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On the Instability of Banking and other Financial Intermediation

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DISCUSSION PAPERS

ON THE INSTABILITY OF BANKING
AND OTHER FINANCIAL INTERMEDIATION[*]

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Abstract

Are financial intermediaries inherently unstable? If so, why? What does this suggest about government intervention? To address these issues we analyze whether model economies with financial intermediation are particularly prone to multiple, cyclic, or stochastic equilibria. Four formalizations are considered: a dynamic version of Diamond-Dybvig banking incorporating reputational considerations; a model with delegated investment as in Diamond; one with bank liabilities serving as payment instruments similar to currency in Lagos-Wright; and one with Rubinstein-Wolinsky intermediaries in a decentralized asset market as in Duffie et al. In each case we find, for different reasons, financial intermediation engenders instability in a precise sense.

JEL Classification Numbers: D02, E02, E44, G21

Keywords: Banking, Financial Intermediation, Instability, Volatility

Banks, as several banking crisis throughout history have demonstrated, are fragile institutions. This is to a large extent unavoidable and is the direct result of the core functions they perform in the economy. *Finance Market Watch Program @ Re-Define, Banks: How they Work and Why they are Fragile.*

Introduction

It is often said that banks, or more generally financial intermediaries, are inherently unstable and prone to volatility. This seems to be based on the notion that financial institutions are special compared to, say, producers or middlemen in retail. Keynes (1936), Kindleberger (1978) and Minsky (1992) are names associated with such a position, with Williams (2015) providing a recent perspective (see also Akerlof and Shiller 2009 or Reinhart and Rogoff 2009). Rolnick and Weber (1986) provide evidence of the widespread acceptance of this view when they say: "Historically, even some of the staunchest proponents of laissez-faire have viewed banking as inherently unstable and so requiring government intervention." As a leading case, Friedman (1960) defended unfettered markets in virtually all contexts, but advocated bank regulation in his program for monetary stability. As additional evidence, consider the voluminous literature dedicated to the study of bank runs.[1]

We share an interest in the questions with which Gorton and Whinton (2002) start their survey: "Why do financial intermediaries exist? What are their roles? Are they inherently unstable? Must the government regulate them?" While there are different ways to proceed, our approach is to build formal models of the institutions and see if they are particularly prone to multiple equilibria or volatile dynamics, including cyclic, chaotic or stochastic outcomes that entail fluctuations even if fundamentals are constant. Central to this approach, by models *of* intermediation we

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[1] For now we discuss bank runs, panics, financial crises, etc. without defining these formally. As Rolnick and Weber (1986) put it, "There is no agreement on a precise definition of inherent instability in banking. However, the conventional view is that it means that general bank panics can occur without economy-wide real shocks." They add "The usual explanation... involves a local real economic shock that becomes exaggerated by the actions of incompletely informed depositors," and suggest this is consistent with Friedman and Schwartz's (1963) view. In terms of models, Chari and Jagannathan (1988) have withdrawals by informed depositors lead to withdrawals by others, while Gu (2011) formalizes this as rational herding. Our approach is different, and avoids fixating only on runs, but does focus squarely on volatility "without economy-wide real shocks."

mean more than models *with* intermediation. It does not suffice to assert, say, that households lend to banks and banks lend to firms but households do not lend to firms – that may be a model *with* banking but not *of* banking.[2]

While there is much research on financial intermediaries, there is no generally-accepted, all-purpose model. This is because the institutions perform a myriad of functions that are difficult to capture in a single setup: they serve as middlemen between savers and borrowers or asset sellers and buyers; they find, screen and monitor investment opportunities on behalf of depositors; they issue liabilities like demand deposits that facilitate third-party transactions; they provide liquidity insurance or maturity transformation; they are safe keepers of cash and other valuables; and they maintain privacy about their assets and/or their customers. Different approaches are used to model these diverse activities, and, in this tradition, we consider several distinct specifications. All of these are constructed using building blocks taken from off-the-shelf models, although the ways in which we combine and apply them are novel, and lead to new insights.

The first formulation extends Diamond and Dybvig's (1983) model of liquidity insurance (or maturity transformation) to an infinite horizon, to highlight bankers' reputation as in Gu et al. (2013a), which is itself based on Kehoe and Levine (1993). The second features fixed costs of exploiting investment opportunities, similar to Diamond (1984) and Huang (2017). The third, an adaptation of Nosal et al. (2017), puts intermediaries like those in Rubinstein and Wolinsky (1987) into an OTC asset market similar to Duffie et al. (2005). The fourth has bank liabilities serving as payment instruments, similar to currency in Lagos and Wright (2005) or Berentsen et al. (2007), in an environment where bank liabilities are less susceptible to loss or

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[2]The declaration that households lend to banks and banks lend to firms but households do not lend to firms is reminiscent of monetary economics following Clower (1965), who said money buys goods and goods buy money but goods do not buy goods. While once a popular shortcut, it is hard to argue that Clower (cash-in-advance) constraints constitute the last word in monetary theory, and we feel similarly about banking (see Wright 2017 for more on this). Of course it is not necessary for every study to make everything endogenous – e.g., Debreu (1959) made progress using a theory *with* firms and households but not a theory *of* firms and households – but it is crucial, we think, to have financial institutions emerge endogenously when asking if they are unstable as "the direct result of the core functions they perform" (from the epigraph).

theft, as in He et al. (2007) and Sanches and Williamson (2010), or less sensitive to information, as in Andolfatto and Martin (2013), Dang et al. (2017) and others mentioned below.

We find in each case that financial intermediation can indeed engender instability: an economy with these institutions is more likely to have multiple equilibria or volatile dynamics than the same economy without them. In some cases there is a unique equilibrium, and it is stable, without intermediation, but multiple or volatile equilibria with it; in other cases both can have multiple or volatile equilibria, but intermediation expands the set of parameters for which this is the case. Further, while the economic logic differs across models, in each case the results are directly related to the *raison d'etre* for intermediation.

As the literature on intermediation is vast, we refer to standard sources (e.g., Freixas and Rochet 2008; Calomiris and Haber 2014; Vives 2016). We do mention Shleifer and Vishny (2010), which has a similar motivation, even if our methods are different, coming mainly from monetary theory (e.g., Lagos et al. 2017; Rocheteau and Nosal 2017). Thus, we use infinite-horizon models, since we are interested in economic dynamics, and, moreover, finite-horizon models are ill suited for capturing salient features of financial activity, including unsecured credit, reputational considerations and the use of money. We also use general equilibrium, in the sense of logically closed systems, making as few as possible exogenous restrictions on prices, contracts or behavior. This is to see if instability arises from intermediation *per se* and not extraneous features like noise traders, sticky prices, irrational expectations etc. To be clear, we have frictions like limited commitment, imperfect information/communication, and spatial/temporal separation, but those are imposed on the environment, not on prices, contracts or behavior. Thus, we think, what follows are models *of* financial intermediation, not just *with* financial intermediation.[3]

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[3]Anticipating some readers might find four models more than they want to see in one paper, to rationalize this, the point is that the results are robust to different ways of formalizing the roles of financial intermediaries. We revisit that point in the Conclusion, where we also discuss the common thread across the different environments. Moreover, as the presentation of each model is self contained, readers could pick and choose without loss of continuity.

Model 1: Insurance

Our various specifications are novel, but all make use of standard building blocks from the literature that have been deemed relevant for banking. The first one extends Diamond and Dybvig's (1983) popular model to an infinite horizon. As in Gu et al. (2013a), this lets us incorporate reputational considerations à la Kehoe and Levine (1993).

Time is discrete and infinite. Some agents live forever, while at each date a $[0, 1]$ continuum of other agents are around only for that period – a simple way to make them care less about reputation (in principle, they can be around for any $N < \infty$ periods, but $N = 1$ is obviously easiest). Each period has two subperiods. The short-lived agents are homogeneous *ex ante* but face idiosyncratic shocks: they are impatient with probability π and patient with probability $1 - \pi$, where impatient (patient) agents derive utility only from consumption in the first (second) subperiod. Given the shock, which is private information, they have utility $u_j(c_j)$, $j = 1, 2$, where c_j is consumption in subperiod j, with $u'_j > 0$ and $u''_j < 0$.[4] Infinite-lived agents have period utility $v(c)$ for c in either subperiod, with $v' > 0$, $v'' \leq 0 = v(0)$.

Short-lived agents have an endowment of 1; infinitely-lived agents have 0. The standard technology is this: a unit of the good invested at the start of the first subperiod yields $R > 1$ units in the second subperiod; or, the investment can be terminated at the end of the first subperiod to get back the input.[5] The good can also be stored one-for-one across subperiods. As in any Diamond-Dybvig model, to insure against the shocks, the short-lived agents can form a coalition that resembles a banking arrangement. Thus, they design a contract (c_1, c_2) to solve

$$\max_{c_1, c_2} \{\pi u_1(c_1) + (1 - \pi) u_2(c_2)\} \tag{1}$$

$$\text{st } (1 - \pi) c_2 = (1 - \pi c_1) R \tag{2}$$

$$c_2 \geq c_1, \tag{3}$$

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[4]Many applications of Diamond-Dybvig assume $u_1(\cdot) = u_2(\cdot)$, but not all (e.g., Peck and Shell 2003). The flexibility of the general version is useful for constructing examples.

[5]See Andolfato et al. (2019) for a version of Diamond-Dybvig with a security market, where early termination entails a cost, which similarly implies it reduces the return on investment.

where (2) is feasibility and (3) is a truth-telling constraint (if $c_2 < c_1$ patient agents would claim to be impatient, get c_1 and store it to the next subperiod). There are also nonnegativity constraints omitted to save space.

This problem is well understood. One result is: assuming $u_1'(1) > u_2'(R)R$ and $u_1'(c) \leq u_2'(c)R$ at $c = R/(1 - \pi + \pi R)$, we get $1 < c_1^* < c_2^* < R$, so (3) is not binding, and full insurance/efficiency obtains. However, this requires commitment; otherwise, when they learn they are patient and are supposed to make transfers to the impatient, agents will renege. Given our short-lived agents cannot commit, naturally, there emerges a role for long-lived agents as bankers who accept deposits, invest them, and pay off depositors on demand at terms to be determined. Importantly, bankers do not have exogenous commitment ability – it is endogenous and based on reputation. Thus, bankers honor obligations lest they get identified as renegers, whence they are punished to autarky, which is a credible threat because there are many perfect substitutes for any given banker.

However, a banker may be tempted to misbehave as in the "cash diversion" models in Biais et al. (2007) or DeMarzo and Fishman (2007): if he misappropriates d deposits, he gets payoff λd, where λ is not too big, so this is socially inefficient, but he might do it opportunistically. As in Gu et al. (2013a,b), the risk is that he gets caught, and punished, with probability $\mu \leq 1$, where one interpretation is that μ is the probability one generation of depositors can communicate his misbehavior to the next generation.[6] Now depositors may set $d < 1$, and invest $1 - d$ on their own, to reduce bank incentives to misbehave, different from most papers that simply assume $d = 1$, but similar to Peck and Setayesh (2019). In addition to d, the contract now specifies payouts per deposit contingent on withdrawal time r_j, $j = 1, 2$, and the banker's income $b \in [0, d]$, which he invests for utility $v(bR)$.

Since there is more than one long-lived agent, the short-lived agents can choose any of them to act as banker, and in the spirit of Diamond-Dybvig they make this choice as a coalition. However, we assume they can choose only one, to avoid

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[6]While $\mu = 1$ is fine, it does not simplify things much, and it is known from other applications that $\mu < 1$ can be interesting (e.g., the extension of Kocherlakota 1998 in Gu et al. 2016).

determining the optimal number of bankers, something we do in Model 2, but would be a distraction here; it can be rationalized by assuming that it is too costly to monitor more than one banker. Still, since they can choose any one, for reasons often summarized as Bertrand competition the contract maximizes the expected utility of the depositors. Yet a banker may get a positive surplus – a rent on his option to act opportunistically – because the contract must give him incentives to not misuse deposits for his own gain.

The banker's incentive constraint is

$$v\left(b_t R\right) + \beta V_{t+1} \geq \lambda d_t + \beta\left(1-\mu\right) V_{t+1}, \tag{4}$$

where β is his discount factor, V_t is his equilibrium payoff, and the RHS is the deviation payoff, including λd for sure and V_{t+1} iff he is not caught. Note that V_{t+1} is his valuation next period, facing a new generation of depositors, and hence is taken as given when designing a contract at t. Also note that bankers do not misuse d on the equilibrium path, but if one were to, he would get λd but not $v\left(Rb\right) + \lambda\left(d-b\right)$.

This leads to the contracting problem

$$\max_{d_t, r_{1t}, r_{2t}, b_t} \left\{ \pi u_1\left(d_t r_{1t} + 1 - d_t\right) + \left(1-\pi\right) u_2\left[d_t r_{2t} + \left(1-d_t\right) R\right] \right\} \tag{5}$$

$$\text{st } \left(1-\pi\right) d_t r_{2t} = \left(d_t - b_t - \pi d_t r_{1t}\right) R \tag{6}$$

$$r_{2t} \geq r_{1t} \tag{7}$$

$$\lambda d_t - v\left(b_t R\right) \leq \phi_t, \tag{8}$$

where (8) rewrites (4) using $\phi_t \equiv \beta \mu V_{t+1}$. Note that ϕ_t is a bank's *franchise value*, capturing the banker's reputation for trustworthiness. Substituting (6) into (5) to eliminate r_2, ignoring t subscripts for now, and taking FOC's wrt $\left(r_1, d, b\right)$ we get

$$r_1 \quad : \quad d\left\{\pi\left[u_1'\left(c_1\right) - Ru_2'\left(c_2\right)\right] - \eta_1\left(1 - \pi + R\pi\right)\right\} r_1 = 0$$

$$d \quad : \quad \left\{\left(r_1 - 1\right)\pi\left[u_1'\left(c_1\right) - Ru_2'\left(c_2\right)\right] + \eta_1\left[R - \left(1-\pi+R\pi\right)r_1\right] - \eta_2 \lambda\right\} d = 0$$

$$b \quad : \quad \left[-u_2'\left(c_2\right) - \eta_1 + \eta_2 v'\left(bR\right)\right] b = 0,$$

where $c_1 = dr_1 + 1 - d$ and $c_2 = dr_2 + \left(1-d\right) R$, while η_1 and η_2 are multipliers for constraints (7) and (8).

These FOC's yield two critical values, $\phi^* > 0$ and $\hat{\phi} < \phi^*$, delineating three regimes: (i) If $\phi \geq \phi^*$ then (8) is slack, and $b = 0$, since the franchise value keeps the banker honest without $b > 0$. In this case there is a continuum of contracts achieving the full-insurance outcome, because depositors can have the bank invest a lot or a little, and in the latter case invest the rest on their own (exactly as in Peck and Setayesh 2019). (ii) If $\phi \in [\hat{\phi}, \phi^*)$ we must either lower $d < d^*$ or raise $b > 0$ to satisfy (8). While lowering d from d^* means less-than-full insurance, this is a second-order cost by the envelope theorem, so the contract sets $d = \phi/\lambda$ and keeps $b = 0$. (iii) If $\phi < \hat{\phi}$, lowering d further entails too much risk, so it sets $b > 0$. In case (i), one of the payoff-equivalent contracts has $r_1 = r_2$, and (ii)-(iii) the unique contract has $r_1 = r_2$; hence wlog we set $r_1 = r_2 = r$ from now on.

In regime (iii), (d, r, b) satisfies

$$b = d/R\left[R - (1 - \pi + R\pi)r\right] \tag{9}$$

$$\phi = \lambda d - v(bR) \tag{10}$$

$$\frac{u_2'(c_2)}{u_1'(c_1) - Ru_2'(c_2)} = \frac{\pi}{1 - \pi + R\pi}\left[\frac{(R-1)(1-\pi)}{\lambda}v'(bR) - 1\right] \tag{11}$$

with $c_1 = 1 - d + (d - b)R/(1 - \pi + R\pi)$, $c_2 = (1 - d)R + (d - b)R/(1 - \pi + R\pi)$. These and the analogs from regimes (i)-(ii) characterize the contract given ϕ, and in particular, one can easily check $b'(\phi) < 0$ $\forall \phi < \hat{\phi}$, which is important below. This is shown in Fig. 1 for the following parameterization:[7]

Example 1: *Let* $v(b) = Bb$,

$$u_1(c_1) = A_1\frac{(c_1 + \varepsilon)^{1-\sigma_1} - \varepsilon^{1-\sigma_1}}{1 - \sigma_1} \text{ and } u_2(c_2) = A_2\frac{(c_2 + \varepsilon)^{1-\sigma_2} - \varepsilon^{1-\sigma_2}}{1 - \sigma_2},$$

where $B = 0.95$, $\sigma_1 = \sigma_2 = 2$, $\varepsilon = 0.01$, $A_1 = 1$, $A_2 = 0.1$, $R = 2.1$, $\mu = 0.7$, $\pi = 0.25$, $\lambda = 0.6$ *and* $\beta = 0.99$.

As mentioned, the contract takes ϕ as given. To embed this in general equilibrium, use $\phi_t \equiv \beta\mu V_{t+1}$ to write $V_t = v(b_t R) + \beta V_{t+1}$ as a dynamical system,

$$\phi_{t-1} = f(\phi_t) \equiv \beta\mu v[b(\phi_t)R] + \beta\phi_t, \tag{12}$$

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[7]Notice $\hat{\phi} > 0$ here (in fact, for the example $\hat{\phi} = 0.3257$ and $\phi^* = 0.600$); the case $\hat{\phi} < 0$ is less interesting because it never has banking in steady state. In terms of primitives, one can show that $\hat{\phi} > 0$ iff $\pi[u_1'(1) - Ru_2'(R)][(R-1)(1-\pi)v'(0) - \lambda] > u_2'(R)(1 - \pi + R\pi)\lambda$.
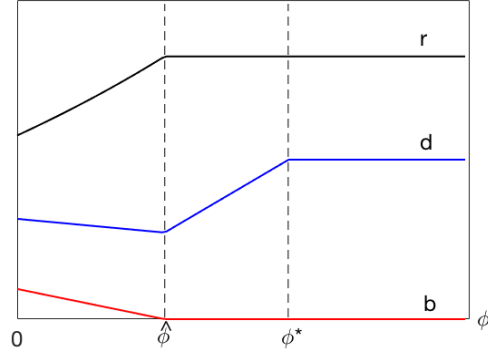
Figure 1: Model 1, bank contract vs ϕ

where $b(\phi_t)$ comes from the contracting problem. Equilibrium is defined as a nonnegative, bounded path for ϕ_t solving (12), from which the other endogenous variables follow using the FOC's. A stationary equilibrium, which is the same as a steady state here, solves $\bar{\phi} = f(\bar{\phi})$. The nature of steady state depends on whether $\hat{\phi} \leq 0$ or $\hat{\phi} > 0$ (conditions for which are given in fn. 7). Appendix A proves:

Proposition 1 *If $\hat{\phi} \leq 0$ the unique steady state has no banking, $d = 0$. If $\hat{\phi} > 0$ the unique steady state has $\bar{\phi} \in (0, \hat{\phi})$ and banking, $d > 0$.*

For dynamic equilibria, first note from (12) that $f(\phi_t)$ has a linear term that is increasing and a nonlinear term that is decreasing because $b'(\phi) < 0$. If the net effect implies $f'(\phi_t) < 0$ over some range the system can exhibit nonmonotone dynamics. For Example 1, Fig. 2a shows f and f^{-1}, which cross on the 45° line at $\bar{\phi} = 0.3215$. In this case the system is monotone and there is a unique equilibrium, which is the steady state, because that is the only bounded path solving (12). To see this is not true in general consider:

Example 2: *Same as above except $\sigma_1 = 10$ and $\mu = 1$.*

As Fig. 2b shows, now $f'(\bar{\phi}) < -1$, and so f and f^{-1} intersect not only on the 45°, but also off it, at (ϕ_L, ϕ_H) and (ϕ_H, ϕ_L) with $\phi_H = 0.0696$ and $\phi_L = 0.0689$. As is standard (see Azariadis 1993), this means there is a two-cycle equilibrium where ϕ_t oscillates deterministically between ϕ_L and ϕ_H. It also means there are sunspot

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equilibria where ϕ_t fluctuates randomly between values close to ϕ_L and ϕ_H (see Appendix B). Thus we can get deterministic or stochastic volatility with banking and not without it. That does not mean banking is a bad idea, as it provides insurance to agents who cannot insure each other, due to commitment issues. It does mean banking can engender instability.
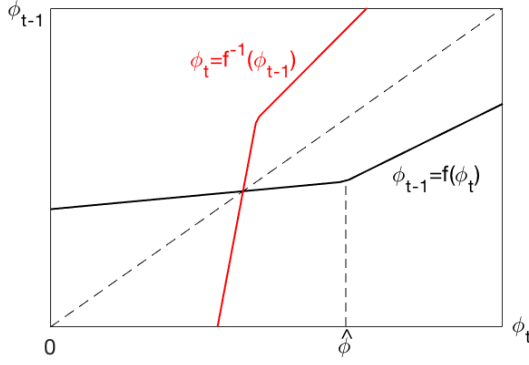

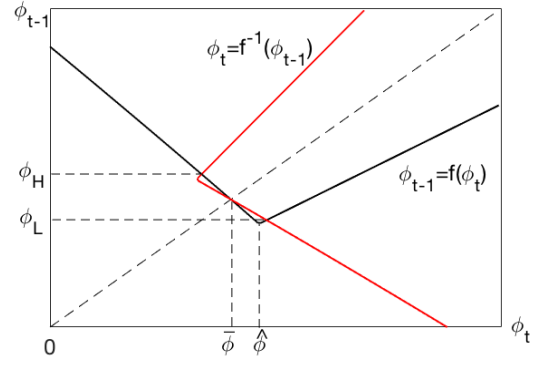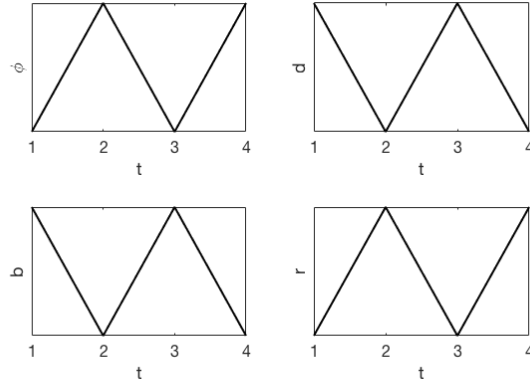Figure 2a: Model 1, monotone f Figure 2b: Model 1, nonmonotone f

Figure 3: Model 1, time series for a two-cycle

The intuition is straightforward: if next period V_{t+1} is high then this period ϕ_t is high and we can discipline bankers with low b_t; but that makes the current V_t and hence ϕ_{t-1} low. This induces a tendency towards oscillations, but for a cycle the

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effect has to dominate the linear term in $f(\phi_t)$, which is why parameters matter. Fig. 3 plots time series of (ϕ, d, b, r) over the cycle in Example 2. Notice r moves with ϕ and b against ϕ. Whether d moves with or against ϕ depends on parameters, but here it is the latter. While the point is not to take this example seriously in a quantitative sense, it is worth noting that the theory does make qualitative predictions, and does not say "anything goes."
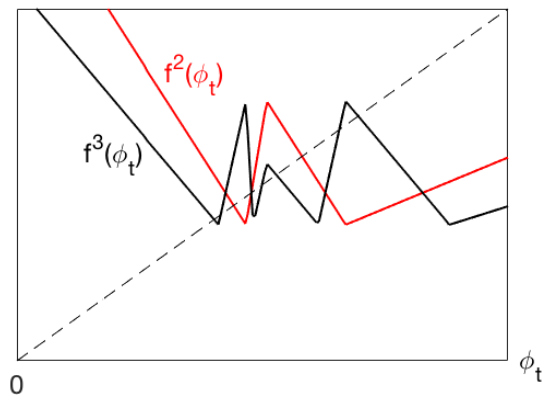

Figure 4: Model 1, two- and three-cycles

Fig. 4 displays the existence of two-cycles in a different way, as fixed points of the second iterate $f^2 = f \circ f$, for another parameterization:

Example 3: $B = 1$, $\sigma_1 = 14$, $\sigma_2 = 1.5$, $\varepsilon = 0.01$, $A_1 = 1$, $A_2 = 0.075$, $R = 2.2$, $\mu = 1$, $\pi = 0.28$, $\lambda = 0.75$ and $\beta = 0.76$.

Notice f^2 has three fixed points, $\bar{\phi}$, plus ϕ_L and ϕ_H from the two-cycle. Also shown is f^3, which has seven fixed points, $\bar{\phi}$ plus a pair of three-cycles. Standard results (again see Azariadis 1993) say the existence of three-cycles implies the existence of $n-$cycles for any integer n, plus chaos, which is basically a cycle with $n = \infty$.

To summarize, banking can introduce many equilibria, including deterministic, stochastic and chaotic dynamics, directly attributable to the idea that banks depend on trust, and at least to some extent that is a self-fulfilling prophecy.

Model 2: Delegated Investment with Fixed Costs

The next formulation has intermediation originating from economies of scale, based on Diamond (1984) and Huang (2017) (see also Leland and Pyle 1976 or Boyd and Prescott 1986 on the bigger picture). Time is discrete and continues forever as in Model 1, but here all agents are infinitely lived. Also, they are now spatially separated – say, across a large number of islands – and randomly relocated at the end of each period, following a literature on banking including Champ et al. (1996), Bencivenga and Smith (1991), Smith (2002) and Bhattacharya et al. (2005).[8] Economies of scale are captured as follows: agents must pay a fixed cost κ, in terms of goods, to locate/evaluate/monitor investment projects, after which any project returns R per unit invested.

Period utility is $u(x) - c(d)$, where x is consumption and d investment (say, output produced one-for-one with labor) where $u', c' > 0$ and $c'' \geq 0 > u''$. Also, $u'(0) R > c'(0)$, so that agents invest if $\kappa = 0$. Normalize $u(0) = c(0) = 0$. If $\kappa > 0$ the payoff is

$$W_1 = \max_{x,d} \{u(x) - c(d)\} \text{ st } x = Rd - \kappa, \tag{13}$$

from investing on one's own (omitting nonnegativity constraints as above). Suppose κ is too high to support this, so $W_1 < 0$, while the autarky payoff is $W_0 = 0$. Now consider agents forming a coalition where some, that we call depositors, delegate their investment to others, that we call bankers, to share the fixed cost.

As is standard in models with nonconvexities, the coalition uses a lottery to chose a subset of members to act as bankers.[9] Thus, ω_t is the probability of being a banker, equal to the measure of bankers if the island population is normalized to 1.

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[8]The main function of random relocation here is to let us avoid long-term contracting considerations, which are interesting but complicated (e.g., in Gu et al. 2013a, bankers' rewards can be backloaded over multi-period contracts). Elsewhere in the paper we avoid those issues using short-lived agents, but here we want all agents to be long-lived, so that *ex ante* anyone can potentially be a banker. In any case, it is important to emphasize that these are not restrictions on contracting *per se*, but assumptions on the environment that impinge on the contract. Does it matter? Yes, because without making them explicit one cannot know, in general, how these assumptions impinge on all endogenous variables.

[9]This is similar to, e.g., Rogerson's (1988) indivisible-labor model, except unlike his, our agents cannot commit, so our contracts must be incentive compatible before and after the lottery.

As in Model 1, bankers have the option to misbehave, with λ and μ playing similar roles. The relevant incentive condition is therefore

$$\beta V_{t+1} \geq \frac{\lambda (1 - \omega_t) x_t}{\omega_t} + (1 - \mu)\beta V_{t+1}, \tag{14}$$

where the RHS is the deviation payoff, given each depositor is promised x_t and each banker controls $(1 - \omega_t) x_t / \omega_t$ of the resources.[10] The trade-off, as emphasized in Huang (2017), is that having fewer banks saves on fixed costs but raises their temptation to misbehave, because they must be larger, given total deposits.

The contract maximizes the payoff to the representative agent on an island

$$W (\phi) = \max_{\omega, X, D, x, d} \left\{ \omega \left[u (X) - c (D) \right] + (1 - \omega) \left[u (x) - c (d) \right] \right\} \tag{15}$$

$$\text{st } \omega X + (1 - \omega) x = R \left[\omega D + (1 - \omega) d \right] - \kappa \omega \tag{16}$$

$$u (x) - c (d) \geq 0 \tag{17}$$

$$\frac{1 - \omega}{\omega} x \leq \phi, \tag{18}$$

where $\phi_t \equiv \mu \beta V_{t+1} / \lambda$, while (X, D) and (x, d) are the consumption/investment allocations of bankers and depositors. Here (16) is the resource constraint, (17) is the incentive constraint for depositors, and (18), which rewrites (14), is the incentive constraint for bankers.

Substituting (16) into (15) to eliminate X, and letting η and γ be multipliers, we get the FOC's:

$$D \quad : \quad u' (X) R - c' (D) = 0$$

$$d \quad : \quad (1 - \omega) \left[u' (X) R - c' (d) \right] - \eta c' (d) = 0$$

$$x \quad : \quad (1 - \omega) \left[u' (x) - u'(X) \right] + \eta u' (x) - \gamma \frac{1 - \omega}{\omega} = 0$$

$$\omega \quad : \quad \omega \left\{ u (X) - c (D) - \left[u (x) - c (d) \right] + u' (X) \frac{x - Rd}{\omega} + \gamma \frac{x}{\omega^2} \right\} = 0$$

One can check $W' (\phi) \geq 0$. Moreover, $W (0) = 0$, so we get no banking at $\phi = 0$. In the limit as $\phi \to \infty$ we get $\omega \to 0$, which means very few banks but they are

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[10]Here we assume that a deviating banker receives $u(X) - c(D)$ in addition to $\lambda[(1 - \omega_t)x_t / \omega_t]$, slightly different from Model 1. In both cases, these assumptions are made mainly for convenience, and neither one is unambiguously better.

huge. Also as $\phi \to \infty$ we get $W(\phi) \to W^* \equiv \max_{x,d}[u(x) - c(d)]$ st $x = Rd$, which totally dissipates the fixed cost (i.e., delivers the same payoff as $\kappa = 0$).

Fig. 5 shows the contract given ϕ for the following parameterization:

Example 4: *Let*

$$u(x) = A\frac{(x+\varepsilon)^{1-\sigma} - \varepsilon^{1-\sigma}}{1-\sigma} \text{ and } c(d) = Bd,$$

with $A = \varepsilon = 0.001$, $\sigma = 2$, $B = 0.1$, $\kappa = 230$, $R = 1.2$, $\beta = 0.76$, $\mu = 0.95$ *and* $\lambda = 9$.

Notice that there is a cutoff $\tilde{\phi}$, which is $\tilde{\phi} = 0.0182$ in this example, and banking is viable iff $\phi \geq \tilde{\phi}$.
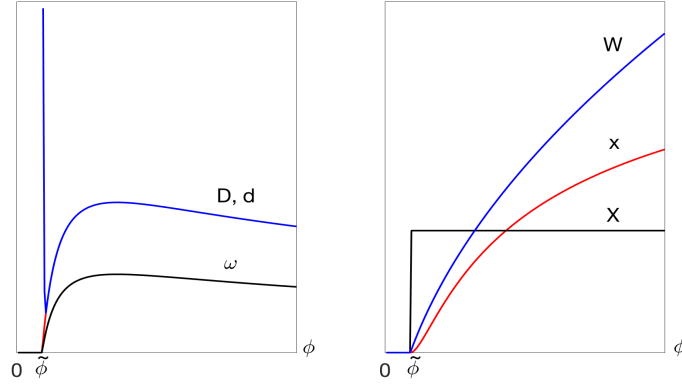

Figure 5: Model 2, bank contract vs ϕ

To embed this in equilibrium use $V_t = W(\phi_t) + \beta V_{t+1}$ and emulate the methods from Model 1 to get

$$\phi_t = f(\phi_{t+1}) \equiv \frac{\beta\mu}{\lambda}W(\phi_{t+1}) + \beta\phi_{t+1}. \tag{19}$$

Equilibrium is a bounded, nonnegative solution to (19). Notice $f(\phi) = \beta\phi$ for $\phi \leq \tilde{\phi}$, and $f(\phi) < \phi$ for big ϕ due to the fact that $W \leq W^*$. Then we have:

Proposition 2 *There is a steady state at* $\bar{\phi} = 0$, *without banking. There can be*

14

steady states with banking, generically an even number that alternate between stable and unstable.

Fig. 6 shows Example 4 has three steady states, $\phi = 0$, plus two with banking, $\phi_2 > \phi_1 > 0$. This is different from Model 1, which has a unique steady state $\bar{\phi}$, and has nonstationary equilibria iff $f'\left(\bar{\phi}\right) < -1$. Now $f'\left(\phi\right) > 0$, so deterministic cycles are impossible, but if there are multiple steady states we can use a different approach to construct sunspot equilibria around the stable ones.[11] Appendix B shows there are equilibria where ϕ fluctuates between ϕ_A and ϕ_B for any $\phi_A \in \left(\phi_0, \phi_1\right)$ and $\phi_B \in \left(\phi_1, \phi_2\right)$. In particular, $\phi_A < \tilde{\phi}$ means we switch stochastically between $d_t > 0$ and $d_t = 0$ – i.e., random episodes of crises, where deposits dry up and banking shuts down, due to sunspots, which are fundamentally irrelevant events. This is again different from Model 1, where d_t can fluctuate, but only with $d_t > 0 \; \forall t$.
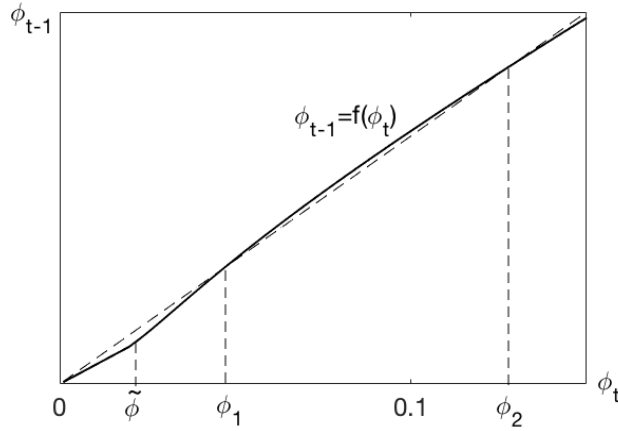

Figure 6: Model 2, monotone f with multiple
steady states

While Models 1 and 2 are different, in terms of economics and mathematics, Appendix C presents an environment that integrates elements of both. It has two agents on each island, one that is infinitely lived and one that is only around for

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[11] To give credit where credit is due, in Model 2 we use the method in Azariadis (1981), while in Model 1 we use the method in Azariadis and Guesnerie (1986).

one period, who negotiate the contract using generalized Nash bargaining (having just two is simpler, but we also considered many depositors and one banker, with multilateral bargaining, and got similar results). There are gains to delegating investment due to $\kappa > 0$, as in Model 2, but only long-lived agents can act as bankers, as in Model 1. Letting θ denote bankers' bargaining power, we get a dynamical system $\phi_t = f\left(\phi_{t+1}\right)$ that can be nonmonotone for $\theta < 1$. Appendix C shows we can have multiple steady states, with $f'\left(\phi\right) > 0$ around the stable ones and hence sunspot equilibria as in the benchmark Model 2, as well as $f'\left(\phi\right) < -1$ around the unstable steady states and hence cycles and sunspots as in Model 1.

The reason $f\left(\phi\right)$ is decreasing in Appendix C is the following well-known (see Kalai 1977) feature of Nash bargaining: agents with bargaining power $\theta < 1$ can get a smaller surplus when the bargaining set expands. Here this is manifest in bankers' surplus falling with ϕ, similar to $b'\left(\phi\right) < 0$ in Model 1. That does not happen in the baseline Model 2, where agents in the coalition are *ex ante* identical and hence treated symmetrically, so they all get a bigger surplus when ϕ increases. Details aside, the point is that there are distinct ways to formalize how banking might engender instability in models based on reputation/trust.[12]

Model 3: Asset Market Intermediation

Banks are not the only interesting financial intermediaries. Work following Duffie et al. (2005) studies asset markets using search theory, where agents may trade with each other, or with middlemen/dealers that buy from those with low asset valuation and sell to those with high valuation. We pursue this with a few changes in their environment. In particular, most papers following Duffie et al. (2005) give middlemen continuous access to a frictionless interdealer market (with some exceptions, e.g., Weill 2007, but they do not study the issues analyzed here). Hence, their intermediaries do not hold assets in inventory. Our middlemen are more like those in Rubinstein and Wolinsky (1987), who buy goods from producers and hold

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[12]Model 1 can be interpreted as bargaining where banks have $\theta = 0$. Hence, one may conjecture that dynamics like those in Appendix C emerge in Model 1 if we allow $\theta \in (0, 1)$, but then Model 1 becomes intractable; the setup in Appendix C is relatively quite tractable.

inventories until they randomly sell to consumers. However, here they trade assets and not goods, which matters. The following presentation builds on Nosal et al. (2017), although we amend their setup in many ways, including: adding heterogenous projects; modifying the market composition conditions (see fn. 13); and switching from continuous to discrete time. All of these affect the results.

There are large numbers of three risk-neutral types, B, S and M, for asset buyers, sellers and middlemen. Type M agents stay in the market forever, while type S and B stay for one period (we also studied alternatives, like letting everyone stay forever, with similar results). Upon exit S and B are replaced by "clones" to maintain stationarity (a device borrowed from Burdett and Coles 1997). Type B agents, sometimes called end users, want to acquire an asset – let's call it capital – to implement a project for profit $\pi > 0$, where π is observable when agents meet, but random across end users with CDF given by $F(\pi)$. The originators of capital, type S, if they enter the market each bring 1 indivisible unit; those that stay out put their capital to alternative uses, defining their opportunity cost of participation, denoted $\kappa_s > 0$, which for simplicity is the same for all S.

Type M agents, who are always in the market, can acquire capital from S, but as usual in these models their inventories are restricted to $k \in \{0, 1\}$ (with exceptions, e.g., Lagos and Rocheteau 2009, but they do not study the issues analyzed here). Let n_t be the measure of type M at t with $k = 1$. Capital held by M depreciates by disappearing each period with probability $\delta \geq 0$, but while he holds it M gets a return $\rho > 0$. His crucial choice is then, if he has $k = 1$ and meets B, should he trade or keep k for himself? This depends on fundamentals, of course, including the end user's π, but as we show below, it can also depend on beliefs.

Market composition is determined as follows: the measures n_m and n_b of types M and B are fixed, but entry by S makes n_s endogenous.[13] Given this, the meeting

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[13]Entry by S is nice because it lets us compare economies with the same entry conditions with and without middlemen. Still, results for entry by M are given in Appendix D; entry by B is less interesting and hence omitted. These alternatives are all better than Nosal et al. (2017), where agents choose to be either type M or S. That is awkward because in cyclic equilibria they switch back and forth over time between being M and S. Here no one switches, but participation by a

technology is standard: each period everyone in the market contacts someone with probability α, and each contact is a random draw from the participants. In particular, if N_t is the total measure of participants then types M and S both meet type B with probability $\alpha n_b / N_t$, so M has no advantage over S in that regard.[14] When B and S meet they trade for sure since this is S's only chance and cost κ_s is sunk. Similarly, when S meets M with $k = 0$ they trade for sure. When M with $k = 1$ meets B, however, they may or may not trade.

As regards prices, if type j gives i capital the latter pays p_{ij} (in terms of transferable utility) determined by bargaining. Thus, if Σ_{ij} is the total surplus available when i and j meet, as long as $\Sigma_{ij} > 0$ they trade, and type i's surplus is $\theta_{ij} \Sigma_{ij}$, where $\theta_{ij} \in [0, 1]$ is his bargaining power. To flesh this out, let $V_{s,t}$ and $V_{b,t}$ be value functions for types S and B; let $V_{k,t}$ be the value function for M when he has $k \in \{0, 1\}$; and let $\Delta_t = V_{1,t} - V_{0,t}$ be M's gain from having inventory. Then

$$\Sigma_{bs,t} = \pi, \ \Sigma_{ms,t} = (1 - \delta)\beta \Delta_{t+1}, \ \Sigma_{bm,t} = \pi - (1 - \delta)\beta \Delta_{t+1},$$

where $\beta \in (0, 1)$ is M's discount factor.[15] Bargaining then yields

$$p_{bs,t} = \theta_{sb}\pi, \ p_{ms,t} = \theta_{sm} (1 - \delta)\beta \Delta_{t+1}, \ p_{bm,t} = \theta_{mb}\pi + \theta_{mb} (1 - \delta)\beta \Delta_{t+1}. \quad (20)$$

When M with $k = 1$ and B with project π meet, they trade with probability $\tau_t = \tau(\pi, R_t)$, where

$$\tau(\pi, R) = \begin{cases} 0 & \text{if } \pi < R \\ [0, 1] & \text{if } \pi = R \\ 1 & \text{if } \pi > R \end{cases} \quad (21)$$

and $R_t \equiv (1 - \delta)\beta \Delta_{t+1}$ is the *reservation value* of a project making $\Sigma_{mb} = 0$. Hence, the market payoff for B with project π is

$$V_{b,t}(\pi) = \frac{\alpha n_{s,t}}{N_t}\theta_{bs}\pi + \frac{\alpha n_t}{N_t}\tau(\pi, R_t)\theta_{bm}[\pi - (1 - \delta)\beta \Delta_{t+1}]. \quad (22)$$

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type can vary, as in conventional search theory (e.g., Pissarides 2000).

[14]This is different from the original Rubinstein-Wolinsky setup, which is based entirely on M being better than S at search; we could allow that, too, but do not need it.

[15]Note there are no continuation values or threat points for S and B, as they are in the market for just one period, but while that simplifies some algebra it is not otherwise important.

The first term on the RHS is the probability B meets S, times his share of the surplus; the second is the probability he meets M with $k = 1$, times the probability they trade, times his share of the surplus; and note prices do not appear since they were eliminated using (20). Similarly, the market payoff for S is

$$V_{s,t} = \frac{\alpha n_b}{N_t} \theta_{sb} \int_0^\infty \pi dF(\pi) + \frac{\alpha(n_m - n_t)}{N_t} \theta_{sm}(1 - \delta)\beta\Delta_{t+1}. \tag{23}$$

The payoff for M depends on inventory. Using $R_t = (1 - \delta)\beta\Delta_{t+1}$, we have

$$V_{0,t} = \frac{\alpha n_{s,t}}{N_t}\theta_{ms}R_t + \beta V_{0,t+1} \tag{24}$$

$$V_{1,t} = \rho + \frac{\alpha n_b}{N_t}\theta_{mb}\int_{R_t}^\infty (\pi - R_t)\, dF(\pi) + (1 - \delta)\beta V_{1,t+1} + \delta\beta V_{0,t+1}. \tag{25}$$

Subtracting and simplifying with integration by parts, we get

$$R_{t-1} = (1 - \delta)\beta\left\{\rho + R_t + \frac{\alpha n_b \theta_{mb}}{N_t}\int_{R_t}^\infty [1 - F(\pi)]\, d\pi - \frac{\alpha n_{s,t}\theta_{ms}}{N_t}R_t\right\}, \tag{26}$$

giving the evolution of R over time. The evolution of inventories held by M is

$$n_{t+1} = n_t(1 - \delta)\left[1 - \frac{\alpha n_b}{N_t}\mathbb{E}\tau(\pi, R)\right] + \frac{(n_m - n_t)\alpha n_{s,t}(1 - \delta)}{N_t}, \tag{27}$$

where $\mathbb{E}\tau(\pi, R) = prob(\pi > R)$ is the unconditional probability that M and B trade. The first term on the RHS is current n times the probably a unit of k does not depreciate or get traded; the second is current $n_m - n$ times the probability M acquires k and it does not depreciate.

We can eliminate N_t in (26)-(27) using S's entry condition, $V_{s,t} = \kappa_s$, which reduces to

$$n_{s,t} = \frac{\alpha n_b \theta_{sb}\mathbb{E}\pi + \alpha(n_m - n_t)\theta_{sm}R_t}{\kappa_s} - n_b - n_m. \tag{28}$$

What's left is a two-dimensional dynamical system that is compactly written as

$$\begin{bmatrix} n_{t+1} \\ R_{t-1} \end{bmatrix} = \begin{bmatrix} f(n_t, R_t) \\ g(n_t, R_t) \end{bmatrix}. \tag{29}$$

Given an initial n_0, equilibrium is a nonnegative, bounded path for (n_t, R_t) solving (29).[16]

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[16] A distinction between this model and others in the paper is that this system is two dimensional:

With no intermediaries, $n_m = 0$, the equilibrium is basically static and it is easy to check that it is unique. With $n_m > 0$, first note that the locus of points satisfying $n = f(n, R)$, called the n-curve, and the locus satisfying $R = g(n, R)$, called the R-curve, both slope up in (n, R) space. Then, to develop some intuition, consider the special case where $\pi = \bar{\pi}$ is constant. As shown in Fig. 7, for this case there are three possible regimes: (i) $R < \bar{\pi}$, so M with $k = 1$ and B trade with probability $\tau = 1$; (ii) $R > \bar{\pi}$, so they trade with probability $\tau = 0$; and (iii) $R = \bar{\pi}$, so they trade with probability $\tau \in (0, 1)$. Appendix A proves:

Proposition 3 *With $\pi = \bar{\pi}$ there exist $\tilde{\rho} > 0$ and $\hat{\rho} > \tilde{\rho}$ such that: (i) if $\rho \in [0, \tilde{\rho})$ there is a unique steady state and it has $R < \bar{\pi}$; (ii) if $\rho \in (\hat{\rho}, \infty)$ there is a unique steady state and it has $R > \bar{\pi}$; (iii) if $\rho \in (\tilde{\rho}, \hat{\rho})$ there are three steady states, $R < \bar{\pi}$, $R > \bar{\pi}$, and $R = \bar{\pi}$.*
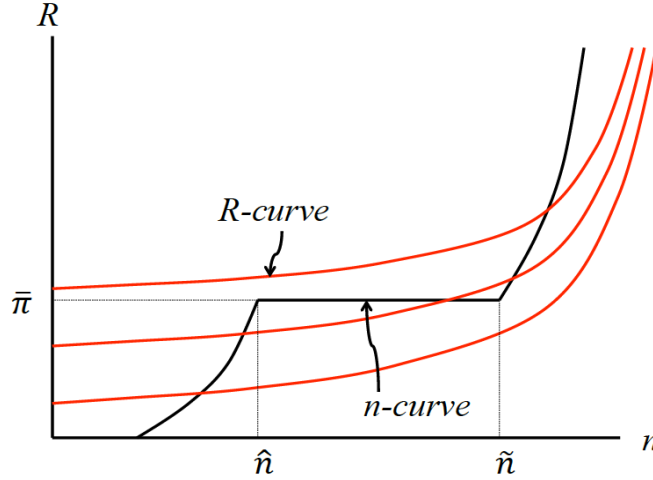

Figure 7: Model 3, phase plane

R is a jump variable, like ϕ in the previous sections, while n is a (predetermined) state variable, so transitions are nontrivial. The version of Model 3 in Appendix D, with entry by M instead of S, is different: there one can solve a univariate system $R_{t-1} = G(R_t)$ to get the path for R_t, after which n_t, N_t etc. follow from simple conditions. Intuitively, with entry by M (entry by S) the model is (is not) block recursive, as discussed in Shi (2009). Hence, Appendix D delivers more results, including chaotic dynamics, but we still prefer entry by S as a benchmark model.

For several reasons we prefer a nondegenerate $F(\pi)$.[17] So, consider a smooth mean-preserving spread of the degenerate case, leading to the shown in phase plane Fig. 8.

Example 6: *Let*

$$F(\pi) = \begin{cases} \pi_1 \pi / \pi_0 & \text{if } 0 \le \pi \le \pi_0 \\ \pi_1 + (\pi_3 - \pi_1)(\pi - \pi_0)/(\pi_2 - \pi_0) & \text{if } \pi_0 < \pi \le \pi_2 \\ \pi_2 + (1 - \pi_3)(\pi - \pi_2)/(\pi_4 - \pi_2) & \text{if } \pi_2 < \pi \le \pi_4 \end{cases} \quad (30)$$

with $\pi_0 = 0.99$, $\pi_1 = 0.05$, $\pi_2 = 1.01$, $\pi_3 = 0.95$ and $\pi_4 = 2$. Also, let $\alpha = 1$, $\kappa_s = 0.1$, $n_b = 0.05$, $n_m = 0.5$, $\theta_{sm} = 0.5$, $\theta_{sb} = 1$, $\theta_{mb} = 0.7$, $\beta = 1/1.04$, $\delta = 0.008$ and $\rho = 0.2$.

Fig. 8 is similar to Fig. 7, except the slope of the n-curve is always positive. Hence the results are similar to Proposition 3, including multiplicity.
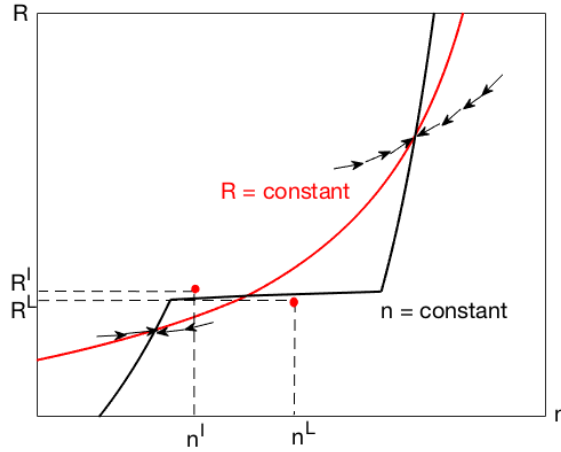

Figure 8: Model 3, phase plane with two-cycle

Here is the intuition. First suppose R is low, so M trades k to B with a high probability. Then the probability M has $k = 0$ is high, which is good for sellers,

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[17] For the nondegenerate $F(\pi)$ studied below, the flat portion of the n-curve in Fig. 7 is eliminated. Then in any steady state M and B are indifferent to trade only in the rare event $\pi = R$, in contrast to the mixed-strategy equilibrium in the degenerate case, where they are always indifferent. Moreover, with nondegenerate $F(\pi)$, if R varies across pure-strategy steady states intermediation activity does too, but not necessarily to the extreme extent of the degenerate case, where it is either $\tau = 1$ or $\tau = 0$. Similarly, for real-time dynamics, cycles with nondegenerate $F(\pi)$ have fluctuations in intermediation activity but not necessarily between $\tau = 0$ and $\tau = 1$.

so n_s is high. That makes it easy for M to get k and rationalizes a low R. Now suppose R is high, so M trades k to B with a low probability. Then the probability M has $k = 0$ is low, which is bad for sellers, so n_s is low. That makes it hard for M to get k and rationalizes a high R. Moreover, market liquidity – i.e., the ease with which agents can buy and sell k – is high (low) if R is low (high). Multiplicity means liquidity is not pinned down by fundamentals, a recurring theme in monetary theory (e.g., Kiyotaki and Wright 1989), but the intuition here is different. In monetary economies, whether an agent accepts something as medium of exchange depends on what others accept. Here, whether type M agents trade away k depends on n_s, and n_s depends on whether type M trade away k, which is a different idea. In particular, our result requires endogenous market composition, not true in monetary models.

Now consider a two-cycle oscillating between a liquid regime with low R and an illiquid regime with high R, or $\left(R^L, n^L\right)$ and (n^I, R^I) solving

$$\begin{bmatrix} n^I \\ R^I \end{bmatrix} = \begin{bmatrix} f(n^L, R^L) \\ g(n^L, R^L) \end{bmatrix} \text{ and } \begin{bmatrix} n^L \\ R^L \end{bmatrix} = \begin{bmatrix} f(n^I, R^I) \\ g(n^I, R^I) \end{bmatrix}. \tag{31}$$

One solution is $\left(R^L, n^L\right) = (0.9862, 0.4504)$ and $\left(R^H, n^H\right) = (1.0103, 0.4312)$. Hence, we have real-time dynamics (not just multiple steady states) with excess volatility in liquidity, trade volume, prices and quantities. Fig. 9 shows the times series. In the liquid regime: R is low, making M more inclined to trade with B; n is high, because M and B traded less last period; and n_s is low, because low R and high n discourage entry by S. The illiquid regime has the opposite properties.
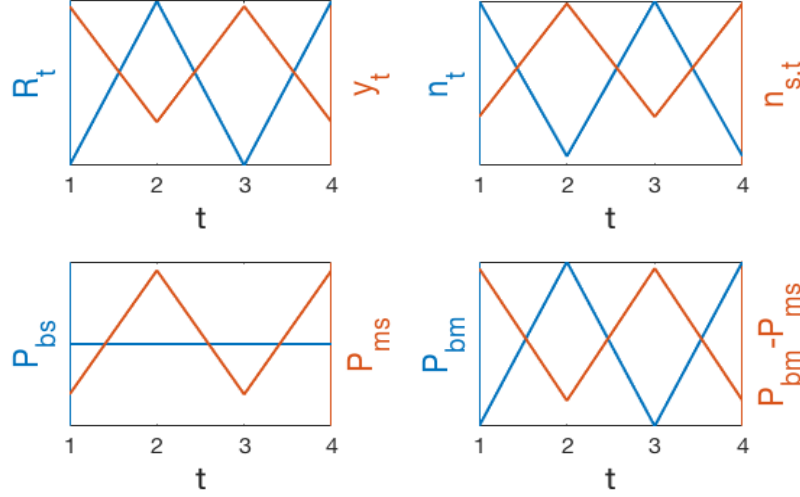
Figure 9: Model 3, time series for a two-cycle

We do not claim that actual data are best explained by a two-cycle, but suggest if such a simple model can deliver equilibria where endogenous variables vary over time, as self-fulfilling prophecies, it lends credence to the notion that intermediated asset markets in the real world might be prone to similar instability.[18]

Model 4: Safety and Secrecy

An important role of banks is the issuance of liabilities that facilitate third-party transactions. Indeed, for the general public, that is virtually their defining characteristic: "banks are distinguished from other kinds of financial intermediaries by the readily transferable or 'spendable' nature of their IOUs, which allows those IOUs to serve as a means of exchange, that is, money... Commercial bank money today consists mainly of deposit balances that can be transferred either by means of paper orders known as checks or electronically using plastic 'debit' cards" (Selgin

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[18]Prices are also shown in Fig. 9 (averaged over π when B trades). The price B pays S is constant over time, as it depends only on fundamentals, but the price M pays S or B pays M moves with R. The spread can go either way, but here it moves against R. This is all broadly consistent with the data discussed in Comerton-Forde et al. (2010), and other stylized facts (e.g., inventories are volatile than output). While this is obviously not a calibration, the finding that it is qualitatively more consistent with observations may lend further credence to the story.

2018). We pursue this idea in a model with an explicit need for payment instruments, building on the New Monetarist framework surveyed by Lagos et al. (2017) and Rocheteau and Nosal (2017), where we introduce banks in two related ways.

In Model 4a, bank liabilities are safe relative to other assets in the sense that they are less susceptible to theft or loss (similar to He et al. 2007; see also Sanches and Williamson 2010). Traveler's checks are a case in point but not the only one – in general, it is obviously worse to have your cash lost or stolen than your checkbook or debit card. Also, if merchandise turns out to be fraudulent or defective, which is similar to theft, it is typically easier to stop payment if you use a check or credit card than if you use cash.[19] Model 4b builds on the idea that payment instruments originating with banks can be, as Dang et al. (2017) put it, *informationally insensitive* when these institutions act as secret keepers; an earlier exposition of this is Gorton and Pennachi (1990) while Andolfatto and Martin (2013), Andolfatto et al. (2014), Loberto (2017), and Monnet and Quintin (2017) are versions that go deeper into the microfoundations.[20]

While there are different approaches to modeling media of exchange, one based on Lagos and Wright (2005) is convenient for both Models 4a and 4b. In that setup, in each period of discrete time two markets convene sequentially: a decentralized market, or DM, with frictions detailed below; and a frictionless centralized market, or CM. There are two types of infinitely-lived agents, a measure 1 of buyers and a measure n of sellers. Their roles differ in the DM, but they are similar in the CM, where they all trade a numeraire consumption good x and labor ℓ for utility $U(x) - \ell$, with $U' > 0 > U''$. They also trade assets in the CM, like the trees in the standard Lucas (1978) model, giving off a dividend $\rho > 0$ in the CM in numeraire.

[19]Safety was a critical feature of banks historically. Consider the British goldsmiths: "At first [they] accepted deposits merely for *safe keeping*; but early in the 17th century their deposit receipts were circulating in place of money" (Encyclopedia Britannica, quoted in He et al. 2005; emphasis added). Also, "In the 17th century, notes, orders, and bills (collectively called demandable debt) acted as media of exchange that spared the costs of *moving, protecting and assaying* specie" (Quinn 1997; emphasis added). Safety was also crucial for earlier bankers, including the Templars (Sanello 2003), who specialized in moving purchasing power over dangerous territory.

[20]Other models of bank liabilities circulating as media of exchange that are similar in spirit but very different in detail include Cavalcanti and Wallace (1999a,b) and Cavalcanti et al. (1999).

All agents discount by $\beta \in (0,1)$ between one CM and the next DM, but wlog they do not discount between the DM and CM.

In the DM agents meet bilaterally, where sellers can provide a good q (different from x) that buyers want. Let α be the probability a buyer meets a seller, so that α/n is the probability a seller meets a buyer. In any meeting, if a seller produces for a buyer the former incurs cost $c(q)$ and the latter gets utility $u(q)$, where $c(0) = u(0) = 0$, $c', u' > 0$ and $c'' \geq 0 > u''$. Also, let q^* satisfy $u'(q^*) = c'(q^*)$. Goods q and x are nonstorable, so they cannot serve as commodity money. Credit is not viable because there is limited commitment and DM trading is anonymous. Hence, as is standard in these models, sellers only produce if they get assets in exchange.

Let the terms of trade be given by a generic mechanism, as in Gu and Wright (2016), meaning this: for a buyer to get q, he must give the seller assets worth $v(q)$ in CM numeraire, for some function with $v(0) = 0$ and $v'(q) > 0$. A simple example is Kalai's (1977) proportional bargaining solution, $v(q) = \theta c(q) + (1 - \theta) u(q)$. Generalized Nash is similar but the formula for $v(q)$ is more complicated when liquidity constraints are operative. For a fairly general class of mechanisms, Gu and Wright (2016) show this: if a buyer has enough assets to make his liquidity constraint slack, he gets the efficient $q = q^*$ and pays $p^* = v(q^*)$; but if he has assets worth $p < p^*$, he gives them all to the seller and gets $q = v^{-1}(p) < q^*$.

In Model 4a, assets can be held in forms that differ in safety and liquidity, where safety is captured by the probability of being stolen (or lost), and liquidity is captured by whether it can be used as means of payment in the DM. To maintain stationarity, any assets that are stolen (or lost) return to the system next period, say because thieves (or finders) bring them to the CM. Let $\mathbf{a} = (a_1, a_2)$ be a buyer's portfolio: a_1 denotes assets held in a safe but illiquid form, say hidden in one's basement, meaning that it cannot be stolen (or lost) but also cannot be used in the DM; and a_2 denotes assets held in a liquid form, which means they are brought to the DM, where can be used as payment instruments, but there is a probability $\delta > 0$ of being stolen (or lost).

The *ex dividend* price of the asset in terms of numeraire is ψ independent of whether it is held as a_1 or a_2. A buyer's CM value function is $W(A)$ where $A = (\psi + \rho)\Sigma_j a_j$ is wealth. His DM value function is $V(\mathbf{a})$, which depends on his portfolio and not just its value. A buyer's CM problem is then

$$W_t(A_t) = \max_{x_t, \ell_t, \hat{\mathbf{a}}_t} \{U(x_t) - \ell_t + \beta V_t(\hat{\mathbf{a}}_t)\} \text{ st } x_t = A_t + \ell_t - \psi_t \Sigma_j \hat{a}_{j,t}$$

where $\hat{\mathbf{a}} = (\hat{a}_1, \hat{a}_2)$ is his updated portfolio, and the CM real wage is 1 assuming that x is produced one-for-one with ℓ. Given an interior solution, several standard results are immediate: (i) $x_t = x^*$ solves the FOC $U'(x^*) = 1$; (ii) $\hat{\mathbf{a}}_t$ solves the FOC's $\beta \partial V_{t+1}/\partial \hat{a}_{j,t} \leq \psi_t, = 0$ if $\hat{a}_{j,t} > 0$, which is independent of $\mathbf{a}_t$, so all buyers exit the CM with the same portfolio; and (iii) $W_t'(A_t) = 1$, so $W_t(A_t)$ is linear in wealth. A seller's CM problem (not shown) is similar, with a CM payoff again linear in wealth.

A buyer's value function is

$$V_{t+1}(\hat{\mathbf{a}}_t) = (1-\delta)\left\{\alpha\left[u(q_{t+1}) - v(q_{t+1})\right] + W_{t+1}(\hat{A}_{t+1})\right\} + \delta W_{t+1}[(\psi_{t+1} + \rho)\hat{a}_{1,t}]$$

where $\hat{A}_{t+1}$ is the wealth implied by $\hat{\mathbf{a}}_t$, with q_{t+1} solving $v(q_{t+1}) = (\psi_{t+1} + \rho)\hat{a}_2$ if $(\psi_{t+1} + \rho)\hat{a}_2 < v(q^*)$, and $v(q_{t+1}) = v(q^*)$ otherwise. The buyer's surplus in a DM transaction is $u(q) - v(q)$, because of the result that $W(\cdot)$ is linear. Equilibrium is described by the Euler equations, which come from inserting the derivatives of V into the FOC's from the CM:

$$0 = \hat{a}_{1,t}\left[\beta\left(\psi_{t+1} + \rho\right) - \psi_t\right] \tag{32}$$

$$0 = \hat{a}_{2,t}\left\{\beta\left(\psi_{t+1} + \rho\right)(1 - \delta)\left[1 + \alpha\lambda(q_{t+1})\right] - \psi_t\right\}, \tag{33}$$

where $\lambda(q) = u'(q)/v'(q) - 1 > 0$ is the *liquidity premium* on assets in the DM.

If we normalize the aggregate asset supply to 1, the dynamical system implied by the model is described as follows. At any t, there are three possible regimes: (i) $\hat{a}_{2,t} = 0$; (ii) $0 < \hat{a}_{2,t} < 1$; and (iii) $\hat{a}_{2,t} = 1$. In regime (i), inserting $\hat{a}_{1,t} = 1$ and $\hat{a}_{2,t} = 0$ into (32) and (33), we get $\psi_t = \beta(\psi_{t+1} + \rho)$ and $(1 - \delta)[1 + \alpha\lambda(0)] \leq 1$,

with the latter equivalent to

$$\delta \geq \hat{\delta} \equiv \frac{\alpha\lambda(0)}{1 + \alpha\lambda(0)}. \tag{34}$$

Thus, agents bring no assets to the DM if the probability of theft is high.[21] If (34) holds, the DM shuts down, in which case the only possible equilibrium outcome has $\psi_t = \psi^F \ \forall t$, where $\psi^F \equiv \beta\rho/(1-\beta)$ may be called the *fundamental price* of the asset.[22]

Now assume $\delta < \hat{\delta}$, and consider regime (ii), where agents hold some but not all their assets in liquid form. Inserting $\hat{a}_{1,t}, \hat{a}_{2,t} > 0$ into (32) and (33), we get $\psi_t = \beta(\psi_{t+1} + \rho)$ and $(1-\delta)[1 + \alpha\lambda(q_{t+1})] = 1$, which means $q_{t+1} = \tilde{q}$ where

$$\alpha\lambda(\tilde{q}) = \frac{\delta}{1-\delta}. \tag{35}$$

One can show regime (ii) obtains iff $\psi_{t+1} + \rho > \hat{a}_{2,t}(\psi_{t+1} + \rho) = v(\tilde{q})$ and $\delta < \hat{\delta}$.

Finally, consider regime (iii). Inserting $\hat{a}_{1,t} = 0$ and $\hat{a}_{2,t} = 1$ into (32) and (33), we get $\psi_t \geq \beta(\psi_{t+1} + \rho)$ and

$$\psi_t = \beta(\psi_{t+1} + \rho)(1-\delta)[1 + \alpha\lambda(q_{t+1})], \tag{36}$$

where $q_{t+1} = v^{-1}(\psi_{t+1} + \rho) < \tilde{q}$. This last condition is equivalent to $\psi_{t+1} \leq \tilde{\psi} \equiv v(\tilde{q}) - \rho$. Hence if $\delta < \hat{\delta}$ the dynamic system is $\psi_t = f(\psi_{t+1})$ where:

$$f(\psi) \equiv \begin{cases} \beta(\psi + \rho)(1-\delta)[1 + \alpha\lambda \circ v^{-1}(\psi + \rho)] & \text{if } \psi < \tilde{\psi} \\ \beta(\psi + \rho) & \text{if } \psi \geq \tilde{\psi} \end{cases} \tag{37}$$

Equilibrium is a nonnegative and bounded path for $\psi_t = f(\psi_{t+1})$.

Proposition 4 *Steady state exists, is unique, and is described as follows. Define* $\tilde{\delta} \in [0, \hat{\delta})$ *by*

$$\tilde{\delta} = \frac{\alpha\lambda \circ v^{-1}(\psi^F + \rho)}{1 + \alpha\lambda \circ v^{-1}(\psi^F + \rho)}. \tag{38}$$

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[21]With Kalai bargaining and the Inada condition $u'(0)/c'(0) = \infty$, this reduces to $\hat{\delta} = \alpha\theta/(1 - \theta + \alpha\theta)$, so $\hat{\delta} = 1$ if $\theta = 1$ and $\hat{\delta} < 1$ otherwise

[22]One might argue that $(1-\delta)\beta(\psi + \rho)$, and not $\beta(\psi + \rho)$, is the fundamental price, since an asset holder only gets the return when it is not stolen. A rebutal is that *someone* always gets the payoff, even if it is the thief. To avoid this semantic issue one can simply interpret ψ^F as notation for $\beta\rho/(1-\beta)$.

Then (i) $\delta \geq \hat{\delta}$ implies $\hat{a}_1 = 1$, $\hat{a}_2 = 0$ and $\bar{\psi} = \psi^F$; (ii) $\delta \in (\tilde{\delta}, \hat{\delta})$ implies $\hat{a}_1 > 0$, $\hat{a}_2 > 0$ and $\bar{\psi} = \psi^F$; and (iii) $\delta \leq \tilde{\delta}$ implies $\hat{a}_1 = 0$, $\hat{a}_2 = 1$ and $\bar{\psi} > \psi^F$.

Fig. 10 shows how steady state depends on δ. In regime (i) the DM is inactive and $\bar{\psi} = \psi^F$, because assets are not safe enough to use as payment instruments. In regime (ii) the DM is active at $q = \tilde{q} > 0$, but since some wealth is parked in illiquid $\hat{a}_1$, we again have $\bar{\psi} = \psi^F$, with $\partial q / \partial \delta < 0$. Thus DM output goes down with δ because it reduces output per trade $\tilde{q}$, as well as the number of trades $(1 - \delta) \alpha$. In regime (iii), which it maximizes "cash in the market" with $\hat{a}_2 = 1$, we get $\bar{\psi} > \psi^F$. Here $\partial q / \partial \delta < 0$ not because $\hat{a}_2$ falls, but because ψ falls, with δ.
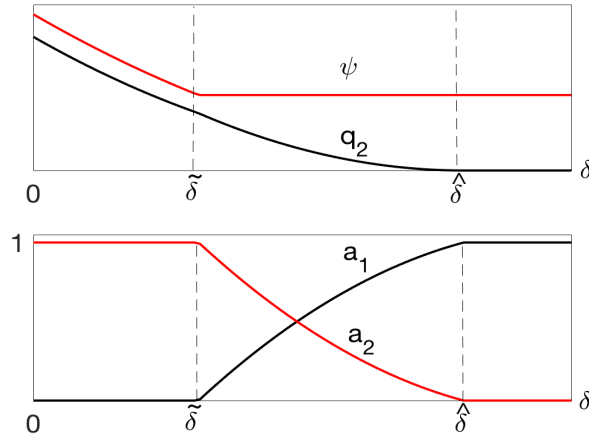

Figure 10: Model 4a, regimes of steady state

A steady state can either be on the linear or the nonlinear branch of $f(\cdot)$. In the former case, $\bar{\psi} = \psi^F > \tilde{\psi}$ and steady state is the only equilibrium (any other path for ψ_t is unbounded). In the latter case, $\psi^F < \bar{\psi} < \tilde{\psi}$, and we potentially have cyclic, chaotic or stochastic equilibria where ψ_t oscillates around $\bar{\psi}$. As usual, this occurs when $f'(\bar{\psi}) < -1$. So the economy may have asset prices above their fundamental value, and these prices may exhibit excess volatility, even without banks, as is true in many monetary models.
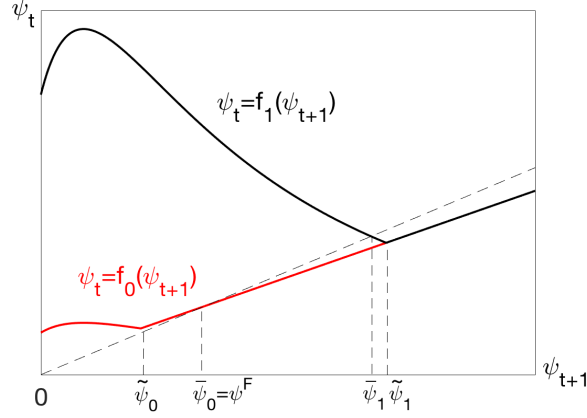
Figure 11: Model 4a, dynamic equilibrium

Now introduce banks that take assets as deposits and issue receipts, claims on these deposits. Let a_3 denote assets on deposit, and assume they are safer than a_2 but still liquid – i.e., as described in the above quotation from Selgin (2018), bank liabilities (the receipts) are *spendable*.[23] However, deposits may entail a lower yield than the original assets, since banks may have operating costs and depositors may be willing to sacrifice return for safety. If ι is the interest rate on deposits then bank profit is

$$\Pi(a_3) = a_3(\rho - \iota) - k(a_3), \tag{39}$$

where $k(a_3)$ is the cost of managing deposits, with $k', k'' \geq 0 = k(0)$. Maximization of Π equates the spread $\rho - \iota$ to marginal cost $k'(a_3)$.

Now $\hat{a}_j > 0 \ \forall j$ is possible because there are three asset characteristics – namely, liquidity, safety and rate of return. However, to begin, suppose deposits are perfectly liquid and safe, and set $k(a_3) = 0$ so that $\iota = \rho$ (we return to the general case below). Then a_3 strictly dominates a_2, and the economy looks like one without

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[23]We can make deposits less than perfectly liquid – i.e., some sellers do not accept receipts or accept them only up to a limit – using private information as in Lester et al. (2012) or Li et al. (2012). We can also make deposits less than perfectly safe if bankers might abscond with them, get robbed or otherwise fail, but it is reasonable to say they are relatively safe.

banking where $\delta = 0$. This is shown in Fig. 11, where f_1 and f_0 are the dynamical systems with and without banking. Adding banks shifts up in the nonlinear branch of $f(\cdot)$, which increases $\tilde{\psi}$ and $\bar{\psi}$, since agents can now keep assets in a safe place and still use them for DM transactions.

Banking increases DM output because it increases the size and number of trades. What does it do for volatility? Starting without banks, suppose steady state is on the linear branch of $f(\cdot)$, so there is a unique equilibrium $\psi_t = \psi^F \; \forall t$. Then introducing banks can shift $f(\cdot)$ up by enough that the new steady state is on the nonlinear branch. Thus, banking can make possible cyclic, chaotic and stochastic equilibria that were impossible without it. For some parameters such outcomes are also possible without banking, but if the economy has a unique equilibrium with banking the same is true without banking.

Fig. 11 is drawn for the following specification:

Example 7: *Let* $k(a_3) = 0$, $c(q) = q$,

$$u(q) = \frac{A}{1 - \sigma} \left[(q + \varepsilon)^{1-\sigma} - \varepsilon^{1-\sigma} \right],$$

and use bargaining with $\theta = 1$. *Also set* $A = 0.15$, $\sigma = 3.1$, $\varepsilon = 0.16$, $\rho = 0.033$, $\beta = 0.8333$, $\delta = 0.85$ *and* $\alpha = 1$.

Without banks there is a unique equilibrium, the steady state $\bar{\psi} = \psi^F = 0.1650$. With banks there is a steady state $\bar{\psi} = 0.3183 > \psi^F$ plus a two-cycle where $\psi_L = 0.3193$ and $\psi_H = 0.3502$.

While this example makes our main point, it is worth asking what else the model can do. We now show it generates something realistic but uncommon in economic theory: the concurrent circulation of assets and bank liabilities as payment instruments. So that $\hat{a}_3$ does not strictly dominate $\hat{a}_2$, consider a more general cost function $k(a_3)$. Then bank's FOC defines a supply curve that is increasing in ι, which is endogenous in equilibrium but taken as given by individuals. Equilibrium is characterized by (32)-(33) with

$$v(q_{t+1}) = \left(\psi_{t+1} + \rho \right) \hat{a}_2 + \left[\psi_{t+1} + \iota(\hat{a}_3) \right] \hat{a}_3,$$

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since DM purchases now use a_2 and a_3. The demand for a_3 satisfies

$$0 = \hat{a}_{3,t} \left\{ \beta \left(\psi_{t+1} + \iota_t \right) \left[1 + \delta \alpha \lambda \left(q'_{t+1} \right) + (1 - \delta) \alpha \lambda \left(q_{t+1} \right) \right] - \psi_t \right\} \quad (40)$$

where $v \left(q'_{t+1} \right) = \left[\psi_{t+1} + \iota \left(\hat{a}_3 \right) \right] \hat{a}_3$ and q'_{t+1} is the DM purchase when a_2 is stolen.

Consider the following example:

Example 8: *Let $A = 2.5$, $\sigma = 2.5$, $\varepsilon = 0.001$ $\rho = 0.04$, $\beta = 0.8$, $\delta = 0.01$, $\alpha = 1$, and $k \left(a_3 \right) = 0.03 a_3$.*

There is a unique steady state in which $\bar{\psi} = 1.3125$, $\hat{\mathbf{a}} = (0, 0, 1)$ and $\iota = 0.01$. There is also a two-cycle with $\psi_L = 1.2128$, $\hat{\mathbf{a}}_L = (0, 0, 1)$, $\psi_H = 1.4760$ and $\hat{\mathbf{a}}_H = (0.0384, 0.2293, 0.7323)$. In the L state, ψ is low, all assets are deposited, and only bank liabilities are used in the DM; in the H state, ψ is high, assets are held in all three forms, with both a_2 and a_3 used in the DM. Fig. 12 shows the price ψ, deposits a_3, their value $(\psi + \iota) a_3$, and the surplus $u(q) - c(q)$ over the cycle.
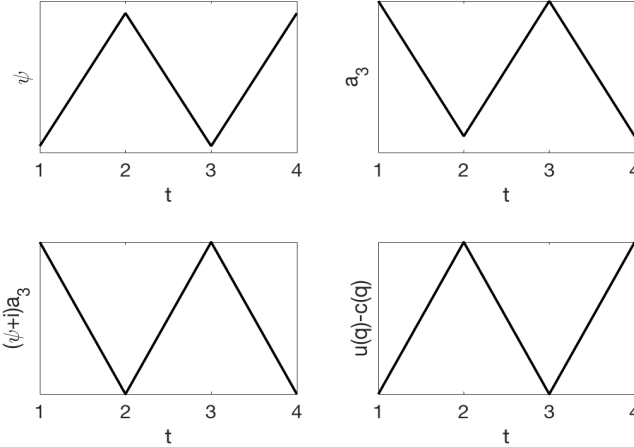

Figure 12: Model 4a, time series for a two-cycle

Now consider Model 4b, based on secrecy rather than safety. First, following in Hu and Rocheteau (2015) or Lagos and Zhang (2019), assume Lucas trees die (disappear) with probability δ at the beginning of each CM.[24] To maintain stationarity,

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[24]For an individual, having one's asset disappear is similar to having it stolen, so Models 4a and 4b are similar, although the details differ. Moreover, they share the alternating CM-DM structure, the use of a generic trading mechanism $v(q)$, and other components, which is why we treat Models 4a and 4b as special cases of the same general environment.

dead trees are replaced by new ones, distributed across agents as lump sum transfers from nature. Also, this is an aggregate shock (all or no assets survive each period). Moreover, information about the shock in the next CM is revealed in the current DM, before agents trade, which is a hindrance to having assets serve as media of exchange. This specification is extreme, in that the asset value drops to 0 after a shock; all we really need, however, is that it goes down.

The CM problem is

$$W_t(a_t) = \max_{x_t, \ell_t, \hat{a}_t} \{U(x_t) - \ell_t + \beta V_{t+1}(\hat{a}_t)\} \text{ st } x_t = (\psi_t + \rho)a_t + \ell_t - \psi_t \hat{a}_t + T$$

where T denotes transfers. Here the asset is the only DM means of payment, and it is only usable when it is revealed that it will survive to the next CM. Hence,

$$V_{t+1}(\hat{a}_t) = (1 - \delta)\{\alpha[u(q_{t+1}) - v(q_{t+1})] + W_{t+1}(\hat{a}_t)\} + \delta W_{t+1}(0)$$

where, as in Model 4a, $v(q_{t+1}) = (\psi_{t+1} + \rho)\hat{a}_t$ if $(\psi_{t+1} + \rho)\hat{a}_t < v(q^*)$ and $v(q_{t+1}) = v(q^*)$ otherwise. Again, we get $\psi_t = f_0(\psi_{t+1})$, where the subscript 0 indicates there are no banks for now, and

$$f_0(\psi) = \beta(1 - \delta)(\psi + \rho)\left[1 + \alpha\lambda \circ v^{-1}(\psi + \rho)\right]. \tag{41}$$

Now introduce banks that take assets on deposit and issue receipts. These deposits are not insured – they are claims to the asset, and if the asset dies the claim is worthless. By design, the role of banks in this formulation is not to provide insurance, but to capture secrecy as follows: while an agent holding an asset can see if it will die in the next CM, once he deposits it in a bank he cannot, and although the banker holding the asset can see it, he may or may not inform people. This is the idea in the literature, discussed above, where some assets are more informationally insensitive than others and banks' role as secret keepers. Agents like to use bank liabilities as DM payment instruments, rather than the original assets, since the former trade at their expected value rather than their realized value. This bank money provides a steadier stream of liquidity.

With banks, the DM value function is

$$V_{t+1}(\hat{a}_t) = \alpha \left[u\left(q_{t+1} \right) - v\left(q_{t+1} \right) \right] + (1 - \delta)W_{t+1}\left(\hat{a}_t \right) + \delta W_{t+1}\left(0 \right)$$

This leads to $\psi_t = f_1\left(\psi_{t+1} \right)$, where

$$f_1\left(\psi \right) = \beta \left(1 - \delta \right)\left(\psi + \rho \right)\left\{ 1 + \alpha\lambda \circ v^{-1}\left[(1 - \delta)\left(\psi + \rho \right) \right] \right\}. \tag{42}$$

As $\lambda\left(\cdot \right)$ is decreasing, f_1 lies above f_0 on the nonlinear branch, and hence f_1 reaches a higher steady state. It can be shown that the liquidity provided by deposits in steady state is lower than that provided by the asset when the asset does not die, but of course is higher when it dies. On net, banking can improve welfare, but it can also engender instability.

This is shown in Fig.13 for the following parameterization:

Example 9 *Same as Example 7 except* $A = 0.5$, $\sigma = 3.5$, $\varepsilon = 0.15$, $\rho = 0.5$, $\beta = 0.9$ *and* $\delta = 0.5$.

Without banking, the unique equilibrium is a steady state where $\psi = 0.4091$, and $q = q^* = 0.6703$ if the asset survives while $q = 0$ otherwise. With banking, there is a steady state where $\psi = 0.7187$ and $q = 0.6093$, and welfare is higher, but there is also a two-cycle where $\psi_L = 0.6081$ and $\psi_H = 0.8514$. The time series (not shown) in this case is simple since all variables move with ψ. Thus, banking eliminates fundamental cycles induced by information about realized asset values, but introduces volatility as a self-fulfilling prophecy.
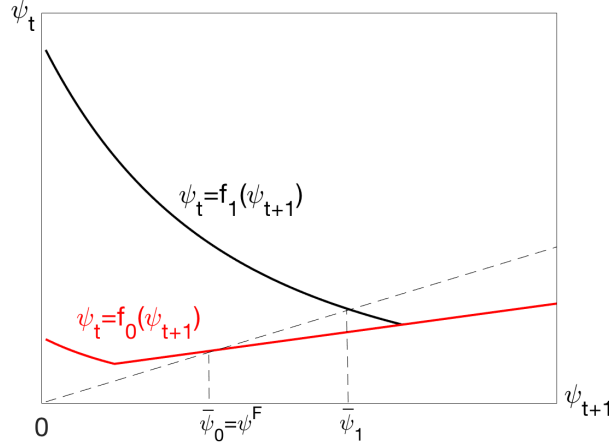
Figure 13: Model 4b, dynamic equilibrium

The first part of Proposition 5 below says banks may engender volatility. The second part says they cannot eliminate volatility due to self-fulfilling prophecies, since if there is a unique equilibrium $\psi_t = \psi^F$ $\forall t$ with banking, there is also a unique equilibrium with $\psi_t = \psi^F$ $\forall t$ without it.

Proposition 5 *When the steady state $\bar{\psi} = \psi^F$ is the unique equilibrium without banking, introducing banks can introduce nonstationary equilibria. When the steady state $\bar{\psi} = \psi^F$ is the unique equilibrium with banking, steady state is the unique equilibrium without banking.*

Models 4a and 4b, based on safety and secrecy, respectively, deliver similar results and can be understood by similar intuition. First, in the presence of liquidity considerations, the system $\psi_t = f\left(\psi_{t+1}\right)$ has two terms: one reflects a store-of-value component making price today monotone increasing in the price tomorrow; the other reflects a medium-of-exchange component making price today generally non-monotone in the price tomorrow. If the second term is decreasing and dominates the first, $f'\left(\bar{\psi}\right) < -1$ is possible, and hence endogenous dynamics can occur. In Model 4a, without banks we have $f_0\left(\psi\right)$, and with banks we have $f_1\left(\psi\right) = f_0\left(\psi\right)/(1-\delta)$ on the nonlinear branch. This explains how we can get $f_0'\left(\psi\right) > -1$ without banks

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and $f_1'(\psi) < -1$ with banks at any given ψ. In addition, steady state moves from $\bar{\psi}_0$ without banking to $\bar{\psi}_1$ with banking, which can also make $f_1'(\bar{\psi}) < -1$ more likely.

In Model 4a banks make the asset better as a store of value and as a medium of exchange by protecting from theft. In Model 4b banks do not make the asset better as a store of value, because there is no way to avoid the loss if the tree dies, but they make it a better medium of exchange if the bank keeps information secret. Hence, in Model 4b agents unambiguously put more weight on the nonmonotone medium-of-exchange component, so it is more likely that $f_1'(\psi) < -1$. In general, although the details are different in the two versions, in both banking can engender volatility.

Conclusion

This paper explored the idea that financial intermediaries are inherently unstable. The approach involved building formal models of these institutions, and then asking if they make it more likely that there will be multiple equilibria and/or excess volatility. We showed that they do, in several theoretical environments designed to capture various facets of intermediation. Models 1 and 2 involved trust, as seems natural in a theory of banking, but they differed in the basic reason for its emergence: one concerned insurance; the other featured fixed costs of investing. Model 3 was built to capture not banks but middlemen in OTC markets. Models 4a and 4b concerned the use of bank liabilities as payment instruments. The analysis used tools from mechanism design, search, bargaining, contract and monetary theory. While many of the ingredients appear in previous studies, the ways in which we combine and apply them are novel and generate new insights.

Although the specification differ economically and mathematically they have similar implications for multiplicity and volatility. This lends support to the notion that financial intermediation engenders instability, because it appears to transcend technical modeling details. However, we again emphasize this does not make it a

bad idea, since intermediation may well improve welfare.[25] We used examples a lot because the claim is only that financial intermediation *may* lead to instability, not that it *must*; future work may ask what happens at realistically calibrated parameters. To reiterate our other claim, it is not that we think actual data are best explained by cycles or sunspots, but that when rudimentary models have equilibria where liquidity, prices, quantities and welfare vary as self-fulfilling prophecies, it seems more likely that this can happen in the real world.

One can naturally ask what the four types of models presented above have in common and why they are all in one paper. What they have in common is this: Building models of financial intermediation requires environments with explicit frictions, including limited commitment, spatial or temporal separation, and imperfect information or communication. This can give rise to an endogenous role for intermediation, like it can give rise a role for money and other institutions whose purpose is ameliorating said frictions. At the same time, frictions mean that there can be multiple Pareto-ranked equilibria and belief-based dynamics. This is related to what Shell (1992) calls the Philadelphia Pholk Theorem: in all models where equilibria are not efficient one can find multiple equilibria or excess volatility, where he was thinking of sunspots, but the reasoning also applies to cycles. It is hard to prove this, in general, because it concerns *all* models, so corroboration consists of showing it works in a series of environments. Our conclusion is that the frictions actuating endogenous intermediation also lead to multiplicity and volatility. The reason to have four models is to show the idea is robust – it applies in various environments, with different frictions, meant to capture different features of financial intermediation in the real world.

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[25]Rajan (2005) argues that volatility, which he takes to be self-evidently bad, has emerged from recent financial innovation. This is consistent with our theoretical findings, but we tend to agree with Summers' comment: Financial innovations are like improvements in transportation technology, which have an overwhelmingly positive impact on welfare even if they increase the possibility of, say, plane crashes. Clearly, financial markets, like the airline industry, may need some regulation, but too much can be counterproductive (e.g., see Lacker 2015). In future work we plan to explore optimal policy intervention.

Appendix A: Proofs of Nonobvious Results

Proposition 1: If $\hat{\phi} \leq 0$ then (12) reduces to $\phi_{t-1} = \beta \phi_t$ and the only equilibrium is the steady state with $\bar{\phi} = 0$. If $\hat{\phi} > 0$ then $f(0) > 0$ and $f(\hat{\phi}) = \beta \hat{\phi} < \hat{\phi}$ implies $\bar{\phi} \in (0, \hat{\phi})$ exists. To see it is unique, first solve (12) for $\phi = \beta \mu v (bR) / (1 - \beta)$ and substitute it into (10) to get $\lambda d = (1 - \beta + \beta \mu) v (bR) / (1 - \beta)$. This implies d is increasing in b. But (11) implies d is decreasing in b, so if they have a solution $(\bar{b}, \bar{d})$ it is unique. ∎

Proposition 3: First, for uniqueness, note that when $\pi = \bar{\pi}$ the equations for the R-curve and n-curve are defined by

$$\left(\frac{r + \delta}{1 - \delta} + \alpha \theta_{ms}\right) R - \rho - \frac{\alpha n_b \theta_{mb} \tau (\bar{\pi} - R) + \alpha (n_b + n_m) \theta_{ms} R}{N} = 0 \quad (43)$$

$$\delta n + n (1 - \delta) \frac{\alpha n_b \tau}{N} - (n_m - n)(1 - \delta) \alpha \left(1 - \frac{n_b + n_m}{N}\right) = 0 \quad (44)$$

where

$$N = \frac{\alpha n_b \theta_{sb} \bar{\pi} + \alpha (n_m - n) \theta_{sm} R}{\kappa_s}.$$

In the region where $R > \bar{\pi}$, where $\tau = 0$ combine (43) and (44) to eliminate N,

$$\left(r + \delta + \frac{\theta_{ms} \delta n}{n_m - n}\right) R = \rho (1 - \delta) \quad (45)$$

This implies

$$\frac{\partial R}{\partial n} = -\frac{\theta_{ms} \delta n_m}{(n_m - n)^2 (r + \delta) + (n_m - n) \theta_{ms} \delta n} < 0.$$

Thus we transform the system (43)-(44) to (45)-(44). As (45) is downward sloping and (44) is upward sloping, there exists at most one steady state with $R > \bar{\pi}$.

In the region where $R < \bar{\pi}$, where $\tau = 1$, combine (43) and (44) to get

$$\left(\frac{r + \delta}{1 - \delta} + \alpha \theta_{ms}\right) R = \rho + \frac{n_b \theta_{mb} (\bar{\pi} - R) + (n_b + n_m) \theta_{ms} R}{(1 - \delta) n_m (n_b + n_m - n)} [(n_m - n)(1 - \delta) \alpha - n\delta].$$

This implies

$$\frac{\partial R}{\partial n} = -\frac{n_b \theta_{mb} (\bar{\pi} - R) + (n_b + n_m) \theta_{ms} R}{r + \delta + (\theta_{ms} + n_b \theta_{mb})(1 - \delta) \alpha / N} \frac{\delta (n_b + n_m) + (1 - \delta) \alpha n_b}{n_m (n_b + n_m - n)^2} < 0 \quad (46)$$

Again, since (46) is downward and (44) upward sloping, there is at most one steady state with $R < \pi$. Similarly, when $R = \bar{\pi}$ and $\tau \in (0, 1)$, the n-curve is flat and R-curve is upward sloping. Hence, there again is at most one steady state.

For existence, first, it is easily verified that the R- and n-curve are upward sloping. At $n = 0$ the R-curve implies $R > 0$ and the n-curve implies $n > 0$. At $R = \infty$ the R-curve implies $n = n_m$ and the n-curve implies $n = \bar{n} \equiv \alpha n_m (1 - \delta) / [\delta + (1 - \delta) \alpha] < n_m$. Hence the curves cross at least once, and generically an odd number of times. Since we already established that there cannot be multiple steady states in the same regime , if there is a steady state at $R = \bar{\pi}$, there must exist two other steady states, one with $R < \bar{\pi}$ and one with $R > \bar{\pi}$. Routine calculation implies $\partial R / \partial \rho > 0$, and so there exist $\tilde{\rho}, \hat{\rho} \geq 0$ with the properties specified in Proposition 3. ∎

Appendix B: Sunspot Equilibria

A dynamical system allows for a two-state sunspot equilibrium solves

$$\phi_{s,t-1} = \zeta_s f\left(\phi_{s,t}\right) + (1 - \zeta_s) f\left(\phi_{-s,t}\right) \tag{47}$$

where $s = A, B$ denotes two states in the sunspot equilibrium, $\zeta_s \in (0, 1)$ is the probability of staying in the same state, and f is the dynamical system in the deterministic case. We seek a pair of probabilities $(\zeta_A, \zeta_B) \in (0, 1)^2$ satisfying (47) in stationary equilibrium.

To proceed, rewrite (47) as

$$\zeta_A = \frac{f\left(\phi_B\right) - \phi_A}{f\left(\phi_B\right) - f\left(\phi_A\right)} \text{ and } \zeta_B = \frac{\phi_B - f\left(\phi_A\right)}{f\left(\phi_B\right) - f\left(\phi_A\right)}$$

Consider wlog $\phi_B > \phi_A$. If f is decreasing on (ϕ_A, ϕ_B), the denominator is negative. Then $\zeta_A, \zeta_B \in (0, 1)$ iff $f(\phi_A) > \phi_B > \phi_A > f(\phi_B)$, which implies that f crosses the 45 degree line from above and $[f(\phi_A) - f(\phi_B)] / (\phi_A - \phi_B) < -1$. Therefore, in Model 1 where f is decreasing around the steady state, there exist sunspot equilibria if $f(\bar{\phi}) < -1$.

Similarly, if f is increasing on (ϕ_A, ϕ_B), the denominator is positive. Then $\zeta_A, \zeta_B \in (0, 1)$ iff $f(\phi_B) > \phi_B > \phi_A > f(\phi_A)$, which implies f crosses the 45° line from below on $[\phi_A, \phi_B]$. Therefore, in Model 2 where f is increasing, there exist sunspot equilibria around a stable steady state ϕ_1 for any $\phi_A \in (0, \phi_1)$ and $\phi_B \in (\phi_1, \phi_2)$.

Appendix C: Bargaining in Model 2

There are two agents on each island, one who lives for one period and one who lives forever, so the former should be the depositor and the latter the banker. Assume

the cost κ is too high for them to invest individually. If the banker's bargaining power is θ, the generalized Nash problem is

$$W(\phi) = \max_{X,x,D,d}[U(X) - C(D)]^\theta [u(x) - c(d)]^{1-\theta} \qquad (48)$$

$$\text{st } X + x = R(D + d) - \kappa \qquad (49)$$

$$u(x) - c(d) \geq 0 \qquad (50)$$

$$x_t \leq \phi_t. \qquad (51)$$

The last constraint is from the banker's incentive condition $\beta V_{t+1} \geq \lambda x_t + (1 - \mu)\beta V_{t+1}$ rewritten using $\phi_t \equiv \beta\mu V_{t+1}/\lambda$. Notice $W'(\phi) > 0$ if (51) binds, and there is a cutoff $\tilde{\phi}$ above which banking is viable and below which it is not.

Denote the solution ignoring (50) and (51) by (X^*, x^*, D^*, d^*). Further, consider the case $u(x^*) > c(d^*)$ and let $\phi^* = x^*$. Substituting (49) into the objective function and taking FOC's, we get

$$D : \quad U'(X) R - C'(D) = 0$$

$$d : \quad \theta U'(X) R[u(x) - c(d)] - (1 - \theta) c'(d)[U(X) - C(D)] - \eta_1 c'(d) = 0$$

$$x : \quad -\theta U'(X)[u(x) - c(d)] + (1 - \theta) u'(x)[U(X) - C(D)] + \eta_1 u'(x) - \eta_2 = 0$$

where η_1 and η_2 are multipliers. From this one can see the banker's surplus may decrease with ϕ at least close to ϕ^*:

$$\left.\frac{\partial[U(X) - C(D)]}{\partial\phi}\right|_{\phi\to\phi^*} = \frac{(1-\theta)U'c''(U-C)\left(R^2 U'' - C''\right)}{(C'' - R^2 U'')[C'c' + (1-\theta)c''(U-C)] - \theta R^2 U'' C''(u-c)} < 0.$$

The banker's value function is $V_t = U(X_t) - C(D_t) + \beta V_{t+1}$, and using $\phi_t = \beta\mu V_{t+1}/\lambda$ we have

$$\phi_{t-1} = \frac{\beta\mu}{\lambda}[U(X_t) - C(D_t)] + \beta\phi_t. \qquad (52)$$

Now (52) can be written as

$$\phi_{t-1} = \begin{cases} \beta\phi_t & \text{if } \phi_t < \tilde{\phi} \\ \frac{\beta\mu}{\lambda}[U \circ X(\phi_t) - C \circ D(\phi_t)] + \beta\phi_t & \text{if } \tilde{\phi} \leq \phi_t < \phi^* \\ \frac{\beta\mu}{\lambda}[U(X^*) - C(D^*)] + \beta\phi_t. & \text{if } \phi_t \geq \phi^* \end{cases}$$

Fig. AC shows the dynamical system for the following parameterization:

Example AC: *Let $U(x) = u(x) = Ax$ and $C(d) = c(d) = Bd^\gamma/\gamma$, where $A = 1$, $B = 0.5$, $\gamma = 5$, $R = 2$, $k = 1.5$, $\theta = 0.01$, $\lambda = 0.01$, $\mu = 1$ and $\beta = 0.35$.*

There are three steady states, $\phi = 0$ and $\phi_2 > \phi_1 > 0$, with f crossing the 45° line from below at ϕ_1 and from above at ϕ_2. Hence there are sunspot equilibria around ϕ_1 fluctuating between any $\phi_A \in (0, \phi_1)$ and $\phi_B \in (\phi_1, \phi_2)$, similar to the baseline version of Model 2, and since $f'(\phi_2) < -1$ there is a two-cycle with periodic points ϕ_L and ϕ_H, plus sunspot equilibria for any $\phi_A \in (\phi_L, \phi_2)$ and $\phi_B \in (\phi_2, \phi_H)$, similar to Model 1.
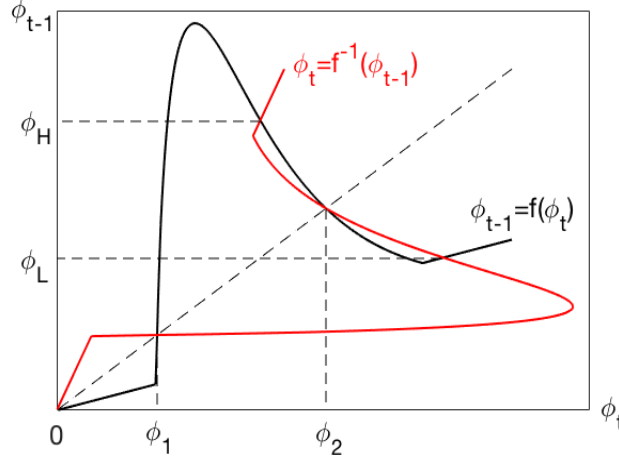

Figure AC: Nash bargaining

Appendix D: Entry by Type M in Model 3

Consider entry by M instead of S. The equations (23)-(27) are the same but now n_s is fixed while $n_{m,t}$ is endogenous. Also $V_{s,t} = \kappa_s$ is replaced by $V_{0,t} = \kappa_m$. Then (24) yields N_t in terms of R_t,

$$N_t = \frac{\alpha n_s \theta_{ms} R_t}{(1 - \beta) \kappa_m}. \tag{53}$$

From (53), N_t depends only on R_t, while with entry by S, it depends on R_t and n_t. Substituting (53) into (26), after some algebra we get $R_{t-1} = G(R_t)$, where

$$G(R) \equiv \beta (1 - \delta) \left\{ \rho + R + \frac{(1 - \beta) \kappa_m n_b \theta_{mb}}{n_s \theta_{ms} R} \int_R^\infty [1 - F(\pi)] d\pi - (1 - \beta) \kappa_m \right\}.$$

From this, R_{t-1} depends only on R_t, while in the version with entry by S, it depends on R_t and n_t.

Hence we now get a univariate system $R_{t-1} = G(R_t)$, which determines the path for R_t, after which N_t follows from (53), n_t from (27), etc. Given a fixed point

$$R = G(R),$$

$$
\begin{aligned}
N &= \alpha n_s \theta_{ms} R_t / (1 - \beta) \kappa_m \\
n_m &= \frac{\alpha n_s \theta_{ms} R_t}{(1 - \beta) \kappa_m} - n_s - n_b \\
n &= \frac{n_s (1 - \delta) [\alpha n_s \theta_{ms} R - (n_b + n_s)(1 - \beta) \kappa_m]}{\delta n_s \theta_{ms} R + (1 - \delta) [n_b \tau(R) + n_s](1 - \beta) \kappa_m}
\end{aligned}
$$

To guarantee the fixed point is a steady state we must check $n_m, n \geq 0$, both of which hold iff $R \geq \underline{R} \equiv (n_s + n_b)(1 - \beta) \kappa_m / \alpha n_s \theta_{ms}$ (we also need $n \leq n_m$ but that never binds). Hence, a solution to $R = G(R) \geq \underline{R}$ is a steady state with type M active; otherwise, there is no intermediation.
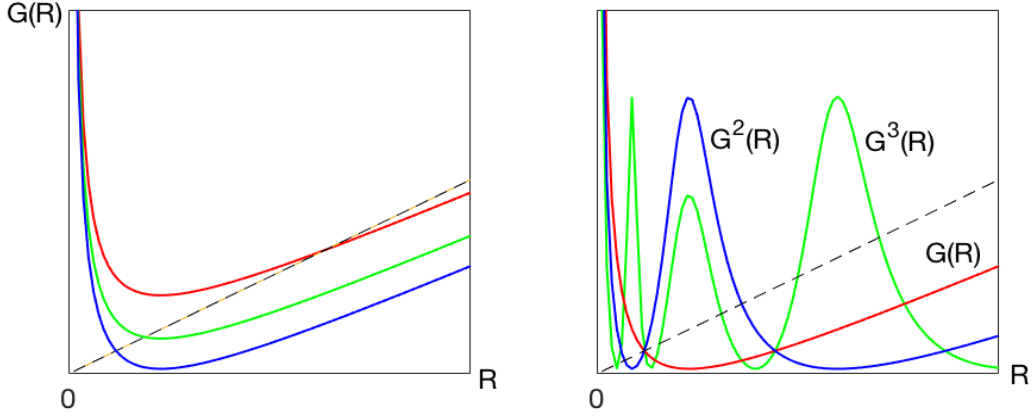

Figure AD: Model 3, cycles with entry by M

One can check $G(0) = \infty$, $G'(R) < 1$ and $G''(R) \geq 0$. Also, $\forall R > \max(\pi)$, G is linear with slope $\beta(1 - \delta)$. This is shown in the left panel of Fig. AD, from which it is clear that there exists a unique fixed point, say $\hat{R}$. In any case, we can have $\hat{R} > \max(\pi)$ on the linear part of $G(R)$ or $\hat{R} < \max(\pi)$, on the nonlinear part of $G(R)$. If $G'(\hat{R}) < -1$ then $\hat{R}$ is locally stable, and there exist cycles and sunspots. There is a threshold ρ_1 such that $G'(\hat{R}) < -1$ iff $\rho < \rho_1$. We do not know if $\rho_1 > 0$ or $\rho_1 < 0$, in general, but all our examples gave $\rho_1 < 0$. Still we have to verify $R \geq \underline{R}$, as discussed above. Is $G'(\hat{R}) < -1$ and $\hat{R} \leq \underline{R}$ possible? Yes, as we now show by way of example.

Example AD: *Consider* $\alpha = 1$, $\delta = 0.01$, $\beta = 0.99$, $n_b = n_s = 1$, $\theta_{mb} = \theta_{ms} = 0.5$, $\kappa_m = 0.1$, $\rho = -0.1$, *and use the* $F(\pi)$ *in (30), with* $\pi_0 = 4.95$, $\pi_1 = 0.05$, $\pi_2 = 5.05$, $\pi_3 = 0.95$ *and* $\pi_4 = 10$.

This is the example used in Fig. AD, where it can be easily checked that $G'(\hat{R}) < -1$ and $\hat{R} < \underline{R}$. Hence this admits a two-cycle. Note that $\rho < 0$ in this example. If we lower ρ a little more, we can get higher-order cycles and chaotic dynamics. This is shown in the right panel of Fig. AD, where we plot $G^3(R)$ and see that there exist fixed points other than $\hat{R}$, namely a pair of three cycles. Hence, we can explicitly construct higher order cycles. Finally, one more result is that $\rho < 0$ implies M and B must trade for some π, $\Pr(R < \pi) > 0$, if M is in the market – just like in the other version, a buy-and-hold-forever strategy is never a good idea at $\rho < 0$.

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