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Optimal Fiscal Policy and Private Sector Borrowing Constraints

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Abstract

When the transmission channel between savers and borrowing firms is disturbed, firms may find themselves borrowing-constrained. I study the optimal fiscal policy response to a tightening borrowing constraint in a simple two-period model. I find that it is not optimal to subsidize firms, although this would relax the constraint and help firms directly. Instead, the optimal response exploits the distortion caused by the borrowing constraint and reduces existing tax distortions. This result is robust to when endogenous government spending and investment are part of the government's set of instruments.

JEL class: E62, H21

Keywords: Optimal fiscal policy, borrowing constraints.

1   Introduction

In this paper, I consider a scenario in which producing firms want to borrow more funds in order to increase production and households are willing to lend additional funds. Yet, they cannot: the transmission channel between savers and borrowers is disturbed. For example, such a situation may be the result of a credit crunch during a financial recession where banks are uncertain about the future and stop lending. Usually, it does not take much time until the public calls for government subsidies or asks the government to step in and pick up the drop in demand in the form of higher government consumption or investment. In such a scenario, is it the optimal fiscal policy response to relax the borrowing constraint by giving firms a subsidy? I set up a simple two-period model and investigate the optimal policy response. First, I answer this question for a given amount of government expenditures. Second, I let government consumption and investment be endogenous and be part of the optimal policy response.

The model lasts for two periods and features a representative household that can save in government bonds and can lend to a representative firm. The firm, facing a borrowing constraint, borrows from the household in the first period to build capital which produces the output good in the second period. The exogenous borrowing constraint is ad hoc and represents a disruption in the financing channel from private savers (households) to borrowers (firms). In addition to issuing public debt, the government levies distortionary labor taxes and savings income taxes on the household. This tax revenue enables the government to finance exogenously fixed government spending and a subsidy transfer to the firm. The possibility of the subsidy allows the government to address the borrowing constraint directly by relaxing the firm's borrowing constraint. Modeled as a lump-sum transfer, the transfer takes the simplest form and imposes no additional distortions.

In a later extension, I allow for endogenous government consumption and investment. Government consumption enters the household's utility function and public investment raises the productivity of private capital. This extension allows addressing whether the government should step in by increasing public spending.[1] Then, the optimal policy of the government does not only consist of setting tax rates in order to finance a given path of government consumption but also of choosing and adjusting the path of government spending.

I study the impact of a tightening borrowing constraint on the Ramsey policy and allocation. I find that if the government does not change its policy, private capital and thus production and consumption are depressed in the second period, as firms are forced to borrow less and build less capital. The household saves less and thus shifts her consumption profile by consuming relatively more in the first period. In equilibrium, the interest rate falls.[2] At the same time, the marginal product of capital increases and a wedge between the

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[1]For example, it is often thought that productive government investment would not only have short-lived demand effects but also long-term benefits via an improvement in the public capital stock. Aschauer (1989) finds that public capital has a significant impact on the productivity of private capital. The International Monetary Fund (2014) suggests that in times of low growth, infrastructure projects would pay for itself in countries with low borrowing rates and public investment needs.

[2]The described scenario could also reflect a savings glut that has been discussed extensively recently.

cost of borrowing and the return to capital appears. This wedge reflects the disruption of the transmission channel: Borrowers and lenders would like to borrow and lend at a slightly higher rate, but cannot because of the borrowing constraint. Without the restriction of the borrowing constraint, firms could increase their profits and households would receive a higher return on their savings.

How should the government respond to a tightening of the borrowing constraint? It seems plausible that the optimal policy involves addressing the distortion caused by the tighter borrowing constraint: By paying a subsidy to the firm, the government could relax or even undo the borrowing constraint such that it is slack. However, I find that increasing subsidies is not optimal. Rather, it is optimal to decrease distortionary labor taxes and increase the tax on the household's return to savings.

The reasoning is as follows. The Ramsey planner can exploit the distortion of the borrowing constraint and the resulting wedge. The Ramsey policy implements a higher savings income tax rate which leads to an increase in the borrowing rate for capital and closes the wedge. Importantly, the increase in the savings tax has no distortionary effect. Using this new revenue, the government could pay out a subsidy to relax the borrowing constraint. However, I show that it is optimal to decrease distortionary labor taxes instead. Only when the borrowing constraint is relatively tight and private capital is sufficiently suppressed it is optimal to pay out a subsidy and address the distortion caused by the borrowing constraint. However, it is never optimal to fully relax the borrowing constraint.

This example provides a more general insight. If a new distortion, here in form of the borrowing constraint, affects the economy, the existence of other distortions matters for the optimal policy response. The government could loosen the borrowing constraint, but the government would have to levy additional distortionary taxes which would worsen the existing tax distortion.

In the extension where government consumption and investment is endogenous, the same mechanism takes place. Until the borrowing constraint is not sufficiently tight, it is not optimal to pay out a subsidy. Instead, the Ramsey policy again involves lower labor taxes and higher savings taxes. The optimal policy also leads to higher government consumption and government investment in the first period.

There are a few other papers that investigate borrowing constraints and optimal policy. Azzimonti and Yared (2017) show in a model with heterogeneous agents and lump-sum taxes that it is optimal to keep some households borrowing-constrained. This enables the government to decrease consumption inequality between agents. Similarly, in Bhandari, Evans, Golosov, and Sargent (2017), welfare can be higher in an equilibrium with borrowing-constrained households. Then the government is the sole supplier of lending and can extract monopoly rent by issuing debt at a lower interest rate. In Yared (2013), building on the Diamond and Dybvig (1983) framework, agents can invest in short- and long-term assets and are subject to liquidity shocks. Private debt is not enforceable, contrary to public debt which the government can pay back by enforcing lump-sum taxes.[3] Accordingly, government

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Real interest rates are historically low even though the return has not diminished much, see e.g. Gomme, Ravikumar, and Rupert (2015). One primary policy advice is usually that the government should take advantage of low interest rates to borrow and invest.

[3]There is a rich literature where public debt provides liquidity and relaxes private borrowing constraints,

debt can undo borrowing limits of the agents who are hit by liquidity shocks. However, fully relaxing the borrowing limit would lead to a sub-optimal investment in the short-term asset. In these three papers, households trade with each other because they have different exogenously given productivities or are hit by liquidity shocks (in Yared, 2013). Government policy then determines if and how individual households are borrowing-constrained. In contrast, I study how the extent of an exogenously given borrowing constraint affects optimal fiscal policy when taxes are distortionary and households are alike.

Biljanovska (2017) investigates optimal policy in a collateral-constrained economy. The inefficiencies caused by the collateral constraint can be undone by corrective (distortionary) and lump-sum taxes that can deliver the first-best allocation. The government does not levy taxes to finance an exogenously given amount of spending, but only to affect the collateral constraint and to redistribute. In response to a tightening of the collateral constraint, it is optimal to decrease capital income and payroll taxes to stimulate production. The fact that distortionary taxes are only present to undo the distortion of the collateral constraint is the main difference compared to my set up, where the borrowing constraint helps to mitigate *existing* tax distortions. Finally, in contrast to the mentioned papers, I study an extension where government spending and investment is endogenous.

In terms of methodology, this paper connects to the literature of optimal taxation (the Ramsey plan) where a government needs to finance expenditures with distortionary taxes (e.g. Lucas and Stokey, 1983; Chari, Christiano, and Kehoe, 1994). The extension in Section 5 also connects to the literature on fiscal policy where government spending and investment is endogenous. Government expenditures increase the productivity of private capital and can, therefore, drive growth (see, e.g., Barro, 1990; Turnovsky, 1997).

The paper is organized as follows. Section 2 introduces the model and defines the competitive equilibrium. Section 3 sets up the Ramsey planner's problem and characterizes optimal policy. In Section 4, I discuss the effect of a tightening borrowing constraint on the Ramsey policy. Section 5 presents the extension with endogenous government expenditures. Section 6 concludes.

2 Model and equilibrium

2.1 Model overview

Time is discrete and the economy lasts two periods, $t = 0, 1$. There are three agents: a representative household, a representative firm and a government.

A. Household sector

The representative household has preferences over consumption c_t and leisure x_t:

$$u(c_0, x_0) + \beta u(c_1, x_1).$$

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e.g. Woodford (1990), Aiyagari and McGrattan (1998) and Holmstrom and Tirole (1998).

Her budget constraints are given by

$$t = 0: \quad c_0 + b_1 + a_1 = (1 - \tau_0^w)(1 - x_0)w_0, \tag{1}$$

$$t = 1: \quad c_1 = (1 - \tau_1^w)(1 - x_1)w_1 + (1 - \tau_1^b)R_1^b b_1 + (1 - \tau_1^a)R_1^a a_1 + \pi_1. \tag{2}$$

The household is endowed with one unit of time and therefore supplies labor $1 - x_t$. In period 0, the wage (or productivity) is exogenous and given by w_0.[4] In the second period, w_1 is determined endogenously. The labor tax rate at time t is τ_t^w. The household can buy government bonds b_1 that yield a gross return R_1^b and lend a_1 to private firms at a gross return R_1^a. The tax rates on her savings income are given by τ_1^b and τ_1^a, respectively. Finally, the household receives the firm profits π_1 in the form of a lump-sum transfer.[5]

The household's optimality conditions are

$$u_c(0) = \beta u_c(1)R_1^b(1 - \tau_1^b), \tag{3}$$

$$u_c(0) = \beta u_c(1)R_1^a(1 - \tau_1^a), \tag{4}$$

$$u_x(0)/u_c(0) = (1 - \tau_0^w)w_0, \tag{5}$$

$$u_x(1)/u_c(1) = (1 - \tau_1^w)w_1. \tag{6}$$

The two Euler equations (3) and (4) give rise to a no-arbitrage condition between the net returns of lending to the firm and the government, respectively: $(1 - \tau_1^b)R_1^b = (1 - \tau_1^a)R_1^a$. Equations (5) and (6) are the standard labor supply equations that reflect the intratemporal tradeoff between consumption and leisure.

B. Firm sector

The representative firm has an initial endowment ω_0. In period $t = 0$, it can borrow a_1 from households to form private capital k_1. In addition the firm may receive a subsidy $s_0 \geq 0$ from the government:

$$k_1 = \omega_0 + a_1 + s_0. \tag{7}$$

In period $t = 1$ the firm maximizes profits π_1 with a Cobb-Douglas technology that uses k_1 and labor L_1 as inputs:

$$f(k_1, L_1) = A k_1^\alpha L_1^\zeta.$$

The parameters α and ζ are the output elasticities of capital and labor, respectively.[6] The firm hires labor L_1 at wage w_1 and pays back the loan a_1 with interest R_1^a. Its profits are

$$\pi_1 = f(k_1, L_1) - R_1^a a_1 - w_1 L_1 + (1 - \delta)k_1, \tag{8}$$

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[4]I abstract from firm production in period 0 in order to keep the problem as simple as possible. I assume that labor productivity is predetermined by actions in a previous period and is hence fixed in $t = 0$ when the borrowing constraint tightens. This simplification ensures a partial closed-form solution.

[5]There is no lump-sum tax on the household. Doing so would make the problem trivial. Following a tightening borrowing constraint, the government could increase the household lump-sum tax and finance a higher subsidy payment, leaving the allocation the same. The household would save less, however, she would receive more profit payments under the optimal policy. See Appendix C.1 for more details.

[6]Whether the production function features increasing, constant or decreasing returns to scale plays no role.

where δ is the depreciation rate on capital. Maximizing profits, the firm chooses borrowing a_1, capital k_1 and labor L_1. Importantly, the borrowing choice of the firm a_1 may be limited by a borrowing constraint $a_1 \leq \bar{a}$. Replacing $a_1 = k_1 - \omega_0 - s_0$ the problem of the firm is

$$\max_{k_1, L_1} f(k_1, L_1) - R_1^a(k_1 - \omega_0 - s_0) - w_1 L_1 + (1-\delta)k_1$$

$$\text{s.t. } k_1 - \omega_0 - s_0 \leq \bar{a}.$$

First order conditions are

$$f_L(k_1, L_1) = w_1, \tag{9}$$

$$f_k(k_1, L_1) + 1 - \delta = R_1^a + \theta_0^f, \tag{10}$$

where $\theta_0^f \geq 0$ is the multiplier associated with the borrowing constraint of the firm. The accompanying complementary slackness condition is

$$\theta_0^f[k_1 - \omega_0 + s_0 - \bar{a}] = 0, \ \theta_0^f \geq 0. \tag{11}$$

Equations (9) and (10) reflect the firm's demand for labor and capital, respectively. While the former is standard, the latter reflects the role of the borrowing constraint. When the constraint is slack, we have $\theta_0^f = 0$ and $f_k + 1 - \delta = R_1^a$: The marginal product of capital equals the cost of borrowing. When the constraint is binding, we have $k_1 = \omega_0 + s_0 + \bar{a}$ and $\theta_0^f > 0$. Then, the constraint is costly and $R_1^a < f_k + 1 - \delta$: The firm would like to borrow more as this would increase its profits. There is a wedge between the firm's borrowing cost and the marginal product of capital.

Remark (borrowing constraint). The ad hoc borrowing constraint can be interpreted in various ways. For example, the borrowing constraint could be a result of limited enforcement by the government as in Bhandari et al. (2017). The government imposes an arbitrarily high punishment on agents if they default on any debt *less* than $\bar{a}$, but does not enforce any contracts where debt exceeds $\bar{a}$. Alternatively, the borrowing constraint could be a result of limited commitment as in Itskhoki and Moll (2014). Here, the firm can default and keep a fraction $0 < \frac{1}{\rho} < 1$ of borrowing a_1 but would lose its endowment. This would lead to a borrowing constraint of $a_1 \leq \rho\omega_0$ and ρ then determines the maximum leverage ratio.

C. Government sector

The government issues debt b_1, levies labor income at tax rates τ_t^w, savings incomes at tax rates τ_1^a and τ_1^b, pays a transfer s_0 to firms and purchases government consumption g_0, g_1. It has an endowment ω_0^g. Its budget constraints are

$$t = 0 : \ g_0 + s_0 = \tau_0^w(1-x_0)w_0 + b_1 + \omega_0^g,$$
$$t = 1 : \ g_1 + R_1^b b_1 = \tau_1^w(1-x_1)w_1 + \tau_1^a R_1^a a_1 + \tau_1^b R_1^b b_1.$$

The government follows a feasible policy $\mu = \{\tau_0^w, \tau_1^w, \tau_1^a, \tau_1^b, b_1, g_0, g_1, s_0\}$ that satisfies its budget constraints. Note that conditional on prices and allocation, not all elements of μ can be set independently of each other in order to fulfill all equilibrium conditions.

Remark (other taxes). The savings income tax and the labor tax allow the Ramsey planner to affect the intertemporal and the intratemporal margin, respectively. Neither a

consumption tax in period 1 nor a profit tax changes the result qualitatively (a consumption tax in period 0 is redundant for the Ramsey policy). Whether the savings income tax is paid by the household or the firm makes no difference. See Appendix C.1 for more details.

D. Market Clearing

In equilibrium, labor market clearing implies

$$L_1 = 1 - x_1. \tag{12}$$

Combining the various dynamic budget constraints yields the following resource constraints:

$$c_0 + g_0 + k_1 = (1 - x_0)w_0 + \omega_0 + \omega_0^g, \tag{13}$$

$$c_1 + g_1 = f(k_1, 1 - x_1) + (1 - \delta)k_1. \tag{14}$$

In the first period, firm and government endowments and labor income are used for private and government consumption, and investment. In the second period, production and left-over capital equal household and public consumption.

2.2 Equilibrium

Equations (1)-(14) characterize the general equilibrium. The government budget constraints are implied by the resource constraints, and the household and firm budget constraints by Walras' Law. This gives a system of 20 variables and 14 equations and leaves 6 free variables for the government to choose. For example, with $\mu = \{g_1, s_0, \tau_1^b, \tau_1^a, \tau_0^w, \tau_1^w\}$, the remaining policy instruments g_0 and b_1 are determined endogenously.

Definition. A competitive equilibrium conditional on a government policy μ is a set of quantities $\{c_0, c_1, x_0, x_1, L_1, k_1, a_1, b_1, \pi_1\}$, and a vector of prices $P = (R_1^a, R_1^b, w_1)$ such that:

1. Given prices and government policy μ the household chooses $\{b_1, a_1, c_0, c_1, x_0, x_1\}$ optimally to solve her maximization problem.

2. Given prices and government policy μ the firm chooses $\{L_1, a_1\}$ optimally to solve its maximization problem subject to the borrowing constraint.

3. The markets for goods and labor clear, i.e. equations (12)-(14) hold and P is the market clearing vector of prices for assets and labor.

Without loss of generality, I set $\tau_1^b = 0$ and thus $R_1^b = R_1^a(1 - \tau_1^a)$ (implied by the no-arbitrage condition from the household's problem). In equilibrium, a change in τ_1^b requires a one-to-one change in R_1^b since household only cares about the return net of taxes. At the same time, in $t = 1$ the government's debt payment net its tax revenue on that debt is also determined by $R_1^b(1 - \tau_1^b)$.

3 The Ramsey problem

In this section, I first present the problem of the Ramsey planner and derive the conditions for a first-best allocation. For an arbitrary government policy, there may be a wedge be-

tween the interest rate R_1^a the firm pays, and the marginal product of capital $f_k + 1 - \delta$ (see equation (10)) of the borrowing-constrained firm. I show that under the optimal Ramsey policy, there is no such wedge.

The implementability constraints. The Ramsey planner maximizes the household's welfare. He chooses an allocation and associated policy while respecting all competitive equilibrium conditions. These conditions can be summarized by the following implementability constraints (see Appendix A.1 for a derivation):

$$u_c(0)c_0 + \beta u_c(1)c_1 = u_x(0)(1 - x_0) + \beta u_x(1)(1 - x_1) + \dots$$
$$+ \beta u_c(1) \underbrace{[(1 - \zeta)f(k_1, 1 - x_1) + (1 - \delta)k_1 - \frac{u_c(0)}{\beta u_c(1)(1 - \tau_1^a)}(k_1 - \omega_0 - s_0)]}_{=\pi_1}, \quad (15)$$

$$c_0 + g_0 + k_1 = (1 - x_0)w_0 + \omega_0 + \omega_0^g, \quad (16)$$

$$c_1 + g_1 = f(k_1, 1 - x_1) + (1 - \delta)k_1, \quad (17)$$

$$k_1 - \omega_0 - s_0 \leq \bar{a}, \quad (18)$$

$$u_c(0) \leq \beta u_c(1)(f_k + 1 - \delta)(1 - \tau_1^a), \quad (19)$$

$$0 \leq s_0. \quad (20)$$

Equation (15) is the household intertemporal budget constraint where the net interest rates and net wages have been replaced by the household's optimality condition for consumption and labor. The term in the square brackets is the firm's profits, where the wage and interest rate have been replaced with the firm's optimality condition for labor and the household's marginal rate of substitution, respectively. Equations (16) and (17) are the resource constraints of the economy. The borrowing constraint is given by (18), where private borrowing a_1 has been replaced by the firm budget constraint. Equation (19) is the firm's optimality condition for capital k_1, where the interest rate has been replaced by the household's marginal rate of substitution. This inequality reflects the possible wedge: the interest rate may be smaller than the marginal product of capital in case the firm is borrowing constrained. Finally, the subsidy constraint (20) ensures that subsidies are positive.

The optimal Ramsey allocation and policy is the one that maximizes

$$u(c_0, x_0) + \beta u(c_1, x_1)$$

subject to the implementability constraints (15)-(20). The following proposition discusses the conditions for the first-best allocation. A direct corollary to this proof will show that under the Ramsey policy there is no wedge, i.e. equation (19) holds with equality.

Proposition 1. *The first-best allocation results if and only if neither the borrowing nor the subsidy constraint is binding.*

Proof. Forming the Lagrangian, the Kuhn-Tucker first-order conditions for s_0 and τ_1^a are

$$s_0: \quad \lambda u_c(0)/(1 - \tau_1^a) = \theta_0 + \theta_2, \quad (21)$$

$$\tau_1^a: \quad \lambda \frac{u_c(0)}{(1 - \tau_1^a)^2}(k_1 - \omega_0 - s_0) = \theta_1 \beta u_c(1)(f_k + 1 - \delta), \quad (22)$$

8

where λ is the multiplier associated with (15) and $\theta_0, \theta_1, \theta_2$ are the multipliers associated with (18)-(20). By definition, the shadow values $\theta_0, \theta_1, \theta_2$ of the inequality constraints are nonnegative. If neither constraint (18) nor (20) is binding, then $\theta_0 = \theta_2 = 0$ which implies $\lambda = 0$ by (21). Equation (22) then implies $\theta_1 = 0$. The set of relevant implementability constraints then collapses to the two resources constraints and the first-best allocation of the social planner results. On the contrary, when one of the two constraints is binding, then $\lambda \neq 0$ and the Ramsey planner has to resort to distortionary taxation and the first-best social planner allocation is not attainable. $\square$

The first-best allocation results, if, for example, ω_0^g is large enough and the government can finance its expenditures with its initial endowment. Alternatively, the first-best can be attained when there is no lower bound on subsidy payments, such that the planner finances the government's expenditures through a lump-sum tax on firm borrowing.[7] As I will show, the interesting case is where the borrowing constraint binds in the presence of distortionary taxes.

Corollary 1. *When the economy is not in the first-best allocation, equation (19) holds with equality.*

Proof. This corollary follows directly from equations (21) and (22). A binding subsidy or borrowing constraint implies that $\lambda \neq 0$ and therefore $\theta_1 \neq 0$. The inequality constraint (19) holds with equality. $\square$

The implication of Corollary 1 is that under the Ramsey policy, there is no wedge between the marginal product of capital and the cost of capital for the firm: $f_k + 1 - \delta = R_1^a$. From the Ramsey planner's point of view, when the inequality constraint $R_1^a \leq f_k + 1 - \delta$ is *just* binding, his choice set is no more constrained than when it holds with a strict inequality. Indeed, the Ramsey planner implements an allocation where $R_1^a = f_k + 1 - \delta$. I explain in more detail in 4.3, why this is optimal.

The result of Corollary 1 allows the substitution of the tax rate τ_1^a in the profit function using equation (19).[8] The maximization problem of the Ramsey planner can be rewritten as

$$
\begin{aligned}
\max \ & u(c_0, x_0) + \beta u(c_1, x_1) \\
\text{s.t.} \ & u_c(0)c_0 + \beta u_c(1)c_1 = u_x(0)(1 - x_0) + \beta u_x(1)(1 - x_1) + \\
& + \beta u_c(1) \underbrace{[(1 - \alpha - \zeta)f(k_1, 1 - x_1) + (f_k + 1 - \delta)(\omega_0 + s_0)]}_{= \pi_1}, \\
& (1 - x_0)w_0 + \omega_0 + \omega_0^g = c_0 + g_0 + k_1, \\
& f(k_1, 1 - x_1) + (1 - \delta)k_1 = c_1 + g_1, \\
& \bar{a} \geq k_1 - \omega_0 - s_0, \\
& s_0 \geq 0.
\end{aligned}
$$

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[7]The statement holds as long as the subsidy payments do not cause further distortions. If, for example, the subsidy carries wasteful tax collection costs, the first-best allocation does not result.

[8]In the first-best allocation the inequality constraint (19) also holds with equality. The borrowing constraint for the firm is not binding and therefore $f_k + 1 - \delta = R_1^a = u_c(0)/[\beta u_c(1)(1 - \tau_1^a)]$.

The first order conditions of this problem together with the implementability constraints, characterize the optimal policy and allocation. See Appendix A.2 for the full list.

4 A special case

In this section, I develop the main insight: Following a tightening of the borrowing constraint, it is not optimal to relax the borrowing constraint, but rather to exploit this new constraint and reduce existing tax distortions. I explore this result with a parametrization that has a (partial) closed-form solution. The results remain robust to other parametrizations.

4.1 Specification and result

Let household preferences be

$$u(c_t, x_t) = \log c_t - \varphi(1 - x_t),$$

and the production function be

$$f(k_1, L_1) = Ak_1^\alpha,$$

i.e. labor is not productive in the second period and therefore there is no labor supply in period 1. There is full depreciation $\delta = 1$ and the firm and the government have no endowment, that is $\omega_0 = \omega_0^g = 0$. The government has to finance an exogenous amount $\bar{g}_0 > 0$, whereas $g_1 = 0$. Therefore the government budget constraints simplify to

$$\bar{g}_0 + s_0 = \tau_0^w(1 - x_0)w_0 + b_1,$$
$$R_1^b b_1 = \tau_1^a R_1^a a_1,$$

where $R_1^b = (1 - \tau_1^a)R_1^a$ because of the no-arbitrage condition between the two assets. Together with $s_0 \geq 0$, these conditions ensure that, independent of whether the borrowing constraint is binding or not, the government needs to levy distortionary taxes to finance $\bar{g}_0$ and the subsidy s_0. The government can either do this by levying labor taxes today, issue debt that is financed by savings income taxes tomorrow or a combination of both. The capital stock is determined by $k_1 = a_1 + s_0$.

Remark. The details of the parametrization are not important and are only chosen to achieve a partial closed-form solution. Preferences are a special case of $u(c, x) = \frac{c^{1-\sigma}}{1-\sigma} - \varphi\frac{(1-x)^{1+\nu}}{1+\nu}$ where $\sigma \to 1$ and $\nu = 0$. I conduct a number of numerical simulations where I vary parameters σ and ν. The observations of the comparative statics exercise remain the same.

Figure 1 graphs the equilibrium allocation and policy for parameter values $w_0 = 2.5$, $\alpha = 2/3$, $\varphi = 3$, $A = 1$, $\beta = 0.98$ and $\bar{g}_0 = 0.1$. The figure shows the evolution of the variables under the optimal policy as a function of the borrowing constraint $\bar{a}$. The way to read the four panels is as follows: On every panel, moving from right to left (along the x-axis) implies a tightening of the borrowing constraint. At the very right of each panel, the borrowing constraint is not yet binding. At this point, as $a_1 < \bar{a}$, a decreasing $\bar{a}$ has no effect on the equilibrium allocation and policy and it is optimal to finance $\bar{g}_0$ with labor taxes τ_0^w only. Savings income taxes τ_1^a are set to zero, thus $b_1 = 0$ and no subsidy is necessary as the firm can finance itself without restriction.
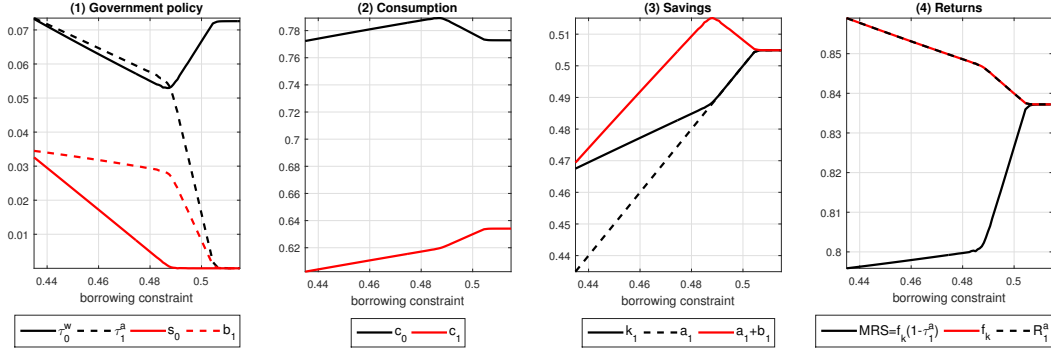
Figure 1: Comparative statics: a special case

Moving right to left, eventually ($\bar{a} = 0.505$), the borrowing constraint becomes binding and the Ramsey planner adjusts the tax rates. However, it is not optimal to increase the subsidy and relax the borrowing constraint. The subsidy s_0 remains zero under the optimal policy. Instead, it is optimal to issue debt, financed by an increase in the tax rate on savings τ_1^a (see Panel 1 in Figure 1). At the same time, it is optimal to decrease the labor income tax rate. The lower labor tax rate allows for higher consumption c_0 in $t = 0$ and leads to lower consumption c_1 (see panel 2).

Then, moving further to the left, there is a kink ($\bar{a} = 0.488$): When the borrowing constraint is sufficiently tight, it becomes optimal to pay out subsidies s_0 to prop up capital k_1. At the same time, labor tax rates increase and consumption c_0 falls again. The fall in c_1 less steep, as the subsidy props up capital and thus production in $t = 1$.

Panel 4 also shows that the wedge is closed under the optimal policy. While the interest rate on government debt falls (black line), the cost of borrowing R_1^a increases along with and is equal to the return to capital f_k.

4.2 Characterization of the optimal policy

In what follows, I characterize optimal policy for the special case considered in Figure 1 from right to left. I distinguish three phases: To the right of the kink, where the borrowing constraint is binding but $\bar{a}$ is relatively high; the kink itself; and to the left of the kink.

A. To the right of the kink. Both constraints are binding.

To the right of the kink I can find a closed-form solution.[9] When the borrowing constraint is not—or only weakly—binding, the subsidy constraint is binding. Because the government has no endowment, it has to raise taxes. It would like to tax the firm by setting $s_0 < 0$ but cannot. It follows that the multiplier θ_2 associated with $s_0 \geq 0$ is positive and $s_0 = 0$. Instead of taxing firms, the government has to levy distortionary labor taxes to finance $\bar{g}_0$.

Thus, when the borrowing constraint becomes binding, we have $k_1 = a_1 = \bar{a}$ since the subsidy continues being set at $s_0 = 0$. Combining the first order conditions leads to the

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[9]See Appendix B.1 for the block of equations that characterize the equilibrium under the optimal policy.

Euler equation

$$\frac{1}{c_0} = \beta \frac{1}{c_1} f_k (1 - \lambda) + \theta_2 = \beta \frac{1}{c_1} f_k - \theta_0, \qquad (23)$$

where λ and θ_0 are the multipliers associated with the implementability constraint derived from the household intertemporal budget constraint and the borrowing constraint $a_1 \leq \bar{a}$, respectively. It follows that in optimum, the tax rate on savings income τ_1^a is positive when the borrowing constraint is binding (i.e. when $\theta_0 > 0$). When both constraints are binding, the optimal allocation and policy is given by

$$1 - x_0 = \frac{1 + \alpha \beta}{\varphi},$$
$$c_0 = (1 + \alpha \beta) w_0 / \varphi - \bar{a} - \bar{g}_0,$$
$$\tau_0^w = 1 - \varphi c_0 / w_0,$$
$$\tau_1^a = 1 - \bar{a} / (\alpha \beta c_0).$$

Finally, $k_1 = \bar{a}$ and $c_1 = A \bar{a}^\alpha$.

These results are reflected in Figure 1. With a tighter borrowing constraint (a falling $\bar{a}$), labor supply is constant and consumption c_0 is increasing.[10] Production and consumption c_1 are decreasing because the subsidy remains at zero. The labor tax rate falls with a more binding borrowing constraint and reflects the lower marginal utility of consumption. The savings income tax rate increases with a tighter borrowing constraint.[11] With consumption higher in $t = 0$ and lower in $t = 1$, the interest rate R_1^b falls strongly. Looking at the government budget constraint,

$$b_1 = \bar{g}_0 - \tau_0^w (1 - x_0) w_0,$$

debt increases. With labor tax revenue falling short of $\bar{g}_0$, the gap is financed with debt and payed back with higher revenue from savings income taxes.

B. Existence of the kink.

As the borrowing constraint tightens (and we move to left in the figure), it eventually becomes optimal to prop up k_1 by increasing the subsidy $s_0 > 0$. At the point at which the subsidy constraint is not binding anymore, the comparative statics of the other variables also change.

Proposition 2. *When borrowing conditions are sufficiently tight, the subsidy constraint is no longer binding.*

Proof. Using the Euler equation (23) for k_1, the shadow value of the subsidy constraint is:

$$\theta_2 = 1/c_0 - \alpha \beta / \bar{a} (2 - w_0 / (c_0 \varphi))$$

with $c_0 = (1 + \alpha \beta) w_0 / \varphi - \bar{a} - \bar{g}_0$. Conditional on the parametrization, we have $\partial \theta_2 / \partial \bar{a} > 0$ and the shadow value is therefore strictly increasing in $\bar{a}$.[12] As the borrowing constraint

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[10]Labor supply decreases when the inverse Frisch elasticity of labor supply is positive.

[11]This follows directly from the fact that the inequality constraint (19) is binding and therefore $c_1 / (\beta c_0) = f_k (1 - \tau_1^a) = \alpha c_1 (1 - \tau_1^a) / \bar{a}$.

[12]$1 + \alpha \beta > 2 \bar{a}$ is a sufficient condition. For a wide range of specifications and parametrizations considered in numerical simulations, the shadow price θ_2 is monotonically decreasing with a falling $\bar{a}$.
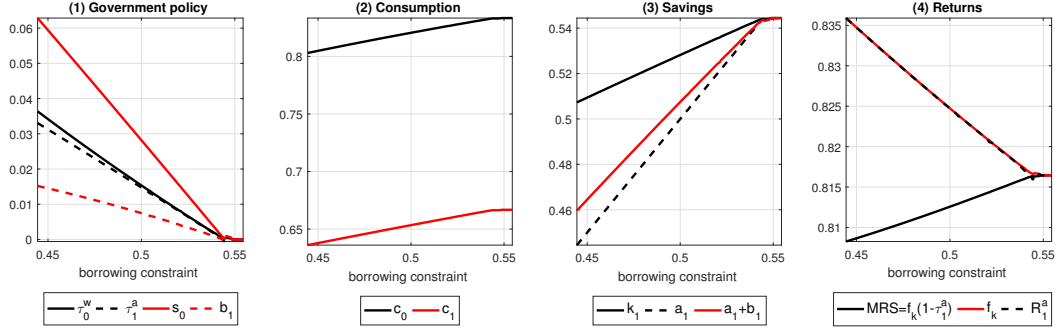
Figure 2: Comparative statics, $\omega_0^g = \bar{g}_0$

tightens and $\bar{a}$ falls, the value of θ_2 approaches zero from above. Setting $\theta_2 = 0$ gives the value $\bar{a}_l$ where the borrowing constraint is no longer binding (is slack):

$$\bar{a}_l = \frac{\alpha\beta(w_0 + 2\alpha\beta w_0 - 2\varphi\bar{g}_0)}{\varphi(1 + 2\alpha\beta)}.$$

□

C. To the left of the kink: Increase of the subsidy and reversal of other variables.

When the subsidy constraint is no longer binding it becomes optimal to set $s_0 > 0$. A closed-form solution is no longer obtainable. The existence of the kink crucially depends on whether the subsidy constraint is (or is not) slack when the borrowing constraint starts to tighten. Helping to understand why the evolution of the variables reverses, Figure 2 illustrates the comparative statics experiment when I start out in a first-best allocation and no distortionary taxes are necessary. For example, this is the case when the government has an endowment $\omega_0^g = \bar{g}_0$ such that it does not need any labor taxes to finance $\bar{g}_0$. This change ensures that the subsidy constraint is slack. Then, without a binding subsidy constraint, the Ramsey planner is constrained by the same set of restrictions as to when we are to the left of the kink in Figure 1. Figure 2 depicts the impact of a tightening borrowing constraint when the subsidy constraint is not binding. When the borrowing constraint is not binding, again at the very right of Figure 2 tax rates τ_0^w and τ_1^a are set to zero. No (distortionary) taxes are necessary because $\bar{g}_0$ can be fully financed with ω_0^g. When the borrowing constraint becomes binding, the transfer s_0 increases with a falling $\bar{a}$ in order to prop up private capital k_1. To finance the transfer, the government increases labor income taxes and issues debt that is paid back based on savings income taxes. The private capital stock k_1 decreases as the constraint becomes tighter. Contrary to before, there is no kink in the evolution of variables. This exercise with $\omega_0^g = \bar{g}_0$ corresponds to the comparative statics of a falling $\bar{a}$ on the left side of the kink in Figure 1 since the Ramsey planner faces the same set of constraints.

Proposition 3. *When the borrowing constraint is binding, all real variables are lower than in the first-best allocation. Government debt and the tax rates are higher.*
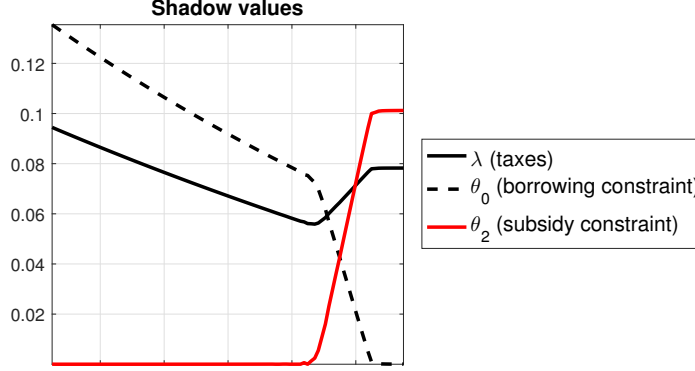
Proof. See Appendix B.2. □

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Figure 3: Lagrange multipliers

Simulations for varying model specifications affirm that when the subsidy constraint is not binding (i.e. we are to the left of the kink), the evolution of variables is monotonous as a function of a tightening borrowing constraint.

4.3 Discussion

I now discuss the optimal policy response and why the evolution of variables changes course as a function of a tightening borrowing constraint. The following discussion does not only correspond to the special case of subsections 4.1–4.2 but to a broad range of parametrizations. As shown above, whether the subsidy constraint is binding plays a key role for the behavior of the optimal policy. As soon as the subsidy constraint is no longer binding, the graphs change their course. I discuss this observation with the help of the Lagrange multipliers that measure the cost of the constraints faced by the Ramsey planner. Figure 3 graphs the values of the three Lagrangian multipliers over the same space. The Lagrange multipliers depict a kink where the subsidy constraint is slack ($\theta_2 = 0$). Moving from right to left, the multiplier λ of the household implementability constraint decreases and then starts to increase again. The multiplier θ_0 of the borrowing constraint shoots up and increases with a lower slope after the kink.

Initially, the appearance of the borrowing constraint has the effect that both the shadow cost of the household implementability constraint λ and the shadow cost of the subsidy constraint θ_2 become smaller. The interdependence between these three multipliers can be seen directly by the first order condition for s_0:

$$\lambda \beta u_c(1) f_k(k_1, L_1) = \theta_0 + \theta_2.$$

Under the optimal policy, marginally increasing the subsidy s_0 has a marginal benefit equal to $\theta_0 + \theta_2$: Increasing the subsidy relaxes both constraints. This benefit is weighed against the cost of having to levy distortionary taxes to finance the subsidy, which is captured by $\lambda \beta u_c(1) f_k(k_1, L_1)$. Even though θ_0 increases with a falling $\bar{a}$, a rapidly falling θ_2 allows for a decrease in λ as well. Once $\theta_2 = 0$ (i.e. the subsidy constraint is slack), λ and θ_0 move in the same direction.[13] I now discuss the optimal policy in more detail from right to left.

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[13]See the Appendix B.1 for closed-form solutions for the multipliers to the right side of the kink.

The cost of the subsidy constraint θ_2 decreases with a falling $\bar{a}$. The intuition is straightforward. Initially, when the borrowing constraint is not yet binding, the Ramsey planner would like to set the subsidy to a negative level to finance $\bar{g}_0$. As private capital becomes depressed, the benefit of doing so becomes smaller because it would depress private capital even further. That is, the Ramsey planner would still like to set a negative s_0, but at a less negative level compared to when the borrowing constraint is not binding. The marginal benefit of relaxing the subsidy constraint θ_2 falls. Eventually, when the borrowing constraint is sufficiently tight, it becomes optimal to implement a positive subsidy s_0: the subsidy constraint is slack ($\theta_2 = 0$).

Tax distortions fall initially. An increase in τ_1^a is not distortionary. As shown above, to the right of the kink the fall in λ is accompanied by a fall in the labor tax rate. In this region, the emergence of a distortion of the borrowing constraint allows for a fall in the distortion caused by taxes. How is this possible? The Ramsey planner can increase the tax rate on savings which has no distortionary effects. This leads to new government revenue which allows for a reduction in distortionary labor tax rates.

A higher tax on savings is not distortionary for the following reason. For illustrative purposes consider that the government would not adjust its policy following a tightening of the borrowing constraint, that is it would keep the same tax rates and debt policy. Ceteris paribus, a wedge would open up between the cost of borrowing R_1^a and the return on capital $f_k + 1 - \delta$: Since the household can save less, consumption is shifted towards the present, and since R_1^a reflects the consumption schedule the interest rate falls. At the same time, the marginal product f_k is higher with a lower capital stock. The Ramsey planner can now "close" the wedge between the interest rate and the marginal product and leave the behavior of the household and firm unchanged. The Ramsey planner implements an equilibrium where both the cost of borrowing R_1^a and the tax on savings τ_1^a increase in a way that leaves the net return of the household $R_1^a(1 - \tau_1^a)$ unchanged. This means the household is still willing to lend $\bar{a}$. At the same time, following the higher interest rate R_1^a, the firm still demands $\bar{a}$ as long as $R_1^a \leq f_k + 1 - \delta$. That is, as long as the wedge is open, the firm demand for capital is completely inelastic. The behavior of both the household and firm does not change even though the tax on savings has increased. It is in this sense that an increase τ_1^a, accompanied with an increase in R_1^a does not lead to additional distortions.

Closing the wedge is optimal because it leads to distortion-free revenue. Importantly, the government has additional tax revenue in $t = 1$ through the increase in τ_1^a (financed by debt in $t = 0$). This additional distortion-free revenue allows the government to reduce distortionary labor taxes, which is reflected by the fall in λ to the right of the kink in Figure 3. Instead of using only distortionary labor taxes, debt and the tax on savings income can be used to finance (some of) the government's expenditures.

Of course, these additional revenues come from somewhere. In fact, the government absorbs and effectively fully "taxes" the additional firm profits that would result from the wedge. By implementing an equilibrium where $R_1^a = f_k$ (opposed to one where it is $R_1^a < f_k$), firm profits decrease (note that in both cases $k_1 = \bar{a}$). Because firm profits are a lump-sum transfer to the household, the partial taxation of these profits does not influence the house-

hold's optimality conditions. From the perspective of the Ramsey planner, the distortion caused by the borrowing constraint allows for an effective lump-sum tax of firm profits by increasing τ_1^a. The emerging distortion by the tightening borrowing constraint leads to an equilibrium where labor tax distortions are lower.

Eventual increase in s_0 and tax distortions. If the borrowing constraint $\bar{a}$ falls sufficiently, it eventually becomes optimal to increase s_0. Then the subsidy constraint $s_0 \geq 0$ is no longer costly to the Ramsey planner and the distortion caused by the subsidy constraint is no longer active. At this point, the distortion of the borrowing constraint is sufficiently strong and the private capital is sufficiently low such that it is optimal to prop up capital k_1 by an increase in the subsidy s_0.

The fact that it becomes optimal to pay out a subsidy has two effects. First, given that the subsidy props up capital, the wedge does not open up as much as before, ceteris paribus (because of a slower increase in the marginal productivity of capital). This gives the government less room to effectively tax firm profits—indeed, the savings income tax rate increases by much less after the kink. Second, by increasing s_0 the government needs new financing. Consequently, labor income tax rates need to increase again. To the left of the kink, the same tradeoffs take place as when no distortionary taxes are needed in the first place, e.g. with a high enough government endowment ω_0^g (see Figure 2). Then, the new distortion of the borrowing constraint cannot reduce labor tax distortions, simply because they are not there in the first place. Thus it is optimal to prop up k_1 with subsidies s_0 which have to be financed with larger (than zero) labor income tax rates.

To sum up, this exercise shows that when a new distortion in the form of a borrowing constraint affects the economy, other existing distortions matter (here, labor tax distortions). It is not optimal to address the borrowing constraint by paying out a subsidy to relax it. This would worsen the existing labor tax distortion since the subsidy has to be financed by additional distortionary labor taxes. Only after the borrowing constraint is sufficiently distortionary, it is optimal to address and relax it somewhat. (It is never optimal to fully relax it as the constraint is always binding.) In fact, the new distortion due to the borrowing constraint can be exploited, since the government has access to distortion-free revenue through which it is first optimal to decrease the already existing labor tax distortion.

Remark. In this exercise, the government does not primarily issue more debt because debt is cheaper (in equilibrium the interest rate R_1^b has fallen with a tightening borrowing constraint). The government mainly issues debt because there is new tax revenue in the future in the form of higher savings income taxes. Debt is used as an instrument to shift taxation from $t = 0$ to $t = 1$.

5 Endogenous government expenditures

The result that it is not optimal to relax the new borrowing distortion but rather to exploit it to reduce another (tax) distortion, translates to richer specifications of the model. I augment the model along two dimensions. First, government spending gives utility to the household

and is thus endogenous under the optimal Ramsey policy. Additionally, there is government investment p_1 which enters the production function of the firm. These two additions allow that the optimal government policy may include higher spending or investment in response to the borrowing constraint.

Let household preferences be

$$u(c_t, x_t, g_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{(1-x_t)^{1+\nu}}{1+\nu} + \eta_t \log g_t.$$

Further, let the production function be

$$f(k_1, p_1, L_1) = A k_1^\alpha p_1^\gamma L_1^\zeta.$$

Public investment p_1 enhances the productivity of both private capital and labor.[14] The public capital stock is financed by the government in period 0. The government's budget constraint are

$$p_1 + g_0 + s_0 = \tau_0(1-x_0)w_0 + b_1,$$
$$g_1 + R_1^b b_1 = \tau_1(1-x_1)w_1 + \tau_1^a R_1^a a_1.$$

The full set of equilibrium conditions that characterize the Ramsey allocation are in Appendix A.2. Compared to before, the question is now not only how to finance a given amount of government spending, but also how large government spending should be. The endogeneity of g and p extends the set of instruments of the government. Instead of lowering the tax rates, it may, for example, invest in public capital to make up for the fall in private capital. Or the government may increase government spending g_1 to pick up low private production in $t = 1$. Figure 4 depicts the optimal policy for parameter values $w_0 = 2.5, \beta = 0.98, A = 3, \alpha = 2/3, \zeta = 1/3, \gamma = 1/3, \delta = 1, \eta = 0.2$ and $\nu = 2$.[15]

By contrast to the special case in 4.1-4.2, labor is productive in period 1 and can thus be taxed. However, the productivity of labor in period 1 and the inclusion of government spending and investment do not affect the main mechanism. The behavior of all variables remains the same. Government spending g_0 and public investment p_1 track private consumption c_0: As a function of a tightening borrowing constraint $\bar{a}$, these variables increase initially and then fall to the left of the kink.

The same mechanism as discussed before is at work. The distortion of the borrowing constraint allows for an increase in the savings income tax rate. The additional revenue allows for a reduction in the labor tax distortion which again leads to higher private consumption in $t = 0$. Due to decreasing marginal utility, the Ramsey planner distributes the additional revenue also across the other two margins that give value to the household: government spending g_0 and public investment p_1.[16] Again, the existing distortion matters for the optimal policy response. In addition, the main reason why it is optimal to increase public spending or investment is not to replace private consumption and production. It is

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[14]Note that with a Cobb-Douglas production function, investment in public capital can also be interpreted as investment in technology A. I have also simulated this exercise with CES production functions, where $f(k_1, p_1, L_1) = A[\alpha k_1^r + (1-\alpha)p_1^r]^{1/r} L_1^\zeta$. The qualitative picture remains the same. I also address the case where public and private investment are perfect substitutes in Appendix C.2.

[15]Again, choosing different parametrizations does not change the result qualitatively.

[16]E.g. the Ramsey planner sets $u_c(0) = u_g(0)$ in equilibrium.
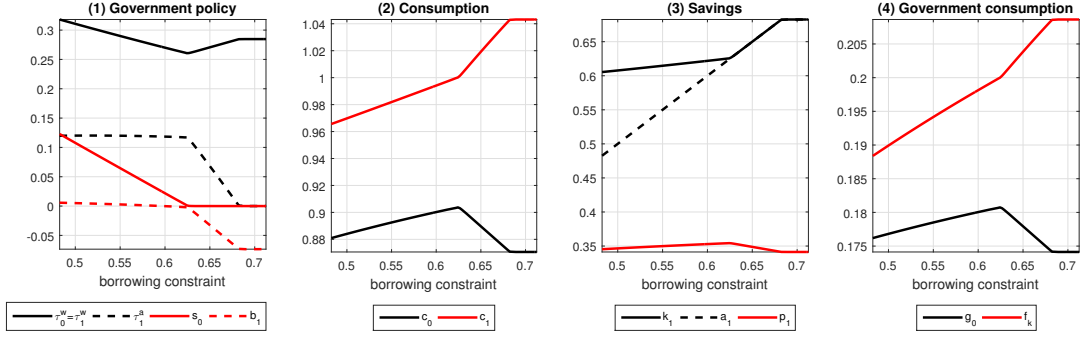
Figure 4: Endogenous government consumption and investment

primarily because the Ramsey planner implements an allocation where the benefit of lower labor taxes is distributed across all margins (private and public consumption and public investment) that give utility to the household.

As the borrowing constraint tightens further, eventually private capital is sufficiently depressed such that it becomes optimal to prop up k_1 with the subsidy s_0. This has to be financed with additional labor tax rates and the course of the other policy variables reverses with a tightening borrowing constraint $\bar{a}$. The additional policy variables g_0 and p_1 still follow c_0 and thus decrease to the left of the kink.

6 Concluding remarks

I analyze the optimal government policy in response to a tightening firm borrowing constraint that may result because of a disturbed transmission channel between savers and borrowers. The optimal policy response of the government depends on whether other distortions are present in the economy. Even though the government could address the new distortion in the economy, I find it is optimal not to. Rather, it is optimal to reduce the existing labor tax distortion. Only if the borrowing constraint is relatively tight it is optimal to loosen the constraint by subsidizing the private firm sector.

While the model is stylized and a government has other tools at hand, the model illustrates in a simple setting that it is not necessarily prudent to tackle an emerging distortion. The model also illustrates that given relatively low interest rates, it is not necessarily optimal for the government to borrow and spend more simply because debt is cheap. Spelling out the borrowing constraint in more detail and adding more periods would be an interesting avenue for future research.

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Appendix

A.1 Derivation of the implementability constraints

I derive the implementability constraint for the augmented model with endogenous government spending and investment that nests the model where government expenditures are held fixed. For the competitive equilibrium conditional on some feasible government policy $\{\tau_0^w, \tau_1^w, \tau_1^a, \tau_1^b, s_0, g_0, g_1, p_1\}$, I have 13 variables $c_0, c_1, x_0, x_1, a_1, k_1, R_1^b, R_1^a, b_1, w_1, \pi_1, L_1, \theta_0^f$ and 14 equations: four budget constraints, six first-order conditions and a market clearing condition for labor $(1 - x_1 = L_1)$, 2 resource constraints and a complementary slackness condition. Fixing 7 of the 8 policy variables then leads to a system of 14 equations and 14 unknowns.

The budget constraints together with the resource constraints yield the government's budget constraints. The Ramsey planner has to respect the following system of equations that implements a competitive equilibrium:

$$c_0 + b_1 + a_1 = (1 - \tau_0^w)(1 - x_0)w_0,$$
$$c_1 = (1 - \tau_1^w)(1 - x_1)w_1 + (1 - \tau_1^b)R_1^b b_1 + (1 - \tau_1^a)R_1^a a_1 + \pi_1,$$
$$u_c(0) = \beta u_c(1)R_1^a(1 - \tau_1^a),$$
$$(1 - \tau_1^b)R_1^b = (1 - \tau_1^a)R_1^a,$$
$$u_x(t)/u_c(t) = (1 - \tau_t^w)w_t, \ t = 0, 1,$$
$$k_1 = \omega_0 + a_1 + s_0,$$
$$\pi_1 = f(k_1, p_1, L_1) - R_1 a_1 - L_1 w_1 + (1 - \delta)k_1,$$
$$c_0 + g_0 + p_1 + k_1 = (1 - x_0)w_0 + \omega_0,$$
$$c_1 + g_1 = f(\cdot) + (1 - \delta)k_1,$$
$$f_{1-x_1} = w_1,$$
$$f_k + 1 - \delta = R_1^a + \theta_0^f,$$
$$\theta_0^f(\bar{a} - a_1) = 0,$$
$$L_1 = 1 - x_1.$$

The FOC for a_1 and the complementary slackness condition can be combined to the inequality constraint $R_1^a \leq f_k + 1 - \delta$ that holds with equality when $a_1 < \bar{a}$ and may be slack when $a_1 = \bar{a}$. Without loss of generality, I can set $\tau_1^b = 0$. L_1 is replaced by $1 - x_1$, w_1 with f_{1-x_1}, and π_1 and a_1 with the firm budget constraints. Combining the household budget constraints eliminates b_1. Replacing $(1 - \tau_t^w)w_t$ with the labor supply equations, R_1^b and $R_1^a(1 - \tau_1^a)$ with the marginal rate of substitution gives rise to the following household implementability constraint:

$$u_c(0) + \beta u_c(1)c_1 = u_x(0)(1 - x_0) + \beta u_x(1)(1 - x_1) + \beta u_c(1)\pi_1$$
$$\Leftrightarrow u_c(0) + \beta u_c(1)c_1 = u_x(0)(1 - x_0) + \beta u_x(1)(1 - x_1) +$$
$$\beta u_c(1)[f(\cdot) - R_1^a(k_1 - \omega_0 + s_0) + (1 - x_1)f_L + (1 - \delta)k_1]$$
$$\Leftrightarrow u_c(0) + \beta u_c(1)c_1 = u_x(0)(1 - x_0) + \beta u_x(1)(1 - x_1) +$$
$$\beta u_c(1)[(1 - \zeta)f(\cdot) - (R_1^a - 1 + \delta)k_1 + R_1^a(\omega_0 + s_0)].$$

Finally, I replace the interest rate on borrowing R_1^a with the household's optimality condition for a_1, $R_1^a = \frac{u_c(0)}{\beta u_c(1)(1 - \tau_1^a)}$:

$$u_c(0) + \beta u_c(1)c_1 = u_x(0)(1 - x_0) + \beta u_x(1)(1 - x_1) +$$
$$\beta u_c(1)[(1 - \zeta)f(\cdot) + (1 - \delta)k_1 + \frac{u_c(0)}{\beta u_c(1)(1 - \tau_1^a)}(\omega_0 + s_0 - k_1)].$$

In addition to the constraints that result from the competitive equilibrium conditions, the Ramsey planner has to respect the borrowing constraint and the subsidy constraint.

Thus the full list of implementability constraints is

$$u_c(0) + \beta u_c(1)c_1 = u_x(0)(1 - x_0) + \beta u_x(1)(1 - x_1) +$$
$$\beta u_c(1)[(1 - \zeta)f(\cdot) + (1 - \delta)k_1 + \frac{u_c(0)}{\beta u_c(1)(1 - \tau_1^a)}(\omega_0 + s_0 - k_1)],$$
$$c_0 + g_0 + p_1 + k_1 = (1 - x_0)w_0 + \omega_0,$$
$$c_1 + g_1 = f(\cdot) + (1 - \delta)k_1,$$
$$f_k + 1 - \delta \geq \frac{u_c(0)}{\beta u_c(1)(1 - \tau_1^a)},$$
$$k_1 - \omega_0 + s_0 \leq \bar{a},$$
$$s_0 \geq 0.$$

A.2 Characterization of the Ramsey policy and allocation

Setting up the Lagrangian leads to the following list conditions that describe the Ramsey allocation. Here I present the list with endogenous government spending and investment, which nests the specification with a given sequence of government expenditures. The La-

grangian writes

$$\mathcal{L} = u(c_0, x_0, g_0) + \beta u(c_1, x_1, g_1)$$
$$+ \lambda[u_c(0)c_0 + \beta u_c(1)c_1 - u_x(0)(1 - x_0) - \beta u_x(1)(1 - x_1)$$
$$- \beta u_c(1)\{(1 - \alpha - \zeta)f(k_1, p_1, 1 - x_1) + (f_k + 1 - \delta)(\omega_0 + s_0)\}]$$
$$+ \mu_0[(1 - x_0)w_0 + \omega_0 + \omega_0^g - c_0 - g_0 - k_1 - p_1]$$
$$+ \beta\mu_1[f(k_1, p_1, 1 - x_1) + (1 - \delta)k_1 - c_1 - g_1]$$
$$+ \theta_0[\bar{a} - k_1 + \omega_0 + s_0]$$
$$+ \theta_2[s_0 - 0].$$

The first-order conditions are

$c_0:$ $\quad u_c(0) + \lambda[u_{cc}(0)c_0 + u_c(0) - u_{xc}(0)(1 - x_0)] = \mu_0$

$c_1:$ $\quad u_c(1) + \lambda[u_{cc}(1)c_1 + u_c(1) - u_{xc}(1)(1 - x_1)$
$\quad\quad - u_{cc}(1)((1 - \alpha - \zeta)f(k_1, 1 - x_1) + (f_k + 1 - \delta)(\omega_0 + s_0))] = \mu_1$

$x_0:$ $\quad u_x(0) + \lambda[u_x(0) - u_{xx}(1 - x_0) + u_{cx}(0)c_0] = \mu_0 w_0$

$x_1:$ $\quad u_x(1) + \lambda[u_x(1) - u_{xx}(1 - x_1) + u_{cx}(1)c_1 - u_{cx}(1)\{(1 - \alpha - \zeta)f(\cdot) + (f_k + 1 - \delta)(\omega_0 + s_0)\}$
$\quad\quad - u_c(1)((1 - \alpha - \zeta)f_{L_1}(-1) + f_{k_1 L_1}(\omega_0 + s_0)(-1))] = \mu_1 f_{L_1}$

$g_0:$ $\quad u_g(0) + \lambda[u_{cg}(0)c_0 - u_{xg}(0)(1 - x_0)] = \mu_0$

$g_1:$ $\quad u_g(1) + \lambda[u_{cg}(1)c_1 - u_{xg}(1)(1 - x_1) - u_{cg}(1)\{(1 - \alpha - \zeta)f(\cdot) + (f_k + 1 - \delta)(\omega_0 + s_0)\}] = \mu_1$

$k_1:$ $\quad -\mu_0 + \beta\mu_1(f_k + 1 - \delta) - \lambda\beta u_c(1)[(1 - \alpha - \zeta)f_k + f_{kk}(\omega_0 + s_0)] - \theta_0 = 0$

$p_1:$ $\quad -\mu_0 + \beta\mu_1 f_{k_g} - \lambda\beta u_c(1)[(1 - \alpha - \zeta)f_{k_g p} + f_{kk_g}(\omega_0 + s_0)] = 0$

$s_0:$ $\quad -\lambda\beta u_c(1)(f_k + 1 - \delta) + \theta_0 + \theta_2 = 0.$

These first order conditions together with the implementability constraints characterize the optimal policy and allocation.

B.1 A special case: closed-form solution of the simplifying specification when the subsidy constraint is binding

I can find a closed-form solution for the right of the kink. The block of equations that characterize the equilibrium under the optimal policy is

$$c_0: \quad 1/c_0 = \mu_0$$

$$c_1: \quad 1/c_1 + \lambda/c_1^2((1-\alpha)f(\cdot) + f_k s_0) = \mu_1$$

$$x_0: \quad \varphi(1+\lambda) = \mu_0 w_0$$

$$k_1: \quad -\mu_0 + \beta\mu_1 f_k - \lambda\beta/c_1[(1-\alpha)f_k + f_{kk}s_0] - \theta_0 = 0$$

$$s_0: \quad \lambda\beta/c_1 f_k = \theta_0 + \theta_2$$

$$BC: \quad k_1 - s_0 \leq \bar{a} \quad \text{(borrowing constraint)}$$

$$SC: \quad s_0 \geq 0 \quad \text{(subsidy constraint)}$$

$$IC: \quad 1 + \beta = \varphi(1-x_0) + \beta/c_1[(1-\alpha)f(\cdot) + f_k s_0] \quad \text{(implementability constraint)}$$

$$RC_0: \quad c_0 + k_1 + \bar{g}_0 = (1-x_0)w_0 \quad \text{(resource constraint)}$$

$$RC_1: \quad c_1 = f(\cdot) \quad \text{(resource constraint)}$$

where $\theta_0, \theta_2 \geq 0$.

When the subsidy constraint is binding (the right side of the kink), it is $s_0 = 0$ and therefore $k_1 = \bar{a}$. From the IC it follows directly $1 - x_0 = (1+\alpha\beta)/\varphi$. From the RC_0 follows

$$c_0 = (1+\alpha\beta)w_0/\varphi - \bar{g}_0 - \bar{a}.$$

Then from the FOC for x_0 it is $\lambda = w_0/(c_0\varphi) - 1$. RC_1 gives $c_1 = A\bar{a}^\alpha$. The Euler equation is given by

$$1/c_0 = \beta\frac{1}{c_1}f_k(1-\lambda) + \theta_2$$

or

$$1/c_0 = \beta\frac{1}{c_1}f_k - \theta_0.$$

From $c_1/(\beta c_0) = f_k(1-\tau_1^a)$ follows that $\tau_1^a = 1 - \bar{a}/(\alpha\beta c_0)$ and $\theta_2 = 1/c_0 - (\alpha\beta)/\bar{a}(2 - w_0/(c_0\varphi))$. The inflection point (where $\theta_2 = 0$) is then:

$$\bar{a}_l = \frac{\alpha\beta(w_0 + 2\alpha\beta w_0 - 2\varphi g_0)}{\varphi(1+2\alpha\beta)}.$$

To find the point where the borrowing constraint becomes binding, one can use the Euler equation (23) with $\theta_0 = 0$: Then it must be that $1/c_0 = \alpha\beta\bar{a}$ or $\bar{a}_h = \alpha\beta[(1+\alpha\beta)w_0/\varphi - \bar{g}_0 - \bar{a}_h]$ or $\bar{a}_h = \frac{\alpha\beta}{1+\alpha\beta}[(1+\alpha\beta)w_0/\varphi - \bar{g}_0]$.

B.2 A special case: Derivation of comparative statics results when the subsidy constraint is not binding

The full list of conditions that describe the Ramsey allocation when the borrowing constraint is binding is

$$c_0: \quad 1/c_0 = \mu_0$$
$$c_1: \quad 1/c_1 + \lambda/c_1^2((1-\alpha)f(\cdot) + f_k(\omega_0 + s_0)) = \mu_1$$
$$x_0: \quad \varphi(1+\lambda) = \mu_0 w_0$$
$$k_1: \quad -\mu_0 + \beta\mu_1 f_k - \lambda\beta/c_1[(1-\alpha)f_k + f_{kk}(\omega_0 + s_0)] - \theta_0 = 0$$
$$s_0: \quad -\lambda\beta/c_1(f_k + 1 - \delta) + \theta_0 = 0$$
$$BC: \quad k_1 - \omega_0 - s_0 = \bar{a}$$
$$IC: \quad 1 + \beta = \varphi(1-x_0) + \beta/c_1[(1-\alpha)f(\cdot) + f_k(\omega_0 + s_0)]$$
$$RC0: \quad c_0 + k_1 + \bar{g}_0 = (1-x_0)w_0 + \omega_0$$
$$RC1: \quad c_1 = f(\cdot)$$

9 equations, 9 unknowns.

Using the FOC for s_0 in the one for k_1 (replacing θ_0), I have

$$-\mu_0 + \beta\mu_1 f_k - \lambda\beta/c_1 f_k(1 - (\alpha - 1)\bar{a}/k_1) = 0.$$

Combining with the FOCs for c_0 and c_1 gives

$$-1/c_0 + \beta f_k/c_1 - \lambda\beta/c_1 f_k\bar{a}/k_1 = 0$$

and can be rewritten as

$$1/c_0 = \beta/c_1 f_k(1 - \lambda\bar{a}/k_1)$$

where $\tau_1^a = \lambda\bar{a}/k_1$. It follows, with distortionary taxes ($\lambda \neq 0$), savings income taxes are positive (it is always $\lambda \geq 0$. Only when $\bar{a} = 0$ it is $\tau_1^a = 0$).

Proof of Proposition 3. *When the borrowing constraint is binding, all real variables are lower than in the first-best allocation. Government debt and the tax rates are higher.*

When the borrowing constraint is not binding, $\lambda = 0$ and the first-best allocation results (see Proposition 1). When it is binding, $\lambda > 0$. In the following, I show that the value of the variables are lower (higher for debt and the tax rates) when $\lambda > 0$ compared to when $\lambda = 0$.

The Euler equations for private capital reads $1/c_0 = \beta/c_1 f_k(1 - \lambda\bar{a}/k_1)$ which implies $\tau_1^a = \lambda\bar{a}/k_1$. When the borrowing constraint is binding, $\lambda > 0$ (otherwise $\lambda = 0$). From the FOC for x_0 we have $c_0 = \frac{w_0}{\varphi(1+\lambda)}$: Consumption c_0 is lower the more costly the implementability constraint is (the higher λ).

Combining the Euler equation with the resource constraint for period 1 gives $k_1 = \alpha c_0(1 - \lambda\bar{a}/k_1)$. With $\lambda > 0$ the RHS is necessarily smaller than when $\lambda = 0$. It follows that the capital stock and thus production in period 1 is lower compared to the first-best case. The resource constraint $c_0 + k_1 + \bar{g}_0 = (1-x_0)w_0$ then implies that labor supply is also lower.

The labor tax rate is higher (greater than zero): $(1 - \tau_0^w)w_0 = c_0\varphi$. The savings tax rate $\tau_1^a = \lambda\bar{a}/k_1$ is now positive (it is zero in the first-best). The same is true for debt: $b_1 = \tau_1^a\bar{a}R_1^a/R_1^b$. Finally, the transfer $s_0 = \tau_0^w(1 - x_0)w_0 + b_1 - \bar{g}_0$ is higher: Tax revenue from labor income taxes and savings income taxes (via b_1) has increased (from zero) and the expenditures on public capital are lower. $\square$

C.1 Other taxes

I investigate how robust my results are when I consider other linear taxes. Some taxes are redundant, others are not, but the picture does not change qualitatively. A special case is an excise tax on firm borrowing paid by the firm.

Profit Taxes. Say the household pays a fraction τ_1^π of the profits received to the government. Without any upper bound on τ_1^π, it would be optimal to set $\tau_1^\pi = 1$ as this tax is not distortionary. Given an upper bound on the tax rate τ_1^π, the optimal policy is to set the tax rate as high as possible (to the upper bound). The comparative statics of a tightening borrowing constraint lead to the same result qualitatively.

Consumption Excise Tax. Say the household pays $(1 + \tau_t^c)$ to consume one unit of consumption in t. The household implementability constraint is then

$$u_c(0)c_0 + \beta u_c(1)c_1 = u_x(0)(1 - x_0) + \beta u_x(1)(1 - x_1) + \frac{\beta u_c(1)}{1 + \tau_1^c}\pi_1.$$

The firm demand optimality condition is

$$f_k + 1 - \delta \geq \frac{u_c(0)}{\beta u_c(1)}\frac{1 + \tau_1^c}{1 + \tau_0^c}\frac{1}{1 - \tau_1^a} = R_1^a.$$

With the tax on savings τ_1^a available, this inequality will again be binding in equilibrium, and the term R_1^a can be replaced by $f_k + 1 - \delta$ in the profit function. Whereas τ_0^c disappears and is thus redundant (the Ramsey plan determines the intratemporal wedge $(1 - \tau_0^w)/(1 + \tau_0^c) = u_x(0)/[u_c(0)w_0]$), the tax rate τ_1^c does not disappear if there are firm profits.

The tax rate τ_1^c can take over the role of τ_1^a. By increasing τ_1^a the interest rate R_1^a increases under the optimal policy and closes the wedge. An increase in τ_1^c achieves the same result. Conducting the comparative statics exercise and setting $\tau_1^a = 0$ leads to the same result. With a higher tax on consumption in period 1, the tax on labor falls, in order that the intratemporal wedge remains the same, i.e. the fraction $(1 - \tau_1^w)/(1 + \tau_1^c)$. To sum up, a consumption tax does lead to a slightly different allocation, because it impacts the valuation of firm profits, however, the qualitative picture and the kink remain the same.

Tax on firm return to capital. Say the firm maximizes profits $\pi = f(\cdot) - (1 + \tau_1^k)R_1^a - w_1L_1 + (1 - \delta)k_1$. Its first order condition then is

$$f_k + 1 - \delta = (1 + \tau_1^k)R_1^a + \theta_0^f.$$

When a wedge opens up, $\theta_0^f > 0$, the government can close it by increasing the tax rate τ_1^k.

This tax rate enters the implementability constraints only through the firm optimality conditions. In fact, the condition reads

$$f_k + 1 - \delta \geq (1 + \tau_1^k)R_1^a = (1 + \tau_1^k)\frac{u_c(0)}{\beta u_c(1)(1 - \tau_1^a)}.$$

The Ramsey planner can close the wedge by making borrowing more expensive. For the household, everything else equal, nothing changes.

Similar to τ_1^c before, τ_1^k takes over the role of τ_1^a. Setting $\tau_1^a = 0$ leads to the exact same Ramsey allocation. These tax instruments all affect the intertemporal margin and can be used to equate the cost of borrowing for the firm to the marginal product on capital and thus closing the wedge.

Excise tax on firm borrowing. Say the firm's budget constraint in period 0 is

$$k_1 = \omega_0 + (1 + \tau_0^z)a_1 + s_0.$$

The government can proportionally give a subsidy per unit borrowing. The firm's borrowing constraint is then

$$a_1 = (1 + \tau_0^z)^{-1}(k_1 - \omega_0 - s_0) \leq \bar{a}$$

or

$$k_1 - \omega_0 - s_0 \leq (1 + \tau_0^z)\bar{a}.$$

The government can also mitigate the borrowing constraint by paying out a subsidy $\tau_0^z \bar{a}$. The firm's optimality condition is $f_k + 1 - \delta = R_1^a/(1 + \tau_0^z) + \theta_0^f$ or $(f_k + 1 - \delta)(1 + \tau_0^z) \geq R_1^a$. Replacing R_1^a with the household optimality condition for a_1, the implementability constraint is $u_c(0) \leq \beta u_c(1)(f_k + 1 - \delta)(1 - \tau_1^a)(1 + \tau_0^z)$. The Ramsey planner can simply undo the borrowing constraint by increasing τ_0^z. He finances this increase with debt and pays it back by increasing τ_1^a. The interest rate R_1^a increases. However, this is fine, as the firm wants to equate the interest rate with $f_k(1 + \tau_0^z)$. If I constrain τ_0^z from above, eventually this is not possible and the usual optimal adjustment process will take place with the resulting kink.

The household's savings remain the same, initially. She simply exchanges lending to the firm with saving in government bonds. The government uses these proceeds to finance $\tau_0^z a_1$ and pays it back with additional $\tau_1^a R_1^a a_1$. The combination of τ_0^z and τ_1^a is such that $(1 + \tau_0^z)(1 - \tau_1^a) = 1$.

If the inequality constraint is binding, I can again replace the firm optimality condition into the profit function. As long as τ_0^z is unconstrained, the Ramsey planner can undo the borrowing constraint and implement the same allocation.

Implementability constraints: The only implementability constraints that change are the household intertemporal budget constraint, the firm optimality condition, and the borrowing constraint. That is

$$+\lambda \left(\ldots - \beta u_c(1)[(1 - \zeta)f(\cdot) - \frac{u_c(0)}{\beta u_c(1)(1 - \tau_1^a)(1 + \tau_0^z)}[k_1 - \omega_0 - s_0] + (1 - \delta)k_1 \right)$$

$$+\theta_0[\bar{a} - (1 + \tau_0^z)^{-1}(k_1 - \omega_0 - s_0)]$$

$$+\theta_1[\beta u_c(1)(f_k + 1 - \delta)(1 - \tau_1^a)(1 + \tau_0^z) - u_c(0)]$$

Assuming an interior solution, the FOC for τ_1^a again ensures that the constraint associated with θ_1 is binding. This means that the term $u_c(0)/[\beta u_c(1)(f_k + 1 - \delta)(1 - \tau_1^a)(1 + \tau_0^z)]$ can be replaced by $f_k + 1 - \delta$. The tax τ_1^a then disappears in the set of implementability constraints, but τ_0^z remains. Then, without restriction on τ_0^z the problem then is not bounded: The FOC for τ_0^z would read $\bar{a}\theta_0 = 0$. Unless $\bar{a} = 0$ it must be that $\theta_0 = 0$ and the constraint is not relevant to the Ramsey planner. When the tax rate is bounded, say $\tau_0^z \leq \kappa$, the borrowing constraint is binding when the tax rate hits that ceiling. When the tax instrument τ_0^z can freely be adjusted, the Ramsey planner can fully undo the borrowing constraint. Once this ceiling is hit, the normal adjustment process takes place, where initially it is optimal not to increase s_0 but it is optimal to decrease labor taxes.

Lump-sum household tax. With a lump-sum tax t_0 on the household's side the problem becomes trivial. The first order condition for t_0 is

$$\lambda u_c(0) = 0.$$

Assuming an interior solution, we have $\lambda = 0$ and the household implementability constraint is not costly to the Ramsey planner. By Proposition 1, both the subsidy constraint and the borrowing constraint cannot be costly and the first-best allocation results, even when the borrowing constraint tightens and $\bar{a}$ falls. The Ramsey planner can implement the first-best allocation in the following way: The Ramsey planner can increase lump-sum taxes on the household, who accordingly saves less in a_1 one to one. The higher lump-sum tax finances a subsidy to the firm. For the household, the loss in savings income is fully compensated by higher profit transfers have fewer savings income, but higher profit transfers make up for the lump-sum tax.

Tax collection cost in the subsidy. I assume the tax collection cost is paid for by the firm, in the sense that a part of the subsidy is wasted.

$$k_1 = \omega_0 + a_1 + s_0 - \chi \frac{s_0^2}{2},$$

where $\chi > 0$. When $s_0 > 0$, the private capital stock increases less than one to one with the subsidy payment. The qualitative properties remain the same. Initially, it is not optimal to increase the subsidy s_0. After the kink, when private capital is sufficiently depressed, the subsidy payment increases less quickly compared to the baseline specification, because raising the subsidy leads to resource losses in the form of payment or collection costs. Introducing a distortion in the collection or payment of the subsidy does not change the picture qualitatively.

C.2 Alternative production function: perfect substitutes

Say the capital stocks k_1 and p_1 are perfect substitutes, i.e. $f(k_1, p_1, L_1) = A(k_1 + p_1)^\alpha L_1^\zeta$. A tightening of the borrowing constraint leads to the same qualitative picture as above. Initially it is not optimal to increase the subsidy s_0, but rather to decrease the labor income tax rate by issuing new debt (see Figure 5). Since k_1 and p_1 are perfect substitutes, it is also
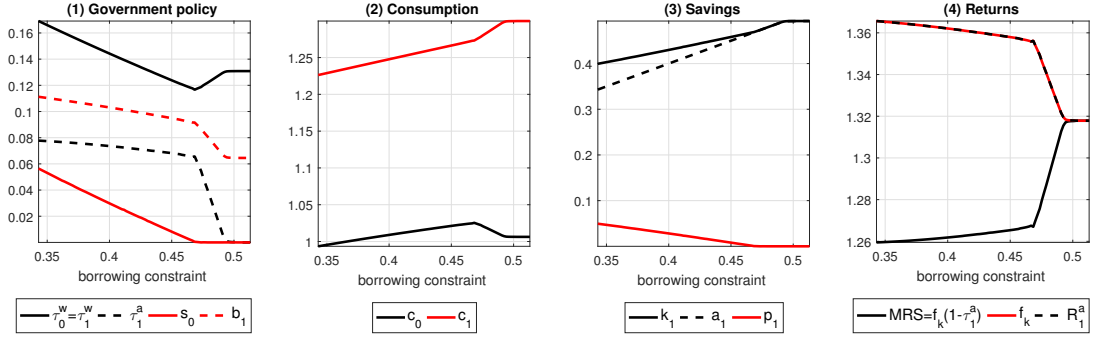
Figure 5: Public and private capital are perfect substitutes, $\bar{g}_0 = 0.3$

optimal not to increase p_1. Once it becomes optimal to increase the subsidy, the composition of k_1 and p_1 is irrelevant. Leaving $s_0 = 0$ and $k_1 = \bar{a}$, and instead increasing p_1 would result in the same allocation.