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Optimal Selling Mechanisms under Imperfect Commitment: Extending to the Multi-Period Case*

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Abstract

This paper studies the optimal mechanism for a seller (she) that sells, in a sequence of periods, an indivisible object per period to the same buyer (he). Buyer's willingness to pay remains constant along time and is his private information. The seller can commit to the current period mechanism but not to future ones. Our main result is that a seller cannot do better than posting a price in every period. We give a complete characterization of the optimal mechanism and equilibrium payoffs for every prior. Also, we show that, when agents are arbitrarily patient, the seller does not learn about buyer's type except in extreme cases, posting a price equal to the minimum buyer's willingness to pay in every period. This result is a reminiscence of the Coase's conjecture, where a monopolist cannot exert her monopoly power due to the lack of long-term commitment.

Keywords: asymmetric information, dynamics, optimal mechanism, imperfect commitment.
JEL codes: D82

1 Introduction

Beccuti (2014) proves that, in a two-period game, price posting is optimal when both players have the same discount factor but not when they are different. In this paper, we extend that model to a finite number of periods when the discount factor is arbitrarily large and equal for both players.

Intuitively, allowing more than two periods provides a richer environment because the seller can now engage in a strategy of gradual learning. More formally, continuation values at any moment in time of a multi-period game may be a non-linear function in the prior for either the buyer or the seller. Moreover, in a static framework, price posting is an optimal mechanism when value functions are linear and it is not when they are not linear.[1] However, linearity (or piecewise linearity) in the prior on value functions is not a sufficient condition for optimality of price posting in an dynamic framework as it can be seen in the two-period setting. Then, it is reasonable to conjecture that price posting might not be optimal in a multi-period game.

We prove two things. First, the seller cannot do better than posting a price in every period as the selling mechanism without loss of generality. Second, in general along the equilibrium path the

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[1] See Chapter 2 in Börgers (mimeo) or Chapter 2 in Bolton and Dewatripont (2005).

seller posts a price equal to the minimum buyer willingness to pay, i.e. the maximum competitive price. Discrimination between types is optimal only when the seller is extremely optimistic about facing a high-type consumer. In other case, learning, albeit possible, is so costly for the seller that it is not optimal. When the seller has the possibility of learning, her profits are reduced due to the strategic behavior of the buyer. We also give a complete characterization of the optimal mechanism and equilibrium payoffs for every prior.

As in Coase's conjecture, the monopolist cannot use a price above the competitive one to discriminate among buyers. Coase (1972) conjectured that a monopolist uses this price from the beginning when she has a durable good to sell in a finite numbers of periods. A solution to this conjecture is renting the durable good. This result implicitly assumes that the monopolist cannot track past buyer's decisions. However, in our framework the monopolist cannot commit to ignore the information disclosed by the buyer. Our model implies that there is no mechanism that solves the Coase's conjecture as a consequence of this lack of commitment.

This has also implications regarding the ratchet effect. In an arbitrary long game and if a discount factor is not too small, a privately informed buyer knows that in case of revealing his valuation in the current period he will not get any information rent thereafter (the ratchet effect). Then, the seller cannot induce him to reveal his information. Schmidt (1993) shows the presence of the ratchet effect on a repeated bargaining model, producing much pooling in all the equilibria of the game. In his work, the buyer (who has the bargaining power in his model) offers a price to a seller. As soon as a price higher than her production cost is accepted (revealing her type) the buyer will not give her any additional rents. This is true even if the price offered by the buyer in the current period is not the optimal one for him. Learning process, when it occurs, is always extreme. In our model, the seller (who has the bargaining power) can offer a more complex selling mechanism than price posting. For example, the seller can propose a menu of contracts such that if the high-type buyer buys the good in the current period, he is not completely revealing his valuation. In other words, in the following period the seller will not be certain about facing a high-type buyer. Therefore, she has to give him rents again if she wants to continue with her learning process. In contrast with Schmidt (1993), the seller can now propose mechanisms that allow her to learn gradually. Since we prove that these mechanisms are suboptimal, the seller cannot break the ratchet effect in equilibrium.

Skreta (2005) shows that her results at Skreta (2006) hold for the multi-period case. As we mentioned for the two-period setting, she studies a different framework: she considers the durable good case.

To solve the model we use a dynamic mechanism design approach following the procedure proposed in Bester and Strausz (2001), which provides a modified version of the revelation principle when there is imperfect commitment.

The rest of the paper is organized as follows. Section 2 provides a general set up of the problem and reviews the Bester and Strausz (2001) revelation principle for this kind of environment. Section 3 analyzes the problem with two types for any finite T periods game. Finally, Section 4 concludes. Those proofs considered relevant for the general understanding of the model are included in the main text while the rest can be found in the Appendix.

2 General Setup

Let's consider a multi-period game with $r = \{1, 2, ...T\}$ and $T < \infty$, where r is the number of periods remaining at the beginning of the current period. There is one risk neutral seller (the principal) and one risk neutral buyer (the agent) facing each other repeatedly. Both players discount the future at the same rate $\delta \in (0, 1]$. At every period, the seller can produce at zero cost a non-storable object that puts for sale to the buyer.[2] This buyer has valuation θ_i for the good, where $\theta_i \in \Theta = \{\theta_L, \theta_H\}$. We call θ_L (θ_H) the low-type buyer (high-type buyer) and sometimes we denote it by the subscript L (H). This valuation remains constant over time and is his private information. The initial probability of facing a high-type buyer is denoted by $p_{H,T+1}$, and for a low-type buyer by $p_{L,T+1} = 1 - p_{H,T+1}$. We refer to this as the prior of the seller.

A mechanism Γ_r in period r specifies a message set M_r and a decision function $y_r = (x_r, w_r)$, where $x_r : M_r \to [0,1]$ is the allocation rule and $w_r : M_r \to \mathbb{R}$ is the payment rule. Then, each element $m_r \in M_r$ commits the seller to implement the allocation rule $x_r(m_r)$ and requires for the buyer the payment $w_r(m_r)$.

The seller has imperfect commitment. This is, at every period the seller can commit herself to a mechanism for the current period but not for future ones. So, at the beginning of period r the seller chooses a mechanism $\Gamma_r \in \Upsilon$ given her prior $p_{H,r+1}$ about facing a high-type buyer, where Υ is the space of mechanisms. Next, the buyer observes Γ_r. His strategy specifies the probability $q_i(m_r)$ with which the agent sends each message m_r, where $q_i : M_r \to [0,1]$, for $i \in \{L, H\}$ and that verifies $\sum_{m_r \in M_r} q_i(m_r) = 1$. The buyer can always choose not to participate in the mechanism Γ_r.[3] In this case he gets zero instant payoffs but he can accept future ones. Next, the seller observes m_r and updates her beliefs about facing a high-type buyer. We denote it by $p_{H,r}(m_r)$ and is updated following a mapping $p_{H,r} : M_r \to [0,1]$. Beliefs constitute the state variable for the next period, i.e., $r-1$. In the following, we use $p_{L,r}(m_r)$ to indicate $1 - p_{H,r}(m_r)$ and $p_r(m_r)$ to indicate the vector of posteriors $(p_{L,r}(m_r), p_{H,r}(m_r))$ when a message m_r is sent.

We denote by $v_r(m_r)$ and $u_{i,r}(m_r)$ to the seller's and buyer's *instant* payoff, respectively, when the buyer with valuation θ_i sends the message m_r, i.e.

$$\begin{aligned} v_r(m_r) &= w_r(m_r), \\ u_{i,r}(m_r) &= x_r(m_r)\theta_i - w_r(m_r), \end{aligned}$$

$V_{r-1} : [0,1]^2 \to \mathbb{R}$ and $U_{i,r-1} : [0,1]^2 \to \mathbb{R}$ represent the continuation values for each player.[4]

Consequently, given the vector of priors $p_{r+1} \equiv (p_{L,r+1}, p_{H,r+1})$, the seller's problem at period r is to choose (q_r, p_r, Γ_r) that maximizes:

$$\sum_{i \in \Theta} \sum_{m_r \in M_r} p_{i,r+1} q_i(m_r) \left(v_r(m_r) + \delta V_{r-1}(p_r(m_r)) \right), \tag{1}$$

where $q_r \equiv (q_r(m_r))_{m_r \in M_r}$ ($q_r(m_r)$ indicates the vector $(q_L(m_r), q_H(m_r))$), and $p_r \equiv (p_r(m_r))_{m_r \in M_r}$, is subject to the following constraints:

- The buyer's Incentive Compatibility ($IC_{i,r}$): the buyer chooses his optimal reporting strategy,

[2] All our results hold for any constant production cost strictly less than the minimum possible willingness to pay of the buyer.

[3] Note that our definition of the mechanism requires participation. We take the usual convention that the buyer can decide whether to participate or not, getting zero payoffs in the last case. This convention is discussed later, when we talk about the individual rationality constraint (IR). Alternatively, it is possible to include a message in M_r that represents no participation.

[4] Continuation values depends on the vector of priors at the beginning of the period. Since there are two types, the vector of priors is completely determined by the prior about facing a high-type buyer, i.e. $p_{H,r+1}$. Then, later in the paper, and with some abuse of notation, continuation values will be represented as depending only in that prior, which we will denoted as p. We also will denote $p(m_r)$ to its posterior after observing m_r.

i.e.,

$$\sum_{m_r \in M_r} q_i(m_r)\left(u_{i,r}(m_r) + \delta U_{i,r-1}(p_r(m_r))\right) \geq \\ \sum_{m_r \in M_r} q_i'(m_r)\left(u_{i,r}(m_r) + \delta U_{i,r-1}(p_r(m_r))\right) \tag{2}$$

for $i \in \{L, H\}$, and for all $q_i'(m_r)$.

- The buyer's Individual Rationality $(IR_{i,r})$: The buyer's individual rationality constraint has to be satisfied for all types to which the seller assigns positive probability:

$$p_{i,r+1}\left[\sum_{m_r \in M_r} q_i(m_r)\left(u_{i,r}(m_r) + \delta U_{i,r-1}(p_r(m_r))\right) - \delta \bar{U}_{i,r-1}\right] \geq 0 \tag{3}$$

for $i \in \{L, H\}$, where $\bar{U}_{i,r-1}$ is the continuation value when the buyer choose not to participate in the mechanism Γ_r. Although there is no loss of generality in assuming that the buyer participates with probability one, we have to warranty that he does not do better staying out. This implies $\bar{U}_{i,r-1} \geq 0$. We assume $\bar{U}_{i,1} = 0$ since it is the less restrictive in (3) and, as we will show later, this is the case at the optimal contract (given we can assume any belief for the out-of-equilibrium message).

- And finally, for each message, the seller's updated belief $p_{i,r}(m_r)$ has to be consistent with Bayes' rule (BR_r) whenever possible:

$$p_{i,r}(m_r) \sum_{j \in \Theta} p_{j,r+1} q_j(m_r) = p_{i,r+1} q_i(m_r). \tag{4}$$

It follows that the seller's problem with imperfect commitment is given by:

$$V_r(p_{r+1}) = \underset{\{q_r, p_r, \Gamma_r\}}{Max} \sum_{i \in \Theta} \sum_{m_r \in M_r} p_{i,r+1} q_i(m_t)\left(v_r(m_r) + \delta V_{r-1}(p_r(m_r))\right), \tag{5}$$

subject to $(2) - (4)$.

We say that the outcome (q_r, p_r, Γ_r) is *incentive feasible* if it satisfies (2)-(4) for all $\theta_i \in \Theta$. Additionally, it is *incentive efficient* if it satisfies (5), i.e. the seller chooses the best outcome among all of the incentive feasible ones. An *optimal mechanism* is a mechanism Γ_r that belongs to an incentive efficient outcome (q_r, p_r, Γ_r). Finally, (q_r, p_r, Γ_r) and (q_r', p_r', Γ_r') are *payoffs equivalent* if they leave the seller and the buyer (of every possible type) with the same payoffs, i.e.

$$\sum_{i\in\Theta}\sum_{m_r\in M_r} p_{i,r+1}q_i\left(m_r\right)\left(v_r(m_r)+\delta V_{r-1}\left(p_r\left(m_r\right)\right)\right)=$$

$$\sum_{i\in\Theta}\sum_{m_r'\in M_r'} p_{i,r+1}q_i'\left(m_r'\right)\left(v_r(m_r')+\delta V_{r-1}\left(p_r'\left(m_r'\right)\right)\right),$$

$$\sum_{m_r\in M_r} q_i\left(m_r\right)\left(u_{i,r}(m_r)+\delta U_{i,r-1}\left(p_r\left(m_r\right)\right)\right)=$$

$$\sum_{m_r'\in M_r'} q_i'\left(m_r'\right)\left(u_{i,r}(m_r')+\delta U_{i,r-1}\left(p_r'\left(m_r'\right)\right)\right),\quad i\in\{L,H\}.$$

Following the revelation principle provided by Bester and Strausz (2001) we can restrict to direct mechanisms. Additionally, as it was explain for the two-period case, posteriors are always determined by Bayes' rule and it is enough to consider a subset of all possible q_r (in particular, $q_H \geq q_L$). Generalization of these results to the multi-period setting is straightforward.

3 Optimal Selling Mechanism

3.1 Road Map

In this section we solve the seller's problem at (5), proving that price posting (see Beccuti (2014) for its definition) is the optimal selling mechanism for every period when $r > 2$.

First, we simplify the problem at (5) as in the two-period case (Lemma 1). We show that $IC_{H,r}$ and $IR_{L,r}$ are binding at the optimum, that $IR_{H,r}$ is redundant and that $IC_{L,r}$ can be replaced by a new constraint (SMC_r) which is more useful in the analysis.

Second, we define the continuation values when the discount factor is arbitrarily large. Next, we prove they are well defined (Lemma 2 and Lemma 3) and that they have some particular properties that are going to be useful to solve the seller's problem (from Lemma 4 to Lemma 7).

Finally, we show that the optimal mechanism follows these continuation values and, at the same time, that price posting is the optimal selling mechanism (Theorem 1 and Corollary 1).

3.2 Analysis

To solve the seller's problem at (5), it is useful to simplify it first. Next lemma establishes the equivalence between (5) after simplifications of Section 2.1 and a reduced program.

Lemma 1 *At any period r, the seller's problem at (5) is equivalent to*

$$\underset{\{q_r,\Gamma_r\}}{Max}\sum_{i\in\Theta}\sum_{m_r=l,h} p_{i,r+1}q_i(m_r)\left[v_r\left(m_r\right)+\delta V_{r-1}\left(p_r\left(m_r\right)\right)\right],\quad \textit{subject to,} \tag{6}$$

$$IC^*_{H,r}:\quad u_{H,r}(h)+\delta U_{H,r-1}\left(p_r\left(h\right)\right)=u_{H,r}(l)+\delta U_{H,r-1}\left(p_r\left(l\right)\right),$$

$$IR^*_{L,r}:\quad u_{L,r}(l)+\delta U_{L,r-1}\left(p_r\left(l\right)\right)=0,$$

$$SMC_r:\quad x_r(h)-x_r(l)\geq\frac{\delta}{\Delta\theta}\left[U_{H,r-1}\left(p_r(l)\right)-U_{H,r-1}\left(p_r(h)\right)\right],\quad \textit{with equality if } q_L>0,$$

$$BR_r:\quad p_{i,r}\left(m_r\right)=\frac{p_{i,r+1}q_i\left(m_r\right)}{\sum_{k=L,H}p_{k,r+1}q_k\left(m_r\right)},\ m_r=l,h,$$

$$x_r \in [0,1]\,,\ q_H > 0,\ q_L < 1,\ q_H \geq q_L.$$

Proof. Proof for Lemma 2 at Beccuti (2014) can be generalized to the multi-period setting. For this reason, it is omitted. ■

The interpretation of (6) is the same than the one for the reduced program in Beccuti (2014).

Since we consider the case with only two types, the vector $p_r(m_r)$ is completely determined by $p_{H,r}(m_r)$. From now on, and when it is not explicitly indicated in a different way, we refer as p to the prior of observing a high-type buyer at period r, and $p(m_r)$ to its posterior when a message m_r is sent.

One further simplification is to substitute $w_r(h)$ and $w_r(l)$ into (6) using $IR^*_{L,r}$ and $IC^*_{H,r}$. This is, we substitute

$$\begin{aligned}
&w_r(l) = x_r(l)\theta_L + \delta U_{L,r-1}(p_r(l))\,,\\
&w_r(h) = (x_r(h) - x_r(l))\,\theta_H + x_r(l)\theta_L + \delta U_{L,r-1}(p_r(l)) + \delta U_{H,r-1}(p_r(h)) - \delta U_{H,r-1}(p_r(l))\,,
\end{aligned}$$

into the seller's problem and we get:

$$\begin{aligned}
&\underset{\{q_r,x_r\}}{Max}\ W_r(x_r, q_r, p, p(m_r)) \ \text{ subject to,}\\
&SMC_r,\ BR_r,\\
&x_r \in [0,1]\,,\ q_H > 0,\ q_L < 1,\ q_H \geq q_L.
\end{aligned}\tag{7}$$

where

$$\begin{aligned}
W_r(x_r, q_r, p, p(m_r)) \ = \ & x_r(l)\theta_L + \rho_H(x_r(h) - x_r(l))\,\theta_H + \delta U_{L,r-1}(p_r(l)) + \\
& \delta\rho_H\left[U_{H,r-1}(p(h)) - U_{H,r-1}(p(l))\right] + \delta\rho_H V_{r-1}(p(h)) + \delta(1-\rho_H)V_{r-1}(p(l)),
\end{aligned}$$

and ρ_H is equal to $(pq_H + (1-p)q_L)$.

3.2.1 Continuation Values

We propose some functions for the seller $\tilde{V}_r(p)$ and for the high-type buyer $\tilde{U}_r(p)$, defining them recursively. For low-type buyer, we propose a function which is equal to zero for every p. We show later that they correspond with the equilibrium continuation values.

From Beccuti (2014) let $\tilde{V}_r(p)$ and $\tilde{U}_r(p)$ equal to $V_r(p)$ and $U_r(p)$ respectively, for periods $r = 1$ and $r = 2$. We denote $\tau_0 = 0$, $\tau_1 = \frac{\theta_L}{\theta_H}$, $\tau^*_1 = \frac{\theta_L}{\theta_H}$ (denotes the priors at last period that are the boundaries between semi-separation and separation, and pooling and separation price posting, respectively)[5], $\tau_2 = \frac{\theta_L[\theta_H + \delta\Delta\theta]}{\theta_H[\theta_L + \delta\Delta\theta]}$ and $\tau^*_2 = \frac{\theta_L}{\theta_H}$ (idem for $r = 2$). Then, let:

$$\tilde{V}_r(p) \equiv \begin{cases} p \geq \tau_r & \bar{q}_r(p)p\left(\theta_H + \delta\tilde{V}_{r-1}(1)\right) + (1 - \bar{q}_r(p)\,p)\,\delta\tilde{V}_{r-1}(\tau_{r-1}) \\ p \in [\tau^*_r, \tau_r) & q^*_r(p)p\left(\theta_H + \delta\tilde{V}_{r-1}(1)\right) + (1 - q^*_r(p)p)\,\delta\tilde{V}_{r-1}\left(\tau^*_{r-1}\right) - pq^*_r(p)\delta^{r-1}\Delta\theta \\ p \in [0, \tau^*_r) & \theta_L + \delta\tilde{V}_{r-1}(p) \end{cases},$$

for all $r > 2$;

$$\tilde{U}_r(p) \equiv \begin{cases} p \geq \tau_r & (1 - \bar{q}_r(p)p)\,\delta\tilde{U}_{r-1}(\tau_{r-1}) \\ p \in [\tau^*_r, \tau_r) & (1 - q^*_r(p)p)\,\delta\tilde{U}_{r-1}\left(\tau^*_{r-1}\right) + \delta^{r-1}\Delta\theta \\ p \in [0, \tau^*_r) & \theta_L + \delta\tilde{U}_{r-1}(p) \end{cases},$$

for all $r > 2$, where,

[5] Notice that there is not a semi-separation price posting in the last period. For mathematical convinience when conjecturing continuation values, we give it this particular value.

- τ_r is the value of $p \in (\tau_{r-1}, 1)$ such that first two lines of $\tilde{V}_r(p)$ coincides and τ_r^* is the value of $p \in \left(\tau_{r-1}^*, 1\right)$ such that last two lines of of $\tilde{V}_r(p)$ coincides.[6]

- $q_r(p, q_L) \equiv \frac{p-\tau_{r-1}}{p(1-\tau_{r-1})} + \frac{(1-p)q_L\tau_{r-1}}{p(1-\tau_{r-1})}$ $\forall p \in (\tau_{r-1}, 1)$, i.e., suppose a low-type buyer is sending a message h with probability q_L, then $q_r(p, q_L)$ is the probability that a high-type buyer sends a message h such that the seller's posterior, when she observes a message l, is equal to τ_{r-1}.

- $\bar{q}_r(p) \equiv \frac{p-\tau_{r-1}}{p(1-\tau_{r-1})}$ $\forall p \in (\tau_{r-1}, 1)$, i.e., the previous probability for the particular case of a low-type buyer sending a message h with zero probability ($q_L = 0$).

- $q_r^*(p) \equiv \frac{p-\tau_{r-1}^*}{p\left(1-\tau_{r-1}^*\right)}$ $\forall p \in \left(\tau_{r-1}^*, 1\right)$, for $r > 2$ and $q_2^*(\tau_2^*) = 1$.

Next figure illustrates seller's value functions, as we shall show later. Intervals $p \in \left[0, \frac{\theta_L}{\theta_H}\right]$, $p \in [\tau_r^*, \tau_r]$ and $p \in [\tau_r, 1]$ are linear in p. The interval $p \in \left[\frac{\theta_L}{\theta_H}, \tau_r^*\right]$ is piecewise linear in p. The figure also illustrates cutoffs τ_r^* and τ_r. These points guarantee that $\tilde{V}_r(p)$ is continuous.

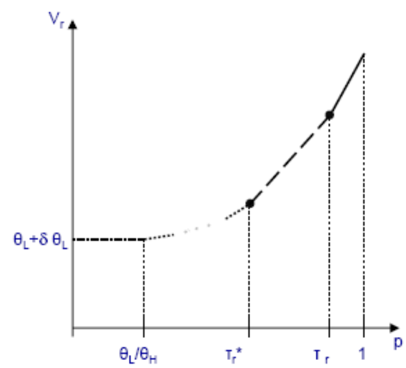

Figure 1: Solid-Line: first line in definition of $\tilde{V}_r(p)$; Dash-Line: second line in definition of $\tilde{V}_r(p)$; Dot-Line: third line in definition of $\tilde{V}_r(p)$.

Previous definition of $\tilde{V}_r(p)$ requires, to be complete, that τ_r^* and τ_r exist and are unique. The following two lemmas prove these properties.

Lemma 2 $\tau_r^* = \frac{\theta_L}{\theta_H} \sum_{i=0}^{r-2} \left(\frac{\Delta\theta}{\theta_H}\right)^i$ *and verifies* $\tau_r^* = \frac{\theta_L}{\theta_H q_r^*(\tau_r^*)}$ $\forall r > 1$.

Proof. See the Appendix. ■

Lemma 3 *Solution* τ_r *exists and it is unique.*

[6] This is, τ_r is the value of $p \in (\tau_{r-1}, 1)$ such that

$$\bar{q}_r(p)p\left(\theta_H + \delta\tilde{V}_{r-1}(1)\right) + (1 - \bar{q}_r(p)\,p)\,\delta\tilde{V}_{r-1}(\tau_{r-1}) = q_r^*(p)p\left(\theta_H + \delta\tilde{V}_{r-1}(1)\right) + (1 - q_r^*(p)p)\,\delta\tilde{V}_{r-1}(\tau_{r-1}^*) - pq_r^*(p)\delta^{r-1}\Delta\theta,$$

and τ_r^* is the value of $p \in (\tau_{r-1}^*, 1)$ such that

$$q_r^*(p)p\left(\theta_H + \delta\tilde{V}_{r-1}(1)\right) + (1 - q_r^*(p)p)\,\delta\tilde{V}_{r-1}(\tau_{r-1}^*) - pq_r^*(p)\delta^{r-1}\Delta\theta = \theta_L + \delta\tilde{V}_{r-1}(p)\,.$$

Then, points τ_r and τ_r^* guarantee continuity of $\tilde{V}_r(p)$ on p.

Proof. See the Appendix. ■

Notice that τ_r^* and τ_r are increasing in r.

Next, we propose a functional form for our conjecture of the continuation values. The proof is by induction.

Lemma 4 *$\tilde{U}_r(p)$ and $\tilde{V}_r(p)$ verify:*

$$\tilde{U}_r(p) = \Delta\theta\left(\sum_{i\in\Omega_r(p)} \delta^i + \mathbf{I}_{[0,\tau_r)}(p)\delta^{r-1}\right),$$

$$\tilde{V}_r(p) = \theta_L \sum_{i\in\Omega_r(p)} \delta^i + p\theta_H \sum_{i\in\bar{\Omega}_r(p)} \hat{q}_{r-i}(p)\delta^i + \theta_L \mathbf{I}_{[0,\tau_r)}(p)\delta^{r-1} + p\theta_H \mathbf{I}_{[\tau_r,1]}(p)\delta^{r-1},$$

where

$$\hat{q}_{r-i}(p) \equiv \begin{cases} \bar{q}_{r-i}(p) & if \quad p \geq \tau_r \\ q^*_{r-i}(p) & o.w. \end{cases},$$

$$\Omega_r(p) \equiv \left\{i \in \{0,1,...,r-2\} : p \in \left[0, \tau^*_{r-i}\right)\right\},$$

$$\bar{\Omega}_r(p) \equiv \left\{i \in \{0,1,...,r-2\} \backslash \Omega_r(p)\right\}.$$

Proof. See the Appendix. ■

As we will show, the set $\Omega_r(p)$ is the set of periods up to $r = 2$ in which the seller sells with probability one no matter the message observed. Its complementary $\bar{\Omega}_r(p)$ is when this does not happen. In particular, $\bar{\Omega}_r(p)$ is the set of periods in which the seller only sells to the high-type buyer with probability $\hat{q}_{r-i}(p)$.

The next lemma ensures that $\tau_r > \tau_r^*$.

Lemma 5 *If δ is sufficiently closed to 1, then $\tau_r^* \in (\tau_{r-2}, \tau_{r-1})$ $\forall r > 2$.*

Proof. See the Appendix. ■

Besides,

Lemma 6 *Suppose $U_{r-1}(p) = \tilde{U}_{r-1}(p)$ and $V_{r-1}(p) = \tilde{V}_{r-1}(p)$. If $\delta \in (\delta^*(T), 1)$, then either $\Omega_{r-1}(p\,(h)) = \Omega_{r-1}(p\,(l))$ or $\Omega_{r-1}(p\,(h)) = \Omega_{r-1}(p\,(l))\backslash \max\{i \in \Omega_{r-1}(p\,(l))\}$, where $\delta^*\,(T)$ is the unique solution in $(0,1)$ to $\delta^{T-2}\,(1+\delta) = 1$.*

Proof. See the Appendix. ■

Lemma 6 follows from the facts that τ_r^* is increasing in r and that δ is arbitrarily large. The former implies that $\Omega_r(p)$ is decreasing in p and $\Omega_{r-1}(p\,(l)) \geq \Omega_{r-1}(p\,(h))$ since $p\,(h) \geq p\,(l)$. The latter implies $\Omega_{r-1}(p\,(h)) = \Omega_{r-1}(p\,(l))$ or $\Omega_{r-1}(p\,(h)) = \Omega_{r-1}(p\,(l))\backslash \max\{i \in \Omega_{r-1}(p\,(l))\}$ in order to verify the SMC.

Definition 1 *A mechanism at period r induces significant* learning *when $\tilde{U}_{r-1}\,(p(l)) - \tilde{U}_{r-1}\,(p(h)) \neq 0$.*

As in the two-period setting, we say that a mechanism induces significant learning (from now on, just *learning*) when the buyer's continuation values are different for each message. This is, learning becomes relevant when it induces the seller to propose in the future a different mechanism for each message observed in the current period. This implies that buyer's payoffs are different for each message. Notice that this definition is with respect to our conjecture on continuation values.

Since $p\,(h) \geq p\,(l)$ and $\tilde{U}_r(p)$ is decreasing in p by definition, *learning* means $\tilde{U}_{r-1}\,(p(l)) > \tilde{U}_{r-1}\,(p(h))$. We distinguish the following cases of *learning* and *no-learning* that correspond with Lemma 6.

Lemma 7 Learning *can arise in the following cases:*

- Learning-a*: if* $\Omega_{r-1}(p(l)) = \Omega_{r-1}(p(h))$*,* $p(h) \geq \tau_{r-1}$ *and* $p(l) \in \left[\tau^*_{r-1}, \tau_{r-1}\right)$*.*
- Learning-b*: if* $\Omega_{r-1}(p(h)) = \Omega_{r-1}(p(l)) \backslash \max\{i \in \Omega_{r-1}(p(l))\}$ *and* $\Omega_{r-1}(p) = \Omega_{r-1}(p(h))$*.*
- Learning-c*: if* $\Omega_{r-1}(p(h)) = \Omega_{r-1}(p(l)) \backslash \max\{i \in \Omega_{r-1}(p(l))\}$ *and* $\Omega_{r-1}(p) = \Omega_{r-1}(p(l))$*.*

Besides, if there is no-learning*, then* $\Omega_{r-1}(p(l)) = \Omega_{r-1}(p(h))$*.*

Proof. See the Appendix ■

By application of Lemma 4 and Lemma 7 we have the following remark.

Remark 1 *In* learning-a $\tilde{U}_{r-1}(p(h)) = 0$ *and* $\tilde{U}_{r-1}(p(l)) = \delta^{r-2}\Delta\theta$*. In* learning-b *and* learning-c*,* $\tilde{U}_{r-1}(p(l)) - \tilde{U}_{r-1}(p(h)) = \delta^j \Delta\theta$ *where* $j = \max\{i \in \Omega_{r-1}(p(l))\}$*.*

3.2.2 Optimality

Next, we solve the problem at (7) using our conjecture of continuation values and we show that the optimal solution follows that conjecture. At the same time, we prove that the optimal selling mechanism is price posting.

Then, the seller's problem is

$$\begin{aligned}
&\underset{\{q_r, x_r\}}{Max}\ \tilde{W}_r(x_r, q_r, p, p(m_r)) \text{ subject to,} \\
&SMC_r,\ BR_r, \\
&x_r \in [0,1],\ q_H > 0,\ q_L < 1,\ q_H > q_L.
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
\tilde{W}_r(x_r, q_r, p, p(m_r)) &= x_r(l)\theta_L + \rho_H(x_r(h) - x_r(l))\theta_H + \delta\rho_H\left[\tilde{U}_{r-1}(p(h)) - \tilde{U}_{r-1}(p(l))\right] + \\
&\quad + \delta\rho_H \tilde{V}_{r-1}(p(h)) + \delta(1-\rho_H)\tilde{V}_{r-1}(p(l)),
\end{aligned}$$

and ρ_H is equal to $(pq_H + (1-p)q_L)$.

We split (8) into two subproblems. We consider the two variables maximization problem as a maximization problem in which the seller chooses first q_r and next x_r.[7] This is, fixing q_r, we maximize with respect to x_r. Since seller's payoff are increasing in $x_r(h)$ and the increment of $x_r(h)$ relaxes the SMC_r, then the optimal $x_r(h)$ is 1. On the other hand, the optimal allocation for message l depends on $\rho_H = pq_H + (1-p)q_L$, i.e. $x_r(l) = \hat{x}_r(l, q_r)$ where

$$\hat{x}_r(l, q_r) = \begin{cases} 0 & if \quad \rho_H \geq \frac{\theta_L}{\theta_H} \\ \mu & if \quad \rho_H < \frac{\theta_L}{\theta_H} \end{cases}, \tag{9}$$

[7] We are using the general property $\underset{\{x,y\}}{Max} f(x,y) = \underset{\{x\}}{Max}\left(\underset{\{y\}}{Max} f(x,y)\right)$.

with $\mu = \min\left\{1, 1 - \delta\frac{\tilde{U}_{r-1}(p(l))}{\Delta\theta} + \delta\frac{\tilde{U}_{r-1}(p(h))}{\Delta\theta}\right\}$ when $q_L = 0$,[8] and

$$\hat{x}_r(l, q_r) = 1 - \delta\frac{\tilde{U}_{r-1}\left(p\left(l\right)\right)}{\Delta\theta} + \delta\frac{\tilde{U}_{r-1}\left(p\left(h\right)\right)}{\Delta\theta} \tag{10}$$

when $q_L \neq 0$.

Now, we have to solve the seller's maximization problem with respect to q_r, i.e.

$$\begin{aligned} & \underset{\{q_r\}}{max}\ \tilde{W}_r\left(\hat{x}_r(l, q_r), q_r, p, p(m_r)\right), \text{ subject to,} \\ & p(h) = \frac{pq_H}{pq_H + (1-p)q_L}, \\ & p(l) = \frac{p(1-q_H)}{p(1-q_H) + (1-p)(1-q_L)}, \\ & q_H \in (0,1]\,,\ q_L \in [0,1)\,,\ q_H > q_L. \end{aligned} \tag{11}$$

To solve the second subproblem, we differentiate those cases where $x_r(l) = 0$ and where $x_r(l) \neq 0$.

Definition 2 *We say that a mechanism has* SMC non-binding *if* $x_r(l) = 0$ *and* SMC binding *if* $x_r(l) \neq 0$*.*

In both cases, it is possible to have *learning* or *no-learning*. Since $x_r(h) = 1$ and δ is arbitrarily large, is not possible to have $x_r(l) = 0$ at (10). It follows that the allocation $x_r(l) = 0$ occurs only when $q_L = 0$ and $\rho_H \geq \frac{\theta_L}{\theta_H}$ from (9). On the other hand, $x_r(l) \neq 0$ occurs either when $q_L = 0$ and $\rho_H < \frac{\theta_L}{\theta_H}$, or when $q_L \neq 0$. In both cases, by (9) or (10), respectively, $x_r(l) = 1 - \delta^{r-1}$ when there is *learning-a*, $x_r(l) = 1 - \delta^{j+1}$ when there is *learning-b* or *learning-c*, and $x_r(l) = 1$ when there is *no-learning*.

We can use previous terminology to distinguish eight subcases: *SMC non-binding with no-learning* (*SMC+NL*), *SMC non-binding with learning* (*SMC+L*) of cases a, b and c (*SMC+La*, *SMC+Lb* and *SMC+Lc*), *SMC binding with no-learning* (*SMC*+NL*), and *SMC binding with learning* (*SMC*+L*) of cases a, b and c (*SMC*+La*, *SMC*+Lb* and *SMC*+Lc*). Some of them could be empty for some prior. To analyze each subcase we assume that continuation values have the functional form proposed at Lemma 4. Next, we prove that the optimal mechanisms give payoffs that indeed follows our proposal. We also characterized the optimal mechanism for any prior. This is stated in the following theorem.

Theorem 1 *For any* $r > 2$ *and for any* $\delta \in (\delta^*(T), 1)$, *the continuation payoffs associated to the optimal selling mechanism are such that* $U_r(p) = \tilde{U}_r(p)$ *and* $V_r(p) = \tilde{V}_r(p)$*. The optimal selling mechanism is characterized by:*

[8] The optimal allocation for next period is

$$\hat{x}_{r-1}(l, q_{r-1}) = \begin{cases} 0 & if \quad \rho_{H,r-1} > \frac{\theta_L}{\theta_H} \\ \alpha_{r-1} & if \quad \rho_{H,r-1} = \frac{\theta_L}{\theta_H} \\ \mu & if \quad \rho_{H,r-1} < \frac{\theta_L}{\theta_H} \end{cases}$$

with $\alpha_{r-1} \in [0, \mu]$. Bester and Strausz specification allows the possibility of giving to the seller the option, at period r, of choosing α_{r-1}. Incluiding this action for the seller complicates the model without upsetting our result. We assume $\alpha_{r-1} = 0$. Given this assumption, we can also assume without loss of generality that $\hat{x}_r(l, q_r) = 0$ when $\rho_H = \frac{\theta_L}{\theta_H}$ at period r.

- *if* $p \geq \tau_r$, (SMC non-binding with no-learning) *satisfies that* $x_r(h) = 1$, $x_r(l) = 0$, $w_r(h) = \theta_H$, $w_r(l) = 0$, $q_H = \bar{q}_r(p)$, *and* $q_L = 0$.

- *if* $p \in [\tau_r^*, \tau_r)$, (SMC non-binding with learning) *satisfies that* $x_r(h) = 1$, $x_r(l) = 0$, $w_r(h) = \theta_H - \delta^{r-1}\Delta\theta$, $w_r(l) = 0$, $q_H = q_r^*(p)$, *and* $q_L = 0$.

- *if* $p \in [0, \tau_r^*)$, (SMC binding with no-learning) *satisfies that* $x_r(h) = x_r(l) = 1$, $w_r(h) = w_r(l) = \theta_L$, $q_H = q_L \neq 0$.

Proof. We start by assuming that continuation values for period $r-1$ are $\tilde{U}_{r-1}(p)$ and $\tilde{V}_{r-1}(p)$ for high-type buyer and for the seller respectively. We assume zero continuation value for low-type buyer.

We proceed as follow. First, in each of the following claims we get payoffs for the optimal mechanism in each subcase, indicating under which priors the subcase is not empty. These payoffs are either linear or piecewise linear in p. Second, we show that *SMC+NL* and *SMC+L* give the same payoffs at prior $p = \tau_r$ and that the former is steeper than the latter. Third, we show that *SMC*+La*, *SMC+L* and *SMC*+NL* give the same payoffs at $p = \tau_r^*$. By slope comparison, we prove that *SMC*+La* is either dominated by *SMC+L* or by *SMC*+NL*. Finally, *SMC*+Lb* and *SMC*+Lc* are dominated by *SMC*+NL*.

Claim 1 *Optimization of (11) subject to the additional constraint* SMC non-binding with no-learning *(*SMC+NL*) verifies that*

$$\begin{aligned} U_r(p) &= 0, \\ V_r(p) &= p\theta_H \sum_{i=0}^{r-2} \delta^i \bar{q}_{r-i}(p) + \delta^{r-1} p\theta_H, \end{aligned} \tag{12}$$

with $q_H = \bar{q}_r$ *and* $q_L = 0$. *Moreover, it is defined for* $p \geq p^*$ *where* $p^* = \tau_{r-1} + (1 - \tau_{r-1})\frac{\theta_L}{\theta_H}$.

Proof of Claim 1. In this case, $\tilde{U}_{r-1}(p(l)) - \tilde{U}_{r-1}(p(h)) = 0$ by no-learning and $x_r(l) = 0$ by non-binding. This allocation implies $q_L = 0$ and $\rho_H \geq \frac{\theta_L}{\theta_H}$ from (9), requiring $q_H \geq \frac{\theta_L}{\theta_H p}$. On the other hand, $p(h) = 1$ by BR_r and, from the functional form of continuation values at Lemma 4, to have no-learning it must be that $p(l) \geq \tau_{r-1}$, requiring $q_H \leq \bar{q}_r$ by definition of $\bar{q}_r$. Hence $p \geq p^*$ where $p^* = \tau_{r-1} + (1 - \tau_{r-1})\frac{\theta_L}{\theta_H}$. Since $p(l) \geq \tau_{r-1}$ then $p(l) > \tau_{r-1}^*$ and $\Omega_{r-1}(p(h)) = \Omega_{r-1}(p(l)) = \emptyset$. Using previous information, we can get agent's continuation values after substituting it in their functional form at Lemma 4. Plugging them into (11) and after some simplifications, the seller maximizes her payoffs with $q_H = \bar{q}_r$ (the maximum q_H such that $p(l) = \tau_{r-1}$), getting (12). ■

Claim 2 *Optimization of (11) subject to the additional constraint* SMC non-binding with learning *(*SMC+L*) verifies that*

$$\begin{aligned} U_r(p) &= 0, \\ V_r(p) &= p\theta_H \sum_{i=0}^{r-2} \delta^i q^*_{r-i}(p) + \delta^{r-1} \theta_L, \end{aligned} \tag{13}$$

with $q_H = q_r^*$ *and* $q_L = 0$. *Moreover, it is defined for* $p \geq \tau_r^*$ *and only* learning-a *is feasible.*

Proof of Claim 2. In this case, $\tilde{U}_{r-1}(p(l)) - \tilde{U}_{r-1}(p(h)) > 0$ by learning and $x_r(l) = 0$ by non-binding. This implies $q_L = 0$ and $\rho_H \geq \frac{\theta_L}{\theta_H}$ from (9), requiring $q_H \geq \frac{\theta_L}{\theta_H p}$. By BR_r, $p(h) = 1$

(i.e. $p(h) > \tau_{r-1}$). *Learning-a* is the only learning case which is feasible with $p(h) > \tau_{r-1}$, i.e. $\Omega_{r-1}(p(h)) = \Omega_{r-1}(p(l) = \emptyset$ and $p(l) \in [\tau^*_{r-1}, \tau_{r-1})$. Then, $q_H \leq q^*_r$ by definition of q^*_r, and jointly with $q_H \geq \frac{\theta_L}{\theta_H p}$, implies that p must be larger or equal to $\tau^*_{r-1} + (1-\tau^*_{r-1})\frac{\theta_L}{\theta_H}$ which it turns to be equal to τ^*_r by Lemma 2. After substituting previous conditions in the functional form of continuation values at Lemma 4, plugging them into (11) and after some simplifications, the seller maximizes her payoffs with $q_H = q^*_r$ (the maximum q_H such that $p(l) = \tau^*_{r-1}$), getting (13). ■

Claim 3 *Optimization of (11) subject to the additional constraint* SMC binding with no-learning *(*SMC*+NL*) verifies that*

$$\begin{aligned} U_r(p) &= \Delta\theta + \delta\tilde{U}_{r-1}(p), \\ V_r(p) &= \theta_L + \delta\tilde{V}_{r-1}(p), \end{aligned} \tag{14}$$

with $q_H = q_L \neq 0$.

Proof of Claim 3. In this case, $\tilde{U}_{r-1}(p(l)) - \tilde{U}_{r-1}(p(h)) = 0$ and $\Omega_{r-1}(p(l)) = \Omega_{r-1}(p(h))$ by *no-learning*. Also, *binding* with *no-learning* means $x_r(l) = 1$ either by (9) when $q_L = 0$ and $\rho_H < \frac{\theta_L}{\theta_H}$ or by (10) when $q_L \neq 0$.
Since $\Omega_{r-1}(p(l)) = \Omega_{r-1}(p(h))$, operating with the definition of seller's continuation values, we get that $\rho_H\tilde{V}_{r-1}(p(h)) + (1-\rho_H)\tilde{V}_{r-1}(p(l))$ is equal to $\tilde{V}_{r-1}(p)$.[9] Hence, substituting previous conditions into (11) and after some simplification, we get that payoffs are equal to (14). The seller can choose any q_H and q_L subject to *SMC*+NL*. In particular, let $q_H = q_L \neq 0$ which give $\tilde{V}_{r-1}(p(h)) = \tilde{V}_{r-1}(p(l)) = \tilde{V}_{r-1}(p)$. ■

Claim 4 *Optimization of (11) subject to the additional constraint* SMC binding with learning-a *(*SMC*+La*) verifies that the seller's expected payoffs are bounded above by*

$$\theta_L + \sum_{i\in\Omega_{r-1}(p)} \delta^{i+1}\theta_L + pq_H\theta_H \sum_{i\in\bar{\Omega}_{r-1}(p)} \delta^{i+1} q^*_{r-1-i}(p(h)) \\ + p(1-q_H)\theta_H \sum_{i\in\bar{\Omega}_{r-1}(p)} \delta^{i+1} q^*_{r-1-i}(p(l)) + \delta^{r-1} pq_H\theta_H,$$

and bounded below by

$$\theta_L + \sum_{i\in\Omega_{r-1}(p)} \delta^{i+1}\theta_L + pq_H\theta_H \sum_{i\in\bar{\Omega}_{r-1}(p)} \delta^{i+1} \bar{q}_{r-1-i}(p(h)) \\ + p(1-q_H)\theta_H \sum_{i\in\bar{\Omega}_{r-1}(p)} \delta^{i+1} \bar{q}_{r-i-1}(p(l)) + \delta^{r-1} pq_H\theta_H.$$

for the optimal q_H such that $p(h) \geq \tau_{r-1}$ and $p(l) \in [\tau^*_{r-1}, \tau_{r-1})$. This mechanism is defined for $p \geq \tau^*_{r-1}$. Moreover, when $p = \tau^*_r$ seller's expected payoffs are equal to (14) with $q_H = q^*_r(\tau^*_r)$ and $q_L = 0$.

Proof of Claim 4. Now $\tilde{U}_{r-1}(p(l)) - \tilde{U}_{r-1}(p(h)) \neq 0$ by *learning* and $x_r(l) \neq 0$ by *binding*. As consequence $\hat{x}_r(l, q_r) < 1$ from (9) when $q_L = 0$ and $\rho_H < \frac{\theta_L}{\theta_H}$ or, from (10) when $q_L \neq 0$. Since we

[9] *No-learning* implies that $\hat{q}_{r-1-i}(p(h))$ and $\hat{q}_{r-1-i}(p(l))$ are either equal to $\bar{q}_{r-1-i}(p(h))$ and to $\bar{q}_{r-1-i}(p(l))$ respectively, or equal to $q^*_{r-1-i}(p(h))$ and to $q^*_{r-1-i}(p(l))$. Then,

$$\rho_H p(h)\,\hat{q}_{r-1-i}(p(h)) + (1-\rho_H)p(l)\,\hat{q}_{r-1-i}(p(l)) = p\hat{q}_{r-1-i}(p).$$

are considering *learning-a*, $\Omega_{r-1}(p(l)) = \Omega_{r-1}(p(h))$, $p(h) \geq \tau_{r-1}$ and $p(l) \in [\tau^*_{r-1}, \tau_{r-1})$, giving $\tilde{U}_{r-1}(p(h)) = 0$ and $\tilde{U}_{r-1}(p(l)) = \delta^{r-2}\Delta\theta$, i.e. $x_r(l) = 1 - \delta^{r-1}$.
By definition, $p(h) \geq \tau_{r-1}$ and $p(l) \in [\tau^*_{r-1}, \tau_{r-1})$ implies that $\hat{q}_{r-1-i}(p(h))$ is equal to $\bar{q}_{r-1-i}(p(h))$ and $\hat{q}_{r-1-i}(p(l))$ to $q^*_{r-1-i}(p(l))$. From $\Omega_{r-1}(p(l)) = \Omega_{r-1}(p(h))$ it follows $\bar{\Omega}_{r-1}(p(l)) = \bar{\Omega}_{r-1}(p(h))$ and $\Omega_{r-1}(p) = \Omega_{r-1}(p(h))$. We can get continuation values form Lemma 4, and after substituting them at (11) and some simplifications, the seller has to choose (q_H, q_L) to maximize,

$$\theta_L + \sum_{i\in\Omega_{r-1}(p)} \delta^{i+1}\theta_L + pq_H\theta_H \sum_{i\in\bar{\Omega}_{r-1}(p)} \delta^{i+1}\bar{q}_{r-1-i}(p(h)) \tag{15}$$

$$+ p(1-q_H)\theta_H \sum_{i\in\bar{\Omega}_{r-1}(p)} \delta^{i+1}q^*_{r-1-i}(p(l)) + \delta^{r-1}pq_H\theta_H,$$

$$\text{subject to } p(h) \geq \tau_{r-1}\ ,\ p(l) \in [\tau^*_{r-1}, \tau_{r-1}).$$

Notice that, since $p(h) \geq p \geq p(l)$, this mechanism can only be defined for $p \geq \tau^*_{r-1}$, which implies $\Omega_{r-1}(p) = \emptyset$.
Since $\bar{q}_r(\cdot) < q^*_r(\cdot)$ by definition, (15) is bounded above when replacing $\bar{q}_{r-1-i}(p(h))$ by $q^*_{r-1-i}(p(h))$.[10] On the other hand, (15) is bounded below when replacing $q^*_{r-i-1}(p(l))$ with $\bar{q}_{r-i-1}(p(l))$.

Moreover, when $p = \tau^*_r$, the seller maximizes (15) choosing $q_H = q^*_r(\tau^*_r)$ (in order to $p(l) = \tau^*_{r-1}$), and $q_L = 0$ (to get $p(h) = 1$ while $\rho_H < \frac{\theta_L}{\theta_H}$) making

$$\theta_L + \theta_L \sum_{i\in\bar{\Omega}_{r-1}(\tau^*_r)} \delta^{i+1} + \tau^*_r(1 - q^*_r(\tau^*_r))\theta_H \sum_{i\in\bar{\Omega}_{r-1}(\tau^*_r)} \delta^{i+1}\frac{\tau^*_{r-1} - \tau^*_{r-2-i}}{\tau^*_{r-1}\left(1 - \tau^*_{r-2-i}\right)} + \delta^{r-1}\theta_L.$$

Using the relation of τ^*_r with τ^*_{r-1} implicit in Lemma 2 we get that

$$\theta_L + \tau^*_r(1 - q^*_r(\tau^*_r))\theta_H \frac{\tau^*_{r-1} - \tau^*_{r-2-i}}{\tau^*_{r-1}\left(1 - \tau^*_{r-2-i}\right)} = \tau^*_r\theta_H q^*_{r-1-i}(\tau^*_r).$$

Then, seller's maximum payoffs can be written as

$$\theta_L + \tau^*_r\theta_H \sum_{i\in\bar{\Omega}_{r-1}(\tau^*_r)} \delta^{i+1}q^*_{r-1-i}(\tau^*_r) + \delta^{r-1}\theta_L.$$

This last expression is equivalent to seller's payoff at (14) when we replace in it the functional form of $\tilde{V}_{r-1}(\tau^*_r)$ from Lemma 4. By the definition of $\tilde{V}_r(p)$, it is also equal to (13) for $p = \tau^*_r$. ■

Claim 5 *Optimization of (11) subject to the additional constraint* SMC binding with learning-b *(*SMC*+Lb*) verifies that the seller expected payoffs are equal to (14) with (q_H, q_L) such that q_H is equal to* $\frac{p-\tau^*_{r-j-2}}{p(1-\tau^*_{r-j-2})} + \frac{(1-p)q_L\tau^*_{r-j-2}}{p(1-\tau^*_{r-j-2})}$, *where* $j = \max\{i \in \Omega_{r-1}(p(l))\}$. *This mechanism is defined for* $p < \tau_{r-1}$.

Proof of Claim 5. $\tilde{U}_{r-1}(p(l)) - \tilde{U}_{r-1}(p(h)) \neq 0$ by *learning* and $x_r(l) \neq 0$ by *binding*. As

[10] To simplify (15) we use

$$\begin{aligned}\rho_{H,}p(h)\hat{q}_{r-1-i}(p(h)) + (1-\rho_H)p(l)\hat{q}_{r-1-i}(p(l)) &= \\ &= pq_H\hat{q}_{r-1-i}(p(h)) + p(1-q_H)\hat{q}_{r-1-i}(p(l)) \\ &= p\hat{q}_{r-1-i}(p),\end{aligned}$$

consequence $\hat{x}_r(l, q_r) < 1$ from (9) when $q_L = 0$ and $\rho_H < \frac{\theta_L}{\theta_H}$ or, from (10) when $q_L \neq 0$. Since we are considering *learning-b*, $|\Omega_{r-1}(p\,(l))| - |\Omega_{r-1}(p\,(h))| = 1$ and $\Omega_{r-1}(p) = \Omega_{r-1}(p\,(h))$, with $p\,(h) < \tau_{r-1}$ and $p\,(l) < \tau_{r-1}$, giving $\tilde{U}_{r-1}\,(p(l)) - \tilde{U}_{r-1}\,(p(h)) = \delta^j \Delta\theta$ where $j = \max\{i \in \Omega_{r-1}(p\,(l))\}$, i.e. $x_r(l) = 1 - \delta^{j+1}$.
Since $\tau_{r-1} > p\,(h) \geq p\,(l)$ and $p\,(h) \geq p \geq p\,(l)$, this mechanism is defined for $p < \tau_{r-1}$. Additionally, $\tau_{r-1} > p\,(h) \geq p\,(l)$ implies $\hat{q}_{r-1-i}(p(\cdot)) = q^*_{r-1-i}\,(p\,(\cdot))$ by definition. Let j to be the larger $i \in \Omega_{r-1}\,(p\,(l))$, i.e.$\Omega_{r-1}\,(p\,(l)) = \{0, 1, ..., j\}$ and $\bar{\Omega}_{r-1}\,(p\,(l)) = \{j+1, ..., r-3\}$. By definition of $\Omega_{r-1}\,(p\,(l))$, it must be that $p\,(l) \in \left[\tau^*_{r-2-j}, \tau^*_{r-1-j}\right)$, and since $|\Omega_{r-1}\,(p\,(l))| = |\Omega_{r-1}\,(p\,(h))| + 1$, $p\,(h) \in \left[\tau^*_{r-1-j}, \tau^*_{r-j}\right)$ with $\Omega_{r-1}\,(p\,(h)) = \{0, 1, ..., j-1\}$ and $\bar{\Omega}_{r-1}\,(p\,(h)) = \{j, j+1, ..., r-3\}$. Continuation values for $r-1$ are given by Lemma 4. Substituting continuation values and allocations at (11) and after some simplifications,[11] the seller chooses (q_H, q_L) to maximize

$$\theta_L + \sum_{i \in \Omega_{r-1}(p(h))} \theta_L \delta^{i+1} + \delta^{j+1} \left(p q_H - \frac{(1-p) q_L \tau^*_{r-j-2}}{1 - \tau^*_{r-j-2}} \right) \theta_H + $$
$$+ p\theta_H \sum_{i \in \bar{\Omega}_{r-1}(p(h)) \setminus \{j\}} q^*_{r-1-i}\,(p)\,\delta^{i+1} + \theta_L \delta^{r-1}$$
$$\text{subject to } p\,(h) \in \left[\tau^*_{r-1-j}, \tau^*_{r-j}\right),\ p\,(l) \in \left[\tau^*_{r-2-j}, \tau^*_{r-1-j}\right).$$

These payoffs are maximized with q_H equal to $\frac{p - \tau^*_{r-j-2}}{p(1-\tau^*_{r-j-2})} + \frac{(1-p) q_L \tau^*_{r-j-2}}{p(1-\tau^*_{r-j-2})}$ (which is the maximum q_H such that $p\,(l) \in \left[\tau^*_{r-2-j}, \tau^*_{r-1-j}\right)$, i.e. $p\,(l) = \tau^*_{r-2-j}$), making

$$\theta_L + \theta_L \sum_{i \in \Omega_{r-1}(p)} \delta^{i+1} + p\theta_H \sum_{i \in \bar{\Omega}_{r-1}(p)} q^*_{r-1-i}\,(p)\,\delta^{i+1} + \theta_L \delta^{r-1}.$$

These payoffs are equal to the expression at (14) when we replace $\tilde{V}_{r-1}(p)$ by its functional form defined for $p < \tau_r$ at Lemma 4. ■

Claim 6 *Optimization of (11) subject to the additional constraint* SMC binding with learning-c *(*SMC*+Lb*) verifies that the seller expected payoffs are equal to*

$$\theta_L + \theta_L \sum_{i \in \Omega_{r-1}(p) \setminus \{j\}} \delta^{i+1} + p\theta_H \sum_{i \in \bar{\Omega}_{r-1}(p) \cup \{j\}} q^*_{r-1-i}\,(p)\,\delta^{i+1} + \theta_L \delta^{r-1}, \tag{16}$$

with (q_H, q_L) *such that* q_H *is equal to* $\frac{p - \tau^*_{r-j-2}}{p(1-\tau^*_{r-j-2})} + \frac{(1-p) q_L \tau^*_{r-j-2}}{p(1-\tau^*_{r-j-2})}$, *where* $j = \{\max i \in \Omega_{r-1}(p\,(l))\}$. *This mechanism is defined for* $p < \tau_{r-1}$.

Proof of Claim 6. $\tilde{U}_{r-1}\,(p(l)) - \tilde{U}_{r-1}\,(p(h)) \neq 0$ by *learning* and $x_r(l) \neq 0$ by *binding*. As consequence $\hat{x}_r(l, q_r) < 1$ from (9) when $q_L = 0$ and $\rho_H < \frac{\theta_L}{\theta_H}$ or, from (10) when $q_L \neq 0$. Since we are considering *learning-c*, $|\Omega_{r-1}(p\,(l))| - |\Omega_{r-1}(p\,(h))| = 1$ and $\Omega_{r-1}(p) = \Omega_{r-1}(p\,(l))$, with $p\,(h) < \tau_{r-1}$ and $p\,(l) < \tau_{r-1}$, giving $\tilde{U}_{r-1}\,(p(l)) - \tilde{U}_{r-1}\,(p(h)) = \delta^j \Delta\theta$ where $j = \max\{i \in \Omega_{r-1}(p\,(l))\}$, i.e. $x_r(l) = 1 - \delta^{j+1}$.

[11] Notice that

$$\rho_{H,} p\,(h)\, q^*_{r-j-1}\,(p\,(h)) = p q_H - \frac{(1-p) q_L \tau^*_{r-j-2}}{1 - \tau^*_{r-j-2}}.$$

Since $\tau_{r-1} > p(h) \geq p(l)$ and $p(h) \geq p \geq p(l)$, this mechanism is defined for $p < \tau_{r-1}$.
Let j to be the larger $i \in \Omega_{r-1}(p(l))$. By definition of $\Omega_r(p)$, $p(l) \in \left[\tau^*_{r-2-j}, \tau^*_{r-1-j}\right)$ and, since $\Omega_{r-1}(p) = \Omega_{r-1}(p(l))$, also $p \in \left[\tau^*_{r-2-j}, \tau^*_{r-1-j}\right)$. Following the same procedure than in previous point, seller's maximum payoffs are equal to (16).
■

We have the optimal mechanisms for each subcase. We proceed now to compare them. Notice that (12) and (13) are linear on p and, that (14), payoffs at Claim 4 and (16) are piecewise linear in p with slopes increasing in p.[12]

Notice that (12) is the functional form at Lemma 4 defined for $p \geq \tau_r$ and (13) the one for $p < \tau_r$, i.e. $V_r(p) = \tilde{V}_r(p)$ and $U_r(p) = \tilde{U}_r(p)$. Then, they follow our definition of $\tilde{V}_r(p)$ and $\tilde{U}_r(p)$ for $p \geq \tau_r$ and $p \in [\tau^*_r, \tau_r)$ respectively. By this definition, they are equal at $p = \tau_r$. Finally, (12) is steeper than (13) due to $\frac{1}{1-\tau_{r-1-i}} \geq \frac{1}{1-\tau^*_{r-1-i}}$. Then, (12) dominates (13) when $p \geq \tau_r$ and the opposite when $p < \tau_r$.

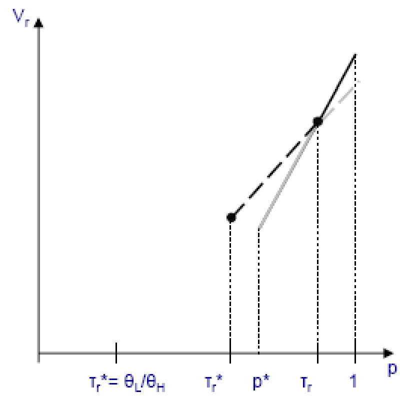

Figure 2: Maximum Seller's payoffs. Dash-Line: SMC non-binding with learning; Solid-Line: SMC non-binding with no-learning.

Payoffs at (14) have the functional form at Lemma 4 defined for $p < \tau^*_r$, i.e. $V_r(p) = \tilde{V}_r(p)$ and $U_r(p) = \tilde{U}_r(p)$. Then by the definition of $\tilde{V}_r(p)$ and $\tilde{U}_r(p)$, (14) and (13) are equal at $p = \tau^*_r$. From Claim 4, (14) and (13) are also equal to seller's payoffs under *SMC binding with learning-a* at $p = \tau^*_r$.

When $p \in (\tau^*_{r-1}, \tau_r)$, the slope of (14) is bounded above by $\sum_{i=1}^{r-2} \delta^i \theta_H \frac{1}{(1-\tau^*_{r-1-i})}$ which is lower than the one of (13), equal to $\theta_H \frac{1}{(1-\tau^*_{r-1})} + \sum_{i=1}^{r-2} \delta^i \theta_H \frac{1}{(1-\tau^*_{r-1-i})}$. When $p < \tau^*_{r-i}$ for $i \in \{1, ..., r-2\}$ the slope of (14) is decreasing in i.[13] Then, the current mechanism dominates the one under *SMC binding with learning* when $p < \tau^*_r$ and the opposite when $p \in [\tau^*_r, \tau_r]$. When $p > \tau_r$, the slope of (14) is now equal to $\theta_H \sum_{i=1}^{r-2} \delta^i \frac{1}{(1-\tau_{r-1-i})} + \delta^{r-1}\theta_H$, which is lower than the one of (12).

[12] When $p \in \left[\tau^*_{r-2-j}, \tau^*_{r-1-j}\right)$, $\bar{\Omega}_{r-1}(p) = \{j+1, ..., r-3\}$ by definition. Applying functional form for continuation values at Lemma6, $\tilde{V}_{r-1}(p)$ at (14) has slope $\sum_{i=j+1}^{r-3} \delta^i \theta_H \frac{1}{1-\tau^*_{r-1-i}}$. On the other hand, when $p \in \left[\tau^*_{r-1-j}, \tau^*_{r-j}\right)$, now $\bar{\Omega}_{r-1}(p) = \{j, ..., r-3\}$, and $\tilde{V}_{r-1}(p)$ has a larger slope equal to $\delta^j \theta_H \frac{1}{1-\tau^*_{r-1-j}} + \sum_{i=j+1}^{r-3} \delta^i \theta_H \frac{1}{1-\tau^*_{r-1-i}}$. The same argument can be used to check that slopes are increasing in p in payoffs at Claim 4 and at (16).

[13] When $p < \tau^*_2$, the slope of (14) is zero.

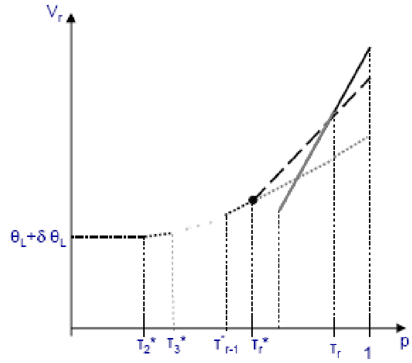

Figure 3: Maximum Seller's payoffs. Dot-Line: SMC binding with no-learning; Dash-Line: SMC non-binding with learning.

From Claim 4, the upper bound of seller's payoffs has a maximum slope equal to

$$\theta_H \sum_{i=0}^{r-3} \delta^{i+1} \frac{1}{\left(1-\tau_{r-2-i}^*\right)} + \delta^{r-1}\theta_H,$$

when assuming that, in the maximization of (11), the seller could choose $q_H = 1$ such that $p(h) \geq \tau_{r-1}$, $p(l) \in \left[\tau_{r-1}^*, \tau_{r-1}\right)$. This slope is lower to the one at (13) which is $\theta_H \sum_{i=0}^{r-2} \delta^i \frac{1}{\left(1-\tau_{r-1-i}^*\right)}$.[14] On the other hand, the lower bound of seller's payoffs has a minimum slope equal to

$$\theta_H \sum_{i=0}^{r-3} \delta^{i+1} \frac{1}{(1-\tau_{r-2-i})} + \delta^{r-1}\theta_H,$$

when assuming that, in the maximization of (11), the seller could choose $q_H = 1$ such that $p(h) \geq \tau_{r-1}$, $p(l) \in \left[\tau_{r-1}^*, \tau_{r-1}\right)$. This slope is larger than the one of (14) (bounded above by $\theta_H \sum_{i=1}^{r-2} \delta^i \frac{1}{(1-\tau_{r-1-i})}$) when $p < \tau_r^*$. Then, when $p > \tau_r^*$, a mechanism *SMC binding with learning-a* is dominated by a mechanism *SMC non-binding with learning* and, when $p < \tau_r^*$, it is dominated by *SMC binding with no-learning.*

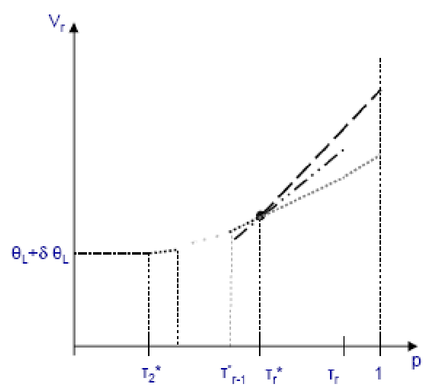

Figure 4: Maximum Seller's payoffs. SMC binding with learning-a (Dash-Double Dot-Line) dominated by SMC binding with no-learning (Dot-Line) and SMC non-binding with learning (

[14] $\frac{1}{\left(1-\tau_{r-1}^*\right)} + \frac{\delta}{\left(1-\tau_{r-2}^*\right)} + ... + \frac{\delta^{r-2}}{\left(1-\tau_1^*\right)} > \frac{\delta}{\left(1-\tau_{r-2}^*\right)} + ... + \delta^{r-1}$ due to $\frac{1}{\left(1-\tau_{r-1}^*\right)} > 1$.

Dash-Line)

From Claim 5 *SMC binding with learning-b* gives the same payoffs than *SMC binding with no-learning*. From Claim 6, and *SMC binding with learning-c* is weakly dominated by *SMC binding with no-learning* since payoffs at (16) are lower than payoffs at (14) due to $pq^*_{r-1-j}(p) < \frac{\theta_L}{\theta_H}$ when $p < \tau^*_{r-1-j}$ by Lemma 2.

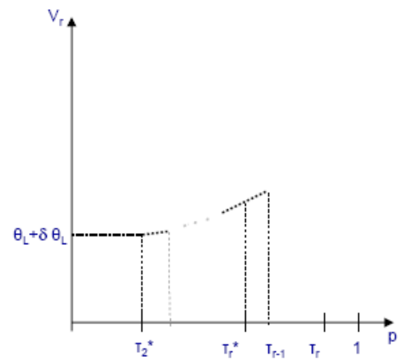

Figure 5: SMC binding with no-learning coincides with SMC binding with learning-b for $p < \tau_{r-1}$.

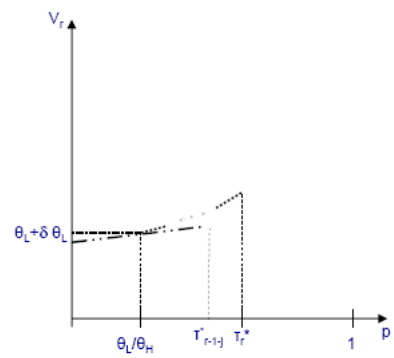

Figure 6: Dot-Line: SMC binding with no-learning; Dash-Line: SMC binding with learning-c.

Concluding, the optimal mechanism is a *SMC non-binding with no-learning* when $p \geq \tau_r$, a *SMC non-binding with learning* when $p \in [\tau^*_r, \tau_r)$ and a *SMC binding with no-learning* when $p \in [0, \tau^*_r)$. Optimal allocations are

$$x_r(h) = 1, \;\; x_r(l) = \begin{cases} 0 & \text{if } p \geq \tau^*_r \\ 1 & \text{if } p < \tau^*_r \end{cases} .$$

Optimal payments are obtained by replacing, for each case, allocations and continuation values at $IR^*_{L,r}$ and $IC^*_{H,r}$ and solving for $w_r(l)$ and $w_r(h)$,

$$w_r(h) = \begin{cases} \theta_H & \text{if } p \geq \tau_r \\ \theta_H - \delta^{r-1}\Delta\theta & \text{if } p \in [\tau^*_r, \tau_r) \\ \theta_L & \text{if } p < \tau^*_r \end{cases} , \quad w_r(l) = \begin{cases} 0 & \text{if } p \geq \tau^*_r \\ \theta_L & \text{if } p < \tau^*_r \end{cases} .$$

Notice that low type payoffs are zero with previous $w_r(l)$ given that we assumed zero continuation values for him. ■

The argument of the proof relies on the following: the optimal payoffs for each subcase are either linear or piecewise linear functions of p. The upper envelope of these functions only contains *SMC non-binding with no-learning* (when $p \geq \tau_r$), *SMC non-binding with learning-a* ($p \in [\tau_r^*, \tau_r)$) and *SMC binding with no-learning* ($p \in [0, \tau_r^*)$). Then, this upper envelope characterizes the optimal mechanism for every prior and it is equal to the definition of $\tilde{V}_r(p)$. It is summarized in Figure 1.

Optimal mechanisms in Theorem 1 are direct mechanisms with allocation $x_r(l) \in \{0, 1\}$. We state in the following corollary that the optimal direct mechanism can be implemented by a price posting, which is an indirect mechanism. To do that, we propose an alternative outcome $(\hat{q}_r, \hat{p}_r, \hat{\Gamma}_r)$ where $\hat{\Gamma}_r$ is a price posting mechanism and we check that this outcome is payoff equivalent to the incentive efficient outcome (q_r, p_r, Γ_r) that solves (6) and contains the optimal selling mechanism characterized in the theorem. Since the proof is mechanic, we relegate it to the Appendix.

Corollary 1 *When $r > 2$, the optimal selling mechanism can be implemented by a price posting equal to*

i) θ_H when $p \geq \tau_r$, the high-type buyer randomizes and the low-type buyer never buys;

ii) $\theta_H - \delta^{r-1}\Delta\theta$ when $p \in [\tau_r^*, \tau_r)$, The high-type buyer always buys and the low-type buyer never buys and;

iii) θ_L when $p < \tau_r^*$, both types always buy.

Proof. See the Appendix. ■

When the seller is optimistic ($p \geq \tau_r^*$), she offers a price posting that separates types. This is, only the high-type buyer buys with positive probability. In case of being extremely optimistic ($p \geq \tau_r$), the seller offers a price posting equal to θ_H. The high-type buyer randomizes and, in case of not buying, the seller will ask for a price equal to θ_H in the following period again. Then, she exploits the buyer extracting all his surplus in every period. This exploiting case corresponds with *SMC non-binding with no-learning.* In case of being moderately optimistic ($p \in [\tau_r^*, \tau_r)$), the seller offers a price posting equal to $\theta_H - \delta^{r-1}\Delta\theta$. Now, the seller is bribing the high-type buyer to induce him to reveal his type. This bribe is equal to his future discounted losses by being discriminated in the current period. This bribing case corresponds with *SMC non-binding with learning.* Finally, when the seller is pessimistic ($p < \tau_r^*$), she offers a price equal to θ_L. This is the pooling case, when both buyer types always buy, which corresponds with *SMC binding with no-learning.*

3.2.3 Belief's Dynamic

Figure 7 indicates how beliefs evolve. Starting at an optimistic prior (i.e. $p \geq \tau_r^*$), the seller's beliefs are updated gradually as information is revealed when the buyer does not buy. On the other hand, when the buyer buys, she quickly learns that she is facing a high-type consumer with certainty. Starting at $p \in \left[\frac{\theta_L}{\theta_H}, \tau_r^*\right)$, seller's beliefs are not updated up to some period $r - i$ where $p \geq \tau_{r-i}^*$. When $p < \frac{\theta_L}{\theta_H}$, seller's beliefs are never updated.

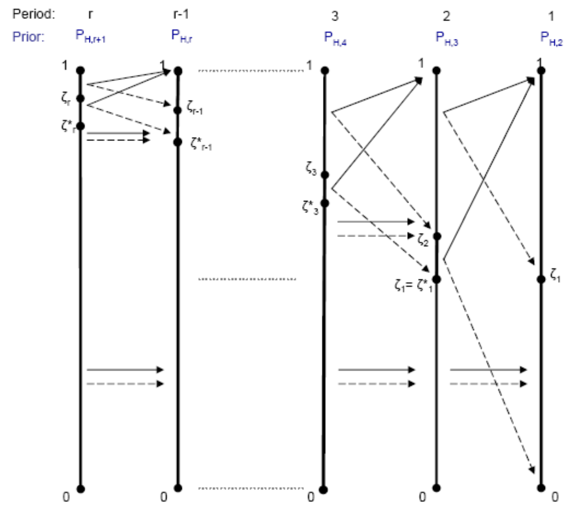

Figure 7: Belief dynamic under different priors for T>2 periods.. A full line shows how beliefs evolve when the buyer buys the good. The dash line is when he does not buy.

4 Concluding Remarks

This paper generalizes the model at Beccuti (2014) for many periods when both players have the same discount factor. It proves that within this framework the optimal selling procedure is to post a price in every period. The paper also gives a complete characterization of equilibrium payoffs.

A natural extension is to study which is the optimal mechanism when discount factors are different but close to one.

5 Appendix

5.1 Proof of Lemma 2

Proof. We proceed by induction.

From initial conditions, $\tau_2^* = \frac{\theta_L}{\theta_H q_2^*(\tau_2^*)}$.

For $r > 2$, assume that $\tau_{r-1}^* = \frac{\theta_L}{\theta_H q_{r-1}^*(\tau_{r-1}^*)}$.

From definition of τ_r^*

$$q_r^*(\tau_r^*)\tau_r^*\left(\theta_H + \delta\tilde{V}_{r-1}(1) - \delta^{r-1}\Delta\theta\right) + (1 - q_r^*(\tau_r^*)\tau_r^*)\,\delta\tilde{V}_{r-1}\left(\tau_{r-1}^*\right) = \theta_L + \delta\tilde{V}_{r-1}(\tau_r^*)\,. \tag{17}$$

Now, let's define $\Psi_{r-1}(p)$ as

$$\Psi_{r-1}(p) \equiv q_{r-1}^*(p)p\left(\theta_H + \delta\tilde{V}_{r-1}(1)\right) + \left(1 - q_{r-1}^*(p)p\right)\delta\tilde{V}_{r-1}\left(\tau_{r-1}^*\right) - pq_{r-1}^*(p)\delta^{r-2}\Delta\theta, \quad \forall p.$$

Since $q_{r-1}^*(p) = \frac{(p-\tau_{r-2}^*)}{p(1-\tau_{r-2}^*)} \Rightarrow q_{r-1}^*(1) = 1$, and $\Psi_{r-1}(1) = \left(\theta_H + \delta\tilde{V}_{r-2}(1)\right) - \delta^{r-2}\Delta\theta$. By definition of $\tilde{V}_{r-1}(p)$ follows that $\tilde{V}_{r-1}(1) = \theta_H + \delta^{r-1}\tilde{V}_{r-2}(1)$. Then, we can write $\tilde{V}_{r-1}(1)$ as equal to $\Psi_{r-1}(1) + \delta^{r-2}\Delta\theta$ and (17) as

$$q_r^*(\tau_r^*)\tau_r^*\left(\theta_H + \delta\Psi_{r-1}(1)\right) + (1 - q_r^*(\tau_r^*)\tau_r^*)\,\delta\tilde{V}_{r-1}\left(\tau_{r-1}^*\right) = \theta_L + \delta\tilde{V}_{r-1}(\tau_r^*)\,, \tag{18}$$

where $\tilde{V}_{r-1}\left(\tau_{r-1}^*\right)$ and $\tilde{V}_{r-1}(\tau_r^*)$ are, by definition,

$$q_{r-1}^*(\tau_{r-1}^*)\tau_{r-1}^*\left(\Psi_{r-1}(1)\right) + \left(1 - q_{r-1}^*(\tau_{r-1}^*)\tau_{r-1}^*\right)\delta\tilde{V}_{r-2}\left(\tau_{r-2}^*\right),$$

and

$$q_{r-1}^*(\tau_r^*)\tau_r^*\left(\Psi_{r-1}(1)\right) + \left(1 - q_{r-1}^*(\tau_r^*)\tau_r^*\right)\delta\tilde{V}_{r-2}\left(\tau_{r-2}^*\right),$$

respectively.

Since $q_r^*(\tau_r^*)\tau_r^* + (1 - q_r^*(\tau_r^*)\tau_r^*)\, q_{r-1}^*(\tau_{r-1}^*)\tau_{r-1}^*$ is equal to $q_{r-1}^*(\tau_r^*)\tau_r^*$ and $\left(1 - q_{r-1}^*(\tau_{r-1}^*)\tau_{r-1}^*\right)(1 - q_r^*(\tau_r^*)\tau_r^*)$ is equal to $\left(1 - q_{r-1}^*(\tau_r^*)\tau_r^*\right)$, the LHS of (18) reduces to $q_r^*(\tau_r^*)\tau_r^*\theta_H + \delta\tilde{V}_{r-1}(\tau_r^*)$. As consequence, $q_r^*(\tau_r^*)\tau_r^*\theta_H = \theta_L$, proving the last part.

Finally, using the definition of $q_r^*(\tau_r^*)$, then $\tau_r^* = \frac{\theta_L}{\theta_H}\left(1 - \tau_{r-1}^*\right) + \tau_{r-1}^*$. Suppose $\tau_{r-1}^* = \frac{\theta_L}{\theta_H}\sum_{i=0}^{r-3}\left(\frac{\Delta\theta}{\theta_H}\right)^i$, then $\tau_r^* = \frac{\theta_L}{\theta_H}\sum_{i=0}^{r-2}\left(\frac{\Delta\theta}{\theta_H}\right)^i$. ■

5.2 Proof of Lemma 3

Proof. From initial conditions, $\tau_2 = \frac{\theta_L[\theta_H + \delta\Delta\theta]}{\theta_H[\theta_L + \delta\Delta\theta]}$ and $\tau_2^* = \frac{\theta_L}{\theta_H}$. From definition of τ_r,

$$\begin{aligned}\bar{q}_r(\tau_r)\tau_r\left(\theta_H + \delta\tilde{V}_{r-1}(1)\right) + (1 - \bar{q}_r(\tau_r)\tau_r)\,\delta\tilde{V}_{r-1}(\tau_{r-1}) = \\ q_r^*(\tau_r)\tau_r\left(\theta_H + \delta\tilde{V}_{r-1}(1) - \delta^{r-1}\Delta\theta\right) + (1 - q_r^*(\tau_r)\tau_r)\,\delta\tilde{V}_{r-1}\left(\tau_{r-1}^*\right).\end{aligned} \tag{19}$$

The limit of the LHS at (19) for $\tau_3 \to 1$ is $\theta_H + \delta\tilde{V}_2(1)$ and the one for the RHS is equal to $\theta_H + \delta\tilde{V}_2(1) - \delta^2\Delta\theta$, which is lower than the LHS. On the other hand, the limit for $\tau_3 \to \tau_2$ is $\delta\tilde{V}_2(\tau_2)$ for LHS and $q_3^*(\tau_2)\tau_2\left(\theta_H + \delta\tilde{V}_2(1) - \delta^2\Delta\theta\right) + (1 - q_3^*(\tau_2)\tau_2)\,\delta\tilde{V}_2(\tau_2^*)$ for RHS. From solutions for the two period case we know that,

$$\tilde{V}_2(\tau_2) = \tau_2\theta_H + \delta\theta_L,$$
$$\tilde{V}_2(1) = \theta_H + \delta\theta_H,$$
$$\tilde{V}_2(\tau_2^*) = \theta_L + \delta\theta_L,$$

so we get that the limit for RHS is larger than the one to the LHS and equal to $\tau_2\theta_H + \delta\theta_L + \delta^2\theta_L$. Since LHS and RHS are both continuous, then there exists at least one point such that they are equal. The derivatives of the LHS and RHS w.r.t. τ_r are constant then, the solution of (19) for τ_3 must be unique.

For $r > 3$, assume that the solution of (19) for τ_{r-1} exists and it is unique.

Taking the limit for the LHS at (19) for $\tau_r \to 1$, we find that it is equal to $\theta_H + \delta\tilde{V}_{r-1}(1)$, and the one for RHS is $\theta_H + \delta\tilde{V}_{r-1}(1) - \delta^{r-1}\Delta\theta$. Notice, that the limit for the LHS is larger than the one for RHS.

On the other hand, taking the limit of the LHS at (19) for $\tau_r \to \tau_{r-1}$, we get $\delta\tilde{V}_{r-1}(\tau_{r-1})$. For the RHS we get $q_r^*(\tau_{r-1})\tau_{r-1}\left(\theta_H + \delta\tilde{V}_{r-1}(1) - \delta^{r-1}\Delta\theta\right) + (1 - q_r^*(\tau_{r-1})\tau_{r-1})\,\delta\tilde{V}_{r-1}\left(\tau_{r-1}^*\right)$, which follows the definition of $\tilde{V}_r(p)$ for $p \in [\tau_r^*, \tau_r)$ when $p = \tau_{r-1}$, i.e. $\tilde{V}_r(\tau_{r-1})$. As $\tilde{V}_r(p)$ is increasing in r, the limit for $\tau_r \to \tau_{r-1}$ of the LHS is now lower than the limit of the RHS.

Since LHS and RHS are both continuous, then there exists at least one point such that they are equal. The derivatives of the LHS and RHS w.r.t. τ_r are constant then, the solution of (19) for τ_r must be unique. ∎

5.3 Proof of Lemma 4

Proof. Suppose $p \geq \tau_r$. Definitions of continuation values for this range of beliefs

$$\begin{aligned}\tilde{V}_r(p) &= \bar{q}_r(p)p\left(\theta_H + \delta\tilde{V}_{r-1}(1)\right) + (1 - \bar{q}_r(p)p)\,\delta\tilde{V}_{r-1}(\tau_{r-1}),\\ \tilde{U}_r(p) &= (1 - \bar{q}_r(p)p)\,\delta\tilde{U}_{r-1}(\tau_{r-1}).\end{aligned}$$

Applying the functional form to $\tilde{V}_{r-1}(1)$, $\tilde{V}_{r-1}(\tau_{r-1})$ and $\tilde{U}_{r-1}(\tau_{r-1})$,

$$\tilde{V}_{r-1}(1) = \theta_H\sum_{i=0}^{r-2}\delta^i,$$
$$\tilde{V}_{r-1}(\tau_{r-1}) = \tau_{r-1}\theta_H\sum_{i=0}^{r-3}\delta^i\bar{q}_{r-1-i}(\tau_{r-1}) + \delta^{r-2}\tau_{r-1}\theta_H,$$
$$\tilde{U}_{r-1}(\tau_{r-1}) = 0.$$

Plugging them into $\tilde{V}_r(p)$ and $\tilde{U}_r(p)$, and after some operations,

$$\tilde{V}_r(p) = \bar{q}_r(p)p\theta_H + \bar{q}_r(p)p_{t-1}\theta_H\sum_{i=1}^{r-2}\delta^i + (1 - \bar{q}_r(p)p)\,\tau_{r-1}\theta_H\sum_{i=1}^{r-2}\delta^i\bar{q}_{r-i}(\tau_{r-1}) + \delta^{r-1}p\theta_H.$$
$$\tilde{U}_r(p) = 0.$$

For $\tilde{V}_r(p)$, since $\bar{q}_r(p)p + (1 - \bar{q}_r(p)p)\,\tau_{r-1}\bar{q}_{r-i}(\tau_{r-1}) = p\bar{q}_{r-i}(p)$, then we can write it as

$$\tilde{V}_r(p) = \bar{q}_r(p)p\theta_H + p\theta_H\sum_{i=1}^{r-2}\delta^i\bar{q}_{r-i}(p) + \delta^{r-1}p\theta_H.$$

Both, $\tilde{V}_r(p)$ and $\tilde{U}_r(p)$, follow the functional form for $p \geq \tau_r$, with $\Omega_r(p) = \varnothing$.

Suppose $p \in [\tau_r^*, \tau_r)$. Now, from definitions of continuation values,

$$\begin{aligned}
\tilde{V}_r(p) &= q_r^*(p)p\left(\theta_H + \delta\tilde{V}_{r-1}(1)\right) + (1 - q_r^*(p)p)\,\delta\tilde{V}_{r-1}\left(\tau_{r-1}^*\right) - pq_r^*(p)\delta^{r-1}\Delta\theta, \\
\tilde{U}_r(p) &= (1 - q_r^*(p)p)\,\delta\tilde{U}_{r-1}\left(\tau_{r-1}^*\right) + \delta^{r-1}\Delta\theta.
\end{aligned}$$

Again, applying the functional form to $\tilde{V}_{r-1}(1)$, $\tilde{V}_{r-1}\left(\tau_{r-1}^*\right)$ and and $\tilde{U}_{r-1}\left(\tau_{r-1}^*\right)$

$$\begin{aligned}
&\tilde{V}_{r-1}(1) = \theta_H \sum_{i=0}^{r-2} \delta^i, \\
&\tilde{V}_{r-1}\left(\tau_{r-1}^*\right) = \tau_{r-1}^*\theta_H \sum_{i=0}^{r-3} \delta^i q_{r-1-i}^*(\tau_{r-1}^*) + \delta^{r-2}\theta_L, \\
&\tilde{U}_{r-1}\left(\tau_{r-1}^*\right) = \delta^{r-2}\Delta\theta.
\end{aligned}$$

Plugging them into $\tilde{V}_r(p)$ and $\tilde{U}_r(p)$, and using $q_r^*(p)p + (1 - q_r^*(p)p)\,\tau_{r-1}^* q_{r-i}^*(\tau_{r-1}^*) = pq_{r-i}^*(p)$, we get,

$$\begin{aligned}
&\tilde{V}_r(p) = q_r^*(p)p\theta_H + p\theta_H \sum_{i=1}^{r-2} \delta^i q_{r-i}^*(p) + \delta^{r-1}\theta_L, \\
&\tilde{U}_r(p) = \delta^{r-1}\Delta\theta,
\end{aligned}$$

following the functional forms of continuation values for $p \in [\tau_r^*, \tau_r)$, again with with $\Omega_r(p) = \varnothing$.

Finally, suppose $p < \tau_r^*$. From definitions of continuation values,

$$\begin{aligned}
\tilde{V}_r(p) &= \theta_L + \delta\tilde{V}_{r-1}(p), \\
\tilde{U}_r(p) &= \theta_L + \delta\tilde{U}_{r-1}(p).
\end{aligned}$$

Applying the functional form to $\tilde{V}_{r-1}(p)$ and $\tilde{U}_{r-1}(p)$,

$$\begin{aligned}
&\tilde{V}_{r-1}(p) = \theta_L \sum_{i\in\Omega_{r-1}(p)} \delta^i + p\theta_H \sum_{i\in\bar{\Omega}_{r-1}(p)} \hat{q}_{r-1-i}(p)\delta^i + \delta^{r-2}\theta_L, \\
&\tilde{U}_{r-1}(p) = \Delta\theta \sum_{i\in\Omega_{r-1}(p)} \delta^i + \delta^{r-2}\Delta\theta,
\end{aligned}$$

and plugging them into $\tilde{V}_r(p)$ and $\tilde{U}_r(p)$,

$$\begin{aligned}
\tilde{V}_r(p) &= \theta_L \sum_{i\in\Omega_r(p)} \delta^i + p\theta_H \sum_{i\in\bar{\Omega}_r(p)} \hat{q}_{r-i}(p)\delta^i + \delta^{r-1}\theta_L, \\
\tilde{U}_r(p) &= \Delta\theta \sum_{i\in\Omega_r(p)} \delta^i + \delta^{r-1}\Delta\theta,
\end{aligned}$$

following the functional form of continuation values for $p < \tau_r^*$. ■

5.4 Proof of Lemma 5

Proof. We first show that $\tau_r^* = \tau_{r-1}\forall r \geq 2$, when $\delta = 1$.

We proceed by induction. The result is direct for $r = 2$ since by definition $\tau_2^* = \frac{\theta_L}{\theta_H}$ and $\tau_1 = \frac{\theta_L}{\theta_H}$. It follows that $q_3^*(p) = \bar{q}_2(p)$ by their definition.

For $r > 2$, suppose $\tau_{r-i}^* = \tau_{r-1-i}\ \forall i \in \{1, ..., r-2\}$, then, from their definitions it must be $q_{r+1-i}^*(p) = \bar{q}_{r-i}(p)$. Additionally, from definitions of τ_r and $\tilde{V}_r(p)$, applying Lemma 4 and after some simplifications, we get

$$\tau_{r-1}\theta_H \sum_{i=0}^{r-3} \delta^i \left[\bar{q}_{r-1-i}(\tau_{r-1}) - q^*_{r-1-i}(\tau_{r-1})\right] = \delta^{r-2}\theta_L - \delta^{r-2}\tau_{r-1}\theta_H. \tag{20}$$

This expression, when $\delta = 1$, and using that $q^*_{r+1-i}(p) = \bar{q}_{r-i}(p)$ (due to $\tau^*_{r-i} = \tau_{r-1-i}$ by assumption) becomes $\tau_{r-1}\bar{q}_{r-1}(\tau_{r-1}) = \frac{\theta_L}{\theta_H}$. From Lemma 2, $\tau^*_r q^*_r(\tau^*_r) = \frac{\theta_L}{\theta_H}$, and since $q^*_r(p) = \bar{q}_{r-1}(p)$ (due to $\tau^*_{r-1} = \tau_{r-2}$ for $i = 1$ by assumption), it follows that $\tau^*_r = \tau_{r-1}$.

Now, let's consider the case $\delta \to 1$.

Again, we proceed by induction. For $r = 2$, $\tau^*_2 = \tau_1 = \frac{\theta_L}{\theta_H}$ and $\tau_0 = 0$ from initial conditions. For $r = 3$, $\tau^*_3 = \frac{\theta_L}{\theta_H}\left(1 + \frac{\Delta\theta}{\theta_H}\right)$ from Lemma 2 and $\tau_2 = \frac{\theta_L[\theta_H + \delta\Delta\theta]}{\theta_H[\theta_L + \delta\Delta\theta]}$ from initial conditions. Value of τ_2 is larger than τ^*_3 for $\delta < 1$. It follows that $q^*_4(p) > \bar{q}_3(p)$ by their definition.

For $r > 2$, we first show that $\frac{\partial\tau_{r-1}}{\partial\delta} < 0$. Suppose $\tau^*_{r-i} < \tau_{r-1-i}$ $\forall i \in \{1, ..., r-2\}$, then $q^*_{r-1-i}(p) > \bar{q}_{r-2-i}(p)$ from their definitions. Let's also assume that $\frac{\partial\tau_{r-1-i}}{\partial\delta} < 0$ $\forall i \in \{1, ..., r-2\}$.

Expression (20) can be written as

$$\tau_{r-1}\theta_H \left(1 - \sum_{i=0}^{r-3} \delta^{i-r+2} \left[q^*_{r-1-i}(\tau_{r-1}) - \bar{q}_{r-1-i}(\tau_{r-1})\right]\right) - \theta_L = 0.$$

LHS is a function of δ, τ_{r-1} and τ_{r-1-i}.[15] Let's call it $F(\delta, \tau_{r-1}, \tau_{r-1-i})$, and let's apply the implicit function theorem, i.e.

$$\frac{\partial\tau_{r-1}}{\partial\delta} = \frac{-\frac{\partial F}{\partial\delta} - \frac{\partial F}{\partial\tau_{r-2-i}}\frac{\partial\tau_{r-2-i}}{\partial\delta}}{\frac{\partial F}{\partial\tau_{r-1}}}.$$

As $(i - r + 2) < 0$ $\forall i \in \{0, ..., r-3\}$ then $\frac{\partial F}{\partial\delta} > 0$. Also, $\frac{\partial F}{\partial\tau_{r-2-i}} < 0$ (due to $\frac{\partial\bar{q}_{r-1-i}(\tau_{r-1})}{\partial\tau_{r-2-i}} < 0$) and, since $\frac{\partial\tau_{r-2-i}}{\partial\delta} < 0$ by assumption, then the numerator is negative. On the other hand, $\frac{\partial F}{\partial\tau_{r-1}} > 0$ (i.e. the denominator is positive) because, first

$$\left(1 - \sum_{i=0}^{r-3} \delta^{i-r+2} \left[q^*_{r-1-i}(\tau_{r-1}) - \bar{q}_{r-1-i}(\tau_{r-1})\right]\right)$$

has to be positive to have (20) equal to zero (τ_{r-1} for $r > 2$, θ_H, and θ_L are all positive) and, second

$$\frac{\partial q^*_{r-1-i}(\tau_{r-1})}{\partial\tau_{r-1}} - \frac{\partial\bar{q}_{r-1-i}(\tau_{r-1})}{\partial\tau_{r-1}} < 0,$$

by definitions of q^*_r and $\bar{q}_r$, and using the assumption $\tau^*_{r-1-i} < \tau_{r-2-i}$ and that $\tau^*_{r-2-i} < \tau^*_{r-1-i}$ from Lemma 2. It follows that $\frac{\partial\tau_{r-1}}{\partial\delta} < 0$.

As $\tau^*_r = \tau_{r-1}$ when $\delta = 1$, then $\tau^*_r \in (\tau_{r-2}, \tau_{r-1})$ when $\delta \to 1$ by continuity. ■

5.5 Proof of Lemma 6

Proof. Since $p(h) \geq p \geq p(l)$, and since $\Omega_r(p)$ is increasing in p by definition, it follows that $|\Omega_{r-1}(p(h))| \leq |\Omega_{r-1}(p(l))|$.

When $q_L \neq 0$, the SMC_t is binding and as consequence $x_r(l) = 1 + \delta\frac{\tilde{U}_{r-1}(p(h))}{\Delta\theta} - \delta\frac{\tilde{U}_{r-1}(p(l))}{\Delta\theta}$.

Using the functional forms for continuation values,

$$x_t(l) = 1 - \textstyle\sum_{i\in\Omega_{r-1}(p(l))\setminus\Omega_{r-1}(p(h))} \delta^i - \delta^{r-1}\mathbf{I}_{(p(l),p(h)]}(\tau_{r-1}).$$

[15] Although we do not write it explicitely, τ_{r-1} and τ_{r-1-i} depends on δ. By definition, $\bar{q}_{r-i}(\tau_{r-1})$ depends on $\tau_{r-1-i}(\delta)$. On the other hand, $q^*_{r-1-i}(\tau_{r-1})$ depends on τ^*_{r-1-i} which does not change with δ.

In order to keep $x_r(l) \geq 0$, $\Omega_{r-1}(p\,(l))\backslash\Omega_{r-1}(p\,(h)) = \emptyset$ when $\mathbf{I}_{(p(l),p(h)]}(\tau_{r-1})$ is equal 1, and at most 1 when $\mathbf{I}_{(p(l),p(h)]}(\tau_{r-1})$ is equal 0. Then, $|\Omega_{r-1}(p\,(l))\backslash\Omega_{r-1}(p\,(h))| \leq 1$.

When $q_L = 0$, the allocation for low type message can also be $x_r(l) = 0$ ($\rho_H > \frac{\theta_L}{\theta_H}$) or $x_r(l) = 1$ ($\rho_H < \frac{\theta_L}{\theta_H}$). Under $x_r(l) = 0$ ($x_r(l) = 1$) it must be that $|\Omega_{r-1}(p\,(l))\backslash\Omega_{r-1}(p\,(h))| \leq 1$ ($|\Omega_{r-1}(p\,(h))| = |\Omega_{r-1}(p\,(l))|$), otherwise the difference between the continuation values for each message violates the SMC_r. To restore the SMC_r and make it binding, $x_r(l) = 1 + \delta\frac{\tilde{U}_{r-1}(p(h))}{\Delta\theta} - \delta\frac{\tilde{U}_{r-1}(p(l))}{\Delta\theta}$ which is the case explained above.

To see that $\Omega_{r-1}(p\,(l))\backslash\Omega_{r-1}(p\,(h)) = \max i \in \Omega_{r-1}(p\,(l))$ when $|\Omega_{r-1}(p\,(l))\backslash\Omega_{r-1}(p\,(h))| = 1$, let $\Omega_{r-1}(p\,(l)) = \{0, 1, ..., j\}$, $\Omega_{r-1}(p\,(h)) = \{0, 1, ..., k\}$ with $k \leq j$ for $j, k \in \{0, 1, ..., r-2\}$. Then, it must be that $k = j - 1$ and $\Omega_{r-1}(p\,(l))\backslash\Omega_{r-1}(p\,(h)) = j$. Otherwise $|\Omega_{r-1}(p\,(l))\backslash\Omega_{r-1}(p\,(h))| > 1$. ■

5.6 Proof of Lemma 7

Proof. The proof is by application of Lemma 4 for each case.

When $\Omega_{r-1}(p\,(l)) = \Omega_{r-1}(p\,(h))$, *learning* is possible only if $p\,(h) \geq \tau_{r-1}$ and $p\,(l) \in [\tau^*_{r-1}, \tau_{r-1})$. If $p\,(h) \geq \tau_{r-1}$ and $p\,(l) \geq \tau_{r-1}$ we are in *no-learning*. If $p\,(h) < \tau_{r-1}$ either $\Omega_{r-1}(p\,(l)) \neq \Omega_{r-1}(p\,(h))$ (contradiction) or $\Omega_{r-1}(p\,(l)) = \Omega_{r-1}(p\,(h))$ with $\tilde{U}_{r-1}\,(p(l)) = \tilde{U}_{r-1}\,(p(h))$ and we are in *no-learning* again.

When $\Omega_{r-1}(p\,(h)) = \Omega_{r-1}(p\,(l))\backslash \max\{i \in \Omega_{r-1}(p\,(l))\}$ (i.e. $|\Omega_{r-1}(p\,(l))| - |\Omega_{r-1}(p\,(h))| = 1$), we have $\tilde{U}_{r-1}\,(p(l)) > \tilde{U}_{r-1}\,(p(h))$. Since $p\,(h) \geq p \geq p\,(l)$ and $|\Omega_{r-1}\,(p\,(l))| - |\Omega_{r-1}\,(p\,(h))| = 1$, the set $\Omega_{r-1}(p)$ must equal to $\Omega_{r-1}\,(p\,(h))$ or to $\Omega_{r-1}\,(p\,(l))$. In both cases, it must be $p\,(h) < \tau_{r-1}$ and $p\,(l) < \tau_{r-1}$. Otherwise, since $\delta \in (\delta^*(T), 1)$, the SMC_r does not hold for any $x_r(l) \in [0, 1]$.

If $|\Omega_{r-1}(p\,(l))| - |\Omega_{r-1}(p\,(h))| > 0$, then $\tilde{U}_{r-1}\,(p(l)) - \tilde{U}_{r-1}\,(p(h)) \neq 0$. Hence, in order to have *no-learning*, it must be that $\Omega_{r-1}(p\,(l)) = \Omega_{r-1}(p\,(h))$. Additionally, it must be either $p\,(h)\,, p\,(l) \in [\tau_{r-1}, 1]$, or $p\,(h)\,, p\,(l) \in [0, \tau_{r-1})$. Otherwise, $\tilde{U}_{r-1}\,(p(l)) - \tilde{U}_{r-1}\,(p(h)) \neq 0$. ■

5.7 Proof of Corollary 1

Proof. Consider a message set M_r with two possible messages $\{"take-it", "leave-it"\}$, a mechanism with an allocation given by

$$x_r(m_r) = \begin{cases} 1 & if \quad m_r = take-it, \\ 0 & if \quad m_r = leave-it, \end{cases} \quad , \; m_r \in M_r,$$

probabilities of observing each message defined by

$$\hat{q}_i(take-it) \equiv q_i x_r(h) + (1-q_i)x_r(l),$$
$$\hat{q}_i(leave-it) \equiv 1 - \hat{q}_i(take-it),$$

and the posteriors of facing a high-type buyer when observing "$take-it$", $\hat{p}(take-it)$, and the one when observing "$leave-it$", $\hat{p}(leave-it)$, are given by Baye's rule.

When $p < \tau^*_r$ the optimal direct selling mechanism has allocations $x_r(h) = x_r(l) = 1$, then, by definition, $\hat{q}_H(take-it) = 1$, $\hat{q}_L(take-it) = 1$ and $\hat{p}(take-it) = p$. It follows that continuation values with the price posting are equal than under the direct mechanisms, i.e. $U_{i,r-1}(\hat{p}(take-it)) = U_{i,r-1}(p(h))$ for both types and $V_{r-1}(\hat{p}(take-it)) = V_{r-1}(p(h))$. Using a price $\hat{w}_r(take-it) = \theta_L$, also instant payoffs under both mechanisms are equal for every player.

When $p \geq \tau^*_r$ the optimal direct selling mechanism has payments $w_r(h) = \theta_H$ and $w_r(l) = 0$, or $w_r(h) = \theta_H - \delta^{r-1}\Delta\theta$ and $w_r(l) = 0$, with allocations $x_r(h) = 1$ and $x_r(l) = 0$. It follows that, $\hat{q}_H(take-it) = q_H$ and $\hat{q}_L(take-it) = q_L$ and $\hat{p}(take-it) = p(h)$ and $\hat{p}(leave-it) = p(l)$. Again, continuation values are equal for both mechanisms, i.e. $U_{i,r-1}(\hat{p}(take-it)) = U_{i,r-1}(p(h))$, $U_{i,r-1}(\hat{p}(leave-it)) = U_{i,r-1}(p(l))$, $V_{r-1}(\hat{p}(take-it)) = V_{r-1}(p(h))$ and $V_{r-1}(\hat{p}(leave-it)) = V_{r-1}(p(l))$. Using $\hat{w}_r(take-it) = w_r(h)$, also instant payoffs under both mechanisms are equal for every player.

Then, for every prior, it is possible to implement an outcome $(\hat{q}_r, \hat{p}_r, \hat{\Gamma}_r)$, where $\hat{\Gamma}_r$ is a price posting mechanism, which is payoff equivalent to the incentive efficient outcome (q_r, p_r, Γ_r) that solves (6)

■

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