Interplay of nuclear physics, effective field theories, phenomenology, and lattice QCD in neutrino physics

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Abstract. Experiments in neutrino physics cover a wide range—from deep inelastic scattering, over long base-line oscillation experiments and low-energy coherent neutrino–nucleus scattering (CEνNS), to searches for neutrinoless double β decay ($0\nu\beta\beta$)—yet in all cases a key aspect in interpreting the results concerns understanding neutrino–nucleus interactions. If the neutrino energy is sufficiently low, the required matrix elements can be constrained in a systematic way by the interplay of effective field theories, phenomenology, and lattice QCD. In these proceedings, we illustrate this strategy focusing on the CEνNS and $0\nu\beta\beta$ processes.

1 Introduction

Neutrinos provide a window into physics beyond the Standard Model (BSM), primarily via the mechanism that generates their masses, and at the same time arguably constitute the sector of the SM that is currently least well understood. In particular, the ordering of neutrino masses and the size of CP violation yet need to be determined. This situation arises as neutrinos are notoriously hard to detect, requiring large-scale detectors to observe a signal. Accordingly, measurement and detection occur in a nuclear/hadronic environment, and the prediction of cross sections requires control over hadronic and nuclear-physics input. While for sufficiently large neutrino energies only data-driven methods are available, for low-energy observables a more systematic approach exists: decomposing the cross section into its different components by means of effective field theory (EFT),

$$\sigma \simeq (\text{short-distance/BSM}) \otimes (\text{hadronic}) \otimes (\text{nuclear}), \tag{1}$$

the (nuclear) matrix elements can be determined by an interplay of phenomenology and lattice QCD, while the dependence on the short-distance part is made explicit in terms of Wilson coefficients. A general overview of the methods that apply for different neutrino energies is provided in Ref. [1]. Here, we focus on CEνNS [2] and $0\nu\beta\beta$ [3], see Sects. 2 and 3, respectively, to illustrate how the required matrix elements can be determined from phenomenology, lattice QCD, or a combination thereof.

2 Coherent elastic neutrino–nucleus scattering

CEνNS was first predicted in 1974 as a consequence of the weak neutral current [4], but only observed recently by the COHERENT collaboration [5, 6]. In analogy to Eq. (1), the

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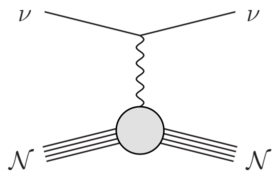

Figure 1. Elastic scattering of a neutrino ν off a nucleus $\mathcal{N}$.

different scales that contribute to the process are most efficiently taken into account by an EFT decomposition [7]

$$\text{Rate} = \text{(B)SM couplings} \otimes \text{hadronic matrix elements} \otimes \text{nuclear structure} \otimes \text{neutrino flux}, \quad (2)$$

with the rate determined by short-distance couplings, hadronic matrix elements, and nuclear structure factors, which are finally convolved with the neutrino flux (either from a reactor or of astrophysical origin). In case a light mediator is present, a similar decomposition applies, provided the dependence on the momentum transfer is adjusted accordingly. Moreover, the EFT formalism [7] was first developed for the direct detection of dark matter [8–15], which involves the same nuclear responses as become relevant in CEνNS.

The effective operators that describe CEνNS at the level of quarks and gluons are

$$\begin{aligned}
\mathcal{L}^{\text{SM}} &= \sum_q \Big(C_q^V \bar{\nu}\gamma^\mu P_L \nu\, \bar{q}\gamma_\mu q + C_q^A \bar{\nu}\gamma^\mu P_L \nu\, \bar{q}\gamma_\mu\gamma_5 q \Big), \\
\mathcal{L}^{\text{BSM}} &= C_F \bar{\nu}\sigma^{\mu\nu} P_L \nu F_{\mu\nu} \\
&\quad + \sum_q \Big(C_q^T \bar{\nu}\sigma^{\mu\nu} P_L \nu\, \bar{q}\sigma_{\mu\nu} q + C_q^S \bar{\nu} P_L \nu\, m_q \bar{q} q + C_q^P \bar{\nu} P_L \nu\, m_q \bar{q} i\gamma_5 q \Big) + \cdots ,
\end{aligned} \quad (3)$$

where we separated the (axial-)vector operators from dipole, tensor, and (pseudo-)scalar interactions, as the former are already present in the SM, while the latter may occur in SM extensions. To be able to calculate a rate, these quark-level operators must be converted to the level of hadrons, which due to confinement requires non-perturbative input. Possible strategies to determine these hadronic matrix elements include: (i) Direct phenomenological determinations are possible for physical flavor combinations of (axial-)vector currents, an example for which will be given in Sect. 2.1. (ii) EFT constraints can yield powerful relations, e.g., the Cheng–Dashen theorem [16, 17] for the scalar–isoscalar operator yields a connection between a scalar matrix element and observables, see Sect. 2.2. (iii) Ward identities allow one to relate (pseudo-)scalar matrix elements to (axial-)vector ones by current conservation [18–21], while (iv) unitarity, e.g., constrains the momentum dependence of tensor matrix elements [22–25]. In principle, all matrix elements are accessible in (v) lattice QCD [26], but of course benchmarks in cases in which alternative methods are available are critical.

	$g_A^{u,p}$	$g_A^{d,p}$	$g_A^{s,p}$
Ref. [28]	0.842(12)	−0.427(13)	−0.085(18)
Ref. [29]	0.847(37)	−0.407(24)	−0.035(9)
Ref. [30]	0.777(39)	−0.438(35)	−0.053(8)

Table 1. Axial-vector charges of the nucleon, from lattice QCD [29, 30] and phenomenology [28].

2.1 Axial-vector matrix elements

The conventional decomposition of the nucleon matrix elements of the axial-vector, pseudoscalar, and gluon currents into form factors reads

$$\begin{aligned}
\langle N(p')|\bar{q}\gamma^\mu\gamma_5 q|N(p)\rangle &= \bar{u}(p')\Big[\gamma^\mu\gamma_5 G_A^{q,N}(t) + \gamma_5\frac{q^\mu}{2m_N}G_P^{q,N}(t) + \frac{i\sigma^{\mu\nu}}{2m_N}q_\nu\gamma_5 G_T^{q,N}(t)\Big]u(p),\\
\langle N(p')|m_q\bar{q}i\gamma_5 q|N(p)\rangle &= m_N\bar{u}(p')i\gamma_5 G_5^{q,N}(t)u(p),\\
\langle N(p')|\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}_a^{\mu\nu}|N(p)\rangle &= 2m_N\bar{u}(p')i\gamma_5 G^N_{G\tilde{G}}(t)u(p),
\end{aligned}\tag{4}$$

written as a function of momentum transfer $q^2 = t$, $q = p' - p$. The direct axial-vector form factor defines the axial-vector charges $G_A^{q,N}(0) \equiv g_A^{q,N} \equiv \Delta q^N$, the induced pseudoscalar form factor is labeled by $G_P^{q,N}(t)$, and the second-class tensor form factor $G_T^{q,N}(t)$ can induce G-parity-breaking corrections for β decays [27], but plays no role in the following. The pseudoscalar, $G_5^{q,N}(t)$, and gluon, $G^N_{G\tilde{G}}(t)$, form factors are related to the axial-vector ones by the Ward identity

$$\begin{aligned}
\partial_\mu\bar{q}\gamma^\mu\gamma_5 q &= 2im_q\bar{q}\gamma_5 q - \frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}_a^{\mu\nu},\\
G_A^{q,N}(t) + \frac{t}{4m_N^2}G_P^{q,N}(t) &= G_5^{q,N}(t) - G^N_{G\tilde{G}}(t),
\end{aligned}\tag{5}$$

in particular, the charges—$G_5^{q,N}(0) = g_5^{q,N}$, $\tilde{a}_N = 2m_N G^N_{G\tilde{G}}(0)$—fulfill the relation $g_A^{q,N} = g_5^{q,N} - \frac{\tilde{a}_N}{2m_N}$.

The current knowledge of the axial charges $g_A^{q,N}$ is summarized in Table 1. First, at the current level of precision one can assume isospin symmetry

$$g_A^{u,p} = g_A^{d,n}, \qquad g_A^{d,p} = g_A^{u,n}, \qquad g_A^{s,p} = g_A^{s,n}. \tag{6}$$

The triplet component is already known very precisely from neutron β decay, $g_A^{u,p} - g_A^{d,p} = g_A = 1.27641(56)$ [31], although yet higher precision is required in the context of first-row CKM unitarity [32]. The octet component can be constrained from hyperon decays, while the singlet needs to be extracted from spin structure functions [28]. The comparison to different lattice-QCD calculations [29, 30], as shown in Table 1, reflects the current uncertainties in our knowledge of the axial-vector charges of the nucleon. Finally, to determine the pseudoscalar charges $g_5^{q,N}$ via the Ward identity, input for the gluon coupling $\tilde{a}_N$ is required. Currently, only large-N_c estimates are available, $\tilde{a}_N = -0.39(12)$ GeV [33], so that a direct lattice-QCD calculation would be highly welcome.

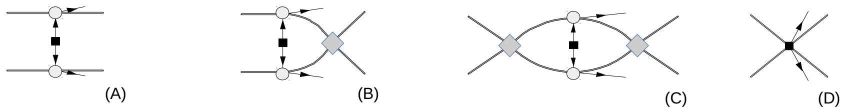

Figure 2. Diagrams for $nn \to ppe^-e^-$ in chiral EFT, with the double lines referring to nucleons, single lines to leptons, and the black square to the insertion of $m_{\beta\beta}$. The contact term (D) is required for the renormalization of diagram (C).

2.2 Scalar matrix elements

The scalar matrix elements are defined by

$$\langle N(p')|m_q\bar{q}q|N(p)\rangle = m_N f_q^N(t)\bar{u}(p')u(p), \qquad f_q^N \equiv f_q^N(0), \tag{7}$$

with charges often expressed in terms of σ terms, $\sigma_{\pi N} = m_N(f_u^N + f_d^N)$, $\sigma_s = m_N f_s^N$. In this case, the Ward identity does not help, as only off-diagonal charges are related to the vector matrix elements [21]. Alternative strategies include the Cheng–Dashen theorem [16, 17], which establishes a connection of $\sigma_{\pi N}$ to a pion–nucleon (πN) scattering amplitude extrapolated to subthreshold kinematics, as well as chiral perturbation theory (ChPT) for σ_s to obtain a relation to baryon masses [34]. While the latter requires SU(3) assumptions in the baryon sector, subject to large higher-order corrections that are difficult to control, the connection of $\sigma_{\pi N}$ to πN scattering has allowed for precision extractions [35, 36], making use of dispersion relations to control the analytic continuation [37–39] as well as precise input on the scattering lengths from pionic atoms [40–44], including effects from isospin-breaking corrections [45–48]. Further, matching to ChPT determines the low-energy constants c_i that are required to evaluate two-body corrections in the axial-vector current [49, 50], which, in turn, becomes relevant at subleading orders for CEνNS as well [7].

The phenomenological values for $\sigma_{\pi N}$ obtained in this way can then be compared to lattice QCD [26], serving as an important benchmark for nucleon matrix elements. So far, most lattice-QCD calculations have produced significantly smaller values, both using the Feynman–Hellmann theorem [51–53] and direct calculations of the three-point function [54, 55]. Recently, a possible solution in terms of larger-than-expected excited-state contamination has been put forward [56, 57], supported by an analysis of such excited-state effects in ChPT, but improved lattice-QCD calculations are required to render this explanation conclusive.

3 Neutrinoless double β decay

The search for $0\nu\beta\beta$ decay is particularly interesting as it is the only experimental probe that can distinguish between a Majorana and a Dirac mass term and thus shed light on the origin of the neutrino mass. In case a signal were observed and the process mediated by a light Majorana-neutrino exchange, the measured $0\nu\beta\beta$ half life $T_{1/2}$ would also allow one to extract the combination of neutrino masses

$$m_{\beta\beta} = \bigg|\sum_k m_k U_{ek}^2\bigg| = \Big|m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i(\alpha_2-\alpha_1)} + m_3|U_{e3}|^2 e^{-i(\alpha_1+2\delta)}\Big|, \tag{8}$$

and thereby become sensitive to the neutrino mass ordering as well as the Majorana phases α_i. The sensitivity of next-generation experiments is summarized, e.g., in Ref. [58].

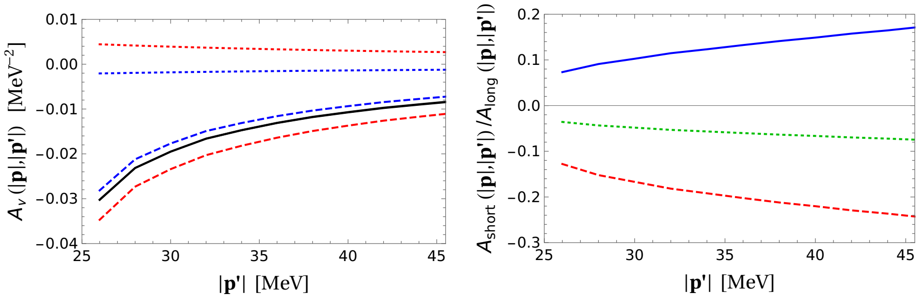

Figure 3. $nn \to ppe^-e^-$ amplitude A_ν as a function of the final-state NN relative momentum $|\boldsymbol{p}'|$, for initial-state relative momentum $|\boldsymbol{p}| = 25\,\text{MeV}$. Left: long-distance (dashed), short-distance (dotted), and total (solid) contribution for two different regulators. Right: ratio of short-to-long distance amplitudes for different regulators. Figures taken from Ref. [79].

3.1 Light Majorana-neutrino exchange

To get from $T_{1/2}$ to $m_{\beta\beta}$, however, the nuclear matrix elements need to be provided as input. Recent years have witnessed significant progress in ab-initio nuclear theory [59–61], which helps control the nuclear-structure aspects. At the same time, a systematic EFT treatment revealed that even for the light Majorana-neutrino exchange a new contact term is required at leading order in the chiral expansion to ensure renormalization of the two-nucleon amplitude $nn \to ppe^-e^-$ [62–64], see Fig. 2. Diagram (D) involves a new low-energy constant $\tilde{C}_1$, whose value is a priori not known. While work is ongoing to determine this amplitude in lattice QCD [65, 66], here we summarize a phenomenological estimate using the Cottingham approach [67].

The basic idea amounts to starting from the amplitude $T_{\mu\nu}$ involving two external currents interacting with a hadronic system, and then closing the loop by connecting the currents. In the electromagnetic case, this strategy allows one to estimate self energies of mesons [68–73] and nucleons [74–77], with numerical results typically dominated by elastic intermediate states. In Refs. [78, 79] this strategy was generalized to the two-nucleon (NN) system, including the case of two weak currents. In the end, this approach allows one to incorporate the known momentum dependence of single-nucleon form factors and the NN amplitude.

A similar contact term describes charge-independence breaking (CIB) in NN scattering, which thus presents an opportunity to validate the results for the $0\nu\beta\beta$ contact operator. Indeed, the CIB combination of NN scattering lengths is reproduced at the quoted level of precision,

$$a_{\text{CIB}} = \frac{a_{nn} + a_{pp}^C}{2} - a_{np} \overset{\text{exp}}{=} 10.4(2)\,\text{fm}, \qquad a_{\text{CIB}}|_{\text{Cottingham}} = 15.5^{+4.5}_{-4.0}\,\text{fm}, \tag{9}$$

where the experimental number for a_{CIB} comes from the combination of Refs. [80–84]. The result for $\tilde{C}_1$ depends on renormalization scale and scheme, in such a way that to be able to incorporate this short-distance effect into nuclear-structure calculations a matching needs to occur at the level of renormalized quantities. To this end, synthetic data for the $nn \to ppe^-e^-$ amplitude were provided in Ref. [79] on a grid of points, see Fig. 3. A first implementation in ^{48}Ca indicates an increase in the matrix element by $\simeq 40\%$ [85], reflecting an enhanced effect from short-range contributions due to a node in the nuclear wave function [63, 64].

	$\langle\pi^+\vert O_1\vert\pi^-\rangle$ $[10^{-4}\,\mathrm{GeV}^4]$	$\langle\pi^+\vert O_2\vert\pi^-\rangle$ $[10^{-2}\,\mathrm{GeV}^4]$	$\langle\pi^+\vert O_3\vert\pi^-\rangle$ $[10^{-2}\,\mathrm{GeV}^4]$	$\langle\pi^+\vert O_4\vert\pi^-\rangle$ $[10^{-2}\,\mathrm{GeV}^4]$	$\langle\pi^+\vert O_5\vert\pi^-\rangle$ $[10^{-2}\,\mathrm{GeV}^4]$
Ref. [86]	1.0(1)(2)	−2.7(3)(5)	0.9(1)(2)	−2.6(8)(8)	−11(2)(3)
Ref. [87]	0.93(5)	−1.89(16)	0.62(6)	−1.89(13)	−7.81(54)
Ref. [88]	0.50(2)	−1.44(8)	0.39(3)	−1.48(10)	−6.26(33)

Table 2. Short-distance matrix elements for $\pi^- \to \pi^+ e^- e^-$ at scale $\mu_{\overline{\mathrm{MS}}} = 3\,\mathrm{GeV}$, from lattice QCD [87, 88] and using SU(3) relations [86].

3.2 Heavy mediators

In the case of heavy mediators the quark-level operators take the form [86]

$$O_1 = \bar{q}_L^\alpha \gamma^\mu \tau^+ q_L^\alpha\, \bar{q}_L^\beta \gamma_\mu \tau^+ q_L^\beta, \qquad O_2 = \bar{q}_R^\alpha \tau^+ q_L^\alpha\, \bar{q}_R^\beta \tau^+ q_L^\beta, \ldots \tag{10}$$

Contrary to the light Majorana-neutrino exchange, estimates for the $nn \to ppe^-e^-$ matrix elements are not yet available, but results exist for the pion analog $\pi^- \to \pi^+ e^- e^-$. In Table 2, we compare the results from Ref. [86], obtained from K^0–$\bar{K}^0$ and $K \to \pi\pi$ matrix elements by means of SU(3) symmetry, with more recent lattice-QCD calculations [87, 88]. In most cases there is reasonable agreement within the assigned SU(3) uncertainties, while at the quoted level of precision tensions become visible between the two lattice-QCD calculations, especially for the matrix element of O_1.

4 Conclusions

In these proceedings we discussed two examples in which the required theory input for the interpretation of neutrino experiments can be determined from a combination of EFTs, phenomenology, and lattice QCD. First, the EFT decomposition for CEνNS depends on a set of nucleon matrix elements, and we reviewed their determination for axial-vector and scalar operators, highlighting the need for benchmarking lattice-QCD calculations wherever possible, as for the case of non-standard interactions in many cases alternative determinations are difficult. A similar EFT decomposition applies to $0\nu\beta\beta$ decays, in this case, the most uncertain few-nucleon matrix elements parameterize the role of short-range contributions. They are, in principle, accessible in lattice QCD, but extracting $nn \to ppe^-e^-$ at the physical point is a challenging endeavor. As a benchmark for future such calculations, we discussed a phenomenological estimate based on the Cottingham approach, validated with CIB effects in the NN scattering lengths.

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