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Confidence and the Financial Accelerator*

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Abstract

We introduce financial frictions in the spirit of Bernanke, Gertler, and Gilchrist (1999) into a standard RBC model and use the heterogeneous-prior framework of Angeletos, Collard, and Dellas (2018) to accommodate confidence-driven business cycle fluctuations. We show that financial frictions strongly amplify the response to confidence shocks—more strongly than the response to fundamental shocks. Furthermore, we show that in the presence of financial frictions, prolonged episodes of unfounded optimism cause boom-bust cycles in investment and to a lesser extent in output. In particular, the financial state of the economy deteriorates severely after the initial boom, which leaves the economy more vulnerable to adverse shocks.

JEL class: E32, E44

Keywords: Confidence, sentiments, financial accelerator, financial frictions, higher-order beliefs, higher-order uncertainty, business cycle.

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1 Introduction

Chuck Prince, the CEO of Citigroup famously observed in July 2007: "[...] As long as the music is playing, you've got to get up and dance." This famous analogy underscores the widespread opinion that the economy, and in particular financial markets, are affected by forces similar to "animal spirits".[1] Consider, for instance, the decision of an investor to invest in the mortgage market. The return on her investment does not only depend on her own behavior, but also on the current and future decisions of other economic agents. Since investors do not know what the other investors believe and how they behave, fads and rumors about the strength of the mortgage market affect the investment decision, irrespective of whether or not these fads and rumors are well founded. Moreover, effects of fads and rumors are particularly strong if investors are leveraged. If an investor believes that other investors are pulling back from the mortgage market, she believes the return on investment falls, which also leads to a lower valuation of her investment projects. This drop in the valuation of the investment project is particularly dire for a leveraged investor since it reduces the investor's equity and thus impairs her ability to raise funds for investment. Hence, a leveraged investor reacts stronger to fads and rumors. This reaction is reinforced even further when other investors are leveraged as well, since investors believe that other leveraged investors cut back investment more strongly, putting even more pressure on the mortgage market. Eventually, the reaction of investors to fads and rumors spill over to the real, non-financial part of the economy.

In this paper, we study the interaction between financial markets and fads and rumors from a macroeconomic perspective. In particular, we explore how financial frictions affect so-called *confidence*-driven fluctuations—fluctuations driven by aggregate waves of fads and rumors *unrelated to economic fundamentals*.[2] To this end, we augment a standard RBC model along two dimensions. First, we introduce financial frictions along the lines of the widely used financial accelerator framework of Bernanke et al. (1999) (henceforth BGG). In BGG, entrepreneurs leverage their net worth and undertake risky investment projects. Second, we use the heterogeneous-prior framework of Angeletos et al. (2018) (henceforth ACD) to accommodate a notion of aggregate waves of unfounded fads and rumors, or equivalently, aggregate waves of unfounded optimism and pessimism.

The paper is related to different strands of the macroeconomic literature on incomplete information. Our methodology builds upon the literature that introduces forces akin to animal spirits in unique-equilibrium DSGE models. As in Angeletos and La'O (2013), we depart from the complete information assumption. This departure allows for non-fundamental driven fluctuations in a unique equilibrium environment. In Angeletos and La'O (2013), information frictions prevent agents from forming exactly the same expectations about economic activity. Then, non-fundamental shocks—so-called sentiment shocks—can influence equilibrium expectations and thus economic activity, even in unique-equilibrium rational-

[1] Here we refer to the interpretation of Keynes (1936), that human behavior can be driven by spontaneous instincts and emotions.

[2] By fundamentals we refer to factors that constitute the structure of an economy, including peoples' abilities and preferences, firms' know-how, government policies, and any news thereof.

expectation models.[3] More precisely, an adverse sentiment shock rationalizes the pessimism of an agent by making her receive signals about the pessimism of others. However, introducing dispersed private information into rational-expectations models complicates the computation of the model solution considerably once dynamic choices are involved. Hence, ACD develop a heterogeneous-prior framework that accommodates forces akin to animal spirits in unique-equilibrium DSGE models, while maintaining a tractable environment.

ACD build on the key insight of Angeletos and La'O (2013) that deviations of higher-order beliefs from first-order beliefs about fundamentals help to accommodate autonomous variations in equilibrium expectations. Their heterogeneous-prior approach enables them to engineer deviations of first-order beliefs from higher-order beliefs while bypassing the computational complications that arise from noisy learning and heterogeneity. While ACD use their framework to study the impact of so-called confidence shocks—the analog of sentiment shocks in Angeletos and La'O (2013)—in various DSGE models, they do not consider models with financial frictions, which is our focus.

There are only a few papers in the macroeconomic literature on incomplete information with a focus on the interaction between financial markets and a notion of animal spirits. Examples are Angeletos, Lorenzoni, and Pavan (2010), Goldstein, Ozdenoren, and Yuan (2013) and Benhabib, Liu, and Wang (2016) who study models where information spillovers lead to a two-way feedback between the financial and the real sector. In Angeletos et al. (2010), the impact of noise shocks—correlated errors across entrepreneurs' information about fundamentals, which can be interpreted as shocks to confidence—on equilibrium outcomes is strongly amplified. Key for the strong amplification is that information spillovers from the real to the financial sector generate a strategic complementarity in the investment decision. Goldstein et al. (2013) focus on information spillovers from the financial sector to the real sector. Correlated errors in the signals to speculators may give rise to trading frenzies, i.e. episodes in which many speculators rush to trade in the same direction. Such episodes lead to a strong pressure on prices. Both contributions focus on static models and can therefore not investigate the role of correlated errors about fundamentals on the dynamics of aggregates. Benhabib et al. (2016) analyze a similar set up as Goldstein et al. (2013), but in addition, they study dynamic effects of sentiment shocks in an overlapping generations model.

In contrast to Angeletos et al. (2010), Goldstein et al. (2013), and Benhabib et al. (2016), we do not focus on information spillovers between economic agents. We study the interaction of confidence and credit market frictions—credit market frictions that make the availability of outside finance, e.g. debt, relevant. In this regard, our paper is closely related to La'O (2010) who investigates a model with a Kiyotaki and Moore (1997) style collateral constraint under dispersed information. In La'O (2010), tighter collateral constraints amplify the impact of noise shocks strongly, whereas the impact of fundamental shocks is muted. Key to this result is that agents over-weight public information when they try to infer aggregate activity (similar to Morris and Shin, 2002). A positive noise shock that is common knowledge leads agents to believe aggregate activity increases strongly. The demand for

[3] In a similar framework, Benhabib, Wang, and Wen (2015) also show that information frictions give rise to fluctuations, which are unrelated to changes in the economy's fundamentals. But in contrast to Angeletos and La'O (2013), confidence can lead to multiple equilibria in their model.

collateral increases and pushes up the collateral's value, which relaxes the credit constraint and enables a strong expansion of production. The contribution of La'O (2010) offers some insights regarding the amplification of a similar notion of confidence shocks; however, in a static set-up. In contrast, our model is a dynamic business cycle model and allows us to study the dynamic aspects of confidence-driven fluctuations as well.

Our paper also relates to the literature on sunspots (see, e.g., Woodford, 1986; Barinci and Chéron, 2001; Bosi, Magris, and Venditti, 2007; Liu and Wang, 2014). In this literature, financial frictions give rise to multiple equilibria and allow for non-fundamental sources of fluctuations, so-called sunspots. Sunspot shocks serve as a device for agents to coordinate on a specific equilibrium. They are also viewed as a possible theoretical justification of Keynes' animal spirits allegory about economic instability. Adverse realizations of sunspot shocks cause agents to coordinate on welfare-inferior equilibria. Such coordination failures can then be interpreted as some form of pessimism. In contrast, our formalization of confidence does not rely on the existence of multiple equilibria.

Our model economy consists of multiple islands. Each island is inhabited by a basic RBC economy with a financial market structure as in BGG. On each island, agents interact in competitive markets and produce a specialized local good. Islands are connected with each other through a centralized market on a mainland where the specialized goods and a final good are traded. The specialized goods are complements in the production of the final good, which takes place on the mainland. The final good is then used for consumption and investment. Importantly, the employment decision on the local island takes place before aggregate production and the agents' real income streams are observed. Following ACD, there is no communication between islands, which impedes islands from reaching common beliefs about aggregate production. Each agent receives a signal about the economy's fundamentals before trade between islands takes place. All agents on an island receive the same signal, which they know is unbiased. However, agents believe the signals of the agents on the other islands to be biased by an exogenous variable ξ_t. Following ACD, we engineer waves of optimism and pessimism through the variable ξ_t, the *confidence shock*.

The basic mechanism of a confidence shock works as follows. A positive innovation in confidence induces agents to believe that the agents on all other islands believe in, say, a technological improvement.[4] Since a positive technology shock leads to higher labor input, agents believe that the production of all other islands will increase. The complementarity in the final good production translates these beliefs to beliefs about improved terms of trade for the domestic island. In turn, beliefs about improved terms of trade stimulate labor input and lead to an effective increase in the domestic island's production. Hence, due to the strategic complementarity in islands' production choices, a confidence shock leads to an increase in production. Because all islands are hit by the same confidence shock, production increases on all islands.

Our main result is that financial frictions amplify the response of economic activity to confidence shocks strongly, more strongly than to other fundamental shocks which are usually

[4]Following ACD, in our baseline specification, the confidence shock affects beliefs about technology. We address confidence shocks that affect beliefs about other fundamentals in 4.3.

considered in the literature.[5] The strong amplification occurs mainly because financial frictions strengthen the strategic complementarity between the production choices of islands. Financial frictions of BGG strengthen the strategic complementarity as follows: The friction introduces a risk premium on the expected return on capital. This risk premium is inversely related to the net worth of entrepreneurs, as a higher level of net worth mitigates agency problems. An increase in the production on all other islands leads to a domestic improvement in terms of trade and raises the level of net worth of entrepreneurs on the domestic island. A higher level of net worth reduces the risk premium, which has a similar effect as a reduction in an investment tax and hence leads to higher production. Thus, if agents on an island believe that production on all other islands will increase, the production of the agents' own island rises more strongly because of this financial accelerator. In contrast, the strengthening of strategic complementarity stemming from financial frictions does not affect the response to *fundamental* shocks. As is common to all complete information rational expectation models, the true value of the current fundamentals is common knowledge. Therefore, agents know that agents on every island behave alike in face of a fundamental shock and that no beliefs about improved terms of trade emerge *in equilibrium*. It is as if only one island exists.

Another interesting finding concerns the propagation of confidence shocks in the presence of financial frictions. With financial frictions, a positive innovation in confidence leads to a boom-bust pattern in investment, which is followed by a slow recovery in investment in the aftermath of the bust. The boom-bust pattern in investment also translates to a lesser extent to the response of output. Without financial frictions, no boom-bust cycle in investment emerges. The significant difference in the propagation of confidence shocks is related to the entrepreneurs' borrowing decisions. When confidence is persistent, an innovation in confidence leads agents to persistently overestimate the return on investment. Based on these erroneous beliefs, an entrepreneur wants to borrow more and is willing to pay higher interest payments than she would if she knew the realized aggregate return on investment of the next period. The, from an ex-post perspective, too high interest payments and higher default rates have a negative effect on the entrepreneur's net worth. As the confidence shock is persistent, the over-borrowing of entrepreneurs continues and net worth eventually falls below its long-run trend. Even after unfounded optimism has vanished, the level of net worth remains low and recovers only slowly. The low level of net worth depresses investment, and thus real activity for a long time after confidence has vanished.

The existence and extent of the boom-bust cycle depend on the persistence of confidence. The more persistent a confidence shock, the longer and stronger entrepreneurs overestimate the return on investment. Consequently, entrepreneurs borrow too heavily against their net worth for a longer time and hence the fall of entrepreneur net worth is larger and longer-lasting.

The remainder of our paper is organized as follows. Section 2 introduces the model. Section 3 describes the equilibrium characterization and the solution method. In Section 4, we investigate the effect of financial frictions on the propagation of confidence shocks within a calibrated version of our model. In Section 5, we apply our model to the US Great

[5] As standard fundamental shocks we consider TFP, news, government expenditure, discount factor and (persistent) investment-specific shocks.

Recession. We conduct counterfactual exercises to determine the role that the interaction of confidence and financial frictions played during the Great Recession. Section 6 concludes.

2 The model

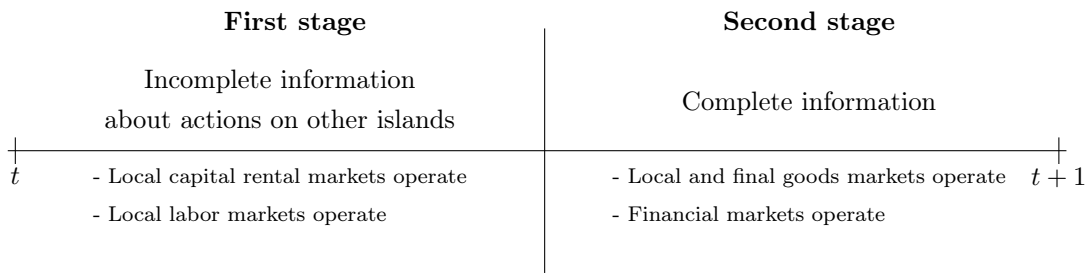
In this section, we present our model economy. First, we describe the physical environment of the economy and the timing of the events. Second, we characterize the non-financial part of the economy. Third, we describe the financial sector and detail the agency problems associated with financial intermediation. Finally, we describe the shocks and the structure of beliefs.

Geography and timing. There is a continuum of islands indexed by i and a mainland. Each island is populated by a continuum of identical households, firms, capital producers and entrepreneurs that interact on perfectly competitive local markets for labor, capital, and credit. Islands are identical except for the specialized good the local firms produce. These specialized local goods are shipped to the mainland to produce a final good. This final good is then sold on a competitive market and is used for consumption and investment on the islands.

Figure 1 illustrates when the different markets operate within a period. Each period is divided into two stages. In the first stage, firms hire labor from households and rent capital from entrepreneurs. These choices take place under commitment. There is no communication across islands and therefore the employment and capital rental decision are made in anticipation of aggregate economic activity within the same period. In the second stage, mainland markets operate and aggregate economic activity is revealed. Households consume the final good and deposit savings in banks. Capital producers combine the final good and old depreciated capital bought from entrepreneurs in order to build new capital. Entrepreneurs repay previous loans and ask for a new loan from banks to buy new capital from capital producers which they rent to local firms in the next period, $t + 1$.

The timing of events and the incomplete information between islands reflects the fact that, in reality, firms cannot communicate with all potential customers before making their employment and capital rental decisions. Hence, incomplete information allows for deviations in the beliefs about economic activity across agents. This opens the door for fluctu-

Figure 1: Timeline: first and second stage



ations in aggregates that may be driven by some form of optimism or pessimism that is unrelated to fundamentals.

We now turn to the problems faced by individual agents on an arbitrary island i. In what follows, we use the expectation operator $\mathbb{E}_{it}$ to denote agents' expectations formed in the first stage of period t, while $\mathbb{E}'_{it}$ refers to expectations formed in the second stage of period t. Without loss of generality, we normalize the prize of the final good $P_t = 1$.

Households. The representative household maximizes lifetime utility over consumption and leisure. Preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t \left(\log c_{it} - \frac{h_{it}^{1+\nu}}{1+\nu} \right), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, c_{it} is consumption, h_{it} are hours worked, and ν is the inverse of the Frisch elasticity of labor supply. The household maximizes (1) subject to a sequence of budget constraints

$$c_{it} + b^h_{i,t+1} = w_{it} h_{it} + R_{i,t-1} b^h_{it} + \Omega_{it}, \qquad t \geq 0.$$

The household earns a predetermined safe interest rate $R_{i,t-1}$ on her deposits b^h_{it} and a wage w_{it} for supplying labor. The variable Ω_{it} summarizes firm profits that are transferred to the household and transfers between the household and entrepreneurs. The household's optimality conditions are

$$h_{it}^{\nu} = \mathbb{E}_{it} \left[\frac{1}{c_{it}} \right] w_{it}, \tag{2}$$

$$\frac{1}{c_{it}} = \beta R_{it} \mathbb{E}'_{it} \left[\frac{1}{c_{it+1}} \right]. \tag{3}$$

The first optimality condition (2) is the usual labor supply equation, but augmented with an expectation operator in consumption. When hours are chosen in the first stage, agents form expectations about consumption which is determined in the second stage. The expectation operator $\mathbb{E}'_{it}$ in the Euler equation (3) reflects that households choose consumption in the second stage only.

Local firms. The representative local firm chooses labor h_{it} and capital k_{it} to maximize profits π_{it}

$$\pi_{it} = p_{it} y_{it} - w_{it} h_{it} - r^k_{it} k_{it},$$

where y_{it} denotes the local specialized good and p_{it} its price. Firms produce with the Cobb-Douglas technology

$$y_{it} = A_t k_{it}^{\alpha} (h_{it})^{1-\alpha},$$

where A_t is aggregate total-factor productivity (TFP). The first order conditions are

$$w_{it} = (1-\alpha) \frac{\mathbb{E}_{it} [p_{it} y_{it}]}{h_{it}}, \tag{4}$$

$$r^k_{it} = \alpha \frac{\mathbb{E}_{it} [p_{it} y_{it}]}{k_{it}}. \tag{5}$$

Conditions (4) and (5) are the firm's optimal labor and capital demand decisions. Both are determined in the first stage.

Capital producers. A representative local capital producer operates in the second stage. He buys the final good and depreciated left-over capital from the entrepreneurs, which he combines to produce capital for the next period (k_{it+1}). He maximizes profits π^c_{it}

$$\pi^c_{it} = \bar{q}_{it} k_{it+1} - q_{it} k_{it}(1-\delta) - i_{it},$$

where $\delta \in (0,1)$ is the depreciation rate, $\bar{q}_{it}$ is the price of new capital, q_{it} the price of left over old capital and i_{it} is the final good the capital producer buys as investment. New capital goods are produced according to the technology

$$k_{it+1} = (1-\delta)k_{it} + i_{it}\left(1 - \Phi\left(\frac{i_{it}}{k_{it}}\right)\right),$$

where $\Phi\left(\frac{i_{it}}{k_{it}}\right)$ captures capital adjustment costs. The presence of capital adjustment costs leads to a varying price of capital. The optimal investment decision is given by

$$\frac{1}{\bar{q}_{it}} = 1 - \Phi\left(\frac{i_{it}}{k_{it}}\right) - \Phi'\left(\frac{i_{it}}{k_{it}}\right)\frac{i_{it}}{k_{it}}.$$

Perfect competition among capital producers leads to zero profits and thus

$$\bar{q}_{it} = \frac{q_{it}(1-\delta) + \frac{i_{it}}{k_{it}}}{(1-\delta) + \frac{i_{it}}{k_{it}}\left(1 - \Phi\left(\frac{i_{it}}{k_{it}}\right)\right)}. \tag{6}$$

Since the capital producer is only active in the second stage and only faces a static optimization problem, he operates under complete information about the actions of others. The inclusion of capital producers is a modeling device that forces entrepreneurs to (re)purchase the entire capital stock in each period, as in BGG. The repurchase of the entire capital stock ensures that leverage restrictions or other financial constraints apply not only to marginal investment, but to the entire stock of capital, i.e. to the entrepreneur as a whole. Furthermore, the presence of capital producers ensures that capital adjustment costs are external for entrepreneurs, so that entrepreneurs take the price of capital as given.

Final good sector. A representative final good firm produces a final good Y_t using local goods y_{it} as inputs. The good is produced with a Cobb-Douglas technology which encompasses strategic complementarity: $\log Y_t = \int_0^1 \log y_{it} di$. The final good firm's problem is

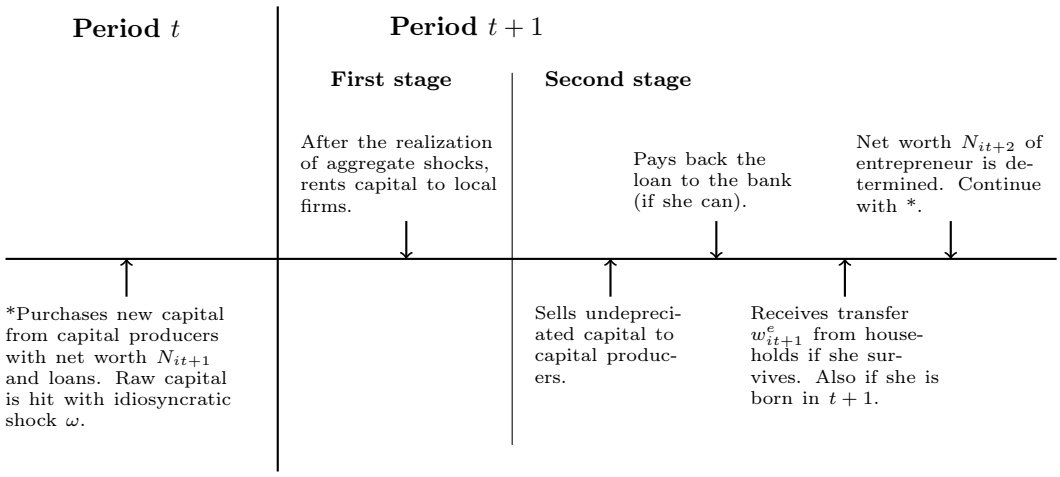
$$\max_{y_{it}} Y_t - \int_0^1 p_{it} y_{it} di.$$

Optimal behavior of agents leads to a demand function that satisfies

$$p_{it} y_{it} = Y_t. \tag{7}$$

Entrepreneurs and banks. The modeling of entrepreneurs follows BGG and Christiano, Motto, and Rostagno (2003). On each island, there is a continuum of entrepreneurs

Figure 2: One period in the life of an entrepreneur



of size one which are risk-neutral. The entrepreneurs face a constant probability of death $(1-\gamma)$ in each period. Further, $(1-\gamma)$ new entrepreneurs are born in each period, such that the population of entrepreneurs stays constant. Having a finite horizon for entrepreneurs ensures that entrepreneurs never accumulate enough net worth to be fully self-financing. Surviving and newly born entrepreneurs receive a transfer w^e_{it} from households, which ensures that bankrupt and newly born entrepreneurs have non-zero wealth and can buy new capital. Surviving entrepreneurs do not consume anything: their objective is to maximize their expected end-of-life consumption. Dying entrepreneurs consume a fraction Θ of their equity (i.e. accumulated wealth) and transfer a fraction $(1-\Theta)$ to the households. In order to abstract from the distinction between household and entrepreneurial consumption on aggregate, we follow Christiano et al. (2003) and let Θ go to zero.

In order to detail the relationship of entrepreneurs with the non-financial and financial business sector, we consider one period in the life of an arbitrary entrepreneur j on island i. We start at the end of period t, after production on the mainland and transfers from and to households have taken place (see Figure 2). At this time, the state of the entrepreneur is fully summarized by her level of net worth N^j_{it+1}. The density of entrepreneurs with net worth N^j_{it+1} is denoted by $f_t(N^j_{it+1})$, and aggregate net worth by $N_{it+1} = \int N^j_{it+1} f_{it}(N_{it+1}) dN^j_{it+1}$. For better readability, we refer to an entrepreneur with net worth N^j_{it+1} as an N^j-type entrepreneur. Each N^j-type obtains a loan $b^{N^j}_{it+1}$ from a bank, and together with her net worth the entrepreneur buys new end of period raw capital $k^{N^j}_{it+1}$ to be used in period $t+1$ at the market price $\bar{q}_{it}$ from the capital producers

$$\bar{q}_{it} k^{N^j}_{it+1} = N^j_{it+1} + b^{N^j}_{it+1}.$$

After the purchase of raw capital, the entrepreneur is hit by an idiosyncratic shock ω which turns her raw capital into effective units of capital $\omega k^{N^j}_{it+1}$. The shock reflects idiosyncratic risk in actual business ventures and is drawn independently by entrepreneurs and independently across time. The random variable $\omega \geq 0$ has mean one and follows a log-normal distribution. The cross-sectional standard deviation of $\log(\omega)$ is $\sigma_{\omega,t}$. In the following pe-

riod $t+1$, in the first stage, the entrepreneur rents her effective capital $\omega k_{it+1}^{N^j}$ at rate r_{it+1}^k to the local firm. Together, the N^j types supply $k_{it+1}^{N^j}$ since the random variable ω has mean one and all N^j-type entrepreneurs buy the same level of capital at the end of period t. In the second stage in $t+1$, after the production of the local good, entrepreneurs sell left-over capital $(1-\delta)\omega k_{it+1}$ at price q_{it+1} to the capital producer. Thus, the return for the entrepreneur is ωR_{it+1}^k, where R_{it+1}^k is the aggregate return on capital in the island economy

$$R_{it+1}^k = \frac{r_{it+1}^k + q_{it+1}(1-\delta)}{\bar{q}_{it}}.$$

The loan contract which N^j-type entrepreneurs choose in period t takes the form of a state-contingent standard debt contract specified by the gross borrowing rate $z_{it+1}^{N^j}$ and the leverage $l_{it}^{N^j} = \bar{q}_{it} k_{it+1}^{N^j} / N_{it+1}^j$. The repayment rate $z_{it+1}^{N^j}$ is contingent on the realized state of the economy in period $t+1$. Depending on the realization of the idiosyncratic risk shock ω, an N^j-type entrepreneur may or may not be able to repay the loan. The threshold realization for ω, below which N^j-type entrepreneurs cannot repay the loan, is defined by $\bar{\omega}_{it+1}^{N^j}$. The threshold is given by

$$\bar{\omega}_{it+1}^{N^j} R_{it+1}^k \bar{q}_{it} k_{it+1}^{N^j} = b_{it+1}^{N^j} z_{it+1}^{N^j} \tag{8}$$

and depends on the realized state of the economy in period $t+1$. Given the definition of the threshold, the standard debt contract can equivalently be expressed by $(\bar{\omega}_{it+1}^{N^j}, l_{it}^{N^j})$. An entrepreneur who cannot repay the loan declares bankruptcy. The bank undertakes costly monitoring of the entrepreneur and confiscates her assets.

Entrepreneurs maximize the next period's expected net worth:[6]

$$\begin{aligned} &\mathbb{E}'_{it}\left[\Lambda_{it,t+1} \int_{\bar{\omega}_{t+1}^{N^j}}^{\infty} \left[\omega R_{it+1}^k \bar{q}_{it} k_{it+1}^{N^j} - b_{it+1}^{N^j} z_{it+1}^{N^j}\right] f_t(\omega) d\omega\right] \\ =&\mathbb{E}'_{it}\left[\Lambda_{it,t+1}\left[1-\Gamma_t(\bar{\omega}_{it+1}^{N^j})\right] R_{it+1}^k l_{it}^{N^j} N_{it+1}^j\right], \end{aligned} \tag{9}$$

where $\Gamma_t(\bar{\omega}_{it+1}) \equiv [1-F_t(\bar{\omega}_{it+1})]\bar{\omega}_{it+1} + G_t(\bar{\omega}_{it+1}), G_t \equiv \int_0^{\bar{\omega}_{it+1}} \omega f_t(\omega) d\omega$ and $\Lambda_{it,t+1} = \beta u'(c_{it+1})/u'(c_{it})$ is the stochastic discount factor. To obtain the right-hand side (RHS) of equation (9), we replace the gross interest payment using (8). The RHS of equation (9) states that the expected next period's net worth of an N^j-type corresponds to the expectation about the share $1-\Gamma_t(\bar{\omega}_{it+1}^{N^j})$ of entrepreneurial earnings $R_{it+1}^k \bar{q}_{it} k_{it+1}^{N^j}$ that the N^j-type entrepreneur receives. As shown by BGG, $\Gamma(\cdot)$ is strictly increasing. Thus, the return is strictly decreasing in $\bar{\omega}_{it+1}$ for a given return spread R_{it+1}^k / R_{it} and a given leverage level.

We now describe the banking sector and the contracts banks offer. Without loss of generality, one can think of banks as specializing to lend only to entrepreneurs with the same level of net worth. That is, for each level of entrepreneurial net worth N^j, there exists a large number of N^j-type banks. Each N^j-type bank holds a perfectly diversified portfolio of loans to N^j-type entrepreneurs. At the end of period t, after the production of the final

[6] This follows BGG. Dmitriev and Hoddenbagh (2017) show that, in a first-order approximation, it does not matter whether entrepreneurs maximize the expected end-of-life consumption or the next period's expected net worth.

good has taken place, the N^j-type bank receives savings $b_{it+1}^{N^j}$ from households for which it promises a safe gross nominal rate R_{it}. The savings are then lent to entrepreneurs. To ensure that the bank is able to pay back the safe rate in $t+1$ in each possible state of the economy, banks offer a schedule of state-contingent contracts. The bank's participation constraint ensures that in each state in period $t+1$, the funds received from entrepreneurs are at least as large as the funds that have to be paid to the households:

$$[1 - F_t(\bar{\omega}_{it+1}^{N^j})]z_{it+1}^{N^j} b_{it+1}^{N^j} + (1-\mu)\int_0^{\bar{\omega}_{it+1}^{N^j}} \omega f_t(\omega) R_{it+1}^k \bar{q}_{it} k_{it+1}^{N^j} \geq b_{it+1}^{N^j} R_{it}. \tag{10}$$

The first term on the left-hand side denotes the payments from the non-defaulting portion of the entrepreneurs. The second term is what the bank can seize from defaulting entrepreneurs, net of a monitoring cost. The monitoring cost is proportional to the assets seized, i.e. a fraction μ of the assets.

Because there is a large number of N^j-type banks and they are perfectly competitive, they make zero profits and equation (10) holds with equality. Using the definition of the threshold $\bar{\omega}_{it+1}^{N^j}$, the break-even constraint of the banks is

$$\Gamma_t(\bar{\omega}_{it+1}^{N^j}) - \mu G_t(\bar{\omega}_{it+1}^{N^j}) = \frac{l_{it}^{N^j} - 1}{l_{it}^{N^j}} \frac{R_{it}}{R_{it+1}^k}. \tag{11}$$

Condition (11) implies that for a chosen leverage by the entrepreneurs, the threshold $\bar{\omega}_{it+1}^{N^j}$ is contingent on the state and the realization of R_{it+1}^k. This zero profit condition therefore defines the set of standard debt contracts $(\bar{\omega}_{it+1}^{N^j}, l_{it}^{N^j})$ offered by the banks. The entrepreneur maximizes her expected return (9) subject to the zero profit condition (11), i.e. knowing that $\bar{\omega}_{it+1}^{N^j}$ will depend on the ex-post realization of R_{it+1}^k. The entrepreneur takes as given the distribution of aggregate and idiosyncratic risk to the return on capital, the stochastic discount factor, the price of capital and the quantity of net worth that she has as collateral. The first-order condition is:[7]

$$\begin{aligned}
&\mathbb{E}'_{it}\left\{ A_t^j \left[\frac{1 - \Gamma_t(\bar{\omega}_{it+1}^{N^j})}{1 - \frac{R_{it+1}^k}{R_{it}}\left(\Gamma_t(\bar{\omega}_{it+1}^{N^j}) - \mu G_t(\bar{\omega}_{it+1}^{N^j})\right)} \times \right.\right. \\
&\left.\left. \frac{R_{it+1}^k}{R_{it}}\left(1 - F_t(\bar{\omega}_{it+1}^{N^j}) - \mu \bar{\omega}_{it+1}^{N^j} F_t'(\bar{\omega}_{it+1}^{N^j})\right)\right]\right\} = \mathbb{E}'_{it}\left\{ A_t^j \left[1 - F_t(\bar{\omega}_{it+1}^{N^j})\right]\right\}, \\
&\text{with } A_t^j \equiv \frac{\Lambda_{it,t+1} R_{it+1}^k}{1 - \frac{R_{it+1}^k}{R_{it}}\left(\Gamma_t(\bar{\omega}_{it+1}^{N^j}) - \mu G_t(\bar{\omega}_{it+1}^{N^j})\right)}.
\end{aligned} \tag{12}$$

As the safe rate and the return R_{it+1}^k are independent of the N^j-type, entrepreneurs choose the same $(\bar{\omega}_{it+1}^{N^j}, l_{it}^{N^j})$ contract, i.e. $(\bar{\omega}_{it+1}^{N^j}, l_{it}^{N^j}) = (\bar{\omega}_{it+1}, l_{it})\ \forall N^j$, independent of their net worth.

Aggregation and market clearing of the final good. In equilibrium, the total amount of produced raw capital k_{it+1} must equal the total amount of raw capital k_{it+1}

[7] Since N_{it+1}^j is already known and taken as given, it can be taken outside of the expectation operator and therefore drops from the entrepreneur's optimality condition. Note that in the linearized solution, A_t^j drops from the entrepreneur's optimality condition.

purchased by entrepreneurs in period t

$$k_{it+1} = \int_0^\infty k_{it+1}^{N^j} f_{it}(N_{it+1}^j) dN_{it+1}^j. \tag{13}$$

Likewise, the aggregate supply of effective capital by entrepreneurs in period t must equal the aggregate demand for capital services by firms

$$k_{it} = \int_0^\infty \int_0^\infty \omega k_{it}^{N^j} f_{t-1}(\omega) d\omega f_{it-1}(N_{it}^j) dN_{it}^j = \int_0^\infty k_{it}^{N^j} f_{it-1}(N_{it}^j) dN_{it}^j, \tag{14}$$

where the second-last equality uses the fact that the mean of ω is unity.

Using the law of large numbers, we obtain the average profits of an N^j-type entrepreneur in period $t+1$

$$\int_0^\infty \max\left\{0, \omega f_t(\omega) R_{it+1}^k \bar{q}_{it} k_{it+1}^{N^j}\right\} d\omega = \left[1 - \Gamma_t(\bar{\omega}_{it+1})\right] R_{it+1}^k \bar{q}_{it} k_{it+1}^{N^j}. \tag{15}$$

Hence, aggregate profits of all entrepreneurs in period $t+1$ are $\left[1 - \Gamma_t(\bar{\omega}_{it+1})\right] R_{it+1}^k \bar{q}_{it} k_{it+1}$. At the end of the period, only a fraction γ of entrepreneurs survives. The newborn and surviving entrepreneurs then receive a transfer w_{it}^e from households. Overall, aggregate net worth on island i evolves according to

$$n_{it+1} = \gamma[1 - \Gamma_{t-1}(\bar{\omega}_{it})] R_{it}^k \bar{q}_{it-1} k_{it} + w_{it}^e. \tag{16}$$

The final good is used for consumption, investment and monitoring costs. Therefore the market clearing condition is

$$Y_t = c_{it} + i_{it} + \mu G_{t-1}(\bar{\omega}_{it}) R_{it}^k \bar{q}_{it-1} k_{it}. \tag{17}$$

TFP shock. The TFP shock impacts A_t and follows a random walk process

$$\log(A_t) = \log(A_{t-1}) + \varepsilon_{a,t},\ \varepsilon_{a,t} \sim \mathcal{N}(0, \sigma_a).$$

Confidence shock and higher-order beliefs. To introduce confidence-driven fluctuations, we adopt the heterogeneous-prior framework of ACD. First, we remove the common-knowledge assumption of A_t to allow for fluctuations driven by non-fundamentals. Each island observes an island-specific signal x_{it} about aggregate TFP $a_t \equiv \log A_t$:

$$x_{it} = a_t + \varepsilon_{it}.$$

Second, we depart from the common-prior assumption. Agents on each island think that the signal on their own island is unbiased, i.e. $\varepsilon_{it} \sim \mathcal{N}(0, \sigma^2)$. However, all agents believe the other islands' signal is biased by a common stationary exogenous random variable ξ_t—the confidence shock—and believe the error ε_{jt} follows $\mathcal{N}(\xi_t, \sigma^2), \forall j \neq i$. The priors and ξ_t are common knowledge. Agents agree to disagree. Intuitively, a positive innovation in ξ_t affects an agent's behavior by making him believe that the agents on all other islands believe

in a technological improvement. However, agents know that no change in the economic fundamentals has taken place. The confidence shock follows an AR(1) process

$$\xi_t = \rho_\xi \xi_{t-1} + \zeta_t,$$

where $\zeta_t \sim \mathcal{N}(0, \sigma_\zeta^2)$ and $\rho_\zeta \in (0, 1)$.

Following ACD, we focus on the limit case where $\sigma \to 0$ to abstract from noisy learning and heterogeneity. That is, all islands receive the true signal about TFP A_t. From a technical viewpoint, ξ_t introduces deviations of higher-order beliefs from first-order beliefs about TFP. It is exactly this gap in beliefs which underlies the confidence driven fluctuations in incomplete information unique-equilibrium rational expectations environments such as Angeletos and La'O (2013) and Huo and Takayama (2015).

3 Equilibrium characterization and solution method

This section characterizes the equilibrium and describes the solution method.

3.1 Equilibrium characterization

In the absence of confidence shocks, the equilibrium characterization of our model is the same as in any other standard complete information rational expectations model. Agents already know in the first stage that all islands behave the same in equilibrium, i.e. they know $y_{it} = y_{jt}\ \forall i, j$ and $p_{it} = 1 (= P_t)$. In the presence of confidence shocks ($\xi \neq 0$), agents believe in the first stage that $y_{it} \neq y_{jt}$ and that terms of trade $p_{it} (= \frac{p_{it}}{P_t})$ behave according to $p_{it} y_{it} = Y_t$. The equilibrium in our model can be characterized as below. Since all islands are identical in equilibrium, we drop the subscript i.

A general equilibrium in our model is a sequence of first-stage quantities $\mathcal{L}_1 = \{H_t, K_t\}_{t=0}^{\infty}$, a sequence of second-stage quantities $\mathcal{L}_2 = \{K_{t+1}, N_{t+1}, B_{t+1}, C_t, I_t, Y_t, \omega_t\}_{t=0}^{\infty}$, a sequence of first-stage beliefs about aggregate production $\mathcal{B} = \{\mathbb{E}_t[Y_t]\}_{t=0}^{\infty}$, and sequences of first-stage prices $\mathcal{P}_1 = \{R_t^k, W_t\}_{t=0}^{\infty}$ and second-stage prices $\mathcal{P}_2 = \{R_t, Q_t, \bar{Q}_t, R_t^k, Z_t\}_{t=0}^{\infty}$ such that

(1) Given a sequence of first-stage prices $\mathcal{P}_1$ and of first-stage beliefs about aggregate demand $\mathcal{B}$, the sequence of first-stage quantities $\mathcal{L}_1$ solves the first-stage problem of the household, the firm and the entrepreneur.

(2) Given a sequence of first-stage quantities $\mathcal{L}_1$ and given first-stage beliefs about aggregate demand $\mathcal{B}$, the sequence of first-stage prices $\mathcal{P}_1$ clears the first-stage markets.

(3) Given a sequence of first-stage quantities $\mathcal{L}_1$ and a sequence of second-stage prices $\mathcal{P}_2$, the sequence of second-stage quantities $\mathcal{L}_2$ solves the second-stage problem of the household, the firm, the entrepreneur, the bank and the capital producer.

(4) Given a sequence of second-stage quantities $\mathcal{L}_2$, the sequence of second-stage prices $\mathcal{P}_2$ clears all second-stage markets.

(5) The sequence of first-stage beliefs about aggregate production $\mathcal{B}$ is consistent with the heterogenous-prior belief formulation and the sequence of second-stage production choices.

(6) The perceived law of motion of the exogenous states of the economy coincides with the objective law of motion of the exogenous states.

(7) The second-stage perception of the next period's level of endogenous states coincides with their actual level of endogenous states.

The differences to the standard complete information rational expectation model lie in conditions (5)–(7). In complete information models, not only the perceived law of motion of exogenous states but also the perceived law of motion of endogenous states coincides with the objective law of motion of the states. That is, condition (6) holds for endogenous states, too. In the presence of confidence, only in the second stage the perceived level of the endogenous states net worth and capital in the next period (N_{t+1}, K_{t+1}) coincides with the objective level. In the first stage, the perceived level of net worth and capital in the next period is subject to a bias.

Further, confidence introduces a gap between beliefs about aggregate production and actual production outcomes (condition 5). In complete information models, no heterogenous-prior can exist. Hence, first-stage beliefs about aggregate production coincide with second-stage production and condition (5) becomes trivial.

3.2 Solution method

We log-linearize the equilibrium equations around the deterministic steady state (see Appendix Section A.1). To solve for the linearized equilibrium of our incomplete information heterogenous-prior model, we apply the solution method developed by ACD. Generally, in standard complete information rational expectations DSGE models, the linearized equilibrium dynamics can be written as

$$(Y_t, X^b_{t+1}) = \Gamma_X X^b_t + \Gamma_Z Z_t.$$

The policy functions for vectors Y_t and X^b_{t+1} can be expressed as linear functions of the backward-looking state vector X^b_t and a vector of shocks Z_t. The heterogenous-prior framework departs from the rational expectations solution concept by introducing a systematic bias in the beliefs which agents form about the equilibrium impact of ξ_t on macroeconomic outcomes.[8] The linearized equilibrium dynamics obtained from the ACD solution method then is

$$(Y_t, X^b_{t+1}) = \Gamma_X X^b_t + \Gamma_Z Z_t + \Gamma_\xi \xi_t,$$

where the matrices Γ_X and Γ_Z are the same as those in the standard complete information rational expectation DSGE model. Even though the heterogenous-prior set-up relaxes some restrictions of the common-prior rational expectation setting, the deviations due to the confidence shock ξ are not free of restrictions. The information structure of the model connects the beliefs about the endogenous objects with each other. Fluctuations due to ξ are disciplined by cross-equation restrictions, which is reflected by the fact that Γ_ξ is obtained by solving an equation which contains Γ_X and Γ_Z.[9]

[8] ACD emphasize that in common-prior settings under rational expectations, similar waves in higher-order beliefs can be obtained without introducing a systematic bias in beliefs. The "bias" in the heterogenous-prior approach corresponds to "rational confusion" in common-prior settings.

[9] Details about the solution algorithm can be found in ACD.

4 Quantitative analysis

In this section, we explore the interaction between financial markets and aggregate waves of optimism and pessimism. More specifically, we analyze how financial frictions influence the propagation of confidence shocks in a calibrated version of our model.

4.1 Calibration

The parameter values of our baseline calibration are summarized in Table 1. The parameters of the non-financial block are set as follows: The discount factor β is set such that the steady state annual risk-free return is 3%. The capital share α is set to 0.3 and the depreciation rate of capital δ is set to 0.012. The inverse Frisch elasticity ν is set to 0.15 to ensure that the relative volatility between total hours and output is similar to the US between 1962Q1–2015Q1.[10] With regard to the capital adjustment costs $\Phi\left(\frac{x_{it}}{k_{it}}\right)$, we follow BGG. We define $\Phi(\delta) = \Phi'(\delta) = 0, \Phi(\delta)\delta^2 = \phi > 0$, and set the capital adjustment cost parameter ϕ (elasticity of the capital price with respect to the investment capital ratio) to 0.2.[11] Turning to the parameters of the financial block, the idiosyncratic productivity shock ω_{t+1} to entrepreneurs in the end of period t follows a log-normal distribution with standard deviation σ_ω and mean $-0.5\sigma_\omega^2$ (so that $\mathbb{E}\omega = 1$). Following Christiano, Motto, and Rostagno (2014) (henceforth CMR), we set the standard deviation σ_ω to 0.26. Similarly to BGG and CMR, the monitoring costs, the fraction of net worth transferred to households as well as the transfer from households to entrepreneurs are chosen to match a steady state leverage ratio of 2, an annual bankruptcy rate of 2.3% and an annualized return spread of 2.8% points. This calibration strategy implies monitoring costs of 0.20, a transfer rate of 3% of the net worth of entrepreneurs to households and a transfer rate of households to entrepreneurs of 5.73% of total output.

Technology follows a random walk and the confidence shock follows an AR(1) process. The auto-regressive parameter for the confidence shock ρ_ξ is set to 0.6. The standard deviations of the shock processes are calibrated by minimizing the distance between the model-implied standard deviations of GDP, consumption, investment and hours and those of the data. Table 7 in the Appendix Section A.3 reports business cycle moments implied by our calibrated model and the moments of the observed data.[12] Overall, our model appears well suited for business cycle analysis. The model is able to replicate the volatility of the non-financial aggregates and the correlations between both non-financial and financial aggregates remarkably well. An important shortcoming of our model is that it does not generate sufficient volatility in credit, net worth and the interest spread. Given our focus on the role that the interaction between confidence and financial frictions plays on the real, non-financial part of the economy, no further steps are undertaken to raise these volatilities.

[10] For details about the data, see Appendix Section A.2. The value of the Frisch elasticity is rather high because capital adjustment cost dampen the response of labor considerably.

[11] BGG consider a range between 0 and 0.5 as meaningful. Recent empirical studies by Hall (2004) and Cooper and Haltiwanger (2006) are in line with that range.

[12] We describe the data in more detail in Appendix Section A.2. To obtain the model-implied moments, we simulate our model and apply a band-pass filter (Christiano and Fitzgerald, 2003) to consider only frequencies between 6 and 32 quarters. For a discussion of the size of the standard deviations, see Appendix Section A.4.

Table 1: Calibration

Parameter	**Role**	**Value**
Non-financial block		
β	Discount rate	$1.03^{-\frac{1}{4}}$
ν	Inverse Frisch elasticity of labor supply	0.15
α	Capital share in production	0.3
δ	Depreciation rate	0.012
ϕ	Capital adjustment costs	0.2
Financial block		
μ	Monitoring costs	0.20
μ_ω	Mean of log ω	$-0.5\sigma_\omega^2$
σ_ω	Standard deviation of log ω	0.26
$(1-\gamma)$	Death rate of entrepreneurs	0.03
w^e/Y	Household transfer to entrepreneurs	0.0573
Shock processes: technology shock (a) and confidence shock (ξ)		
ρ_a	Persistence of a	1
ρ_ξ	Persistence of ξ	0.60
σ_a	Standard deviation of a	0.70
σ_ξ	Standard deviation of ξ	10.08

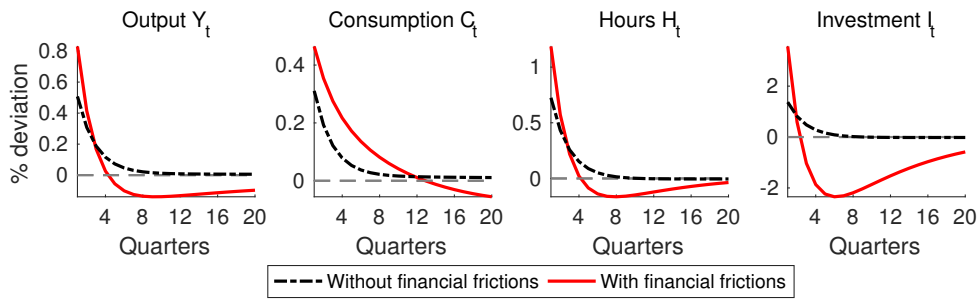
Shutting down the financial accelerator. Given our interest in the interaction of financial frictions with confidence, we consider a version of our model without financial frictions, where $\mu = 0$. Financial frictions introduce a wedge between the expected return on capital and the rate at which households can save. This wedge distorts the consumption-savings decision of households. Without monitoring costs ($\mu = 0$), the agency problem between lenders and borrowers disappears and the wedge vanishes in a first-order approximation ($\mathbb{E}'_{it}[\hat{R}^k_{it+1}] = \hat{R}_{it}$). Therefore, with $\mu = 0$, the equilibrium outcome of our linearized model is the same as the one of the underlying RBC model without banks and entrepreneurs. The financial accelerator becomes a sideshow in equilibrium. In other words, in equilibrium, the non-financial part of the economy is independent of financial market aggregates such as the leverage level, the level of net worth and the interest spread.

For the model specification without financial frictions, we use the same parameter values described in Table 1 but set μ to zero—which implies a return spread of 0% points—and adjust the household's transfer to entrepreneurs w^e to 17.57% of output. The change in w^e ensures that the steady state leverage ratio and the bankruptcy rate remains the same.

4.2 Financial frictions and the propagation of confidence shocks

We now turn to the discussion of our two main results, illustrated by Figure 3. The figure depicts the responses of non-financial aggregates to a one-standard-deviation confidence shock in the baseline model ($\mu = 0.20$) and in the absence of financial frictions ($\mu = 0$). The responses are measured in percentage deviations to the steady state. First, in 4.2.1,

Figure 3: IRFs of non-financial aggregates to a confidence shock



Note: Innovation size is σ_ξ.

we discuss the strong amplification effect of financial frictions on confidence shocks. In the presence of financial frictions, output, consumption, hours and investment react much stronger to a confidence shock on impact. Second, in 4.2.2, we explore how financial frictions lead to a boom-bust pattern in investment in the sense that the response of investment eventually falls below its steady state after a few quarters. The responses of hours and output exhibit a similar pattern, however, the fall below their steady states is not as pronounced.

4.2.1 Amplification effect

Financial frictions strongly amplify the responses to confidence shocks. On impact, financial frictions raise the response of output and hours from 0.51 to 0.83 percent and from 0.72 to 1.19 percent above steady state—this change in the responses corresponds to an amplification effect of more than 64 percent. The response of investment is amplified even more: The response on impact more than doubles from 1.37 to 3.56 percent. These numbers are large compared to other fundamental shocks. The flexible-price version of the BGG model, which we adopt, is known to amplify responses to non-financial fundamental shocks only weakly (see, e.g., Dmitriev and Hoddenbagh, 2017). Table 2 reports the amplification effect of financial frictions on the impact response of output to other fundamental shocks. For the technology shock, for example, the amplification effect of financial frictions on output is only 5.6%. For all fundamental shocks, the amplification effect of financial frictions is much weaker compared to the confidence shock.[13]

Table 2: Amplification effect on output (on impact)

Confidence	Technology	Discount	Investment-specific	News
64%	5.7%	6.5%	-29%	-11%

Note: News shocks are shocks to TFP that are anticipated one year in advance. Technology and investment-specific technology shocks follow random walks. The discount factor shock is modeled as an AR(1) process. The persistence is chosen to maximize the amplification effect and leads to $\rho_{disc} = 0$.

[13]We do not assess the amplification effect on the response to financial shocks because, in the absence of financial frictions, financial shocks have no influence on output.

The strong amplification is caused by the strengthening of the strategic complementarity between the islands' production choices. This strengthening is best illustrated with a beauty contest interpretation. As we will show below, the equilibrium impact response of output can be represented in a way that resembles a static beauty contest.

An illustrative example: beauty contest. For illustrative purposes, the following presents a static beauty contest model that is derived from a static model similar to our baseline model. This static model features similar financial frictions and the same multiple island structure as our baseline model. We also consider the same heterogeneous-prior framework as in our baseline model, i.e. the same information and shock structure. Details can be found in Appendix Section A.5.1. Each island of this model can be represented by a local fictitious planner that chooses production on his local island. The optimal production choice of an island i is given by the following linear decision rule:

$$\hat{y}_{it} = \kappa \mathbb{E}_{it}[\hat{A}_t] + \chi \underbrace{\mathbb{E}_{it}[\hat{p}_{it} - \hat{P}_t]}_{=\mathbb{E}_{it}[\hat{Y}_t - \hat{y}_{it}]}, \quad \kappa > 0, \chi > 0, \tag{18}$$

where the hat symbol denotes log-deviations from the steady state of the corresponding variable. The coefficient κ measures the strength of island i's output response to a technology shock $\hat{A}_t$ for given expectations about terms of trade ($\mathbb{E}_{it}[\hat{p}_{it} - \hat{P}_t]$). The coefficient χ measures the production response of an island to expectations about terms of trade for a given level of technology. Because the terms of trade (ToT_i) are equivalent to the difference between aggregate and island specific production ($\mathbb{E}_{it}[\hat{Y}_t - \hat{y}_{it}]$), the model can be represented as a static beauty contest. Hence, χ can be interpreted as a measure of the strategic complementarity between the production choices of islands.[14]

Proposition 1. *The solution to the beauty contest defined in equation (18) is given by*

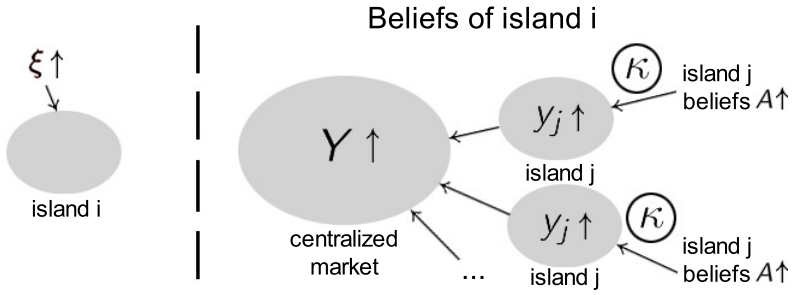
$$\hat{y}_{it} = \kappa \hat{A}_t + \kappa \chi \hat{\xi}_t.$$

Proof: See Appendix Section A.5.2.

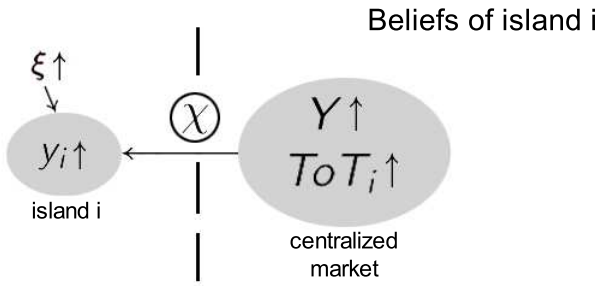
The response of production to a technology shock $\hat{A}_t$ is determined completely by κ. Additionally, κ also affects the strength of the response to a confidence shock. The intuition behind this result is shown in Figure 4. A positive innovation in ξ_t induces island i to believe that all other islands believe in a technological improvement (Figure 4a). Hence, island i believes that all other islands $j \neq i$ increase production y_j. The size of the increase in the production of all other islands depends on κ because κ determines the strength of the production response to a technological improvement. The stronger the increase of the other islands' production and hence aggregate production (Y), the stronger is the improvement in terms of trade of island i, and the more strongly island i raises output (Figure 4b). In turn, the strength with which island i reacts to beliefs about an improvement in terms of trade is determined by χ. Therefore, how strongly production y_i of an island i reacts to a confidence shock depends on both κ, the response to a technology shock, and χ, the response to a change in expected terms of trade.

[14] For convenience, we define the beauty contest slightly different than is usually the case. Usually, the

Figure 4: Mechanism of a confidence shock

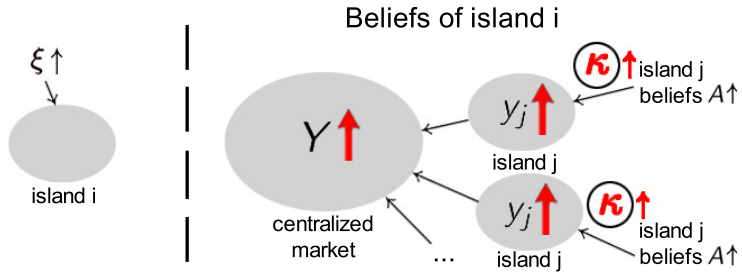


(a) Beliefs about response of other islands to a confidence shock

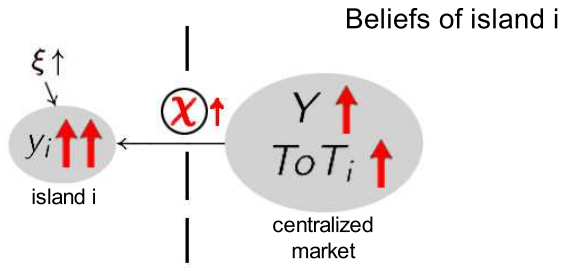


(b) Response to an increase of production on all other islands

Figure 5: Amplification of the response to confidence shock



(a) Standard financial accelerator mechanism



(b) Strengthening of the strategic complementarity

Financial frictions have two effects on this beauty contest representation of our static model (see Appendix Section A.5.3) and we illustrate these effects in Figure 5. First, financial frictions strengthen the response to technology shocks, i.e. financial frictions increase κ (Figure 5a). If an island believes that the others believe in a technological improvement, the island believes that all other islands increase production by more. Second, financial frictions strengthen the strategic complementarity between islands, i.e. financial frictions raise χ (Figure 5b). For a given expected change in terms of trade, the island increases production more strongly. Both effects together explain the strong amplification of the response to confidence shocks.

The reason why the strengthening of the strategic complementarity χ does not matter for the amplification of fundamental shocks is as follows. As in all complete information rational expectation models, all agents know the true value of the current fundamentals and know that all other agents do so as well. Hence, facing a fundamental shock, all islands know that every island chooses exactly the same production level and there are no beliefs about changes in terms of trade. It is as if only one island exists. Thus, the degree of strategic complementarity does not matter for the equilibrium response to fundamental shocks. It follows that the difference between the amplification effect for fundamental and confidence shocks is explained completely by the strengthening of the strategic complementarity.

Amplification in the baseline model. The intuition gained from the illustrative static beauty contest translates to our baseline model. Because the island set-up and information structure in our baseline model are the same, the basic mechanism of a confidence shock is also the same. Hence, the response to a confidence shock is determined by the reaction of an island's production to a technology shock and by the strategic complementarity. In addition, financial frictions also strengthen both, the response to a technology shock and the strategic complementarity.

The intuition of why the financial accelerator of BGG strengthens the response to technology shocks and the strategic complementarity is as follows. Financial frictions introduce a risk premium Ψ between the safe rate R_{it}, at which households save, and the expected return on capital $\mathbb{E}_{it}[R^k_{it+1}]$:

$$\mathbb{E}_{it}[R^k_{it+1}] = \Psi(n_{it+1}, \cdot)R_{it}. \tag{19}$$

The risk premium Ψ is inversely related to today's net worth of entrepreneurs n_{it+1}. Everything else equal, a higher net worth mitigates the agency problem between entrepreneurs and banks and hence reduces the risk premium. The reduction in the risk premium is comparable to a reduction in investment taxes and therefore stimulates output. If net worth moves pro-cyclically with output, the risk premium moves counter-cyclically and amplifies business cycle movements. A technology shock causes such a pro-cyclical movement in net worth: An improvement in technology raises the return on capital, which improves the profits and hence the net worth position of entrepreneurs. In turn, the improved net worth position reduces the risk premium which stimulates investment and, thereby, output. We refer to

beauty contest is written as $\hat{y}_{it} = \bar{\kappa}\mathbb{E}_{it}[\hat{A}_t] + \bar{\chi}\mathbb{E}_{it}[\hat{Y}_t]$. Hence our definition of the strategic complementarity χ is equal to $\frac{\bar{\chi}}{1-\bar{\chi}}$, where $\bar{\chi}$ is the standard definition of the strategic complementarity.

this strengthening effect as the "standard financial accelerator" effect—"standard" because the influence of the financial accelerator on the response to fundamental shocks has been studied thoroughly in the literature.

Regarding the strengthening of the strategic complementarity, a similar argumentation applies. Following an increase in the production on all other islands, terms of trade improve which raise the return on capital. In the presence of financial frictions, the larger return on capital translates to an improved net worth position of entrepreneurs, which strengthens the response of the island's production to an increase in the terms of trade—or equivalently, to an increase in aggregate production on all other islands. Hence, just like financial frictions amplify the response of production to a technology shock, financial friction strengthen the strategic complementarity between islands.

We now quantify the contribution of the standard financial accelerator effect and of the strengthening of the strategic complementarity for the overall amplification of the response to confidence shocks *on impact*. On impact, our model's equilibrium response of output can be represented in a way that resembles a static beauty contest:

Proposition 2. *The equilibrium response of island* i*'s output* on impact *can be written as:*

$$\hat{y}_{it} = \tilde{\kappa}\hat{A}_t + \tilde{\chi}\mathbb{E}_{it}[\hat{Y}_t - \hat{y}_{it}], \tag{20}$$

where $\tilde{\kappa}$ denotes the strength with which production on island i reacts to a fundamental technology shock in equilibrium.

Proof: See Appendix Section A.6.

Because $\tilde{\chi}$ represents the strategic complementarity in the context of a static beauty contest, we use $\tilde{\chi}$ as a measure for the strategic complementarity of our model *on impact* of a given shock. To decompose the overall amplification effect for the response to confidence shocks we make use of the following corollary.

Corollary 2.1. *The equilibrium response of island i* on impact *of a given shock can be written as:*

$$\hat{y}_{it} = \tilde{\kappa}\hat{A}_t + \tilde{\kappa}\tilde{\chi}\xi_t$$

Proof: See Appendix Section A.6.

Using Corollary 2.1, the amplification effect of financial frictions to a confidence shock is given by

$$\frac{\hat{y}_{it}^{\text{ff}}}{\hat{y}_{it}} = \frac{\tilde{\kappa}^{\text{ff}}\tilde{\chi}^{\text{ff}}}{\tilde{\kappa}\tilde{\chi}},$$

where the superscript ff denotes the objects of the model with financial frictions. The amplification effect can thus be decomposed into the "standard" financial accelerator effect ($\tilde{\kappa}^{\text{ff}}/\tilde{\kappa}$) and the strengthening of strategic complementarity ($\tilde{\chi}^{\text{ff}}/\tilde{\chi}$).

Quantitatively, the strengthening of the strategic complementarity is strong—much stronger than the standard financial accelerator effect. The introduction of financial frictions raises

the strategic complementarity ($\tilde{\chi}$) by about 54%, whereas $\tilde{\kappa}$ increases only by 5.7%. Hence, the strengthening of the strategic complementarity accounts for the bulk of the amplification of the output's response to confidence shocks by 64%.

4.2.2 Boom-bust pattern in investment

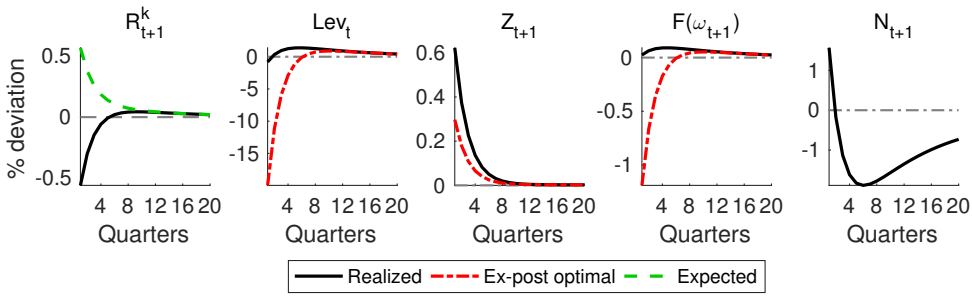
Figure 3 shows that the introduction of financial frictions also changes the propagation of confidence shocks after impact. Especially the boom-bust like pattern of investment stands out. After a strong boom on impact, investment drops much faster and eventually below its long-run trend in the presence of financial frictions. Further, its trough response is more than two percent below its steady state value and takes place one and a half years after impact. Thereafter the investment activity recovers only slowly. Similarly, output and hours also revert back to their trend values quicker than in the absence of financial frictions and fall (slightly) below their steady states after a few quarters. This pattern stands in stark contrast to the case without financial frictions, where all non-financial aggregates revert back to their steady state values (almost) monotonically. The boom-bust like pattern in investment is a specific feature of the interaction of confidence with financial frictions. With other standard frictions or model add-ons like sticky prices, investment adjustment costs or habit persistence, investment barely drops below its steady state after a positive innovation in confidence has occurred.

The boom-bust pattern in investment occurs because entrepreneurs leverage their net worth too much under the bias of a positive confidence shock. To give an intuition for this boom-bust pattern, Figure 6 depicts the responses to a confidence shock for various variables that are crucial for the entrepreneur's decision making. The first panel depicts the realized (black solid line) and the expected next period's return on capital (green dashed line). The second to fourth panels depict the realized (black solid line) and the ex-post optimal (red dashed-dotted line) leverage choice lev^{expo}, interest rate on bank loans Z^{expo} and bankruptcy probability $F(\bar{\omega}^{expo})$. We define lev^{expo} as follows. As shocks hit the economy every period, agents make errors when they forecast the return on capital. Hence, retrospectively, an entrepreneur has chosen a too high (low) leverage level that is accompanied by too high (low) costs of borrowing—as revenues from investment projects are lower (higher) than expected—compared to the optimal choice that would have maximized the entrepreneur's net worth in the next period. The ex-post optimal leverage choice defines the leverage an entrepreneur would choose if she foresaw the actual next period's equilibrium return on capital. In defining lev^{expo}, we assume that only a small group of entrepreneurs chooses the leverage level lev^{expo}—so small that the group has no influence on aggregate outcomes.[15] In the same manner, we define Z^{expo} and $F(\bar{\omega}^{expo})$. Z^{expo} and $F(\bar{\omega}^{expo})$ are the interest rate and the bankruptcy probability, a small group of entrepreneurs would face if they chose the ex-post optimal leverage level lev^{expo}. Details of the computation can be found in Appendix Section A.7.

The boom-bust pattern hinges critically on the financial accelerator mechanism and the persistent errors in forecasting the return on capital. A positive innovation in confidence

[15] More precisely, to compute the ex-post optimal variables, we also assume that the small group represents the full distribution of idiosyncratic productivity levels ω. See Appendix Section A.7 for more details.

Figure 6: IRFs to confidence: boom-bust

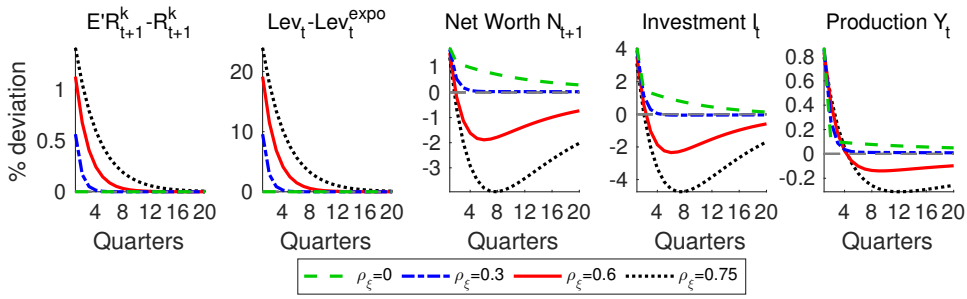


Note: From left to right: return on capital, leverage, interest rate on borrowing, bankruptcy rate and net worth. The red dash-dotted IRFs depict the leverage an entrepreneur would choose from an ex-post perspective and the accompanying interest rate and bankruptcy rate. The green dashed IRF depicts next period's expected return on capital in the second stage.

in period t induces entrepreneurs to be too optimistic about the terms of trade in period $t+1$. Hence, agents are too optimistic about the return on capital in period $t+1$ (Figure 6, first panel). Based on these erroneous beliefs, entrepreneurs leverage their net worth in period t too strongly. If an entrepreneur would have known the next period's return on capital in advance, she would have chosen a lower leverage level (Figure 6, second panel) that would have gone along with a lower interest rate on her loans (Figure 6, third panel) and a lower bankruptcy rate (Figure 6, fourth panel) in period $t+1$. Stated differently, the erroneous beliefs makes entrepreneurs willing to pay higher interest rates and induces an overly risky behavior with regard to the bankruptcy probability. The too high interest rate and too high bankruptcy rate result in a worse financial position compared to what is achievable for an entrepreneur who chooses lev^{expo}. Even though in period $t+1$ the return on capital is lower than expected, entrepreneurs are again too optimistic about the next period's return on capital. Just as in period t, entrepreneurs leverage again too much in $t+1$, which again has an inferior effect on aggregate net worth in period $t+2$, and so on. Therefore, net worth drops below the steady state after its initial boom (Figure 6, fifth panel). The trough reaction of aggregate net worth is reached one and a half year after impact. At this stage, confidence has vanished almost completely. The low level of net worth persists as households refinance entrepreneurs only slowly. The boom-bust pattern in net worth translates to a boom-bust pattern in investment. As net worth drops strongly after the initial boom, the risk premium also deteriorates strongly after an initial boom. With the deteriorating risk premium, the costs of investment increase strongly, which explains the boom-bust pattern in investment. In turn, the boom-bust pattern in investment translates to a lesser extent to output and hours.

In the absence of financial frictions, no boom-bust pattern in investment emerges. The behavior of net worth no longer affects the cost of investment because there is no risk premium. Hence, even though entrepreneurs persistently over-estimate the return on capital in face of a confidence shock, the risk premium does not deteriorate after an initial improvement. Because the risk premium does not deteriorate, investment does no longer drop below its steady state after a few periods. Also, no boom-bust pattern in investment emerges when the errors in forecasting the return on capital are purely transitory. Figure 7 explains the

Figure 7: IRFs to confidence: persistence



Note: The first panel depicts the difference between the expected return on capital and the realized return on capital. The second panel depicts the difference between the leverage choice and the leverage choice entrepreneurs would choose from an ex-post perspective.

logic behind this claim. The figure depicts the response of the (second-stage) forecast error about the next period's return on capital (difference between $\mathbb{E}'_t[R^k_{t+1}]$ and R^k_{t+1}), the difference between the actual and the ex-post optimal leverage choice, aggregate net worth, investment and production to a confidence shock for different levels of persistence. In the case of a purely transitory confidence shock process ($\rho_\xi = 0$), agents' forecast errors are purely transitory. An innovation in confidence in period t only leads to a bias in the first-stage expectations of agents on the impact of the shock. From the second stage onwards, agents perfectly forecast the future behavior of the economy. Hence, when entrepreneurs make their leverage choice, they correctly anticipate the next period's return on capital. Because the chosen leverage levels (apart from the one in period $t-1$) are optimal, net worth and investment monotonically revert back to their long-run trend after the initial boom. In contrast, if the confidence shock is persistent, entrepreneurs persistently overestimate the return on capital.[16] The more persistent the confidence shock, the longer the overestimation of entrepreneurs persists. Hence, the drop in net worth is stronger and more prolonged for more persistent confidence shock processes. For example, if the confidence shock exhibits a half-life of two and a half quarters ($\rho_\xi = 0.75$), the trough reaction of investment is about 4.7% below its long-run trend. This trough reaction is twice as strong as in our baseline calibration ($\rho_\xi = 0.6$, a half-life of 1.3 quarters). To sum up, the extent and existence of the boom-bust pattern in our baseline model crucially depend on the persistence of the confidence shock.

4.2.3 The role of confidence for business cycle fluctuations and welfare

The strong influence of financial frictions on the propagation of confidence shocks is also reflected in the role of confidence for business cycle fluctuations and for welfare. The role of confidence for business cycle fluctuations is shown in Table 3. The table reports the forecast error variance (FEV) contribution of an innovation in confidence at different horizons in the presence and in the absence of financial frictions. With financial frictions, confidence plays an important role for the fluctuations in the economy, especially in the very short run. On

[16] Note that the forecast errors induced by fundamental shocks are always *iid*, irrespective whether the fundamental shock process is persistent or not.

Table 3: FEV contribution of confidence in % for different horizons

	Y	C	H	I	Y/H	N	B	Z/R
Model with financial frictions								
Impact	48.61	37.35	96.70	62.41	23.92	65.39	55.36	65.49
1 year	16.61	15.79	89.33	47.68	5.53	71.99	94.10	88.08
Model without financial frictions								
Impact	28.32	19.51	95.74	42.96	9.95	46.80	95.93	47.27
1 year	8.58	5.44	85.58	15.59	2.32	42.30	99.27	78.41

Table 4: Welfare cost of fluctuations due to confidence

λ^{ff}	λ	$\lambda^{\text{ff}}/\lambda$
0.07%	0.04%	1.6

Note: Welfare costs are measured in terms of steady state consumption.

impact, with the exception of the FEV of productivity, confidence explains between roughly half to almost all of the FEV of both non-financial and financial aggregates on impact. The contribution of confidence to the FEV of the non-financial variables vanishes quickly as the horizon increases. For financial variables, confidence has a longer lasting influence.

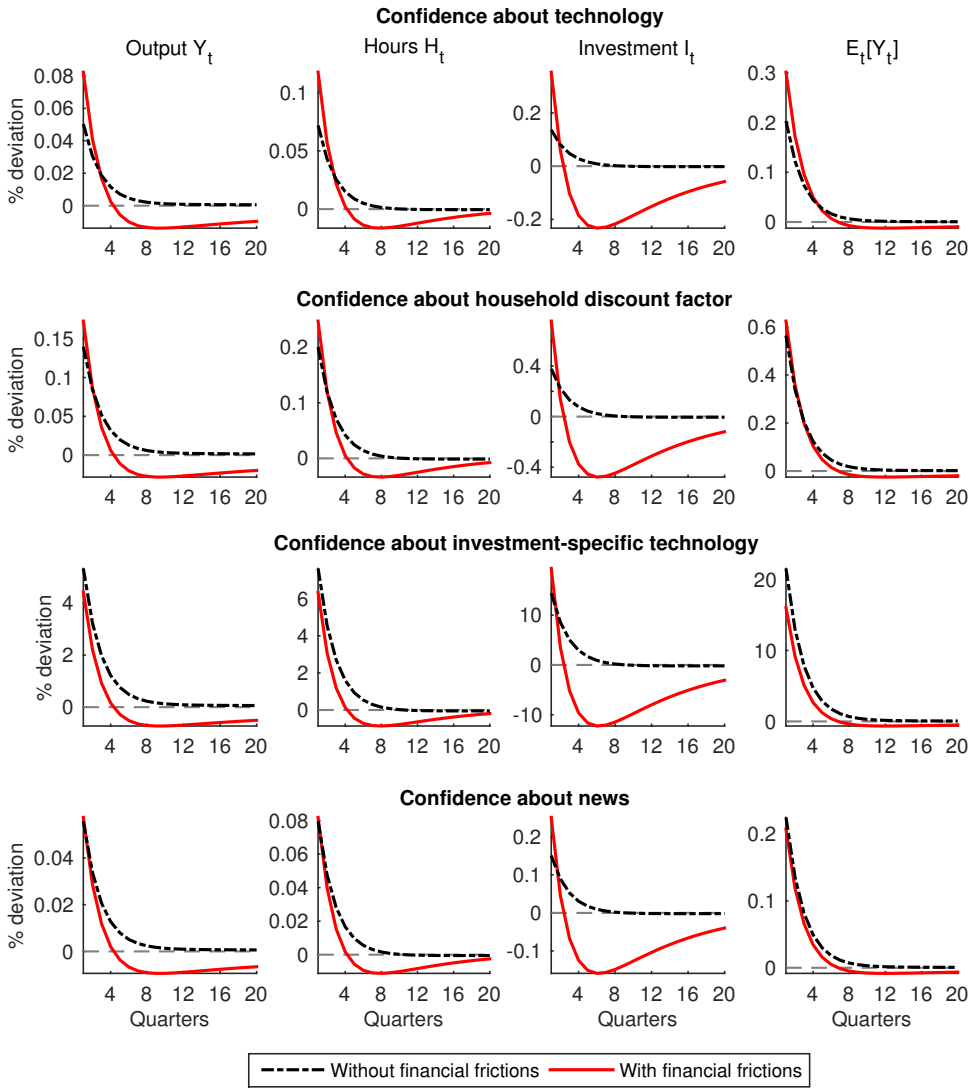
For the model without financial frictions, the influence of confidence on the FEV is, with the exception of credit, weaker on impact. For example, for output, the FEV contribution on impact is more than 70% larger in the presence of financial frictions. At the one-year horizon, the contribution to the FEV of output is almost twice as large. Hence, financial frictions also increase the persistence of confidence's influence on the FEV. Overall, these observations highlight that financial frictions reinforce the role of confidence for business cycle fluctuations.

The influence of financial frictions on the welfare costs of confidence-driven fluctuations is shown in Table 4. The table depicts the welfare costs λ of fluctuations induced by confidence in the presence (superscript ff) and the absence of financial frictions. The welfare costs are measured in terms of steady state consumption that agents would sacrifice to avoid the fluctuations. Details for the computation can be found in Appendix Section A.8. In the presence of financial frictions, the welfare costs are about 60% larger. The pronounced increase in the welfare costs of confidence-driven fluctuations underlines the strong influence financial frictions exert on the propagation of confidence shocks.

4.3 Confidence about other fundamentals

Given the importance of technology shocks for generating business cycles, we focus on confidence about technology in our baseline specification. In this section, we consider confidence about other (non-financial) fundamentals and find that the interaction of financial frictions

Figure 8: IRF to confidence about different fundamentals



and confidence remains the same.[17] Figure 8 graphs the response of production, hours, investment and first-order beliefs about aggregate production to an innovation in confidence about technology, the household discount factor, investment-specific technology and news about technology.[18] The red solid line depicts the responses in the presence of financial frictions ($\mu = 0.2$), the black dashed-and-dotted line depicts the responses in the absence of

[17] We do not consider confidence about financial fundamentals. The reason is that in the absence of financial frictions, financial shocks have no influence on the real, non-financial part of the economy. Hence, confidence about financial fundamentals also has no influence on the economy in the absence of financial frictions.

[18] We have also looked at other underlying fundamental shocks, e.g. government expenditure shocks. The qualitative results remain the same.

Table 5: Amplification decomposition: confidence about different fundamentals

	Technology	Discount	Investment-specific	News
Amplification confidence	64%	24%	-17%	3%
Amplification fundamental	5.7%	6.5%	-29%	-11%
Strengthening strategic compl.	54%	16%	16%	16%

Note: News shocks are shocks to TFP that are anticipated one year in advance. Technology and investment-specific technology shocks follow random walks. The discount factor shock is modeled as an AR(1) processe. The persistence is chosen to maximize the amplification effect and leads to $\rho_{disc} = 0$.

financial frictions ($\mu = 0$). Each confidence shock exhibits the same persistence as in our baseline model. For the specifications of the underlying fundamental shock processes, see Table 2.

Two observations stand out. First, up to a constant factor, the responses across the different confidence shock specifications are exactly the same. And this is true for both cases, with and without financial frictions. Hence, financial frictions have the same influence on the dynamics of the responses to the different confidence shocks: Irrespective of whether we consider confidence about technology or other fundamentals, financial frictions introduce a boom-bust pattern and a slow recovery phase into the response of output, hours and investment. The reason for the similarity in the responses and in the role of financial frictions is as follows. A confidence shock affects an agent's behavior by making him believe the other islands' agents change their behavior. The only channel how the other islands' behavior can affect the domestic island is via the centralized market. A higher production on all other islands raises the demand for the own island's specialized good and thereby improves the domestic island's terms of trade. Hence, in the context of our model, an innovation in confidence about *any* fundamental can be interpreted as a shock to the perceived future path of aggregate production, or similarly, terms of trade. Because of this similarity in the mechanism of all confidence shocks, the responses and the influence of financial frictions are the same up to a constant factor.

Second, while financial frictions exert the same influence on the dynamics of the responses, the amplification effect on impact differs across the different confidence shocks. Table 5 presents the same decomposition of the amplification as in 4.2.1. The first line depicts the overall amplification effect on the response of output on impact ($\hat{y}_{it}^{\text{ff}}/\hat{y}_{it}$) to the different confidence shocks. The second line describes the "standard" financial accelerator effect, i.e. how strongly financial frictions amplify the response of production to the underlying fundamental shock ($\tilde{\kappa}^{\text{ff}}/\tilde{\kappa}$). The third line reports the strengthening of the strategic complementarity as measured by $\tilde{\chi}^{\text{ff}}/\tilde{\chi}$.

In our set-up, the amplification is weaker for confidence shocks about other fundamentals than technology. Yet, across the board, we still observe a strengthening of the strategic complementarity. For example, the response of output to an innovation in confidence about the discount factor is amplified by 24%, whereas the response to an innovation in confidence about investment-specific technology is even dampened by 17%. The difference in the overall amplification effects mainly stems from the "standard financial accelerator" effect. Financial

frictions amplify the output's response to a discount factor shock by 6.5%, while they dampen the response to an investment-specific technology shock by 29%. Yet, financial frictions always strengthen the strategic complementarity by 16%, three times as strong as financial frictions amplify the response of production to technology shocks.

5 Empirical application: confidence and the Great Recession

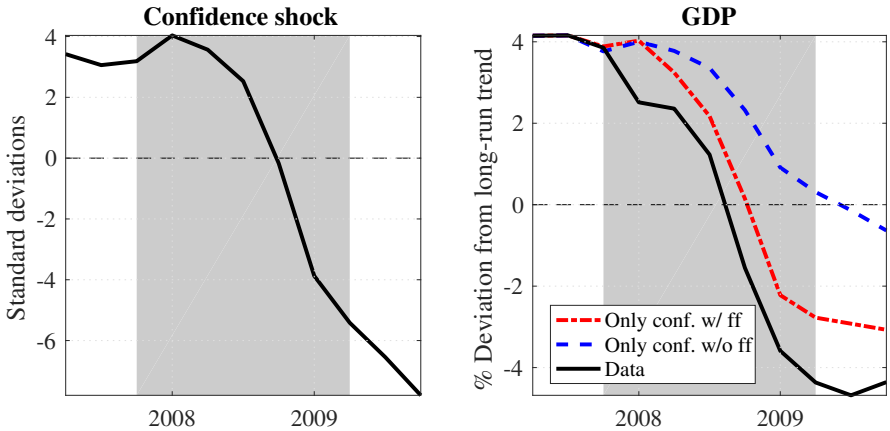
In this section, we explore the role of confidence and its interaction with financial frictions during the Great Recession in the US. Pursuing this task, we first augment our model and introduce a financial shock, as financial shocks are considered to be an important source for the Great Recession. More specifically, we include risk shocks—shocks on the dispersion of the idiosyncratic productivity level of entrepreneurs—as in CMR. Then we take our extended model to the data and identify the innovation pattern that explains the Great Recession. To identify the innovations, we use the Kalman filter and the Kalman smoother on linearly detrended GDP, total hours worked, and consumption from 1962–2015.[19] We choose our observable series for the following reasons. First, we use real GDP as a measure for business cycle activity. Second, we use hours such that together with real GDP, information about labor productivity is included. The information about labor productivity helps to disentangle technology and confidence shocks because labor productivity decreases in response to a confidence shock, whereas it has a positive response to a technology shock. Third, we use consumption to distinguish between risk shocks and the other shocks. In contrast to technology and confidence shocks, risk shocks lead to a counter-cyclical behavior in consumption. We also considered data on investment instead of consumption as an observable series, but the qualitative implications remain the same.

The left panel of Figure 9 depicts the identified confidence shock pattern. The gray shaded area denotes the time span between the NBER peak and trough dates. The presented confidence shock series is scaled by its standard deviation. Through the lens of our model, the US economy was characterized by a wave of unfounded optimism at the end of 2007 and the beginning of 2008. In 2008, unfounded optimism vanished quickly and turned into strong unfounded pessimism. The right panel of Figure 9 shows the empirical path and two counterfactual paths of US GDP. The red counterfactual depicts the evolution of GDP if only innovations in confidence would have hit the economy, beginning at the start of the recession. That is, from 2007Q4 onwards, the other two shocks are set to zero. The comparison to the empirical evolution (black line) tells us the contribution of confidence shocks to the fall in GDP. From the NBER peak to the trough date, GDP dropped from 3.9% above to 4.4% below trend. If only innovations in confidence would have occurred, the drop would have been only slightly milder. At the NBER trough date, GDP would have been 2.8% below trend. Hence, the drop in confidence can account for about 80% of the drop of US GDP.

The blue line depicts the counterfactual path of the same experiment but without fi-

[19] For the data description, see Appendix Section A.2. For the calibration of the risk shock, see Appendix Section A.9.

Figure 9: Confidence shock and counterfactual path of GDP



Note: Right panel: Counterfactual evolution of GDP when only confidence shocks hit the economy (red) and when, additionally, there are no financial frictions (blue).

nancial frictions from 2007Q4 onwards. The comparison between the first and the second counterfactual experiment indicates the importance of the interaction between confidence and financial frictions for the role of confidence for the Great Recession. In the absence of financial frictions, the influence of confidence on GDP would have been smaller. The drop in confidence would have only led to a drop in GDP to 0.3% above its long-run trend in 2009Q2. Hence without the interaction with financial frictions, confidence can only account for about 40% of the drop of US GDP.

Through the lens of our model, an unfounded wave of aggregate optimism in 2007 and the beginning of 2008 led entrepreneurs to leverage their net worth too strongly and take excessive risk. After an initial boom, the excessive lending and risk-taking caused the net worth of entrepreneurs to fall as the return on investment was lower than expected and more entrepreneurs had to declare bankruptcy. Eventually, the unfounded optimism turned into unfounded pessimism. The net worth of entrepreneurs dropped further and inferior credit conditions led to a strong reduction in aggregate investment which also reduced activity in the real, non-financial part of the economy. In the absence of financial frictions, the wave of optimism would not have improved the credit conditions and, hence, entrepreneurs would not have borrowed as much. Additionally, during the downturn, the drop in net worth would not have magnified the adverse effects on investment and the downturn would have been much milder.

This rationale fits well with the narrative that financial institutions invested too heavily in the mortgage market and subsequently suffered from large losses or even had to declare bankruptcy. In our model, entrepreneurs can be interpreted as both, firms in the non-financial business sector, or as financial firms that hold a non-diversified portfolio of loans to risky non-financial firms.[20] Hence, financial institutions heavily exposed to the mortgage market such as Bear Stearns or Lehman Brothers should be rather interpreted as entrepreneurs than banks in the context of our model.

[20]See CMR and Christiano and Ikeda (2011) for a more thorough discussion of the interpretation of entrepreneurs.

6 Concluding remarks

Recent events such as the Great Recession have highlighted feedbacks between confidence and financial markets. Understanding this interaction is essential to conduct the appropriate policy intervention and prevent future crises. In this paper, we make progress along this frontier by augmenting an RBC economy with financial frictions under incomplete information. We show that financial frictions interact with confidence and strongly amplify the effect of confidence shocks. Our mechanism also helps to explain the observed boom-bust pattern and the slow recovery in economic activity which are typically associated with financial recessions. Following a positive confidence shock, entrepreneurs are over-leveraged and end up with a depressed level of net worth which is accompanied by a prolonged fall in investment activity. An interesting avenue for further research is to specify a housing sector, as the mortgage market is suspected to be a crucial element of various crises, including the Great Recession. This extension would allow studying how debt and leverage decisions of households interact with confidence.

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A Appendix

A.1 The model

In the following, the general equilibrium conditions, the steady state conditions as well as the log-linearized general equilibrium conditions of our model are presented in a condensed form. In contrast to the main body of the text, we assume that firms can adjust the utilization of capital at a cost. Hence, the firms maximization problem writes:

$$\max_{k_{it},h_{it},u_{it}} \pi_{it} = p_{it}y_{it} - w_{it}h_{it} - r^k_{it}k_{it} - \psi(u_{it})k_{it},,$$
$$\text{s.t. } y_{it} = A_t(k_{it}u_{it})^{\alpha}h_{it}^{1-\alpha},$$
$$\psi(u_{it}) = \psi_0(u_{it}^{\frac{1}{1-\psi}} - u_{st.st.}^{\frac{1}{1-\psi}}), \psi_0 > 0$$

The case of fixed capital utilization is nested within this formulation and is obtained when $\psi = 1$. The list of general equilibrium equations is:

Local representative households optimality conditions:

$$w_{it} = (1-\alpha)\frac{\mathbb{E}_{it}[p_{it}y_{it}]}{h_{it}} \quad \Rightarrow \quad w_{it} = (1-\alpha)\frac{\mathbb{E}_{it}[Y_t]}{h_{it}} \tag{21}$$

$$r^k_{it} = \alpha\frac{\mathbb{E}_{it}[p_{it}y_{it}]}{k_{it}} - \mathbb{E}_{it}[\psi(u_{it})] \tag{22}$$

$$\alpha\frac{Y_t}{k_{it}} = \psi'(u_{it})u_{it} \tag{23}$$

Local representative capital producers optimality conditions:

$$\frac{1}{\bar{q}_{it}} = 1 - \Phi\left(\frac{i_{it}}{k_{it}}\right) - \Phi'\left(\frac{i_{it}}{k_{it}}\right)\frac{i_{it}}{k_{it}} \quad \text{investment demand} \tag{24}$$

$$\bar{q}_{it} = \frac{q_{it}(1-\delta) + \frac{i_{it}}{k_{it}}}{(1-\delta) + \frac{i_{it}}{k_{it}}\left(1 - \Phi\left(\frac{i_{it}}{k_{it}}\right)\right)} \quad \text{zero profit condition} \tag{25}$$

Local representative households optimality conditions:

$$c_{it}^{-\sigma_c} = \beta R_{it}\mathbb{E}'_{it}[c_{i,t+1}^{-\sigma_c}] \tag{26}$$

$$h^{\nu}_{it} = \mathbb{E}_{it}[c_{it}^{-\sigma_c}]w_{it} \tag{27}$$

Exogenous evolution of households transfers to entrepreneurs (w/Y exogenous):

$$w^e_{it} = \frac{w}{Y}p_{it}y_{it} \quad \Rightarrow \quad w^e_{it} = \frac{w}{Y}Y_t \tag{28}$$

Entrepreneurs rate of return:

$$R^k_{it+1} = \frac{r_{it+1} + q_{it+1}(1-\delta)}{\bar{q}_{it}} \tag{29}$$

Entrepreneurs optimality condition:

$$\mathbb{E}'_{it}\left\{A^j_t\left[\frac{1-\Gamma_t(\bar{\omega}^{N^j}_{it+1})}{1-\frac{R^k_{it+1}}{R_{it}}\left(\Gamma_t(\bar{\omega}^{N^j}_{it+1})-\mu G_t(\bar{\omega}^{N^j}_{it+1})\right)}\times\right.\right.$$
$$\left.\left.\frac{R^k_{it+1}}{R_{it}}\left(1-F_t(\bar{\omega}^{N^j}_{it+1})-\mu\bar{\omega}^{N^j}_{it+1}F'_t(\bar{\omega}^{N^j}_{it+1})\right)\right]\right\} = \mathbb{E}'_{it}\left\{A^j_t\left[1-F_t(\bar{\omega}^{N^j}_{it+1})\right]\right\},$$

$$\text{with } A^j_t \equiv \frac{\Lambda_{it,t+1}R^k_{it+1}}{1-\frac{R^k_{it+1}}{R_{it}}\left(\Gamma_t(\bar{\omega}^{N^j}_{it+1})-\mu G_t(\bar{\omega}^{N^j}_{it+1})\right)},\ \Lambda_{it,t+1} = \beta u'(c_{it+1})/u'(c_{it}), \tag{30}$$

$$F_t(\bar{\omega}_{t+1}) = \Phi\left(\frac{\ln(\bar{\omega}_{t+1})-\mu}{\sigma_{\omega,t}}\right),\ \mu = -\frac{\sigma^2_{\omega,t}}{2},\ G_t(\bar{\omega}_{t+1}) = \Phi\left(\frac{\ln\bar{\omega}_{t+1}-\frac{1}{2}\sigma^2_{\omega,t}}{\sigma_{\omega,t}}\right), \tag{31}$$

$$\Gamma_t(\bar{\omega}_{t+1}) = \Phi\left(\frac{\ln\bar{\omega}_{t+1}-\frac{1}{2}\sigma^2_{\omega,t}}{\sigma_{\omega,t}}\right) + \bar{\omega}_{t+1}\left(1-\Phi\left(\frac{\ln\bar{\omega}_{t+1}+\frac{1}{2}\sigma^2_{\omega,t}}{\sigma_{\omega,t}}\right)\right) \tag{32}$$

Aggregate zero profit condition of banks:

$$\frac{\bar{q}_{it}k_{it+1}}{n_{it+1}} = \frac{1}{1-\frac{R^k_{t+1}}{R_t}\left(\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})\right)} \tag{33}$$

Aggregate net worth:

$$n_{it+1} = \gamma[1-\Gamma_{t-1}(\bar{\omega}_{it})]R^k_{it}\bar{q}_{it-1}k_{it} + w^e_{it} \tag{34}$$

Final goods market clearing:

$$Y_t = c_{it} + i_{it} + \psi(u_{it})k_{it} + \mu G_{t-1}(\bar{\omega}_{it})R^k_{it}\bar{q}_{it-1}k_{it} \tag{35}$$

Law of motion of raw capital:

$$k_{i,t+1} = (1-\delta)k_{it} + i_{it}\left(1-\Phi\left(\frac{i_{it}}{k_{it}}\right)\right) \tag{36}$$

Local goods producers production function:

$$y_{it} = a_t h^{1-\alpha}_{it}(u_{it}k_{it})^\alpha \tag{37}$$

Some further useful definitions:

Aggregate credit:

$$b_{it+1} = \bar{q}_{it}k_{it+1} - n_{it+1}$$

Interest on credit:

$$z_{it+1} = R^k_{it+1}\bar{\omega}_{it+1}\frac{\bar{q}_{it}k_{it+1}}{b_{it+1}}$$

The relevant steady state values of the endogenous variables are:

$$R = 1/\beta$$
$$\bar{q} = 1$$
$$q = 1$$
$$R^k = R\frac{1 - F(\bar{\omega})}{(1 - \Gamma(\bar{\omega}))(1 - F(\bar{\omega}) - \mu\bar{\omega}F'(\bar{\omega})) + (1 - F(\bar{\omega}))(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))}$$
$$\frac{k}{Y} = \frac{\alpha}{R^k - (1 - \delta)}$$
$$\frac{k}{n} = \frac{1}{1 - \frac{R^k}{R}(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))}$$
$$\frac{n}{Y} = \frac{k/Y}{k/N}$$
$$\frac{x}{Y} = \delta\frac{k}{Y} \text{ (also with investment adjustment costs)}$$
$$\frac{c}{Y} = 1 - \frac{x}{Y} - \mu G(\bar{\omega})R^k\frac{k}{Y}$$
$$h = \left[\frac{1 - \alpha}{c/Y}\right]^{\frac{1}{1+\nu}}$$
$$\frac{b}{Y} = \frac{k}{Y} - \frac{n}{Y}$$

The list of log-linearized general equilibrium equations is:

$$(I) \quad \hat{y}_{it} = \alpha(\hat{u}_{it} + \hat{k}_{it}) + (1 - \alpha)\hat{h}_{it} + \hat{a}_t$$
$$(II) \quad \hat{\bar{q}}_{it} = \varphi(\hat{\imath}_{it} - \hat{k}_{it}) \Rightarrow$$
$$(III) \quad \hat{r}_{it} = \hat{Y}_t - \hat{k}_{it} - \hat{u}_{it}$$
$$(IV) \quad \hat{Y}_t - \hat{k}_{it} = \frac{1}{1 - \psi}\hat{u}_{it}$$
$$(V) \quad \hat{R}^k_{it} = \frac{\alpha\frac{Y}{k}}{R^k}(\mathbb{E}_{it}[\hat{Y}_t] - \hat{k}_{it} - \mathbb{E}_{it}[\hat{u}_{it}]) + \frac{1 - \delta}{R^k}\hat{q}_{it} - \hat{\bar{q}}_{it-1}, \quad \hat{\bar{q}}_{it} = (1 - \delta)\hat{q}_{it}$$
$$\Rightarrow \quad \hat{R}^k_{it} = \frac{\alpha\frac{Y}{k}}{R^k}(\mathbb{E}_{it}[\hat{Y}_t] - \hat{k}_{it} - \mathbb{E}_{it}[\hat{u}_{it}]) + \frac{1}{R^k}\hat{\bar{q}}_{it} - \hat{\bar{q}}_{it-1}$$

$$(VI) \quad \frac{R^k}{R}\left((1-F(\bar{\omega})-\mu\bar{\omega}F'_{\bar{\omega}}(\bar{\omega}))(1-\Gamma(\bar{\omega}))+...\right.$$
$$\left.+(1-F(\bar{\omega}))(\Gamma(\bar{\omega})-\mu G(\bar{\omega}))\right)(\mathbb{E}'_{it}[\hat{R}^k_{it+1}]-\hat{R}_{it})+\frac{R^k}{R}\left(F'_{\bar{\omega}}(\bar{\omega})\frac{R}{R^k}+...\right.$$
$$(-F'_{\bar{\omega}}(\bar{\omega})-\mu\bar{\omega}F''_{\bar{\omega},\bar{\omega}}(\bar{\omega})-\mu F'_{\bar{\omega}}(\bar{\omega}))(1-\Gamma(\bar{\omega}))+(1-F(\bar{\omega})-\mu\bar{\omega}F'_{\bar{\omega}}(\bar{\omega}))(-\Gamma'_{\bar{\omega}}(\bar{\omega}))+...$$
$$+(-F'_{\bar{\omega}}(\bar{\omega}))(\Gamma(\bar{\omega})-\mu G(\bar{\omega}))+(1-F(\bar{\omega}))(\Gamma'_{\bar{\omega}}(\bar{\omega})-\mu G'_{\bar{\omega}}(\bar{\omega})))\,\bar{\omega}\mathbb{E}'_{it}[\hat{\bar{\omega}}_{it+1}]+...$$
$$...\frac{R^k}{R}\left(F'_{\sigma_\omega}(\bar{\omega})\frac{R}{R^k}+(-F'_{\sigma_\omega}(\bar{\omega})-\mu\bar{\omega}F''_{\bar{\omega},\sigma_\omega}(\bar{\omega}))(1-\Gamma(\bar{\omega}))+...\right.$$
$$(1-F(\bar{\omega})-\mu\bar{\omega}F'_{\bar{\omega}}(\bar{\omega}))(-\Gamma'_{\sigma_\omega}(\bar{\omega}))+(-F'_{\sigma_\omega}(\bar{\omega}))(\Gamma(\bar{\omega})-\mu G(\bar{\omega}))+...$$
$$\left.(1-F(\bar{\omega}))(\Gamma'_{\sigma_\omega}(\bar{\omega})-\mu G'_{\sigma_\omega}(\bar{\omega}))\right)\sigma_\omega\hat{\sigma}_{\omega,t}$$

$$(VII) \quad \frac{n}{Y}\hat{n}_{it+1}=\gamma(1-\Gamma(\bar{\omega}))R^k\frac{k}{Y}(\hat{R}^k_{it}+\hat{\bar{q}}_{it-1}+\hat{k}_{it})-...$$
$$\gamma\Gamma'_{\bar{\omega}}(\bar{\omega})R^k\frac{k}{Y}\bar{\omega}\hat{\bar{\omega}}_{it}-\gamma\Gamma'_{\sigma_\omega}(\bar{\omega})R^k\frac{K}{Y}\sigma_\omega\hat{\sigma}_{\omega,it-1}+\frac{w^e}{Y}\hat{w}^e_{it}$$

$$(VIII) \quad \hat{w}^e_{it}=\hat{Y}_t$$

$$(IX) \quad \frac{n}{Y}(\hat{\bar{q}}_{it-1}+\hat{k}_{it})-\frac{R^k}{R}(\Gamma(\bar{\omega})-\mu G(\bar{\omega}))\frac{k}{Y}(\hat{R}^k_{it}-\hat{R}_{it-1})-\frac{R^k}{R}\frac{K}{Y}(\Gamma'_{\bar{\omega}}(\bar{\omega})-\mu G'_{\bar{\omega}}(\bar{\omega}))\bar{\omega}\hat{\bar{\omega}}_{it}$$
$$....-\frac{R^k}{R}\frac{K}{Y}(\Gamma'_{\sigma_\omega}(\bar{\omega})-\mu G'_{\sigma_\omega}(\bar{\omega}))\sigma_\omega\hat{\sigma}_{\omega,it-1}=\frac{n}{Y}\hat{n}_{it}$$

$$(X) \quad -\sigma_c\hat{c}_{it}=\hat{R}_{it}-\sigma_C\mathbb{E}'_{it}[\hat{c}_{it+1}]$$

$$(XI) \quad (1+\nu)\hat{h}_{it}=-\sigma_c\mathbb{E}_{it}[\hat{c}_{it}]+\mathbb{E}_{it}[\hat{Y}_t]$$

$$(XII) \quad \frac{c}{Y}\hat{c}_{it}+\frac{x}{Y}\hat{x}_{it}+\alpha\hat{u}_{it}+\mu G'_{\bar{\omega}}(\bar{\omega})\bar{\omega}R^k\frac{k}{Y}\hat{\bar{\omega}}_{it}+\mu G'_{\sigma_\omega}(\bar{\omega})\sigma_\omega R^k\frac{K}{Y}\hat{\sigma}_{\omega,it-1}+...$$
$$\mu G(\bar{\omega})R^k\frac{k}{Y}(\hat{R}^k_{it}+\hat{k}_{it}+\hat{\bar{q}}_{it-1})=\hat{Y}_t$$

$$(XIII) \quad \hat{k}_{it+1}=(1-\delta)\hat{k}_{it}+\delta\hat{x}_{it}$$

$$(XIV) \quad a_t=\rho_a a_{t-1}+\varepsilon_{a,t}$$

$$(XV) \quad \sigma_{\omega,t}=\rho_{\sigma_\omega}\sigma_{\omega,t-1}+\varepsilon_{\omega,t}$$

Additional equations not needed to close model (definitions):

$$\frac{b}{Y}\hat{b}_{it+1}=\frac{k}{Y}(\hat{\bar{q}}_{it}+\hat{k}_{it+1})-\frac{n}{Y}\hat{n}_{it+1}$$
$$\hat{z}_{it}=\hat{R}^k_{it}+\hat{\bar{q}}_{it-1}+\hat{k}_{it}+\hat{\bar{\omega}}_{it}-\hat{b}_{it}$$
$$\hat{Lev}_{it}=\hat{\bar{q}}_{it}+\hat{k}_{it+1}-\hat{n}_{it+1}$$
$$\hat{risk spr}_{it}=\hat{z}_{it}-\hat{R}_{it}$$

A.2 Data

We use quarterly observations for 7 macroeconomic US series covering the period from 1962Q1–2015Q1. The series for GDP, consumption, investment and hours worked are standard in the empirical analysis of aggregate data. The other 3 series are measures for the financial variables credit, net worth, and the interest spread. All series but the interest spread are measured in real, per capita terms.[21] We use the same measure for credit B_{t+1} in period t as CMR, i.e. we use data on credit to non-financial firms taken from the flow of funds dataset constructed by the US Federal Reserve Board. The period t measure for net worth is built using the S&P 500 stock market index. The indicator for the credit spread is built similarly to CMR, i.e. the credit spread is measured by the difference between the interest rate on BAA-rated corporate bonds and the federal funds rate. The data series are illustrated in table 6.

[21] In order to account for the jumps in the US population, we take the HP-filtered trend of the civilian noninstitutional population over 16 series when we deflate the variables by population growth.

Data	**Formula**
GDP	$GDP = NGDP/(GDPDEF \times CNP16OV)$
Consumption	$C = (NCND + NCS)/(GDPDEF \times CNP16OV)$
Investment	$I = (NCD + NFPI + NDI)/(GDPDEF \times CNP16OV)$
Hours	$H = AWH * CEVO16/CNP16OV$ (similar to CMR)
Net worth	$N = Stock/(GDPDEF \times CNP16OV)$
Credit	B=$(CNF + CNFA)/(GDPDEF \times CNP16OV)$
Interest Spread	ZsR=$((1 + (NZ - NFED)/100)^{\frac{1}{4}} - 1) \times 100$
Data	**Description and source**
NGDP	US nominal GDP, seasonally adjusted, Index 2009. Bureau of Economic Analysis (BEA)
GDPDEF	US GDP implicit price deflator, seasonally adjusted, Index 2009. BEA
CNP16OV	Civilian noninstitutational population over 16. Bureau of Labor Statistics (BLS)
CEVO16	Civilian employment over 16. BLS
NCND	Nominal personal consumption expenditure: nondurable goods, seasonally adjusted. BEA
NCS	Nominal personal consumption expenditure: services, seasonally adjusted. BEA
NCD	Nominal personal consumption expenditure: durable goods, seasonally adjusted. BEA
NFPI	Nominal fixed private investment, seasonally adjusted. BEA
NDI	Nominal change in private inventories, seasonally adjusted. BEA
AWH	Average weakly hours, nonfarm business, seasonally adjusted, Index 2009. BLS
Stock	S&P 500 stock market index.
CNF	Credit market instruments, total net borrowing and lending of nonfarm, nonfinancial corporate businesses, seasonally adjusted. Federal Reserve, US
CNFA	Credit market instruments, total net borrowing, and lending of nonfarm, noncorporate businesses, seasonally adjusted. Federal Reserve, US
NZ	Daily interest rates Moodys seasoned BAA rated corporated bonds. Federal Reserve, US
NFED	Daily Federal Funds Effective Rate. Federal Reserve, US

Table 6: Description of the data

A.3 Business cycle moments

For the data analysis, the logarithm of all series except the interest spread is taken and all series are Bandpass-filtered at frequencies corresponding to 6-32 quarters.[22] Thus, the filtered data reflects log-deviations from the trend, i.e. all series can now be compared to their theoretical counterparts in terms of deviations from steady state.

St. dev.:	Y	C	H	I	B	N	$\frac{Z}{R}$	$\frac{Y}{H}$			
Data	1.44	0.86	1.28	5.28	2.04	9.58	0.33	0.63			
Model	1.29	0.82	1.07	5.38	1.15	3.04	0.04	0.77			
Corr.:	C,Y	H,Y	I,Y	B,Y	N,Y	$\frac{Z}{R}$,Y	C,H	I,H	C,I	Y,$\frac{Y}{H}$	H,$\frac{Y}{H}$
Data	0.87	0.90	0.95	0.21	0.62	-0.63	0.86	0.89	0.80	0.45	0.01
Model	0.89	0.79	0.90	-0.19	0.76	-0.63	0.62	0.80	0.61	0.44	-0.17

Table 7: Bandpass-filtered moments (6-32 quarters)

A.4 Size of σ_ξ

In our baseline model, σ_ξ is much larger than σ_a. Intuitively, if the uncertainty faced by agents regarding the variability in an economic fundamental (payoff uncertainty) is much smaller than the uncertainty faced by agents due to higher-order beliefs about that fundamental (higher-order uncertainty), there are doubts about the plausibility of the formulated mechanism for higher-order uncertainty.[23] However, because of the persistence of shocks and the forward-looking aspects exhibited by our model, we cannot simply compare the standard deviations of the innovations to assess the payoff uncertainty faced by agents relative to higher-order uncertainty. For a more sophisticated assessment, we apply the present-value metric proposed by ACD: Let F_t^T be the present value of first-order beliefs about TFP from period t to period $t+T$, and S_t^T be the present value of the second-order belief about TFP from period t to period $t+T$. Further let $B_t^T \equiv S_t^T - F_t^T$ be the difference of these measures, which arises due to the confidence shock ξ_t:

$$F_t^T = \mathbb{E}_t\left[\sum_{j=0}^{T}\beta^j a_{t+j}\right],\ S_t^T = \mathbb{E}_t\left[\sum_{j=0}^{T}\beta^j (a_{t+j}+\xi_{t+j})\right],\ B_t^T = \mathbb{E}_t\left[\sum_{j=0}^{T}\beta^j \xi_{t+j}\right].$$

In order to get a rough measure for the higher-order uncertainty faced by agents relative to the payoff uncertainty, we consider the standard deviation of B_t^T relative to F_t^T from the perspective in period $t-1$. Thus, for an agent in period $t-1$, the ratio $\chi_{a,\xi}$ gives us the rough measure:

$$\chi_{a,\xi} = \frac{V(B_t^T|t-1,t-2,...)}{V(F_t^T|t-1,t-2,...)}.$$

[22] The bandpass filter is preferable to the simpler HP filter because it removes all high-frequency "noise" and low-frequency trends; see Stock and Watson (1999).

[23] ACD emphasize that for static common-prior settings, methods exist to obtain a bound on the size of the higher-order uncertainty as a function of the underlying payoff uncertainty, developed in Bergemann and Morris (2013) and Bergemann, Heumann, and Morris (2015). However, an analogous method does not yet exist for dynamic settings.

Under the baseline calibration we observe a ratio of 1.87 for $T = 8$ years and a ratio of 0.09 for $T = \infty$. Thus, the implied higher-order uncertainty relative to the payoff uncertainty of our calibration is relatively high. However, as soon as we introduce variable capital utilization into our model these ratios drop strongly. For example if we assume $\psi = 0.3$ as in ACD, the same calibration exercise delivers $\sigma_\xi = 4.02$, $\sigma_a = 0.62$. The corresponding $\chi_{a,\xi}$ ratios drop to 0.30 for $T = 8$ years and to 0.01 for $T = \infty$. Hence, with variable capital utilization, the higher-order uncertainty relative to the payoff uncertainty is rather modest. In the main body of the text, we abstract from variable capital utilization to keep the explanation of the amplification effects as simple as possible. The introduction of capital utilization does not change the qualitative insights and only mildly affects the quantitative insights.

A.5 A model with input financing constraints

A.5.1 Model description

Consider a model economy that has the same island and information structure as our baseline model. The model economy consists of a continuum of islands, indexed by i, and a mainland. Islands are connected with each other via the production of a final good. Each island i produces a specialized good, that is used as a complement in the final good production. The only difference between the model economy and the baseline model is that the island specific economies are different. The representative household on island i maximizes the following objective

$$\max_{c_{it}, L_{it}, h_{it}} \frac{\left(c_{it} - \frac{h_{it}^{1+\nu}}{1+\nu}\right)^{1-\rho}}{1-\rho} + \frac{L_{it}^{1-\gamma}}{1-\gamma}$$
$$s.t.\ P_t c_{it} + q_{it} L_{it} \leq w_{it} h_{it} + \pi_{it},$$

where c_{it} is consumption, L_{it} is land, h_{it} is hours worked, P_t the price of consumption goods (which we normalize to 1), q_{it} is the price of land and π_{it} is the share of profits of a household. The households optimality conditions are:

$$h_{it}^{\nu} = w_{it},$$
$$\mathbb{E}_{it}[\lambda_{it}] q_{it} = L_{it}^{-\gamma},$$
$$\lambda_{it} = \left(c_{it} - \frac{h_{it}^{1+\nu}}{1+\nu}\right)^{-\rho}.$$

Households make their labor supply and the land demand decision in the first-stage. The consumption decision takes place in the second-stage.

The representative firms maximization problem is as follows:

$$\max_{h_{it}} \pi_{it} = p_{it} y_{it} - w_{it} h_{it} \left(1 + \tau\left(\frac{w_{it} h_{it}}{q_{it} L_{it}}\right)\right) + q_{it} L_{it}$$
$$\tau(st.st.) = \tau'(st.st.) = 0,\ \tau''(st.st.) \left(\frac{wh}{qL}\right)^2 = \varphi > 0$$
$$y_{it} = A_t h_{it}^{1-\alpha}$$

The representative firm earns revenues from two sources. First, it produces a specialized good y_{it} that it sells at price p_{it} on the mainland. Second, firms are endowed with land

L_{it}, which they sell at price q_{it}. The costs of hiring workers takes a special form. This form, in a reduced form way, is designed to capture financial frictions of the financing of labor input through the function τ. The idea is that firms need to finance labor input $w_{it}h_{it}$ before they receive revenues. Land serves as a collateral. For a given value of the collateral $q_{it}L_{it}$, a higher amount of lending $w_{it}h_{it}$ raises the financing costs. In contrast, the larger the value of the collateral, the lower are the additional cost generated by financial frictions. The optimal labor choice satisfies:

$$(1-\alpha)\frac{\mathbb{E}_{it}[Y_t]}{h_{it}} = w_{it}\left(1+\tau\left(\frac{w_{it}h_{it}}{q_{it}L_{it}}\right)+\tau'\left(\frac{w_{it}h_{it}}{q_{it}L_{it}}\right)\frac{w_{it}h_{it}}{q_{it}L_{it}}\right)$$

The model is completely characterized by the following set of general equilibrium equations:

$$\begin{aligned}
&h_{it}^{\nu} = w_{it}\\
&(1-\alpha)\frac{\mathbb{E}_{it}[Y_t]}{h_{it}} = w_{it}\left(1+\tau\left(\frac{w_{it}h_{it}}{q_{it}\bar{L}_i}\right)+\tau'\left(\frac{w_{it}h_{it}}{q_{it}L_{it}}\right)\frac{w_{it}h_{it}}{q_{it}\bar{L}_i}\right)\\
&\mathbb{E}_{it}[\lambda_{it}]q_{it} = \bar{L}_i^{-\gamma}\\
&\lambda_{it} = \left(c_{it}-\frac{h_{it}^{1+\nu}}{1+\nu}\right)^{-\rho}\\
&y_{it} = A_t h_{it}^{1-\alpha}\\
&c_{it} = Y_t - w_{it}h_{it}\tau\left(\frac{w_{it}h_{it}}{q_{it}\bar{L}_i}\right)
\end{aligned}$$

In order to transform the static model into a linear model, we take a log-linear approximation around its steady state:[24]

$$\begin{aligned}
&\nu\hat{h}_{it} = \hat{w}_{it}\\
&\mathbb{E}_{it}[\hat{Y}_t] - \hat{h}_{it} = \hat{w}_{it} + \varphi\left(\hat{w}_{it}+\hat{h}_{it}-\hat{q}_{it}\right)\\
&\mathbb{E}_{it}[\hat{\lambda}_{it}] = -\hat{q}_{it}\\
&\hat{\lambda}_{it} = -\rho\frac{Y}{\lambda^{\frac{-1}{\rho}}}\hat{Y}_t + \rho\frac{\frac{h^{1+\nu}}{1+\nu}}{\lambda^{\frac{-1}{\rho}}}(1+\nu)\hat{h}_{it}\\
&\hat{y}_{it} = \hat{A}_t + (1-\alpha)\hat{h}_{it}\\
&\hat{Y}_t = \frac{c}{Y}\hat{c}_{it} + \left(wh\tau\left(\frac{wh}{q\bar{L}}\right)+wh\tau'\left(\frac{wh}{q\bar{L}}\right)\frac{wh}{q\bar{L}}\right)\left(\hat{w}_{it}+\hat{h}_{it}\right) - wh\tau\left(\frac{wh}{q\bar{L}}\right)\hat{q}_{it}
\end{aligned}$$

[24] To have a steady state, we need to assume that the technology shock is stationary.

A.5.2 Beauty contest representation and solution

We can obtain the same equilibrium outcome from a beauty contest between islands:

$$
\begin{aligned}
&\mathbb{E}_{it}[\hat{Y}_t] = (1+\nu)(1+\varphi)\hat{h}_{it} - \varphi\rho\frac{Y}{\lambda^{\frac{-1}{\rho}}}\mathbb{E}_{it}[\hat{Y}_t] + \varphi\rho\frac{\frac{h^{1+\nu}}{1+\nu}}{\lambda^{\frac{-1}{\rho}}}(1+\nu)\hat{h}_{it} \\
&\Rightarrow \hat{y}_{it} = \mathbb{E}_{it}[\hat{A}_t] + B\mathbb{E}_{it}[\hat{Y}_t], \\
&B \equiv \frac{\left(1+\varphi\rho\frac{Y}{\lambda^{\frac{-1}{\rho}}}\right)(1-\alpha)}{(1+\nu)\left(1+\varphi\left(1+\rho\frac{\frac{h^{1+\nu}}{1+\nu}}{\lambda^{\frac{-1}{\rho}}}\right)\right)}, \\
&Y = A^{1+(1-\alpha)(\nu+\alpha)}(1-\alpha)^{(1-\alpha)(\nu+\alpha)}, \\
&h = [(1-\alpha)A]^{\nu+\alpha}, \\
&\lambda = \left(Y - \frac{h^{1+\nu}}{1+\nu}\right)^{-\rho}.
\end{aligned}
$$

B denotes the standard definition of the strategic complementarity between island's production choices. In the form used in the main body we have:

$$
\begin{aligned}
\hat{y}_{it} =& \kappa\mathbb{E}_{it}[\hat{A}_t] + \chi\mathbb{E}_{it}[\hat{Y}_t - \hat{y}_{it}], \\
\kappa \equiv& \frac{1}{1-B},\ \chi \equiv \frac{B}{1-B}.
\end{aligned}
$$

Proof of Proposition 1. The solution of the beauty contest

$$\hat{y}_{it} = \mathbb{E}_{it}[\hat{A}_t] + B\mathbb{E}_{it}[\hat{Y}_t]$$

can be obtained by forward iteration and applying the heterogeneous prior setting:

$$
\begin{aligned}
\hat{y}_{it} &= \mathbb{E}_{it}[\hat{A}_t] + B\mathbb{E}_{it}[\hat{Y}_t] \\
&= \mathbb{E}^1_{it}[\hat{A}_t] + B\mathbb{E}^2_{it}[\hat{A}_t] + B^2\mathbb{E}^3_{it}[\hat{A}_t] + B^3\mathbb{E}^4_{it}[\hat{A}_t]...
\end{aligned}
$$

where $\mathbb{E}^k_{it}[\cdot]$ is the kth-order belief about the fundamental:

$$\mathbb{E}^2_{it} = \mathbb{E}_{it}[\bar{\mathbb{E}}_t[\hat{A}_t]],\ \mathbb{E}^3_{it} = \mathbb{E}_{it}[\bar{\mathbb{E}}_t[\bar{\mathbb{E}}_t[\hat{A}_t]]],\ ...;\quad \bar{\mathbb{E}}_t[\cdot] = \int \mathbb{E}_{jt}[\cdot]dj.$$

For a fundamental shock, all agents share the same information and therefore the law of iterated expectations applies. However, in the case of confidence shocks the information sets are heterogenous. Hence, the law of iterated expectations is no longer applicable. The advantage of the heterogenous prior setting is that it simplifies the structure of higher-order beliefs. Under the heterogenous prior setting, we assume f_t and ξ_t are perfectly observed. Beliefs satisfy

$$
\begin{aligned}
&\mathbb{E}_{it}[\hat{A}_t] = \hat{A}_t \\
&\mathbb{E}_{it}[\mathbb{E}_{jt}[\hat{A}_t]] = \mathbb{E}_{it}[\hat{A}_t] + \xi_t = \hat{A}_t + \xi_t
\end{aligned}
$$

where j indicates another island $j \neq i$. Since any island i knows that all other islands are alike, we have $\mathbb{E}_{jt}[\cdot] = \bar{\mathbb{E}}_t[\cdot]$ from perspective of island i. Ongoing iteration leads to:

$$\begin{aligned}
\mathbb{E}^1_{it}[\hat{A}_t] &= \hat{A}_t \\
\mathbb{E}^2_{it}[\hat{A}_t] &= \mathbb{E}_{it}[\bar{\mathbb{E}}_t[\hat{A}_t]] = \hat{A}_t + \xi_t \\
\mathbb{E}^3_{it}[\hat{A}_t] &= \mathbb{E}_{it}[\bar{\mathbb{E}}_t[\bar{\mathbb{E}}_t[\hat{A}_t]]] = \hat{A}_t + \xi_t + \xi_t \\
&= ... \\
\mathbb{E}^k_{it}[\hat{A}_t] &= \hat{A}_t + (k-1)\xi_t.
\end{aligned}$$

Therefore in equilibrium, output reacts as follows to fundamental and confidence shocks:

$$\begin{aligned}
y_{it} =& \mathbb{E}^1_{it}[\hat{A}_t] + B\mathbb{E}^2_{it}[\hat{A}_t] + B^2\mathbb{E}^3_{it}[\hat{A}_t] + B^3\mathbb{E}^4_{it}[\hat{A}_t]... \\
=& \hat{A}_t + B\hat{A}_t + B^2\hat{A}_t + B^3\hat{A}_t + ... \\
& ... + B\xi_t + B^2\xi_t + B^3\xi_t + ... \\
& ...B^2\xi_t + B^3\xi_t + ... \\
& ...B^3\xi_t + ... \\
y_{it} =& \frac{1}{1-B}\hat{A}_t + \frac{B}{(1-B)^2}\xi_t.
\end{aligned}$$

Using the definition of κ and χ, we obtain:

$$\hat{y}_{it} = \kappa\hat{A}_t + \kappa\chi\xi_t.$$

□

A.5.3 Amplification effect

In order for financial frictions ($\varphi > 0$)to amplify the response to a technology shock we need $\kappa(\varphi > 0) > \kappa(\varphi = 0)$. This condition is equivalent to a larger B when $\varphi > 0$ compared to $\varphi = 0$:

$$\begin{aligned}
& B(\varphi > 0) > B(\varphi = 0) \\
& \frac{\left(1 + \varphi\rho\frac{Y}{\lambda^{\frac{-1}{\rho}}}\right)(1-\alpha)}{(1+\nu)\left(1 + \varphi\left(1 + \rho\frac{\frac{h^{1+\nu}}{1+\nu}}{\lambda^{\frac{-1}{\rho}}}\right)\right)} > \frac{1-\alpha}{1+\nu} \\
& \varphi\rho\frac{Y - \frac{h^{1+\nu}}{1+\nu}}{\lambda^{\frac{-1}{\rho}}} > \varphi \\
& \rho > 1
\end{aligned}$$

Hence, a necessary and sufficient condition is $\rho > 1$.

If $B(\varphi > 0) > B(\varphi = 0)$, we furthermore have that $\chi(\varphi > 0) > \chi(\varphi = 0)$. Hence, financial frictions strengthen the strategic complementarity between islands.[25]

[25]The standard definition of the strategic complementarity parameter in a beauty contest model is $\frac{\chi}{1+\chi}$, which is increasing in χ.

A.6 Quantifying the amplification effect

Proof of Proposition 2. The policy rule for production on island i is:

$$\hat{y}_{it} = a_k \hat{k}_{it} + a_n \hat{n}_{it} + \tilde{\kappa}\hat{a}_t + a_\xi \xi_t.$$

First order beliefs about aggregate output are:

$$\mathbb{E}_{it}[\hat{Y}_t] = \mathbb{E}_{it}[\hat{y}_{jt}] = a_k \underbrace{\mathbb{E}_{it}[\hat{k}_{jt}]}_{=k_{it}=K_t} + a_n \underbrace{\mathbb{E}_{it}[\hat{n}_{jt}]}_{=n_{it}=N_t} + \tilde{\kappa} \underbrace{\mathbb{E}_{it}[\mathbb{E}_{jt}[\hat{a}_t]]}_{=\hat{a}_t+\xi_t} + a_\xi \underbrace{\mathbb{E}_{it}[\mathbb{E}_{jt}[\hat{\xi}_t]]}_{=\xi_t}$$

It follows that first order beliefs about terms of trade can be rewritten as function of the confidence shock only:

$$\mathbb{E}_{it}[\hat{Y}_t - \hat{y}_{it}] = \tilde{\kappa}\xi_t$$

Hence, the response of production on island i can be written as:

$$\hat{y}_{it} = a_k \hat{k}_{it} + a_n \hat{n}_{it} + \tilde{\kappa}\hat{a}_t + \tilde{\chi}\mathbb{E}_{it}[\hat{Y}_t - \hat{y}_{it}],\ \tilde{\chi} \equiv \frac{a_\xi}{\tilde{\kappa}}$$

Finally, capital k_{it} and net worth n_{it} are predetermined. Hence, for the response of output on impact it is $\hat{k}_{it} = \hat{n}_{it} = 0$. □

Proof of Corollary 2.1. From the proof of Proposition 2 follows $a_\xi = \tilde{\kappa}\tilde{\chi}$. Hence, using the policy rule and using $\hat{k}_{it} = \hat{n}_{it} = 0$ delivers:

$$\hat{y}_{it} = \tilde{\kappa}\hat{a}_t + \tilde{\kappa}\tilde{\chi}\xi_t.$$

□

A.7 Ex-post optimal leverage choice, interest rate and bankruptcy probability

The optimal leverage decision of entrepreneurs is obtained from maximizing the entrepreneurs' objective subject to the zero profit condition of banks. To compute the ex-post optimal leverage decision, we simply impose the realized instead of the equilibrium expectations about the next period's return on capital. In log-linearized terms:

$$\widehat{lev}_{it} = \upsilon \left(\mathbb{E}'_{it}[\hat{R}^k_{it+1}] - \hat{R}_{it} \right),$$
$$\widehat{lev}^{expo}_{it} = \upsilon \left(\hat{R}^k_{it+1} - \hat{R}_{it} \right),$$

where $\upsilon = \left(C + D\frac{A}{B}\right) > 0$, and where steady state terms A, B, C and D are given by

$$\begin{aligned}
A \equiv & \frac{R^k}{R}\left((1 - F(\bar{\omega}) - \mu\bar{\omega}F'_{\bar{\omega}}(\bar{\omega}))(1 - \Gamma(\bar{\omega})) + (1 - F(\bar{\omega}))(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))\right), \\
B \equiv & \frac{R^k}{R}\Bigg(F'_{\bar{\omega}}(\bar{\omega})\frac{R}{R^k} + (-F'_{\bar{\omega}}(\bar{\omega}) - \mu\bar{\omega}F''_{\bar{\omega},\bar{\omega}}(\bar{\omega}) - \mu F'_{\bar{\omega}}(\bar{\omega}))(1 - \Gamma(\bar{\omega})) + ... \\
& (1 - F(\bar{\omega}) - \mu\bar{\omega}F'_{\bar{\omega}}(\bar{\omega}))(-\Gamma'_{\bar{\omega}}(\bar{\omega})) + ... \\
& +(-F'_{\bar{\omega}}(\bar{\omega}))(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})) + (1 - F(\bar{\omega}))(\Gamma'_{\bar{\omega}}(\bar{\omega}) - \mu G'_{\bar{\omega}}(\bar{\omega})))\,\bar{\omega}, \\
C \equiv & \frac{Y}{n}\frac{R^k}{R}(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))\frac{k}{Y}, \\
D \equiv & \frac{Y}{n}\mu G'_{\bar{\omega}}(\bar{\omega}))\bar{\omega}.
\end{aligned}$$

Thus, the ex-post optimal leverage decision takes the perspective of a small group of entrepreneurs—small enough such that they have no influence on the aggregate outcomes but at the same time the group represents the full distribution of idiosyncratic productivity levels ω—that know in advance the next period's return on capital. Hence, $\widehat{lev}_{it}^{expo}$ is an off-equilibrium concept. The small group assumption is needed in order that the bank can offer the state-contingent contract defined by equation (11). The schedule of state-contingent contract of banks implicitly assumes that always a continuum of entrepreneurs makes the same leverage choice. Hence, the contract schedule of banks is based on the assumption that banks can diversify away the idiosyncratic risk. If we consider only a single entrepreneur that makes a specific leverage choice, banks can not diversify away the idiosyncratic risk component and the contract schedule would take a different form.

The ex-post optimal interest payment and bankruptcy probability is obtained in a similar manner. To obtain the optimal bankruptcy probability, we first compute the ex-post optimal bankruptcy threshold $\bar{\omega}^{expo}$. We obtain $\bar{\omega}^{expo}$ by imposing the next period's realized return on capital in entrepreneurs optimal contract decision:[26]

Entrepreneurs optimal contract choice:

$$A\left(\mathbb{E}'_{it}[\hat{R}^k_{it+1}] - \hat{R}_{it}\right) + B\mathbb{E}'_{it}[\widehat{\omega}_{it+1}]$$

Entrepreneurs optimal contract choice given the return on capital:

$$\widehat{\omega}^{expo}_{it+1} = \frac{A}{B}\left(\hat{R}^k_{it+1} - \hat{R}_{it}\right)$$

Ex-post optimal bankruptcy probability:

$$\widehat{F(\bar{\omega}^{expo}_{it+1})} = f(\bar{\omega})\bar{\omega}\hat{\bar{\omega}}^{expo}_{it+1}$$

If the small group of entrepreneurs that knows next period's return on capital in advance chooses the ex-post optimal leverage level, it faces a bankruptcy probability that equals $F(\bar{\omega}^{expo}_{t+1})$.

The ex-post optimal interest rate z^{expo}_{t+1} is obtained by the definition for the ex-post optimal threshold $\bar{\omega}_{t+1}$, the realized return on capital, the ex-post optimal leverage choice and the ex-post optimal bankruptcy threshold:

$$\hat{z}^{expo}_{it+1} = \hat{\bar{\omega}}^{expo}_{it+1} + \hat{R}^k_{it+1} - \frac{1}{lev-1}\hat{lev}^{expo}_{it}.$$

z^{expo}_{t+1} is the interest payment the small group of entrepreneurs would face if they chose the ex-post optimal leverage level.

A.8 Welfare loss computation

We measure the welfare loss λ of fluctuations induced by confidence in percentage terms of steady state consumption:

$$\sum_{t=0}^{\infty}\beta^t\left[\log(C(1-\lambda)) - \frac{H^{1+\nu}}{1+\nu}\right] = \sum_{t=0}^{\infty}\beta^t\left[\log(\tilde{C}_t) - \frac{\tilde{H}_t^{1+\nu}}{1+\nu}\right],$$

$$\lambda = 1 - \exp\left((1-\beta)\sum_{t=0}^{\infty}\beta^t\left[\log(\tilde{C}_t) - \log(C) - \left(\frac{\tilde{H}_t^{1+\nu}}{1+\nu} - \frac{H^{1+\nu}}{1+\nu}\right)\right]\right)$$

[26] Note that in steady state $\bar{\omega}^{expo} = \bar{\omega}$.

$\tilde{C}_t$ and $\tilde{H}_t$ denote consumption and employment in an economy driven by confidence shocks, whereas C and H denote consumption and employment in the steady state.

We compute the average welfare loss of confidence driven fluctuations as follows. We generate 1000 confidence shock time series of the length of 500 years. For each time series, we use our log-linearized decision rules to compute a series of consumption and hours and compute λ:

$$\lambda = 1 - \exp\left((1-\beta)\sum_{t=0}^{\infty}\beta^t\left[\hat{\tilde{c}}_t - \left(\frac{\exp((1+\nu)(\hat{\tilde{h}}_t + \log(H))}{1+\nu} - \frac{\exp((1+\nu)\log(H))}{1+\nu}\right)\right]\right)$$

The solution approach does not allow for a higher-order approximation of the model solution. The welfare loss of confidence driven fluctuations is then computed by taking the average over the 1000 values of λ.

A.9 Great Recession counterfactual

The risk shock $\sigma_{\omega,t}$ is modelled as the time-varying standard deviation of the idiosyncratic productivity shock ω to the entrepreneur's raw capital. We assume the logarithm of $\sigma_{\omega,t}$ follows an AR(1) process with mean $\log(\bar{\sigma}_\omega)$:

$$\log(\sigma_{\omega,t}) = \rho_s \log(\sigma_{\omega,t-1}) + (1-\rho_s)\log(\bar{\sigma}_\omega) + \varepsilon_{s,t},\ \varepsilon_{s,t} \sim \mathcal{N}(0, \sigma_s^2).$$

As all other fundamental shocks, $\sigma_{\omega,t}$ hits the economy in the beginning of the first-stage of a period. The persistence of the risk shock is set to $\rho_s = 0.97$, based on the estimates of CMR. All other model parameters, but the standard deviations of the shock, remain the same as in our baseline model. To calibrate the standard deviations of all shocks, we follow the same calibration procedure as outlined in Section 4.1. This procedure leads us to $\sigma_a = 0.70$, $\sigma_s = 0.03$, $\sigma_\xi = 10.08$.