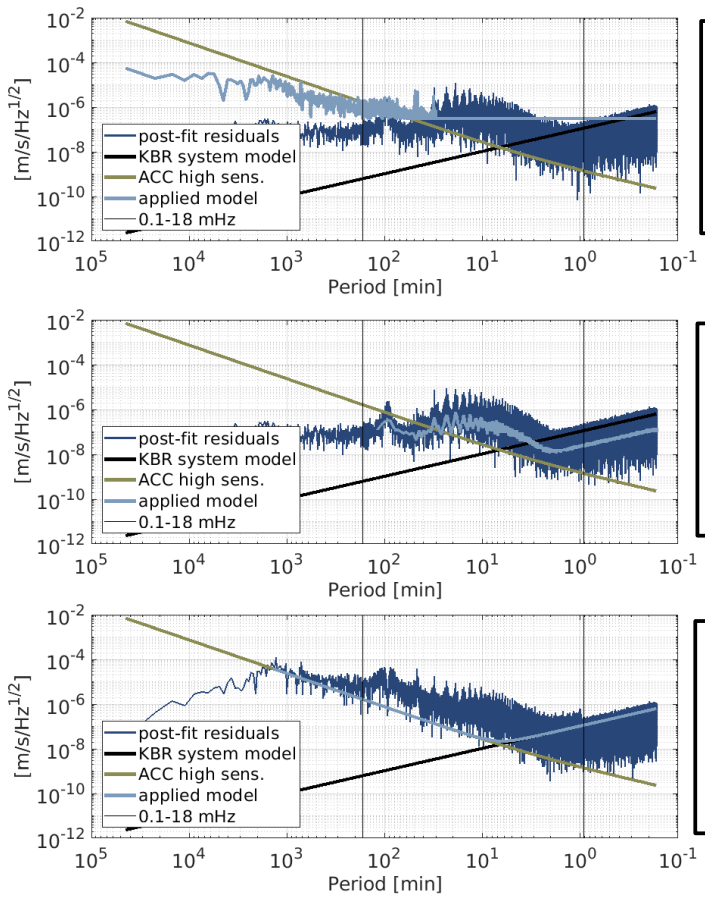


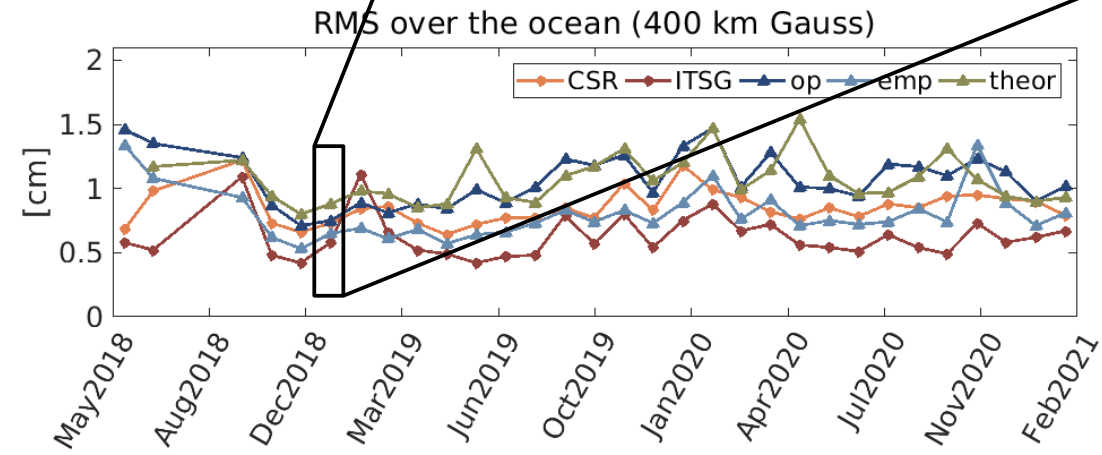
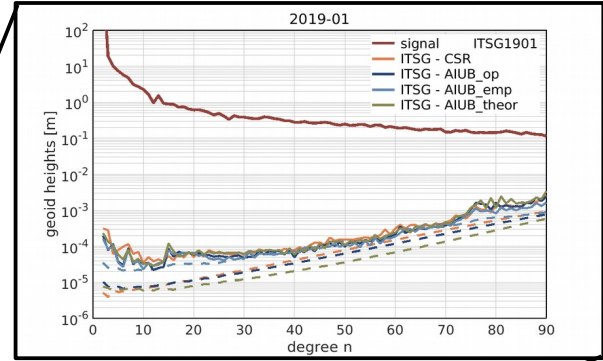
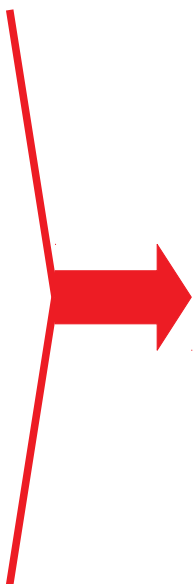
# Comparison of empirical noise models for GRACE Follow-On derived with the Celestial Mechanics Approach



15 min piecewise-constant accelerations  
 «op»

empirical noise model based on post-fit residuals  
 «emp»

theoretical noise models for KBRR and ACC [Kim, 2000]  
 «theor»



**CONCLUSIONS**

- Best performance for «emp»
- No severe degradation for «theor»
- Smallest residuals in «op»



# **Comparison of empirical noise models for GRACE Follow-On derived with the Celestial Mechanics Approach**

Martin Lasser, Ulrich Meyer, Daniel Arnold, Adrian Jäggi

*Astronomical Institute, University of Bern, Switzerland*

EGU General Assembly 2021

Apr. 19-30, 2021

Online



# Introduction

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Correctly modelling the measurement noise is crucial for recovering high quality gravity fields:

- AIUB\_op («op»): piecewise-constant accelerations (PCA)
- AIUB\_emp («emp»): empirical noise model based on post-fit residuals
- AIUB\_theor («theor»): theoretical noise models for GRACE [Kim, 2000]

# Introduction

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Correctly modelling the measurement noise is crucial for recovering high quality gravity fields:

- AIUB\_op («op»): piecewise-constant accelerations (PCA)
- AIUB\_emp («emp»): empirical noise model based on post-fit residuals
- AIUB\_theor («theor»): theoretical noise models for GRACE [Kim, 2000]

## **Celestial Mechnaics Approach (CMA) with**

Gravity field	Internal AIUB static GRACE field
Astromomic bodies	JPL DE421 (all planets + Pluto)
Mean pole	Linear
Solid Earth tides, Solid Earth pole tides, Relativistic effects	IERS2010
Ocean tides	FES2014b (+ admittances from ITSG)
Ocean pole tides	Desai
Atmospheric tides, Atmospheric & oeanic dealiasing	AOD RL06
Non-conservative forces	Accelerometer L1b (from ITSG)

# Noise models – piecewise-constant accelerations

«op»

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

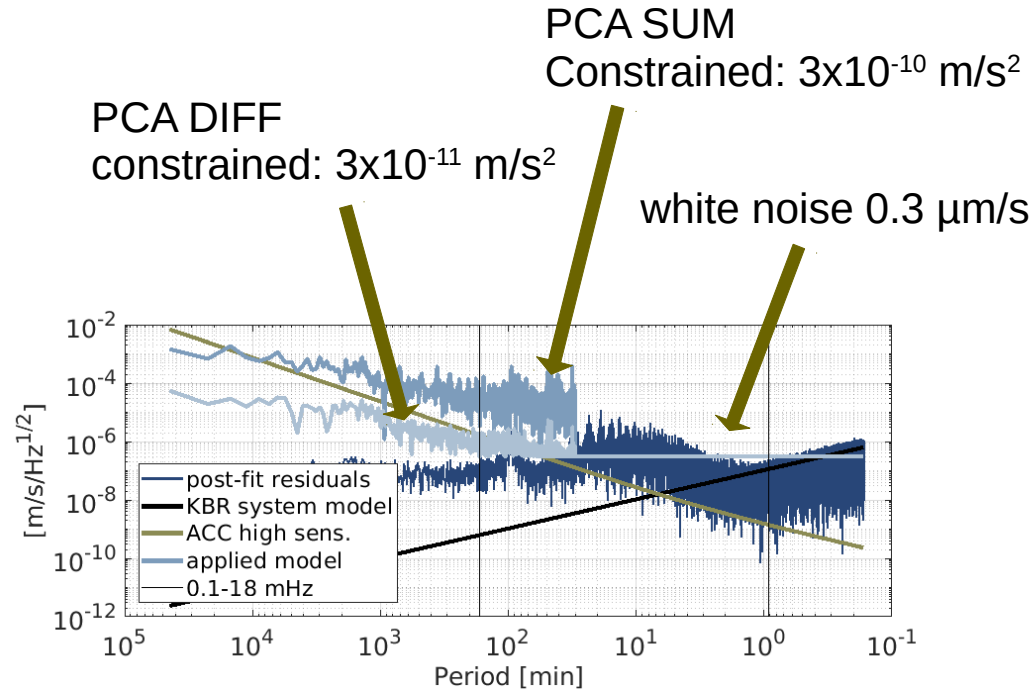
noise model:

- KBRR white noise 0.3  $\mu\text{m/s}$
- 15 min PCA per satellite in
  - radial 2x(96)
  - along track 2x(96)
  - Cross-track 2x(96)

parameters per arc 576

in daily arcs (30 days):

- 18000 parameter,
- 17280 for the noise model
- + gravity field



# Noise models – piecewise-constant accelerations

«op»

basic parametrisation:  
• initial conditions 2x(6)

PCA SUM

• orbit transformation (described in Beutler et al. 2010):  
instead of estimating parameters for GF1 & GF2  
transform orbit parameters to:

$\frac{GF1+GF2}{2}$  → SUM: referring to the mean point in space between GF1 & GF2 (driven by GPS)

$\frac{GF1-GF2}{2}$  → DIFF: referring to the difference between GF1 & GF2 (driven by K-band)

in daily arcs (30 days):  
• 18000 parameter,  
• 17280 for the noise model  
• + gravity field

M. Lasser, U. Meyer, D. Arnold, A. Jäggi: Comparison of empirical noise models for GRACE Follow-On derived with the Celestial Mechanics Approach, EGU General Assembly 2021, 30 April, 2021

# PCA - performance

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

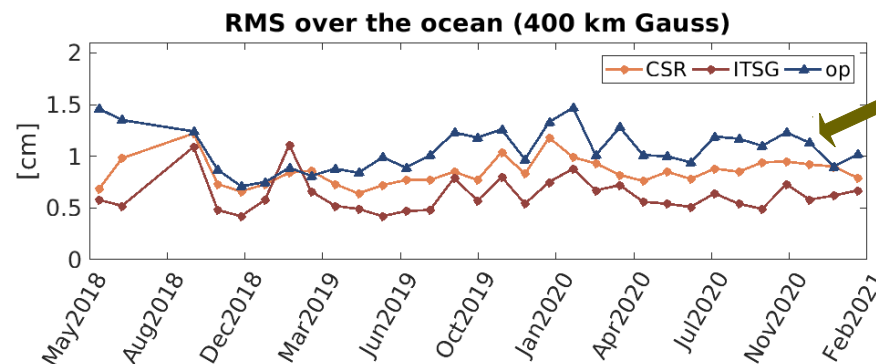
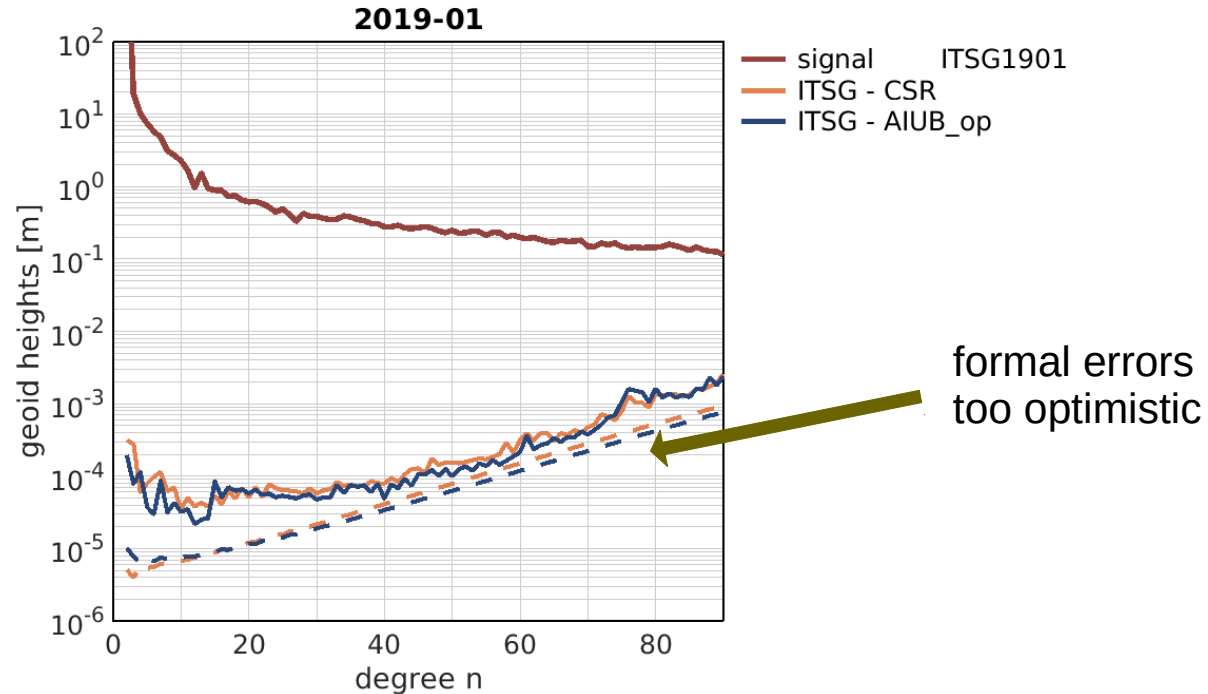
noise model:

- KBRR white noise 0.3  $\mu\text{m/s}$
- 15 min PCA per satellite in
  - radial 2x(96)
  - along track 2x(96)
  - Cross-track 2x(96)

parameters per arc 576

in daily arcs (30 days):

- 18000 parameter,
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«op»

M. Lasser, U. Meyer, D. Arnold, A. Jäggi: Comparison of empirical noise models for GRACE Follow-On derived with the Celestial Mechanics Approach, EGU General Assembly 2021, 30 April, 2021



# PCA – conclusions

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model:

- KBRR white noise 0.3  $\mu\text{m/s}$
- 15 min PCA per satellite in
  - radial 2x(96)
  - along track 2x(96)
  - Cross-track 2x(96)

parameters per arc 576

in daily arcs (30 days):

- 18000 parameter,
- 17280 for the noise model
- + gravity field

- «classical» Celestial Mechanics Approach
- no iterations required
- published at ICGEM

«op»

The screenshot shows the ICGEM website interface. The main heading is 'Gravity Field Solutions for dedicated Time Periods'. Below this, it states 'The following gravity field time series are presently available:'. There are two main sections: 'GRACE and Grace-FO solutions from the Science Data System centers CSR, GFZ and JPL' and 'GRACE / CHAMP solutions from other groups'. The 'GRACE / CHAMP solutions from other groups' section is circled in red and includes entries for AIUB (Astronomical Institute University Bern) and CNES (Centre national d'études spatiales).

- extension of parameter space demands higher memory
- and CPU capacity (inversion)
- constraints need to be determined (manually, VCE)



# Noise models - from post-fit residuals

«emp»

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model:

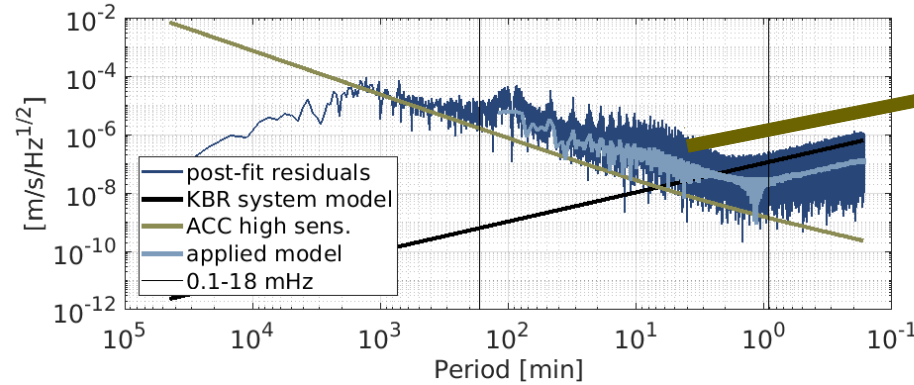
- empirical covariances based on post-fit residuals

no additional parameters

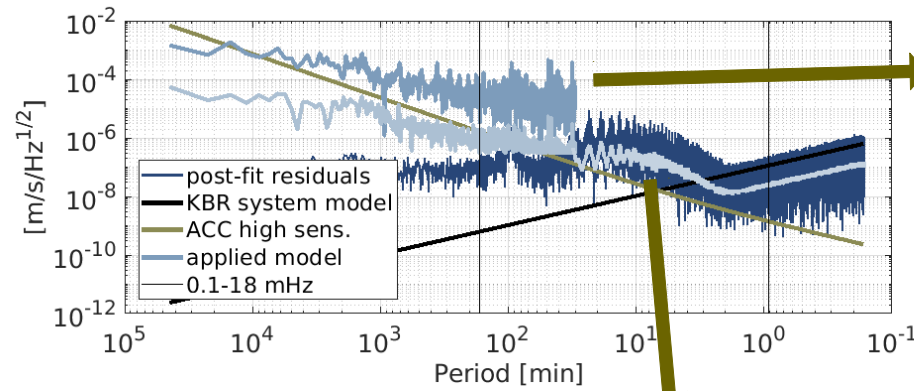
- requires iterative procedure
- assumption of stationarity

in daily arcs (30 days):

- 720 parameters
- + gravity field



empirical model applied for 3 hours



empirical model applied for 3 hours

noise model:

- KBRR white noise 0.3  $\mu\text{m/s}$
- 15 min PCA per satellite in
  - radial 2x(96)
  - along track 2x(96)
  - Cross-track 2x(96)

parameters per arc 576

M. Lasser, U. Meyer, D. Arnold, A. Jäggi: Comparison of empirical noise models for GRACE Follow-On derived with the Celestial Mechanics Approach, EGU General Assembly 2021, 30 April, 2021

# Empirical covariances

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model:

- empirical covariances based on post-fit residuals

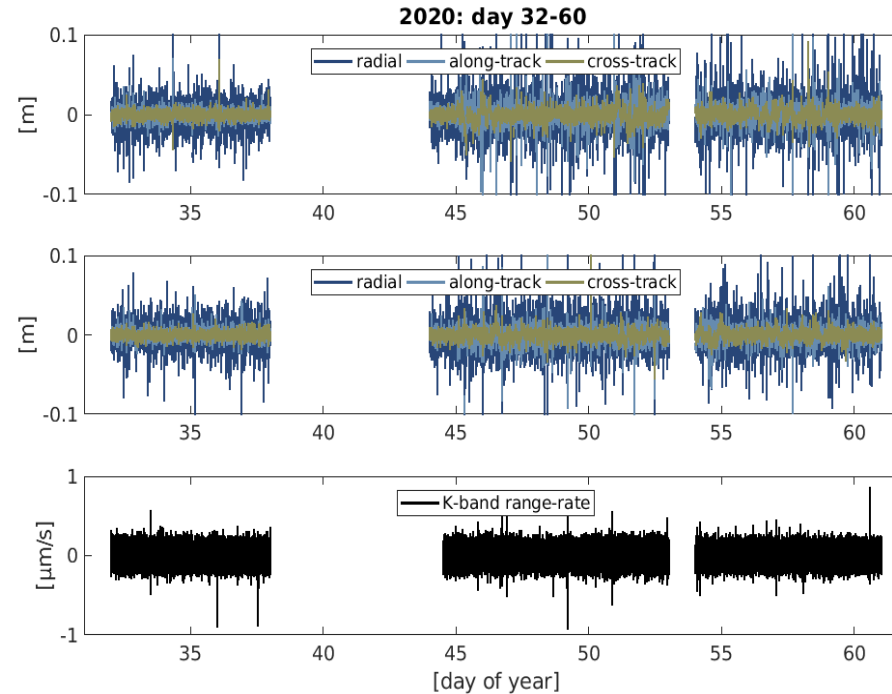
no additional parameters

- requires iterative procedure
- assumption of stationarity

in daily arcs (30 days):

- 720 parameters
- + gravity field

«emp»



auto/cross-correlation



$P_{\text{KIN,GF1}}$



$P_{\text{KIN,GF2}}$



$P_{\text{Kb}}$

# Empirical covariances

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model:

- empirical covariances based on post-fit residuals

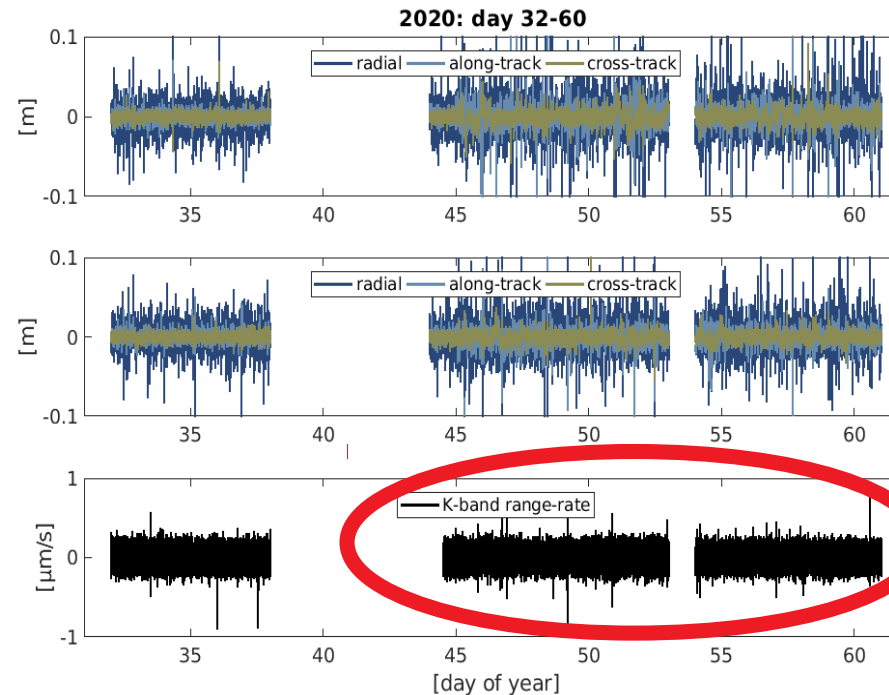
no additional parameters

- requires iterative procedure
- assumption of stationarity

in daily arcs (30 days):

- 720 parameters
- + gravity field

«emp»



auto/cross-correlation



$P_{KIN,GF1}$



$P_{KIN,GF2}$



$P_{Kb}$

stationary?

# Empirical covariance function

«emp»

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model:

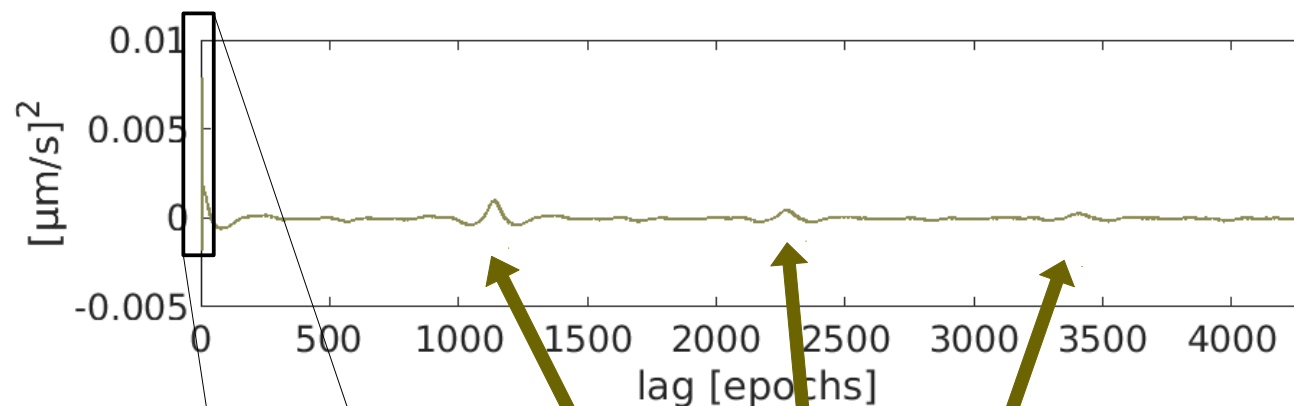
- empirical covariances based on post-fit residuals

no additional parameters

- requires iterative procedure
- assumption of stationarity

in daily arcs (30 days):

- 720 parameters
- + gravity field



multiple per revolution

# Empirical covariances - performance

«emp»

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model:

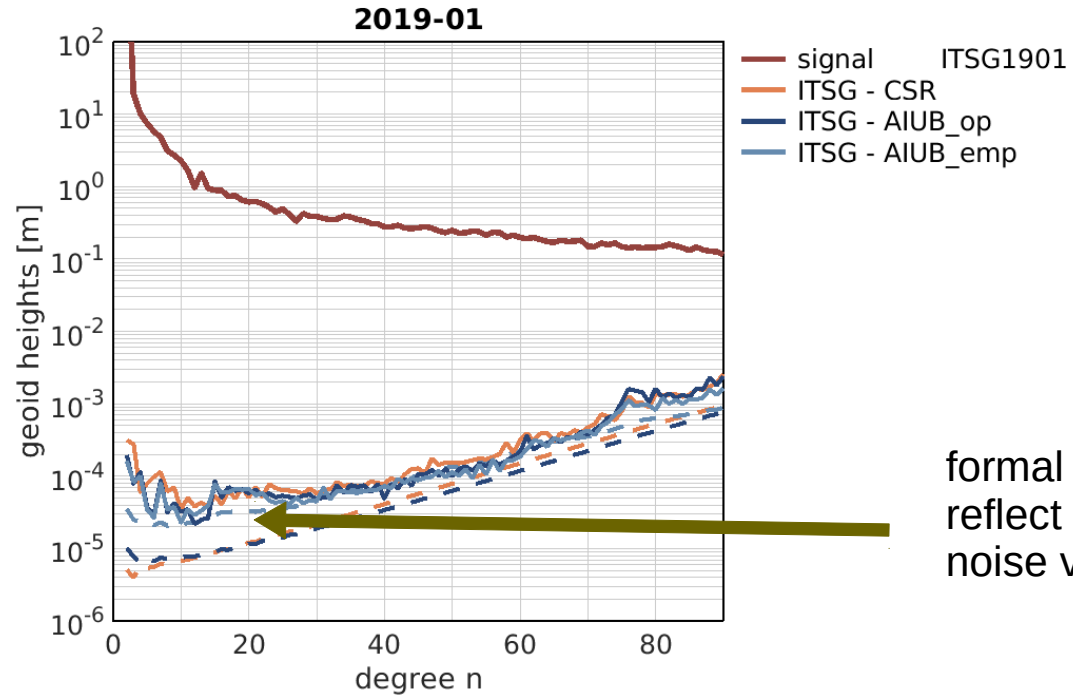
- empirical covariances based on post-fit residuals

no additional parameters

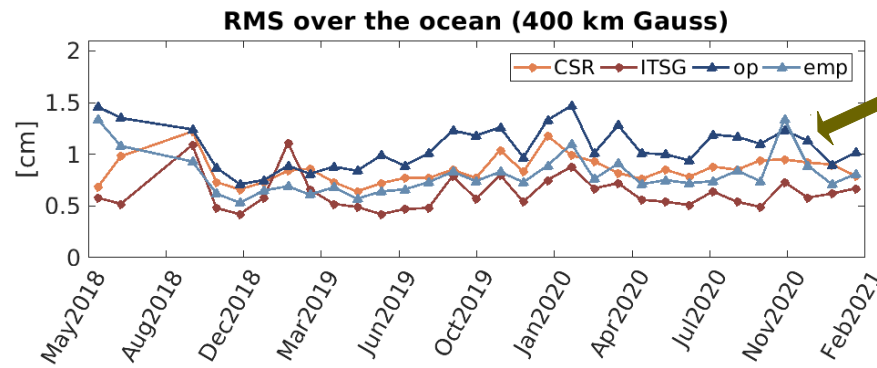
- requires iterative procedure
- assumption of stationarity

in daily arcs (30 days):

- 720 parameters
- + gravity field



formal errors reflect assessed noise very well



clear improvement of solution quality

# Empirical covariances - performance

«emp»

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model:

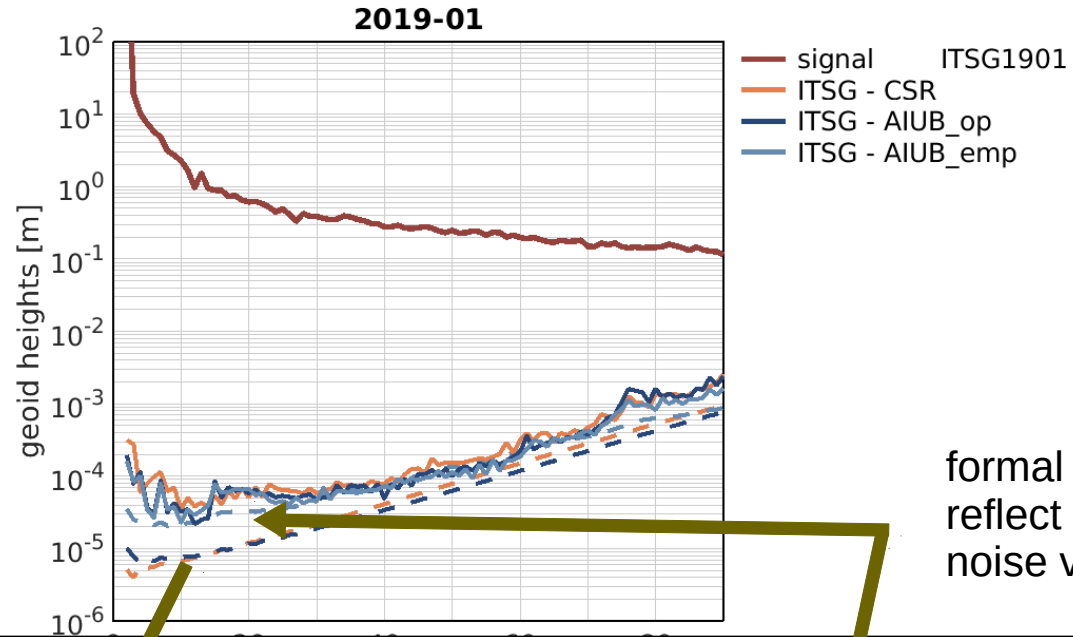
- empirical covariances based on post-fit residuals

no additional parameters

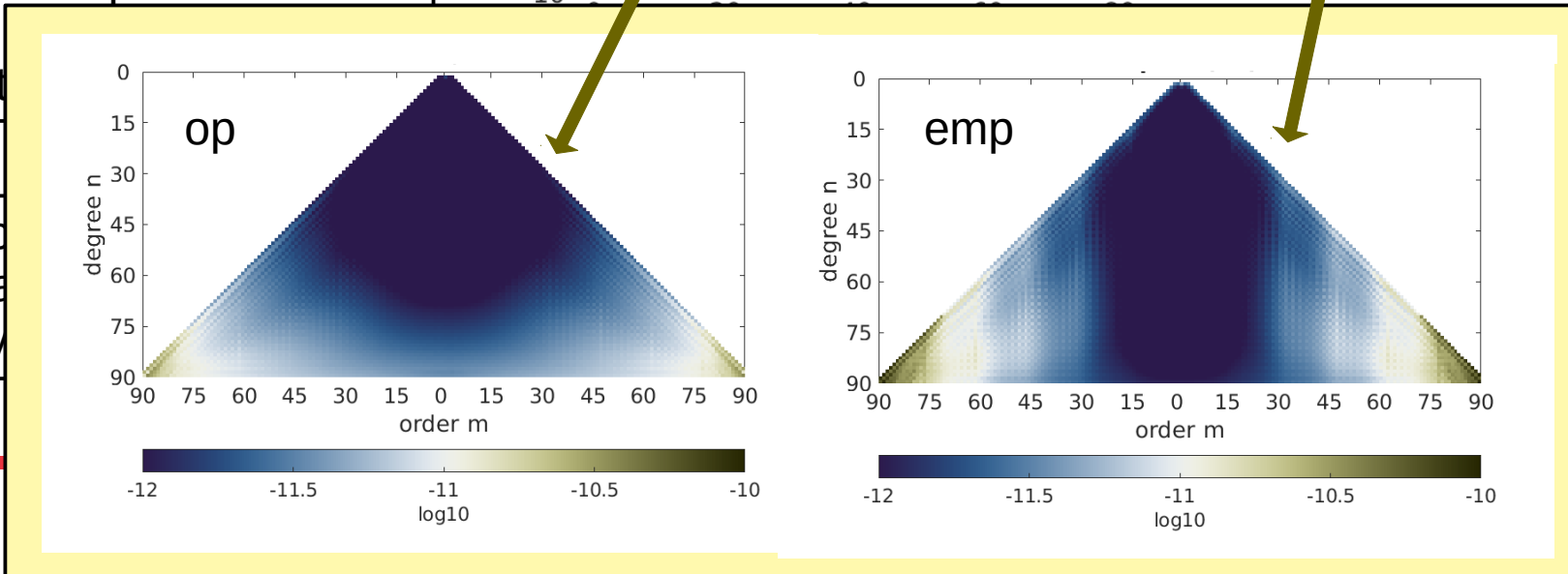
- requires
- assumption

in daily arc

- 720 parameters
- + gravity



formal errors reflect assessed noise very well



improvement of model quality

# Empirical covariances - conclusions

«emp»

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model:

- empirical covariances based on post-fit residuals

no additional parameters

- requires iterative procedure
- assumption of stationarity

in daily arcs (30 days):

- 720 parameters
- + gravity field

- possible on any (stationary) residuals time series
- additional parameters can be reduced as stationary behaviour can be absorbed
- formal errors become much more realistic and show resonance orders (if correlation length > 3 h)
- no constraints needed
- no/few a priori knowledge needed
  
- iterations required (might be time consuming)
- memory consumption and inversion time dependent on length of auto/cross-correlation



# Noise models - theoretical (pre-launch)

«theor»

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model [Kim, 2000]:

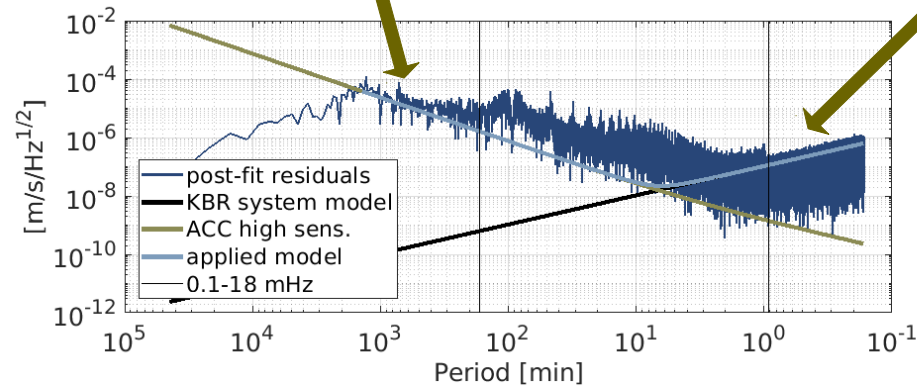
- ACC high sensitivity axis  
 $(1 + 0.005/f) \times 10^{-20} \text{ m/s}^2$
- KBR white noise  
1  $\mu\text{m}$  range  
(differentiation to KBRR)

in daily arcs (30 days):

- 720 parameters
- + gravity field

ACC noise (red noise):  
PSD:  $(1 + 0.005/f) \times 10^{-20} \text{ m/s}^2$

ACC noise (violet noise):  
1  $\mu\text{m}$  range



# Noise models - theoretical (pre-launch)

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- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model [Kim, 2000]:

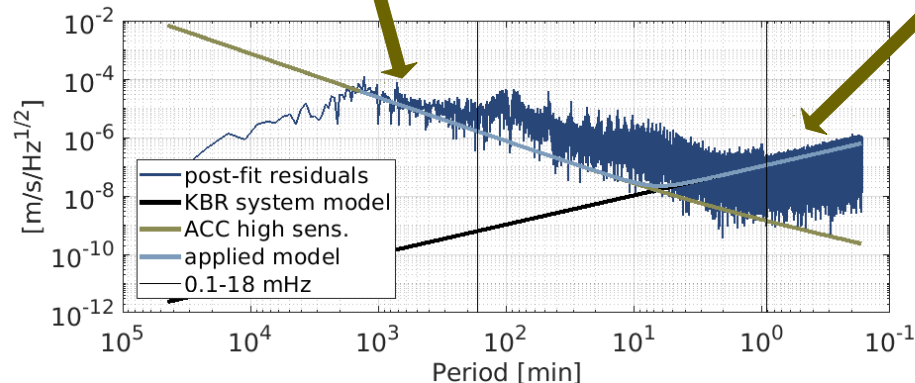
- ACC high sensitivity axis  $(1 + 0.005/f) \times 10^{-20} \text{ m/s}^2$
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in daily arcs (30 days):

- 720 parameters
- + gravity field

ACC noise (red noise):  
PSD:  $(1 + 0.005/f) \times 10^{-20} \text{ m/s}^2$

ACC noise (violet noise):  
1  $\mu\text{m}$  range



- coloured noise ~stationary
- sum at least jointly stationary

→ Wiener-Khinchin theorem to derive auto-correlation from PSD

# Theoretical (pre-launch) - performance

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

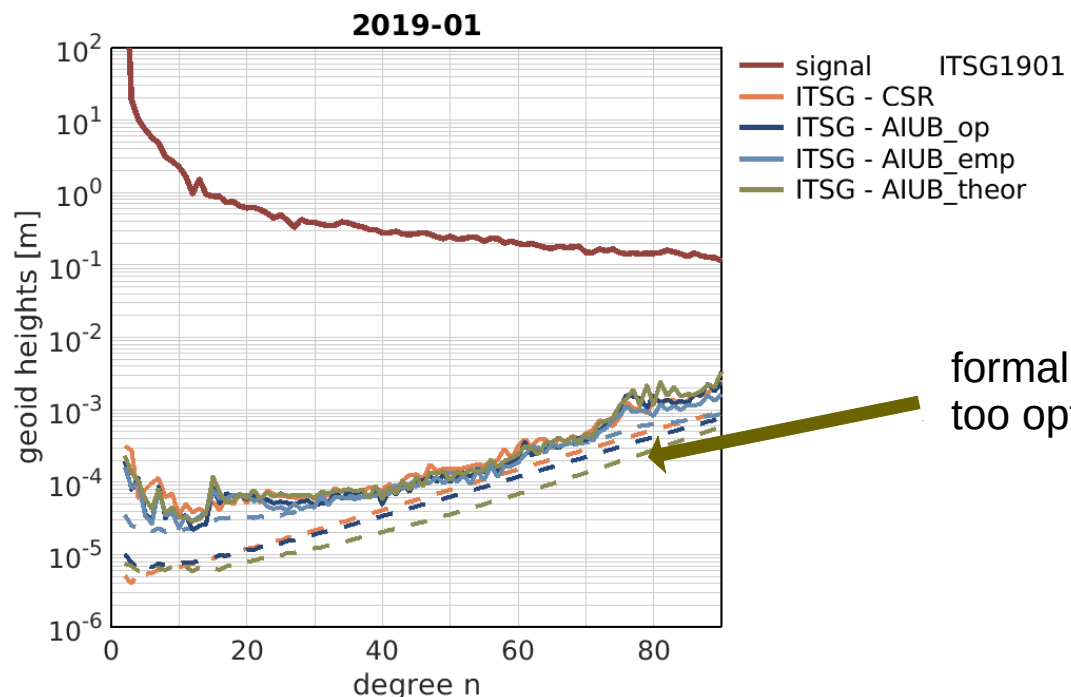
noise model [Kim, 2000]:

- ACC high sensitivity axis  $(1 + 0.005/f) \times 10^{-20} \text{ m/s}^2$
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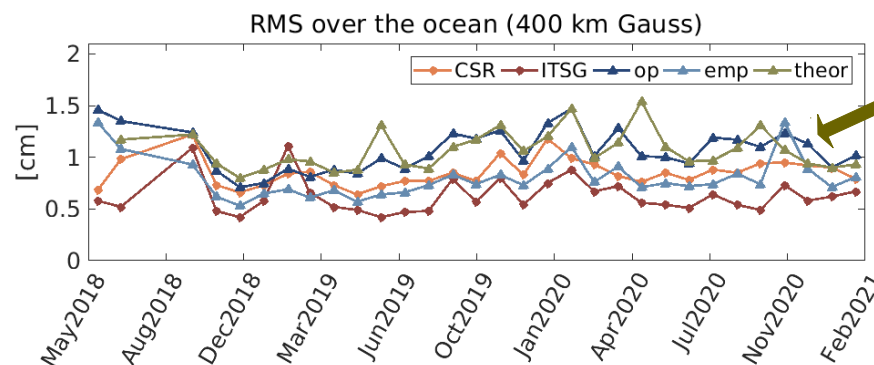
in daily arcs (30 days):

- 720 parameters
- + gravity field

«theor»



formal errors too optimistic



solution quality acceptable (!) with room for improvement

# Theoretical (pre-launch) - conclusions

«theor»

basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

noise model [Kim, 2000]:

- ACC high sensitivity axis  
( $1 + 0.005/f$ )  $\times 10^{-20}$  m/s<sup>2</sup>
- KBR white noise  
1  $\mu$ m range  
(differentiation to KBRR)

in daily arcs (30 days):

- 720 parameters
- + gravity field

- based on priori knowledge
- additional parameters can be reduced
- provides a good solution in case observations act as a priori models state
- no constraints needed
- no iterations needed
- requirements on memory and CPU low
  
- might not reflect actual noise (e.g., without ACT from ITSG)
- formal errors too optimistic
- not all error sources included (e.g., background models)

# References

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Beutler, G., Jäggi, A., Mervart, L. and Meyer, U. [2010]: The celestial mechanics approach: theoretical foundations. Journal of Geodesy, vol. 84(10), pp. 605-624. <https://doi.org/10.1007/s00190-010-0401-7>

Kim, J. [2000]: Simulation Study of A Low-Low Satellite-to-Satellite Tracking Mission, PhD-Thesis, Center for Space and Research, Texas, USA.  
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