UNIVERSITÄT
BERN

Faculty of Business, Economics
and Social Sciences

Department of Economics

Selection upon Wage Posting

Silvio Sticher

13-11

November 2013

DISCUSSION PAPERS

Schanzeneckstrasse 1
Postfach 8573
CH-3001 Bern, Switzerland
http://www.vwi.unibe.ch

Selection upon Wage Posting*

Silvio Sticher†

University of Bern

November 8, 2013

Abstract

We discuss a model of a job market where firms announce salaries. Thereupon, they decide through the evaluation of a productivity test whether to hire applicants. Candidates for a job are locked in once they have applied at a given employer. Hence, such a market exhibits a specific form of the bargain-then-ripoff principle. With a single firm, the outcome is efficient. Under competition, what might be called "positive selection" leads to market failure. Thus our model provides a rationale for very small employment probabilities in some sectors.

JEL codes: D83, J21, J31

Keywords: directed search, selection, wage posting

*I am grateful for comments from Marc Blatter, Lutz Dümbgen, Winand Emons, Marion Fischer, Marc Möller, and seminar participants at the University of Bern.

†silvio.sticher@vwi.unibe.ch

1 Introduction

Why do most job interviews end up without an agreement? In a recent study on the Swiss market for skilled labor, Blatter, Muehlemann, and Schenker (2012) establish that firms interview an average of 4.8 applicants to fill a single vacancy. Similar results are available for other countries.[1] In areas such as finance and consulting, the percentage of applicants that are actually offered positions is even considerably smaller. As Rivera (2011) points out, "elite professional service firms often receive thousands or even tens of thousands of applications for fewer than two hundred spots, yielding admissions ratios at the most prestigious firms that are more competitive than that of any Ivy League College." Related to this, Moen (2003) summarizes that it is a "shared belief in the labor market segmentation literature [. . .] that too few high wage (primary sector) jobs are created in the market."

Observed high-end salaries, on the other hand, suggest that "high-end labor" is a very scarce resource. Within a simple Walrasian framework, the salaries prevailing in the mentioned industries indicate a steep supply curve, that is, a small number of job candidates.

How can these two facts, small acceptance rates and high salaries, fit together?

We introduce a model where firms first announce their salaries. Thereupon, candidates of differing productivities apply. Finally, each firm determines a threshold regarding productivity signals, above which it is willing to hire an applicant. These signals or "scores" can be interpreted as bits of information which emerge, for instance, in the course of an entry talk.

Using parametric assumptions, we simulate the firms' selection process, which in turn determines the applicants' employment probabilities. We show that, compared to a Pareto optimal allocation, applicants approach firms with salaries too high and employment probabilities too low. In particular, we find an explanation why most encounters between employers and applicants end up without agreements, even though—or rather because—salaries are high.

[1] In an earlier study on the US labor market, Barron, Bishop, and Dunkelberg (1985) report an average of 6.3 interviewed applicants to fill a position.

What are the key ingredients of a job market of which the equilibrium features the afore-mentioned attributes?

Most of the modern labor-market literature is along the lines of Mortensen and Pissarides (1994). The matching approach which is applied there stands in a rich tradition of models that include bargaining as a mean of wage determination.[2] In contrast to simplifying supply-and-demand considerations, this branch of search theory is suitable to address issues such as the extent of unemployment. However, it does not provide a sufficiently explicit characterization of the matching and wage formation process itself. Furthermore, it lacks to determine the division of the surplus endogenously.

In addition to these theoretical concerns, models with ex-post bargaining are repeatedly contested by empirical work. Hall and Krueger (2008), for instance, estimate that between a quarter and a half of the workers in the United States are employed in jobs where wages are posted. Similar results can also be found in earlier literature.[3] Hence, although models with announced wages exhibit the disputable feature of firms being committed to ignore counteroffers, empirical evidence suggests that wage posting takes place to a significant extent.

Theoretical work uses the term "directed search" to describe labor markets with posted wages. In directed-search models, workers typically face trade-offs between higher wages and higher probabilities of getting a job. Thereby, credible announcing of wages generally ensures constrained-efficient market outcomes, whereas the "constraints" arise from the fact that workers cannot coordinate their application behavior and, as a result, are forced to randomly pick some employer. By introducing such coordination failures, frictions arise in these models as well, but workers are directed towards effi-

[2] An overview of these models is given, for instance, in Rogerson, Shimer, and Randall (2005).

[3] By showing that higher wages attract longer queues of applicants, the study of Holzer, Katz, and Krueger (1991) implicitly implies that there is at least partial commitment on behalf of the firms to refuse any kind of bargaining.

Along the same lines is the finding in Wial (1991) where it is described that chronic excess supply of adequate candidates often coincides with jobs that are ex-ante considered to be good ones, whereas the notion of "good" includes pay and is often passed on interpersonally.

cient applying behavior, since firms implicitly reveal their hiring probabilities by announcing wages.[4]

There exists, however, a branch within this literature which stresses potential market failure, arising from the existence of heterogenous applicants. A series of papers within the framework of Lang and Manove (2003) works out discretely separated employment rates which are due to negligible differences in productivity. Similarily, Moen (2003) mentions unconditional wage announcements and worker-firm specific productivities as a source of inefficiently low aggregate production. More specifically, Moen shows that low-productivity workers jeopardize high-productivity workers to such an extent that firms are forced to raise their posted wages in order to screen the market for superior productivities. Although they are capable of doing so, in a separating equilibrium, the high-type candidates' employment probabilities turn out to be well below efficiency.

We adopt a similar idea in the following. But in contrast to the aforementioned work, we introduce two innovations. First, we do not assume that firms can only hire one applicant. Second, and more importantly, we take into consideration that, upon being contacted, employers are not fully informed about their candidates' productivity. As opposed to existing work on directed search, we proceed on the assumption that, once a firm is contacted, the hiring process is not yet at its end. However, unlike work within the matching tradition, we do not assume that the remaining interaction consists of negotiating on the payment. Instead, we find it a more convincing assumption that job talks are meant to evaluate the applicants' abilities.

Imagine the following scenario. Firms post their salaries, say on the Internet or in a newspaper. These binding offers are noticed by a large number

[4] By assuming that firms are price-takers, Montgomery (1991) shows for the case of a large labor market with identical applicants that employers endogenize the workers' trade-off between search intensity and wage, which ensures efficiency. Going one step further, but omitting welfare considerations, Peters (1991) takes into account the effect of unilaterally deviating firms by considering the limit case of an atomic economy. Based on these two papers, Moen (1997) and Shimer (2001) introduce heterogeneity concerning the productivity of vacant jobs and workers, respectively, whereby the equilibrium efficiency features of the earlier developed models are maintained.

Analogous results, also concerning multiple worker types, are obtained in Coles and Eeckhout (2000) and Michelacci and Suarez (2006), where firms endogenously opt for posted wages as opposed to auctions and bargaining.

of potential employees who reflect on their relative prospects at the different firms. Their productivities are private knowledge, but the candidates know that part of this information becomes revealed once job interviews take place. Hence, after the candidates choose a firm to submit their applications, employers obtain additional information about a candidate's type. A job interview may be interpreted as a series of tasks, where a firm specifies a performance level which the candidate needs to pass. Only successful applicants are hired. They receive the salary which was promised at the outset.

Before firms use job talks as a sorting device, applicants on their part potentially reveal their type by selecting a particular employer. Thus to determine the beliefs of firms prior to job interviews, we have to examine a signaling game. Accordingly, the inclusion of incomplete knowledge comes at the cost of off-the-equilibrium-path beliefs which we need to construct. If firms deduce that lower salaries are rather selected by unproductive candidates, no applicant would head to a low-paying job. But why should a firm—in our case off the equilibrium path—make a conclusion of that kind? In the course of this paper, we provide an argument which rationalizes such a structure of beliefs.

Once employers expect that applicants uniformly approach the best-paying firms, they are willing to outbid their opponents as long as they will find a mixture of applicants which justifies such sizable salaries. But the higher the promised payment is, the more thoroughly a firm needs to scrutinize its applicants. As only the best are worth their price, most applicants are left without an employment. Thus we find an explanation for the low hiring rates we referred to at the beginning.

We organize the rest of the article as follows. In Section 2, we describe the assumptions of the model. In Section 3, we look at a single firm (a "monopsonist") to illustrate the relation between salaries and the employment rate. In Section 4, we present the results for the competitive case. We discuss a more general specification of the interviewing process in Section 5. In Section 6, we provide a comparative statics analysis, contrast the market outcomes with Pareto optimal allocations, and briefly address policy measures. In Section 7, we endogenize the firms' off-the-equilibrium-path beliefs, which we impose up to there. Section 8 concludes. All proofs are relegated to the appendix.

2 Model

We look at a continuum of applicants (candidates) with a total mass of 1. $\theta_i \in \{0, 1\}$ denotes the value of the marginal product of applicant i. Thus each candidate is either productive ($\theta_i = 1$) or completely unproductive ($\theta_i = 0$). Both types have an outside option of 0. There are J identical firms. To circumvent issues of market power, we let $J \to \infty$.[5] We assume both applicants and firms to be risk-neutral maximizers of their expected monetary payoffs.

The timing of the game is as follows. In period 1, nature chooses each applicant's productivity $\theta_i \in \{0, 1\}$. With probability $q_1 = 1 - q_0$, an individual applicant's productivity is 1. In period 2, each firm posts an unconditional salary w_j at which it obliges itself to pay accepted candidates. In period 3, candidates simultaneously select firms. If the expected payoff is the same at several firms, we assume that one of the most promising offers is picked randomly. In period 4, each applicant's score value s_i is realized. In period 5, the contacted firms decide on which applicants to employ.

Regarding period 4, we differentiate between two specifications:

Scenario 1. $s_i = \theta_i + \varepsilon_i$, whereas $\varepsilon_i \sim$ i.i.d. $\mathcal{N}(0, \sigma^2)$.

Scenario 2. $s_i = \sum_{k=1}^{n} t_k$, whereas $n \in \mathbb{N}^+$ and $t_k \in \{0, 1\}$. We define $p_i := \mathbb{P}\left[t_k = 1 | \theta_i\right] \in (0, 1)$, and assume that $p_1 > p_0$.

We discuss Scenario 1 in Sections 3 and 4. Normally distributed score values enable the firms to accurately control their composition of the workforce. In fact, from the law of large numbers, this can be done to an arbitrarily precise extent. In order to overcome this artificiality, in Section 5 we refer to Scenario 2, where we consider binomially distributed score values. By doing so, we interpret $\{t_k\}_{k=1}^{n}$ as a series of tasks which are examined in the course of a job talk. From the number of successfully mastered tasks, a firm

[5] As previously mentioned, we also look at the case of a single firm in the preliminary Section 3, so as to separately study the role of job interviews.

draws conclusions about an applicant's type. As the binomial distribution converges to a normal distribution, Scenario 1 constitutes a special case of Scenario 2 where $n \to \infty$. Hence Scenario 2 is the more general framework.

By $\mu(w_j)$, we denote the firms' posterior belief that a candidate who approaches firms with payment w_j is of type 1. If candidates behave homogeneously, posterior beliefs of the firms equal their prior beliefs, thus $\mu(w_j) = q_1$. When candidates behave heterogeneously, we assume firms to conclude that the highest accepted offer is only chosen by type-1 applicants, while any other offer is only chosen by type-0 applicants. For the moment, we take this (obviously strong) assumption as given. In Section 7, however, we show that it endogenously arises as the consequence of a standard refinement criterion ("D1"). By denoting by I the set of firms which are chosen by either applicant type with positive probability, we summarize the structure of beliefs in Assumption 1.

Assumption 1.

$$\begin{aligned} \forall j, l \in I : w_j = w_l \quad &\Rightarrow \quad \mu(w_j) = \mu(w_l) = q_1, \\ \exists l \in I : w_l < \max_{j \in I}\{w_j\} \quad &\Rightarrow \quad \mu(\max_{j \in I}\{w_j\}) = 1 \text{ and } \mu(w_l) = 0. \end{aligned}$$

3 The Monopsony Case

We first consider Scenario 1 with a single employer. In this case the analysis of the applicants' behavior becomes redundant since they have no alternative to approaching the "monopsonist". Accordingly, all applicants apply at the monopsonist, as long as it offers $w^m \geq 0$. In a subgame-perfect equilibrium, the monopsonist sets $w^m = 0$, and all candidates end up being employed. Although this result follows straightforwardly, we proceed by conducting a step-by-step analysis of the game. The central aim of the following backwards induction is to focus on the employer's selection process, and to illustrate the relation between salaries and employment probabilities.

The Hiring Decision At the final stage, the monopsonist decides on which applicants to accept. Given it hires an applicant with a score value $\underline{s}^m$, it also hires applicants with higher score values, because $\partial\mathbb{P}[\theta_i = 1|s_i]/\partial s_i > 0$. Therefore, the monopsonist's hiring problem can be reduced to finding the profit-maximizing $\underline{s}^m$.

Independently of $w^m \geq 0$, the employer expects an applicant to be of type 1 with probability q_1 since, prior to the realization of the score value, nothing about an applicant's type has transpired. Hence, we can state the firm's ex-post profit maximization problem as

$$\begin{aligned}\underline{s}^{m*} =&\ \arg\max_{\underline{s}^m}\{\pi(\underline{s}^m)\}\\ :=&\ \arg\max_{\underline{s}^m}\{(1-\Phi_1[\underline{s}^m])\,q_1 - w_j\,[(1-\Phi_1[\underline{s}^m])\,q_1\\ &+\ (1-\Phi_0[\underline{s}^m])\,q_0]\}\,, \end{aligned} \tag{1}$$

where $\Phi_{\theta_i}[\cdot]$ denotes the cumulative distribution function of $s_i(\theta_i)$. The first-order condition of problem (1) is

$$q_1\phi_1(\underline{s}^m) \overset{!}{=} w^m(q_1\phi_1[\underline{s}^m] + q_0\phi_0[\underline{s}^m]), \tag{2}$$

where $\phi_{\theta_i}[\cdot]$ denotes the density function of $s_i(\theta_i)$.

The left-hand side of equation (2) is the marginal benefit from increasing $\underline{s}^m$: the productive candidate's density at $\underline{s}^m$ times the ex-ante probability q_1 of such an applicant. The right-hand side of (2) is the marginal cost of increasing $\underline{s}^m$: the marginal probability of hiring *any* applicant, $q_1\phi_1[\underline{s}^m] + q_0\phi_0[\underline{s}^m]$, multiplied by the salary w^m.

By applying $s_i \sim$ i.i.d. $\mathcal{N}(\theta_i, \sigma^2)$ on (2), we obtain the monopsony's optimal threshold $\underline{s}^{m*}$.

Lemma 1.

$$\underline{s}^{m*} = \frac{1}{2} + \ln\left(\frac{q_0}{q_1}\frac{w^m}{(1-w^m)}\right)\sigma^2 \tag{3}$$

Suppose, for instance, that $q_1 = q_0 = 0.5 = w^m = 0.5$. In this case, the monopsony obtains $\theta_i - w^m = 0.5$ per type-1 applicant and -0.5 per type-0 applicant. Therefore, as long as the marginal probability of hiring a type-1 applicant is above the marginal probability of hiring a type-0 applicant, the firm wishes to decrease its lower bound $\underline{s}^m$ to accept more candidates. The monopsonist reaches its optimum by setting the marginal probabilities equal to each other. This is reflected in (3), which for the current example reads $\underline{s}^{m*} = 1/2 + \ln(1)\sigma^2 = 1/2$.

The Choice of the Payment When choosing its payment offer, the monopsonist takes two effects into account. Ceteris paribus, an increasing payment lowers the monopsonist's profit. Through $\underline{s}^{m*}(w^m)$, however, it also forces the firm to adjust its composition of the workforce. By scaling up $\underline{s}^m$, the monopsonist increases its proportion of type-1 applicants. Equivalently,

$$\lim_{\underline{s}^m \to \infty} q_1\phi_1[\underline{s}^m] + q_0\phi_0[\underline{s}^m] = q_1\phi_1[\underline{s}^m] \tag{4}$$

because $\lim_{\underline{s}^m} \phi_1[\underline{s}^m]/\phi_0[\underline{s}^m] = \lim_{\underline{s}^m} e^{(2\underline{s}^m-1)/2\sigma^2} = \infty$. Equation (4) and an analogous statement for $\underline{s}^m \to -\infty$ imply that the monopsonist's first-order condition (2) has an interior solution whenever $0 < w^m < 1$. In Corollary 1 of Lemma 1, we rephrase this condition by turning our attention to corner solutions. Furthermore, we state an analogous result regarding the impact of the ex-ante distribution of types.

Corollary 1. *In (3),* $\lim_{q_1 \searrow 0} \{\underline{s}^{m*}(\cdot)\} = \lim_{w^m \nearrow 1} \{\underline{s}^{m*}(\cdot)\} = \infty$, *as there are either only type-0 candidates applying or* w^m *is prohibitively high. Likewise,* $\lim_{q_1 \nearrow 1} \{\underline{s}^{m*}(\cdot)\} = \lim_{w^m \searrow 0} \{\underline{s}^{m*}(\cdot)\} = -\infty$.

Of course, if the firm raises $\underline{s}^m$, it does not only affect the mixture of its workforce but also its size. If the monopsonist sets $w^m = 0$, its marginal cost, the right-hand side of equation (2), equals 0. This allows it to set $\underline{s}^m$ to $-\infty$, thus it accepts every applicant. As a result, the monopsonist realizes $\pi^{m*} = q_1$. We state this result in Corollary 2 of Lemma 1.

Corollary 2. *The monopsony sets* $w^{m*} = \arg\max_{w^m}\{\pi^m(w^m)\} = 0$.

Welfare Analysis and Policy From a Utilitarian point of view, welfare increases in the number of employed type-1 applicants, which in turn decreases with $\underline{s}^m$.[6] By reconsidering equation (3), we see that

$$\frac{\partial \underline{s}^{m*}}{\partial w^m} = \frac{\sigma^2}{w^m(1-w^m)} > 0. \tag{5}$$

Hence the monopsonist's behavior is in line with the one of a social planner. The total surplus is q_1.

Before shifting the focus of the analysis to a market with many employers, we briefly look at potential government interventions in the monopsony case.

Minimum Salary From equation (5), a minimum salary lowers the amount of hired type-1 applicants. Utilitarian welfare is reduced.

Maximum Salary Since the monopsonist chooses $w^{m*} = 0$, any restriction from above is without consequences.

4 The Competitive Case

As opposed to the analysis of the previous section, we now consider the case with $J \to \infty$ employers. As a consequence, we have to deal with a combination of a screening game and a signaling game. Screening occurs at the outset: firms try to influence the composition of their workforce by offering their respective salaries. Signaling occurs subsequently: each applicant faces an array of salary offers; thus by approaching a particular firm, it becomes possible to convey a signal. Respecting this setting of incomplete information, we use the notion of a Perfect Bayesian Equilibrium (henceforth PBE): at each stage of the game, agents maximize their payoffs given their beliefs, which are consistent in equilibrium. Regarding off-the-equilibrium-path beliefs, we impose Assumption 1, which we endogenize in Section 7. As before, we proceed by solving the game backwards.

The Hiring Decision At the final stage, each firm deliberates on which applicants it accepts to employ. Firm j's decision depends on its salary offer, w_j, and its belief on its applicants type, $\mu(w)$. Thereby, $w := (w_j, w_{-j})$

[6]The cumulative distribution function of a normally distributed variable is strictly increasing in its argument.

denotes the vector of all salaries offered in the first period. That is, upon observing the entire array of salary offers and receiving a certain amount of applicants, firm j assesses its applicants to be of type 1 with probability $\mu(w)$ and of type 2 with probability $1-\mu(w)$.

Apart from this, the analysis of the hiring decision in the competitive case mirrors the one in case of a monopsonistic employer, discussed in Section 3. As a result, we obtain an individual firm's optimal threshold $\underline{s}_j^*$.

Lemma 2.

$$\underline{s}_j^* = \frac{1}{2} + \ln\left(\frac{(1-\mu(w))}{\mu(w)}\frac{w_j}{(1-w_j)}\right)\sigma^2 \tag{6}$$

According to Lemma 2, $\underline{s}_j^*$ increases in the fraction of type-0 applicants, in firm j's offered salary, as well as in the variance of the signal distribution.[7] Also referring to Section 3, we summarize the limit behavior of $\underline{s}_j^*$ as follows.

Corollary 3. *In (6),* $\lim_{\mu(w)\searrow 0}\left\{\underline{s}_j^*(\cdot)\right\} = \lim_{w_j\nearrow 1}\left\{\underline{s}_j^*(\cdot)\right\} = \infty$, *as either j expects only type-0 candidates applying or w_j is prohibitively high. Likewise,* $\lim_{\mu(w)\nearrow 1}\left\{\underline{s}_j^*(\cdot)\right\} = \lim_{w_j\searrow 0}\left\{\underline{s}_j^*(\cdot)\right\} = -\infty$.

The Selection of a Firm To determine $\mu(w)$, we have to consider the applicants' selection of employers in period 3. Each candidate i selects firm j^* which maximizes his or her expected payoff. That is,

$$(j^*|\theta_i) = \arg\max_j \left\{w_j \mathbb{P}[s_i \geq \underline{s}_j^*(\cdot)|\theta_i]\right\}. \tag{7}$$

In (7), the probability that firm j accepts applicant i, $\mathbb{P}[s_i \geq \underline{s}_j^*(\cdot)|\theta_i]$, depends on the employers belief $\mu(w)$ (through $\underline{s}_j^*$). From Assumption 1, we have that i is believed to be a type-0 applicant whenever i approaches an employer with an offer below the maximum offer among all selected firms. Furthermore, from Corollary 3, such a firm will never accept a positive amount of applicants whenever it offers a positive salary. As a result, the applicants only choose among the employers with the highest salary offers.

[7] As an exception, $\underline{s}_j^*$ is independent of σ^2 if $w_j(1-\mu(w)) = (1-w_j)\mu(w)$. In this case, $\underline{s}_j^* = 1/2$.

Lemma 3. *For $\theta_i \in \{0, 1\}$,*

$$(j^*|\theta_i) \in K : \forall k \in K : w_k = \max_j \{w_j\},$$

and each candidate randomly picks one of the firms in K.

Given the other applicants go for a firm with the highest salary offer, no individual applicant wants to be exposed as being of the low productivity type 0. Similarly, if some candidates select a firm which does not belong to the set of firms with the highest offer, there is an incentive for each applicant to go for the highest offer and to be declared as being of type 1.

The Choice of the Offered Salary Being aware of the subsequent equilibrium behavior, in the initial state each firm j offers salary w_j in order to maximize its expected profit. Proposition 1 states that in equilibrium the maximum offered salary cannot be smaller than 1.

Proposition 1. *In the PBE, it holds that*

$$\nexists \hat{w} < 1 : \forall j \in J : w_j \leq \hat{w}.$$

The proof of Proposition 1 is somewhat cumbersome.[8] In order to understand its message, it suffices to consider a counterfactual example where the highest offered salary is, say, 0.99. For the sake of the argument, assume that all firms offer this salary. As a result, and since we focus on $J \to \infty$, an individual employer obtains 0. By increasing its offer, for example, to 0.991, an individual firm l can achieve to become the only employer contacted by the candidates (see Lemma 3). Does such a deviation pay off? In order to see why this is true, be aware that once the workers have contacted firm l, according to (6), l sets

$$\underline{s}_l^* = \frac{1}{2} + \ln\left(\frac{q_0}{q_1}\frac{0.991}{0.009}\right)\sigma^2,$$

[8]Whereas this proof and its counterpart regarding Proposition 2 treat firms as only being able to choose among pure strategies, extensions to mixed strategies such as done in the proof of Lemma 3 would be straightforward. We impose the restriction to pure strategies to ease notation, and because the results are unaffected.

which is approximately 5.2 for $q_0 = q_1$ and $\sigma^2 = 1$. As a consequence,

$$\mathbb{P}[s_i \geq \underline{5}.2|\theta_i] \simeq \begin{cases} 0.0000001 & \text{for } \theta_i = 0, \\ 0.0000134 & \text{for } \theta_i = 1. \end{cases}$$

Thus, by choosing $\underline{s}_l^* = 5.2$, firm l ensures that its ratio between productive and unproductive workers is roughly 134:1. That is, for each type-0 worker which leaves the firm with a loss of 0.991, it hires 134 type-1 applicants, on each of which it obtains a margin of 0.009. As a result, l's profit per 135 employed candidates is $134 \times 0.009 - 0.901 = 0.215$.

That is, for $w_j < 1$, it is always feasible to steer the ratio between accepted type-1 and type-0 applicants towards the desired direction. This has, however, detrimental effects on the employment rate, as we point out in the following corollary.

Corollary 4. *In the PBE, we have* $\max_j \{w_j\} = 1$*, and both the employment rate and profits are 0.*

Of course, in its present form, this result is too crude to match the observed pattern. In the following, we show that the reason for this lies in the continuos space of the score values. With discrete score values, any employment rate is possible, though we approach the result of Corollary 4 the more we refine the value space.

5 Generalization of the Competitive Case

In contrast to above, we now assume that only coarse signals about productivity are transmitted. We accordingly modify all so-far conducted steps.

The Hiring Decision Once a firm expects to face $\mu(w)$ type-1 candidates and $1 - \mu(w)$ type-0 ones, it is confronted with the same trade-off as in Section 4. If the minimum-requirement level $\underline{s}_j$ is excessively high, a lot of productivity is lost because both applicant types are rejected. If it is modest, the firm employs too many type-0 candidates on which it loses. Correspondingly, the higher is w_j, the higher $\underline{s}_j$ has to be. In fact, in the following we show that in a PBE each firm's salary offer w_j is insomuch high that only the fulfillment of the highest possible threshold $\underline{s}_j = n$ gives rise to an employment.

As a first step to seeing that, consider that, in contrast to the above case, the distinguishing assumption of Scenario 2 implies that the signal of each applicant can only take on integer values within a bounded set. Specifically, signal $s_i(\theta_i)$ follows a binomial distribution and thus assumes the probability mass function $\mathbb{P}[s_i = k|\theta_i] = \binom{n}{k} p_{\theta_i}(1-p_{\theta_i})^{n-k}$, where p_{θ_i} denotes the constant probability that an single of n assessed tasks is successfully handled by candidate i with $\theta_i \in \{0,1\}$. In analogy to (1), we state the ex-post profit maximization problem of firm j as

$$\underline{s}_j^* = \arg\max_{\underline{s}_j \in \mathbb{N}_0} \left\{ \sum_{k=\underline{s}_j}^{n} \binom{n}{k} \Big[(1-w_j)\mu(w) p_1^k (1-p_1)^{n-k} \right. \\ \left. - w_j \left(1-\mu(w)\right) p_0^k (1-p_0)^{n-k} \Big] \right\}. \tag{8}$$

Due to the discreteness of problem (8), first-order conditions are of no help here. Nevertheless, Lemma 4 provides a sufficient condition for $\underline{s}_j^* = n$, whereas in the course of the proof of Proposition 2 we point out that the requested condition always holds.

Lemma 4. *Assume there exists a unique solution for $\underline{s}_j^*$. Then, for $\mu(w) \in (0,1)$, we have*

$$\forall j \in J : \frac{w_j}{1-w_j} \geq \frac{\mu(w)}{1-\mu(w)} \left(\frac{p_1}{p_0}\right)^n \;\; \Rightarrow \;\; \underline{s}_j^* = n. \tag{9}$$

Thus the obligation of paying a high salary w_j forces firm j to investigate more thoroughly whether an applicant is of type 1. Furthermore, if the values of p_1 and p_0 are close together, and if there are relatively few type-1 applicants expected, it is particularly important to let a candidate accomplish as many assessment tasks as possible.

The Selection of a Firm Regarding the choice of a firm, the former analysis involving an unbounded and continuous signal space carries over to the present setting with bounded and discrete test scores. A replicated proof of Lemma 3 would merely adjust the probability of being accepted at firm j in the case of homogenous behavior of the applicants to

$$\mathbb{P}[s_i \geq n | s_i \sim \text{Binomial}(n, p_{\theta_i})] = p_{\theta_i}^n \in (0,1), \tag{10}$$

leaving the remainder of the discussion unaffected. Therefore we omit a repetition of the proof.

The Choice of the Offered Salary Again, by choosing a salary w_j, firm j has to take the sequential consequences on $\mu(w)$ and $\underline{s}_j^*(\cdot)$ into account. Proposition 2 provides a simple decision rule for the firms in period 1. It states that, in contrast to the case of a continuous signal space, the maximum salary chosen in the initial period can be, and generally is, below 1, even though it approaches 1 for $n \to \infty$. Since the proof of Proposition 2 makes use of Lemma 4, we need to scrutinize the condition stated in (9) each time we apply the lemma, irrespective of whether it is on or off the equilibrium path.[9]

Proposition 2. *In the PBE, we have*

$$\max_j \{w_j\} = \frac{q_1 p_1^n}{q_1 p_1^n + q_0 p_0^n} =: w^*. \tag{11}$$

In the proof of Proposition 2, we show that in any constellation where the highest offered salary is not equal to w^* there are incentives for at least one firm to either increase or decrease its offer. Here, we only sketch out that there actually exists a PBE which involves a maximum salary as described in (11). To see this, note that

$$\frac{w^*}{1-w^*} = \frac{q_1 p_1^n}{q_0 p_0^n} = \frac{\mu(w)}{1-\mu(w)} \left(\frac{p_1}{p_0}\right)^n,$$

whereas the later equality follows from the random assignment of candidates to jobs that offer w^* and Assumption 1 about the structure of beliefs. It follows from condition (9) in Lemma 4 that, upon offering w^*, firm j requires its candidates to pass the maximum number of tasks, n. From the binomial

[9] Otherwise, we would risk running into a circular argument.

distribution of the score values, the odds of passing n tasks and becoming employed is $p^n_{\theta_i}$, $\theta_i \in \{0,1\}$. Accordingly, each firm's profit is

$$(1-w^*)q_1 p_1^n - w^* q_0 p_0^n = 0.$$

Given all competing firms offer w^*, there is no incentive to deviate by announcing a higher salary: all applicants would accept the deviating firm's offer, and the latter would end up with a loss. Similarly, cutting the offered salary is useless, as Assumption 1 guarantees that all applicants approach the best-paying firm in order to not reveal themselves as type-0 candidates.

As Proposition 2 indicates, not all firms necessarily make identical offers. As long as the highest offered salary equals w^*, each firm obtains zero profit, regardless of whether it attracts applicants or not. However, for instance under the restriction of symmetric firm behavior, w^* is the unique offered salary in a PBE. Hence, for simplicity, in the remainder of this paper we assume that in equilibrium $w_j = w^*$ for all firms $j \in J$.

We close this section with Corollary 5 of Proposition 2 regarding the employment rate and expected payoffs.

Corollary 5. *In the PBE, the probability of an applicant of type θ_i to become employed is $p^n_{\theta_i}$. Accordingly, the population-wide employment rate is $(q_1 p_1^n + q_0 p_0^n)/(q_1+q_0)$, and the expected payoff of an applicant with productivity θ_i is $(p^n_{\theta_i} q_1 p_1^n)/(q_1 p_1^n + q_0 p_0^n)$.*

6 Welfare and Policy

Regarding total surplus, it solely matters how many workers are employed in equilibrium. Irrespective of informational issues, full employment of productive candidates can be achieved by enforcing $\underline{s}_j = \min_i \{s_i\}$, alongside $w_j \in [0,1]$, for each firm j. Such constrained-efficient allocations are also Pareto optimal, as is, among others, the monopsonist's solution.

In Table 1, we compare values of the model's key variables for both considered scenarios with the outcome of the monopsonist's problem. In both scenarios a monopsonist j chooses a salary as low as possible, and at the same time it hires as many applicants i with $\theta_i = 1$ as possible. Respecting

	Outcomes		
	Scenario 1	Scenario 2	Monopsony
Maximum salary	1	$\in \left(\frac{q_1}{q_1+q_0}, 1\right)$	0
Employment rate	0	$\in (0,1)$	1
Fraction of type-1 employees	1 (limit)	$\in \left(\frac{q_1}{q_1+q_0}, 1\right)$	$\frac{q_1}{q_1+q_0}$
Producer surplus (applicants)	0	$\in (0, q_1)$	0
Consumer surplus (firms)	0	0	q_1

Table 1: Comparative statics

the candidates' individual-rationality constraints which impose $w_j \geq 0$, these two objectives are not competing. With $w_j = 0$, all applicants apply at j. Thereupon, due to $\underline{s}_j = \min_i \{s_i\}$, all candidates, including the productive ones, are employed. Regarding the profit maximizing threshold, we have $\underline{s}_j^* = -\infty$ in Scenario 1 with a continuous and unbounded signal space, and $\underline{s}_j^* = 0$ in Scenario 2 with an atomic, bounded distribution. Ignoring equity considerations due to transfer payments, the monopsony maximizes overall welfare.

Under competition, the main difference between the two scenarios is that in Scenario 2 the firms do not have illimitable means to sort out type-0 applicants by raising $\underline{s}_j$. Anticipating this, they are not willing to set their salary offers arbitrarily close to 1 in the initial period. Therefore, the odds of becoming employed are non-zero for both types of candidates. With discrete score values, the applicants' surplus is

$$\begin{aligned} & w^* \left(q_1 \mathbb{P}\left[s_i \geq n | \theta_i = 1\right] + q_0 \mathbb{P}\left[s_i \geq n | \theta_i = 0\right]\right) \\ = \ & \frac{q_1 p_1^n}{q_1 p_1^n + q_0 p_0^n} \left(q_1 p_1^n + q_0 p_0^n\right) = q_1 p_1^n. \end{aligned}$$

Accordingly, overall welfare is higher for values of p_1 closer to 1 and a lower number of testing criteria n. That is, the coarser the firms' framework for the assessment of the candidates, the more likely it is that candidates withstand even the highest classification requirement. This is reflected in a higher employment rate.

Policy Since n and, through the difficulty level of the assessment tasks, p_1 are parameters which are generically determined within firms, they are no adequate means of regulation. It is, however, plausible that policy-makers restrict the range of w_j. In fact, it is easy to see that by upwards restricting $w_j \leq \overline{w}$, any measure of unemployment can be eliminated by appropriately choosing $\overline{w}$. Thus in our stylized model, lowering $\overline{w}$ from its equilibrium value leads to a Pareto improvement, because firms become enabled to extract a positive profit, whereas an applicant of type θ_i obtains $\overline{w}\mathbb{P}[s_i \geq \underline{s}_j^*(\overline{w})|\theta_i]$.

The preference ordering of firms and applicants over possible values of $\overline{w}$, however, is not aligned. Consider Scenario 1. Regarding firm j's profit,

$$\pi_j(\overline{w}) \;= \left(1 - \Phi_1[\underline{s}_j(\overline{w})]\right)\frac{q_1}{J} - \overline{w}\left[\left(1 - \Phi_1[\underline{s}_j(\overline{w})]\right)\frac{q_1}{J} + \left(1 - \Phi_0[\underline{s}_j(\overline{w})]\right)\frac{q_0}{J}\right],$$

we apply the regular envelope theorem to see that the impact of an increase in $\overline{w}$ on j's payoff is

$$\left.\frac{\partial \pi_j(\cdot)}{\partial \overline{w}}\right|_{s_j(\overline{w})=s_j^*(\overline{w})} = -\left[\left(1 - \Phi_1[\underline{s}_j^*(\overline{w})]\right)\frac{q_1}{J} + \left(1 - \Phi_0[\underline{s}_j^*(\overline{w})]\right)\frac{q_0}{J}\right],$$

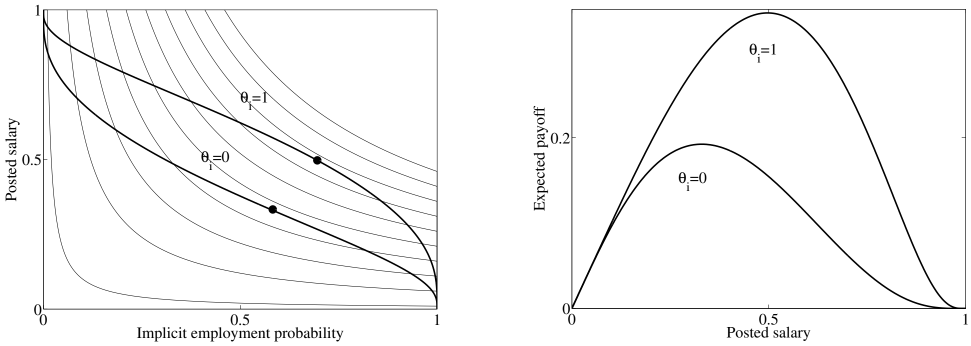
thus strictly negative for all $\overline{w} \in [0, 1)$. As opposed to this, for both candidate types there exists a clear-cut and positive optimal upper bound on salaries.

Proposition 3. *When salaries are restricted by $w_j \leq \overline{w}$, for each applicant type $\theta_i \in \{0, 1\}$ there exists a unique $\overline{w}_{\theta_i}$ which satisfies*

$$\overline{w}_{\theta_i} = \arg\max_{\overline{w}} \left\{\overline{w}\mathbb{P}[s_i \geq \underline{s}_j^*(\overline{w})|\theta_i]\right\}. \tag{12}$$

Furthermore, $\overline{w}_1 > \overline{w}_0$; and type-$\theta_i$ applicants' payoffs are strictly increasing in $\overline{w}$ if and only if $0 \leq \overline{w} < \overline{w}_{\theta_i}$.

Consider Figure 1. Each value of $\overline{w}$ implicitly defines a candidate's employment probability. The thick lines in panel (a) thus constitute "budget lines" for both candidate types. Crucially, the implicit employment probability decreases in the posted salary, but faster so for type-0 applicants. Therefore, with any $\overline{w}$, type-1 applicants have higher expected payoffs than type-0 applicants, as we show in panel (b). In addition, the highest payoff is



(a) Posted salaries, implicit employment probabilities, and common indifference curves

(b) Expected payoffs as functions of posted salaries

Figure 1: Determination of $\overline{w}_1$ and $\overline{w}_0$

achieved at a higher value of $\overline{w}$ for type-1 applicants, as can be seen in both panels.

Somewhat surprisingly, when it comes to determining $\overline{w}$, the preference order of type-0 workers is better aligned with the preference order of firms. The only party which remotely benefits from not having a salary ceiling is the type-1 applicants.

7 Endogenization of Beliefs

In both Scenario 1 and Scenario 2, the signaling behavior of the applicants clearly drives the main results. Specifically, from Assumption 1 it follows that all candidates necessarily approach the best-paying firm, because otherwise they would—correctly or mistakenly—reveal themselves as being of the unproductive type 0. Obviously, this is a strong assumption, which we imposed axiomatically up to now. Thus we owe it to the reader to provide an appropriate motivation.

In the following, we consider Scenario 1. We examine to what extent two established refinements for signaling games apply. To begin with, consider the equilibrium outcome where both candidate types choose a firm k with $w_k = \max_j \{w_j\} = 1$.

Refinement 1. The "Intuitive Criterion" by Cho and Kreps (1987) eliminates equilibria where firms asses the probability that an off-the-equilibrium-path message is sent by an applicant for whom sending it is dominated by the equilibrium strategy to be non-zero. Accordingly, the equilibrium profile survives the Intuitive Criterion if and only if the remaining type of applicant does not strictly benefit from sending the off-the-equilibrium-path message.

In the present model, it only pays off for an applicant to deviate from the equilibrium strategy (that is, heading for $w_l < w_k$ instead of $w_k = 1$) if this induces a higher probability of becoming employed. From Lemma 2, the employment probability of a type-θ_i applicant is

$$1 - \Phi_{\theta_i}\left[\frac{1}{2} + \ln\left(\frac{(1-\mu(w))}{\mu(w)}\frac{w_l}{(1-w_l)}\right)\sigma^2\right], \tag{13}$$

which is positive for $\mu(w) > 0$ and $w_l < 1$. Since in equilibrium both candidate types have a payoff of 0, the firms cannot exclude either of them upon receiving such a message. Hence the proposed equilibrium does not violate the Intuitive Criterion.

Refinement 2. The '"D_1 Criterion" by Banks and Sobel (1987) is more restrictive in as much as it constrains the firms to eliminate types which are "less likely" than others to send particular off-the-equilibrium-path messages. That is, upon observing an applicant to approach a firm offering $w_l < w_k$, the firms deduce that the deviating candidate is of the type for whom deviating is profitable in a higher number of cases, characterized by $\mu(w)$.

For $w_l = 0$, irrespectively of $\mu(w)$, it does never pay off to deviate for either type. For $0 < w_l < 1$, (13) implies that, for both types, the set of $\mu(w)$ which rewards departure from $w_k = 1$ is given by $\mu(w) \in (0,1)$. Thus $M_1 = M_0$, and neither is a proper superset of the other. Therefore, we cannot further restrict off-the-equilibrium-path beliefs, which leaves the D_1 Criterion—as well as the Intuitive Criterion—without bite.

So far we focussed on the case where the highest offered salary is 1 and thus no applicant is employed in equilibrium. However, there are many more subgames to consider, each of them being a signaling game with a fixed distribution of $\{w_j\}_{j\in J}$. For each of these subgames, we now show that the beliefs imposed in Assumption 5 necessarily arise under the D_1 Criterion.[10]

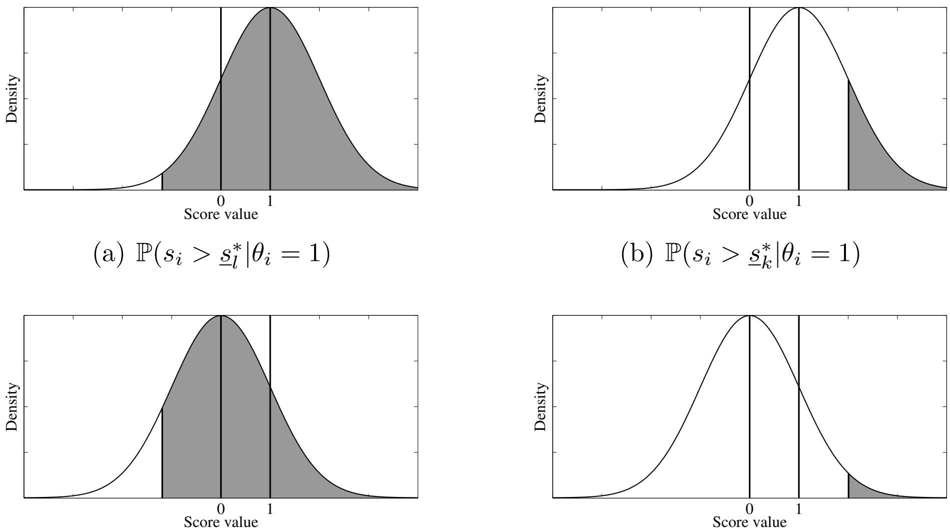
Lemma 5. *Denote by k a firm with $w_k = \max_j \{w_j\}$ and by l a firm with $w_l < w_k$ in any subgame. Let M_θ be the set of beliefs $\mu(w)$ for which*

$$\mathbb{P}\left[s_i \geq \underline{s}_l^*(\mu(w), w_l)|\theta_i = \theta\right] w_l > \mathbb{P}\left[s_i \geq \underline{s}_k^*(w_k)|\theta_i = \theta\right] w_k, \tag{14}$$

where $\underline{s}_k^*(w_k)$ is the equilibrium value complying with Lemma 3 and the further course of the game. Then $M_1 \subset M_0$.

That is, each $\mu(w)$ which gives type-1 applicants an incentive to deviate from their putative equilibrium strategies also induces type-0 applicants to do so. The same, however, does not apply the other way around.

Consider Figure 2. In order for $\mu(w) \in M_1$, w_l must be associated with a



(a) $\mathbb{P}(s_i > \underline{s}_l^*|\theta_i = 1)$ (b) $\mathbb{P}(s_i > \underline{s}_k^*|\theta_i = 1)$

(c) $\mathbb{P}(s_i > \underline{s}_l^*|\theta_i = 0)$ (d) $\mathbb{P}(s_i > \underline{s}_k^*|\theta_i = 0)$

Figure 2: Relative probabilities of becoming employed

sufficient increase in the likelihood of a type-1 applicant to become employed. Such a candidate compares the relation of hiring probabilities, indicated in panels (a) and (b), with the relative wage w_k/w_l. Specifically, in the proof of

[10] Accordingly, they satisfy the Intuitive Criterion as well.

Lemma 5 we show that if the proportion $\mathbb{P}[s_i \geq \underline{s}_l^* | \theta_i = 1]/\mathbb{P}[s_i \geq \underline{s}_k^* | \theta_i = 1]$ exceeds w_k/w_l, the same must be true for $\mathbb{P}[s_i \geq \underline{s}_l^* | \theta_i = 0]/\mathbb{P}[s_i \geq \underline{s}_k^* | \theta_i = 0]$. In terms of Figure 2, this means that whenever the shaded area in panel (a) is sufficiently large as compared with the shaded area in panel (b), the same necessarily holds with regard to the relation between the indicated areas in panels (c) and (d). Therefore, each $\mu(w)$ which satisfies (14) with respect to $\theta = 1$ also satisfies (14) with respect to $\theta = 0$, but the opposite is not true. As a result, firm l which offers the lower salary w_l infers that its applicants are likelier to be of type 0. The D_1 Criterion then requires firm l to rule out the possibility of being approached by a type-1 candidate. From Corollary 3 of Lemma 2, it eventually follows that the probability of being hired at firm l is 0. Accordingly, there are no incentives for either candidate to apply at lower-paying firms. This justifies the structure of the off-the-equilibrium-path beliefs which we impose in Assumption 1.

8 Conclusion

In contrast to the existing literature on directed job search, the equilibria we presented in this paper exhibit tremendous inefficiencies. Due to "D_1 considerations", applicants approach only the best-paying firms. Therefore, firms incur Bertrand-style outbidding. In retrospect, this forces them to excessively raise their hiring thresholds, which leads to unemployment.

To be clear, our results are far too extreme for the purpose of interpreting them in a "literal" manner. For the sake of highlighting a subtle but potentially crucial emergence, we made a number of simplifications. Regarding future research, it thus would be of interest to address some of the following issues in greater detail.

First, we assume homogenous firms. This may not pose a problem concerning the analysis of a specific market. However, within a more comprehensive framework, applicants would differ with respect to their outside opportunities including alternative job offers and unemployment benefits. Once we endogenized their participation constraints, the mechanism which deteriorates expected employment probabilities might be mitigated.

Secondly, if we abandoned the imposed dichotomy regarding the applicants' productivity, we would also have to adjust the firms' hiring rules and the specification of their beliefs. Discrete jumps in the value of the marginal product, which are essential to the discussed phenomena, are more probable to arise at the less densely populated upper part of the distribution. Therefore we may suppose that the examined effect plays a more important role within this range.

Of course, there exists a wide array of other possible extensions, such as on-the-job search (Delacroix and Shi, 2006), endogenized waging-rule determination (Michelacci and Suarez, 2006), job destruction, multiple applications, multi-period problems, learning processes, and many more. Incorporating these in a model of incomplete knowledge issues serious challenges. Replacing the normally distributed score values by some easier tractable random variables could be an option to counteract formal difficulties.

The above points make clear that our model is intended to serve as a starting point, indicating a potential direction of research, and not as a tool with predictive power. In its current state, it is neither appropriate for policy advice. In particular, implications of measures such as a salary ceiling strongly depend on institutional factors such as the international environment.

We showed, however, that a job-market model with imperfect observability of type is analytically tractable, and the arising PBE exhibits some properties which are well worth noting. In contrast to most of the existing literature, we pointed out that a directed-search equilibrium can be far from efficient, and that both job-seekers and—even more—firms may be keen on taking some action. The fact that our model's "pathetic" equilibrium survives the D_1 Criterion indicates our analysis is more than a mathematical exercise.

References

Banks, Jeffery S. and Joel Sobel. 1987. "Equilibrium Selection in Signaling Games." *Econometrica* 55 (3):647–661.

Barron, John M., John Bishop, and William C. Dunkelberg. 1985. "Employer Search: The Interviewing and Hiring of New Employees." *Review of Economics and Statistics* 67 (1):43–52.

Blatter, Marc, Samuel Muehlemann, and Samuel Schenker. 2012. "The Costs of Hiring Skilled Workers." *European Economic Review* 56:20–35.

Cho, In-Koo and David M. Kreps. 1987. "Signaling Games and Stable Equilibria." *The Quarterly Journal of Economics* 102 (2):179–222.

Coles, Melvyn G. and Jan Eeckhout. 2000. "Efficient Job Allocation." Manuscript, available at `http://www.econ.upenn.edu/system/files/00-17.pdf`.

Delacroix, Alain and Shouyong Shi. 2006. "Directed Search on the Job and the Wage Ladder." *International Economic Review* 47 (2):651–699.

Hall, Robert E. and Alan B. Krueger. 2008. "Wage Formation Between Newly Hired Workers and Employers: Survey Evidence." *NBER Working Papers* 14329.

Holzer, Harry J., Lawrence F. Katz, and Alan B. Krueger. 1991. "Job Queues and Wages." *The Quarterly Journal of Economics* 106 (3):739–768.

Lang, Kevin and Michael Manove. 2003. "Wage Announcements with a Continuum of Worker Types." *Annals of Economics and Statistics* 71/72:223–244.

Michelacci, Claudio and Javier Suarez. 2006. "Incomplete Wage Posting." *The Journal of Political Economy* 114 (6):1098–1123.

Moen, Espen R. 1997. "Competitive Search Equilibirium." *The Journal of Political Economy* 105 (2):385–411.

———. 2003. "Do Good Workers Hurt Bad Workers – or Is It the Other Way Around?" *International Economic Review* 44 (2):779–800.

Montgomery, James D. 1991. "Equilibrium Wage Dispersion and Interindustry Wage Differentials." *The Quarterly Journal of Economics* 106 (1):163–179.

Mortensen, Dale T. and Christopher A. Pissarides. 1994. “Job Creation and Job Destruction in the Theory of Unemployment.” *Review of Economic Studies* 61 (3):397–415.

Peters, Michael. 1991. “Ex Ante Price Offers in Matching Games Non-Steady States.” *Econometrica* 59 (5):1425–1454.

Rivera, Lauren A. 2011. “Ivies, Extracurriculars, and Exclusion: Elite Employers’ Use of Educational Credentials.” *Research in Social Stratification and Mobility* 29:71–90.

Rogerson, Richard, Robert Shimer, and Wright Randall. 2005. “Search-Theoretic Models of the Labor Market: A Survey.” *Journal of Economic Literature* 43 (4):959–988.

Shimer, Robert. 2001. “The Assignment of Workers to Jobs in an Economy with Coordination Frictions.” *NBER Working Paper* 8501 (4):959–988.

Wial, Howard. 1991. “Getting a Good Job: Mobility in a Segmented Labor Market.” *Industrial Relations* 30 (3):396–416.

Appendix

A.1 *Proof of Lemma 1*

From

$$\phi_1(\underline{s}^m) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\underline{s}^m - 1)^2}{2\sigma^2}} \text{ and } \phi_0(\underline{s}^m) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\underline{s}^m)^2}{2\sigma^2}},$$

we write (2) as

$$q_1(1 - w_j) e^{-\frac{(\underline{s}^m - 1)^2}{2\sigma^2}} = w_j q_0 e^{-\frac{-(\underline{s}^m)^2}{2\sigma^2}}.$$

Upon taking logarithms and rearranging, we obtain (3). For $w^m \in (0,1)$ and $q_1 \in (0,1)$, $\underline{s}^m(\cdot)$ is well-defined and continuous in its arguments. Furthermore, the expected profit is positive at $\underline{s}^{m*}$, zero for $\underline{s}^m(\cdot) \to \infty$, and negative for $\underline{s}^m(\cdot) \to -\infty$. Therefore condition (3) is not only necessary but also sufficient for $\underline{s}^{m*}$ being a global maximizer.

A.2 *Proof of Corollary 2*

The monopsonist maximizes

$$\pi^m(w^m) = (1 - \Phi_1[\underline{s}^m(w^m)])q_1 - w_j \left[(1 - \Phi_1[\underline{s}^m(w^m)])q_1 + \ (1 - \Phi_0[\underline{s}^m(w^m)])q_0\right].$$

From $w^m \geq 0$, it is sufficient to show that $\pi^m(w^m)$ decreases in w^m once we impose $\underline{s}^m(w^m) = \underline{s}^{m*}(w^m)$ (from subgame perfection). By applying the regular envelope theorem, we observe that, as w^m increases, the change in the monopsonist's profit is

$$\left.\frac{\partial \pi^m(w^m, \underline{s}^m(w^m))}{\partial w^m}\right|_{\underline{s}^m(w^m) = \underline{s}^{m*}(w^m)} = -\left(1 - \Phi_1[\underline{s}^{m*}(w^m)]\right) q_1 - \left(1 - \Phi_0[\underline{s}^{m*}(w^m)]\right) q_0,$$

which is strictly negative for any $w^m \in [0,1)$.

A.3 *Proof of Lemma 3*

Suppose to the contrary that for either $\theta_i = 0$ or $\theta_i = 1$ (or both) it holds that $(j^*|\theta_i) \notin K$ with some positive probability p. If a firm $l \in L = J \backslash K$ gets contacted by such a candidate, it infers from Assumption 1 that $\mu(w) = 0$. From (6), l sets $\underline{s}_l^* = \infty$. Thus it holds for the deviating applicant's expected payoff that

$$p \times w_l \mathbb{P}[s_i > \infty | \theta_i] + (1 - p) \times w_k = (1 - p) \times w_k < w_k.$$

On the other hand, if an individual candidate opts for $k \in K$, the off-the-equilibrium-path beliefs of firm k ensure that $\underline{s}_k^* = -\infty$ and the expected payoff of the applicant is w_k.

Alternatively, suppose that all candidates approach firms within L. In this case, applicant i's expected payoff is

$$w_l \mathbb{P}[s_i > \underline{s}_l^*(\cdot) | \theta_i] \leq w_l < w_k.$$

If applicant i deviates to firm k, he or she would (wrongly or rightly) be identified as being of type 1, thus earning w_k for sure.

Finally, it does not pay to deviate from the suggested equilibrium, as only downwards deviations are possible. By doing so, a candidate would be interpreted as being of type 0, thus earning nothing.

A.4 Proof of Proposition 1

Case 1. Suppose to the contrary that $\forall j \in J : w_j = \hat{w} < 1$. For a finite number J of firms, it holds from (6) that each firm's profit is

$$\begin{aligned}\pi_j(\hat{w}) = &\ (1-\hat{w})\left(1-\Phi_1\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{\hat{w}}{1-\hat{w}}\right)\sigma^2\right]\right)\frac{q_1}{J}\\ - &\ \hat{w}\left(1-\Phi_0\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{\hat{w}}{1-\hat{w}}\right)\sigma^2\right]\right)\frac{q_0}{J}.\end{aligned}$$

As $J \to \infty$, we have $\pi_j(\hat{w}) \to 0$ for any $\hat{w} \in (0,1)$. Hence, with a continuum of firms, we solely remain to show that it is possible for a single firm to get a positive profit by deviating regarding the offered salary. Since $\hat{w} < 1$, there exists an $\varepsilon > 0$ such that $1-\varepsilon > \hat{w}$. Therefore, it is possible for a single firm l to set $w_l = 1-\varepsilon$ and thereby achieve a profit of

$$\begin{aligned}\pi_l(w_l) &= \varepsilon\left(1-\Phi_1\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{1-\varepsilon}{\varepsilon}\right)\sigma^2\right]\right)q_1\\ &- (1-\varepsilon)\left(1-\Phi_0\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{1-\varepsilon}{\varepsilon}\right)\sigma^2\right]\right)q_0\Bigg].\\ &=: \varepsilon(1-\Phi_1[\cdot])q_1-(1-\varepsilon)(1-\Phi_0[\cdot])q_0. \qquad \text{(A.1)}\end{aligned}$$

We next show that $\pi_l(w_l) > 0$, or, from (A.1),

$$\frac{q_1}{q_0}\frac{\varepsilon}{1-\varepsilon} > \frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]}, \qquad \text{(A.2)}$$

for $\varepsilon > 0$ sufficiently small. Since $\lim_{\varepsilon\searrow 0}\{q_1\varepsilon/q_0(1-\varepsilon)\} = 0$, it is *necessary* for (A.2) that

$$\lim_{\varepsilon\searrow 0}\left\{\frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]}\right\} = 0. \qquad \text{(A.3)}$$

In order to show (A.3), we apply l'Hôpital's rule, as in the limit both the numerator and the denominator of the fraction at hand are 0, because for $\theta \in \{0,1\}$ it holds that $\lim_{\varepsilon\searrow 0}\Phi_\theta[\cdot] = 1$. According to l'Hôpital,

$$\lim_{\varepsilon\searrow 0}\left\{\frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]}\right\} = \lim_{\varepsilon\searrow 0}\left\{\frac{\frac{\partial(1-\Phi_0[\cdot])}{\partial\varepsilon}}{\frac{\partial(1-\Phi_1[\cdot])}{\partial\varepsilon}}\right\} = \lim_{\varepsilon\searrow 0}\left\{\frac{\phi_0\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{1-\varepsilon}{\varepsilon}\right)\sigma^2\right]}{\phi_1\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{1-\varepsilon}{\varepsilon}\right)\sigma^2\right]}\right\}, \qquad \text{(A.4)}$$

By employing $s_i \sim$ i.i.d. $\mathcal{N}(\theta_i, \sigma^2)$, we rewrite (A.4) as

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{\exp\left(- \frac{\left[\ln\left(\frac{q_0}{q_1}\frac{(1-\varepsilon)}{\varepsilon}\right)\sigma^2 + \frac{1}{2}\right]^2}{2\sigma^2} \right)}{\exp\left(- \frac{\left[\ln\left(\frac{q_0}{q_1}\frac{(1-\varepsilon)}{\varepsilon}\right)\sigma^2 - \frac{1}{2}\right]^2}{2\sigma^2} \right)} \right\}$$
$$= \lim_{\varepsilon \searrow 0} \left\{ \exp\left[- \ln\left(\frac{q_0}{q_1} \frac{(1-\varepsilon)}{\varepsilon} \right) \right] \right\} = \lim_{\varepsilon \searrow 0} \left\{ \frac{q_1}{q_0} \frac{\varepsilon}{(1-\varepsilon)} \right\} = 0,$$

which is the required necessary condition. In order to find a sufficient condition for (A.2), we use l'Hôpital's rule a second time. Specifically, we need

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{\partial \frac{q_1}{q_0} \frac{\varepsilon}{1-\varepsilon}}{\partial \varepsilon} \right\} > \lim_{\varepsilon \searrow 0} \left\{ \frac{\partial \frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]}}{\partial \varepsilon} \right\}. \tag{A.5}$$

The left-hand side of (A.5),

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{q_1}{q_0} \frac{1}{(1-\varepsilon)^2} \right\},$$

is positive. Therefore, it is sufficient to show that this does not hold for its right-hand side, which we rewrite as

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{[(-\phi_0(\cdot))(1-\Phi_1[\cdot]) + \phi_1(\cdot)(1-\Phi_0[\cdot])] \frac{1}{\varepsilon(1-\varepsilon)}}{(1-\Phi_1[\cdot])^2} \right\}. \tag{A.6}$$

(A.6) is negative if $\phi_0(\cdot)(1-\Phi_1[\cdot]) > \phi_1(\cdot)(1-\Phi_0[\cdot])$. De novo, we use l'Hôpital's rule to show that

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{\phi_0(\cdot)}{\phi_1(\cdot)} \right\} > \lim_{\varepsilon \searrow 0} \left\{ \frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]} \right\} \Leftrightarrow \lim_{\varepsilon \searrow 0} \left\{ \frac{1}{1-\varepsilon} \right\} > \lim_{\varepsilon \searrow 0} \{\varepsilon\},$$

which completes the proof for the first case.

Case 2. Suppose now that there are multiple offered wages, whereas $\hat{w} := \max_j\{w_j\} < 1$. Consider firm l which offers $\tilde{w}_l$ such that $0 \leq \tilde{w}_l < \hat{w}$. From Lemma 3 it follows that no worker applies at l, leaving l's profit to be zero. However, as it was argued in the course of Case 1, there exists a wage $w_l \in (\hat{w}, 1)$, which yields a positive profit. Therefore, $\tilde{w}_l$ cannot be a best answer from the beginning.

A.5 Proof of Corollary 4

The first fact directly follows from (1). Then, from Lemma 3, applicants of both types $\theta_i \in \{0, 1\}$ choose firm j^* such that

$$(j^*|\theta_i) \in K : \forall k \in K : w_k = 1.$$

In this case, equilibrium belief formation as stated in Assumption 1 ensures that $(1 - \mu(w))/\mu(w) = q_0/q_1$. Furthermore,

$$(\underline{s}_j^*|w_j = 1) = \lim_{w_j \nearrow 1} \left\{ \frac{1}{2} + \ln\left(\frac{q_0}{q_1} \frac{w_j}{(1-w_j)} \right) \sigma^2 \right\} = \infty.$$

Hence in equilibrium no candidate will be hired.

A.6 Proof of Lemma 4

From (8), the difference in firm j's profit between the two cases $\underline{s}_j = s \in \mathbb{N}$ and $\underline{s}_j = s - 1$ is

$$\begin{aligned} &\Delta\pi_j(s,w) := \pi_j(s,w) - \pi_j(s-1,w) \\ =& \binom{n}{s} \left[w_j\,(1-\mu(w))\,p_0^s(1-p_0)^{n-s} \right. \\ -& \left. (1-w_j)\mu(w)p_1^s(1-p_1)^{n-s} \right]. \end{aligned} \tag{A.7}$$

By rearranging (A.7), we obtain that $\Delta\pi_j(s,w)$ is non-negative as long as

$$\frac{w_j}{1-w_j} \geq \underbrace{\frac{\mu(w)}{1-\mu(w)} \left(\frac{p_1}{p_0}\right)^s \left(\frac{1-p_1}{1-p_0}\right)^{n-s}}_{=:\kappa(\cdot)>0}. \tag{A.8}$$

The derivative of the right-hand side of (A.8), $\kappa(\cdot)$ with respect to s is

$$\frac{\partial\kappa(\cdot)}{\partial s} = \ln\left(\frac{p_1}{(1-p_1)} \frac{(1-p_0)}{p_0} \right) \kappa(\cdot),$$

which is positive, since $p_1 > p_0$. Therefore, if (A.8) holds for the highest possible value of s, n, it implicitly holds for lower values of s as well. That is,

$$\frac{w_j}{1-w_j} \geq \frac{\mu(w)}{1-\mu(w)} \left(\frac{p_1}{p_0}\right)^n \;\Rightarrow\; \forall s \in \mathbb{N} \leq n : \Delta\pi_j(s,w) \geq 0. \tag{A.9}$$

From the uniqueness assumption with regard to $\underline{s}_j^*$, (A.9) completes the proof.

A.7 Proof of Proposition 2

Case 1. Suppose to the contrary that $\forall j \in J : w_j = \hat{w} < w^*$. Independently from the choice of $\underline{s}_j^*$, with $J \to \infty$ we have $\pi_j(\hat{w}) \to 0$, since applicants choose among all firms with equal probability.

Since $\hat{w} < w^*$, there exists an $\varepsilon > 0$ such that $w^* - \varepsilon := \tilde{w}_l > \hat{w}$ can be offered by a deviating firm l. From Assumption 1, all applicants select l and thus $\mu(w) = q_1$ and $(1 - \mu(w)) = q_0$ (see Lemma 3). From the profit function (8) with $\underline{s}_j = n$, it then follows that

$$\begin{aligned}\pi_l(\tilde{w}_l) &= (1-\tilde{w}_l)q_1p_1^n - \tilde{w}_l q_0 p_0^n \\ &= (1-w^*+\varepsilon)q_1p_1^n - (w^*-\varepsilon)q_0p_0^n \\ &= \left(\frac{q_0p_0^n}{q_1p_1^n+q_0p_0^n}+\varepsilon\right)q_1p_1^n - \left(\frac{q_1p_1^n}{q_1p_1^n+q_0p_0^n}-\varepsilon\right)q_0p_0^n \\ &= \varepsilon(q_1p_1^n+q_0p_0^n) > 0.\end{aligned}$$

As it is sufficient to show that l obtains a profit by choosing $\underline{s}_j = n$, the result does not hinge on condition (9) in Lemma 4.

Case 2. Suppose now that there are multiple offered salaries with $\hat{w} := \max_j\{w_j\} < w^*$. Consider firm l which offers $w_l < \hat{w}$. As before, it follows Assumption 1 (and Lemma 3) that no applicant chooses firm l, leaving l without a profit. By referring to Case 1, it holds that there exists an offer $\tilde{w}_l \in (\hat{w}, w^*)$ which yields a positive profit. Therefore, w_l cannot be a best answer.

Case 3. Finally suppose that the highest offer is above w^*. By denoting that highest salary by $w_l := w^* + \varepsilon$, it follows that

$$\frac{w_l}{1-w_l} > \frac{w^*}{1-w^*} = \frac{q_1p_1^n}{q_0p_0^n} = \frac{\mu(w)}{1-\mu(w)}\left(\frac{p_1}{p_0}\right)^n, \tag{A.10}$$

whereas $\frac{q_1}{q_0} = \frac{\mu(w)}{1-\mu(w)}$ results from the random assignment of workers to firms offering w_l. According to Lemma 4, the inequality in (A.10) ensures that $\underline{s}_l^* = n$ at the final stage. Let the number of firms which offer w_l be L. Since for each firm $l \in L$ it applies that $\mu(w_l) = \frac{q_1}{L}$ and $(1-\mu(w_l)) = \frac{q_0}{L}$, their equilibrium profit reads

$$\begin{aligned}\pi_l(w_l) &= (1-w^*-\varepsilon)\frac{q_1p_1^n}{L} - (w^*+\varepsilon)\frac{q_0p_0^n}{L} \\ &= \left(\frac{q_0p_0^n}{q_1p_1^n+q_0p_0^n}-\varepsilon\right)\frac{q_1p_1^n}{L} - \left(\frac{q_1p_1^n}{q_1p_1^n+q_0p_0^n}+\varepsilon\right)\frac{q_0p_0^n}{L} \\ &= -\frac{\varepsilon}{L}(q_1p_1^n+q_0p_0^n) < 0.\end{aligned}$$

Since each $l \in L$ could avoid losses by choosing $\tilde{w}_l < w_l$, w_l cannot be a best answer.

A.8 Proof of Proposition 3

With a salary ceiling $\overline{w}$, the threshold for an applicant's score value at any firm j is

$$\underline{s}_j(\overline{w}) = \frac{1}{2} + \ln\left(\frac{q_0}{q_1}\frac{\overline{w}}{(1-\overline{w})}\right)\sigma^2. \tag{A.11}$$

By incorporating the subsequent course of the game, we write (12) as

$$\begin{aligned}\overline{w}_{\theta_i} &= \arg\max_{\overline{w}}\left\{\overline{w}\left(1-\Phi_{\theta_i}\left[\underline{s}_j(\overline{w})\right]\right)\right\} \\ &= \arg\max_{\overline{w}}\left\{\overline{w}\left(1-\Phi_0\left[\underline{s}_j(\overline{w})-\theta_i\right]\right)\right\}.\end{aligned} \tag{A.12}$$

From (A.11), it follows that the first-order condition to problem (A.12) is

$$1 - \Phi_0\left[\underline{s}_j - \theta_i\right] = \phi_0(\underline{s}_j - \theta_i)\frac{\sigma^2}{1-\overline{w}},$$

which we write as

$$\begin{aligned}\frac{\sigma^2}{1-\overline{w}} &= \int_{\underline{s}_j-\theta_i}^{\infty}\frac{\phi_0(t)}{\phi_0(\underline{s}_j-\theta_i)}\mathrm{d}t \\ &= \int_0^{\infty}\frac{\phi_0(\underline{s}_j-\theta_i+u)}{\phi_0(\underline{s}_j-\theta_i)}\mathrm{d}u \\ &= \int_0^{\infty} e^{-\frac{u^2}{2\sigma^2}-\left(\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{\overline{w}}{(1-\overline{w})}\right)\sigma^2-\theta_i\right)\frac{u}{\sigma^2}}\mathrm{d}u. \qquad \text{(A.13)}\end{aligned}$$

The left-hand side of (A.13) equals σ^2 for $\overline{w} = 0$, then continuously and strictly increases, and approaches infinity for $\overline{w} \nearrow 1$. Meanwhile, its right-hand side approaches infinity for $\overline{w} \searrow 0$, then strictly and continuously decreases, and converges to 0 as $\overline{w} \nearrow 1$. Since the objective function is positive at an interior solution, (A.13) uniquely determines $\overline{w}_{\theta_i}$. Since the expected payoff is 0 at both $\overline{w} = 0$ and $\overline{w} = 1$ and its derivative is 0 only at $\overline{w}_{\theta_i}$, it also holds that the objective function increases for values below $\overline{w}_{\theta_i}$ and decreases for values above $\overline{w}_{\theta_i}$. Finally, since the right-hand side of (A.13) increases in θ_i, we have $\overline{w}_1 > \overline{w}_0$.

A.9 Proof of Lemma 5

We express (14) as

$$\frac{\mathbb{P}\left[s_i \geq \underline{s}_l^*(\mu(w), w_l)|\theta_i = \theta\right]}{\mathbb{P}\left[s_i \geq \underline{s}_k^*(w_k)|\theta_i = \theta\right]} =: \frac{1-\Phi_\theta[\kappa]}{1-\Phi_\theta[\kappa+\Delta]} > \frac{w_k}{w_l},$$

where it follows from $w_k > w_l$ that $\Delta > 0$. Thereby,

$$\frac{1-\Phi_0[\kappa]}{1-\Phi_0[\kappa+\Delta]} > \frac{1-\Phi_1[\kappa]}{1-\Phi_1[\kappa+\Delta]} = \frac{1-\Phi_0[\kappa-1]}{1-\Phi_0[\kappa+\Delta-1]} \qquad \text{(A.14)}$$

implies that

$$\mu(w) \in M_1 \ \Rightarrow \ \mu(w) \in M_0.$$

For (A.14) to be true for arbitrary κ and $\Delta > 0$, it is sufficient to show that

$$\frac{\partial \frac{1-\Phi_0[\kappa]}{1-\Phi_0[\kappa+\Delta]}}{\partial \kappa} > 0 \ \Leftrightarrow \ \frac{\partial\left\{\ln\left(1-\Phi_0[\kappa]\right) - \ln\left(1-\Phi_0[\kappa+\Delta]\right)\right\}}{\partial\kappa} > 0.$$

Otherwise put,

$$\frac{\partial \int_\kappa^{\kappa+\Delta}\frac{\partial \ln(1-\Phi_0[t])}{\partial t}\mathrm{d}t}{\partial\kappa} = \frac{\partial\int_0^{\Delta}\frac{\partial\ln(1-\Phi_0[\kappa+u])}{\partial u}\mathrm{d}u}{\partial\kappa} < 0. \qquad \text{(A.15)}$$

(A.15) necessarily holds if

$$\frac{\partial \frac{\partial \ln(1-\Phi_0[\kappa+u])}{\partial u}}{\partial \kappa} = \frac{\partial \frac{-\phi_0[\kappa+u]}{1-\Phi_0[\kappa+u]}}{\partial \kappa} < 0. \tag{A.16}$$

(A.16) needs to be true for any κ, thus equivalently we can show that

$$\frac{\partial \frac{-\phi_0[\kappa]}{1-\Phi_0[\kappa]}}{\partial \kappa} < 0. \tag{A.17}$$

Since in

$$\begin{aligned}
\frac{-\phi_0[\kappa]}{1-\Phi_0[\kappa]} &= -\left(\int_\kappa^\infty \frac{\phi_0(t)}{\phi_0(\kappa)}\mathrm{d}t\right)^{-1} \\
&= -\left(\int_0^\infty \frac{\phi_0(u+\kappa)}{\phi_0(\kappa)}\mathrm{d}u\right)^{-1} \\
&= -\left(\int_0^\infty e^{-\frac{u^2}{2\sigma^2}-\frac{\kappa u}{\sigma^2}}\,\mathrm{d}u\right)^{-1},
\end{aligned}$$

for a fixed $u > 0$, each integrand is strictly decreasing in κ, the same holds for the negative of the inverse of the integral.