

The $\bar{B} \rightarrow X_s \gamma \gamma$ decay: NLL QCD contribution of the Electromagnetic Dipole operator \mathcal{O}_7

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We calculate the set of $O(\alpha_s)$ corrections to the double differential decay width $d\Gamma_{77}/(ds_1 ds_2)$ for the process $\bar{B} \rightarrow X_s \gamma \gamma$ originating from diagrams involving the electromagnetic dipole operator \mathcal{O}_7 . The kinematical variables s_1 and s_2 are defined as $s_i = (p_b - q_i)^2/m_b^2$, where p_b, q_1, q_2 are the momenta of b -quark and two photons. While the (renormalized) virtual corrections are worked out exactly for a certain range of s_1 and s_2 , we retain in the gluon bremsstrahlung process only the leading power w.r.t. the (normalized) hadronic mass $s_3 = (p_b - q_1 - q_2)^2/m_b^2$ in the underlying triple differential decay width $d\Gamma_{77}/(ds_1 ds_2 ds_3)$. The double differential decay width, based on this approximation, is free of infrared- and collinear singularities when combining virtual- and bremsstrahlung corrections. The corresponding results are obtained analytically. When retaining all powers in s_3 , the sum of virtual- and bremsstrahlung corrections contains uncanceled $1/\epsilon$ singularities (which are due to collinear *photon* emission from the s -quark) and other concepts, which go beyond perturbation theory, like parton fragmentation functions of a quark or a gluon into a photon, are needed which is beyond the scope of our paper.

I. INTRODUCTION

Inclusive rare B -meson decays are known to be a unique source of indirect information about physics at scales of several hundred GeV. In the Standard Model (SM) all these processes proceed through loop diagrams and thus are relatively suppressed. In the extensions of the SM the contributions stemming from the diagrams with “new” particles in the loops can be comparable or even larger than the contribution from the SM. Thus getting experimental information on rare decays puts strong constraints on the extensions of the SM or can even lead to a disagreement with the SM predictions, providing evidence for some “new physics”.

To make a rigorous comparison between experiment and theory, precise SM calculations for the (differential) decay rates are mandatory. While the branching ratios for $\bar{B} \rightarrow X_s \gamma$ [1] and $\bar{B} \rightarrow X_s \ell^+ \ell^-$ are known today even to next-to-next-to-leading logarithmic (NNLL) precision (for reviews, see [2, 3]), other branching ratios, like the one for $\bar{B} \rightarrow X_s \gamma \gamma$ discussed in these proceedings, has been calculated before to leading logarithmic (LL) precision in the SM by several groups [4–7] and only recently a first step towards next-to-leading-logarithmic (NLL) precision was presented by us in [8]. In contrast to $\bar{B} \rightarrow X_s \gamma$, the current-current operator \mathcal{O}_2 has a non-vanishing matrix element for $b \rightarrow s \gamma \gamma$ at order α_s^0 precision, leading to an interesting interference pattern with the contributions associated with the electromagnetic dipole operator \mathcal{O}_7 already at LL precision. As a consequence, potential new physics should be clearly visible not only in the total branching ratio, but also in the differential distributions.

As the process $\bar{B} \rightarrow X_s \gamma \gamma$ is expected to be measured at the planned Super B -factories in Japan and Italy, it is necessary to calculate the differential distributions to NLL precision in the SM, in order to fully exploit its potential concerning new physics. The starting point of our calculation is the effective Hamiltonian, obtained by integrating out the heavy particles in the SM, leading to

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^8 C_i(\mu) \mathcal{O}_i(\mu), \quad (1)$$

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where we use the operator basis introduced in [9]:

$$\begin{aligned}
 \mathcal{O}_1 &= (\bar{s}_L \gamma_\mu T^a c_L) (\bar{c}_L \gamma^\mu T_a b_L), & \mathcal{O}_2 &= (\bar{s}_L \gamma_\mu c_L) (\bar{c}_L \gamma^\mu b_L), \\
 \mathcal{O}_3 &= (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q), & \mathcal{O}_4 &= (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T_a q), \\
 \mathcal{O}_5 &= (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho q), & \mathcal{O}_6 &= (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho T_a q), \\
 \mathcal{O}_7 &= \frac{e}{16\pi^2} \bar{m}_b(\mu) (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}, & \mathcal{O}_8 &= \frac{g_s}{16\pi^2} \bar{m}_b(\mu) (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a.
 \end{aligned} \tag{2}$$

The symbols T^a ($a = 1, 8$) denote the $SU(3)$ color generators; g_s and e , the strong and electromagnetic coupling constants. In eq. (2), $\bar{m}_b(\mu)$ is the running b -quark mass in the $\overline{\text{MS}}$ -scheme at the renormalization scale μ . As we are not interested in CP-violation effects in the present paper, we made use of the hierarchy $V_{ub}V_{us}^* \ll V_{tb}V_{ts}^*$ when writing eq. (1). Furthermore, we also put $m_s = 0$.

While the Wilson coefficients $C_i(\mu)$ appearing in eq. (1) are known to sufficient precision at the low scale $\mu \sim m_b$ since a long time (see e.g. the reviews [2, 3] and references therein), the matrix elements $\langle s\gamma\gamma | \mathcal{O}_i | b \rangle$ and $\langle s\gamma\gamma g | \mathcal{O}_i | b \rangle$, which in a NLL calculation are needed to order g_s^2 and g_s , respectively, are not known yet. To calculate the $(\mathcal{O}_i, \mathcal{O}_j)$ -interference contributions to the differential distributions at order α_s is in many respects of similar complexity as the calculation of the photon energy spectrum in $\bar{B} \rightarrow X_s \gamma$ at order α_s^2 needed for the NNLL computation. As a first step in this NLL enterprise, we derived in our paper [8], the $O(\alpha_s)$ corrections to the $(\mathcal{O}_7, \mathcal{O}_7)$ -interference contribution to the double differential decay width $d\Gamma/(ds_1 ds_2)$ at the partonic level. The variables s_1 and s_2 are defined as $s_i = (p_b - q_i)^2/m_b^2$, where p_b and q_i denote the four-momenta of the b -quark and the two photons, respectively.

At order α_s there are contributions to $d\Gamma_{77}/(ds_1 ds_2)$ with three particles (s -quark and two photons) and four particles (s -quark, two photons and a gluon) in the final state. These contributions correspond to specific cuts of the b -quarks self-energy at order $\alpha^2 \times \alpha_s$, involving twice the operator \mathcal{O}_7 . As there are additional cuts, which contain for example only one photon, our observable cannot be obtained using the optical theorem, i.e., by taking the absorptive part of the b -quark self-energy at three-loop. We therefore calculate the mentioned contributions with three and four particles in the final state individually.

We work out the QCD corrections to the double differential decay width in the kinematical range

$$0 < s_1 < 1 \quad ; \quad 0 < s_2 < 1 - s_1.$$

Concerning the virtual corrections, all singularities (after ultra-violet renormalization) are due to **soft gluon** exchanges and/or **collinear gluon** exchanges involving the s -quark. Concerning the bremsstrahlung corrections (restricted to the same range of s_1 and s_2), there are in addition kinematical situations where **collinear photons** are emitted from the s -quark. The corresponding singularities are not canceled when combined with the virtual corrections. We found, however, that there are no singularities associated with collinear photon emission in the double differential decay width when only retaining the leading power w.r.t to the (normalized) hadronic mass $s_3 = (p_b - q_1 - q_2)^2/m_b^2$ in the underlying triple differential distribution $d\Gamma_{77}/(ds_1 ds_2 ds_3)$. Our results of our paper are obtained within this ‘‘approximation’’. When going beyond, other concepts which go beyond perturbation theory, like parton fragmentation functions of a quark or a gluon into a photon, are needed [10].

II. LEADING ORDER AND FINAL RESULTS FOR THE DECAY WIDTH

In $d = 4$ dimensions, the leading-order spectrum (in our restricted phase-space) is given by

$$\frac{d\Gamma_{77}^{(0)}}{ds_1 ds_2} = \frac{\alpha^2 \bar{m}_b^2(\mu) m_b^3 |C_{7,eff}(\mu)|^2 G_F^2 |V_{tb}V_{ts}^*|^2 Q_d^2}{1024 \pi^5} \frac{(1 - s_1 - s_2)}{(1 - s_1)^2 s_1 (1 - s_2)^2 s_2} r_0. \tag{3}$$

where

$$\begin{aligned}
 r_0 &= -48s_2^3 s_1^3 + 96s_2^2 s_1^3 - 56s_2 s_1^3 + 8s_1^3 + 96s_2^3 s_1^2 - 192s_2^2 s_1^2 + 112s_2 s_1^2 - 56s_2^3 s_1 + \\
 &\quad 112s_2^2 s_1 - 96s_2 s_1 + 8s_1 + 8s_2^3 + 8s_2
 \end{aligned}$$

The complete order α_s correction to the double differential decay width $d\Gamma_{77}/(ds_1 ds_2)$ is obtained by adding the renormalized virtual corrections and the bremsstrahlung corrections. Explicitly we obtain

$$\frac{d\Gamma_{77}^{(1)}}{ds_1 ds_2} = \frac{\alpha^2 \bar{m}_b^2(\mu) m_b^3 |C_{7,eff}(\mu)|^2 G_F^2 |V_{tb}V_{ts}^*|^2 Q_d^2}{1024 \pi^5} \times \frac{\alpha_s}{4\pi} C_F \left[\frac{-4 r_0 (1 - s_1 - s_2)}{(1 - s_1)^2 s_1 (1 - s_2)^2 s_2} \log \frac{\mu}{m_b} + f \right], \quad (4)$$

where f can be found explicitly in [8].

The order α_s correction $d\Gamma_{77}^{(1)}/(ds_1 ds_2)$ in Eq. (4) to the double differential decay width for $b \rightarrow X_s \gamma \gamma$ was the main result of our paper [8].

III. SOME NUMERICAL ILLUSTRATIONS

In our procedure the NLL corrections have three sources: (a) α_s corrections to the Wilson coefficient $C_{7,eff}(\mu)$, (b) expressing $\bar{m}_b(\mu)$ in terms of the pole mass m_b and (c) virtual- and real- order α_s corrections to the matrix elements. To illustrate the effect of source (c), which is worked out for the first time in our paper [8], we show in Fig. 1 (by the long-dashed line) the (partial) NLL result in which source (c) is switched off. We conclude that the effect (c) is roughly of equal importance as the combined effects of (a) and (b).

For completeness we show in this figure (by the dotted line) also the result when QCD is completely switched off, which amounts to put $\mu = m_W$ in the LL result.

From Fig. 1 we see that the NLL results are substantially smaller (typically by 50% or slightly more) than those at LL precision, which is also the case when choosing other values for s_2 .

In the numerical discussion above, we have systematically converted the running b -quark mass $\bar{m}_b(\mu)$ in terms of the pole mass m_b . As perturbative expansions often behave better when expressed in terms of the running mass, we also studied the results obtained when systematically converting m_b in terms of $\bar{m}_b(\mu)$. After doing also this version, we observe the following: Generally speaking, NLL corrections are not small for both cases, when taking into account the full range of μ , i.e., $m_b/2 < \mu < 2 m_b$. More precisely, in the $\overline{\text{MS}}$ version they are large for $\mu = m_b/2$ and smaller for larger values of μ , while in the pole mass version they are large for all values of μ .

We stress that the numerically important contributions involving the operator O_2 are not discussed in our paper. Therefore, the issue concerning the reduction of the μ dependence at NLL precision cannot be addressed at this level. Finally, the relevant input parameters that we used in our analysis together with the values of the Wilson coefficient C_7 and the strong coupling α_s at different values of the scale μ are listed in Table I.

Parameter	Value
$m_b(\text{pole})$	4.8 GeV
$m_t(\text{pole})$	175 GeV
M_W	80.4 GeV
M_Z	91.19 GeV
G_F	$1.16637 \times 10^{-5} \text{ GeV}^{-2}$
$V_{tb}V_{ts}^*$	0.04
α^{-1}	137
$\alpha_s(M_Z)$	0.119

	$\alpha_s(\mu)$	$C_{7,eff}^0(\mu)$	$C_{7,eff}^1(\mu)$
$\mu = M_W$	0.1213	-0.1957	-2.3835
$\mu = 2 m_b$	0.1818	-0.2796	-0.1788
$\mu = m_b$	0.2175	-0.3142	0.4728
$\mu = m_b/2$	0.2714	-0.3556	1.0794

TABLE I: **Left:** Relevant input parameters. **Right:** $\alpha_s(\mu)$ and the Wilson coefficient $C_{7,eff}(\mu)$ at different values of the scale μ .

IV. CONCLUDING REMARKS

In the present work we calculated the set of the $O(\alpha_s)$ corrections to the decay process $\bar{B} \rightarrow X_s \gamma \gamma$ originating from diagrams involving \mathcal{O}_7 . To perform this calculation, it is necessary to work out diagrams with three particles (s -quark and two photons) and four particles (s -quark, two photons and a gluon) in the final state. From the technical point of view, the calculation was made possible by the use of the Laporta Algorithm to identify the needed

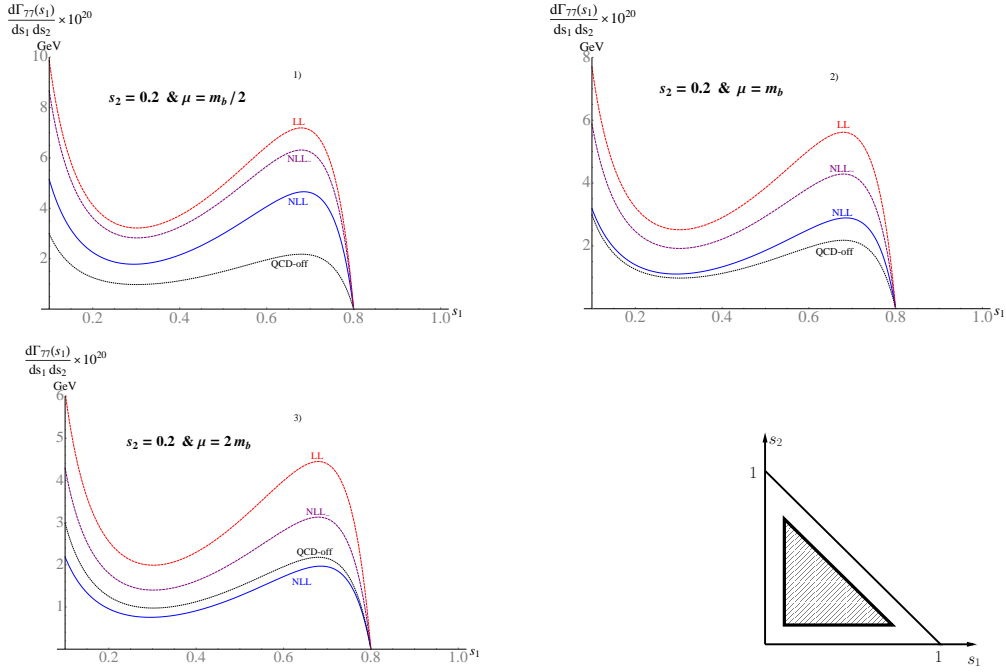


FIG. 1: **Frames 1)-3)**: Double differential decay width $d\Gamma_{77}/(ds_1 ds_2)$ as a function of s_1 for s_2 fixed at $s_2 = 0.2$. The dotted(black), the short-dashed(red) and the solid line(blue) shows the result when neglecting QCD-effects, the LL and the NLL result, respectively. The long-dashed line(purple) represents the (partial) NLL result in which the virtual- and bremsstrahlung corrections worked out in our paper [8] are switched off. In the frames 1), 2) and 3) the renormalization scale is chosen to be $\mu = m_b/2$, $\mu = m_b$ and $\mu = 2 m_b$, respectively. **Down right**: The relevant phase-space region for s_1 and s_2 .

master integrals and by applying the differential equation method to solve the master integrals. When calculating the bremsstrahlung corrections, we take into account only terms proportional to the leading power of the hadronic mass. We find that the infrared and collinear singularities cancel when combining the above mentioned approximated version of bremsstrahlung corrections with the virtual corrections. The numerical impact of the NLL corrections is large: for $d\Gamma_{77}/(ds_1 ds_2)$ the NLL result is approximately 50% smaller than the LL prediction.

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- [1] M. Misiak, H. Asatrian, K. Bieri, M. Czakon, A. Czarnecki, et al., Phys.Rev.Lett. **98**, 022002 (2007), hep-ph/0609232.
 - [2] T. Hurth and M. Nakao, Ann.Rev.Nucl.Part.Sci. **60**, 645 (2010), 1005.1224.
 - [3] A. J. Buras (2011), 1102.5650.
 - [4] H. Simma and D. Wyler, Nucl.Phys. **B344**, 283 (1990).
 - [5] L. Reina, G. Ricciardi, and A. Soni, Phys.Lett. **B396**, 231 (1997), hep-ph/9612387.
 - [6] L. Reina, G. Ricciardi, and A. Soni, Phys.Rev. **D56**, 5805 (1997), hep-ph/9706253.
 - [7] J.-j. Cao, Z.-j. Xiao, and G.-r. Lu, Phys.Rev. **D64**, 014012 (2001), hep-ph/0103154.
 - [8] H. Asatrian, C. Greub, A. Kokulu, and A. Yeghiazaryan, Phys.Rev. **D85**, 014020 (2012), 1110.1251.
 - [9] K. G. Chetyrkin, M. Misiak, and M. Munz, Phys.Lett. **B400**, 206 (1997), hep-ph/9612313.
 - [10] A. Kapustin, Z. Ligeti, and H. D. Politzer, Phys.Lett. **B357**, 653 (1995), hep-ph/9507248.