relaxed in Bayesian estimation. The model does not have to be the correct data generating process for the posterior inference to be valid. Uncertainty about the parameters as well as about the models is reflected in the posterior distributions.[4] It is therefore possible to compute relative probabilities of competing models, even if all models are known to be false.

For estimation purposes the equations (2.2) to (2.16) have to be written into the form

$$\mathbf{\Gamma}_0 \mathbf{X}_{t+1} = \mathbf{\Gamma}_1 \mathbf{X}_t + \mathbf{\Psi} \mathbf{Z}_{t+1} + \mathbf{\Pi} \boldsymbol{\eta}_{t+1}$$

in order to solve for the solution of this rational expectations model using Sims' algorithm (Sims, 2001). Note that we replace the expectation operator $\mathbb{E}_t$ by defining

$$\boldsymbol{\eta}_{t+1} \equiv \mathbf{X}_{t+1} - \mathbb{E}_t \mathbf{X}_{t+1}$$

as the expectation errors. Furthermore, the shocks are collected into

$$\mathbf{Z_t} = \left(\varepsilon_{\pi_h,t}, \varepsilon_{\pi_F,t}, \varepsilon_{q,t}, \varepsilon_{i,t}, \varepsilon_{y^*,t}, \varepsilon_{\pi^*,t}, \varepsilon_{i^*,t}\right)'.$$

The Sims' algorithm provides a solution in the form of a first-order difference equation

$$\mathbf{X}_{t+1} = \mathbf{G}(\mathbf{\Theta}) \mathbf{X}_t + \mathbf{H}(\mathbf{\Theta}) \mathbf{Z}_{t+1}$$

where $\mathbf{G}(\mathbf{\Theta})$ and $\mathbf{H}(\mathbf{\Theta})$ are functions of the parameters of the system. To be able to use the Kalman filter we connect these states to the data using the measurement equation

$$\mathbf{Y}_{t+1} = \mathbf{F} \mathbf{X}_{t+1}$$

with the vector $\mathbf{Y}_{t+1}$ containing the data and the matrix $\mathbf{F}$ selecting the state variables such that they correspond to the observed data. Making distributional assumptions about $\mathbf{Z}_t$ the Kalman filter can be used to evaluate the likelihood function derived from the state-space representation. However, the likelihood $\mathcal{L}(\mathbf{Y}|\mathbf{\Theta})$ will be augmented by a prior distribution to impose information on parameters. This is possible due to the fact that the observation $\mathbf{Y}$ is taken as given from a Bayesian perspective and the parameters $\mathbf{\Theta}$ are treated as random variables which stays in contrast to the classical perspective, where the realization $\mathbf{Y}$ is treated as a random variable and the parameters are treated as fixed. Therefore, we can incorporate a priori views

[4] See for example Canova (2007) or DeJong and Dave (2007).

regarding the parameters by specifying a prior distribution for the parameters $\mathbf{\Theta}$, denoted by $\pi(\mathbf{\Theta})$. Recall that the joint probability of $(\mathbf{Y}, \mathbf{\Theta})$ is given by

$$p(\mathbf{Y}, \mathbf{\Theta}) = \mathcal{L}(\mathbf{Y}|\mathbf{\Theta})\pi(\mathbf{\Theta})$$

or

$$p(\mathbf{Y}, \mathbf{\Theta}) = \mathcal{P}(\mathbf{\Theta}|\mathbf{Y})p(\mathbf{Y})$$

and by plugging in the former into the latter we get the posterior distribution:

$$\mathcal{P}(\mathbf{\Theta}|\mathbf{Y}) = \frac{\mathcal{L}(\mathbf{Y}|\mathbf{\Theta})\pi(\mathbf{\Theta})}{p(\mathbf{Y})} \propto \mathcal{L}(\mathbf{Y}|\mathbf{\Theta})\pi(\mathbf{\Theta})$$

There is no analytical solution available due to the non-linear mapping from the DSGE model parameters to the moments of the data. Therefore, numerical methods are used. The first estimation approach relies on Sims' algorithm `csminwel`[5] to minimize over the objective given by $f(\mathbf{\Theta}) = -\ln \mathcal{L}(\mathbf{Y}|\mathbf{\Theta}) - \ln \pi(\mathbf{\Theta})$. This optimization algorithm proved to be useful in dealing with likelihoods that exhibit discontinuities as it combines a derivative-based optimization method with a simplex algorithm. This approach enables one to retrieve the mode of the parameters. However, to derive the posterior distribution it is necessary to resort to numerical methods that generate draws from the posterior distribution. To this end the Random Walk Metropolis Hastings algorithm and estimation setup as exposed in Lubik and Schorfheide (2006), An and Schorfheide (2007) and Fernández-Villaverde (2009) is used. Furthermore, modified harmonic mean estimates of the marginal densities are computed to compare the models (see Geweke (1999), Rabanal and Rubio-Ramirez (2005) and An and Schorfheide (2007)).

3.2 Data

The model is estimated using quarterly data ranging from the first quarter 1989 to the second quarter 2010.[6] Higher frequency data is transformed to quarterly data by taking averages. Observations on Swiss GDP growth rates, import price inflation, CPI inflation, 3 month LIBOR in CHF, the real exchange rate, GDP growth rates and inflation of OECD countries as well as the interest rate on the EURO is used.[7]

[5] Program code can be found on `http://www.princeton.edu/~sims/`.

[6] The data sources do not provide information on the 3 month LIBOR in CHF prior to 1989.

[7] The EUR interest rate is used as around 78% of imports and around 56% of exports are due to trades with EUR countries (Data for 2010 taken from `http://www.ezv.admin.ch/themen/00504/01506/01533/index.html?lang=de`).

Quarterly data on seasonally adjusted real GDP of Switzerland is taken from the Swiss State Secretariat for Economic Affairs (SECO) and transformed to quarterly growth rates by taking log differences and multiplying by 100. Swiss inflation rates are defined as percentage changes of CPI to the corresponding month of the previous year. The data is taken from the monthly bulletin of the Swiss National Bank (SNB). The monthly import price index is obtained from the SNB and transformed to annualized percentage rates by multiplying the log-differences with 400. The interest rate is defined as the 3-month-LIBOR. The monthly series is obtained from Econstats[8] for observations up to 1998. For later observations, data provided by the SNB is used. Quarterly data on the relative consumer price index supplied by the OECD provides information on the real exchange rate. The series is detrended by regressing it on a constant as well as a trend and multiplying the respective residuals with 100.

As a measure of foreign output the seasonally adjusted quarterly real GDP of OECD countries is taken. The series is detrended in the same manner as the real exchange rate. Detrended data is used instead of growth rates due to the approximation of the foreign economy by a VAR(2).[9] The quarterly series on percentage changes of the consumer price index of OECD countries on the same period of the previous year is the measure for foreign inflation. The foreign interest rate is defined as the quarterly EURO short term interest rates. Data source is the Area Wide Model (AWM) dataset for the years up to 2008 and the short term interest rate of the EURO provided by the OECD for the remaining observations.[10] The mean is subtracted from all series prior to estimation. A detailed overview of the data is given in Table 5 in the appendix.

3.3 Prior Specification

The prior distribution is determined by relying on a priori beliefs coming from past studies and is reported in Table 2. In the first column the parameter symbol is given. The second column reports the chosen prior mean. The third column exhibits the prior standard deviation, while the fourth column specifies the assumed distribution for any given parameter. Since the prior distribution is an important part of the Bayesian estimation some space will

[8] `http://www.econstats.com/r/rlib_em4.htm`

[9] Foreign GDP growth corresponds to $\Delta y_t^* + z_t^*$ in the model. However, foreign productivity is not modeled due to the approximation by a VAR(2).

[10] See Fagan et al. (2001) or `http://www.eabcn.org/area-wide-model` for a description of the AWM dataset.

be devoted to discuss its specification. However, the specification follows closely Beltran and Draper (2008).

3.3.1 Calibrated Parameters

The parameters that are not estimated are discussed prior to the discussion of estimated parameters. As commonly done in the literature, the discount factor, β, is set to 0.9961, implying an annual steady-state real interest rate of 1.6 percent. The share of imported goods in the small open economy's consumption bundle, γ, is fixed at 0.25 as the average ratio of imported goods[11] to the Swiss GDP lies in this region. The coefficient of relative risk aversion, σ, is set to one, which implies log-utility in consumption.[12] Furthermore, for the foreign economy we estimate a VAR(2) process.

3.3.2 Prior Distributions

The degree of habit persistence has to lie between zero and one. Therefore, a beta distribution is assumed. There are a few studies (e.g. Fuhrer (2000) and Christiano et al. (2005)) that find estimates for h around 0.6 and 0.8. However, I use a mean of 0.5 but allow it to vary.

The mean of the inverse labor supply elasticity and the mean of the substitution elasticity between domestic and foreign goods are set to one which seems reasonable as these values are used when linearizing around the steady state. Both parameters are assumed to follow a gamma distribution.

The fraction of firms not able to adjust their prices is set to 0.75 which implies an average duration between price changes of 4 months.[13] Since the price stickiness parameters reflect probabilities, they have to lie between zero and one. This is reflected by the assumption that they follow a beta distribution. Moreover, in one model specification we allow for indexation by firms and retailers. The prior means are set to 0.5 and the parameters are assumed to follow a normal distribution.

Productivity shocks are assumed to be highly persistent. Therefore, the prior mean for the autoregressive coefficient in the exogenous technology

[11] The interpretation that γ corresponds to the share of imported goods is valid as the prior mean of η is set to one.

[12] The results do not depend on this specification. Assuming that σ follows a Gamma distribution with mean 1 and standard deviation 0.5 does not alter the results.

[13] See Kaufmann (2009) for microeconomic evidence.

process is set to 0.9. Furthermore, most empirical studies of monetary policy rules have found a large degree of interest rate smoothing.[14] Hence, we set the prior mean to 0.7. For these autoregressive coefficients only values between zero and one are reasonable and, therefore, a beta distribution is assumed.

Finally, for the case of Switzerland it seems reasonable to assume a rather strong reaction of the monetary authority to inflation and, in comparison, a rather modest reaction to real economic activity. Therefore, the value 1.5 as reaction to inflation and 0.5 as reaction to a measure of real economic activity is chosen. The parameters are assumed to follow a gamma distribution as only positive values seem to be reasonable. Furthermore, the prior mean for the reaction of the monetary authority to changes in the nominal exchange rate is assumed to be 0.25 and follows a normal distribution. Note that this parameter is not present in each Taylor rule of the different models. The prior mean of the indexation parameters is set to 0.5 and assumed to follow a normal distribution with a standard deviation of 0.20.

The term $\phi^*_{yy} \cdot y^*$ approximates the effect of foreign productivity and the foreign natural output on domestic natural output. The prior mean is set to one but the parameter is allowed to vary substantially. Since there is no a priori information on the reaction of flexible terms of trade to foreign output available, a zero mean and a high variance is assumed for $\phi^*_{xz^*}$. For both parameters a normal distribution is assumed. All standard deviations of domestic shocks are assumed to have a mean of 0.1 and follow an inverse gamma distribution.

4 Estimation results

As given in equation 2.11 different Taylor rules are estimated because this equation is based on ad-hoc assumptions. Hence, it is advisable to explore which specification performs best at fitting the data. Five different models containing reasonable Taylor rules are considered. In the first one (BASE) the monetary authority is assumed to react to inflation and the natural output gap. The second model (GROW) stipulates a Taylor rule where the monetary authority reacts to inflation and growth. The third one (DETR) assumes that the monetary authority reacts to inflation and detrended output. The fourth model (EXCH) considers a reaction of the monetary authority to inflation, the natural output gap and the exchange rate. Finally, the model

[14] See for example Clarida et al. (1998), Lubik and Schorfheide (2007) and Justiniano and Preston (2010b)

(IND) has the same specification as model EXCH, but, additionally, allows for indexation in the New Keynesian Phillips curves. An overview of the Taylor rule specifications is provided in Table 1.

Variables \ Model	BASE	GROW	DETR	EXCH	IND
Inflation	+	+	+	+	+
Output Gap	+			+	+
GDP Growth		+			
Detrended GDP			+		
Exchange Rate				+	+
Indexation					+

Table 1: Model overview: The table shows which variables are present in the Taylor rule of the respective model.

4.1 Estimated Parameters

The estimation results are presented in Table 2. The second column reports the specified prior mean as discussed in section 3.3. The third column reports the prior standard distribution. The following columns report the posterior means and posterior standard deviations for the estimated models.

Similar to Beltran and Draper (2008), the degree of habit persistence (h) is high for all considered model specifications. These estimates are a bit higher than usually found in the literature.[15] The parameter of the inverse elasticity of labor supply (φ) is lower compared to other studies for most of the chosen model specifications.[16] This implies that the labor supply is quite responsive to changes in real wage. Only when assuming that the monetary authority reacts to the GDP growth rate we find an estimate of the inverse labor supply elasticity that corresponds to the literature. The elasticity of substitution between domestic and foreign goods (η) is very low while the degree of price stickiness for domestic producers (θ_H) and for domestic retailers (θ_F) is rather high. This implies that firms are able to adjust their prices only about every second year which stays in contrast to microeconomic evidence (Kaufmann, 2009). However, Justiniano and Preston (2010a) find similar results for domestic price stickiness but not for the imported price stickiness using a small open economy model for Canada. The monetary authority

[15]See e.g. Fuhrer (2000), Christiano et al. (2005) and Adolfson et al. (2007)

[16]See e.g. Beltran and Draper (2008), Justiniano and Preston (2010b) and Justiniano and Preston (2010a)

strongly reacts to inflation but not so much to the real economic activity with the exception of the reaction to growth rate (GROW model). It also reacts to changes in the exchange rate when it is allowed to. The indexation parameters are higher than otherwise found in the literature (e.g. Bäurle and Menz (2008) or Justiniano and Preston (2010b)). Finally, technology and the interest rate are highly persistent processes.

4.2 Model comparison

Two approaches are chosen in order to evaluate the performance of the different models at explaining the data. Firstly, the log marginal likelihoods are compared following the approach of An and Schorfheide (2007). Secondly, the best model with respect to the log marginal likelihood is used to compute the correlations between the observable data implied by each of these two models. Densities of these model implied correlations are then compared to the actual correlation in the data to get more information on the model fit.

The log marginal likelihood and the Bayes Factor (BF) are given in the last rows of Table 2. Both values were computed for a range of values. However, only the least favorable Bayes Factor for model IND is reported. Details of the computation are given in the appendix. As in Lubik and Schorfheide (2007) and Justiniano and Preston (2010b) it can be concluded that including a reaction to changes in nominal exchange rate in the Taylor rule is favored by the data. Most importantly, the models that are superior with respect to the log marginal likelihood include the output gap in the Taylor rule. Therefore, estimating the output gap instead of just using the growth rate or detrended GDP improves the explanatory power of the model. The Bayes Factor is given by

$$BF_{IND,j} = \exp\left\{\ln p(\mathbf{Y}|M_{\text{IND}}) - \ln p(\mathbf{Y}|M_j)\right\}$$

where $\ln p(\mathbf{Y}|M_i)$ is the log marginal likelihood of model i. The table reports the Bayes Factor for the comparison of model IND to the other models. Kass and Raftery (1995) suggest that values of $2\ln(BF) > 10$ indicate very strong evidence against model j. Hence, allowing for indexation improves the models ability at predicting data as all Bayes Factors are greater than 10. [17]

[17] The DSGE model was also estimated using the HP-cycle of Swiss GDP as observable instead of the growth rate. The log marginal likelihood using this approach was also lower than for the models with the output gap. However, as observables were exchanged in order to be able to have the HP-cycle in the Taylor rule this result has to be taken with caution.

	Priors			BASE		GROW		DETR		EXCH		IND	
	mean	s.d.	distr.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
h	0.50	0.10	B	0.86	0.04	0.85	0.03	0.84	0.04	0.84	0.03	0.85	0.03
φ	1.00	0.50	G	0.81	0.36	1.11	0.51	0.77	0.41	0.78	0.36	0.75	0.35
η	1.00	0.50	G	0.06	0.03	0.11	0.05	0.06	0.03	0.05	0.03	0.06	0.03
θ_H	0.75	0.10	B	0.96	0.01	0.96	0.01	0.96	0.01	0.96	0.01	0.97	0.01
θ_F	0.75	0.10	B	0.97	0.01	0.90	0.02	0.97	0.01	0.97	0.01	0.96	0.03
ρ_z	0.90	0.05	B	0.73	0.08	0.60	0.06	0.72	0.08	0.74	0.08	0.73	0.08
ρ_i	0.70	0.12	B	0.83	0.03	0.87	0.02	0.88	0.02	0.89	0.02	0.90	0.02
ψ_π	1.50	0.20	G	1.12	0.14	2.16	0.14	1.36	0.17	1.19	0.15	1.26	0.18
ψ_a	0.50	0.15	G	0.26	0.04	0.96	0.17	0.22	0.04	0.33	0.06	0.38	0.08
ψ_s	0.25	0.15	N	-	-	-	-	-	-	0.27	0.08	0.30	0.09
δ_H	0.50	0.20	N	-	-	-	-	-	-	-	-	0.78	0.07
δ_F	0.50	0.20	N	-	-	-	-	-	-	-	-	0.50	0.11
ϕ_{yy^*}	1.00	1.00	N	-0.61	0.41	1.00	1.00	1.01	0.99	-0.81	0.41	-0.82	0.41
ϕ_{xz^*}	0.00	3.00	N	0.34	0.22	-0.01	2.98	0.00	3.01	0.38	0.26	0.35	0.22
σ_{π_H}	0.10	4.00	IG	0.72	0.06	0.75	0.07	0.71	0.06	0.72	0.06	0.46	0.04
σ_{π_F}	0.10	4.00	IG	1.42	0.12	1.54	0.14	1.43	0.12	1.43	0.11	1.32	0.12
σ_q	0.10	4.00	IG	5.07	0.42	5.17	0.50	5.30	0.43	4.67	0.39	4.66	0.40
σ_i	0.1	4	IG	0.10	0.01	0.10	0.01	0.12	0.01	0.10	0.01	0.10	0.01
σ_z	0.1	4	IG	0.59	0.05	0.59	0.05	0.60	0.06	0.60	0.05	0.60	0.05
log marginal likelihood				-1'217		-1'224		-1'228		-1'207		-1'160	
$2\ln(BF)$				115.29		129.69		136.95		93.88		1	

Table 2: Estimation Results: The specified prior distributions are beta (B), gamma (G), normal (N) and inverse gamma (IG) distributions.

Therefore, the following discussion relies on results from the IND model although the results regarding the impulse response functions as well as the shock decomposition of output are robust across the different model specifications. The distribution of model implied standard deviations and correlations is computed using 1000 draws from the posterior distribution. To compute

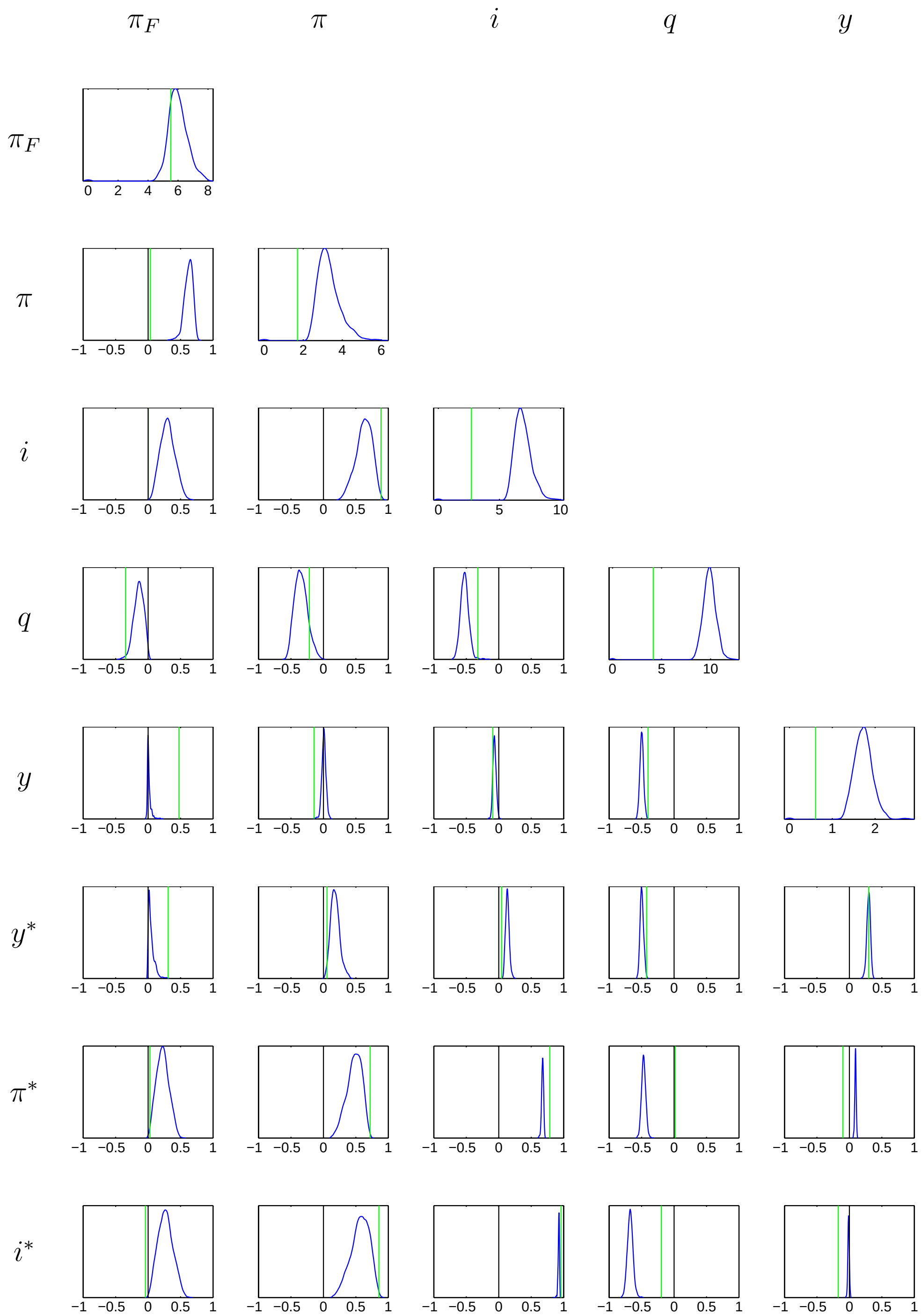


Figure 1: Model IND: Standard deviations of domestic variables on diagonal. Correlations between variables (domestic and foreign) on the off diagonal. The green line depicts standard deviation and correlation of the data, the blue densities are the model based estimates.

the density the Epanechnikov kernel is applied. The comparison of these model implied correlations to the correlation in the data is given in Figure 1.

The standard deviation of imported inflation is captured very well while the model tends to overestimate the remaining standard deviations. Most of the model based correlations are similar to the correlation in the data. The correlation between inflation and real exchange rate as well as the correlation between domestic output and foreign output are captured especially well (2 of totally 25 correlations). The observed correlation of the data lies in the distribution of model based correlations (15 of 25) with a only few exceptions. Of these exceptions 5 model based correlations show the same sign as the correlation in the data. For 3 out of 25 correlations the signs of the correlation are not appropriately estimated. Notably, this only concerns correlations with the foreign variables i^* and π^* which points to problems with the VAR(2) approximation of the foreign economy. Impulse responses are analyzed in the following section to further assess the credibility of the model.

4.3 Impulse Responses

Insights on the behavior of endogenous variables can be gained by examining the impulse response functions of selected variables. The impulse response functions display the median response to a shock of the size of one standard deviation together with a 95-percent credible interval. The credible interval was constructed by randomly drawing parameters from the posterior distribution and computing the impulse responses for each draw. The interval was computed for each horizon separately. The impulse responses of model IND are provided. However, if nothing else is stated the EXCH and even the BASE model exhibit qualitatively the same impulse responses.

Figure 2 displays the responses to a positive domestic productivity shock. It raises output significantly but less pronounced than natural output. The reason is that deviation of marginal costs from constant steady state markup ($\widehat{mc}_t$) reacts negatively to a productivity shock. As prices cannot adjust immediately the effect of a positive productivity shock on output is dampened compared to the effect on natural output. Therefore, the output gap reacts negatively to a productivity shock. The decrease in the interest rate as well as Swiss CPI inflation is statistically significant but due to the small size it is economically irrelevant. While the foreign inflation is exogenous and Swiss CPI inflation hardly reacts the domestic currency depreciates. Hence, the real exchange rate increases. Furthermore, the price of home goods decreases after a productivity shocks while the imported goods become more expensive

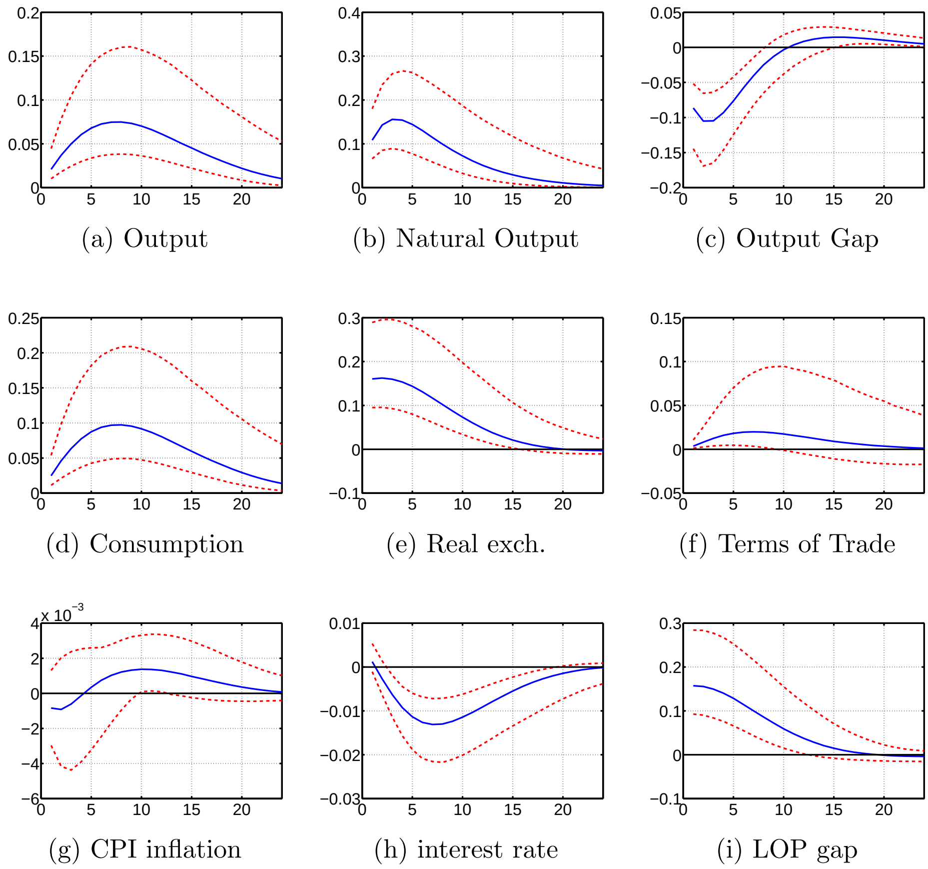
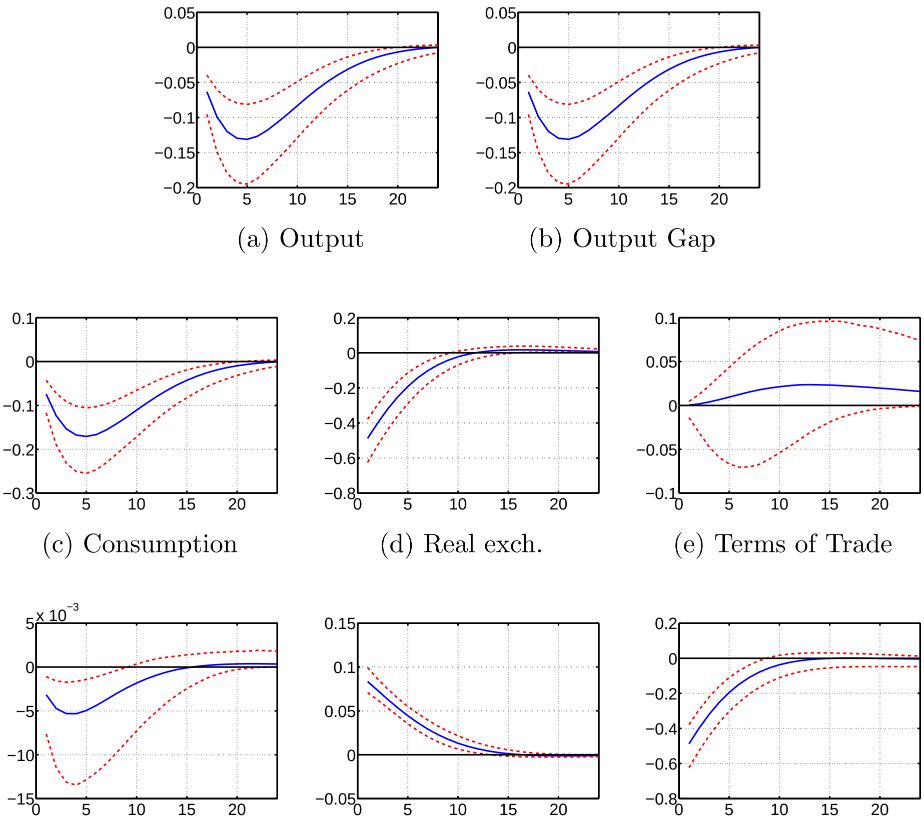


Figure 2: Impulse Responses (model IND) to a One Standard Deviation Productivity Shock with 95-Percent Confidence Interval

as the exchange rate depreciates. Thus, the terms of trade respond positively to a productivity shock which corresponds to a depreciation of terms of trade for Switzerland. The response of the law of one price gap to a domestic productivity shock is positive because foreign prices are exogenous and the increase of the exchange rate dominates the increase of imported goods prices. Consumption increases as the consumption of home goods increases enough to offset the negative effect of terms of trade depreciation.

A positive interest rate shock of the size of one standard deviation leads to a decrease in the real exchange rate as the nominal exchange rate decreases more than the domestic price level (Figure 3). The decrease in the nominal exchange rate and prices for imported goods implies a decrease in the law of one price gap. The inflation goes down as well although the effect dies



(a) Output (b) Output Gap

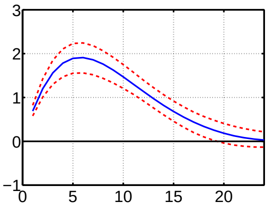
(c) Consumption (d) Real exch. (e) Terms of Trade

(f) CPI inflation (g) interest rate (h) LOP gap

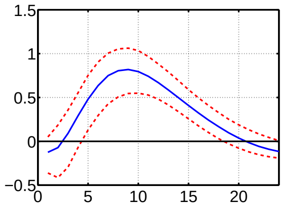
Figure 3: Impulse Responses (model IND) to a One Standard Deviation Interest Rate Shock with 95-Percent Confidence Interval

out rather quickly. The hump shaped response is only achieved by allowing for indexation. The appreciation of domestic currency also leads to a weaker foreign demand for the home good. Thus, consumption as well as output fall after an interest rate shock. Furthermore, the natural level of output does not react to an interest rate shock. Therefore, the reaction of the output gap in response to an unexpected interest rate shock corresponds to the reaction of output. The terms of trades do not significantly react. It is important to note that output, consumption and CPI inflation respond in a way that corresponds to conventional wisdom and to findings of other studies (e.g. Adolfson et al. (2007)).

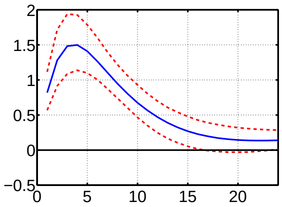
Figure 4 displays the responses to a shock to foreign output. Foreign demand for home goods increases which leads to an increased domestic out-



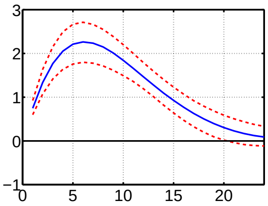
(a) Output



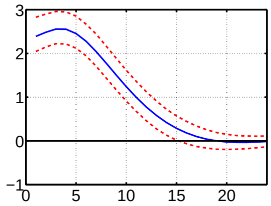
(b) Natural Output



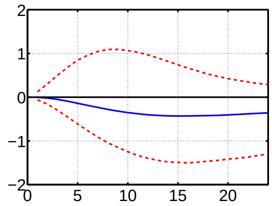
(c) Output Gap



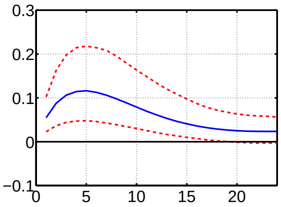
(d) Consumption



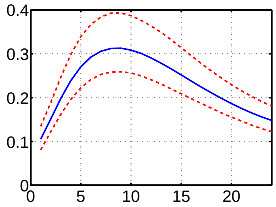
(e) Real exch.



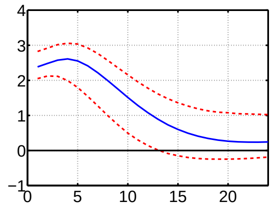
(f) Terms of Trade



(g) CPI inflation



(h) interest rate

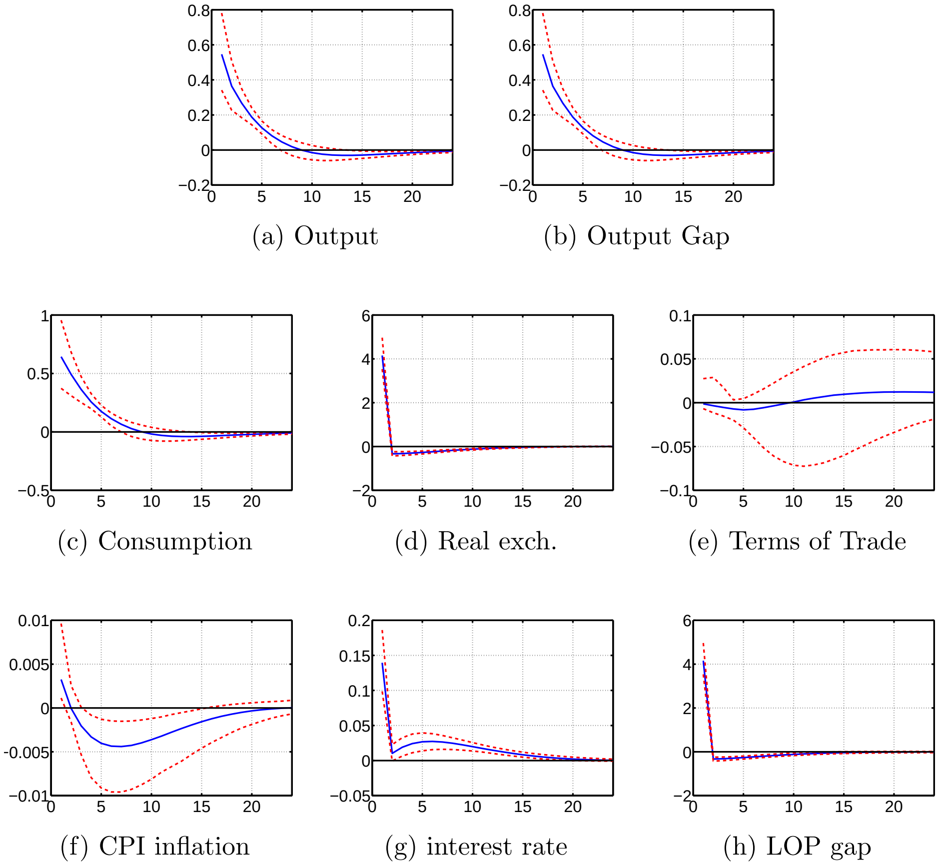


(i) LOP gap

Figure 4: Impulse Responses (model IND) to a One Standard Deviation Foreign Output Shock with 95-Percent Confidence Interval

put and consumption in combination with the positive response of the law of one price gap. Under flexible prices the terms of trade would decrease and this affects natural output which therefore increases only with a lag of 5 quarters. The response of the output gap resembles closely the response of output itself. Somewhat surprisingly, the nominal exchange rate increases and in combination with the increase of the foreign price level this explains the increase of the real exchange rate. The increased demand leads to a rise in CPI inflation which is counteracted with a lagged increase of domestic interest rates. The terms of trade do not significantly react.

The responses to a one standard deviation shock to the uncovered interest rate parity (UIP) are depicted in Figure 5. This shock can be interpreted as a risk premium shock . It has an important effect on the real exchange rate



(a) Output (b) Output Gap

(c) Consumption (d) Real exch. (e) Terms of Trade

(f) CPI inflation (g) interest rate (h) LOP gap

Figure 5: Impulse Responses (model IND) to a One Standard Deviation Shock to UIP condition with 95-Percent Confidence Interval

through the nominal exchange rate. The rise in the real exchange rate coincides with an increase of domestic output and consumption as the domestic economy can benefit from the depreciation of domestic currency. As natural output does not react to an UIP shock, the output gap moves one-to-one with output. Domestic inflation rises on impact but decreases afterwards. The risk premium shock also leads to a rise in the interest rate. The substantial increase of the interest rate in the first quarter is due to the reaction of the monetary authority to changes in the nominal exchange rate. For the remaining quarters the impulse response is similar to the BASE model.

The impulse response function with respect to a shock to home goods inflation, imported price inflation, foreign interest rates as well as foreign inflation are reported in the appendix (Figures 13 to 16). Overall, the impulse

	$\tilde{y}$	HP	BP (4 lags)	BP (12 lags)
$\tilde{y}$	2.24			
HP-filter	0.67	1.20		
BP (4 lags)	0.41	0.76	0.58	
BP (12 lags)	0.64	0.98	0.80	0.85

Table 3: Standard deviations of output gap measures on diagonal, corresponding correlations on off diagonal

response of the law of one price gap is similar to the impulse response of detrended domestic real exchange rate except for a shock to home goods inflation. The response of CPI inflation to this shock dominates the response of the change in nominal exchange rate. Therefore, the real exchange rate reacts negatively where the law of one price gap is dominated by the response of the change in nominal exchange rate as the imported price inflation only weakly responds to this shock.

4.4 Recessions and the Output Gap

Estimates of the output gap were obtained by running the Kalman filter using the estimated posterior mean of the parameters. The best linear prediction of the states X_t given the observation up to and including period t is computed. Figures 6 and 7 compare the estimated output gap to the HP-cycle and the Bandpass-Filter (Baxter King) cycle. Furthermore, in each plot different measures to determine recession periods are used. Table 3 provides an overview on standard deviations of and correlations between these different measures. [18]

In figure 6 recessions are plotted as shaded areas and are defined as two consecutive quarters with a negative growth rate, where the growth rate corresponds to the (demeaned) measure used in the estimation of the DSGE model.[19] This approach is chosen as there are no official recession dates available in contrast to for example the US, where the NBER publishes these dates. The first series depicted in figure 6 is the estimated output gap using the best linear prediction based on the DSGE model. The second series

[18]The time line on the X-axis corresponds to the last day of a quarter. Therefore, the time index 2000 in the figure corresponds to the value of the measure at the end of the last quarter which would be 1999:4 in this example.

[19]Using negative growth rates of GDP (not demeaned) yields similar recession periods (Elmer and Schenker, 2010).

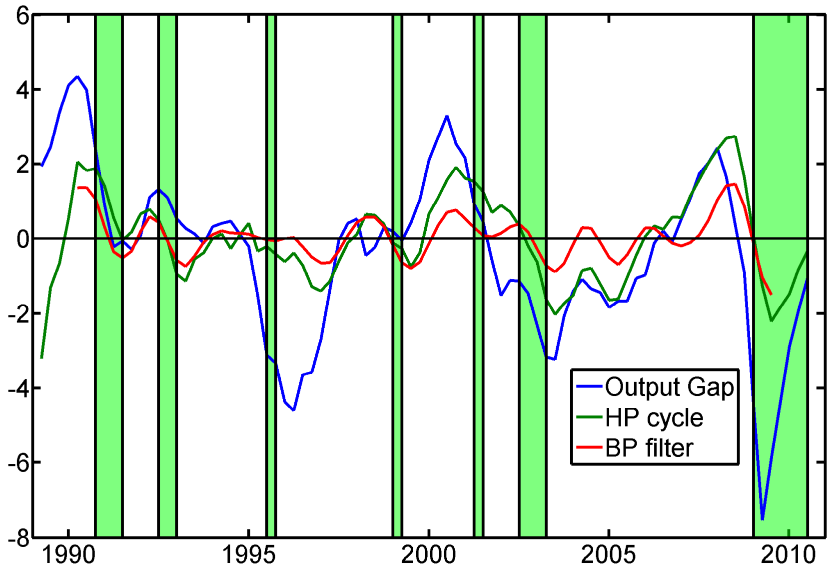


Figure 6: Recessions defined as two consecutive quarters of negative growth (shaded areas), output gap estimate based on model IND, HP or BP filtered cycle series.

is the business cycle derived from the Hodrick-Prescott (HP filter) and the last measure is the Baxter-King Band-Pass (BP) filter. From this figure we can infer that the first two series move in a comparable manner for most of the sample with the exception of the years 1995 to 1997, where the DSGE output gap series deviates from its steady state up to more than -4% while the HP-cycle fluctuates close around zero. However, both series peak in 1990, 2001 as well as in 2008 and exhibit a trough in 2003 which corresponds well to conventional wisdom. The Band-Pass filtered series does not tell us much about the business cycle. However, this is probably due to the chosen lag which was set to four in order to include the last recession. Setting the number of lags to 12 yields a Band-Pass filtered series that exhibits a correlation of 0.98 with the HP-cycle. Therefore, the BP-filtered series is only provided in this figure and excluded from following ones. The following paragraphs provide a detailed discussion of these series and put them in a historical context.

The years 1989 and 1990 were characterized by a worldwide economic rebound and a domestic housing bubble. Therefore, where the SNB was mainly concerned about a strengthening of domestic currency and a slackening of the economy their focus switched to the inflation pressure caused by the economic rebound in combination with a depreciation of the domestic currency (Hildebrand, 2004). Inflation increased up to 6.6% and the years 1991 to 1993 were perceived as a recession (or at least stagnation) in the conventional wisdom. However, this does not conform to the estimated series. Although they sharply decline they do not go into negative numbers.

It is interesting to note that only the DSGE output gap depicts large negative values for the years 1995 to 1997. Although the economy was still weak and the structural crisis in the building and housing sector was still not completely ceased, the SNB sticked to a tight monetary policy up to 1995. Only after the occurrence of the Mexican crisis and the following worldwide economic downturn the SNB lowered interest rates also due to the strong Swiss Franc and the stagnation of the domestic economy (Hildebrand, 2004). The negative values of DSGE output gap correspond well with conventional wisdom that Switzerland faced stagnation or at least weak growth during the 1990s although there is some disagreement about the scale of it in academia (Lambelet and Mihailov (1999), Kehoe and Ruhl (2005) and Abrahamsen et al. (2005)). However, this weak performance of the Swiss economy is only captured by DSGE output gap and not by the HP-cycle.[20] The large drop of the output gap perfectly coincides with the Mexican peso crisis.

Also in accordance with conventional wisdom the HP-cycle and DSGE output gap peak in 2001 after the burst of the technology bubble at the stock exchanges and before the terrorist attacks in the US. According to both measures the economy started to rebound in 2003 and the following boom peaked in 2008, when the financial crisis occured. The dating of the downturn as well as the severity compared to other recessions is consistent with conventional wisdom. Finally, both measures indicate that the economy reached its trough in 2010.

In figure 7 the shaded areas correspond to the low-growth regime from a Markov-switching model allowing for only two regimes. The dating is taken from Siliverstovs (2011). All periods of low-growth correspond to downturns in both business cycle measures.

Figure 8 recessions correspond to the recession regime of a Markov-switching model allowing for three regimes. The dating of the recession regime is again taken from Siliverstovs (2011). While the first two recessions are well captured by both measures only the DSGE output gap series captures an economic downturn in the first quarter of 1995. In contrast to the business cycle dating definition of two consecutive quarters of negative

[20]Figure 17 provides even more compelling evidence that the Swiss economy indeed experienced a difficult time in the 90ies. The deviation of model based DSGE output from its steady state is negative for almost the whole decade.

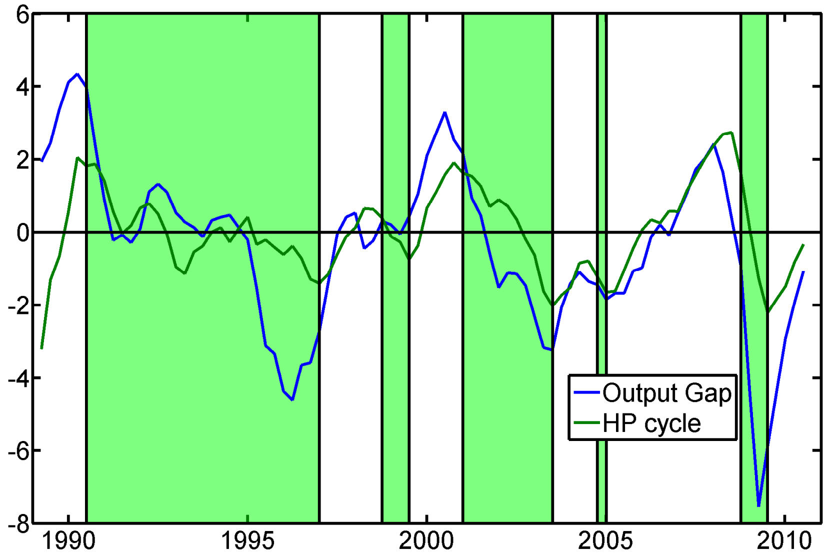


Figure 7: Low growth periods of MS model with two regimes Siliverstovs (2011) (shaded areas) compared to model based output gap (IND) estimate and HP cycle

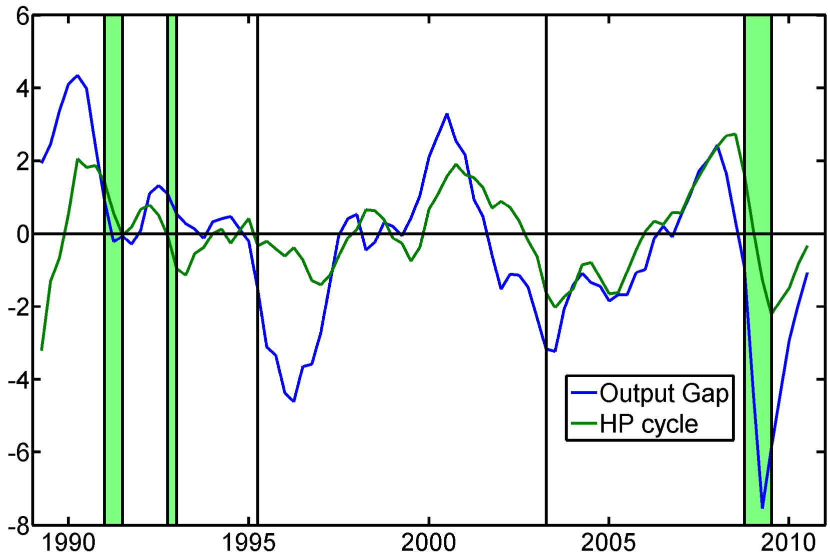


Figure 8: Recessions of MS model with three regimes Siliverstovs (2011) (shaded areas/black lines) compared to model based output gap (IND) estimate and HP cycle

growth, the dating by this Markov switching model implies only a recession for the first quarter of 1995 instead of a recession for the first two quarters. Furthermore, it does not find any recession in 1999 and only a one quarter recession in the first quarter of 2003 instead of a recession of two quarters. Finally, the great depression period is also smaller than in the first recession dating approach presented above. However, if and only if a marked downturn in the DSGE output gap series occurs this last recession dating approach finds a recession.

In light of the discussion in this section we see that both measures perform similarly at measuring business cycles when comparing the estimated series to recession definitions. However, especially for the mid 1990ies the DSGE output gap series seems to outperform the HP-cycle estimate. Therefore, it seems reasonable to conclude that the output gap measure derived from the DSGE model provides an alternative estimate of business cycle. Furthermore, the DSGE based output gap estimate can be used to analyze the recessions regarding their driving forces which cannot be done when using agnostic statistical procedures. This advantage of the DSGE based measure will be exploited in the next section.

4.4.1 Shock Decomposition

The series of output, flexible price (natural) output and the output gap are decomposed into historical shock contributions in order to analyze the Swiss business cycles. The historical shock decomposition of output, natural output and the output gap was computed by switching off all shocks but one. Eight series for each measure corresponding to the evolution of this measure given only the realization of one historical shock series were computed. In a first step these series were aggregated for each quarter and measure and plotted as a stacked bar graph. Since the effect of initial values is neglected, the bars do not add up to the plotted line corresponding to the filtered series with respect to all shocks. However, the effect of initial values vanishes rather quickly and is not of interest.

Figure 9 shows the decomposed DSGE output series. In the beginning of the 1990ies the domestic inflation rate was high compared to the rest of the sample and domestic inflation shocks had a considerable negative impact on output which corresponds well to the historical account on the Swiss economy in chapter 4.4. Moreover, some regularities are worth mentioning. If the output series deviates at least 1.5% from the steady state we see that the shock to foreign output ε_{y^*} acted procyclically. Furthermore, the contribution

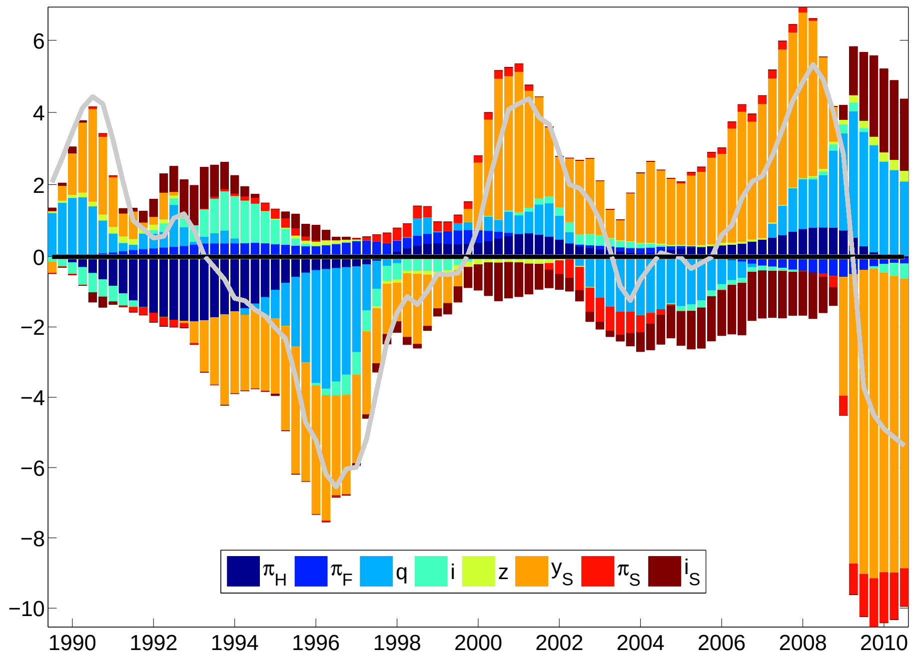


Figure 9: Shock Decomposition of output y (model IND): Contribution of a single shock depicted by the bars. Estimated output series plotted as a grey line.

of the UIP shock also acted procyclically in boom and recession periods with only one exception. Remarkably, this exception occurred during the great recession which has its roots in a financial crisis. This finding will be discussed in detail below.

The shock decomposition of natural output will not be discussed in detail. It does only react to foreign variables and domestic productivity as can be seen in Figure 18. Domestic natural output reacts to foreign interest rates and foreign inflation as the foreign economy is approximated by a VAR and the foreign output level y^* is used to approximate y^{*flex} and z^*.[21] The above mentioned procyclical effect of the UIP shock is even more pronounced with respect to the output gap. However, figure 10 shows also that this procyclical

[21]The results presented here do not depend on this. They still hold when using the structurally defined output gap as given in the appendix but defining $z^* = b_{11} * \varepsilon_{y^*}$ (or $z_t^* = \rho_{z^*} z_{t-1}^* + \varepsilon_{y^*}$), where ε_{y^*} is the identified structural shock from the VAR. Additional to the structural definition of the output gap the equation defining foreign natural output is also implemented in these approaches. However, the results stay qualitatively the same but the log marginal density indicates that the models are unlikely.

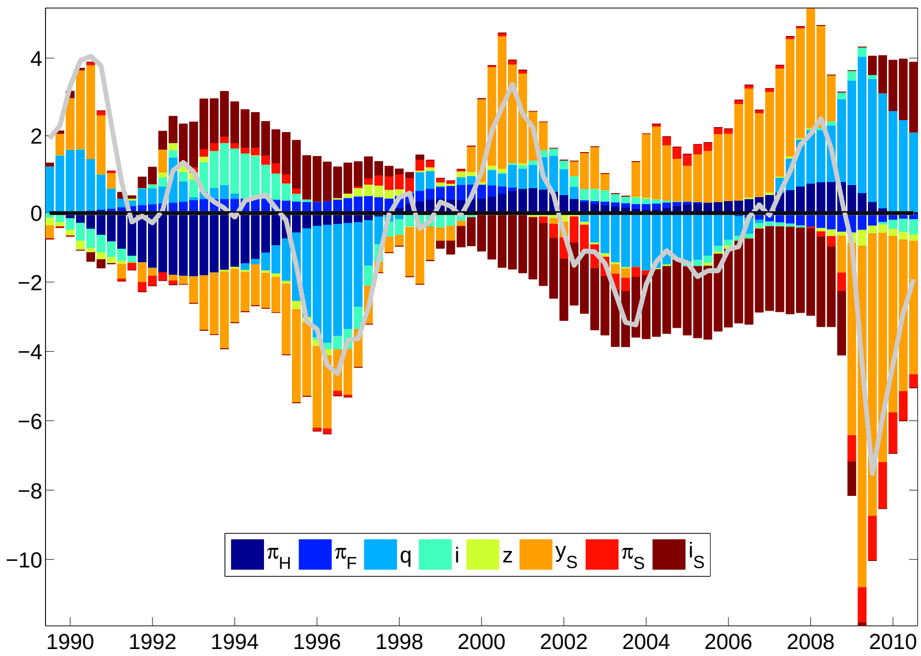


Figure 10: Shock Decomposition of output $\tilde{y}$ (model IND): Contribution of a single shock depicted by the bars. Estimated output gap series plotted as a grey line.

effect of the UIP shock breaks down in the latest recession. Interestingly, although foreign output had a positive effect on the output gap for almost every quarter over the years 2002 to 2006, the output gap remained negative during these years. This is not only due to the procyclical effect of the UIP shock but also due to the huge negative effect of foreign interest rate shocks for the whole first decade of this century. This pronounced negative effect of foreign interest rates on the output gap stems from the negative effect on output itself while natural output would have been positively affected during this period.

Figure 11 makes clear that the output series are mainly dominated by foreign output shocks. This is not a surprise as the Swiss economy heavily depends on international markets. Switzerland not only has a strong export sector but also has a financial sector of high importance. Together with the classical features of a small open economy the domination of output series by foreign output shocks is consistent with conventional wisdom.

The natural output is not depicted in Figure 12 as it does not react to

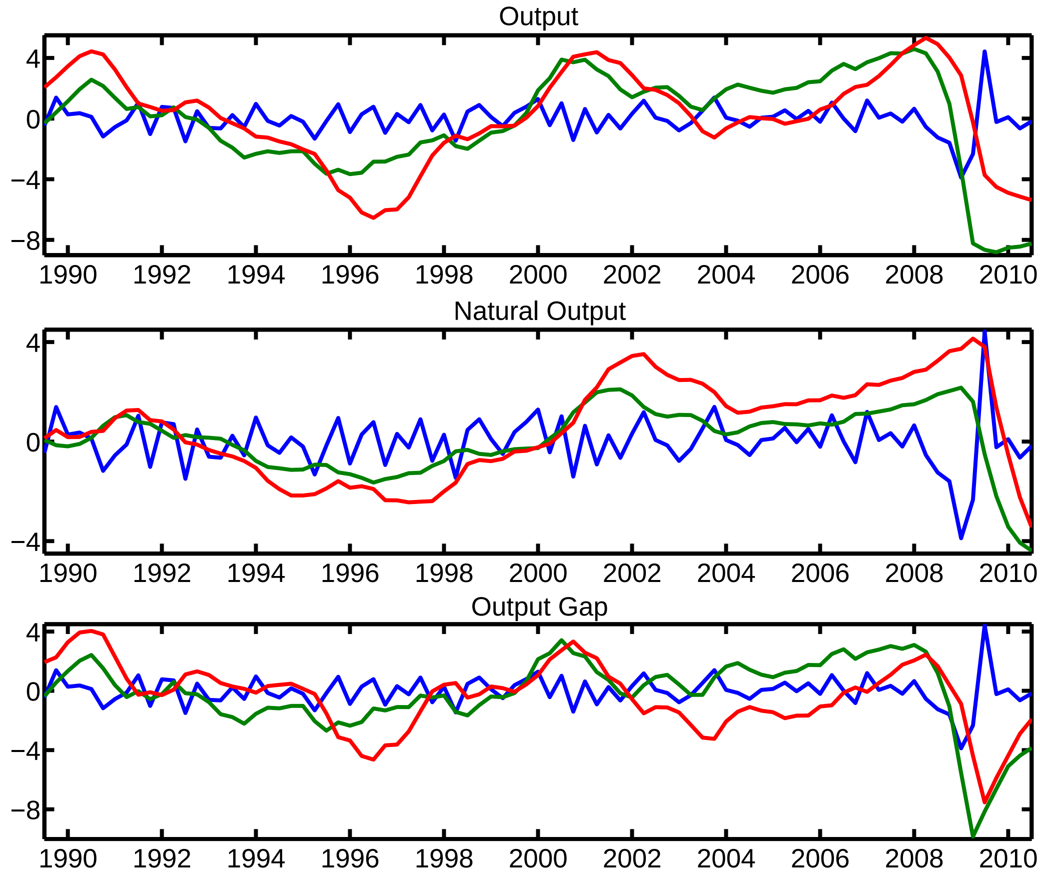


Figure 11: Foreign shock contribution (model IND): The figure shows the historical shock ε_{y^*} (blue), the evolution of the series if only that shock occurred (green) and the filtered series allowing every shock (red).

UIP shocks. The output series and the output gap series exhibit a strong co-movement with the same series only allowing for an UIP shock up to the last recession as already noted above. The breakdown of this strong correlation coincides with a large positive UIP shock. This can be interpreted as a risk premium shock as some parts of the Swiss financial sector came under an especially heavy pressure during the last recession which might have induced the markets to demand higher interest rates. The effect of this demand for an increased risk premium had a positive impact on output and output gap series. Furthermore, it might have dampened the upward pressure on the domestic currency and real exchange rate.

Table 4 confirms the important role of UIP and foreign output shocks for the output gap. While the UIP shock only affects the output gap in the very

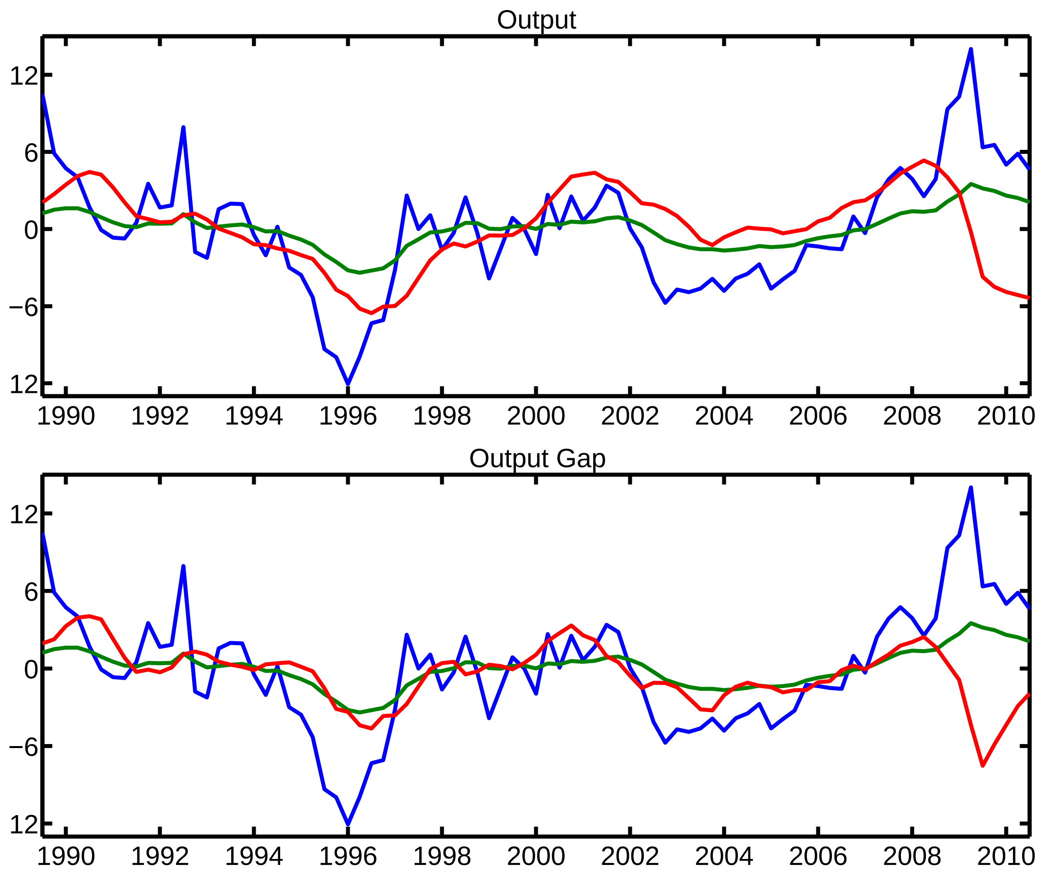


Figure 12: UIP shock contribution (model IND): The figure shows the historical shock ε_q (blue), the evolution of the series if only that shock occurred (green) and the filtered series allowing every shock (red).

short run, the foreign output shock is the main component in the forecast error variance decomposition.

5 Conclusion

Most of the literature relies on pure statistical approaches to estimate the output gap. This has the drawback that the estimate is not based on economic theory. Thus, the estimate based on agnostic statistical approaches does not provide the possibility to analyze the source of the variation in the output gap. Moreover, this estimate will be prone to the Lucas critique.

This paper describes the estimation of the output gap based on a New

h \ shock	ε_{π_H}	ε_{π_F}	ε_q	ε_i	ε_z	ε_{y^*}	ε_{π^*}	ε_{i^*}
1.000	0.000	0.000	0.267	0.004	0.007	0.611	0.004	0.107
4.000	0.003	0.001	0.064	0.005	0.005	0.806	0.012	0.103
8.000	0.011	0.003	0.037	0.007	0.003	0.803	0.008	0.128
12.000	0.019	0.004	0.031	0.007	0.003	0.762	0.012	0.162
16.000	0.022	0.004	0.029	0.007	0.003	0.729	0.017	0.189
20.000	0.022	0.004	0.028	0.007	0.003	0.709	0.020	0.208

Table 4: Forecast error variance decomposition of output gap (model IND) for a selection of horizons (h)

Keynesian small open economy model. This approach has the advantage that it relies on a microeconomic foundation and provides a clear definition of the output gap. The DSGE model exhibits imperfect competition in product markets and sticky prices. Compared to Leist and Neusser (2010) a richer framework was chosen such that external habit persistence is appropriately incorporated and deviations from the law of one price are allowed. Using the output gap in the Taylor rule dominates models where output growth or detrended output is used in the Taylor rule. Hence, output gap measure improves the model fit compared to just using GDP growth or detrended output which contrasts the finding of Adolfson et al. (2008). Furthermore, adding the exchange rate to the Taylor rule and allowing for indexation in the NKPC improves the model fit.

The estimation results of the model with output gap and exchange rate in the Taylor rule as well as indexation suggest that in the absence of price rigidities output would have evolved differently. In the case of Switzerland, the difference between deviation of output from its steady state and deviation of natural output from its steady state lies in the range of approximately -8 to $+4.5$ percent. Booms as well as recessions would have been less pronounced under flexible prices. Furthermore, the estimated output gap corresponds to conventional wisdom about Swiss business cycles and is also partially consistent with some agnostic definitions of recessions coming from a regime switching model or by simply defining a recession as two quarters of negative growth rates in a row.

In contrast to agnostic approaches, the microeconomic foundation of the output gap estimation allows a detailed analysis of recessions. This analysis shows that the output gap is mainly driven by the UIP shock and foreign output shock. The latter finding stays in contrast to Justiniano and Preston (2010a) who find that US disturbances have nearly no effect on Canada and conclude that this contrasts with other empirical work. Therefore they suggest to concentrate on international transmission mechanisms to improve in this dimension. The main difference to their approach lies in the use of a VAR model to approximate the foreign economy instead of using a DSGE model to characterize the foreign economy.

Particularly interesting is the finding that the UIP shock acted procyclically with respect to the output gap over almost the whole sample with the notable exception of the great depression. Therefore, the effect of the increased uncertainty about the large financial sector in Switzerland is reflected in this break up of the comovement of the output gap and the UIP shock. The increased uncertainty actually helped the Swiss economy as it lead to an appreciation of the real exchange rate.

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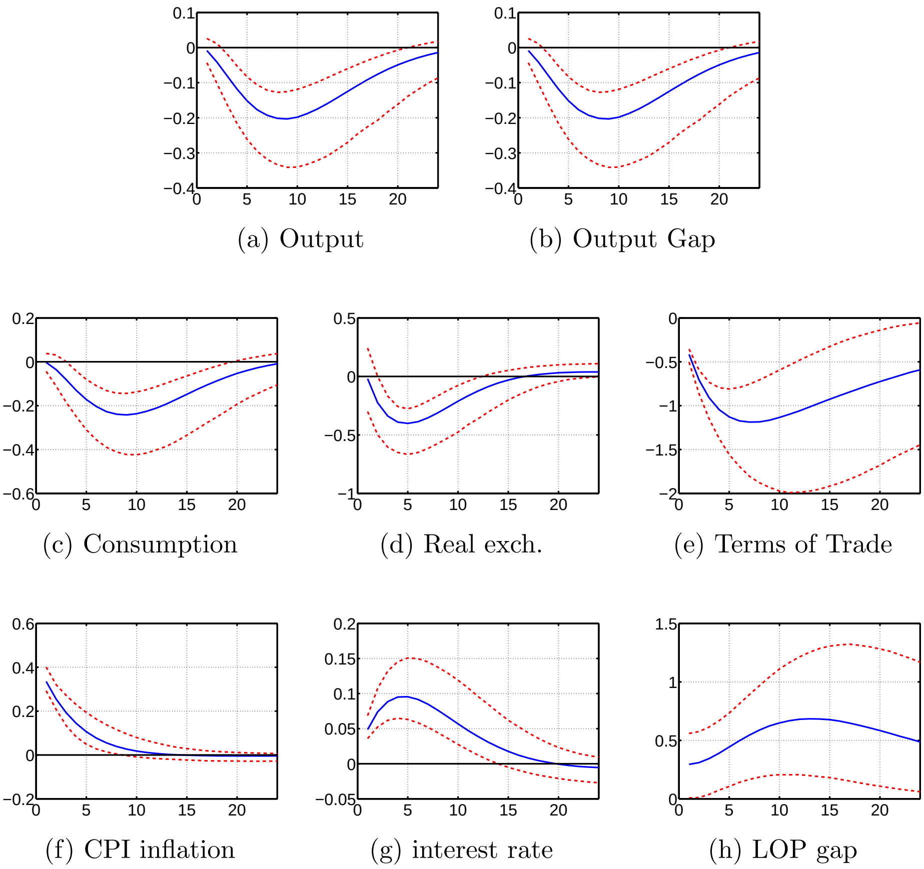
A Data

Series	Sources	Description	Transformation (all data is demeaned after the transformations below)	Relation to model
Import Prices	SNB O2	Monthly index of import prices	$400 \cdot \Delta \log$ averages of monthly data	$4\pi_F$
CPI Inflation	SNB O1_1	Inflation based on CPI Percentage changes to previous year	averages of monthly data	4π
LIBOR	Econstats SNB	3 Month LIBOR CHF	averages of monthly data	$4i$
Real exch. rate	OECD	Relative consumer price indices, 2005=100 Real Effective Exchange Rates (RER)	residuals of regression on a constant and trend multiplied with 100	$-q_t$
GDP	SECO	Chained values of seasonally adjusted quarterly GDP in Mio. Swiss Francs, at prices of preceding year. The reference year is 2000.	$100 \cdot \Delta \log GDP$	$y_t - y_{t-1} + z_t$
GDP OECD	OECD.stat VPVOBARSA	GDP expenditure approach Millions of USD, fixed PPPs OECD reference year, annual levels, seasonally adjusted	residuals of regression on a constant and trend multiplied with 100	y_t^*
Inflation OECD	OECD.stat	Percentage change of CPI on the same period of the previous year		$4\pi^*$
Interests EUR	AWM (STN) OECD.stat	Nominal Short Term Interest Rate		$4i^*$
SNB:	Swiss National Bank			
SECO:	State Secretariat for Economic Affairs SECO			
AWM:	EABCN Area Wide Model			

Table 5: Data

B Impulse Response Functions



(a) Output (b) Output Gap (c) Consumption (d) Real exch. (e) Terms of Trade (f) CPI inflation (g) interest rate (h) LOP gap

Figure 13: Impulse Responses (model IND) to a One Standard Deviation Shock to home goods price inflation with 95-Percent Confidence Interval

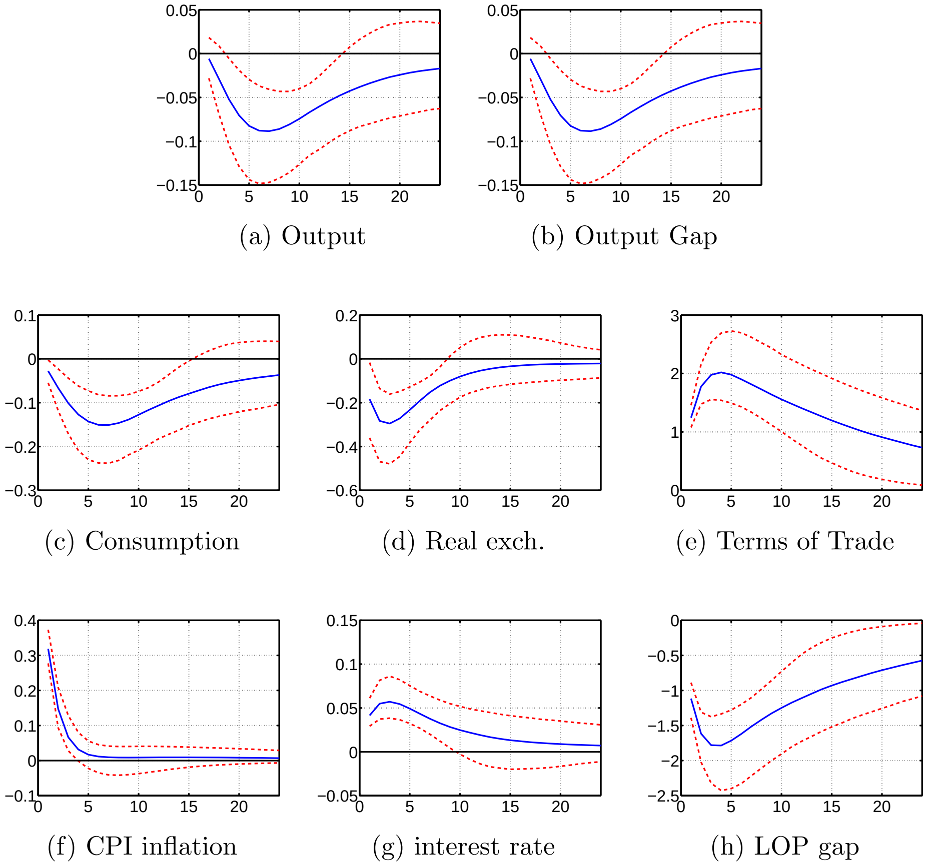
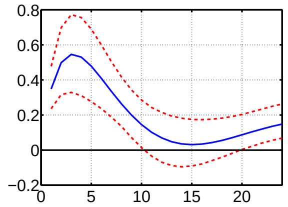
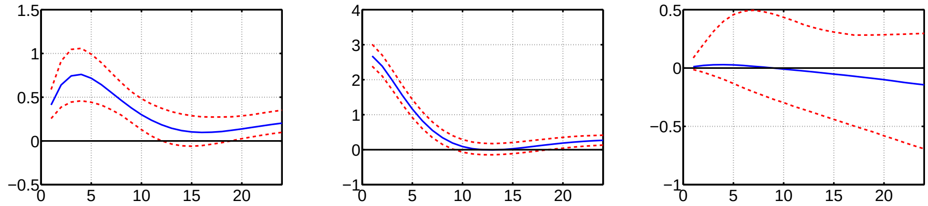


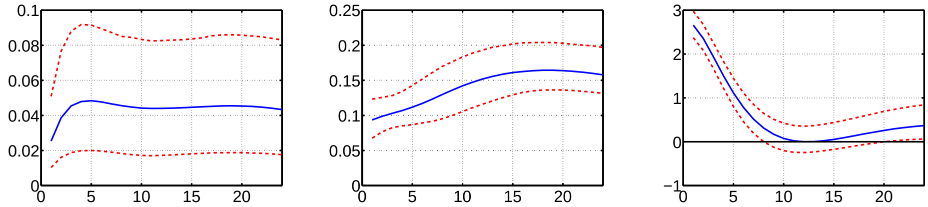
Figure 14: Impulse Responses (model IND) to a One Standard Deviation Shock to imported price inflation with 95-Percent Confidence Interval



(a) Output (b) Natural Output (c) Output Gap

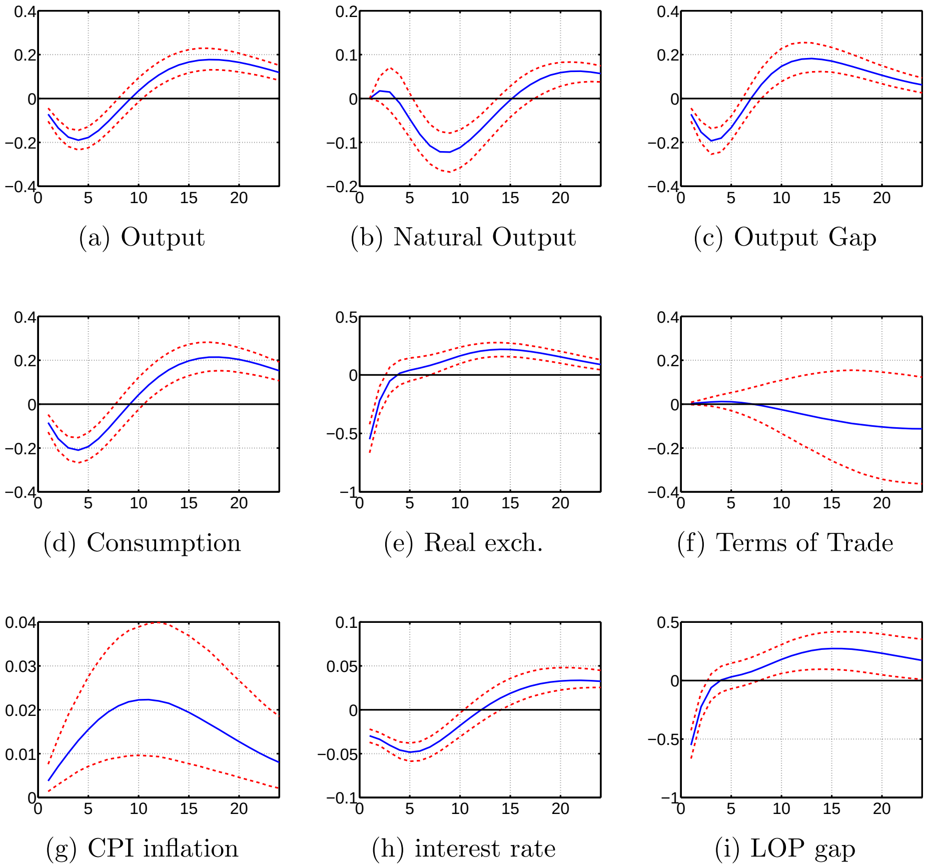


(d) Consumption (e) Real exch. (f) Terms of Trade



(g) CPI inflation (h) interest rate (i) LOP gap

Figure 15: Impulse Responses (model IND) to a One Standard Deviation Shock to foreign interest rates with 95-Percent Confidence Interval



(a) Output (b) Natural Output (c) Output Gap

(d) Consumption (e) Real exch. (f) Terms of Trade

(g) CPI inflation (h) interest rate (i) LOP gap

Figure 16: Impulse Responses (model IND) to a One Standard Deviation Shock to foreign inflation with 95-Percent Confidence Interval

C Recessions and Shock Decompositions

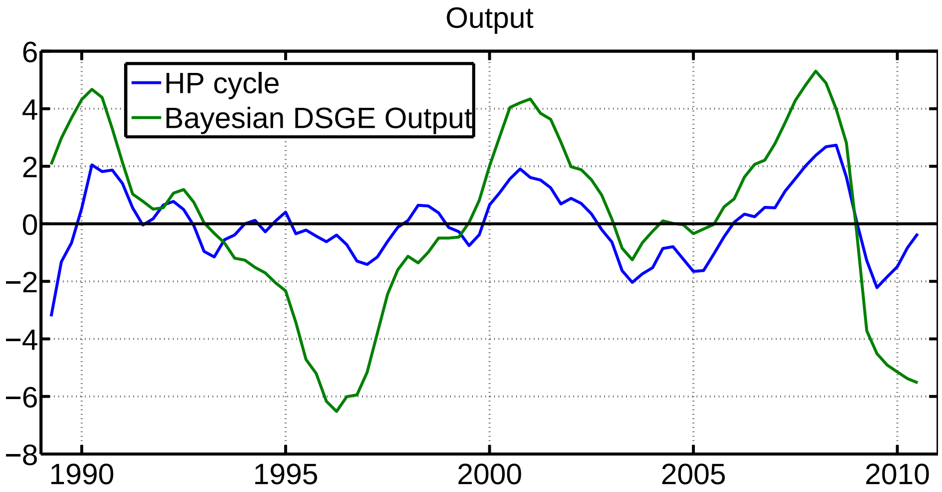


Figure 17: Comparison of cycles (model IND)

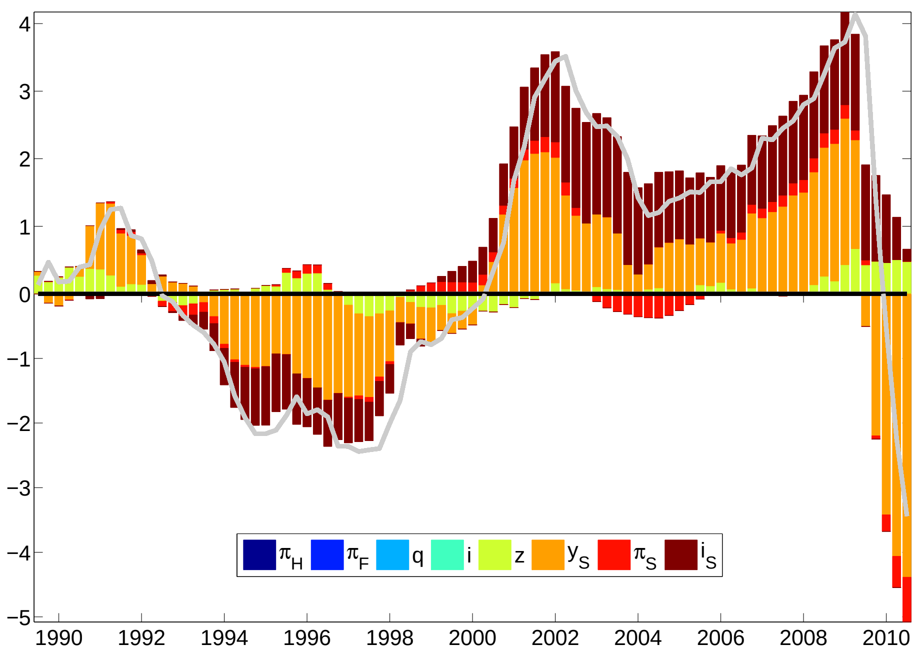


Figure 18: Shock Decomposition of output y^{flex} (model IND): Contribution of a single shock depicted by the bars. Estimated natural output series plotted as a grey line.

D Marginal Likelihood and Bayes Factor

To compute the marginal likelihood draws from the posterior distribution are used. This follows Rabanal and Rubio-Ramirez (2005) and An and Schorfheide (2007) who apply the modified harmonic modified mean approach proposed by Geweke (1999) to compare different DSGE models. The marginal density given a model M_i with i denoting a specific model (BASE, GROW, DETR, EXCH or IND) is defined as:

$$p(\mathbf{Y}|M_i) = \int \mathcal{L}(\mathbf{Y}|\mathbf{\Theta}_i, M_i)\pi(\mathbf{\Theta}_i|M_i)d\mathbf{\Theta}_i$$

It provides information on how well the model is able to reconcile the information given by the data and by the prior distribution. It will be low if the likelihood function does not accord with the prior. The harmonic modified mean approach uses the identity (see Gelfand and Dey (1994))

$$\frac{1}{p(\mathbf{Y}|M_i)} = \int \frac{f(\mathbf{\Theta}_i)}{\mathcal{L}(\mathbf{Y}|\mathbf{\Theta}_i, M_i)\pi(\mathbf{\Theta}_i|M_i)} p(\mathbf{\Theta}_i|\mathbf{Y}, M_i)d\mathbf{\Theta}_i$$

and assumes

$$\begin{aligned}\ln f(\mathbf{\Theta}_i) = & \left[-\ln(\tau) - \frac{K_i}{2}\ln(\frac{2}{\pi}) - \frac{1}{2}\ln(|\mathbf{\Sigma}_{\mathbf{\Theta}_i}|) - \frac{1}{2}\left((\mathbf{\Theta}_i - \bar{\mathbf{\Theta}}_i)'\mathbf{\Sigma}_{\mathbf{\Theta}_i}^{-1}(\mathbf{\Theta}_i - \bar{\mathbf{\Theta}}_i)\right)\right] \\ & \times I\left\{(\mathbf{\Theta}_i - \bar{\mathbf{\Theta}}_i)'\mathbf{\Sigma}_{\mathbf{\Theta}_i}^{-1}(\mathbf{\Theta}_i - \bar{\mathbf{\Theta}}_i) \leq \text{Inv-}\chi^2(\tau, K_i)\right\} \\ & \tau = [0.05; 0.1; 0.25; 0.5; 0.75; 0.9; 0.95];\end{aligned}$$

where K_i is the number of estimated parameters in model i and $\bar{\mathbf{\Theta}}_i$ is the posterior mean. $|\mathbf{\Sigma}_{\mathbf{\Theta}_i}|$ is determinant of the covariance matrix of $\mathbf{\Theta}_i$, the vector containing only the estimated parameters in model i. Inv-$\chi^2(\tau, K)$ is the inverse of a $\chi^2(\tau, K_i)$ distribution with K_i degrees of freedom at the values in τ.

An estimator of the log marginal likelihood is then given by

$$\ln \hat{p}(\mathbf{Y}|M_i) = -\ln\left[\frac{1}{N}\sum_{s=1}^{N}\exp\left\{\ln f(\mathbf{\Theta}_i^{(s)}) - \ln\mathcal{L}(\mathbf{Y}|\mathbf{\Theta}_i^{(s)}) - \ln\pi(\mathbf{\Theta}_i^{(s)})\right\}\right]$$

where N is the number of draws from the posterior distribution.[22] The Bayes

[22] More specifically the implemented estimator is

$$\ln \hat{p}(\mathbf{Y}|M_i) = \tilde{C} - \ln\left[\frac{1}{N}\sum_{s=1}^{N}\exp\left\{\ln f(\mathbf{\Theta}_i^{(s)}) - \ln\mathcal{L}(\mathbf{Y}|\mathbf{\Theta}_i^{(s)}) - \ln\pi(\mathbf{\Theta}_i^{(s)}) + \tilde{C}\right\}\right]$$

where $\tilde{C} \equiv \max(\ln\mathcal{L}(\mathbf{Y}|\mathbf{\Theta}_{\text{EXCH}}))$ is the maximum of the computed log-likelihoods of model EXCH. This term is introduced to make the computation feasible.

Factor comparing model i versus model j with $j \neq i$ is given by:

$$BF_{i,j} = \exp\left\{\ln p(\mathbf{Y}|M_i) - \ln p(\mathbf{Y}|M_j)\right\}$$

If the data provides strong evidence for model i, then the Bayes Factor should be large. Kass and Raftery (1995) provide guidelines based on the natural logarithm to get the same scale as for the likelihood ratio test statistics. They argue that if $(2 \times \ln BF_{i,j}) > 10$ this provides very strong evidence against model j.

E Model Derivation

The model of Monacelli (2005) as presented and extended in Justiniano and Preston (2004) and Beltran (2007) is derived in this section. It is assumed that the small open economy is of negligible size relative to the rest of the world. Therefore, the rest of the world is treated as a closed economy, which implies that output equals consumption and consumer price inflation equals domestic price inflation. Note that for all periods $t \geq 0$ stochastic events $s_t \in S$ are realized. The publicly observable history of realized events up to time t is denote by $s^t = [s_0, s_1, \ldots, s_t]$. The unconditional probability that a particular sequence of events s^t is realized is denoted by $\mathcal{P}(s^t)$ and the conditional probability that event s_{t+1} is realized given the sequence s^t has been realized is given by $\mathcal{P}(s_{t+1}|s^t)$.

E.1 The household

Assume that the representative household in the small open economy and the rest of the world face the same optimization problem. Hence, the optimality condition of the representative household of the domestic world and of the rest of the world correspond to the ones derived below. There is a continuum of infinitely lived households of measure one that consume domestic, $C_H(i)$, and imported, $C_F(i)$, differentiated goods. The composite consumption bundle C is defined as

$$C_t \equiv \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{E.1}$$

where η is the elasticity of substitution between domestic and foreign goods and γ is the share of consumed goods produced by the foreign country. CES aggregators are imposed such that

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{E.2}$$

where ε is the elasticity of substitution between the differentiated goods. To ease the notation the state dependence of these quantities is neglected in the equations above. The preferences of the household are given by

$$\sum_{t=0}^{\infty} \sum_{s_t} \beta^t \mathcal{P}(s_t|s_0) \left[\frac{(C_t(s^t) - H_t(s^t))^{1-\sigma}}{1-\sigma} - \frac{N_t(s^t)^{1+\varphi}}{1+\varphi}\right] \tag{E.3a}$$

or letting $\mathbb{E}_0$ be the mathematical expectations operator[23] denoting $\sum_{s_t} \mathcal{P}(s_t|s_0)$ and using A_t to denote a variable $A_t(s^t)$ we have

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{E.3b}$$

where N_t is labor input, β is the discount factor, σ is the coefficient of relative risk aversion and $H_t = hC_{t-1}$ represents external habit formation. Furthermore, the households face the budget constraint

$$P_t(s^t)C_t(s^t) + \sum_{s_{t+1}} Q_{s^{t+1}|s^t} D_{t+1}(s^{t+1}) \leq W_t(s^t)N_t(s^t) + D_t(s^t) \tag{E.4}$$

where $P_t(s^t)$ is the domestic consumer price index and $W_t(s^t)$ is the nominal wage. $D_{t+1}(s_{t+1})$ denotes the nominal payoff in $t+1$ of the portfolio held at the end of period t and $Q_{s^{t+1}|s^t}$ is the corresponding stochastic discount factor. It is assumed that the household has access to a complete set of contingent claims.

E.1.1 Optimization

The household minimizes costs given any consumption level. Therefore, it first chooses a combination of goods that minimizes costs for a given consumption level.[24] In a next step, as the household is aware of the costs of achieving any level of consumption, he chooses optimally among C_t and N_t. Optimizing implies that the expenditures on all varieties are cost minimizing for any level of the above given consumption bundles $C_{H,t}$ and $C_{F,t}$. Therefore,

$$\min_{C_{H,t}(i)} \int_0^1 \left[P_{H,t}(i) C_{H,t}(i) \right] di$$

$$\text{s.t. } C_{H,t} \leq \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\mathcal{L} = \int_0^1 \left[P_{H,t}(i) C_{H,t}(i) \right] di - P_{H,t} \left(\left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - C_{H,t} \right)$$

[23] Similarly, $\mathbb{E}_t$ is used to denote $\sum_{s_{t+1}} \mathcal{P}(s_{t+1}|s^t)$.

[24] The state dependence can be neglected to ease the exposition.

FOC:

$$P_{H,t}(i) = P_{H,t}\frac{\varepsilon}{\varepsilon-1}\left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{1}{\varepsilon-1}}\frac{\varepsilon-1}{\varepsilon}C_{H,t}(i)^{\frac{-1}{\varepsilon}}$$

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}C_{H,t} \tag{E.5}$$

Plugging this into the CES aggregator of $C_{H,t}$ yields

$$C_{H,t} = \left[\int_0^1 \left(\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}C_{H,t}\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}} \tag{E.6}$$

with $P_{H,t}$ being the index of prices of domestically produced goods. The same arguments yield

$$C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t} \tag{E.7}$$

and

$$P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}.$$

Combining (E.5) with (E.6) we get

$$C_{H,t}(i)P_{H,t}(i) = P_{H,t}(i)\left[\frac{P_{H,t}(i)}{P_{H,t}}\right]^{-\varepsilon}C_{H,t}$$

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di = P_{H,t}C_{H,t}$$

and similarly

$$\int_0^1 P_{F,t}(i)C_{F,t}(i)di = P_{F,t}C_{F,t}.$$

As total expenditures are given by

$$P_tC_t = \int_0^1 \left[P_{H,t}(i)C_{H,t}(i) + P_{F,t}(i)C_{F,t}(i)\right]di = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$$

the optimization

$$\min_{C_{H,t},C_{F,t}} P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$$
$$\text{s.t. } C_t \leq \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$
$$\mathcal{L} = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} - P_t\left(\left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} - C_t\right)$$

yields the following FOC w.r.t. $C_{H,t}$

$$P_{H_t} - P_t[\cdot]^{\frac{1}{\eta-1}}(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{-\frac{1}{\eta}} = 0$$

and using the fact that

$$C_t^{\frac{1}{\eta}} = \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}}$$

we can replace $[\cdot]$ in the FOC to get

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t. \tag{E.8}$$

Similarly, we get:

$$C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

Plugging this into the CES aggregator of C_t we get

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}}\left((1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\right)^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}}\left(\gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

which can be solved for the consumer price index

$$P_t = \left[(1-\gamma)(P_{H,t})^{1-\eta} + \gamma(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{E.9}$$

Maximizing (E.3a) subject to (E.4) and assuming exogenous habit formation we get the following first order conditions

$$\mathcal{P}(s_t|s_0)(C_t(s^t) - H_t(s^t))^{-\sigma} - \lambda_t P_t(s^t) = 0$$
$$-\mathcal{P}(s_t|s_0)N_t(s^t)^{\varphi} + \lambda_t W_t(s^t) = 0$$

where λ_t is the multiplier on the constraint. Combining these first order conditions yields:

$$\frac{N_t(s^t)^{\varphi}}{(C_t(s^t) - H_t(s^t))^{-\sigma}} = \frac{W_t(s^t)}{P_t(s^t)}$$

Furthermore, in the optimum it has to hold that

$$V_{s^{t+1}|s^t} = \beta \left(\frac{(C_{t+1}(s^t) - H_{t+1}(s^t))}{(C_t(s^t) - H_t(s^t))} \right)^{-\sigma} \frac{P_t(s^t)}{P_{t+1}(s^t)} \mathcal{P}(s_{t+1}|s^t)$$

where $V_{s^{t+1}|s^t}$ is the price of an Arrow security in period t. As the households can re-optimize in each period we take expectations conditioned on information available in period t on both sides[25]

$$\frac{N_t^{\varphi}}{(C_t - H_t)^{-\sigma}} = \frac{W_t}{P_t}$$

$$Q_t \equiv \mathbb{E}_t Q_{s^{t+1}|s^t} = \beta \mathbb{E}_t \left(\frac{(C_{t+1} - H_{t+1})}{(C_t - H_t)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{E.10}$$

rearranging the last equation yields

$$1 = \beta R_t \mathbb{E}_t \left(\frac{(C_{t+1} - H_{t+1})}{(C_t - H_t)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{E.11}$$

where $R_t = (1 + i_t) = \frac{1}{Q_t}$ denotes the gross return of a one-period discount bond paying off one unit of domestic currency in $t + 1$.[26]

Taking logs of the intertemporal first order conditions yields

$$\varphi \ln N_t + \sigma \ln (C_t - hC_{t-1}) - \ln W_t + \ln P_t = 0$$
$$\exp (\varphi n_t + \sigma \ln (C_t - hC_{t-1}) - w_t + p_t) = 1$$

and after a first order Taylor approximation we get

$$\varphi \hat{n}_t + \frac{\sigma}{1-h} (\hat{c}_t - h\hat{c}_{t-1}) = \hat{w}_t - \hat{p}_t \tag{E.12}$$

[25] Again, A_t is used to denote a variable $A_t(s^t)$.

[26] The one period gross return is defined as next periods price (including any dividends) divided by the actual price $R_t \equiv (1 + i_t) = \frac{D_{t+1}(s_{t+1})}{\mathbb{E}_t Q_{s^{t+1}|s^t} D_{t+1}(s_{t+1})}$. Therefore, in general we have the consumption based pricing equation in terms of returns given by $\mathbb{E}_t \left[Q_{s^{t+1}|s^t} R_t \right] = 1$. Considering a one-period discount bond paying off one unit of domestic currency in $t + 1$ yields $R_t = \frac{1}{Q_t}$ where R_t is the return of the before mentioned specific (risk-free) bond.

where the hat on the variables denotes deviation from steady state. We proceed similarly for the intratemporal first order condition given by equation (E.11):

$$(\hat{c}_t - h\hat{c}_{t-1}) = E_t\left[\frac{(1-h)}{\sigma}\left(\hat{i}_t - \hat{\pi}_{t+1}\right) + (\hat{c}_{t+1} - h\hat{c}_t)\right] \tag{E.13}$$

Where $\hat{a}_t \equiv \ln\frac{A_t}{A}$ denotes percentage deviation from steady state.

E.2 Firms

Monopolistic competitive firms produce the variety of domestic goods and are owned by consumers. Furthermore, they cannot re-optimize their pricing decision in every period. We assume that they are subject to Calvo-style price setting and their production function is $Y_{H,t}(i) = Z_t N_t(i)$, where Z_t is productivity that is common across all firms. Each firm is able to reset its price with probability $1-\theta_H$ in any given period. If a firm is not able to re-optimize we assume that it sets the price according to the indexation rule

$$P_{H,t}(i) = P_{H,t-1}(i)\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H}$$

taking into account past inflation of domestic goods. We will assume that all firms behave identically and therefore the price index for domestic goods is

$$P_{H,t} = \left[\int_0^{\theta_H} P_{H,t}(i)^{1-\varepsilon}di + \int_{\theta_H}^1 P_{H,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}$$

$$P_{H,t} = \left[\theta_H\left(P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H}\right)^{1-\varepsilon} + (1-\theta_H)\left(\bar{P}_{H,t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{E.14}$$

by equation (E.6)[27] and denoting with $\bar{P}_H$ the price that firms set if they are able to re-optimize. Dividing by $P_{H,t-1}$ yields

$$\Pi_{H,t} = \left[\theta_H\left(P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H}\right)^{1-\varepsilon} + (1-\theta_H)\left(\bar{P}_{H,t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}\frac{1}{P_{H,t-1}} \tag{E.15}$$

[27] $P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}} = \left[\int_0^{\theta_H} P_{H,t}(i)^{1-\varepsilon}di + \int_{\theta_H}^1 P_{H,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}$

and the first order Taylor Approximation around a zero inflation steady state delivers:[28]

$$\pi_{H,t} = \theta_H \delta_H \pi_{H,t-1} + (1-\theta_H)\left(\bar{p}_{H,t} - p_{H,t-1}\right) \tag{E.16}$$

A firm that optimized its price in period t and did not have the opportunity to re-optimize faces the following demand curve in period $t+\tau$

$$\begin{aligned} Y_{H,t+1}(i) &= C_{H,t+1}(i) + C^*_{H,t+1}(i) \\ Y_{H,t+1}(i) &= \left(\frac{P_{H,t+1}(i)}{P_{H,t+1}}\right)^{-\varepsilon}\left(C_{H,t+1} + C^*_{H,t+1}\right) \end{aligned}$$

Now assuming that last re-optimization occured in t

$$Y_{H,t+1|t}(i) = \left(\frac{\bar{P}_{H,t}(i)\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\delta_H}}{P_{H,t+1}}\right)^{-\varepsilon}\left(C_{H,t+1} + C^*_{H,t+1}\right)$$

$$Y_{H,t+\tau|t}(i) = \left(\frac{\bar{P}_{H,t}(i)}{P_{H,t+\tau}}\left(\frac{P_{H,t+\tau-1}}{P_{H,t-1}}\right)^{\delta_H}\right)^{-\varepsilon}\left(C_{H,t+\tau} + C^*_{H,t+\tau}\right) \tag{E.17}$$

where it is assumed that the foreign demand for the home good, $C^*_{H,t+1}$, is the same as demand in home country (equation (E.5)).[29] We will assume that the firms behave identically and consider the symmetric equilibrium. Therefore, the expected discounted profit for a firm that re-optimizes in t is

$$\sum_{\tau=0}^{\infty} \mathbb{E}_t \theta_H^\tau Q_{t,t+\tau}\left(\bar{P}_{H,t+\tau|t}\left(\frac{P_{H,t+\tau-1}}{P_{H,t-1}}\right)^{\delta_H} Y_{H,t+\tau|t} - \underbrace{\frac{W_{t+\tau}}{Z_{t+\tau}} Y_{H,t+\tau|t}}_{TC_t(Y_{t+\tau|t})}\right)$$

$$s.t. \quad Y_{H,t+\tau|t} = \left(\frac{\bar{P}_{H,t}}{P_{H,t+\tau}}\left(\frac{P_{H,t+\tau-1}}{P_{H,t-1}}\right)^{\delta_H}\right)^{-\varepsilon}\left(C_{H,t+\tau} + C^*_{H,t+\tau}\right)$$

The first order condition with respect to $\bar{P}_{H,t}$ yields

$$\begin{aligned} \sum_{\tau=0}^{\infty} \mathbb{E}_t \theta_H^\tau Q_{t,t+\tau}\left((1-\varepsilon)\left(\frac{P_{H,t+\tau-1}}{P_{H,t-1}}\right)^{\delta_H} Y_{H,t+\tau|t} + \varepsilon \frac{\partial TC_t(Y_{t+\tau|t})}{\partial Y_{H,t+\tau|t}} Y_{H,t+\tau|t} \frac{1}{\bar{P}_{H,t}}\right) &= 0 \\ \sum_{\tau=0}^{\infty} \mathbb{E}_t \theta_H^\tau Q_{t,t+\tau} Y_{H,t+\tau|t}\left((1-\varepsilon)\left(\frac{P_{H,t+\tau-1}}{P_{H,t-1}}\right)^{\delta_H} + \varepsilon \frac{1}{\bar{P}_{H,t}} \varsigma_{t+\tau|t}\right) &= 0 \end{aligned}$$

[28] The hat notation on prices and inflation is neglected to ease readability.

[29] $P_{H,t+3}(i) = P_{H,t+2}(i)\left(\frac{P_{H,t+2}}{P_{H,t+1}}\right)^{\delta_H} = \ldots = P_{H,t}(i)\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\delta_H}\left(\frac{P_{H,t+2}}{P_{H,t}}\right)^{\delta_H}$

with the real marginal costs $MC_{t+\tau|t} = \frac{\varsigma_{t+\tau|t}}{P_{H,t+\tau}} = \frac{W_{t+\tau}}{P_{H,t+\tau}Z_{t+\tau}}$ derived from the total cost function $TC \equiv \min_{N_t} W_t N_t$ s.t. $Y_t \leq Z_t N_t$ which yields nominal marginal costs $\varsigma_{t+\tau|t} = \frac{\partial TC_t\tau(Y_{t+\tau|t})}{\partial Y_{t+\tau}} = W_{t+\tau}/Z_{t+\tau}$. Simplifying the expression above yields:

$$\sum_{\tau=0}^{\infty} \mathbb{E}_t \theta_H^\tau Q_{t,t+\tau} Y_{H,t+\tau|t} \left(\frac{\bar{P}_{H,t}}{P_{H,t-1}} \left(\underbrace{\frac{P_{H,t+\tau-1}}{P_{H,t-1}}}_{\equiv \Pi_{H,t-1,t+\tau-1}} \right)^{\delta_H} - \frac{\varepsilon}{\varepsilon - 1} MC_{t+\tau|t} \underbrace{\frac{P_{H,t+\tau}}{P_{H,t-1}}}_{\equiv \Pi_{H,t-1,t+\tau}} \right) = 0 \tag{E.18}$$

In the zero inflation steady state, $\frac{P^*_{H,t}}{P_{H,t-1}} = 1$, $\Pi_{t-1,t+\tau} = 1$ and $P_{H,t} = P_{H,t+\tau}$. From the last information it follows that $Y_{H,t+\tau|t} = Y$ (in combination with equation (E.17)) and $MC_{t+\tau|t} = MC$. Finally, combining these insights with (E.10) evaluated at the steady state we get that $Q_{t,t+\tau} = \beta^\tau$ and for condition (E.18) to hold we have $MC = \frac{\varepsilon - 1}{\varepsilon}$ in steady state. Thus, defining the logarithm of the steady state marginal costs as $\mu \equiv \ln \frac{\varepsilon}{1-\varepsilon} = -mc$ taking logs of the variables on the LHS of equation (E.18) gives

$$\begin{aligned} \sum_{\tau=0}^{\infty} \mathbb{E}_t (\theta_H \beta)^\tau \big[&\exp\left(\bar{p}_{H,t} - p_{H,t-1} + \delta_H \left(p_{H,t+\tau-1} - p_{H,t-1}\right)\right) \\ &- \exp\left(\mu + mc_{t+\tau|t}\right) \exp\left(p_{H,t+\tau}(i) - p_{H,t-1}(i)\right) \big] \end{aligned}$$

and a first order Taylor expansion of the terms in brackets around the zero inflation steady state yields

$$\begin{aligned} \exp\left(\bar{p}_{H,t} - p_{H,t-1} + \delta_H \left(p_{H,t+\tau-1} - p_{H,t-1}\right)\right) &\approx 1 + \bar{p}_{H,t}(i) - p_{H,t-1} \\ &\quad + \delta_H \left(p_{H,t+\tau-1} - p_{H,t-1}\right) \\ \exp\left(\mu + mc_{t+\tau|t} + p^*_{H,t+\tau} - p_{H,t-1}\right) &\approx -1 - \left(\mu + mc_{t+\tau|t} + p_{H,t+\tau} - p_{H,t-1}\right) \end{aligned}$$

and substituting this back gives:

$$\begin{aligned} 0 = \sum_{\tau=0}^{\infty} \mathbb{E}_t (\theta_H \beta)^\tau \big(&\bar{p}_{H,t}(i) - p_{H,t-1} + \delta_H \left(p_{H,t+\tau-1} - p_{H,t-1}\right) \\ &- \left(\mu + mc_{t+\tau|t} + p_{H,t+\tau} - p_{H,t-1}\right) \big) \end{aligned}$$

and therefore[30]

$$0 = \sum_{\tau=0}^{\infty} \mathbb{E}_t (\theta_H \beta)^\tau \Bigg(\bar{p}_{H,t}(i) - p_{H,t-1} + \delta_H \pi_{H,t+\tau} - \left(\mu + mc_{t+\tau|t} + (1-\delta_H) \sum_{k=0}^{\tau} \pi_{H,t+k} \right) \Bigg)$$

where the log of the desired gross mark-up under flexible prices (or no inflation) is $\mu = -mc$. Hence, we can write

$$\bar{p}_{H,t} - p_{H,t-1} = (1-\beta\theta_H) \sum_{\tau=0}^{\infty} (\beta\theta_H)^\tau \mathbb{E}_t \left\{ \delta_H \pi_{H,t+\tau} + \widehat{mc}_{t+\tau|t} + (1-\delta_H) \sum_{k=0}^{\tau} \pi_{H,t+k} \right\} \quad \text{(E.19)}$$

This can be written as[31]

$$\bar{p}_{H,t} - p_{H,t-1} = \sum_{\tau=0}^{\infty} (\beta\theta_H)^\tau \mathbb{E}_t \left\{ (1-\beta\theta_H) \widehat{mc}_{t+\tau} + (1-\delta_H\theta_H\beta) \pi_{H,t+\tau} \right\}$$

using the constant returns to scale of the production function which implies $mc_{t+k|t} = mc_{t+k}$.[32] Rewriting the condition as a difference equation

$$\bar{p}_{H,t} - p_{H,t-1} = (1-\beta\theta_H)\widehat{mc}_t + (1-\delta_H\theta_H\beta)\pi_{H,t} + \beta\theta_H \mathbb{E}_t \left(\bar{p}_{H,t+1} - p_{H,t} \right)$$

[30]Note that

$$\sum_{\tau=0}^{\infty} \delta_H \left(p_{H,t+\tau-1} - p_{H,t-1} \right) - \left(p_{H,t+\tau} - p_{t-1} \right) = \delta_H \sum_{\tau=0}^{\infty} \sum_{k=0}^{\tau} \pi_{H,t+k} - \delta_H \sum_{\tau=0}^{\infty} \pi_{H,t+k} - \sum_{\tau=0}^{\infty} \sum_{k=0}^{\tau} \pi_{H,t+k}$$

[31]

$$\begin{aligned}
\sum_{\tau=0}^{\infty} \left(\delta_H \pi_{H,t+\tau} + (1-\delta_H) \sum_{k=0}^{\tau} \pi_{H,t+k} \right) &= \delta_H \pi_{H,t} + (1-\delta_H)\pi_{H,t} \\
&\quad + \beta\theta(1-\delta_H)\pi_{H,t} + \delta_H \pi_{H,t+1} + \beta\theta(1-\delta_H)\pi_{H,t+1} \\
&\quad + (\beta\theta)^2 (1-\delta_H)\pi_{H,t} \qquad + (\beta\theta)^2 (1-\delta_H)\pi_{H,t+1} \\
&= \frac{\delta_H(1-\beta\theta) + (1-\delta_H)}{1-\beta\theta} \pi_{H,t} + \frac{\delta_H - \beta\theta\delta_H + 1 - \delta_H}{1-\beta\theta} \pi_{H,t+1} \\
&= \sum_{\tau=0}^{\infty} \frac{1-\beta\theta\delta_H}{1-\beta\theta} \pi_{H,t+\tau}
\end{aligned}$$

[32]See e.g. (Galí, 2008, p. 46-47)

and plugging in equation (E.16):

$$\frac{1}{1-\theta_H}\left(\pi_{H,t}-\theta_H\delta_H\pi_{H,t-1}\right)=\Bigg((1-\beta\theta_H)\widehat{mc}_t+(1-\delta_H\theta_H\beta)\pi_{H,t}$$
$$+\beta\theta_H\mathbb{E}_t\frac{1}{1-\theta_H}\left(\pi_{H,t+1}-\theta_H\delta_H\pi_{H,t}\right)\Bigg)$$

Some rearrangement yields:

$$\pi_{H,t}=\delta_H\pi_{H,t-1}+\frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}\widehat{mc}_t+\beta\mathbb{E}_t\left(\pi_{H,t+1}-\delta_H\pi_{H,t}\right) \quad \text{(E.20)}$$

Note that the law of one price is assumed to hold for the export price $P_{H,t}^F$ and that firms can reset it in every period.

E.2.1 Retailers

In contrast to the flexible price assumption on $P_{H,t}^F$ we assume that retail firms importing differentiated foreign goods have some degree of pricing power as we assume monopolistic competition. Therefore, the law of one price does not hold any more as the retail firms importing the foreign goods can price a mark-up over there costs. The wedge introduced by this mark-up is defined as

$$\Psi_{F,t}=\frac{S_t P_{F,t}^*}{P_{F,t}}$$

where S_t is the nominal exchange rate (domestic currency price of one foreign unit). It describes the wedge between prices paid by importing firms ($S_t P_{F,t}^*$) and the price paid by consumers ($P_{F,t}$). Similar to domestic producers the retail firms are subject to Calvo pricing, where θ_F denotes the fraction of firms not able to reset prices. A retailer able to adjust prices in t maximizes

$$\sum_{\tau=0}^{\infty}\mathbb{E}_t\theta_F^{\tau}Q_{t,t+\tau}Y_{F,t+\tau}(i)\left(\bar{P}_{F,t+\tau|t}(i)\left(\frac{P_{F,t+\tau-1}}{P_{F,t-1}}\right)^{\delta_F}-S_{t+\tau}P_{F,t+\tau}^*\right)$$

subject to

$$Y_{F,t+\tau|t}(i)=\left(\frac{\bar{P}_{F,t}(i)}{P_{F,t+\tau}}\left(\frac{P_{F,t+\tau-1}}{P_{F,t-1}}\right)^{\delta_F}\right)^{-\varepsilon}C_{F,t+\tau}$$

which yields the first order condition

$$\sum_{\tau=0}^{\infty}\mathbb{E}_t\theta_H^{\tau}Q_{t,t+\tau}Y_{F,t+\tau}(i)\left((1-\varepsilon)\left(\frac{P_{F,t+\tau-1}}{P_{F,t-1}}\right)^{\delta_F}+\varepsilon S_{t+\tau}P_{F,t+\tau}^*\frac{1}{\bar{P}_{F,t}(i)}\right)=0$$

Since the imports of the small open economy are negligible, we can treat $P^*_{F,t}$ as given. Furthermore, we see that in the zero inflation steady state $\Psi_F = \frac{\varepsilon-1}{\varepsilon}$ must hold for the above condition to be valid. Similar reasonings as for the domestic producers let us derive the hybrid Phillips Curve:

$$0 = \sum_{\tau=0}^{\infty} \mathbb{E}_t(\theta_F\beta)^\tau \left(\bar{p}_{F,t}(i) - p_{H,t-1} + \delta_F\pi_{F,t+\tau} - \left(\psi_{F,t} + (1-\delta_F)\sum_{k=0}^{\tau} \pi_{F,t+k} \right) \right)$$

Similar to the domestic firms we can derive an expression for foreign inflation as in equation (E.16)

$$\pi_{F,t} = \theta_F\delta_F\pi_{F,t-1} + (1-\theta_F)\left(\bar{p}_{F,t} - p_{F,t-1}\right) \tag{E.21}$$

and use this to substitute $\bar{p}_{F,t} - p_{F,t-1}$ out to get

$$\pi_{F,t} = \delta_F\pi_{F,t-1} + \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F}\psi_{F,t} + \beta\mathbb{E}_t\left(\pi_{F,t+1} - \delta_F\pi_{F,t}\right) \tag{E.22}$$

where $\psi_{F,t} = s_t + p^*_{F,t} - p_{F,t}$ is the deviation of the log law of one price gap from its steady state.[33]

E.2.2 Equilibrium

The assumption of complete asset markets implies perfect risk-sharing and therefore the first order condition of the domestic household holds also for the foreign household

$$\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t(s^t)}{P_{t+1}(s^{t+1})} = Q_{s^{t+1}|s^t} = \beta\frac{\lambda^*_{t+1}}{\lambda^*_t}\frac{P_t(s^t)^*S_t(s^t)}{P^*_{t+1}(s^{t+1})S_{t+1}(s^{t+1})}$$

where the price and payoff of the security is denoted in the domestic currency. Hence, we have the exchange rate S in the condition of the foreign household. This implies that

$$\mathbb{E}_t\left(\frac{(C_{t+1}-H_{t+1})}{(C_t-H_t)}\right)^{-\sigma}\frac{P_t}{P_{t+1}} = \mathbb{E}_t\left(\frac{(C^*_{t+1}-H^*_{t+1})}{(C^*_t-H^*_t)}\right)^{-\sigma}\frac{P^*_t}{P^*_{t+1}}\frac{S_t}{S_{t+1}}$$

where the state dependence has again been neglected for notational simplicity. Iterating as given in Chari et al. (2002) or Galí (2008) yields

$$\left(\frac{(C_t-H_t)}{(C_0-H_0)}\right)^{-\sigma}\frac{P_0}{P_t} = \left(\frac{(C^*_t-H^*_t)}{(C^*_0-H^*_0)}\right)^{-\sigma}\frac{P^*_0}{P^*_t}\frac{S_0}{S_t}$$

[33] Again, for readability the hat notation on prices and inflation is neglected.

and some rearrangement gives:

$$S_t \frac{P_t^*}{P_t} = \left(\frac{(C_t^* - H_t^*)}{(C_t - H_t)} \right)^{-\sigma} \underbrace{\left(\frac{(C_0 - H_0)}{(C_0^* - H_0^*)} \right)^{-\sigma} \frac{P_0^*}{P_0} S_0}_{\equiv \kappa}$$

After assuming symmetric initial conditions with zero net foreign asset holdings and ex-ante identical environments (corresponds to setting the constant $\kappa = 1$ (Galí, 2008)) we get

$$\hat{c}_t - h\hat{c}_{t-1} = \hat{c}_t^* - h\hat{c}_{t-1}^* + (1-h)\frac{1}{\sigma}\hat{q}_t \tag{E.23}$$

after log-linearization around the steady state. Moreover, the equilibrium price in domestic currency of a risk free bond paying one unit of foreign currency in the next period denominated in foreign currency is $\frac{1}{1+i_t^*} S_t$ and the expected return is S_{t+1}. Therefore,

$$R_t = \frac{1+i_t^*}{S_t} \mathbb{E}_t \left[S_{t+1} \right]$$

or

$$(1+i_t) = \mathbb{E}_t \frac{S_{t+1}}{S_t} (1+i_t^*)$$

where log-linearization yields the uncovered interest rate parity (UIP) condition

$$\hat{i}_t = \hat{i}_t^* + \mathbb{E}_t \Delta \hat{s}_{t+1}.$$

Let us define the real exchange rate as $\tilde{Q}_t \equiv S_t \frac{P_t^*}{P_t}$. Then, log-linearization yields

$$\hat{q}_t = \hat{s}_t + p_t^* - p_t$$

where $\ln \tilde{Q}_t \equiv q_t$ or

$$\Delta \hat{q}_t = \Delta \hat{s}_t + \pi_t^* - \pi_t$$

which allows us to rewrite the UIP condition as

$$\hat{i}_t = \hat{i}_t^* + E_t \left[\Delta \hat{q}_{t+1} + \pi_{t+1} - \pi_{t+1}^* \right] + \varepsilon_{q,t} \tag{E.24}$$

where $\varepsilon_{q,t}$ is an exogenous risk premium shock. Log-linearization of the CPI around the symmetric steady state under the assumption of $\eta = 1$ such that $P_t = (P_{H,t})^{1-\gamma} (P_{F,t})^{\gamma}$ yields:

$$p_t = (1-\gamma) p_{H,t} + \gamma p_{F,t} \tag{E.25}$$

Note that this allows us to rewrite the definition of the real exchange rate

$$\hat{q}_t = \hat{s}_t + p_t^* - p_t$$

to get

$$\hat{q}_t = \psi_{F,t} - (1-\gamma)\underbrace{(p_{H,t} - p_{F,t})}_{\equiv -\hat{x}_t} \tag{E.26}$$

where x_t is the log terms of trade. Taking the difference we can express it in terms of the inflation rates:

$$\Delta\hat{x}_t = \pi_{F,t} - \pi_{H,t} \tag{E.27}$$

Furthermore, we can rewrite equation (E.23)

$$\hat{c}_t - h\hat{c}_{t-1} = \hat{y}_t^* - h\hat{y}_{t-1}^* + \frac{1-h}{\sigma}\left(\psi_{F,t} + (1-\gamma)\hat{x}_t\right) \tag{E.28}$$

to see that changes in the law of one price gap affect the relative consumption baskets. To derive an expression for the marginal costs we use the log-linearized marginal costs

$$\widehat{mc}_t = \hat{w}_t - p_{H,t} - z_t$$

and equation E.12 as well as the definition of the terms of trade to get:

$$\begin{aligned}
\widehat{mc}_t &= \hat{w}_t - p_{H,t} + (1-\gamma)p_{H,t} + \gamma p_{F,t} - p_{H,t} - z_t \\
\widehat{mc}_t &= \varphi(\hat{y}_t - z_t) + \frac{\sigma}{1-h}\left(\hat{c}_t - h\hat{c}_{t-1}\right) + \gamma x_t - z_t \\
\widehat{mc}_t &= \varphi\hat{y}_t + \frac{\sigma}{1-h}\left(\hat{c}_t - h\hat{c}_{t-1}\right) + \gamma\hat{x}_t - (1+\varphi)z_t
\end{aligned} \tag{E.29}$$

Using equation (E.25) and the definition of terms of trade we can link CPI and domestic inflation:

$$\begin{aligned}
\pi_t &= (1-\gamma)\pi_{H,t} + \gamma\pi_{F,t} \\
\pi_t &= \pi_{H,t} + \gamma\hat{x}_t
\end{aligned} \tag{E.30}$$

Aggregating the market clearing condition $y_t(i) = (1-\gamma)c_{H,t}(i) + \gamma c_{H,t}^*(i)$ yields

$$\hat{y}_t = (1-\gamma)\hat{c}_{H,t} + \gamma\hat{c}_{H,t}^*$$

denoted in deviations from steady state. The log-linearized version of equation (E.8) is given by

$$\hat{c}_{H,t} = -\eta(\hat{p}_{H,t} - \hat{p}_t) + \hat{c}_t$$

and by assuming $\eta = 1$ substituting $p_{H,t} - p_t = p_{H,t} - (1-\gamma)p_{H,t} - \gamma p_{F,t} = \gamma(p_{H,t} - p_{F,t}) = -\gamma \hat{x}_t$ yields:

$$\hat{c}_{H,t} = \gamma\eta\hat{x}_t + \hat{c}_t$$

For the foreign demand of the home good we get:

$$\hat{c}^*_{H,t} = -\eta(p^*_{H,t} - p^*_t) + \hat{c}^*_t$$

As the LOP is expected to hold for export prices we have $p^*_{H,t} = p_{H,t}$, assuming the foreign economy is large $p^*_{F,t} = p^*_t$. P^*_t is multiplied with S_t to denominate the goods in home currency:[34]

$$\hat{c}^*_{H,t} = \eta(\hat{x}_t + \psi_{F,t}) + \hat{c}^*_t$$

By plugging these derived demands for $\hat{c}_{H,t}$ and $\hat{c}^*_{H,t}$ into the aggregate market clearing condition we get:

$$\begin{aligned} \hat{y}_t &= (1-\gamma)(\gamma\eta\hat{x}_t + \hat{c}_t) + \gamma(\eta(\hat{x}_t + \psi_{F,t}) + \hat{c}^*_t) \\ \hat{y}_t &= (1-\gamma)\hat{c}_t + (2-\gamma)\gamma\eta\hat{x}_t + \gamma\eta\psi_{F,t} + \gamma\hat{y}^*_t \end{aligned} \tag{E.31}$$

In order to close the model we assume a simple Taylor rule

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1-\rho_i)(\psi_\pi \pi_t + \psi_{\tilde{y}}\tilde{y}_t) + \varepsilon_{i,t} \tag{E.32}$$

where the natural output gap $\tilde{y}_t$ will be derived below and $\varepsilon_{i,t}$ is an exogenous i.i.d. shock.

E.2.3 Foreign Economy

In what follows, we will assume that preferences and technology in the foreign country are the same as in the home country. Furthermore, we will assume that the foreign economy is large. Therefore, it can be characterized by a

[34] Assuming symmetric preferences for the foreign economy we have

$$C^*_{H,t} = \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t$$

and therefore we have $c^*_{H,t} = \eta\left(p^*_t - p^*_{H,t}\right) - c^*_t = \eta\left(p^*_t + s_t - p_{F,t} + p_{F,t} - p_{H,t}\right) - c^*_t$ where $c^*_{H,t}$ is now denominated in home the currency.

closed economy model ($\pi^*_{H,t} = \pi^*, c^*_t = y^*_t$) with the following equations (see Home economy equations E.13, E.20 and E.29)

$$(\hat{c}^*_t - h\hat{c}^*_{t-1}) = \mathbb{E}_t \left[\frac{(1-h)}{\sigma} \left(\hat{i}^*_t - \hat{\pi}^*_{t+1} \right) + \left(\hat{c}^*_{t+1} - h\hat{c}^*_t \right) \right] \tag{E.33}$$

$$\pi_t = \delta^* \pi^*_{t-1} + \frac{(1-\theta^*)(1-\beta^*\theta^*)}{\theta^*} \widehat{mc}^*_t + \beta \mathbb{E}_t \left(\pi^*_{t+1} - \delta^* \pi^*_t \right) + \varepsilon_{\pi^*,t} \tag{E.34}$$

$$\widehat{mc}^*_t = \varphi \hat{y}^*_t + \frac{\sigma}{1-h} \left(\hat{y}^*_t - h\hat{y}^*_{t-1} \right) - (1+\varphi) z^*_t \tag{E.35}$$

$$\hat{i}^*_t = \rho_i \hat{i}^*_{t-1} + (1-\rho_i)(\psi_\pi \pi^*_t + \psi_{\tilde{y}} \tilde{y}^*_t) + \varepsilon_{i^*,t} \tag{E.36}$$

where $\varepsilon_{\pi^*,t}$ and $\varepsilon_{i^*,t}$ are exogenous i.i.d. shocks. Finally, we have

$$z^*_t = \rho_z z^*_{t-1} + \varepsilon_{z^*,t}$$

and the foreign natural output gap is given by:

$$\begin{aligned} \tilde{y}^*_t &\equiv y^*_t - y^{*flex}_t \\ \hat{y}^{*flex}_t &= C z^*_t + D y^{*flex}_{t-1} \\ C &= \frac{(1+\varphi)(1-h)}{\varphi(1-h)+\sigma} \\ D &= \frac{\sigma h}{\varphi(1-h)+\sigma} \end{aligned}$$

E.2.4 Deriving the domestic natural output gap

This derivation is performed as described in Beltran (2007). Under perfectly flexible prices the deviation of the marginal costs from the constant steady state markup is zero. Therefore, we can write the flexible mc equation as,

$$0 = \varphi \hat{y}^{flex}_t + \frac{\sigma}{1-h} \left(\hat{c}^{flex}_t - h\hat{c}^{flex}_{t-1} \right) + \gamma \hat{x}^{flex}_t - (1+\varphi) z_t \tag{E.37}$$

and

$$\hat{c}^{flex}_t - h\hat{c}^{flex}_{t-1} = \hat{y}^{*flex}_t - h\hat{y}^{*flex}_{t-1} + \frac{1-h}{\sigma}(1-\gamma)\hat{x}^{flex}_t \tag{E.38}$$

and the flexible price market clearing condition as:

$$\hat{y}^{flex}_t = (1-\gamma)\hat{c}^{flex}_t + (2-\gamma)\gamma\eta \hat{x}^{flex}_t + \gamma \hat{y}^{*flex}_t \tag{E.39}$$

Similarly we get the flexible price marginal cost equation for the foreign economy

$$0 = \varphi \hat{y}_t^{*flex} + \frac{\sigma}{1-h}\left(\hat{y}_t^{*flex} - h\hat{y}_{t-1}^{*flex}\right) - (1+\varphi)z_t^* \tag{E.40}$$

Using these equations we can derive the flexible price output gap and the flexible price terms of trade. From equation (E.37), (E.38) and (E.40) the output level under flexible prices can be derived. Equation (E.39) and the equation for the output under flexible prices are used to derive the terms of trade under flexible prices.[35]

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^{flex} \text{ where} \tag{E.41}$$

$$\hat{y}_t^{flex} = \frac{1+\varphi}{\varphi}(z_t - z_t^*) - \frac{\hat{x}_t^{flex}}{\varphi} + \hat{y}_t^{*flex} \tag{E.42}$$

$$\hat{x}_t^{flex} = A\hat{x}_{t-1}^{flex} + B\left[z_t - hz_{t-1} - (z_t^* - hz_{t-1}^*)\right] \tag{E.43}$$

$$A = \frac{h\sigma\left[\varphi\gamma\eta(2-\gamma)+1\right]}{\sigma\left[\varphi\gamma\eta(2-\gamma)+1\right] + (1-h)(1-\gamma)^2\varphi}$$

$$B = \frac{\sigma(1+\varphi)}{\sigma\left[\varphi\gamma\eta(2-\gamma)+1\right] + (1-h)(1-\gamma)^2\varphi}$$

[35] Setting (E.39) equal to (E.42) and solving for x_t^{flex} we get

$$\hat{x}_t^{flex} = -\frac{z_t - z_t^* + z_t\varphi - z_t^*\varphi + \varphi\hat{y}_t^{*flex} - \hat{c}_t^{flex}\varphi + \hat{c}_t^{flex}\varphi\gamma - \gamma\varphi\hat{y}_t^{*flex}}{-1 - 2\gamma\eta\varphi + \gamma^2\eta\varphi}$$

and computing the difference $\hat{x}_t^{flex} - h\hat{x}_{t-1}^{flex}$ we can solve for $\hat{c}_t^{flex} - h\hat{c}_{t-1}^{flex}$

$$\begin{aligned}\hat{c}_t^{flex} - h\hat{c}_{t-1}^{flex} = {} & \frac{\hat{x}_t^{flex} + 2\gamma\eta\hat{x}_t^{flex}\varphi - \gamma^2\eta\hat{x}_t^{flex}\varphi - h\hat{x}_{t-1}^{flex} - 2h\hat{x}_{t-1}^{flex}\gamma\eta\varphi + h\hat{x}_{t-1}^{flex}\gamma^2\eta\varphi - z_t + z_t^*}{\varphi(-1+\gamma)} \\ & + \frac{-z_t\varphi + z_t^*\varphi - \varphi * \hat{y}_t^{*flex} + \gamma\varphi\hat{y}_t^{*flex} + hz_{t-1} - hz_{t-1}^* + hz_{t-1}\varphi - h\varphi z_{t-1}^*}{\varphi(-1+\gamma)} \\ & + \frac{+h\varphi\hat{y}_{t-1}^{*flex} - \gamma\varphi h\hat{y}_{t-1}^{*flex}}{\varphi(-1+\gamma)}\end{aligned}$$

which can be plugged into equation (E.38). Solving for x_t^{flex} then yields equation (E.43).