RESEARCH ARTICLE

Meta-analysis as a system of springs

Theodoros Papakonstantinou*[1] | Adriani Nikolakopoulou[1,2] | Matthias Egger[1,3] | Georgia Salanti[1]

[1]Institute of Social and Preventive Medicine, University of Bern, Bern, Switzerland
[2]Institute of Medical Biometry and Statistics, Faculty of Medicine and Medical Center-University of Freiburg, Freiburg, Germany
[3]Population Health Sciences, Bristol Medical School, University of Bristol, Bristol, U.K.

Correspondence
*Theodoros Papakonstantinou, Institute of Social and Preventive Medicine University of Bern Mittelstrasse 43 CH-3012 Bern, Switzerland. Email: theodoros.papakonstantinou@ispm.unibe.ch

Abstract

Meta-analysis results are usually presented in forest plots, which show the individual study results and the summary effect along with their confidence intervals. In this paper, we propose a system of linear springs as a mechanical analogue of meta-analysis that enables visualisation and enhances intuition. The length of a spring corresponds to a study treatment effect and the stiffness of the spring corresponds to its inverse variance. To synthesise study springs we use two main operations: connection in parallel and connection in series. We show the equivalence between meta-analysis and linear springs for fixed effect and random effects pairwise meta-analysis and we also derive indirect treatment effects. We use examples to illustrate the different meta-analytical schemes using the corresponding system of springs. The proposed visualization tool can serve as an educational plot, especially useful for researchers with no statistical background. The analogy between meta-analysis and springs facilitates intuition for notions such as heterogeneity and the differences between fixed and random effects meta-analysis.

KEYWORDS:
evidence synthesis, visualization, mechanical analogue

1 INTRODUCTION

A forest plot is the most common graphical tool to depict the results of a pairwise meta-analysis. It shows the treatment effects of individual studies, usually as squares whose area is proportional to their weight, along with their confidence intervals. The summary effect is usually lying at the bottom of the forest plot, illustrated as a diamond, the width of which represents its confidence interval. The addition of squares around each study's point estimate, representing study's precision, originates from initial concerns that the eye is drawn to the most imprecise studies, as the confidence intervals are wider[1]. The addition is said to be inspired by modified box plots[2] and proposed in 1983 by Stephen Evans at a Royal Statistical Society medical section meeting at the London School of Hygiene and Tropical Medicine[3]. Researchers have proposed several modifications of the forest plot as well as alternative plots, such as the Galbraith and the Abbe plot, to show meta-analysis results[4,5,6,7]. A vast array of graphical displays for meta-analysis have been recently summarized by Kossmeier et al.[8].

In this paper, we propose a mechanical representation of meta-analysis as linear springs, which shows not only the meta-analysis results, but also the synthesis process itself. We will use two main operations for synthesizing evidence: connection of springs in parallel and connection of springs in series. We show that the equivalence between meta-analysis and linear springs holds for the fixed effect and random effects models and propose a visualization tool to illustrate this equivalence. The visualization tool can serve as an intuitive educational approach for researchers with no statistical background illustrating the analogy between meta-analysis and a system of linear springs.

The rest of the paper is organised as follows. We start by introducing the needed notions from mechanics in 2.1 and then show in 2.2 how a single study can be illustrated as a spring. The analogy of fixed effect pairwise meta-analysis to a system of springs is outlined in 2.3. In 2.4 we make use of serial connection to derive indirect meta-analytic effect estimates and in 2.5 we outline the representation of random effects pairwise meta-analysis as a system of springs. We conclude with a discussion.

2 | METHODS

2.1 | Definitions from mechanics

A spring is a simple mechanical component that can compress and expand in one dimension. An example from everyday life is the coil spring inside certain ballpoint pens. Springs are characterized by two quantities: *natural length* L_{BA}, where B and A refer to spring's ends, and stiffness k. Natural length L_{BA} is the distance between the spring's ends B and A when no forces are applied to it. It holds that $L_{AB} = p_B - p_A = -L_{BA} = p_A - p_B$ where p_A and p_B give the positions of ends A and B respectively. Thus, the definition of L_{BA} involves assuming either of the spring's ends as the reference and L_{BA} can be either positive or negative depending on the relative position of the non-referent end. Let us assume that the first index of L_{BA} indicates the reference treatment, which is assumed to be 'fixed'; the end referring to the non-reference treatment is assumed to be 'open'. Unlike natural length, which is constant for each spring, we also define the spring's current length $l_{BA} \in (-\infty, \infty)$ which is the length when forces are acting on it. For positive natural length, when the current length is greater than the natural the spring is extended and when the current length is smaller than the natural the spring is compressed. The opposite holds when $L_{BA} < 0$. The *displacement* x_{BA} of the spring is defined as the difference of the spring's current length to the spring's natural length: $x_{BA} = l_{BA} - L_{BA}$. As an effective spring we define the substitution of parts of the system into a single spring with the exact same stiffness and length.

A linear spring obeys Hook's law which states that the force f_A needed to displace a spring by x_{BA} is proportional to that displacement x_{BA}. The constant factor k is the spring's stiffness.

$$f_A = kx_{BA} \tag{1}$$

A common use of springs is to store energy. For example, if one compresses a spring of stiffness k by x and locks it in place with a latch the spring has stored energy $U = 1/2kx^2$. This is referred to as potential energy which the spring can give back later. In the example of a ballpoint pen, when we push to open the pen, the spring inside has the potential energy that is needed for it to stay in place. The work is the energy spent to compress the spring and equals the spring's potential energy. In the general case an external force is exerted to the system in order to stay still. In this context we will ignore energy due to motion (kinetic energy) ... ng systems are always static. We thus define a transition from one state to another as a quastistatic process meaning the velocity is infinitesimal and the system remains always in equilibrium. A system equilibrates when either there is no force or all forces acting on it cancel out. From now on we will be using the term energy when referring to *potential energy* **U**.

For a system of $i = 1, \dots, n$ springs the energy of each individual spring's i U_i

$$U_i = \frac{1}{2} k_i x_{i,BA}^2 \tag{2}$$

is the potential energy stored in the spring when its length is displaced by x_{BA} and the total energy **U** is given by

$$\mathbf{U} = \sum_{i=1}^{n} U_i \tag{3}$$

2.2 | Effect : spring representation; single study

Suppose that n studies comparing treatments A and B have been conducted and each contributes an effect size $y_{i,BA}$, $i = 1, ..., n$ with variances $\hat{\upsilon}_i$. Each study is weighted according to its inverse variance, $\hat{w}_i = \frac{1}{\hat{\upsilon}_i}$. The spring representation of a study effect is straightforward. The effect size $y_{i,BA}$ of treatment comparison B vs A with reference treatment B equals the spring's natural length. The stiffness coefficient equals the inverse variance weight, meaning that more precise studies are represented by stiffer

	Meta-analysis		Springs	
	Terminology			
	Study		Spring	
	Treatments		Spring ends	
	Summary estimate		Effective spring	
	Quantity	Symbol	Quantity	Symbol
section 2.2	Study treatment effect	$y_{i,BA}$	Natural length (no force - constant)	$L_{i,BA}$
	Study variance	$\hat{\upsilon}_i$	Compliance	$1/k_i$
	Study weight (or precision)	$\hat{w}_i$	Stiffness (constant)	k_i
	-	-	Current length (variable)	l_{BA}
section 2.3	Summary estimate fixed effect	$\hat{\mu}^D_{BA}$	Effective length of parallel springs	L^D_{BA}
	Variance of summary estimate	$Var\left(\hat{\mu}^D_{BA}\right)$	Compliance of effective spring	$1/k^D$
	Precision of summary estimate	$1/Var\left(\hat{\mu}^D_{BA}\right)$	Stiffness of effective spring	k^D
	-	-	Force on open end A	$f_{i,A} = k_i x_{i,BA}$
	-	$\hat{\mu}^D_{BA}$-$y_{i,BA}$	Displacement (deviation from common effect)	$x_{i,BA} = l^D_{BA} - L_{i,BA}$
	Study contribution on **Q**	$Q_i = \frac{(y_{i,BA}-\hat{\mu}^D_{BA})^2}{\upsilon_i}$	Spring energy*	$U_i = \frac{1}{2}k_i x^2_{i,BA}$
	Q for Heterogeneity	$\mathbf{Q} = \sum_i^n Q_i$	Total energy	$\mathbf{U} = \sum_i^n U_i$
section 2.4	True treatment effects : Transitivity	$\mu_{BA} = \mu_{BC} + \mu_{CA}$	Springs in series	$l_{BA} = l_{BC} + l_{CA}$
	Indirect estimated treatment effect	$\hat{\mu}^I_{BA} = \hat{\mu}^D_{BC} + \hat{\mu}^D_{CA}$	Effective length of springs in series	$L^I_{BA} = L^D_{BC} + L^D_{CA}$
	Variance of indirect treatment effect	$Var\left(\hat{\mu}^I_{BA}\right)$	Compliance of effective spring	$\frac{1}{k^I_{BA}} = \frac{1}{k^D_{BC}} + \frac{1}{k^D_{CA}}$
	Precision of indirect treatment effect	$1/Var\left(\hat{\mu}^I_{BA}\right)$	Stiffness of effective spring	k^I_{BA}
section 2.5	Summary estimate random effects	$\hat{\mu}^{D*}_{BA}$	Summary length	L^{D*}_{BA}
	Variance of summary estimate	$Var\left(\hat{\mu}^{D*}_{BA}\right)$	Effective compliance (summary level)	$1/k^{D*}$
	Precision of summary estimate	$1/Var\left(\hat{\mu}^{D*}_{BA}\right)$	Effective stiffness (summary level)	k^{D*}
	Random effects	δ_i	Random effects spring length	$l_{i,\tau}$
	-	$1/\tau^2$	Stiffness of random effects spring	k_τ
	Heterogeneity	τ^2	Compliance of random effects spring	$1/k_\tau$
	Random effects variance	$\upsilon^*_i = \upsilon_i + \tau^2$	Inverse of effective stiffness (study level)	$\frac{1}{k^I_i} = \frac{1}{k_i} + \frac{1}{k_\tau}$

TABLE 1 Pairwise meta-analysis - Spring system representation. *The spring's energy U_i corresponds to $Q_i/2$.

springs. Thus, the equivalence of a study to a spring is summarised as:

$$L_{i,BA} \equiv y_{i,BA} \tag{4}$$

$$k_i \equiv \frac{1}{\hat{\upsilon}_i} \tag{5}$$

If the effect of A is larger than that of B in study i, both $y_{i,BA}$ and $L_{i,BA}$ are positive and negative otherwise. Usually comparison XR is read X vs R and the reference treatment is the second term R. However, since there is no formal convention, we follow the same notation for springs and effects with the first treatment of the comparison indicating the reference.

In the following we use the illustrative worked example given in Table 14.1 of Borenstein's et al. Introduction to Meta-analysis[9]. The outcome data is continuous and consists of mean values, standard deviations and sample sizes per arm. One of those studies, Peck, is illustrated in figure 1 both in a forest plot (figure 1a) and as a spring (figure 1b). Contrary to the forest plot, where the length of the line shows the confidence interval, the spring's length corresponds to the effect size (mean difference) of the Peck study. In particular, we compute a mean difference of 10 for B vs A for the Peck study, corresponding to a natural length of $L_{1,BA} = 10$ for the spring in 1b. The stiffness coefficient k_1 is equal to $k_1 = \frac{1}{\hat{\upsilon}_1} = \frac{1}{6.04^2} = 0.0274$ and is represented as the thickness of the spring in figure 1b. Defining B as the reference treatment is shown in figure 1b as fixing end B and leaving end A open. As springs are assumed to be linear, only additive effect sizes such as mean difference, standardized mean difference, log odds ratio, log risk ratio, risk difference, etc can be represented. Measures on the ratio scale, such as odds ratio, risk ratio, hazard ratio, cannot be represented.

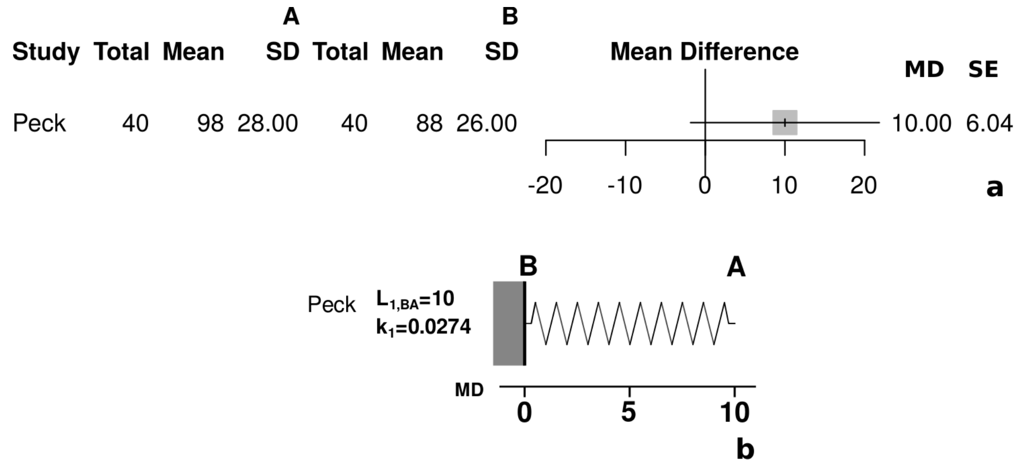

FIGURE 1 Spring representation of a study. Panel **a**: the forest plot of the fictional *Peck* study. Panel **b**: Corresponding spring representation of panel a. Since *B* is the reference treatment it is shown to be fixed. The spring's thickness is proportional to the stiffness coefficent k. MD: mean difference; SD: standard deviation; SE: standard error.

2.3 | Fixed effect pairwise meta-analysis : springs in parallel

The fixed (or common) effect model for pairwise meta-analysis can be formulated as:

$$y_{i,BA} = \mu_{BA} + \varepsilon_i; \quad \varepsilon_i \sim N\left(0, v_i\right) \tag{6}$$

where μ_{BA} is the assumed true effect and ε_i is the random error of study i. The summary estimate $\hat{\mu}^D_{BA}$ (our estimation of the true effect using direct evidence) is given by the following weighted average of the observed effects[9]:

$$\hat{\mu}^D_{BA} = \frac{\sum_{i=1}^{n} \hat{w}_i y_{i,BA}}{\sum_{i=1}^{n} \hat{w}_i} \tag{7}$$

with $\hat{w}_i$ being the inverse variance weights. If the true variances were known, the summary estimate's variance would be $Var\left(\hat{\mu}^D_{BA}\right) = \frac{1}{\sum_{i=1}^{n} w_i}$. In practice, it is common that we estimate it as

$$Var\left(\hat{\mu}^D_{BA}\right) = \frac{1}{\sum_{i=1}^{n} \hat{w}_i} \tag{8}$$

although models that avoid the assumption of within-study normality are also available. Among those, GLMMs is a reliable alternative; it is not currently clear how the representation of meta-analysis using GLMMs can be visualized as a system of springs.

We can represent the observed study treatment effects $y_{i,BA}$ of a fixed effect meta-analysis by a series of parallel open springs where the end of the reference end is fixed for each spring. To do so, we use the same example of[9] as before, using two of the studies, Grant and Peck. Figure 2a shows the forest plot for these two studies, whose synthesis results to a summary mean difference of 7.12.

The observed effects are visualized as springs in figure 2b; their length shows their effect size and more precise studies are represented by stiffer (thicker) springs. The process of synthesizing all *B* vs *A* studies corresponds to forcing all springs to have equal length, l^D_{BA}, by fixing all open ends together. In this position we have to exert force to hold the system in place. We then let the system reach its minimum energy (figure 2c) where no external force is acting on it. The set of springs –all of length l^D_{BA}- can now be replaced with an *effective spring*, of natural length $L^D_{BA} = l^D_{BA}$, which represents the summary meta-analytic effect (figure 2d). The common current length where the energy is minimum is found by replacing $k_i = \hat{w}_i$ and $x_{i,BA} = l^D_{BA} - y_{i,BA}$ (table:1) in equation 2; equation 3 becomes

$$\mathbf{U} = \sum_{i=1}^{n} \frac{1}{2} \hat{w}_i \left(l^D_{BA} - y_{i,BA}\right)^2 \tag{9}$$

And it turns out that the common current length l_{BA}^{D} is the one that satisfies

$$U^{'} = 0 \Rightarrow \sum_{i=1}^{n} f_{i,A} = 0 \Rightarrow \sum_{i=1}^{n} \hat{w}_i \left(l_{BA}^{D} - y_{i,BA} \right) = 0 \tag{10}$$

that is

$$l_{BA}^{D} = \frac{\sum_{i=1}^{n} \hat{w}_i y_{i,BA}}{\sum_{i=1}^{n} \hat{w}_i} \tag{11}$$

In equilibrium all forces on the open end cancel out, which means that the total force applied is 0. In an actual realization of the spring system where springs would have mass and friction would act on them, we could simply release the system after we joined its ends. The friction acting on the system would force the system to stop at equilibrium length by turning the extra potential energy into heat. It holds that $\mathbf{U} = \frac{1}{2}\mathbf{Q}$ where **Q** is the **Q** statistic for heterogeneity for the comparison B vs A. Thus, **Q** for heterogeneity can be interpreted as double the energy that is needed for all studies to get aligned to the same point; that is the effective spring or the summary effect.

The stiffness of the effective spring in (fig:2d) will be derived by synthesising the stiffnesses of the two parallel springs of (fig:2c). For springs with stiffnesses k_1 and k_2 which are connected in parallel, their displacements equal the total displacement $x_{eff} = x_1 = x_2$ and the total force is given as $f_{eff} = f_1 + f_2$ which gives $k_{eff} * x_{eff} = k_1 x_{eff} + k_2 x_{eff}$ and thus $k_{eff} = k_1 + k_2$. So the stiffness of the effective spring is given by the sum of the individual springs in parallel. For the example of fig:2d, the stiffness of the effective spring is $k^D = k_1 + k_2 = 0.0703 + 0.0274 = 0.0977$.

In the general case:

$$k^D = \sum_{i=1}^{n} k_i \tag{12}$$

By replacing $k_i = \hat{w}_i$ it holds that

$$k^D = \frac{1}{\mathrm{Var}(\hat{\mu}_{BA}^{D})} \tag{13}$$

The inverse of the stiffness of the effective spring is equivalent to the variance of the fixed effect meta-analytic summary.

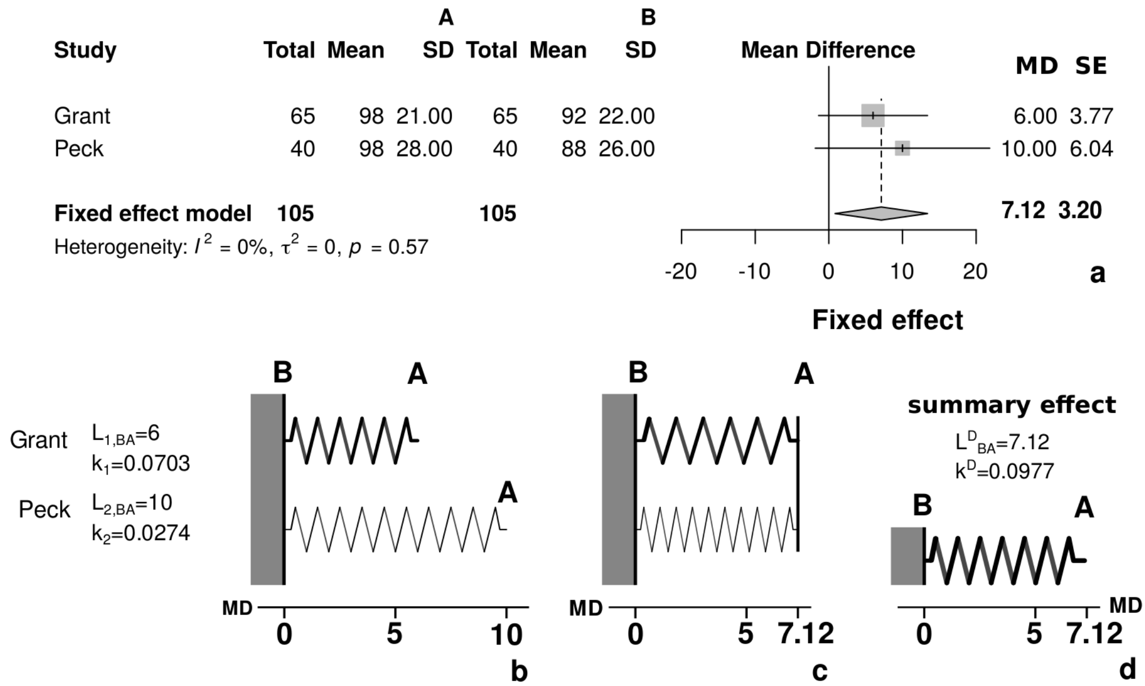

FIGURE 2 Combining studies in parallel. Panel **a**: Forest plot of two fictional studies[9] and their fixed effect summary. Panel **b**: Spring representation of panel a prior to synthesis (as open springs). Panel **c**: Synthesis process by constraining the study springs so as they have the same length which equals the fixed effect summary estimate. Panel **d**: Effective spring of panel c, which represents the fixed effect summary estimate and its variance. MD: mean difference; SD: standard deviation; SE: standard error.

We showed that the parallel spring system in equilibrium is analogous to the fixed effect pairwise meta-analysis model and the summary effect corresponds to the effective spring (fig:2c) that can replace the system of springs. We depict spring's stiffness as thickness. The amount that each spring is compressed or extended depicts how much it differs from the summary treatment effect. This provides intuition to the summary process, since stiff springs are more difficult to move meaning the effect size of a study is more important in the synthesis when the precision is high. An alternative way to visualise $y_{i,BA} = \mu_{BA} + \varepsilon_i$ would be representing μ_{BA} as a stick (or rod) and ε_i as a coil spring attached to that rod. The coil spring representing ε_i would be of zero natural length and stiffness w_i. In such a visualization, it would hold that $L_{i,BA} \equiv \mu_{BA}$. As this approach is less intuitive, we replace $L_{i,BA}$ with its effective spring, which results from synthesizing μ_{BA} and ε_i.

4 | Indirect estimate : springs in series

Network meta-analysis is an increasingly used evidence synthesis tool that combines information from studies comparing more than two treatments [10,11,12,13]. The simplest form of a network meta-analysis is the derivation of an indirect effect [14]; in the fictional example of figure 3a, suppose that we have a series of studies comparing B vs C and C vs A which result to mean differences 4 and 12 respectively. Under the transitivity assumption (table:1), we have

$$\hat{\mu}^I_{BA} = \hat{\mu}^D_{BC} + \hat{\mu}^D_{CA} \quad (14)$$

$$\text{Var}\left(\hat{\mu}^I_{BA}\right) = \text{Var}\left(\hat{\mu}^D_{BC}\right) + \text{Var}\left(\hat{\mu}^D_{CA}\right) \quad (15)$$

where the superscripts I and D refer to indirect and direct summary effects respectively.

Such successive estimates can be represented as a system of effective springs in series, as is shown in figure 3b. In order to

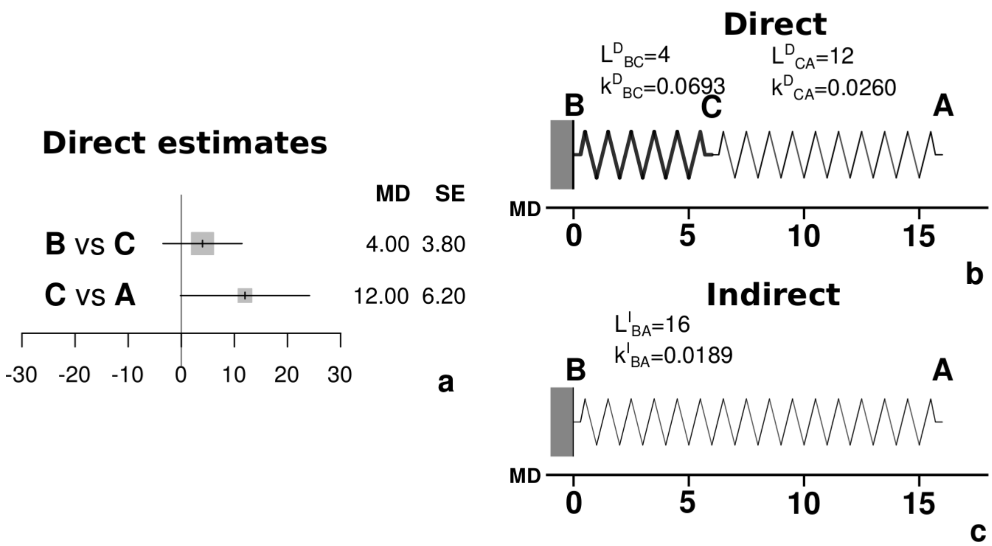

FIGURE 3 Springs in series. Panel **a**. Fictional forest plot for comparisons B vs C and C vs A Panel **b**. The spring representation of panel a. Panel **c**. The effective spring of panel b. MD: mean difference; SE: standard error.

represent these comparisons, we can simply join the springs to their common comparator (C in figure 3b). Then, consistently with equation 14, the synthesis process consists of connecting these two springs in series, which results to an *effective spring* with length

$$L^I_{BA} = L^D_{BC} + L^D_{CA} \quad (16)$$

and means that we can replace the two springs of figure 3b with lengths L^D_{BC} and L^D_{CA} with an effective spring of length L^I_{BA} The stiffness of the effective spring for springs in series is given by:

$$\frac{1}{k^I_{BA}} = \sum_{j=1}^{d} \frac{1}{k_j} \quad (17)$$

where d is the number of springs in serial connection, here 2. The effective spring of length L^I_{BA} and stiffness k^I_{BA} is shown in figure 3c. By replacing $1/k_j$ with $\text{Var}\left(\hat{\mu}^D_{BC}\right)$ and $\text{Var}\left(\hat{\mu}^D_{CA}\right)$ it is easily shown that the effective stiffness (eq:17) corresponds to the inverse variance of the indirect effect (eq:15). Equation 17 implies that k^I_{BA} is smaller than any k_j.

2.5 | Random effects pairwise meta-analysis : springs in parallel and in series

To show the analogy of random effects meta-analysis with a system of linear springs, we need both synthesis operations: connection in parallel and connection in series. The random effects pairwise meta-analysis model is written as:

$$y_{i,BA} = \mu_{BA} + \varepsilon_i + \delta_i \tag{18}$$

$$\varepsilon_i \sim N\left(0, \upsilon_i\right) \tag{19}$$

$$\delta_i \sim N\left(0, \tau^2\right) \tag{20}$$

where $y_{i,BA}$, μ_{BA}, ε_i, $\hat{\upsilon}_i$ are as in section 2.3 , δ_i are the study random effects and τ^2 denotes heterogeneity. The random effects summary estimate $\hat{\mu}^{D*}_{BA}$ and its variance $Var\left(\hat{\mu}^{D*}_{BA}\right)$ are given as in equations 7 and 8 by replacing $\hat{w}_i$ with $\hat{w}^*_i = \frac{1}{\hat{\upsilon}_i+\tau^2}$. Various methods can be used to estimate heterogeneity[15,16,17]. We have so far treated $\hat{\upsilon}_i$ as if they were known and uncorrelated with the effect size. However, the uncertainty around the estimation of υ_i, may impact both on the study-level treatment effects and on the estimation of τ^2. Moreover, the assumption of uncorrelated variances with effect sizes may be unrealistic, especially for standardized mean differences and log odds ratios.

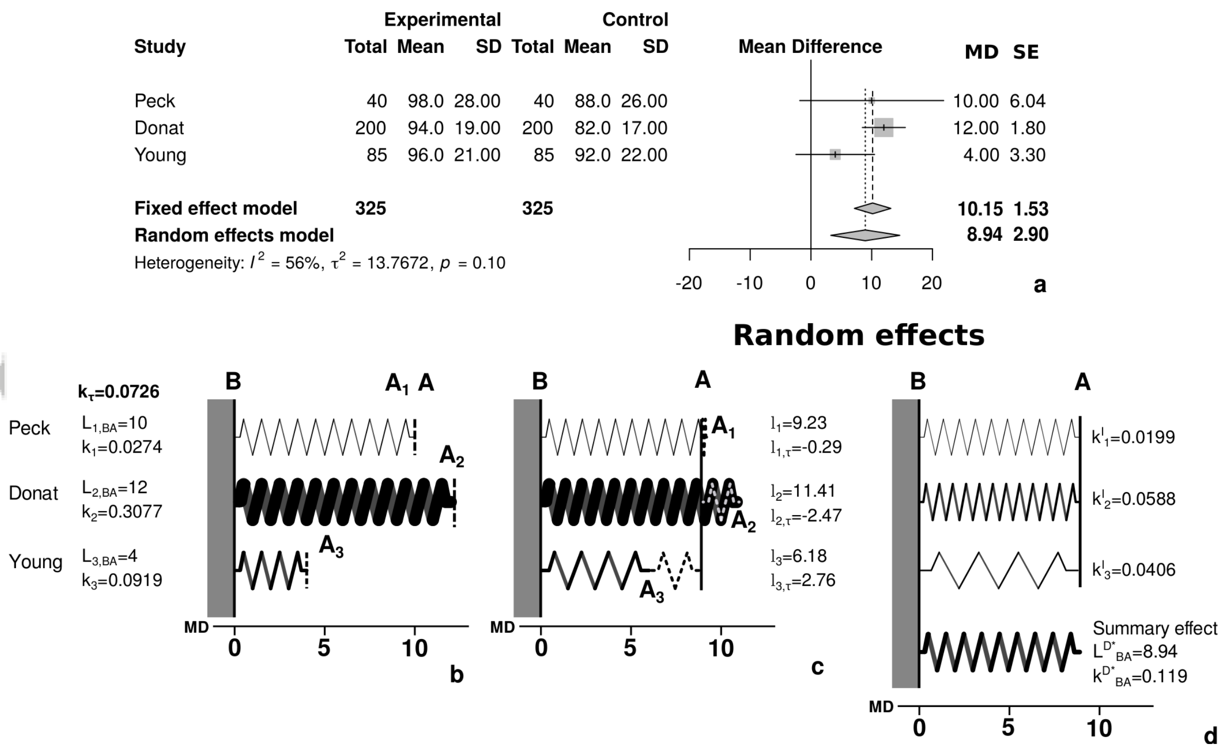

FIGURE 4 Random effects meta-analysis. Panel **a**: Forest plot of 3 studies and their fixed and random effects summaries. Panel **b**: The spring representation of panel a. Dashed lines at the A_i ends of each spring represent random effects springs of zero natural length. Panel **c**: Synthesis process by fixing the length of the system at the A ends of the heterogeneity springs. To the right of each spring individual lengths of each study l_i and heterogeneity spring $l_{i,\tau}$ are given. Panel **d**: Replacing springs of panel **c** by their effective indirect k^I_i. At the bottom is the effective spring of the system giving the summary of the random effects model. The same stiffness to thickness scale is used as to enable comparisons between panels. MD: mean difference; SD: standard deviation; SE: standard error.

Our approach to represent model 18 is adding in the system of springs of figure 2 an extra spring per study to represent δ_i. This extra spring links the free end A_i of each spring L_{i,BA_i} to the common end A. The natural length of the random effects springs is $L_\tau = 0$ and stiffness $k_\tau = 1/\tau^2$.

Let us assume that we have the three studies of figure 4a, Peck, Donat and Young. The heterogeneity for this subset of studies is considerable, estimated at $\tau^2 = 13.8$ using the DerSimonian and Laird method (although estimation with such few studies is problematic). The random effects summary effect is 8.94. The springs in figure 4b represent these three studies, along with their random effects. The random effects springs are drawn as vertical dashed lines to depict that they have zero natural length. As their natural length is zero, they cannot be compressed but they can be stressed to any length either on the right or on the left.

Figure 4c illustrates the synthesis process, which consists of binding all open ends together and is equivalent to solving the system of forces

$$\sum_{i=1}^{n} f_{i,A} = 0 \tag{21}$$

$$\sum_{j=1}^{n} f_{j_i,A_i} = 0 \quad \forall i \tag{22}$$

for points A and A_i. Figure 4c includes the representation of the random effects; their displacement from their zero natural length (the change from A_i to A) shows their contribution to heterogeneity.

Alternatively, one can see the synthesis process as a two stage approach shown in panel d of Figure 4. In the first stage, we replace the springs in series by their effective spring, with the same length and stiffness $k_i^I = \frac{1}{1/k_i + 1/k_\tau}$. This process is equivalent to deriving an indirect effect for the comparison B vs A via an intermediate node A_i. Thus, equations 14 and 15 are used for the calculation of this mock 'indirect' B vs A effect. The second stage of synthesis consists of connecting in parallel the indirect effective springs using equations 12 and 13. Depending on the realization, the natural length for study i would either be equivalent to μ_{BA}, $\mu_{BA} + \varepsilon_i$, $\mu_{BA} + \delta_i$ or $\mu_{BA} + \varepsilon_i + \delta_i$. The representation in figure 4d showcases that although stiffness drops in every spring it affects more precise studies (see Donat). Figure 4d also shows the summary effect from random effects meta-analysis as the effective spring L_{BA}^{D*}.

3 | DISCUSSION

In this paper, we showed that meta-analysis can be represented by a system of linear springs. We used connection in parallel and in series to show synthesis of studies within a pairwise comparison. The analogue of connection of springs in parallel is the inverse variance weighted average, used to estimate the summary effect in fixed effect pairwise meta-analysis. Connection in series is used for synthesizing successive estimates, for example for deriving indirect effects. Random effects pairwise meta-analysis makes use of both synthesis operations parallel and in series to synthesize successively study and random effects and in parallel study effects with random effects weights.

The representation of studies as springs provides insight into the process of meta-analysis, as well as the notions of fixed and random effects models. It is well known that precise studies take relatively more weight under fixed than under random effects meta-analysis[18]. This can be shown by the addition of the heterogeneity spring to each study. Their effective spring has less stiffness than either spring and that drop is more evident in stiff springs.

A physical analogue for network meta-analysis has previously been proposed by Rücker, where the network of studies is represented by an electrical circuit[19]. Rücker's approach allowed methods of graph theory for electrical networks to be applied in meta-analysis[19]. The implementation of the method in the `netmeta` R package has greatly popularised the use of network meta-analysis[20]. Since the appearance in the literature in 2012 of the seminal paper[19] the method has been extended in numerous directions to include methods for investigating inconsistency, ranking treatments and exploring the effects of separate components in network meta-analysis[21,22,23]. The subcase of comparing two treatments, as in pairwise meta-analysis, is also covered by the electrical network and graph theory approach.

We aim to extend the work presented in this paper to represent network meta-analysis, having in mind the equivalence between mechanical networks and electrical networks. In particular, any given mechanical network has a corresponding electrical network with the same mathematical representation. Among their elements, frictional resistance corresponds to electrical resistance, mass to inductance, and compliance to capacitance. Despite the similarities, Rücker's approach uses resistors to represent variance whereas we use stiffness to represent the reciprocal of variance. Thus, parallel connection of resistors uses the reciprocal scale, whereas the stiffness of a parallel connection of springs is the sum of the stiffnesses of the individual springs.

While the tools presented already suffice to produce and depict network estimates, a special manipulation is needed for multi-arm studies. A possible way to overcome this issue is to replace multi-arm studies with a required number of springs corresponding to two-arm studies, for example using the equivalence shown in [19,21]. Extension of the correspondence of quantities and notions is also needed to include measures which apply only to network meta-analysis, such as inconsistency [24]. While electrical networks have been shown to represent most features of network meta-analyses, such analogies are yet to be investigated for mechanical networks. For example, inconsistency may be viewed as the energy spent to align the parallel system of effective springs corresponding to different sources of evidence. We are currently developing open-source software to show pairwise and network meta-analysis to enrich current graphical representations of evidence synthesis results.

Bowden and Jackson proposed another physical analogue for meta-analysis as a seesaw, where the effect estimate corresponds to the centre of the mass [25]. An empirical study comparing the accuracy and effectiveness between physical analogues for meta-analysis as well as conventional graphical methods, such as the forest plot, would be of interest. We anticipate that the various visualisation methods would have different, yet possibly complementary, strengths. However, the accuracy may not be the only criterion for judging the appropriateness of a graph; as Cleveland and McGill point out "The power of a graph is its ability to enable one to take in the quantitative information, organise it, and see patterns and structure not readily revealed by other means of studying the data" [26]. Forest plots certainly meet these requirements, and are thus used in the vast majority of meta-analyses applications. Our springs representation suggestion only aims to complement the already existing approaches.

A limitation of figures 2 and 4 lies on the fact that the information contained in confidence intervals is not shown. This is not necessarily the case; confidence intervals can be added as an extra line centred around the study treatment effect or the summary effect. However, the representation would then contain a lot of information, so the extra line is not an integral part of the figure. This limitation further reinforces the fact that we do not advocate spring representation of meta-analyses as a substitute for a forest plot, but rather as a didactic tool. In fact, a wooden box with springs could be used as a useful educational device in meta-analysis courses to help students understand important concepts, such as the synthesis process, the notion of heterogeneity and the differences between fixed and random effects meta-analysis. The analogy with springs allows these concepts to become intuitive and tangible without loss of rigor. A further limitation of the approach is its inability to be applied to meta-analyses that use likelihood-based or Bayesian methods. We aim to extend the current work to a more general representation where study effects can be represented as nonlinear springs. In summary, we provide a translation of meta-analysis to a corresponding mechanical system of springs, which allows the intuitive visual representation of basic concepts of evidence synthesis.

4 | HIGHLIGHTS

What is already known: Meta-analysis is very commonly applied in several fields including health sciences, education and ecology. However, some concepts, such heterogeneity, are not always easy to grasp.

What is new: We show the analogy between meta-analysis and a system of linear springs and propose a new educational graphical tool to enhance intuition of meta-analytical concepts.

Potential impact for Review Synthesis Methods readers outside the authors' field: The proposed educational device can help researchers understand the synthesis process, the differences between fixed and random effects and heterogeneity in meta-analysis.

5 | DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

References

1. Chalmers I, Hedges LV, Cooper H. A Brief History of Research Synthesis. *Evaluation & the Health Professions* 2002; 25(1): 12-37. PMID: 11868442doi: 10.1177/016327870202500100³

2. McGill R, Tukey JW, Larsen WA. Variations of Box Plots. *The American Statistician* 1978; 32(1): 12–16.

3. Lewis S, Clarke M. Forest plots: trying to see the wood and the trees. *BMJ* 2001; 322(7300): 1479.

4. Anzures-Cabrera J, Higgins JPT. Graphical displays for meta-analysis: An overview with suggestions for practice. *Research Synthesis Methods* 2010; 1(1): 66-80. doi: 10.1002/jrsm.6

5. Galbraith RF. A note on graphical presentation of estimated odds ratios from several clinical trials. *Statist. Med.* 1988; 7(8): 889–894. doi: 10.1002/sim.4780070807

6. Galbraith RF. Some Applications of Radial Plots. *Journal of the American Statistical Association* 1994; 89(428): 1232–1242.

7. L' Abbe K, Detsky A, O'Rourke K. Meta-Analysis in Clinical Research. *Annals of Internal Medicine* 1987; 107(2): 224–233. doi: 10.7326/0003-4819-107-2-224

8. Kossmeier M, Tran US, Voracek M. Charting the landscape of graphical displays for meta-analysis and systematic reviews: a comprehensive review, taxonomy, and feature analysis. *BMC Medical Research Methodology* 2020; 20(1): 26.

9. Borenstein M, Hedges LV, Higgins JPT, Rothstein HR. *Introduction to Meta-Analysis*. Chichester: John Wiley & Sons . 2009

10. Petropoulou M, Nikolakopoulou A, Veroniki AA, et al. Bibliographic study showed improving statistical methodology of network meta-analyses published between 1999 and 2015. *Journal of Clinical Epidemiology* 2017; 82: 20–28. doi: 10.1016/j.jclinepi.2016.11.002

11. Salanti G. Indirect and mixed-treatment comparison, network, or multiple-treatments meta-analysis: many names, many benefits, many concerns for the next generation evidence synthesis tool. *Res Syn Meth* 2012; 3: 80-97. doi: 10.1002/jrsm.1037

12. Jansen JP, Fleurence R, Devine B, et al. Interpreting indirect treatment comparisons and network meta-analysis for health-care decision making: report of the ISPOR Task Force on Indirect Treatment Comparisons Good Research Practices: part 1. *Value in health : the journal of the International Society for Pharmacoeconomics and Outcomes Research* 2011; 14: 417–428.

13. Hoaglin DC, Hawkins N, Jansen JP, et al. Conducting indirect-treatment-comparison and network-meta-analysis studies: report of the ISPOR task force on indirect treatment comparisons good research practices: part 2. *Value Health* 2011; 14: 429-37. doi: 10.1016/j.jval.2011.01.011

14. Higgins JPT, Whitehead A. Borrowing strength from external trials in a meta-analysis. *Statistics in Medicine* 1996; 15(24): 2733–2749. doi: 10.1002/(SICI)1097-0258(19961230)15:24<2733::AID-SIM562>3.0.CO;2-0

15. DerSimonian R, Laird N. Meta-analysis in clinical trials. *Controlled Clinical Trials* 1986; 7(3): 177–188. doi: 10.1016/0197-2456(86)90046-2

16. Paule RC, Mandel J. Consensus Values, Regressions, and Weighting Factors. *Journal of research of the National Institute of Standards and Technology* 1989; 94: 197–203.

17. Harville DA. Maximum Likelihood Approaches to Variance Component Estimation and to Related Problems. *Journal of the American Statistical Association* 1977; 72(358): 320-338. doi: 10.1080/01621459.1977.10480998

18. Borenstein M, Hedges LV, Higgins JPT, Rothstein HR. A basic introduction to fixed-effect and random-effects models for meta-analysis. *Research synthesis methods* 2010; 1: 97–111.

19. Rücker G. Network meta-analysis, electrical networks and graph theory. *Res Syn Meth* 2012; 3: 312-24. doi: 10.1002/jrsm.1058

20. Rücker G, Krahn U, König J, Efthimiou O, Schwarzer G. *netmeta: Network Meta-Analysis using Frequentist Methods*. ; : 2019. https://github.com/guido-s/netmeta http://meta-analysis-with-r.org.

21. Rücker G, Schwarzer G. Reduce dimension or reduce weights? Comparing two approaches to multi-arm studies in network meta-analysis. *Statistics in Medicine* 2014; 33(25): 4353–4369. doi: 10.1002/sim.6236

22. Rücker G, Schwarzer G. Ranking treatments in frequentist network meta-analysis works without resampling methods. *BMC Medical Research Methodology* 2015; 15(1): 58.

23. Rücker G, Petropoulou M, Schwarzer G. Network meta-analysis of multicomponent interventions. *Biometrical Journal* 2019; 62(3): 808-821. doi: 10.1002/bimj.201800167

24. Dias S, Welton NJ, Sutton AJ, Caldwell DM, Lu G, Ades AE. Evidence synthesis for decision making 4: inconsistency in networks of evidence based on randomized controlled trials. *Medical decision making : an international journal of the Society for Medical Decision Making* 2013; 33: 641–656.

25. Bowden J, Jackson C. Weighing Evidence “Steampunk” Style via the Meta-Analyser. *The American Statistician* 2016; 70(4): 385–394. doi: 10.1080/00031305.2016.1165735

26. Cleveland WS, McGill R. Graphical Perception: Theory, Experimentation, and Application to the Development of Graphical Methods. *Journal of the American Statistical Association* 1984; 79(387): 531–554.

How to cite this article: Papakonstantinou T., Nikolakopoulou A., Egger M., and Salanti G. (2020), Meta-analyis as a system of springs, *Res Syn Meth*