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Thermal right-handed neutrino self-energy in the non-relativistic regime*

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Recently the issue of radiative corrections to leptogenesis has been raised. Considering the "strong washout" regime, in which OPE-techniques permit to streamline the setup, we report the thermal self-energy matrix of heavy right-handed neutrinos at NLO (resummed 2-loop level) in Standard Model couplings. The renormalized expression describes flavour transitions and "inclusive" decays of chemically decoupled right-handed neutrinos. Although CP-violation is not addressed, the result may find use in existing leptogenesis frameworks.

1. Introduction

Leptogenesis is currently among the most popular scenarios for explaining the observed cosmological baryon asymmetry. Surprisingly, although the basic mechanism is fairly simple [1], it appears difficult to develop a fully consistent theoretical description of the physics involved. The reason is that many different subtle topics, such as CP-violation, baryon number violation, deviations from thermal equilibrium, as well as resummations necessary for a systematic treatment of relativistic thermal field theory, need all to be put consistently together.

As far as CP-violation is concerned, it could be both of "direct" and "indirect" type, known as "vertex" and "wave function" corrections, respectively [2, 3, 4]. The indirect CP-violation originating from flavour oscillations is non-trivial even at low temperatures where essentially the vacuum formalism can be used [5, 6]. Moreover, collective thermal phenomena and higher-order corrections could be important [7, 8, 9]. At present many approaches remain phenomenological, but efforts towards a more systematic treatment are under way [10]–[14].† Ultimately the goal should be to present theoretically consistent results in terms of Standard Model couplings, in the sense that have recently been obtained for CP-conserving rates both in the "ultrarelativistic" ($m_{\rm top} \lesssim M \ll \pi T$) [18, 19] and "non-relativistic" ($m_{\rm top} \lesssim \pi T \ll M$) [20, 21] regimes. Here we concentrate on the latter regime, called the "strong washout" case in that memory about initial conditions has been lost.

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†The situation is analogous to that for CP-violation in phase transition-based baryogenesis [15, 16, 17].

In efforts towards systematic leptogenesis, the right-handed neutrino self-energy plays an important role [22, 23, 24]. Examples of recent discussions can be found in sec. 3 of ref. [13] and in sec. 4.1 of ref. [14], in both of which the self-energy was handled at leading order in Standard Model couplings. In the temperature regime of interest, right-handed neutrinos are out of equilibrium, but the Standard Model particles are *in* equilibrium, at least as far as CP-conserving reactions are concerned. Therefore the self-energy of the right-handed neutrinos, which reflects the dynamics of the other particles, can be computed with established techniques of thermal field theory, and there is no reason to restrict to leading order.

In this note a result for the thermal right-handed neutrino self-energy in the non-relativistic regime is presented at NLO (partly even NNLO) in Standard Model couplings. Compared with our earlier work [21], where the production rate of right-handed neutrinos was computed, the real part of the self-energy is added here, and the full Lorentz and flavour structures are included. Technically, we work at order $\mathcal{O}(h_\nu^\dagger h_\nu)$ in neutrino Yukawa couplings, thus addressing CP-conserving processes, whereas CP-violation originates at the order $\mathcal{O}(h_\nu^\dagger h_\nu)^2$. As an outlook, the same observable could in principle also be computed in the "relativistic" ($\pi T \sim M$) and ultrarelativistic ($\pi T \gg M$) regimes; its physics is related to "CP even damping and time evolution", as discussed e.g. in secs. 5.2-3 of ref. [25].

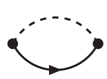
2. Self-energy matrix

We start by computing the Euclidean correlator (the Lagrangian and other conventions are summarized in appendix A)

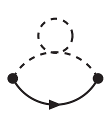
$$\Sigma_E(K) \equiv \int_0^\beta \mathrm{d}\tau \int_{\mathbf{x}} e^{iK\cdot X} \Big\langle (\tilde\phi^\dagger a_{\rm L}\ell)(X)\; (\bar\ell\, a_{\rm R}\tilde\phi)(0) \Big\rangle_T \;, \tag{2.1}$$

where $X = (\tau, \mathbf{x})$; $K = (k_n, \mathbf{k})$, where k_n are fermionic Matsubara frequencies; ℓ is a lepton doublet; and $\tilde\phi \equiv i\sigma_2\phi^*$ is a Higgs doublet. The corresponding retarded correlator is obtained through the analytic continuation $k_n \to -i[k^0 + i0^+]$.

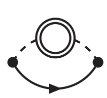
Treating γ^5 in Naive Dimensional Regularization and employing Feynman gauge (other regularizations and gauges were discussed in ref. [21]), we obtain the contributions (dashed line denotes Higgs, solid lepton, double top, wiggly $W^\pm, Z^0$)



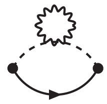
$$= 2 \sum\!\!\!\!\!\!\!\int_P \frac{i a_{\rm L}(\not K - \not P) a_{\rm R}}{P^2 (K-P)^2} \;, \tag{2.2}$$



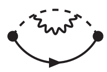
$$= -12\lambda_{\rm B} \sum\!\!\!\!\!\!\!\int_{PQ} \frac{i a_{\rm L}(\not K - \not P) a_{\rm R}}{Q^2 P^4 (K-P)^2} \;, \tag{2.3}$$



$$= -2|h_{t\rm B}|^2 N_{\rm c} \sum\!\!\!\!\!\!\!\int_{P\{R\}} \frac{i a_{\rm L}(\not K - \not P) a_{\rm R} \,\mathrm{Tr}\,[a_{\rm L} \not R\, a_{\rm R} (\not P - \not R)]}{P^4 (K-P)^2 R^2 (P-R)^2} \;, \tag{2.4}$$

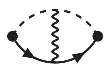


$$= -(g_{1\mathrm{B}}^2 + 3g_{2\mathrm{B}}^2)\,\frac{D}{2}\,\sum\!\!\!\!\!\!\!\int_{PQ} \frac{i a_{\mathrm{L}}(\not{K} - \not{P})a_{\mathrm{R}}}{Q^2P^4(K-P)^2}\;, \tag{2.5}$$



$$= (g_{1\mathrm{B}}^2 + 3g_{2\mathrm{B}}^2)\,\frac{1}{2}\,\sum\!\!\!\!\!\!\!\int_{PQ} \frac{i a_{\mathrm{L}}(\not{K} - \not{P})a_{\mathrm{R}}(P+Q)^2}{P^4Q^2(P-Q)^2(K-P)^2}\;, \tag{2.6}$$

$$= (g_{1\mathrm{B}}^2 + 3g_{2\mathrm{B}}^2)\,\frac{1}{2}\,\sum\!\!\!\!\!\!\!\int_{PQ} \frac{i a_{\mathrm{L}}(\not{K} - \not{P})\gamma_\mu(\not{K} - \not{Q})\gamma_\mu(\not{K} - \not{P})a_{\mathrm{R}}}{P^2(P-Q)^2(K-P)^4(K-Q)^2}\;, \tag{2.7}$$



$$= -(g_{1\mathrm{B}}^2 + 3g_{2\mathrm{B}}^2)\,\frac{1}{2}\,\sum\!\!\!\!\!\!\!\int_{PQ} \frac{i a_{\mathrm{L}}(\not{K} - \not{P})(\not{P} + \not{Q})(\not{K} - \not{Q})a_{\mathrm{R}}}{P^2Q^2(P-Q)^2(K-P)^2(K-Q)^2}\;. \tag{2.8}$$

Here $D = 4 - 2\epsilon$; $\sum\!\!\!\!\!\!\int_P$, $\sum\!\!\!\!\!\!\int_Q$ are bosonic sum-integrals; and $\sum\!\!\!\!\!\!\int_{\{R\}}$ is a fermionic one.

The sum-integrals in eqs. (2.2)–(2.8) are of a "tensor" type [26]; the numerator transforms non-trivially in O(4) rotations. It is known that in an ultrarelativistic plasma, fermion self-energy does not respect Lorentz invariance; the part multiplying γ^0 differs from that multiplying the spatial γ^k [27]. It turns out, however, that in the non-relativistic regime, i.e. $(\pi T)^2 \ll K^2$, in which the problem can be discussed with OPE language [28], the first thermal corrections do respect Lorentz invariance. Perhaps the simplest way to understand this is that thermal corrections of $\mathcal{O}(\pi T)^2$ are represented by the condensate $\langle \phi^\dagger \phi \rangle_T$ [21], and there is no room for tensor structures in this condensate.

In practice, the sum-integrals are of two types. Those proportional to the external four-momentum $\not{K}$ are scalars; results can be found in appendix C of ref. [21]. Those proportional to $\not{P}$, $\not{Q}$ or $\not{R}$ are tensors; Matsubara sums need to be carried out separately for the temporal and spatial components. In the OPE regime this can be done with the techniques introduced in refs. [29, 30]. In each case, only structures proportional to $\not{K}$ survive at $\mathcal{O}(\pi T)^2$.

Concretely, multiplying Σ_E by the renormalization factor related to the neutrino Yukawa couplings, $\mathcal{Z}_\nu \equiv 1 + \frac{1}{(4\pi)^2\epsilon}\big[|h_t|^2 N_{\mathrm{c}} - \frac{3}{4}\,(g_1^2 + 3g_2^2)\big] + \mathcal{O}(g^4)$ where g denotes generic Standard Model couplings and corrections involving neutrino Yukawa couplings were omitted, we obtain

$$\begin{aligned} \mathcal{Z}_\nu\,\Sigma_E(K) \;=\; & a_{\mathrm{L}} i\not{K}\, a_{\mathrm{R}} \bigg\{ \frac{1}{(4\pi)^2}\bigg(\frac{1}{\epsilon} + \ln\frac{\bar\mu^2}{K^2} + 2\bigg) \\ & + \frac{|h_t|^2 N_{\mathrm{c}}}{(4\pi)^4}\bigg(\frac{1}{2\epsilon^2} - \frac{3}{4\epsilon} - \frac{1}{2}\ln^2\frac{\bar\mu^2}{K^2} - \frac{7}{2}\ln\frac{\bar\mu^2}{K^2} - \frac{57}{8}\bigg) \\ & + \frac{g_1^2 + 3g_2^2}{(4\pi)^4}\bigg(-\frac{3}{8\epsilon^2} + \frac{17}{16\epsilon} + \frac{3}{8}\ln^2\frac{\bar\mu^2}{K^2} + \frac{29}{8}\ln\frac{\bar\mu^2}{K^2} + \frac{275}{32} - 3\zeta(3)\bigg) \\ & + \bigg[1 + \frac{6\lambda}{(4\pi)^2}\bigg(\ln\frac{\bar\mu^2}{K^2} + 1\bigg)\bigg]\frac{\mathcal{Z}_m \langle \phi^\dagger\phi\rangle_T}{K^2} \;+\; \mathcal{O}\Big(g^4, \frac{T^4}{K^4}\Big)\bigg\}\;. \end{aligned} \tag{2.9}$$

Here $\mathcal{Z}_m \equiv 1 + \frac{1}{(4\pi)^2\epsilon}\big[6\lambda + |h_t|^2 N_{\mathrm{c}} - \frac{3}{4}\,(g_1^2 + 3g_2^2)\big] + \mathcal{O}(g^4)$ is the renormalization factor related to the Higgs mass parameter (denoted below by m_0^2) and, for $\pi T \gtrsim m_{\rm top}$ [21],

$$
\begin{aligned}
\mathcal{Z}_m \langle \phi^\dagger \phi \rangle_T &= \frac{T^2}{6} - \frac{T^2}{2\pi}\sqrt{\frac{m_{\rm H}^2}{T^2} - \frac{g_1^2 m_{\rm D1} + 3 g_2^2 m_{\rm D2}}{16\pi T}} \\
&+ \frac{T^2}{48\pi^2}\biggl\{ -6\lambda \biggl[\ln\biggl(\frac{\bar\mu e^{\gamma_{\rm E}}}{4\pi T}\biggr) - 3 \biggr] - |h_t|^2 N_{\rm c} \ln\biggl(\frac{\bar\mu e^{\gamma_{\rm E}}}{8\pi T}\biggr) \\
&+ \frac{3(g_1^2 + 3 g_2^2)}{4} \biggl[\ln\biggl(\frac{\bar\mu e^{\gamma_{\rm E}}}{4\pi T}\biggr) - \frac{2}{3} - 2\gamma_{\rm E} - 2\frac{\zeta'(-1)}{\zeta(-1)} + 4 \ln\biggl(\frac{2\pi T}{m_{\rm H}}\biggr) \biggr] \biggr\} + \mathcal{O}(g^3) \,,
\end{aligned}
\tag{2.10}
$$

$$
m_{\rm H}^2 = m_0^2 + \biggl(\frac{\lambda}{2} + \frac{|h_t|^2 N_{\rm c}}{12} + \frac{g_1^2 + 3 g_2^2}{16} \biggr) T^2 + \mathcal{O}(g^4) \,. \tag{2.11}
$$

Here $m_{\rm D1}$, $m_{\rm D2}$ are the $\mathrm{U_Y}(1)$ and $\mathrm{SU_L}(2)$ Debye masses, and chemical potentials have been set to zero. All thermal corrections lie in $\mathcal{Z}_m \langle \phi^\dagger \phi \rangle_T$. *If* we count $(\pi T)^2 \sim K^2$ then eq. (2.9) is "NNLO" because, after analytic continuation, its real part contains terms of $\mathcal{O}(g^0)$, $\mathcal{O}(g^1)$, $\mathcal{O}(g^2)$ (cf. eqs. (2.10), (2.13)), and its imaginary part terms of $\mathcal{O}(g^0)$, $\mathcal{O}(g^2)$, $\mathcal{O}(g^3)$ (cf. eqs. (2.10), (2.14)). However, in the non-relativistic regime it is more natural to count $(\pi T)^2 \sim g^2 K^2$, and then the imaginary part is complete only up to NLO.

For the next steps, a formalism needs to be chosen by which to represent right-handed neutrinos; various possibilities are summarized in appendix A. If the neutrinos are represented as chiral Dirac fermions then, after the analytic continuation $k_n \to -i[k^0 + i0^+]$, eq. (2.9) amounts to a "wave function correction" in their retarded self-energy:

$$
a_{\rm L} \not{\mathcal{K}}\, a_{\rm R} \to a_{\rm L} \not{\mathcal{K}}\, a_{\rm R} \Bigl\{ \mathbb{1} + h_\nu^\dagger h_\nu \left[\phi_{\rm R}(\mathcal{K}^2) + i\, \mathrm{sign}(k^0) \phi_{\rm I}(\mathcal{K}^2) \right] \Bigr\} \,, \tag{2.12}
$$

where $\mathcal{K} = (k^0, \mathbf{k})$, h_ν is a 3×3 matrix of Yukawa couplings (cf. eq. (A.5)), and

$$
\begin{aligned}
\phi_{\rm R}(\mathcal{K}^2) &= \frac{1}{(4\pi)^2} \biggl(\frac{1}{\epsilon} + \ln\frac{\bar\mu^2}{\mathcal{K}^2} + 2 \biggr) \\
&+ \frac{|h_t|^2 N_{\rm c}}{(4\pi)^4} \biggl(\frac{1}{2\epsilon^2} - \frac{3}{4\epsilon} - \frac{1}{2}\ln^2\frac{\bar\mu^2}{\mathcal{K}^2} - \frac{7}{2}\ln\frac{\bar\mu^2}{\mathcal{K}^2} - \frac{57}{8} + \frac{\pi^2}{2} \biggr) \\
&+ \frac{g_1^2 + 3g_2^2}{(4\pi)^4} \biggl(-\frac{3}{8\epsilon^2} + \frac{17}{16\epsilon} + \frac{3}{8}\ln^2\frac{\bar\mu^2}{\mathcal{K}^2} + \frac{29}{8}\ln\frac{\bar\mu^2}{\mathcal{K}^2} + \frac{275}{32} - \frac{3\pi^2}{8} - 3\zeta(3) \biggr) \\
&- \biggl[1 + \frac{6\lambda}{(4\pi)^2} \biggl(\ln\frac{\bar\mu^2}{\mathcal{K}^2} + 1 \biggr) \biggr] \frac{\mathcal{Z}_m \langle \phi^\dagger\phi \rangle_T}{\mathcal{K}^2} + \mathcal{O}\Bigl(g^4, \frac{T^4}{\mathcal{K}^4} \Bigr) \,,
\end{aligned}
\tag{2.13}
$$

$$
\begin{aligned}
\frac{\phi_{\rm I}(\mathcal{K}^2)}{\pi} &= \frac{1}{(4\pi)^2} - \frac{|h_t|^2 N_{\rm c}}{(4\pi)^4} \biggl(\ln\frac{\bar\mu^2}{\mathcal{K}^2} + \frac{7}{2} \biggr) + \frac{g_1^2 + 3g_2^2}{(4\pi)^4} \biggl(\frac{3}{4}\ln\frac{\bar\mu^2}{\mathcal{K}^2} + \frac{29}{8} \biggr) \\
&- \frac{6\lambda}{(4\pi)^2} \frac{\mathcal{Z}_m \langle \phi^\dagger\phi \rangle_T}{\mathcal{K}^2} + \mathcal{O}\Bigl(g^4, \frac{T^4}{\mathcal{K}^4} \Bigr) \,.
\end{aligned}
\tag{2.14}
$$

If we rather employ Majorana spinors, then eq. (A.7) implies that the flavour structure can be "reflected" to the left-handed components:

$$
\not{\mathcal{K}} \to \not{\mathcal{K}} \, \Bigl\{ \mathbb{1} + \left[h_\nu^\dagger h_\nu a_{\rm R} + (h_\nu^\dagger h_\nu)^T a_{\rm L} \right] \left[\phi_{\rm R}(\mathcal{K}^2) + i\, \mathrm{sign}(k^0) \phi_{\rm I}(\mathcal{K}^2) \right] \Bigr\} \,. \tag{2.15}
$$

Given that $h_\nu^\dagger h_\nu$ is Hermitean, we denote $(h_\nu^\dagger h_\nu)^T$ by $(h_\nu^\dagger h_\nu)^*$ in the following. If a time-ordered correlator (Σ_T) is considered rather than a retarded one (Σ_R), then the corresponding self-energy reads

$$\Sigma_T(\mathcal{K}) = \mathrm{Re}\,\Sigma_R(\mathcal{K}) + i\big[1 - 2n_{\rm F}(k^0)\big]\,\mathrm{Im}\,\Sigma_R(\mathcal{K}) \;, \tag{2.16}$$

where $n_{\rm F}$ denotes the Fermi distribution, and Im refers to a cut across the k^0-axis. In the non-relativistic regime, $|k^0| \gg \pi T$, we can simplify $1 - 2n_{\rm F}(k^0) = \mathrm{sign}(k^0)$, and therefore in the time-ordered case it is the combination $\phi_{\rm R}(\mathcal{K}^2) + i\phi_{\rm I}(\mathcal{K}^2)$ that appears.

3. On-shell renormalization

As suggested by eqs. (2.12), (2.15), it is conventional to view the divergences appearing in $\phi_{\rm R}$, eq. (2.13), as being cancelled by wave function renormalization. Wave function normalization being somewhat unphysical, it may be more elegant to discuss only mass renormalization explicitly. This can be achieved by considering renormalization at an on-shell point.

In the literature, various formalisms are being used for addressing right-handed neutrino dynamics in the context of leptogenesis. Within the "Kadanoff-Baym" framework, retarded and advanced self-energies appear directly, and the pole positions of the corresponding propagators can be solved for (cf. e.g. sec. 4.1 of ref. [14]). Within the "canonical" approach, a density matrix between free one-particle states is constructed and evolved [31, 32, 33]. In both cases on-shell particles appear and the considerations below may apply.

Of course, strictly speaking right-handed neutrinos are not on-shell states, because they decay. Nevertheless, the width being parametrically suppressed by $\mathcal{O}(h_\nu^\dagger h_\nu)$, the concept of an on-shell particle may still be a useful practical notion, like for the top quark. In the following, we refer to the pole mass as the real part of the pole position in the k^0-plane.

Let M be the renormalized mass matrix which we choose to be non-negative and diagonal, and write $M_{\rm B} = M + \delta M$. The diagonal elements of M are denoted by m_i: $M = \mathrm{diag}(m_1, m_2, m_3)$. Making use of Majorana spinors, the effective action for time-ordered correlators has the form

$$\begin{aligned} \mathcal{S}_{\rm eff} \;=\; & \int_{\mathcal{X}} \frac{1}{2}\,\bar{\tilde{N}}(\mathcal{X})\Big\{ i\,\partial\!\!\!/ - M - \delta M\, a_{\rm R} - \delta M^* a_{\rm L} \\ & + \big[h_\nu^\dagger h_\nu a_{\rm R} + (h_\nu^\dagger h_\nu)^* a_{\rm L}\big] i\,\partial\!\!\!/\,\big[\phi_{\rm R}(-\partial^2) + i\phi_{\rm I}(-\partial^2)\big]\Big\}\tilde{N}(\mathcal{X}) \;, \quad \mathcal{X} \equiv (t,\mathbf{x}) \;, \end{aligned} \tag{3.1}$$

where δM denotes the mass counterterm. Within the $\mathcal{O}(h_\nu^\dagger h_\nu)$ part we can make use of tree-level equations of motion; denoting

$$\tilde{M} \equiv \mathrm{diag}(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3) \;, \quad \tilde{m}_i \equiv m_i\big[\phi_{\rm R}(m_i^2) + i\phi_{\rm I}(m_i^2)\big] \;, \tag{3.2}$$

and recalling from eq. (A.6) that mass matrices can always be symmetrized, we get

$$\mathcal{S}_{\rm eff} = \int_{\mathcal{X}} \frac{1}{2}\, \bar{\tilde{N}}(\mathcal{X}) \Big\{ i \not{\partial} - M - \delta M\, a_{\rm R} - \delta M^* a_{\rm L} + \frac{1}{2} \big[h_\nu^\dagger h_\nu \tilde{M} + \tilde{M} (h_\nu^\dagger h_\nu)^* \big] a_{\rm R} + \frac{1}{2} \big[(h_\nu^\dagger h_\nu)^* \tilde{M} + \tilde{M} h_\nu^\dagger h_\nu \big] a_{\rm L} \Big\} \tilde{N}(\mathcal{X}) \,. \tag{3.3}$$

In the on-shell scheme δM is chosen to cancel the vacuum part of the $\mathcal{O}(h_\nu^\dagger h_\nu)$ correction; hence, writing $\phi_{\rm R} = \phi_{\rm R}^{(0)} + \phi_{\rm R}^{(T)}$, where $\phi_{\rm R}^{(0)}$ corresponds to the three first lines of eq. (2.13),

$$\delta M_{i_1 i_2} \equiv \frac{1}{2} \Big[(h_\nu^\dagger h_\nu)_{i_1 i_2}\, m_{i_2} \phi_{\rm R}^{(0)}(m_{i_2}^2) + (h_\nu^\dagger h_\nu)_{i_2 i_1}\, m_{i_1} \phi_{\rm R}^{(0)}(m_{i_1}^2) \Big] \,. \tag{3.4}$$

The finite remainder, which we parametrize through

$$\tilde{M}_r \equiv {\rm diag}(\tilde{m}_{r,1}, \tilde{m}_{r,2}, \tilde{m}_{r,3}) \,, \quad \tilde{m}_{r,i} \equiv m_i \big[\phi_{\rm R}^{(T)}(m_i^2) + i \phi_{\rm I}(m_i^2) \big] \,, \tag{3.5}$$

contains thermal "mass corrections" (from $\phi_{\rm R}^{(T)}$) as well as a non-Hermitean part, representing vacuum and thermal "decay widths" (from $\phi_{\rm I}$).

To be concrete, consider the canonical formalism and define a free Hamiltonian as

$$\hat{\mathcal{H}}^{(0)}_{\rm eff}(\mathcal{X}) \equiv \frac{1}{2} : \bar{\hat{\tilde{N}}}(\mathcal{X}) \Big(-i\gamma^k \partial_k + M \Big) \hat{\tilde{N}}(\mathcal{X}) : \,. \tag{3.6}$$

From eq. (3.3), the first-order correction reads

$$\hat{\mathcal{H}}^{(1)}_{\rm eff}(\mathcal{X}) - \frac{i}{2} \hat{\Gamma}^{(1)}_{\rm eff}(\mathcal{X}) \equiv -\frac{1}{2} : \bar{\hat{\tilde{N}}}(\mathcal{X}) \Big(\frac{\mathbb{1}}{2} \big\{ {\rm Re}(h_\nu^\dagger h_\nu), \tilde{M}_r \big\} + \frac{i\gamma^5}{2} \big[{\rm Im}(h_\nu^\dagger h_\nu), \tilde{M}_r \big] \Big) \hat{\tilde{N}}(\mathcal{X}) : \,. \tag{3.7}$$

Defining a state with a specific flavour (i_1), momentum ($\mathbf{k}_1$), and spin (τ_1) formally as

$$|i_1 \mathbf{k}_1 \tau_1\rangle \equiv (2\pi)^{3/2} \hat{a}^\dagger_{i_1 \mathbf{k}_1 \tau_1} |0\rangle \,, \tag{3.8}$$

and inserting the interaction picture field operators from eqs. (A.10), (A.11), matrix elements can be computed. Standard properties of the on-shell spinors (cf. appendix A) lead to

$$\langle i_1 \mathbf{k} \tau_1 |\, \hat{\mathcal{H}}^{(0)}_{\rm eff}(0)\, | i_2 \mathbf{k} \tau_2 \rangle = \delta_{i_1 i_2} \delta_{\tau_1 \tau_2}\, \mathcal{K}^0_{i_1} \,, \qquad \mathcal{K}^0_{i_1} = \sqrt{k^2 + m_{i_1}^2} \,, \tag{3.9}$$

amounting to a free one-particle energy. For $i_1 = i_2$, the $\mathcal{O}(h_\nu^\dagger h_\nu)$ correction yields

$$\langle i_1 \mathbf{k} \tau_1 |\, \hat{\mathcal{H}}^{(1)}_{\rm eff}(0)\, | i_1 \mathbf{k} \tau_2 \rangle = -\frac{m_{i_1}^2}{\mathcal{K}^0_{i_1}}\, {\rm Re}(h_\nu^\dagger h_\nu)_{i_1 i_1}\, \delta_{\tau_1 \tau_2} \phi_{\rm R}^{(T)}(m_{i_1}^2) \,, \tag{3.10}$$

which may be interpreted as a thermal mass correction: $\delta \mathcal{K}^0_{i_1} \approx {\rm Re}(h_\nu^\dagger h_\nu)_{i_1 i_1} \mathcal{Z}_m \langle \phi^\dagger \phi \rangle_T / \mathcal{K}^0_{i_1}$. The decay width is

$$\langle i_1 \mathbf{k} \tau_1 |\, \hat{\Gamma}^{(1)}_{\rm eff}(0)\, | i_1 \mathbf{k} \tau_2 \rangle = \frac{2 m_{i_1}^2}{\mathcal{K}^0_{i_1}}\, {\rm Re}(h_\nu^\dagger h_\nu)_{i_1 i_1}\, \delta_{\tau_1 \tau_2} \phi_{\rm I}(m_{i_1}^2) \,, \tag{3.11}$$

agreeing with refs. [20, 21]. Flavour non-diagonal parts are non-zero and can be worked out from eqs. (A.16), (A.17); in the non-relativistic regime, $k \ll m_i$, the spin-part coming from γ^5 is seen to be suppressed, and the flavour structure is given by the first term of eq. (3.7).

4. Summary

Addressing the full problem of leptogenesis is demanding: although non-perturbative *equations* have been written down, no general systematic *solution* is known even to leading order in Standard Model couplings. However, simpler subproblems can be studied in a controlled fashion, and hopefully a similar progress ultimately permeates the whole topic.

In this note, one simple subproblem has been considered, that of $\mathcal{O}(h_\nu^\dagger h_\nu)$ flavour transitions and inclusive decays of heavy right-handed neutrinos in the non-relativistic regime ($m_\text{top} \lesssim \pi T \ll M$). In particular, the NLO corrections to the right-handed neutrino self-energy matrix have been determined, cf. eqs. (2.13)–(2.15). Technically, this requires considering "tensor-type" sum-integrals [26]. Conceivably, generalizing techniques from ref. [34], a similar computation can in the end also be carried out in the relativistic regime.

The main physics conclusion of this study is that in the non-relativistic regime thermal effects are (only) power-suppressed, but nevertheless infrared safe up to the order studied, both as far as mass corrections and decay widths are concerned. Whether this statement also applies to the actual lepton asymmetry generation is, however, unclear at present. A potential obstacle is that OPE techniques [28], which help to understand the structure of the current results as well as the nature of higher-order corrections, are not trivially applicable. The reason is that, particularly in the resonant case, CP-violating observables contain the heavy mass scale M also in internal propagators, not only in external states.

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Appendix A. Conventions for Majorana spinors

For completeness, we specify in this appendix our conventions for right-handed neutrinos. As usual, they can be represented by two-component Weyl spinors (ξ), four-component Majorana spinors ($\tilde{N}$), or right-handed chiral projections of Dirac spinors ($\nu_\text{R} \equiv a_\text{R}\nu$).

Choosing the Weyl representation for the Dirac matrices,

$$\gamma^0 \equiv \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} , \quad \gamma^k \equiv \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} , \quad \gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} , \tag{A.1}$$

where σ_k are the Pauli matrices, the chiral projectors $a_\text{L,R} \equiv (\mathbb{1} \mp \gamma^5)/2$ become diagonal, and the four-component Dirac spinor can be decomposed as

$$\nu = (a_\text{L} + a_\text{R})\nu = \nu_\text{L} + \nu_\text{R} \; , \quad \nu_\text{R} \equiv \begin{pmatrix} 0 \\ \xi \end{pmatrix} . \tag{A.2}$$

The charge conjugation matrix can be defined to be

$$C \equiv i\gamma^2\gamma^0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} , \tag{A.3}$$

and if we set

$$\tilde{N} \equiv \begin{pmatrix} i\sigma_2\xi^* \\ \xi \end{pmatrix} , \quad \bar{\tilde{N}} = \Big(\xi^\dagger \;\; -\xi^T i\sigma_2\Big) , \tag{A.4}$$

then it is easy to see that $\tilde{N}^c \equiv C\bar{\tilde{N}}^T = \tilde{N}$, i.e. that $\tilde{N}$ is a Majorana spinor.

In terms of Dirac spinors, the bare Lagrangian of the right-handed neutrinos reads

$$\mathcal{L} \equiv \bar{\nu}_{\rm R} i\gamma^\mu\partial_\mu\nu_{\rm R} - \Big(\bar{\ell}\, a_{\rm R}\tilde{\phi}\, h_{\nu{\rm B}}\, \nu_{\rm R} + \frac{1}{2}\, \bar{\nu}^c_{\rm R} M_{\rm B}\nu_{\rm R} + \text{H.c.}\Big) . \tag{A.5}$$

Issues related to γ^5 were briefly recalled in ref. [21]. Both $h_{\nu{\rm B}}$ and $M_{\rm B}$ are matrices in flavour space; the Grassmann nature of the fields and the antisymmetry of C imply that the matrix $M_{\rm B}$ is symmetric, $M_{\rm B}^T = M_{\rm B}$. Making use of Weyl spinors, the Lagrangian can be rewritten as

$$\mathcal{L} = \xi^\dagger i\bar{\sigma}^\mu\partial_\mu\xi - \Big(\ell_{\rm L}^\dagger\, \tilde{\phi}\, h_{\nu{\rm B}}\, \xi - \frac{1}{2}\, \xi^T\, i\sigma_2\, M_{\rm B}\, \xi + \text{H.c.}\Big) , \tag{A.6}$$

where $\bar{\sigma}^\mu \equiv (\mathbb{1}, \sigma_k)$, and $\ell_{\rm L}$ is to be interpreted as a 2-component spinor. Finally, for the Majorana representation, eq. (A.4) implies that

$$\bar{\tilde{N}} i\gamma^\mu\partial_\mu a_{\rm R}\tilde{N} = \xi^\dagger i\bar{\sigma}^\mu\partial_\mu\xi , \qquad \bar{\tilde{N}} i\gamma^\mu\partial_\mu a_{\rm L}\tilde{N} = \xi^T i\bar{\sigma}^{\mu T}\partial_\mu\xi^* , \tag{A.7}$$

$$\bar{\tilde{N}} M_{\rm B} a_{\rm R}\tilde{N} = -\xi^T\, i\sigma_2\, M_{\rm B}\, \xi , \quad \bar{\tilde{N}} M_{\rm B}^* a_{\rm L}\tilde{N} = \xi^\dagger\, i\sigma_2\, M_{\rm B}^*\, \xi^* . \tag{A.8}$$

Omitting a total derivative from the kinetic term (or symmetrizing that in eq. (A.6)),

$$\mathcal{L} = \frac{1}{2}\, \bar{\tilde{N}}(i\gamma^\mu\partial_\mu - M_{\rm B} a_{\rm R} - M_{\rm B}^* a_{\rm L})\tilde{N} - \Big(\bar{\ell}\, a_{\rm R}\tilde{\phi}\, h_{\nu{\rm B}}\, \tilde{N} + \text{H.c.}\Big) . \tag{A.9}$$

A pleasant feature of the Majorana formulation is that the free on-shell condition has the form of the usual Dirac equation, $(i\gamma^\mu\partial_\mu - M_{\rm B} a_{\rm R} - M_{\rm B}^* a_{\rm L})\tilde{N} = 0$. If the mass matrix is diagonal, with eigenvalues $m_i \in \mathbb{R}^+$, then field operators of the interaction picture can be expanded as

$$\hat{\tilde{N}}_i(\mathcal{X}) = \int \frac{{\rm d}^3\mathbf{k}}{\sqrt{(2\pi)^3 2\mathcal{K}_i^0}} \sum_\tau \Big(u_{i\mathbf{k}\tau}\hat{a}_{i\mathbf{k}\tau} e^{-i\mathcal{K}_i\cdot\mathcal{X}} + v_{i\mathbf{k}\tau}\hat{a}^\dagger_{i\mathbf{k}\tau} e^{i\mathcal{K}_i\cdot\mathcal{X}} \Big) , \tag{A.10}$$

$$\hat{\bar{\tilde{N}}}_i(\mathcal{X}) = \int \frac{{\rm d}^3\mathbf{k}}{\sqrt{(2\pi)^3 2\mathcal{K}_i^0}} \sum_\tau \Big(\bar{u}_{i\mathbf{k}\tau}\hat{a}^\dagger_{i\mathbf{k}\tau} e^{i\mathcal{K}_i\cdot\mathcal{X}} + \bar{v}_{i\mathbf{k}\tau}\hat{a}_{i\mathbf{k}\tau} e^{-i\mathcal{K}_i\cdot\mathcal{X}} \Big) , \tag{A.11}$$

where $v_{i\mathbf{k}\tau} \equiv C\bar{u}^T_{i\mathbf{k}\tau}$ and $\bar{v}_{i\mathbf{k}\tau} = u^T_{i\mathbf{k}\tau}C$; τ enumerates the spin states; and $\mathcal{K}_i^0 \equiv \sqrt{k^2 + m_i^2}$. The creation and annihilation operators are assumed to satisfy

$$\{\hat{a}_{i_1\mathbf{k}_1\tau_1}, \hat{a}^\dagger_{i_2\mathbf{k}_2\tau_2}\} = \delta_{i_1 i_2}\delta_{\tau_1\tau_2}\delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_2) , \tag{A.12}$$

and the on-shell spinors obey $(\not{\mathcal{K}}_i - m_i)u_{i\mathbf{k}\tau} = (\not{\mathcal{K}}_i + m_i)v_{i\mathbf{k}\tau} = 0$. Writing

$$u_{i\mathbf{k}\tau} \equiv \frac{\not{\mathcal{K}}_i + m_i}{\sqrt{\mathcal{K}_i^0 + m_i}}\,\eta_\tau \;, \quad \eta_\tau \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} |\tau\rangle \\ |\tau\rangle \end{pmatrix} \;, \tag{A.13}$$

where $|\tau\rangle$ are eigenstates of $\hat{\mathbf{n}}\cdot\sigma$ with $\hat{\mathbf{n}}$ an arbitrary unit vector, on-shell spinors obey the same relations as in the Dirac case:

$$-\bar{v}_{i\mathbf{k}\tau_2} v_{i\mathbf{k}\tau_1} = \bar{u}_{i\mathbf{k}\tau_1} u_{i\mathbf{k}\tau_2} \;=\; 2m_i\,\delta_{\tau_1\tau_2} \;, \quad \bar{u}_{i\mathbf{k}\tau_1} v_{i\mathbf{k}\tau_2} = \bar{v}_{i\mathbf{k}\tau_1} u_{i\mathbf{k}\tau_2} = 0 \;, \tag{A.14}$$

$$\bar{u}_{i\mathbf{k}\tau_1}\gamma^0 u_{i\mathbf{k}\tau_2} = \bar{v}_{i\mathbf{k}\tau_1}\gamma^0 v_{i\mathbf{k}\tau_2} \;=\; 2\mathcal{K}_i^0\delta_{\tau_1\tau_2} \;, \quad \bar{u}_{i\mathbf{k}\tau_1}\gamma^0 v_{i-\mathbf{k}\tau_2} = \bar{v}_{i\mathbf{k}\tau_1}\gamma^0 u_{i-\mathbf{k}\tau_2} = 0 \;. \tag{A.15}$$

Making use of $\sum_\tau \eta_\tau\bar{\eta}_\tau = \frac{1}{2}(\mathbb{1}+\gamma^0)$, the usual completeness relations are readily verified: $\sum_\tau u_{i\mathbf{k}\tau}\bar{u}_{i\mathbf{k}\tau} = \not{\mathcal{K}}_i + \mathbb{1}m_i \;, \quad \sum_\tau v_{i\mathbf{k}\tau}\bar{v}_{i\mathbf{k}\tau} = \not{\mathcal{K}}_i - \mathbb{1}m_i$. For different flavours, the representation of eq. (A.13) can be used for showing that

$$\bar{u}_{i_1\mathbf{k}\tau_1}\,u_{i_2\mathbf{k}\tau_2} \;=\; \delta_{\tau_1\tau_2}\left\{\frac{(\mathcal{K}_{i_1}^0 + m_{i_1})(\mathcal{K}_{i_2}^0 + m_{i_2}) - k^2}{\sqrt{\mathcal{K}_{i_1}^0 + m_{i_1}}\sqrt{\mathcal{K}_{i_2}^0 + m_{i_2}}}\right\} \;, \tag{A.16}$$

$$\bar{u}_{i_1\mathbf{k}\tau_1}\,\gamma^5\,u_{i_2\mathbf{k}\tau_2} \;=\; \langle\tau_1|\sigma\cdot\mathbf{k}|\tau_2\rangle\left\{\sqrt{\frac{\mathcal{K}_{i_2}^0 + m_{i_2}}{\mathcal{K}_{i_1}^0 + m_{i_1}}} - \sqrt{\frac{\mathcal{K}_{i_1}^0 + m_{i_1}}{\mathcal{K}_{i_2}^0 + m_{i_2}}}\right\} \;. \tag{A.17}$$

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