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Competitive Market Segmentation

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Competitive Market Segmentation[*]

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Abstract

In a two-firm model where each firm sells a high-quality and a low-quality version of a product, customers differ with respect to their brand preferences and their attitudes towards quality. We show that the standard result of quality-independent markups crucially depends on the assumption that the customers' valuation of quality is identical across firms. Once we relax this assumption, competition across qualities leads to second-degree price discrimination. We find that markups on low-quality products are higher if consuming a low-quality product involves a firm-specific disutility. Likewise, markups on high-quality products are higher if consuming a high-quality product creates a firm-specific surplus. For either case, we provide second-order approximations of the equilibrium prices.

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1 Introduction

One of the common findings of the literature on horizontal and vertical market segmentation are quality-independent markups: differences in costs are translated one-to-one into differences in prices.[1] This result stands in stark contrast to observed empirical patterns.

For example, Barsky et al. (2003) use scanner data from one of the largest Chicago area supermarket chains to estimate markup ratios between low-quality "no-name" products and high-quality "national brands".[2] Consistently across 19 product categories, they find higher markups on low-quality products than on high-quality products, with estimated ratios ranging from 1.14 (for canned tuna) up to 2.33 (for toothbrushes). This result is mirrored elsewhere, as in the study of Scott Morton and Zettelmeyer (2004) who generically find that retailers "can earn greater net profit[s] from selling the store brands."[3]

In other cases, markups on high-quality products are systematically higher. For the automobile industry, Verboven (1999) provides empirical evidence that premium products have larger absolute markups than base products. Similarly, Barron, Taylor, and Umbeck (2000) estimate the average dealer margin for premium gasoline to be almost 60 percent higher than the average margin for regular gasoline. For the hardware industry, Deltas, Stengos, and Zacharias (2011) show that markups on flagship computers are substantially higher than markups on slower and older machines.

These contrasting examples provide reason for the following conjecture: When the firms' high-quality products are identical, and their respective lower-quality versions differ from each other (such as in retailing), markups on low-quality products tend to be higher. On the other hand, when competitors offer baseline products which are similar, but their premium versions stand out from each other (such as in the automobile industry), high-quality products are seemingly the better deal for firms.

In the present paper, we provide a theory which is in line with this pattern. Contrary to existing theoretical work in this field, we do not assume that a

[1] For comprehensive surveys on the topic, see Armstrong (2006) and Stole (2007).

[2] More precisely, they compute lower bounds on these markup ratios, as marginal cost is not observed.

[3] Analogous results appear, for instance, in Chintagunta, Bonfrer, and Song (2002) and Bonfrer and Chintagunta (2004), where markups on store brands are higher than markups on national brands in 17 of 18 categories. Regarding the music industry, Rabinovich, Maltz, and Sinha (2008) show (among other things) that markups are lower on more popular CDs.

buyer's difference in utility between consuming a high-quality product and a low-quality product is constant across firms.[4]

Existing theories on the subject usually draw on the sequential revelation of information in order to explain quality-related differences in markups. Verboven (1999) and Ellison (2005), for instance, consider "add-on pricing games" where only prices of low-quality products are publicly accessible. The differences between low-quality and high-quality products are interpreted as "add-ons", which provide additional utility if consumed together with the base good. Such a structure allows firms to sell add-ons at the monopoly price, a result which follows from Diamond's (1971) influential paper on price adjustment under learning cost. This, in turn, explains higher markups on premium versions of a product. Such models are certainly relevant concerning questions revolving around two-part tariffs, hidden costs, and the like. In the above examples, however, it is hard to argue that there exists an informational asymmetry between prices of high-quality products and prices of low-quality products. We should, therefore, consider a static "standard pricing game".

Another strand of literature (Armstrong and Vickers, 2001; Rochet and Stole, 2002; Yang and Ye, 2008) studies such "one-shot" models. Unfortunately, for general specifications of demand and cost, these models can only be solved numerically. Under similar assumptions as in the present paper, however, closed-form solutions are available: In a symmetric equilibrium with fully covered markets, prices are cost-plus-fixed-fee, which implies that markups are quality-independent. Verboven (1999), who considers a discrete-choice version of this model, obtains the same result, which he calls "somewhat surprising". In the following, we argue that this finding is actually the result of the fact that in all of these models, the firms basically play two separate competition-on-a-line games à la Hotelling (1929).[5] By considering

[4]Stole (1995), who separately analyzes horizontal and vertical second-degree price differentiation in oligopolistic markets, states that "vertical preferences [...] are harmonious across firms – a customer with a high marginal valuation of quality for one firm will have similar preferences for other firms as well; all firms prefer these customers." While we agree that quality enters utility in a monotonous fashion, we oppose the idea of "harmonious" preferences. Marginal utility may differ from firm to firm.

[5]A notable exception is Bonatti (2011), who integrates brand-specific preferences in a model of competitive market segmentation. In a similar way as Armstrong and Vickers (2001), Rochet and Stole (2002), and Yang and Ye (2008), he allows firms to simultaneously pick quality levels and prices. He obtains a "no distortion at the top" outcome. In the present paper, we consider quality levels as given. This enhances the tractability of the model. Accordingly, we can include horizontal differentiation and (more importantly) explicitly approximate customer prices in the case of uniformly distributed preferences.

alternative distributions of preferences, we investigate departures from this outcome.

In our model, we also describe horizontal differentiation by use of the conventional Hotelling framework. Our novelty concerns the characterization of vertical preferences. We assume the disutility from consuming a low-quality product rather than a high-quality product to be *firm-specific*.[6] As an illustration, consider the case of two retailing firms which both sell a popular brand of a product as well as their respective "no-name" substitutes. The no-name products may differ from the trademark brand in various dimensions, many of which may be called "vertical": technical sophistication, conditions of production, product safety, quality of ingredients, ease of operation, eco-friendliness, durability, and performance are just a few that come to mind. Similarly, consumers are heterogenous as well: Whereas a "gourmet" primarily cares about the sophistication of a meal, a "gourmand" puts more emphasis on its size. In a nutshell, it is possible that the difference between a customer's willingness-to-pay for the two qualities varies from firm to firm; but the sign and magnitude of this difference also varies among customers.

Taking the customers' preferences as given, we look at two firms, each selling a high-quality product and a low-quality product. If the customers' preferences for quality are identical across the firms – an instance which we consider as a *special case* – the firms essentially play two separate Hotelling games: The disutility which a costumer incurs when buying the low-quality product at firm A is the same as it would be at firm B. Thus, alongside the vertical dimension, the firms are in a Bertrand-like situation. In this case, horizontal differentiation remains as the single source of market power, and equilibrium markups are unaffected by vertical preferences.

By contrast, if the customers' vertical preferences are firm-specific, a fundamental asymmetry comes into play: On the one hand, a customer who buys the high-quality product has a constant willingness to pay, regardless of whether his or her vertical disutility (which *would* be incurred by consuming the low-quality product) assumes a low or a high value. On the other hand, a customer who buys the low-quality product has a higher willingness to pay the less the qualitative difference between the two varieties affects his or her utility. That is, the lower the realization of the vertical disutility at firm A, the likelier it becomes that even a B-affine customer starts

[6] Instead of a firm-specific *disutility*, we could think of an *additional utility* which arises from consuming high-quality products. Think of the airline industry, where economy-class compartments are similar across many carriers. Regarding the business class or the first class, one airline focusses on wider seats, whereas another airline serves better food or offers a more up-to-date entertainment system. As we show in the final section of this paper, such a setting tends to induce higher markups on high-quality products.

buying the low-quality product at A. Thus, metaphorically speaking, we show that both firms are able to catch low-quality customers out of their competitor's pond. Accordingly, each firm's low-quality-product customer base becomes to some extent inflated. Usual marginal-versus-infra-marginal-consumers considerations then imply that the firms set higher prices for their lower-quality products.

We organize the rest of the article as follows. In Section 2 we set up a model of spatial competition with two exogenously given qualities. We allow for general distributions, including firm-dependent vertical preferences. As we show in Section 3, the equilibrium relation between markups and quality depends on the distribution of customers. This contrasts with the earlier contributions, but is too general as a result to gain additional insights. Therefore, in the following sections we look at prototypical cases where the customers' taste parameters are uniformly distributed. In Section 4, we impose the standard assumption of perfectly correlated vertical preferences. This leads to quality-independent markups. In Section 5, we put this result into perspective by looking at independently and identically distributed taste parameters. In the case of homogenous high-quality products (abstracting form horizontal preferences), we find higher markups on low-quality products. This result is reversed once we assume that low-quality products are vertically the same, and additional benefits from consuming high-quality products are specific to the firm, as we exemplify in Section 6. Section 7 concludes. Parts of the proofs are relegated to the appendix.

2 Model

Following the standard procedure in the literature, we consider a Hotelling model of horizontal differentiation which we augment by a vertical component. Two profit-maximizing firms, A and B, both offer a low-quality product L and a high-quality product H.[7] For each firm $i \in \{A, B\}$, we denote i's prices for L and H by p_L^i and p_H^i. The constant marginal cost of production is c_L for L and c_H for H. Thus it is the same across firms but generally varies with quality. It is natural to assume that $c_H \geq c_L \geq 0$.

There is a continuous unit mass of consumers, each of which is described by a triplet $\mathbf{d} = (d, d_L^A, d_L^B) \in [0,1]^3$. d is the conventional (horizontal) taste parameter, whereas d_L^i reflects the reduction of a consumer's willingness to pay if he or she buys L instead of H at i. For the moment, we leave it open

[7]In the following, everything which holds for A automatically translates to B, as we assume the firms to be symmetric.

whether $d_L^A = d_L^B$ or not. Each consumer buys only one product at only one firm. If opting for firm i, the utility of consumer $\mathbf{d}$ is

$$u^i(\mathbf{d}) = \begin{cases} v - p_H^i - td^i & \text{if } q_H^i = 1 \\ v - p_L^i - td^i - t_L d_L^i & \text{if } q_L^i = 1, \end{cases} \tag{1}$$

where $d^i = d$ if $i = A$ and $d^i = 1 - d$ if $i = B$. t and t_L are weighting parameters which measure the relative importance of the horizontal and vertical differentiation.[8] We assume v, the base utility of both versions of the product, to be sufficiently high, such that each consumer ends up buying either L or H. Accordingly, we do not have to make any assumptions on a consumer's reservation value. Furthermore, we impose $c_H - c_L \leq t_L$, an assumption which ensures interior equilibria.

Based on (1), we can subdivide the decision of an individual consumer into two parts. First, the consumer chooses firm A if and only if

$$v - \min\{p_H^A, p_L^A + t_L d_L^A\} - td \geq v - \min\{p_H^B, p_L^B + t_L d_L^B\} - t(1-d). \tag{2}$$

Given condition (2), the consumer prefers L over H if and only if

$$p_L^A + t_L d_L^A \leq p_H^A. \tag{3}$$

In analyzing the model, we will look at symmetric subgame-perfect Nash-equilibria for the "pricing game" where each firm i chooses prices p_L^i and p_H^i and consumers subsequently buy their preferred product. These equilibria may or may not involve the selling of both qualities. However, whenever firm i sells both L *and* H in a (putative) equilibrium, we know from (3) that

$$0 \leq x^i := (p_H^i - p_L^i)/t_L < 1. \tag{4}$$

That is, by defining x^i as the threshold value above which d_L^i must lie in order to buy H at i, we know that x^i is located between 0 and 1 in an interior equilibrium.

In the following sections, the structure of the firms' objective functions is generally subject to the relation between prices. Therefore, as long as a firm chooses to remain within an interior symmetric equilibrium, we impose (4) (as well as symmetric prices) in order to construct a firm's objective function. It is clear, however, that these relations can never be used strategically: each firm optimizes with respect to its own prices only.

[8] Hence horizontal preferences are continuously distributed in the compact set $[0, t]$, and vertical preferences are continuously distributed in the compact set $[0, t_L]$.

In Section 3, we derive optimality conditions for arbitrary distributions of (d, d_L^A, d_L^B) and show that, in general symmetric interior equilibria, markups on H and L are not the same but depend on the particular distribution. To illustrate this, Section 4 and Section 5 compare the outcomes of the pricing game for two specific distributions of the consumers' preferences.

In Section 4, we assume that $d_L^A = d_L^B =: d_L$, and $(d, d_L) \sim$ i.i.d. $\mathcal{U}_2[0,1]$. Such a specification is analogous to Verboven (1999), Armstrong and Vickers (2001), and Rochet and Stole (2002), and describes the case where consumers perceive the difference between L and H to the same extent at both firms.

By contrast, in Section 5 we assume that $\mathbf{d} \sim$ i.i.d. $\mathcal{U}_3[0,1]$. There, vertical preferences with respect to the two firms are completely uncorrelated. It might be that at firm A a consumer barely notices any difference between L and H, whereas at firm B the willingness to pay is much higher for H than for L.

3 General Demand Function: Differing Markups on H and L

Before examining the competing firms' equilibrium behavior, we need to determine their demand as a function of all prices.

Demand Function Q_H^A, firm A's total demand for product H, is the (ex-ante) probability that a consumer prefers to buy H at A to buying H or L at B or L at A. Using (1), we have

$$\begin{aligned} Q_H^A = P[q_H^A = 1] = P \Big[& \underbrace{p_H^A + td \le p_H^B + t(1-d)}_{d \le \overline{HH}} \\ \cap\ & \underbrace{p_H^A + td \le p_L^B + t(1-d) + t_L d_L^B}_{d \le \overline{HL}} \\ \cap\ & \underbrace{p_H^A + td < p_L^A + td + t_L d_L^A}_{d_L^A > x^A} \Big]. \end{aligned} \tag{5}$$

In (5), $\overline{X^A X^B}$ refers to the "switching line", below which a consumer's value of d must lie such that he or she prefers to buy $X^A \in \{L, H\}$ at A as compared to buying $X^B \in \{L, H\}$ at B. As defined in (4), x^A is the critical value of d_L^A above which a consumer prefers to buy H at A instead of L at A. Analogically, we write Q_L^A as

$$\begin{aligned} Q_L^A = P[q_L^A = 1] = P\Big[& \underbrace{p_L^A + td + t_L d_L^A \leq p_H^B + t(1-d)}_{d \leq \overline{LH}} \\ & \cap \underbrace{p_L^A + td + t_L d_L^A \leq p_L^B + t(1-d) + t_L d_L^B}_{d \leq \overline{LL}} \\ & \cap \underbrace{p_L^A + td + t_L d_L^A \leq p_H^A + td}_{d_L^A \leq x^A} \Big]. \end{aligned} \tag{6}$$

After solving the first two inequalities in (5) for d, we write the conditional joint probability of $d \leq \overline{HH}$ and $d \leq \overline{HL}$ as

$$P\left[d \leq \overline{HH} \cap d \leq \overline{HL} | d_L^A, d_L^B\right] = \int_0^{\min\{\overline{HH}, \overline{HL}\}} f(d|d_L^A, d_L^B) \mathrm{d}d. \tag{7}$$

For this equality to be true, we need $\min\{\overline{HH}, \overline{HL}\} \in [0, 1]$. In an interior symmetric equilibrium, this condition holds by symmetry and condition (4), provided $0 \leq (p_H^A - p_L^B)/t \leq 1$.[9] That is, prices need to be sufficiently close; and therefore, for each combination of d_L^A and d_L^B, there are both consumers with small values of d who buy H at A and consumers with large values of d who buy any of the products at B. By incorporating the remaining inequality in (5), $d_L^A > x^A$, into (7), we obtain

$$Q_H^A = \int_0^1 \left(\int_{x^A}^1 \left(\int_0^{\min\{\overline{HH}, \overline{HL}\}} f(d, d_L^A, d_L^B) \mathrm{d}d \right) \mathrm{d}d_L^A \right) \mathrm{d}d_L^B. \tag{8}$$

In the upper part of Figure 1, we graphically represent Q_H^A, the region where $d \leq \overline{HH}$, $d \leq \overline{HL}$, and $d_L^A > x^A$. For given prices p_H^A and p_L^A, with $p_H^A > p_L^A$, consumers buy H at A if their value of d is low and their value of d_L^A is high. Furthermore, in order for a consumer to buy H, d_L^B must not be too small.

[9] In Section 4, this additional condition is redundant, as $d_L^A = d_L^B$ is sufficient for $\min\{\overline{HH}, \overline{HL}\} \in [0, 1]$ and $\min\{\overline{LH}, \overline{LL}\} \in [0, 1]$. In Section 5, we first assume the validity of the condition, and subsequently prove that $p_H^A - p_L^B > t$ cannot hold in any symmetric interior equilibrium.

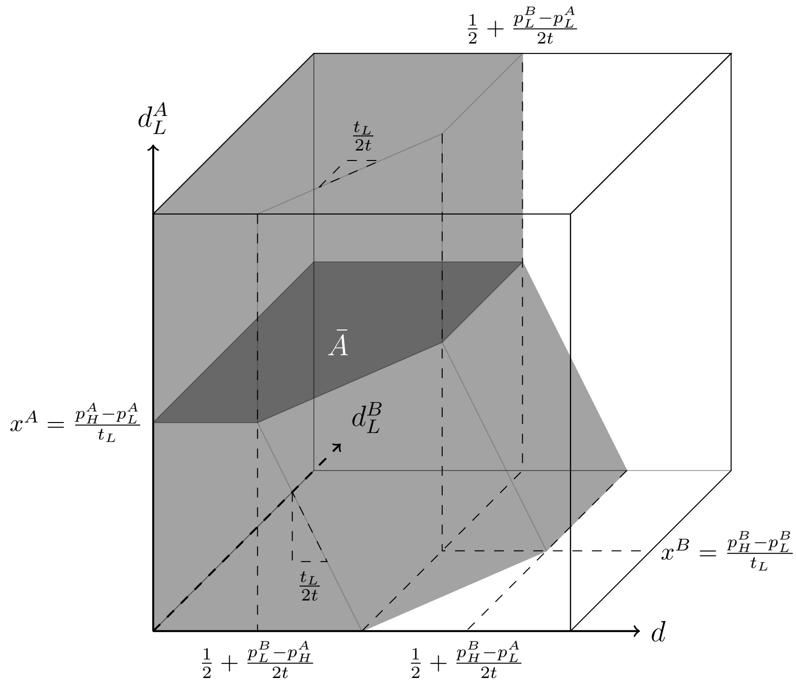

Figure 1: Consumers who buy A's high-quality product, Q_H^A, are displayed in the upper region, where $d_L^A > x^A$. Consumers who buy A's low-quality product, Q_L^A, are displayed in the lower region, where $d_L^A \leq x^A$.

The derivation of Q_L^A is analogous to the derivation of Q_H^A. Consumers buy L at A if this is more favorable than either buying any of H or L at B or buying H at A. Using (6), we obtain

$$Q_L^A = \int_0^1 \left(\int_0^{x^A} \left(\int_0^{\min\{\overline{LH}, \overline{LL}\}} f(d, d_L^A, d_L^B) \mathrm{d}d \right) \mathrm{d}d_L^A \right) \mathrm{d}d_L^B. \tag{9}$$

In the lower part of Figure 1, we graphically represent Q_L^A, the region where $d \leq \overline{LH}$, $d \leq \overline{LL}$, and $d_L^A \leq x^A$. By comparing Q_H^A and Q_L^A, the following asymmetry stands out. Regarding d, the support of Q_H^A is independent of d_L^A, whereas the support of Q_L^A increases for a decreasing d_L^A. In other words, regarding consumers of L, the utility is generally higher the lower d_L^A is. On the other hand, consumers of H all obtain the same level of utility, once we abstract from d. As it later turns out, this asymmetry plays a crucial role for the determination of markups whenever we assume that a change in d_L^A not necessarily implies a change in d_L^B.

Equilibrium In the following, we first determine the prices and profits which occur in symmetric interior equilibria. Later, for each distribution of $\mathbf{d}$ which we analyze, we also check whether firms have incentives to deviate to corner solutions, i.e., by only selling L or H. Furthermore, we demonstrate that symmetric corner solutions cannot constitute equilibria in each of the considered cases.

In an interior equilibrium, firm A maximizes

$$\pi^A = Q_H^A(p_H^A - c_H) + Q_L^A(p_L^A - c_L), \tag{10}$$

with Q_H^A and Q_L^A defined in (8) and (9). Since the integrands in Q_H^A and Q_L^A are continuous functions in p_H^A and p_L^A, (10) is differentiable, and the first-order conditions (FOCs) with respect to p_H^A and p_L^A are necessarily satisfied in an interior equilibrium. As we show in Appendix A.1, after imposing $p_H^A = p_H^B =: p_H$ and $p_L^A = p_L^B =: p_L$ (and thus $x^A = x^B =: x$), we can express the FOC with respect to p_H^A as

$$\begin{aligned}&\left(\int_0^1\int_x^1 m^H \mathrm{d}d_L^A \mathrm{d}d_L^B + \int_0^1 \frac{A^H(d_L^A = x)}{t_L}\mathrm{d}d_L^B\right)(p_H - c_H)\\ =&\underbrace{\int_0^1\int_x^1 A^H \mathrm{d}d_L^A \mathrm{d}d_L^B}_{=Q_H^A} + \int_0^1 \frac{A^H(d_L^A = x)}{t_L}\mathrm{d}d_L^B(p_L - c_L),\end{aligned} \tag{11}$$

where A^H and m^H are defined as follows.

$$A^H := \int_0^{\min\{\overline{HH},\overline{HL}\}} f(\mathbf{d})\mathrm{d}d$$

denotes the amount of H-consumers at A for given values of d_L^A and d_L^B, and

$$m^H := (1/2t)f(\min\{\overline{HH},\overline{HL}\}, d_L^A, d_L^B)$$

is the "margin" of A^H, that is, the subset of A^H which leaves firm A towards firm B after a marginal increase of p_H^A.

How can we interpret equation (11)? Of course, in an interior optimum, the marginal benefits of increasing p_H^A equal the marginal costs. The left-hand side of (11) displays the latter: The first term, where $d_L^A > x$, relates to A's H-consumers which leave A towards B if A raises p_H^A. By doing so, the firm looses $p_H - c_H$ on each of these consumers. It also looses the same markup on consumers who stay at A but start to buy L instead of H. The measure of these intra-firm migrants is $\bar{A}/t_L$, where $\bar{A} := \int_0^1 A^H(d_L^A = x)\mathrm{d}d_L^B$ is shown in Figure 1. The same consumers, however, are also part of the marginal benefit of increasing p_H^A, which we display on the right-hand side of (11). On each such consumer, A gains $p_L - c_L$, the markup on L. The remaining part of the marginal benefit is Q_H^A, which refers to the infra-marginal consumers on whom A benefits by increasing p_H^A.

Analogically, and also in Appendix A.1, we simplify the symmetric-version FOC with respect to p_L^A as

$$\left(\int_0^1 \int_0^x m^L \mathrm{d}d_L^A \mathrm{d}d_L^B + \int_0^1 \frac{A^L(d_L^A = x)}{t_L} \mathrm{d}d_L^B\right)(p_L - c_L)$$
$$= \underbrace{\int_0^1 \int_0^x A^L \mathrm{d}d_L^A \mathrm{d}d_L^B}_{=Q_L^A} + \int_0^1 \frac{A^L(d_L^A = x)}{t_L} \mathrm{d}d_L^B (p_H - c_H), \tag{12}$$

where $A^L := \int_0^{\min\{\overline{LH}, \overline{LL}\}} f(\mathbf{d})\mathrm{d}d$, and $m^L := (1/2t) f(\min\{\overline{LH}, \overline{LL}\}, d_L^A, d_L^B)$ are similarly interpreted as A^H and m^H. In Appendix A.1, we show that $A^L(d_L^A = x) = A^H(d_L^A = x)$. That is, marginal consumers *within* firm A are the same for changes in p_H^A and changes in p_L^A. We highlight these consumers by $\bar{A}$ in Figure 1.

Beyond that, equation (12) again displays two considerations. First, we have the conventional tradeoff between gains on infra-marginal consumers and losses on marginal consumers who leave the firm. Second, regarding $\bar{A}/t_L$, the FOC displays an instance of "intra-firm competition" between H at A and L at A.

Before heading to applications of what we established so far, we state Proposition 1, which sheds some new light on the results of Verboven (1999), Armstrong and Vickers (2001), and Rochet and Stole (2002).

Proposition 1. *Suppose there is a symmetric interior equilibrium of the pricing game, where $p_H - p_L \leq t$. Then the firms' markups on H and L are generally different.*

Proof. Note that $\min\{\overline{HH}, \overline{HL}\} = \overline{HH}$ if and only if $d_L^B > x$. Analogically, $\min\{\overline{LH}, \overline{LL}\} = \overline{LH}$ if and only if $d_L^B > x$. Next, suppose that markups on H and L were identical. Define $\mathcal{M} = p_H - c_H = p_L - c_L$. In this case, adding up equations (11) and (12) yields

$$\frac{1}{2t}\left(\int_0^x \left(\int_0^x f(\overline{LL}, d_L^A, d_L^B)\mathrm{d}d_L^B + \int_x^1 f(\overline{LH}, d_L^A, d_L^B)\mathrm{d}d_L^B\right)\mathrm{d}d_L^A\right.$$
$$\left.+ \int_x^1 \left(\int_0^x f(\overline{HL}, d_L^A, d_L^B)\mathrm{d}d_L^B + \int_x^1 f(\overline{HH}, d_L^A, d_L^B)\mathrm{d}d_L^B\right)\mathrm{d}d_L^A\right)\mathcal{M}$$
$$= Q_L^A + Q_H^A = 1/2,$$

where the second equality is based on the symmetry condition.

We see that, in general, markups on H and L are not identical: Even though a variation in the distribution of either d_L^A or d_L^B could be absorbed by a change of $\mathcal{M}$ (it necessarily holds that $x = (c_H - c_L)/2$), this is generally not possible for a change in both of these distributions. $\square$

As a corollary, Proposition 1 implies that, if $f(\mathbf{d})$ is constant (and therefore equals 1) and there are identical markups on H and L, these markups are $\mathcal{M} = t$. More importantly, the same applies if

$$f(\mathbf{d}) = \begin{cases} 1 & \text{if } d_L^A = d_L^B, \\ 0 & \text{otherwise,} \end{cases} \tag{13}$$

which is the standard case in the literature.

In Section 4, we show that markups are indeed identical (and therefore equal $\mathcal{M}$) in this standard case, and that symmetric equilibria usually exist and are interior. Thereupon, in Sections 5 and 6, we discuss counterexamples where markups on H and L differ.

4 The Benchmark: Correlated Vertical Preferences

Here we consider the case where each consumer perceives the quality difference between L and H to the same extent at both firms. We show that, from a firm's point of view, the presence of a second (vertically differentiated) product is inconsequential, as compared to the standard Hotelling case.

Demand Function Specifically, we assume that $d_L^A = d_L^B =: d_L$, and $(d, d_L) \sim$ i.i.d. $\mathcal{U}_2[0,1]$. The difference in utility between consuming H and L is the same at both firms. In this case, equations (8) and (9) simplify to

$$Q_H^A = \int_{x^A}^1 \left(\int_0^{\min\{\overline{HH}, \overline{HL}\}} \mathrm{d}d \right) \mathrm{d}d_L, \text{ and } Q_L^A = \int_0^{x^A} \left(\int_0^{\min\{\overline{LH}, \overline{LL}\}} \mathrm{d}d \right) \mathrm{d}d_L.$$

We show graphical representations of Q_H^A and Q_L^A in Figures 2 and 3, respectively, which can be considered as cross-sections of Figure 1 for which $d_L^A = d_L^B$.

In Figure 2, each of three shaded areas represents consumers who prefer to buy H at A as compared buying either H at B ($d \leq \overline{HH}$), L at B ($d \leq \overline{HL}$) or L at A ($d_L^A > x^A$). The intersection of these areas is Q_H^A. In a symmetric

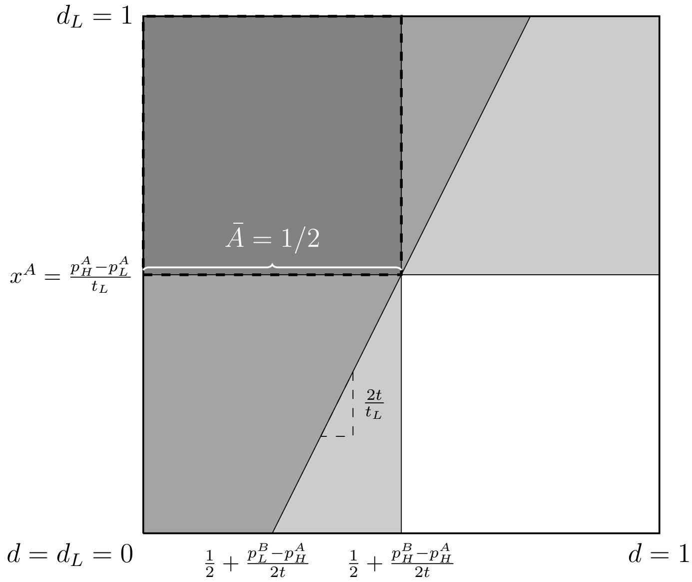

Figure 2: The fraction of consumers which buy firm A's high-quality product, Q_H^A, is given by the intersection of the three shaded areas.

equilibrium, $d \leq \overline{HL}$, which is bordered by the upward-sloping line, never binds: Whenever a consumer prefers buying H at A over buying L at A and H at B, it follows from symmetric prices and $d_L^A = d_L^B$ that this consumer also prefers buying H at A over buying L at B. In other words, there is no direct "inter-firm" competition between H and L.

In Figure 3, we represent Q_L^A. Here, the downward-sloping line borders $d \leq \overline{LH}$, a condition which also never binds, for exactly the same reason.

Equilibrium and Welfare In order determine prices and profits of a symmetric interior equilibrium, we start with the FOCs (11) and (12), which hold in a symmetric interior equilibrium. Thereupon, we show that firms do not have incentives to deviate to a corner solution, that is, by only selling L or H. Furthermore, we show that, in general, a symmetric corner solution is not an equilibrium either.

By applying (13) to (11) and (12), we write the symmetric-version FOCs with respect to p_H^A and p_L^A as

$$\left(\frac{1-x}{2t} + \frac{1}{2t_L}\right)(p_H - c_H) = \frac{1-x}{2} + \frac{1}{2t_L}(p_L - c_L), \tag{14}$$

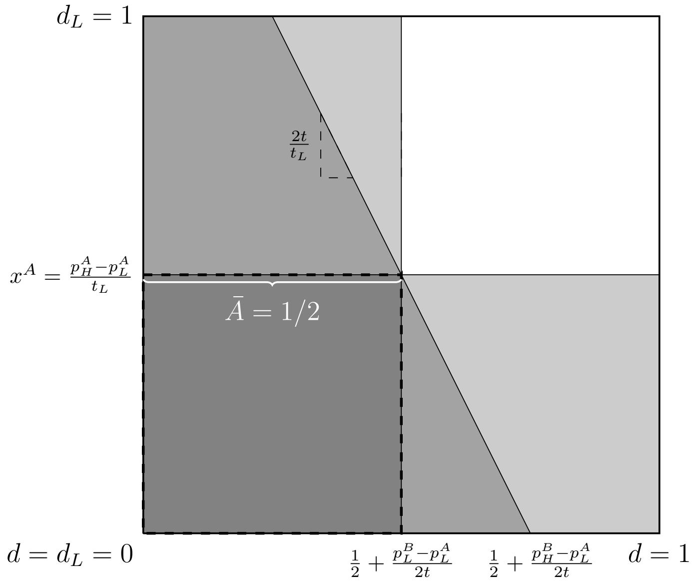

Figure 3: The fraction of consumers which buy firm A's low-quality product, Q_H^A, is given by the intersection of the three shaded areas.

and

$$\left(\frac{x}{2t} + \frac{1}{2t_L}\right)(p_L - c_L) = \frac{x}{2} + \frac{1}{2t_L}(p_H - c_H). \tag{15}$$

Before discussing equations (14) and (15), we use them to establish Lemma 1.

Lemma 1. *If $d_L^A = d_L^B = d_L$, and $(d, d_L) \sim$ i.i.d. $\mathcal{U}_2[0,1]$, symmetric interior equilibria of the pricing game are characterized by $p_H^* = c_H + t$ and $p_L^* = c_L + t$.*

Proof. Note that $x(p_H^*, p_L^*) = (c_H - c_L)/t_L$ and (p_H^*, p_L^*) solves (14) and (15). In the opposite direction, assume to the contrary that $p_H - c_H > p_L - c_L$. If this is the case, (14) implies that $p_H - c_H < t$, while (15) implies that $p_L - c_L > t$. Therefore, $p_L - c_L > t > p_H - c_H$, which contradicts $p_H - c_H < p_L - c_L$. Since $p_H - c_H < p_L - c_L$ yields a similar contradiction, it must hold that $p_H - c_H = p_L - c_L =: \mathcal{M}$. Formulated this way, (14) reads $(1-x)/2t \times \mathcal{M} = (1-x)/2$, or $\mathcal{M} = p_H - c_H = p_L - c_L = t$. In Appendix A.2, we show that there are no unilateral deviations towards corner solutions. $\square$

How can we interpret Lemma 1? In the symmetric equilibrium, each firm serves half the customer base, a fraction x of which buys L and a fraction

$1-x$ buys H. Markups on H and L are identically equal to t, and profits are $\pi(p_H^*, p_L^*) = t/2$. The left-hand side of equation (14) displays the marginal cost of increasing p_H^A: $(1-x)/2t$ consumers (A's H-consumers) leave A towards B; and $1/2t_L$ consumers change the product but not the firm. The right-hand side of (14) shows the marginal benefit of increasing p_H^A: $(1-x)/2$ relates to A's infra-marginal consumers; and $1/2t_L$ start buying L instead of H at A. The interpretation of (15) is analogous. Thus, from identical markups on H and L, there is no need for A to care about customers who stay with A but only swap qualities. It solely remains to trade off between the marginal cost from consumers who leave A and the marginal benefit on infra-marginal ones. That is, regarding H, A and B play a standard Hotelling game with respect to these consumers for whom $d_L > x = (c_H - c_L)/t_L$. Concerning consumers with $d_L \leq x$, A and B play a Hotelling game regarding L. As we have seen earlier, in a symmetric equilibrium, A's version of H does not directly compete with B's version of L. That is, by marginally raising p_H^A, A affects the composition of its own customer base, and A looses some consumers who start buying H at B. From $d_L^A = d_L^B$, however, there is no first-order effect concerning consumers who both change the firm and the product. In Section 5, we drop the assumption of correlated vertical preferences. There, both H and L of B will pose a threat if A increases p_H^A or p_L^A. The alternative assumption $d_L^A \perp d_L^B$ will crucially change the result of Lemma 1.

Before concluding this section with a more general statement which takes into account the possibility of corner solutions, we compare the market outcome with first-best efficient allocations. In the present case, maximizing a utilitarian welfare function is equivalent to minimizing the sum of the firms' production cost and the consumers' disutility from both horizontal and vertical distance to the product which is actually bought. If and only if it is efficient that consumer $\bar{\mathbf{d}}$ buys L at A, it is efficient that consumers with lower values of d and d_L also buy L at A. Likewise, if and only if it is efficient that $\bar{\mathbf{d}}$ buys H at A, it is efficient that consumers with lower values of d and higher values of d_L also buy H at A. Hence, in order to determine a socially efficient allocation, it is sufficient to find cutoff values $\bar{d}^*$ and $\bar{d}_L^*$ which minimize

$$\begin{aligned} W(\bar{d}, \bar{d}_L) = &\int_0^{\bar{d}} \left(\int_0^{\bar{d}_L} c_L + td + t_L d_L \mathrm{d}d_L + \int_{\bar{d}_L}^1 c_H + td \mathrm{d}d_L \right) \mathrm{d}d \\ &+ \int_{\bar{d}}^1 \left(\int_0^{\bar{d}_L} c_L + t(1-d) + t_L d_L \mathrm{d}d_L + \int_{\bar{d}_L}^1 c_H + t(1-d) \mathrm{d}d_L \right) \mathrm{d}d. \end{aligned}$$

The FOCs of $W(\bar{d}, \bar{d}_L)$ with respect to $\bar{d}$ and $\bar{d}_L$ yield $\bar{d}^* = 1/2$ and $\bar{d}_L^* = (c_H - c_L)/t_L$. These cutoff levels comply with the market solutions described in the following proposition. Market allocations are efficient because, abstracting from horizontal preferences, $d_L^A = d_L^B$ implies that firms are in a Bertrand situation, which in turn leads to first-best efficiency.

Before relaxing $d_L^A = d_L^B$, we summarize the main findings of this section in Proposition 2. We present the remaining parts of the proof in Appendix A.3.

Proposition 2. *If $d_L^A = d_L^B = d_L$, and $(d, d_L) \sim$ i.i.d. $\mathcal{U}_2[0,1]$, symmetric equilibria of the pricing game are characterized by $p_H^* = c_H + t$ and $p_L^* = c_L + t$. Consumers with $d \leq 1/2$ ($d > 1/2$) buy at firm A (B). Consumers with $d_L > (c_H - c_L)/t_L$ ($d_L \leq (c_H - c_L)/t_L$) buy H (L). Each firm's equilibrium profit is $\pi(p_H^*, p_L^*) = t/2$, and the market outcome is efficient.*

5 Uncorrelated Vertical Preferences (I): Markups on L May Be Higher

Here we consider the case where the consumer-specific perception of the quality difference between H and L depends on the firm. We show that, in contrast to the outcome of Section 4, the markup on L is higher than the markup on H, equilibrium profits increase, and the market outcome is inefficient.

Demand Function From $\mathbf{d} \sim$ i.i.d. $\mathcal{U}_3[0,1]$, we use the definitions in (8) and (9) with $f(\mathbf{d}) \equiv 1$ to describe Q_H^A and Q_L^A. The difference in utility between consuming H and L is now specific to the firm. For a graphical representation of Q_H^A and Q_L^A, recall Figure 1 in Section 3. There, we hinted at a fundamental asymmetry between Q_H^A and Q_L^A, which we study now in greater detail.

Abstracting from the horizontal characteristic d, consumers with high values of d_L^A (which buy H at A) all obtain the same utility, which equals the utility of consumers who are indifferent between buying L and H. To see this, consider Figure 4, which reproduces Figure 1 and augments it by Q_H^B and Q_L^B (as they are allocated in a symmetric interior equilibrium). Consumer $\mathbf{d^{LH}}$ is indifferent between buying at A and buying at B, but also between buying L at A and buying H at A. Consumers with higher values of d_L^A, such as consumer $\mathbf{d^H}$, are indifferent between the firms, but prefer buying product H once they opt for firm A. The only difference between $\mathbf{d^{LH}}$ and $\mathbf{d^H}$ concerns the realization of d_L^A. For a H-consumer, however, d_L^A is irrelevant, as it only affects the utility of L-consumers. For these, the lower the value of d_L^A, the higher is the utility obtained from buying L. Therefore, consumers who

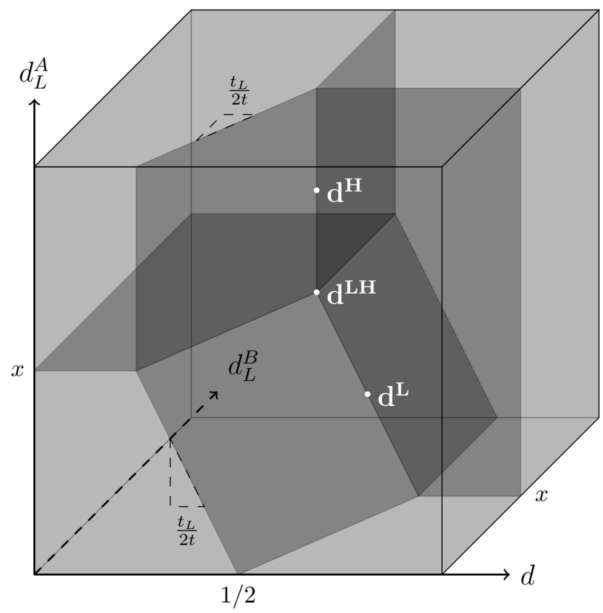

Figure 4: In a symmetric interior equilibrium allocation, consumers with high values of d_L^A and low values of d buy H at A; consumers with low values of d_L^A and low values of d buy L at A; consumers with high values of d_L^B and high values of d buy H at B; and consumers with low values of d_L^B and high values of d buy L at B.

resemble $\mathbf{d^{LH}}$, but exhibit lower values of d_L^A, are strictly better off than $\mathbf{d^{LH}}$ (by buying L at H). Consequentially, even consumers with higher values of d, such as $\mathbf{d^L}$, are equally well off as $\mathbf{d^{LH}}$.

Naturally, we can replicate this thought experiment for each consumer who is indifferent between either of the product versions at A. As we see in Figure 4, this results in an inflated quantity Q_L^A. More precisely, the ratio between A's infra-marginal L-consumers and A's marginal L-consumers (on the boundary towards B) exceeds the same ratio with respect to A's H-consumers. This, in turn, provides an incentive for A to raise p_L^A.

Equilibrium and Welfare As in Section 4, we start with the FOCs (11) and (12), which apply in a symmetric interior equilibrium. Thereupon, we show once more that firms do not have incentives to deviate to a corner solution, and we demonstrate that, in general, a symmetric corner solution is not an equilibrium either.

After imposing $f(\mathbf{d}) \equiv 1$ on (11) and (12), we write the symmetric-version FOCs with respect to p_H^A and p_L^A as

$$\left(\frac{1-x}{2t} + \frac{\bar{A}}{t_L}\right)(p_H - c_H) = (1-x)\bar{A} + \frac{\bar{A}}{t_L}(p_L - c_L), \tag{16}$$

and

$$\left(\frac{x}{2t} + \frac{\bar{A}}{t_L}\right)(p_L - c_L) = x\left(\bar{A} + \frac{p_H - p_L}{4t}\right) + \frac{\bar{A}}{t_L}(p_H - c_H), \tag{17}$$

where $\bar{A} := 1/2 - (t_L/4t)x^2$ represents the frontier between consumers of H and consumers of L at A (see Figure 1).

Equations (16) and (17) help us understand the firms' equilibrium behavior. The left-hand side of (16) displays the marginal cost of increasing p_H^A: A share of A's customers who are indifferent between the two firms, namely the fraction $1-x$ which buys H at A, leaves A towards B; an additional fraction $\bar{A}/t_L$ moves inside the firm to buy L. On both of these groups, A loses $p_H - c_H$ per customer. Regarding the customers which move within A, however, the marginal loss is offset by a marginal profit of $p_L - c_L$ on L. We see this on the right-hand side of (16), alongside with $(1-x)\bar{A}$, A's existing consumers of H, on whom A increases its profit by increasing p_H^A.

The interpretation of (17) is similar. An increase of p_L^A leads to a per-customer loss of $p_L - c_L$ on L, both on consumers leaving A towards B (x) and on the ones moving internally ($\bar{A}/t_L$). Again, part of this loss is retained by a gain on H. The marginal profit on infra-marginal consumers, however, looks different here ($x(\bar{A} + (p_H - p_L)/4t)$, instead of $x\bar{A}$). These additional $x(p_H - p_L)/4t$ consumers refer to a firm's inflated demand for L, as we elaborated in the previous paragraph.

If $p_H^A = p_L^A$, infra-marginal L-consumers, and with them the associated asymmetry, disappear. As we show next, $c_H = c_L$ is a sufficient condition for $p_H^A = p_L^A$ in a symmetric interior equilibrium. In this case, (16) and (17) have a simple solution, which is identical markups on H and L. This, however, only holds for $c_H = c_L$. In a next step, we will thus "perturb" c_H to examine equilibrium markups on H and L in a more general setting where $c_H \geq c_L$.

Lemma 2. *If* $\mathbf{d} \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$*, and* $c_H = c_L$*, in a symmetric interior equilibrium of the pricing game it must hold that* $p_H = p_L$*.*

Proof. If $p_H < p_L$, nobody buys L. If $p_H > p_L$, we know from (16) that

$$(p_H - c_H)/2t < \bar{A}. \tag{18}$$

From (17), $p_H > p_L$ implies

$$(p_L - c_L)/2t > \bar{A} + (p_H - p_L)/4t. \tag{19}$$

By combining (18) and (19), we have $p_L - c_L > 2t\bar{A} + (p_H - p_L)/2 > 2t\bar{A} > p_H - c_H$, a contradiction to $c_H = c_L$ and $p_H > p_L$. $\square$

In words, in the case of identical costs $c_H = c_L$, it cannot be that markups on high-quality products are higher than markups on low-quality products. As we state in (18), if $p_H > p_L$, firms had an incentive to lower p_H and internally move consumers to H, unless there were relatively many infra-marginal consumers of H.[10] Meanwhile, as we state in (19), firms had an incentive to raise p_L, unless there were relatively little infra-marginal consumers of L. High gains on infra-marginal H-consumers and low gains on infra-marginal L-consumers, however, cannot occur simultaneously. The former implies a high amount H-consumers. The latter implies a high amount of L-consumers, as here the ratio between infra-marginal and marginal consumers (marginal with respect to B) decreases with the quantity of low-quality consumers.

Next, we show that, for the special case $c_H = c_L$, the equilibrium prices equal the ones in Section 4.

Lemma 3. *If* $\mathbf{d} \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$, *and* $c_H = c_L$, *the only symmetric interior equilibrium of the pricing game is characterized by* $p_H^* = c_H + t$ *and* $p_L^* = c_L + t$.

Proof. Note that $x(p_H^*, p_L^*) = 0$, $A(p_H^*, p_L^*) = 1/2$, and therefore (p_H^*, p_L^*) solves (16) and (17). In the opposite direction, Lemma 2 requires $p_H - c_H = p_L - c_L =: \mathcal{M}$. Formulated this way, (17) reads $(1/2t + 1/2t_L)\mathcal{M} = 1/2 + (1/2t_L)\mathcal{M}$, implying $\mathcal{M} = p_H - c_H = p_L - c_L = t$. Later, that is, for the general case without restricting to $c_H = c_L$, we show that there are no unilateral deviations towards corner solutions. $\square$

The intuition behind Lemma 3 is simple: Given $p_H = p_L$, the probability that a consumer buys H is 1. Therefore, vertical preferences do not play a role, and the firms essentially play a standard Hotelling game, the familiar outcome of which is $p_H^* = c_H + t$. The according profits are $\pi(p_H^*, p_L^*) = t/2$.

Once we drop the assumption $c_H = c_L$, matters become more difficult. In particular, the analytic solution to (16) and (17) is generally intricate and barely interpretable. Since Lemma 3 reveals a simple solution for $c_H = c_L$, however, we can use "perturbation methods" (see, for instance, Judd, 1996)

[10] "Relatively" refers to the relation between A's $(1-x)A$ infra-marginal H-consumers and the marginal gain $(1-x)(p_H - c_H)$ on consumers who start buying at A if p_H^A decreases.

in order to locally approximate p_H^* and p_L^* for $c_H > c_L$ near $c_H = c_L$. In our case, the appropriate perturbation technique consists of using Taylor's theorem alongside with the implicit function theorem for $\mathbb{R}^2$. In Appendix A.4, we derive second-degree Taylor approximations for p_H^* and p_L^* which we present in Lemma 4.

Lemma 4. *If* $\mathbf{d} \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$*, and* $c_H \geq c_L$ *sufficiently close, the only symmetric interior equilibrium of the pricing game is characterized by*

$$p_H^* = c_H + t + \mathcal{O}\left((c_H - c_L)^3\right)$$

and

$$p_L^* = c_L + t + \frac{(c_H - c_L)^2}{2t} + \mathcal{O}\left((c_H - c_L)^3\right).$$

Sketch of Proof. After defining $\mathbf{p} := (p_H, p_L)$, we write (16) and (17) as $f(\mathbf{p}) = 0$ and $g(\mathbf{p}) = 0$. Next, we define $\mathbf{F}(\mathbf{p}) := (f(\mathbf{p}), g(\mathbf{p}))$. By the implicit function theorem, it holds that

$$[\mathbf{F}_{\mathbf{p}}]_{c_H=c_L}\mathbf{p}' + [\mathbf{F}_c]_{c_H=c_L} = 0, \tag{20}$$

and

$$[\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')]_{c_H=c_L}\mathbf{p}' + [\mathbf{F}_{\mathbf{p}}]_{c_H=c_L}\mathbf{p}'' + 2[\mathbf{F}_{\mathbf{p}c}]_{c_H=c_L}\mathbf{p}' + [\mathbf{F}_{cc}]_{c_H=c_L} = 0, \tag{21}$$

where $\mathbf{p}' := \mathrm{d}\mathbf{p}/\mathrm{d}c_H$, $\mathbf{p}'' := \mathrm{d}^2\mathbf{p}/(\mathrm{d}c_H)^2$, $\mathbf{F}_{\mathbf{p}(c)}$ is the Jacobian of $\mathbf{F}$ with respect to $\mathbf{p}$ (c_H), $\mathbf{F}_{\mathbf{p}c}$ ($\mathbf{F}_{cc}$) is the derivative of $\mathbf{F}_{\mathbf{p}}$ ($\mathbf{F}_c$) with respect to c_H, and $\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')$ is a multiplicative operation of $\mathbf{p}'$ and the Hessian tensor of $\mathbf{F}$ with respect to $\mathbf{p}$, which we explicitly formulate in Appendix A.4. Also in Appendix A.4, we compute $\mathbf{F}_{\mathbf{p}}$ and $\mathbf{F}_c$, and solve (20) for $\mathbf{p}'$. This yields $\mathbf{p}' = (1, 0)$.[11] After plugging this first-order approximation into (21), and computing $\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')$, $\mathbf{F}_{\mathbf{p}c}$, and $\mathbf{F}_{cc}$, we obtain $\mathbf{p}'' = (0, 1/t)$. By Taylor's theorem, we yield the proposed second-order approximation of $\mathbf{p}$. In Appendix A.4, we also show that there are no incentives to unilaterally deviate to selling only one product. □

Lemma 4 states that, once we assume that vertical preferences are uncorrelated, markups on L are higher than on H, at least in a neighborhood of $c_H = c_L$.[12] We discussed the reason for this in the previous paragraph (see Figure 4): In contrast to H-buyers, the consumer rent of L-buyers generally

[11] This first-order approximation perfectly mirrors Lemma 1 in Section 4. There, however, we have $\mathbf{p}'' = 0$, an invalid result in the case of uncorrelated vertical preferences.

[12] This might be different for slightly modified assumptions, as we show in the following section.

increases the further consumers are from $\bar{A}$, the "switching line" between buying H and L. This leads to proportionally more infra-marginal L-consumers, and acts as an incentive to narrow the gap between p_H^A and p_L^A.[13]

Before concluding this section, we make a remark about welfare. As in Section 4, from a utilitarian point of view, we are only interested in the allocation of consumers but not in prices. However, if prices were such that both a social planner and a consumer $\bar{\mathbf{d}} = (\bar{d}, \bar{d}_L^A, \bar{d}_L^B)$ were indifferent between $\bar{\mathbf{d}}$ buying any of the four product versions, these prices were socially optimal regarding all other consumers as well. From identical marginal costs of A and B, it follows that a planner chooses $\bar{d}^* = 1/2$. From (1), the planner's allocation further satisfies

$$v - \bar{d}/2 - c_H = v - \bar{d}/2 - t_L \bar{d}_L^A - c_L \quad \Leftrightarrow \quad \bar{d}_L^{A*} = (c_H - c_L)/t_L.$$

On the other hand, regarding the market solution, we have seen in Lemma 4 that for the vertically indifferent consumer x it holds that

$$x(p_H^*, p_L^*) = \frac{p_H^* - p_L^*}{t_L} \simeq \frac{c_H - c_L}{t_L} - \frac{(c_H - c_L)^2}{2tt_L}.$$

Accordingly, too many consumers buy H. That is, the markup on L is not higher for efficiency reasons, but only because, on top of horizontal competition softening, firm-specific vertical preferences additionally cushion competition.

We summarize the central findings of this section in Proposition 3, which also rules out corner equilibria, except for $c_H = c_L$. We discuss the remaining parts of the proof of Proposition 3 in A.5.[14]

Proposition 3. *If* $\mathbf{d} \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$*, and* $c_H \geq c_L$*, with* c_H *and* c_L *sufficiently close, symmetric equilibria of the pricing game are characterized by* $p_H^* \simeq c_H + t$ *and* $p_L^* \simeq c_L + t + (c_H - c_L)^2/2t$*. A's (B's) consumers buy H if and only if* d_L^A $(d_L^B) > (p_H^* - p_L^*)/t_L$*, and otherwise buy L. Each firm's equilibrium profit is* $\pi(p_H^*, p_L^*) = t/2$ *at* $c_H = c_L$*, and locally increases in* c_H*. From a welfare point of view, too many consumers buy H.*

[13] An interesting corollary to Lemma 4 is that firms generally profit from higher (!) marginal costs on H. Only with costs (and prices) apart, the above asymmetry comes into effect. The markup on L becomes strictly higher than t, and profits reach values above $t/2$.

[14] The proof also includes an argument why $p_H^i - p_L^j \leq t$, $\{i, j\} \in \{A, B\}$, a condition we took for granted so far. A rough intuition is as follows. With prices for H and L far apart, firm A wishes to relocate consumers from L to H by lowering p_H^A, unless there is a large amount of infra-marginal H-consumers. The latter, however, cannot be the case with $p_H^A - p_L^A > t$, as in such a (putative) equilibrium most consumers prefer to buy L.

6 Uncorrelated Vertical Preferences (II): Markups on H May Be Higher

In the previous section, we described vertical preferences by the firm-specific disutility a buyer receives if he or she consumes L instead of H. This implies that, abstracting from horizontal characteristics, the high-quality product H is perceived in exactly the same manner at the two firms. In this final section, we suppose that the relation between H and L is in the opposed direction. Consuming H instead of L now generates additional utility, which we assume to be firm-specific.

To do so, we could stay with the previous utility function, and assume for the distribution of consumers that $\mathbf{d} \sim$ i.i.d. $\mathcal{U}_3\left[(0,1)\times(-1,0)\times(-1,0)\right]$. This would both alter the formulation of Q_H^A and Q_L^A and result in a counter-intuitive interpretation. In particular, L would be the high-quality product, and H would be the low-quality product, which seems somewhat odd.

For the sake of exposition, we thus modify the utility function (1) to

$$u^i(\mathbf{d}') := u^i(d, u_H^A, u_H^B) = \begin{cases} v - p_H^i - td^i + t_H u_H^i & \text{if } q_H^i = 1, \\ v - p_L^i - td^i & \text{if } q_L^i = 1. \end{cases} \tag{22}$$

We remain assuming unit demand, $t > 0$, $t_H > 0$, and $\mathbf{d}' \sim$ i.i.d. $\mathcal{U}_3[0,1]$. Intuitively, instead of seeing L as a (firm-specific) inferior version of H, we interpret the difference between H and L as a firm-specific "add-on" in the sense of Verboven (1999) and Ellison (2005).

Apart from that, our analysis remains the same as in Section 5. For this reason, we do not repeate the above arguments one by one but focus on the fundamental intuition behind the analysis.

From (22), we specify A's demand for H as

$$Q_H^A = \int_0^1 \left(\int_{x^A}^1 \left(\int_0^{\min\{\overline{HH},\overline{HL}\}} \mathrm{d}d \right) \mathrm{d}u_H^A \right) \mathrm{d}u_H^B,$$

and A's demand for L as

$$Q_L^A = \int_0^1 \left(\int_0^{x^A} \left(\int_0^{\min\{\overline{LH},\overline{LL}\}} \mathrm{d}d \right) \mathrm{d}u_H^A \right) \mathrm{d}u_H^B.$$

Except re-labeling (writing u_H^A and u_H^B instead of d_L^A and d_L^B), Q_H^A and Q_L^A are the same as in (8) and (9). What differs, however, are the expressions for $\overline{HH}$, $\overline{HL}$, $\overline{LH}$, and $\overline{LL}$. Recall that we defined $\overline{X^A X^B}$ as the value of d such that a consumer is indifferent between buying X^A at A and X^B

at B. As we see in Figure 5, it is now $\min\{\overline{LH}, \overline{LL}\}$ which is constant in u_H^A, whereas $\min\{\overline{HH}, \overline{HL}\}$ increases in u_H^A. This asymmetry results in an inflated amount of infra-marginal H-consumers, and we expect $p_H^* - c_H$ to exceed $p_L^* - c_L$.

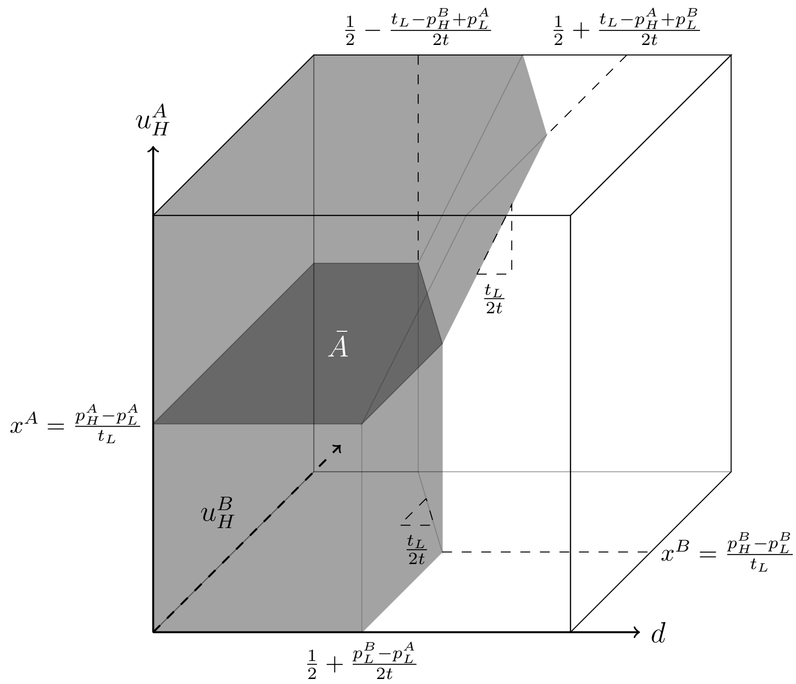

Figure 5: Consumers who buy A's high-quality product, Q_H^A, are displayed in the upper region, where $u_H^A > x^A$. Consumers who buy A's low-quality product, Q_L^A, are displayed in the lower region, where $u_H^A \leq x^A$.

Indeed, by approximating equilibrium prices around $c_H = c_L + t_H$[15] in the same fashion as we did in the previous section, for $c_H \leq c_L + t_H$, we obtain

$$p_H^* = c_H + t + \frac{(c_H - c_L - t_H)^2}{2t} + \mathcal{O}\left(c_H - c_L - t_H)^3\right)$$

and

$$p_L^* = c_L + t + \mathcal{O}\left((c_H - c_L - t_H)^3\right).$$

Hence, markups are higher for H, and too many consumers buy L, as compared to what would socially be efficient.

[15]We need to approximate around $c_H = c_L + t_L$ instead of $c_H = c_L$ in order to avoid non-continuities at $c_H = c_L$. At $c_H = c_L + t_H$, we obtain $Q_H^A = 0$, which is analogous to $Q_L^A = 0$ at $c_H = c_L$ in Section 5. In both instances, departing from this limiting cases, the first- and second-order effects come just into effect, and we can compare the results.

After checking for unilateral deviations and considering putative corner equilibria, we can state Proposition 4, which is analogous to Proposition 3 in Section 5.

Proposition 4. *If the consumers' utility function is given by* (22), $\mathbf{d}' \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$, *and* $c_H \leq c_L + t_H$, *with* c_H *and* $c_L + t_H$ *sufficiently close, symmetric equilibria of the pricing game are characterized by* $p_H^* \simeq c_H + t + (c_H - c_L - t_H)^2/2t$ *and* $p_L^* \simeq c_L + t$. *A's (B's) consumers buy H if and only if* u_H^A $(u_H^B) > (p_H^* - p_L^*)/t_L$, *and otherwise buy L. Each firm's equilibrium profit is* $\pi(p_H^*, p_L^*) = t/2$ *at* $c_H = c_L + t_H$, *and locally increases for a decreasing* c_H. *From a welfare point of view, too many consumers buy L.*

7 Conclusion

General models of horizontal and vertical market segmentation find that, in oligopolistic contexts, markups do not vary with quality.

We qualify this somewhat counterintuitive result by relaxing the assumption that vertical preferences are perfectly harmonious across firms: a high reduction in utility from consuming the low-quality product instead of the high-quality product at one firm not necessarily implies the same at the firm's competitor. In such a setting, we find that markups on low-quality products exceed markups on high-quality products in a symmetric interior equilibrium. To the contrary, if low-quality products are only distinguished by their horizontal characteristics, and supplementary utility from consuming a high-quality product is specific to the firm, we obtain the opposite result.

From a welfare perspective of view, we want horizontal competition to be weak. Although this softens competition, the lack of horizontal competitiveness does not affect horizontal allocative efficiency.[16] In addition, if firms are horizontal substitutes, they differentiate their customers vertically. A social planner wants to prevent this. Hence, regarding horizontal differentiation, the firms' objective is perfectly in line with the objective of the planner. On the other hand, regarding vertical preferences, the objectives of firms and planner are diametrically opposed. If the prices of low-quality products are distorted, we want the marginal valuation of quality to be high, such that more consumers buy the high-quality product. If the prices of high-quality products are distorted, we want the marginal valuation of quality to be low, such that more consumers buy the low-quality product. On the other hand, as we have seen, firms' profits unanimously increase in the extent of price distortions.

[16] This, of course, crucially depends on our assumption of fully covered markets.

In order to test our opposed results of Sections 5 and 6, we could ask for which species of products it is the high-quality version for which the differentiation is specific to the firm, and when it is the low-quality version. According to our theory, it is firm-specific differentiation which opens the door for higher markups.

An interesting theoretical exercise would be to endogenize the firms' choice of the type and degree of differentiation. This, however, brings along intricacies regarding asymmetric departure points in the second stage. As often in the field, simulations could be a viable backdoor strategy.

Finally, our theory could be improved by allowing consumers to buy multiple (or zero) units; by allowing firms to sell more than two product versions; by considering asymmetric equilibria; or by including the entry decision of (additional) firms. These and related considerations indicate that the scope for augmenting and adjusting our model is almost unlimited.

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Appendix

A.1 First-order Conditions

The FOC with respect to p_H^A is

$$\frac{\partial \pi^A}{\partial p_H^A} = \int_0^1 \Bigg(- \int_{x^A}^1 \underbrace{\frac{1}{2t} f\left(\min\{\overline{HH}, \overline{HL}\}, d_L^A, d_L^B\right)}_{=:m^H} \mathrm{d}d_L^A$$
$$- \frac{1}{t_L} \left(\underbrace{\int_0^{\min\{\overline{HH}(d_L^A = x^A), \overline{HL}(d_L^A = x^A)\}} f(d, x^A, d_L^B) \mathrm{d}d}_{=:A^H(d_L^A = x^A)} \right) \Bigg) \mathrm{d}d_L^B (p_H^A - c_H)$$
$$+ \int_0^1 \left(\int_{x^A}^1 \left(\underbrace{\int_0^{\min\{\overline{HH}, \overline{HL}\}} f(d, d_L^A, d_L^B) \mathrm{d}d}_{=:A^H} \right) \mathrm{d}d_L^A \right) \mathrm{d}d_L^B$$
$$+ \int_0^1 \left(\frac{1}{t_L} \underbrace{\int_0^{\min\{\overline{LH}(d_L^A = x^A), \overline{LL}(d_L^A = x^A)\}} f(d, x^A, d_L^B) \mathrm{d}d}_{=:A^L(d_L^A = x^A)} \right) \mathrm{d}d_L^B (p_L^A - c_L) \stackrel{!}{=} 0, \tag{A.1}$$

with

$$\overline{HH} := \frac{1}{2} + \frac{p_H^B - p_H^A}{2t},$$
$$\overline{HL} := \frac{1}{2} + \frac{p_L^B - p_H^A}{2t} + \frac{t_L}{2t} d_L^B,$$
$$\overline{LH} := \frac{1}{2} + \frac{p_H^B - p_L^A}{2t} - \frac{t_L}{2t} d_L^A,$$
$$\overline{LL} := \frac{1}{2} + \frac{p_L^B - p_L^A}{2t} + \frac{t_L}{2t} (d_L^B - d_L^A).$$

In a symmetric equilibrium with $p_L^A = p_L^B =: p_L$, $p_H^A = p_H^B =: p_H$ (and thus $x^A = x^B =: x$), it holds that

$$\min\{\overline{HH}, \overline{HL}\} = \begin{cases} \overline{HH} = 1/2 & \text{if } d_L^B \in (x, 1] \\ \overline{HL} = (1/2t)(t + p_L - p_H + t_L d_L^B) & \text{if } d_L^B \in [0, x], \end{cases} \tag{A.2}$$

and

$$\min\{\overline{LH}, \overline{LL}\} = \begin{cases} \overline{LH} = (1/2t)(t + p_H - p_L - t_L d_L^i\ ) & \text{if } d_L^B \in (x, 1] \\ \overline{LL} = (1/2t)(t + t_L(d_L^B - d_L^A)) & \text{if } d_L^B \in [0, x]. \end{cases} \tag{A.3}$$

Therefore, and from evaluating (A.2) and (A.3) at $d_L^A = x$, we have

$$A^H(d_L^A = x | d_L^B > x) = A^L(d_L^A = x | d_L^B > x) = \int_0^{1/2} f(d, x, d_L^B)\mathrm{d}d,$$

since

$$A^H(d_L^A = x) = \int_0^{\overline{HH}(d_L^A = x)} f(d, x, d_L^B)\mathrm{d}d = \int_0^{1/2} f(d, x, d_L^B)\mathrm{d}d,$$

and

$$A^L(d_L^A = x) = \int_0^{\overline{LH}(d_L^A = x)} f(d, x, d_L^B)\mathrm{d}d = \int_0^{1/2} f(d, x, d_L^B)\mathrm{d}d.$$

Analogically, for $d^B \leq x$, we obtain

$$A^H(d_L^A = x | d_L^A \leq x) = A^L(d_L^A = x | d_L^A \leq x) = \int_0^{\frac{1}{2} + \frac{p_L - p_H}{2t} + \frac{t_L}{2t} d_L^B} f(d, x, d_L^B)\mathrm{d}d.$$

Consequently, we can simplify the FOC (A.1) to

$$\begin{aligned} &\left(\int_0^1 \int_x^1 m^H \mathrm{d}d_L^A \mathrm{d}d_L^B + \int_0^1 \frac{A^H(d_L^A = x)}{t_L} \mathrm{d}d_L^B\right)(p_H - c_H) \\ &= \underbrace{\int_0^1 \int_x^1 A^H \mathrm{d}d_L^A \mathrm{d}d_L^B}_{=Q_H^A} + \int_0^1 \frac{A^H(d_L^A = x)}{t_L} \mathrm{d}d_L^B (p_L - c_L) \end{aligned} \tag{A.4}$$

The FOC with respect to p_L^A is

$$\begin{aligned} \frac{\partial \pi^A}{\partial p_L^A} = \int_0^1 &\Bigg(- \int_0^{x^A} \underbrace{\frac{1}{2t} f\left(\min\{\overline{LH}, \overline{LL}\}, d_L^A, d_L^B\right)}_{=:m^L} \mathrm{d}d_L^A \\ &- \frac{1}{t_L} \Bigg(\underbrace{\int_0^{\min\{\overline{LH}(d_L^A = x^A), \overline{LL}(d_L^A = x^A)\}} f(d, x^A, d_L^B)\mathrm{d}d}_{=A^L(d_L^A = x^A)} \Bigg)\Bigg) \mathrm{d}d_L^B (p_L^A - c_L) \\ &+ \int_0^1 \left(\int_0^{x^A} \Bigg(\underbrace{\int_0^{\min\{\overline{LH}, \overline{LL}\}} f(d, d_L^A, d_L^B)\mathrm{d}d}_{=A^L} \Bigg) \mathrm{d}d_L^A \right) \mathrm{d}d_L^B \end{aligned}$$

$$+\int_0^1 \left(\frac{1}{t_L} \underbrace{\int_0^{\min\{\overline{HH}(d_L^A=x^A),\overline{HL}(d_L^A=x^A)\}} f(d,x^A,d_L^B)\mathrm{d}d}_{=A^H(d_L^A=x^A)} \right) \mathrm{d}d_L^B (p_H^A - c_H) \stackrel{!}{=} 0. \tag{A.5}$$

For the version of (A.5) which applies in a symmetric interior equilibrium, we use $A^L(d_L^A = x) = A^H(d_L^A = x)$ from above. Accordingly, we have

$$\begin{aligned} &\left(\int_0^1 \int_0^x m^L \mathrm{d}d_L^A \mathrm{d}d_L^B + \int_0^1 \frac{A^L(d_L^A = x)}{t_L}\mathrm{d}d_L^B\right)(p_L - c_L) \\ =&\underbrace{\int_0^1 \int_0^x A^L \mathrm{d}d_L^A \mathrm{d}d_L^B}_{=Q_L^A} + \int_0^1 \frac{A^L(d_L^A = x)}{t_L}\mathrm{d}d_L^B (p_H - c_H). \end{aligned} \tag{A.6}$$

A.2 *Proof of Lemma 1 (Completion)*

In absence of corner solutions, the proof is given in the main body of the text. Alternatively, firm A may only sell either H or L. By only selling H, A's objective function is

$$\pi^A = \int_0^1 \min\left\{\frac{1}{2} + \frac{p_H^B - p_H^A}{2t}, \frac{1}{2} + \frac{p_L^B - p_H^A}{2t} + \frac{t_L}{2t}d_L\right\}\mathrm{d}d_L(p_H^A - c_H),$$

the FOC of which is

$$p_H^A - c_H = \int_0^1 \min\left\{t + p_H^B - p_H^A, t + p_L^B - p_H^A + t_L d_L\right\}\mathrm{d}d_L.$$

After plugging in $p_H^B = c_H + t$ and $p_L^B = c_L + t$, we obtain $\tilde{p}_H^A = c_H + t - (c_H - c_L)^2/4t_L$. A's markup on H thus is smaller than or equal to t. As A's equilibrium profit is $t/2$, it remains to be shown $Q_H^A(\tilde{p}_H^A) \leq 1/2$. Formally,

$$\begin{aligned} Q_H^A(\tilde{p}_H^A) =& \int_{\frac{c_H - c_L}{t_L}}^1 \frac{1}{2} + \frac{(c_H + t) - \left(c_H + t - (c_H - c_L)^2/4t_L\right)}{2t}\mathrm{d}d_L \\ &+ \int_0^{\frac{c_H - c_L}{t_L}} \frac{1}{2} + \frac{(c_L + t) - \left(c_H + t - (c_H - c_L)^2/4t_L\right)}{2t} + \frac{t_L}{2t}d_L \mathrm{d}d_L \leq \frac{1}{2}, \end{aligned}$$

from the definition of Q_H^A. This holds for all c_H and c_L, because

$$\begin{aligned} \Leftrightarrow\ & \frac{(c_H - c_L)^2}{4t_L} + \int_0^{\frac{c_H - c_L}{t_L}} t_L d_L \mathrm{d}d_L \leq \int_0^{\frac{c_H - c_L}{t_L}} c_H - c_L \mathrm{d}d_L \\ \Leftrightarrow\ & \frac{(c_H - c_L)^2}{4t_L} + \frac{(c_H - c_L)^2}{2t_L} \leq \frac{(c_H - c_L)^2}{t_L}. \end{aligned}$$

By only selling L, A's objective function is

$$\pi^A = \int_0^1 \min\left\{\frac{1}{2} + \frac{p_H^B - p_L^A}{2t} - \frac{t_L}{2t}d_L, \frac{1}{2} + \frac{p_L^B - p_L^A}{2t}\right\} \mathrm{d}d_L (p_L^A - c_L).$$

From the associated FOC, we obtain $\tilde{p}_L^A = c_L + t - (t_L - c_H + c_L)^2/4t_L$. Again, it is sufficient to show that $Q_L^A(\tilde{p}_L^A) \leq 1/2$. Formally,

$$\begin{aligned} Q_L^A(\tilde{p}_H^A) = &\int_{\frac{c_H - c_L}{t_L}}^1 \frac{1}{2} + \frac{(c_H + t) - \big(c_L + t - (t_L - c_H + c_L)^2/4t_L\big)}{2t} - \frac{t_L}{2t}d_L \mathrm{d}d_L \\ &+ \int_0^{\frac{c_H - c_L}{t_L}} \frac{1}{2} + \frac{(c_L + t) - \big(c_L + t - (t_L - c_H + c_L)^2/4t_L\big)}{2t} \mathrm{d}d_L \leq \frac{1}{2}, \end{aligned}$$

from the definition of Q_L^A. This holds for all considered values of c_H and c_L, because

$$\begin{aligned} &\Leftrightarrow \frac{(t_L - c_H + c_L)^2}{4t_L} + \int_{\frac{c_H - c_L}{t_L}}^1 (c_H - c_L)\mathrm{d}d_L \leq \int_{\frac{c_H - c_L}{t_L}}^1 t_L d_L \mathrm{d}d_L \\ &\Leftrightarrow \frac{t_L - c_H + c_L}{4} + (c_H - c_L) \leq \frac{t_L}{2}\left(1 + \frac{c_H - c_L}{t_L}\right) \\ &\Leftrightarrow c_H - c_L \leq t_L, \end{aligned}$$

which is true by assumption.

A.3 Proof of Proposition 2 (Completion)

From Lemma 1 and condition (4), it follows that symmetric interior equilibria exist if and only if $0 \leq c_H - c_L < t_L$, and that they are characterized by $p_H^* = c_H + t$ and $p_L^* = c_L + t$.

Regarding symmetric corner equilibria, first assume that both firms only sell H. In this case, firm A maximizes

$$\pi^A = \left(\frac{1}{2} + \frac{p_H^B - p_H^A}{2t}\right)(p_H^A - c_H), \tag{A.7}$$

with respect to p_H^A, as in the standard Hotelling case. The (symmetric) solution of (A.7) is ${p^A}^*_H = c_H + t$, and A's profit is ${\pi^A}^* = t/2$. Whenever A finds a price $\tilde{p}_L^A$ such that its total demand increases without decreasing the markup on either of its products, only selling H cannot be an equilibrium. From (2), A steals customers from B by offering L if and only if

$$v - (\tilde{p}_L^A + t_L \times 0) - t/2 > v - (c_H + t) - t/2 \;\Leftrightarrow\; \tilde{p}_L^A < c_H + t. \tag{A.8}$$

Meanwhile, A's markup on either product is not lowered if and only if

$$\tilde{p}_L^A - c_L \geq p_H^A - c_H = t \;\Leftrightarrow\; p_L^A \geq c_L + t. \tag{A.9}$$

(A.8) and (A.9) are simultaneously feasible if and only if

$$c_L + t < c_H + t \;\;\Leftrightarrow\;\; c_L < c_H.$$

For $c_H = c_L$, check that $\tilde{p}_H^A = c_H + t$ and $\tilde{p}_L^A = c_L + t$ solve (14) and (15) for $p_H^B = c_H + t$ and $p_L^B = \infty$. The resulting profit is $\pi^A(\tilde{p}_H^A, \tilde{p}_L^A) = t/2$, hence there is no incentive to offer both H and L in this case.

If both firms only sell L, A objective function is also the one of a standard Hotelling game, because $d_L^A = d_L^B = d_L$. Therefore, ${p^A}^*_L = c_L + t$, and ${\pi^A}^* = 1/2$. In this case, the equivalents to (A.9) and (A.8) yield that A has an incentive to offer H if and only if $c_H - c_L < t_L$. For $c_H = c_L + t_L$, $\tilde{p}_H^A = c_H + t$ and $\tilde{p}_L^A = c_L + t$ solve (A.4) and (A.6) for $p_H^B = \infty$ and $p_L^B = c_L + t$. Again, the resulting profit is $\pi^A(\tilde{p}_H^A, \tilde{p}_L^A) = 1/2$, and there is no incentive to offer both H and L.

A.4 Proof of Lemma 4 (Completion)

We write (16) and (17) as

$$\tilde{f}(\mathbf{p}) := \frac{\bar{A}}{t_L}\left(p_H - c_H - p_L + c_L - t_L(1-x)\right) + \frac{1-x}{2t}(p_H - c_H) = 0,$$

and

$$\tilde{g}(\mathbf{p}) := \frac{\bar{A}}{t_L}\left(p_L - c_L - p_H + c_H - t_L x\right) + \frac{x}{2t}\left(p_L - c_L - \frac{p_H - p_L}{2}\right) = 0.$$

In the following, we use $f(\mathbf{p}) = 2tf(\mathbf{p}) = 0$ and $g(\mathbf{p}) = 2tg(\mathbf{p}) = 0$, which simplifies fractions, as we see later on. Furthermore, by using $\bar{A} = 1/2 - (t_L/4t)x^2$ and $x = (p_H - p_L)/t_L$, we write $f(\mathbf{p}) = 0$ and $g(\mathbf{p}) = 0$ as

$$\begin{aligned} 2t_L f(\mathbf{p}) :=& \left(2tt_L - (p_H - p_L)^2\right)\left(2(p_H - p_L) - c_H + c_L - t_L\right) \\ &+ 2t_L(t_L - p_H + p_L)(p_H - c_H) = 0, \end{aligned} \tag{A.10}$$

and

$$\begin{aligned} 2t_L g(\mathbf{p}) :=& \left(2tt_L - (p_H - p_L)^2\right)\left(2(p_L - p_H) + c_H - c_L\right) \\ &+ 2t_L(p_H - p_L)(p_L - c_L) - t_L(p_H - p_L)^2 = 0. \end{aligned} \tag{A.11}$$

The partial derivatives of (A.10) and (A.11) with respect to p_H, p_L, and c_H are

$$\begin{aligned} 2t_L\frac{\partial f(\mathbf{p})}{\partial p_H} =& -2(p_H - p_L)\left(2(p_H - p_L) - c_H + c_L - t_L\right) \\ &+ \left(2tt_L - (p_H - p_L)^2\right)2 - 2t_L\left(2p_H - p_L - c_H - t_L\right), \end{aligned} \tag{A.12}$$

$$\begin{aligned} 2t_L\frac{\partial f(\mathbf{p})}{\partial p_L} =&\; 2(p_H - p_L)\left(2(p_H - p_L) - c_H + c_L - t_L\right) \\ &- \left(2tt_L - (p_H - p_L)^2\right)2 + 2t_L\left(p_H - c_H\right), \end{aligned} \tag{A.13}$$

$$2t_L\frac{\partial f(\mathbf{p})}{\partial c_H} = -2tt_L + (p_H - p_L)^2 - 2t_L\left(t_L - p_H + p_L\right), \tag{A.14}$$

$$
\begin{aligned}
2t_L\frac{\partial g(\mathbf{p})}{\partial p_H} &= -\,2(p_H-p_L)\left(2(p_L-p_H)+c_H-c_L\right)\\
&\quad -\left(2tt_L-(p_H-p_L)^2\right)2+2t_L(2p_L-p_H-c_L), && \text{(A.15)}\\
2t_L\frac{\partial g(\mathbf{p})}{\partial p_L} &= \;2(p_H-p_L)\left(2(p_L-p_H)+c_H-c_L\right)\\
&\quad +\left(2tt_L-(p_H-p_L)^2\right)2-2t_L\left(3p_L-2p_H-c_L\right). && \text{(A.16)}\\
2t_L\frac{\partial g(\mathbf{p})}{\partial c_H} &= \;2tt_L-(p_H-p_L)^2. && \text{(A.17)}
\end{aligned}
$$

After substituting $p_H = c_H + t$ and $p_L = c_L + t$ into (A.12) through (A.17), we write (20) as

$$\begin{pmatrix} t+t_L & -t \\ -t & t \end{pmatrix}\begin{pmatrix} p'_H \\ p'_L \end{pmatrix} + \begin{pmatrix} -(t+t_L) \\ t \end{pmatrix} = 0,$$

the solution of which is $(p'_H, p'_L) = (1, 0)$. After taking the derivatives of (A.12) through (A.17) with respect to p_H, p_L, and c_H, and after plugging in $p_H = c_H + t$ and $p_L = c_L + t$ once more, we use $(p'_H, p'_L) = (1, 0)$ in order to write (21) as

$$[\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')]_{c_H=c_L}\mathbf{p}' + \begin{pmatrix} t+t_L & -t \\ -t & t \end{pmatrix}\begin{pmatrix} p''_H \\ p''_L \end{pmatrix} + 2\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0. \tag{A.18}$$

Further, we write $[\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')]_{c_H=c_L}\mathbf{p}'$ as $\mathbf{F}_{\mathbf{pp}(1)}\mathrm{vec}(\mathbf{p}'\mathbf{p}'^T)$, where $\mathbf{F}_{\mathbf{pp}(1)}$ is the 1-mode flattening matrix of the Hessian tensor of $\mathbf{F}$ with respect to $\mathbf{p}$. That is,

$$[\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')]_{c_H=c_L}\mathbf{p}' = \begin{pmatrix} f_{HH} & f_{HL} & f_{LH} & f_{LL} \\ g_{HH} & g_{HL} & g_{LH} & g_{LL} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} f_{HH} \\ g_{HH} \end{pmatrix},$$

where $f_{ij} := \left[\partial^2 f(\cdot)/\partial p_i \partial p_j\right]_{c_H=c_L}$ and $g_{ij} := \left[\partial^2 g(\cdot)/\partial p_i \partial p_j\right]_{c_H=c_L}$. From (A.12) and (A.15), we have $f_{HH} = g_{HH} = -1$, and we write (A.18) as

$$\begin{pmatrix} t+t_L & -t \\ -t & t \end{pmatrix}\begin{pmatrix} p''_H \\ p''_L \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix},$$

the solution of which is $(p''_H, p''_L) = (0, 1/t)$.

Alternatively, firm A may only sell either H or L. In order to analyze such unilateral deviations for the general case $c_H \geq c_L$, it is useful to first show that the objective function is concave at $c_H = c_L$. From (A.1) and (A.5) with $f(\mathbf{d}) \equiv 1$, and from the fact that the $\pi^A(p^A_H, p^A_L)$ is twice differentiable, it follows that

$$\left[\frac{\partial^2 \pi^A}{(\partial p^A_H)^2}\right]_{c_H=c_L} = -\frac{1}{t} - \frac{1}{2t_L}, \quad \left[\frac{\partial^2 \pi^A}{(\partial p^A_L)^2}\right]_{c_H=c_L} = -\frac{1}{2t_L},$$

and

$$\left[\frac{\partial^2 \pi^A}{\partial p^A_H \partial p^A_L}\right]_{c_H=c_L} = \left[\frac{\partial^2 \pi^A}{\partial p^A_L \partial p^A_H}\right]_{c_H=c_L} = \frac{1}{2t_L}.$$

π^A is locally strictly concave at $c_H = c_L$ if the Hessian matrix $\mathbf{H}$ of its second derivatives is negative definite. This is the case here, since the leading principal minors of

$$-\mathbf{H} = \begin{pmatrix} \frac{1}{t} + \frac{1}{2t_L} & -\frac{1}{2t_L} \\ -\frac{1}{2t_L} & \frac{1}{2t_L} \end{pmatrix}$$

are $1/t + 1/2t_L$ and $1/2tt_L$, that is, positive.

Next, for the case that $c_H = c_L$, consider firm A's deviation to only selling H. In this case, its objective function is

$$\tilde{\pi}^A = \int_0^1 \min\left\{\frac{1}{2} + \frac{p_H^B - p_H^A}{2t}, \frac{1}{2} + \frac{p_L^B - p_H^A}{2t} + \frac{t_L}{2t} d_L^B\right\} \mathrm{d}d_L^B.$$

We state the according FOC with respect to p_H^A as

$$\frac{\partial \tilde{\pi}^A}{\partial p_H^A} = \int_0^1 \underbrace{\min\left\{t + p_H^B - p_H^A, t + p_L^B - p_H^A + t_L d_L^B\right\}}_{t+p_H^B-p_H^A \leq t+p_L^B-p_H^A+t_L d_L^B \;\Leftrightarrow\; d_L^B \geq (p_H^B - p_L^B)/t_L} \mathrm{d}d_L^B - (p_H^A - c_H) \stackrel{!}{=} 0.$$

Using $p_H^B = c_H + t$ and $p_L^B = c_L + t$, this is

$$\begin{aligned} &\int_0^{\frac{c_H - c_L}{t_L}} (t + c_L + t - p_H^A + t_L d_L^B) \mathrm{d}d_L^B \\ &+ \int_{\frac{c_H - c_L}{t_L}}^1 (t + c_H + t - p_H^A) \mathrm{d}d_L^B = p_A^A - c_H. \end{aligned}$$

From $c_H = c_L$, the first term cancels out, which implies $\tilde{p}_H^A = c_H + t$. Since L is bought with probability 0 at $c_H = c_L$, we have $\pi^A(\tilde{p}_H^A, \infty) = t/2$, and there is no incentive to unilaterally deviate. Furthermore, at $c_H = c_L$, we can equivalently set $(\tilde{p}_H^A, \tilde{p}_L^A)$ to $(c_H + t, c_L + t)$, since there the "no-selling constraint" with respect to L is not binding. From strict concavity and continuity of π^A at $c_H = c_L$, we know that the prices in Lemma 4 are uniquely best answers for $c_H > c_L$ sufficiently close. This makes the no-selling constraint concerning $(\tilde{p}_H^A, \tilde{p}_L^A)$ binding, which proves that deviating to only selling H is unilaterally not beneficial.

Next, consider firm A's deviation to only selling L. Here, the objective function is

$$\begin{aligned} \tilde{\pi}^A = \int_0^1 \int_0^1 \min\Bigg\{&\frac{1}{2} + \frac{p_H^B - p_L^A}{2t} - \frac{t_L}{2t} d_L^A, \\ &\frac{1}{2} + \frac{p_L^B - p_L^A}{2t} + \frac{t_L}{2t}(d_L^B - d_L^A)\Bigg\} \mathrm{d}d_L^A \mathrm{d}d_L^B (p_L^A - c_L). \end{aligned} \tag{A.19}$$

For $c_H = c_L$, we write the according FOC with respect to p_L^A as

$$\int_0^1 \int_0^1 (t + p_H^B - p_L^A - t_L d_L^A) \mathrm{d}d_L^A \mathrm{d}d_L^B \stackrel{!}{=} p_L^A - c_L,$$

which implies $\tilde{p}_L^A = c_L + t - t_L/4$. By plugging $\tilde{p}_L^A$, $p_H^B = c_H + t$, and $p_L^B = c_L + t$ into (A.19), and by imposing $c_H = c_L$, we yield

$$\pi^A(\infty, \tilde{p}_L^A) = \left(\frac{1}{2} - \frac{t_L}{8t}\right)\left(t - \frac{t_L}{4}\right) < \frac{1}{2}t = \pi^A(p_H^*, p_L^*).$$

Since both (p_H^*, p_L^*) and $\tilde{p}_L^A$ are continuous functions in c_H and c_L, unilaterally deviating to only selling L is also not beneficial in the case of $c_H > c_L$ with c_H and c_L sufficiently close.

A.5 Proof of Proposition 3 (Completion)

First, assume that both firms only sell H. In this case, they play a standard Hotelling game, the solution of which is $p_H^* = c_H + t$ and $\pi^A(p_H^*) = t/2$. Consider A's potential deviation to selling both products. The according FOCs are given by (A.1) and (A.5) with $p_L^B = \infty$. By simplifying these we yield (16) and (17), with $\bar{A} := 1/2 + (p_H^B - p_H^A)2t$. For $c_H = c_L$, the according solution is $\tilde{p}_H^A = c_H + t$ and $\tilde{p}_L^A = c_L + t$. The resulting allocation of consumers is the same as in the standard Hotelling case, and $\pi^A(\tilde{p}_H^A, \tilde{p}_L^A) = t/2$. Therefore, both firms only offering H is another equilibrium if $c_H = c_L$. If $c_H > c_L$, we apply the same approximation method as in the proof of Lemma 4. We obtain

$$\tilde{p}_H^A = c_H + t + \frac{(c_H - c_L)^2}{4t_L} + \mathcal{O}\left((c_H - c_L)^3\right)$$

and

$$\tilde{p}_L^A = c_L + t + \left(\frac{1}{t} + \frac{1}{2t_L}\right)\frac{(c_H - c_L)^2}{2} + \mathcal{O}\left((c_H - c_L)^3\right).$$

Since $\tilde{p}_H^A - \tilde{p}_L^A > 0$, firm A seeks to sell L, which in turn eliminates the putative equilibrium.

Consider the opposite case where both firms only sell L. Then, A's objective function is

$$\pi^A = \int_0^1 \int_0^1 \left(\frac{1}{2} + \frac{p_L^B - p_L^A}{2t}\right) \mathrm{d}d_L^A \mathrm{d}d_L^B (p_L^A - c_L).$$

From the associated FOC, and after imposing symmetry, we obtain $p_L^* = c_L + t$ and $\pi^A(p_L^*) = t/2$. In this case, A can increase its profit by only selling H at $\tilde{p}_H^A = c_L + t$. By doing so, A covers the whole market, and its profit is $\pi^A(\tilde{p}_H^A) = t - (c_H - c_L)$. Since $\pi^A(\tilde{p}_H^A) > \pi^A(p_L^*)$ if and only if $t/2 > c_H - c_L$, firm A prefers to sell H if $c_H \geq c_L$ are sufficiently close. This eliminates the second putative equilibrium.

So far, we assumed that $p_H^i - p_L^j \leq t$, $\{i, j\} \in \{A, B\}$. Suppose now, to the opposite, that $p_H^A - p_L^A > t$. From (4), there is is no interior equilibrium if $t > t_L$. Therefore, it remains for us to demonstrate that $t < p_H^* - p_L^* \leq t_L$ cannot be an equilibrium. From $p_H^A - p_L^B > t$ it follows that (8) and (9) no longer represent Q_H^A and Q_L^A, since $\min\{\overline{HH}, \overline{HL}\}$ and $\min\{\overline{LH}, \overline{LL}\}$ lie outside $[0,1]$ for some combinations of $d_L^A \in [0,1]$ and $d_L^B \in [0,1]$. This is illustrated in Figure 6. Therefore, FOCs (16) and (17) need to be adjusted accordingly. The adapted analogue of (16) is

$$\left(\frac{\bar{F}}{2t} + \frac{\bar{A}}{t_L}\right)(p_H - c_H) = \bar{F}\bar{A} + \frac{\bar{A}}{t_L}(p_L - c_L), \tag{A.20}$$

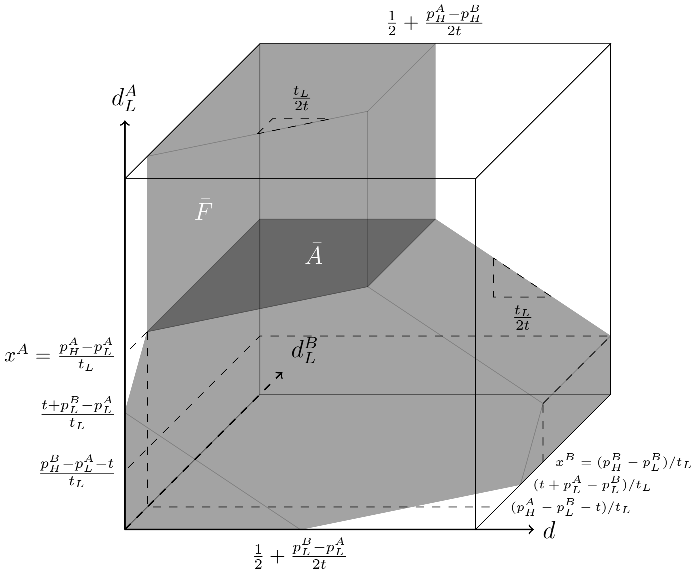

Figure 6: Consumers who buy A's high-quality product, Q_H^A, are displayed in the upper region, where $d_L^A > x^A$. Consumers who buy A's low-quality product, Q_L^A, are displayed in the lower region, where $d_L^A \le x^A$.

where

$$\bar{A} := \frac{1}{2t_L}\left(t_L - p_H + p_L + \frac{t}{2}\right)$$

represents the frontier between consumers of H and consumers of L at A, and

$$\bar{F} := \left(1 - \frac{p_H - p_L - t}{t_L}\right)\left(1 - \frac{p_H - p_L}{t_L}\right)$$

represents the "inter-firm" marginal consumers (see Figure 6). For $c_H \ge c_L$ with c_H and c_L sufficiently close, (A.20) is necessarily violated if

$$\frac{\bar{A}}{t_L}(p_H - p_L) > \bar{F}\bar{A} - \frac{\bar{F}}{2t}(p_H - c_H). \tag{A.21}$$

From $p_H - p_L \le t_L$, we have $\bar{A}/t_L \ge t/4t_L^2$; and from $p_H - p_L > t$, we have $\bar{F} < (t_L - t)/t_L$. Using (A.21), it is thus sufficient to show that

$$\frac{t}{4t_L^2} \times t > \frac{t_L - t}{t_L}\left(\bar{A} - \frac{p_H - c_H}{2t}\right) \tag{A.22}$$

Also from $p_H - p_L > t$, we have $p_H - c_H > t + p_L - c_H$. For c_H sufficiently close to c_L, this implies $p_H - c_H > t$, since $p_L \geq c_L$ in any interior equilibrium. Further, $p_H - p_L > t$ implies that $\bar{A} < (1/2t_L)(t_L - t/2) = 1/2 - t/4t_L$. Using (A.22), it consequently suffices to show that

$$\left(\frac{t}{2t_L}\right)^2 > \frac{t_L - t}{t_L}\left(\frac{1}{2} - \frac{t}{4t_L} - \frac{1}{2}\right) \tag{A.23}$$

Since the left-hand side of (A.23) is positive, and the right-hand side of (A.23) is negative, we have eliminated the third putative equilibrium.