One Year of STACK at the University of Bern

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Abstract: In 2020 we contributed with 50 STACK problems of undergraduate linear algebra to the Abacus Pool. Our goal was twofold: on the one hand we wanted to decrease the workload of the assistants by substituting part of the classical homework by STACK exercises, and on the other hand we wanted to provide a guided support for multi-step algorithmic problems. Inspired by our success from 2020, we wanted to make use of our expertise for addressing the following issue: The rate of first year bachelor students finishing their studies at the same university and in the same study domain as they started is the lowest in case of exact and natural sciences. This is what we experience also at the University of Bern. We would like to increase this number by developing an online self-assessment in mathematics for potential future students.

Keywords: mathematics education, STACK, online self-assessment, automated and customised feedback.

1 Introduction

The start of our project involving the development of STACK exercises at the beginning of 2020 was conveniently aligned with the digitalisation trend in higher education, in general and at the University of Bern, in particular. We learned about it by joining the Abacus E-Assessment Material Bank [A21], where STACK is used to develop computer aided assessments mainly in mathematics and STEM.

In this paper we first present the 50 formative exercises implemented in STACK, that represent our contribution to the Abacus Pool. We proceed to our current work, the outcome of which will be an online self-assessment. This consists of STACK questions integrated into an ILIAS test. Finally, we are going to draw some conclusions and sketch future prospects for both projects.

2 Getting started with STACK

At the beginning of 2020 we became members of the Abacus Material Pool [Ra16]. Abacus is an initiative of 7 Finnish universities, which supports the creation of university level educational materials. Aalto University manages its platform, where one can share the produced content between other members. STACK is the major tool used for creating

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the teaching materials for the Abacus Pool.

STACK was originally conceived for the Moodle learning platform [S21]. As the user tutorials focus on using STACK on Moodle, we first tried out the creation of STACK questions on Moodle. Around April 2020, we switched to ILIAS, which is the learning management system used by the University of Bern.

The 50 STACK problems, that we created for the Abacus Pool can be consulted by the members of the pool on the links: Linear Algebra 1, Linear Algebra 2.

2.1 Context and goal

Our context for creating the 50 STACK problems was a two-semester long Linear Algebra course. The audience was composed of first year students in Mathematics, Computer Science and Physics.

Our goal was twofold:

- on the one hand we wanted to provide a guided support for multi-step algorithmic problems,
- and on the other hand we wanted to decrease the workload of the assistants by substituting part of the classical homework by STACK exercises.

We have started by considering the classical homework and we tried to convert as many exercises as possible into STACK questions.

2.2 Covered topics

As an outcome of our efforts, in the first semester students can choose between 33 STACK problems and practice basic matrix computations, calculate the determinant, the inverse or the rank of a matrix. They can better understand new notions and concepts, such as linear independence, system of generators and basis of a vector space. Furthermore, we provide them with guided algorithms for solving systems of linear equations and for carrying out the Gram-Schmidt orthogonalization. The semester ends with the study of linear transformations. We have created an exercise for the dimension formula or another one in which, given a linear transformation w.r.t. to the canonical bases, we ask the students to write it w.r.t. other bases. We have also designed a GeoGebra app, which shows how a chosen vector behaves under the action of a linear transformation. Based on this, students have to determine the linear transformation. This is related to the notions of eigenvalues and eigenvectors and it constitutes the transition to the 2^{nd} semester.

For the second semester we have created in total 17 different STACK questions. After being able to calculate the eigenvalues and eigenvectors, as an application, students can practice finding the Jordan form of a matrix and diagonalizing symmetric matrices. This is also related to the reduction of quadratic forms and conics to the canonical expression.

2.3 Key aspects of the produced STACK questions

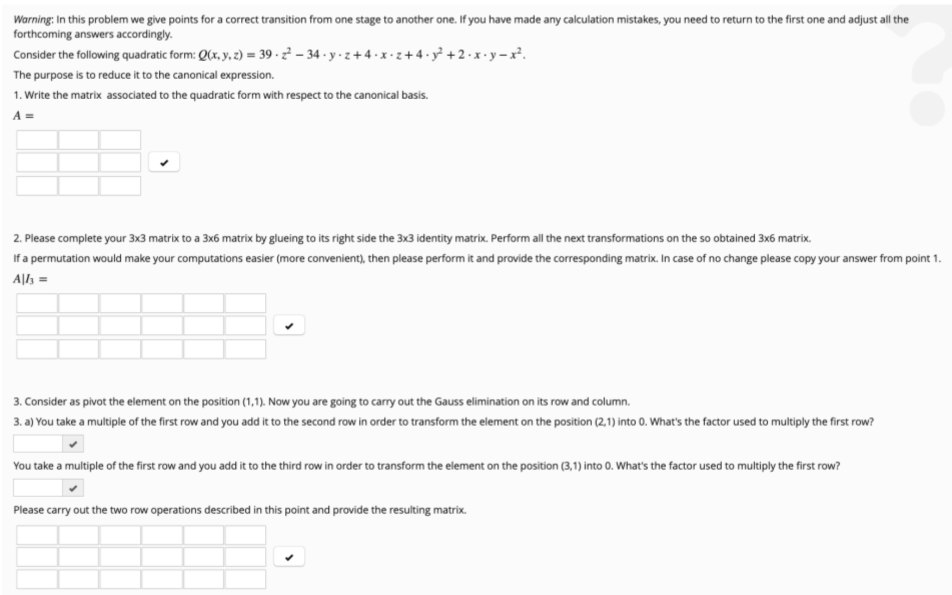
First of all, we have created guided problems to instruct students how to perform longer algorithmic calculations. To support this process even more, our STACK questions are formulated in a detailed, step-by-step manner.

We really liked to explore the possibility that each student can answer the same question, but with different numbers. This is achieved by randomising the parameters of the problem. For randomisation a bottom-up design / a reverse engineering is frequently used. This assures that the partial and final results contain only nice numbers.

2.4 Typical STACK question example

We have selected a representative exercise (see Fig. 1, Fig. 2) to demonstrate the key aspects of the framework that we have been working with.

A classical linear algebra problem is the reduction of a quadratic form to the canonical expression. This is important for the classification of quadratic forms. A random quadratic form is given. Following a guided, step-by-step procedure, the students' objective is to derive its canonical form and signature.


Warning: In this problem we give points for a correct transition from one stage to another one. If you have made any calculation mistakes, you need to return to the first one and adjust all the forthcoming answers accordingly.

Consider the following quadratic form: $Q(x, y, z) = 39 \cdot z^2 - 34 \cdot y \cdot z + 4 \cdot x \cdot z + 4 \cdot y^2 + 2 \cdot x \cdot y - x^2$.

The purpose is to reduce it to the canonical expression.

1. Write the matrix associated to the quadratic form with respect to the canonical basis.

$A =$

2. Please complete your 3x3 matrix to a 3x6 matrix by glueing to its right side the 3x3 identity matrix. Perform all the next transformations on the so obtained 3x6 matrix.

If a permutation would make your computations easier (more convenient), then please perform it and provide the corresponding matrix. In case of no change please copy your answer from point 1.

$A|I_3 =$

3. Consider as pivot the element on the position (1,1). Now you are going to carry out the Gauss elimination on its row and column.

3. a) You take a multiple of the first row and you add it to the second row in order to transform the element on the position (2,1) into 0. What's the factor used to multiply the first row?

You take a multiple of the first row and you add it to the third row in order to transform the element on the position (3,1) into 0. What's the factor used to multiply the first row?

Please carry out the two row operations described in this point and provide the resulting matrix.

Fig. 1: The reduction of a quadratic form to the canonical expression – first part

The main drive for creating such problems is that they are difficult to correct and generate such that each student works on a different exercise, but with convenient numbers.

Another goal was to replicate the steps that a student would make when solving the problem by hand.

An inconvenience was that, depending on the initial quadratic form, these steps may vary. Due to the lack of dynamic behaviour of the text of the problem, we came up with the solution to divide it into several cases. Each case is implemented in a separate STACK question. This way we can construct a test which covers all possible situations, even the most particular ones.

To check all the distinct cases and to try them out, the exercises can be downloaded from the Abacus Pool.

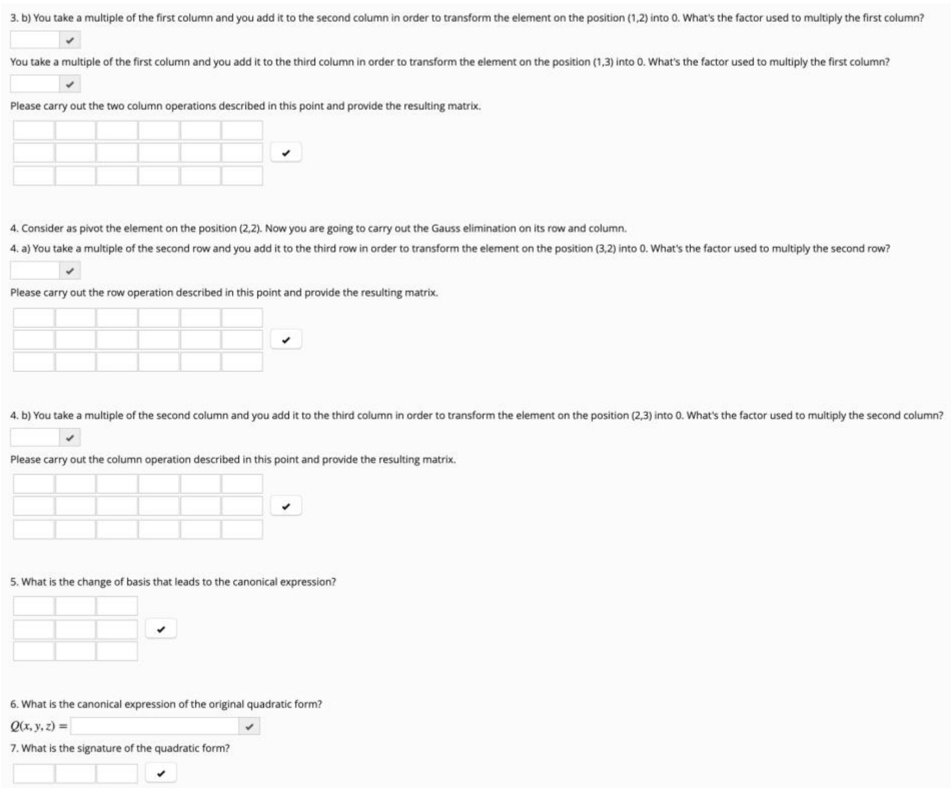


Fig. 2: The reduction of a quadratic form to the canonical expression – second part

At the end of 2020, based on our own experience and on that of our colleagues from other universities, we decided to use STACK and create an online self-assessment at the University of Bern. This is because STACK proved to be a promising and reliable tool. The current project started at the beginning of 2021 and it is described in the next section.

3 Online self-assessment with STACK

Based on Swiss statistics, early drop-outs from university are one of the highest for exact and natural sciences [Ü21]. One of the possible explanations is that high-school-level and university-level mathematics can be quite different. If we can make high school students aware of this fact, their study choice will be more informed. To increase awareness on this matter we have chosen to create an online self-assessment. Consequently our target group consists of potential future students, namely those who are close to graduating.

3.1 Sources of inspiration

The first one is the MINTFIT Hamburg [BS18]. Its outcome test is composed of Basic Skills 1 and Basic Skills 2. For solving each of them 45 minutes are recommended. In total there are 36 questions and many of them are written using STACK. After completing these, students receive a performance review and they are guided towards e-learning materials, needed to close their gaps. The test can be taken by accessing the link in [M21].

The second source of inspiration is the online self-assessment of the University of Bonn [B21]. This influenced us more from the point of view of the content. We have designed for our online self-assessment mathematical problems of a similar structure. Also, as we have seen there, we have included a survey.

A part of our test consists of multiple choice questions. These could be also implemented with other tools than STACK. However, for some specific problems (as it can be seen in the MINTFIT Hamburg) STACK is more suitable. To this second category belong open-ended questions, which need a more sophisticated evaluation, and questions with random parameters. Hence for a unified aspect, we decided to implement everything in STACK.

3.2 Structure of the online self-assessment

Fig. 3: Progress bar of the online self-assessment

The progress bar (see Fig. 3) gives a first visual idea about the structure of the online self-assessment. Now we will describe it in more detail below:

- When opening the link of the test, general information is welcoming participants. We explain that the goal is to support the choice of studies by giving an insight into the mathematical way of thinking and reasoning. We describe here also the structure of the test and we specify the approximate time span needed to solve it. In case of interruption, students can suspend the test and resume it at any later time.
- The test starts with demographic questions, which have a statistical value, providing the profile of future students (e.g. age, where did they graduate, core subjects).

- The next part is about the students' expectations and attitude towards mathematics. Here we ask how much do the students expect calculations, proofs, if-then statements and algorithms to be part of their study. Submitting the answers triggers the appearance of our opinion about each of these aspects.
- The first mathematical part consists of three guided problems. By these we do not want to test the capacity to memorise formulas, but we would like to check whether the participants can follow a complex process of analytical reasoning. In one of these problems we guide students through the process of proving by induction the Cauchy-Bunyakowsky-Schwarz inequality.
- After each guided problem we ask how difficult and interesting the problem was and whether students would like to deal with similar problems in the future. In addition, in a text field they can give us a more detailed feedback.
- A section about the students' interests and motivation follows. Here we ask about their prior interactions and own experience with mathematics.
- After this we get back to testing basic skills. This part of the test is compound of short, independent questions, for which one needs to recall some basic factual knowledge as well. Here we plan to have about 20, predominantly open-ended questions, which rely on the CAS interpretation of the answers available in STACK.
- The last part of the test is reserved for a feedback about the online self-assessment itself. We are interested to find out what message got through to the students, how do they evaluate the test, what did they like or dislike about it and whether the test has influenced their choice of studying mathematics.

3.3 Feedback types

The different feedback types, that we provide for the students during the test, can also be distinguished by the visual solutions used for their implementation.

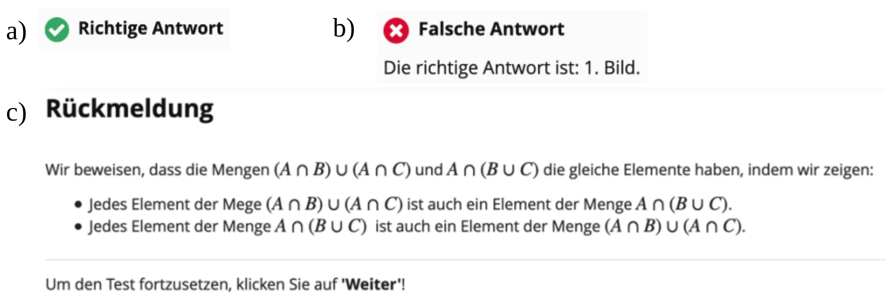


Fig. 4: Feedback types in mathematical questions. a) & b) Feedback received in case of correct and incorrect answer, respectively. c) Detailed solution of the problem.

For mathematical questions, the evaluation of the answer is included in the body of the problem. Its appearance is triggered by submitting the answer. To make it more noticeable, we use some colourful widgets (see Fig. 4 a) & b)).

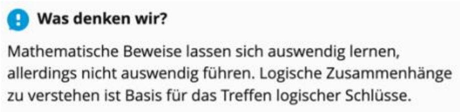


Fig. 5: Feedback given to a survey question about the students' expectations and interests

In the survey part we just give feedback to the students' expectations and interests. Here we do not evaluate the answers, we just provide the students a formative feedback by sharing our opinion. They should be able to draw some conclusions by comparing their point of view with ours.

Finally, we present the planned evolution of our projects in the future and we summarize our interactions with STACK.

4 Future development

4.1 Linear algebra exercises for the Abacus Material Bank

Up to now we implemented the linear algebra problems and we tested them on a small number of students. These exercises were not mandatory. Solving them was just optional and one could earn bonus points this way. Starting with the next academic year, we plan to make them compulsory for all first year students. Furthermore, solving them will be a requirement for entering the exam.

4.2 Online self-assessment

In 2021 the target group consists of potential future students, for whom the completion of the online self-assessment is just recommended. From 2022 on we plan to make the online self-assessment mandatory for all registered to study mathematics. For this set-up our goal is slightly different, namely we would like that students ask themselves the following question: “Before the university year starts, in order for my studies to go smoothly, do I need to invest more time into studying mathematics?” If the answer to this question is “yes”, students can attend a bridge course offered by our university. A more distant possible extension of this project could be the creation of online learning resources. We would like to link them with the test in such a manner that students are guided to the very materials that could be helpful to improve their skills and knowledge.

5 Conclusions

STACK proved to be an effective, reliable and powerful tool for implementing mathematical questions. One of the key advantages is that it scales well to large groups of students (partially due to the fact that the underlying parameters can be randomized). Other positive aspects are the possibility to evaluate complex mathematical properties of the answer and provide customized feedback based on the students' response. Moreover, there is a large user community that continuously develops and shares new materials. Even if, during our experience with STACK, we were confronted with some limitations specific to the plugin for ILIAS, the STACK community was very helpful and supportive.

In the next phase of both projects we plan to collect statistical data and analyze the efficiency of our products. Unfortunately, we have encountered some issues concerning the automatic collection of students' answers given for STACK questions. This refers to the fact that in ILIAS only the points are included in the automatically generated statistics and for the analysis we need precisely the students' responses.

We have observed that linear algebra problems, mostly because of the algorithmic specificity of the subject, are easily "stackifiable". Inspired by the success of the members of the STACK community, we also plan to develop other problems, e.g. with a deeper theoretical content or problems from different branches of mathematics.

6 Acknowledgements

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