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Determination of the light, strange and charm quark masses using twisted mass fermions

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We present results for the light, strange and charm quark masses using $N_f = 2+1+1$ twisted mass fermion ensembles at three values of the lattice spacing, including two ensembles simulated with the physical value of the pion mass. The analysis is done both in the meson and baryon sectors. The difference in the mean values found in the two sectors is included as part of the systematic error. The presentation is based on the work of Ref. [1], where more details can be found.

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1. Introduction

Quark masses are crucial inputs for the phenomenological description of the plethora of phenomena governed by the strong nuclear force. We use several ensembles simulated using the twisted mass fermion action at three values of the lattice spacing and spanning pion masses in the range from about 350 MeV to 135 MeV. This enable us to perform a combined chiral and continuum extrapolation. The properties of the ensembles used in this work are summarized in Table 1. We will refer to the ensembles in Table 1 with the names starting with cA in their names as A ensembles, those starting with cB as B ensembles and those with cC as C ensembles.

In order to avoid undesired $O(a^2)$ mixing of the strange and charm flavours in our physical observables, we adopt a non-unitary lattice setup [2], where the twisted-mass action for non-degenerate strange and charm quarks is employed only in the sea sector, while the valence strange and charm quarks that enter the correlation functions are regularized as exactly flavour-diagonal Osterwalder-Seiler fermions [3].

Ensemble	$L^3 \times T$	$a\mu_\ell$	am_π	af_π	$m_\pi L$	m_N/m_π	m_π [MeV]
$\beta = 1.726$, $c_{SW} = 1.74$, $a\mu_\sigma = 0.1408$, $a\mu_\delta = 0.1521$, $w_0/a = 1.8352\,(35)$							
cA211.53.24	$24^3 \times 48$	0.00530	0.16626 (51)	0.07106 (36)	3.99	–	346.4 (1.6)
cA211.40.24	$24^3 \times 48$	0.00400	0.14477 (70)	0.06809 (30)	3.47	–	301.6 (2.1)
cA211.30.32	$32^3 \times 64$	0.00300	0.12530 (16)	0.06674 (15)	4.01	4.049 (14)	261.1 (1.1)
cA211.12.48	$48^3 \times 96$	0.00120	0.08022 (18)	0.06133 (33)	3.85	5.685 (28)	167.1 (0.8)
$\beta = 1.778$, $c_{SW} = 1.69$, $a\mu_\sigma = 0.1246864$, $a\mu_\delta = 0.1315052$, $w_0/a = 2.1299\,(16)$							
cB211.25.32	$32^3 \times 64$	0.00250	0.10475 (45)	0.05652 (38)	3.35	4.104 (36)	253.3 (1.4)
cB211.25.48	$48^3 \times 96$	0.00250	0.10465 (14)	0.05726 (12)	5.02	4.124 (17)	253.0 (1.0)
cB211.14.64	$64^3 \times 128$	0.00140	0.07848 (10)	0.05477 (12)	5.02	5.119 (36)	189.8 (0.7)
cB211.072.64	$64^3 \times 128$	0.00072	0.05659 (8)	0.05267 (14)	3.62	6.760 (30)	136.8 (0.6)
$\beta = 1.836$, $c_{SW} = 1.6452$, $a\mu_\sigma = 0.106586$, $a\mu_\delta = 0.107146$, $w_0/a = 2.5045\,(17)$							
cC211.20.48	$48^3 \times 96$	0.00200	0.08540 (17)	0.04892 (13)	4.13	4.244 (25)	245.73 (98)
cC211.06.80	$80^3 \times 160$	0.00060	0.04720 (7)	0.04504 (10)	3.78	6.916 (19)	134.3 (0.5)

Table 1: Parameters of the $N_f = 2 + 1 + 1$ ensembles analyzed in this study. In the first column we give the name of the ensemble, in the second the lattice volume, in the third the twisted-mass parameter, $a\mu_\ell$, for the average up/down (light) quark, in the fourth and in the fifth the pion mass am_π and decay constant af_π in lattice units from Ref. [4], in the sixth the pion mass times the lattice spatial length, $m_\pi L$, in the seventh the ratio m_N/m_π as determined in Section 4 and, finally, in the last column the pion mass in physical units, using our determination of the gradient-flow scale w_0 obtained in Ref. [4]. We also include for each set of ensembles with the same lattice spacing the coupling constant β, the clover-term parameter c_{SW}, the parameters of the non-degenerate operator $a\mu_\sigma$ and $a\mu_\delta$, related to the renormalized strange and charm sea quark masses [2], and the value of the gradient-flow scale w_0/a determined at the physical pion mass in Ref. [4].

A new feature of this work is the use of two sets of observables to set the scale and to evaluate the quark masses enabling us to study systematic effects in the determination of the quark masses using different inputs. One set of observables is based on quantities from the meson sector, namely we use the pion mass and decay constant to set the scale and to determine the average up/down quark mass, referred thereafter as light quark mass, and the kaon and D-meson masses for the determination of the mass of the strange and charm quarks, respectively. In the baryon sector, the

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nucleon and pion masses are used to set the scale and light quark mass and the Ω^- and the Λ_c masses are used to determine the strange and charm quark masses, respectively. In our analysis in the baryon sector we restricted ourselves to using gauge ensembles simulated with pion masses less than 300 MeV.

Another feature of this work is the improved determination of the renormalization factor Z_P. In the maximally twisted-mass formulation used here the renormalized quark mass is given by $m_q = \mu_q/Z_P$ and it is, thus, a crucial input for determining the quark masses. Our approach to compute Z_P is described in more detail in Ref. [1] and presented at this conference in Ref. [5].

2. Determination of the lattice spacing

Meson sector. We use the iso-symmetric values of the pion mass and decay constant, given respectively by [6],

$$m_\pi^{isoQCD} = 135.0(2)\text{ MeV} \quad \text{and} \quad f_\pi^{isoQCD} = 130.4(2)\text{ MeV} . \tag{1}$$

NLO SU(2) chiral perturbation theory is employed, to correct for volume effects and take the continuum limit of m_π and f_π in units of w_0. Using w_0/a computed for for each gauge ensemble we extrapolate to the physical pion mass and continuum limit. We find $w_0 =$ 0.17383(63) fm [4] and using this value we determine the three lattice spacings. Details are given in Ref. [4].
Baryon sector. We use the iso-symmetric values of the pion and nucleon mass, m_N^{isoQCD}= 0.938 GeV and SU(2) chiral perturbation theory to one-loop

$$(a_i m_N) = a_i m_N^0 - 4c_1 \frac{(a_i m_\pi)^2}{a_i} - \frac{3g_A^2}{16\pi f_\pi^2}\frac{(a_i m_\pi)^3}{a_i^2}, \tag{2}$$

where a_i are the three lattice spacings, m_N^0 is the nucleon mass at the chiral limit and c_1 is fixed using the value of m_N^{isoQCD}. The axial charge g_A is set to its physical value of $g_A = 1.27641(56)$.

The values of the lattice spacing extracted from the pion sector and from the nucleon mass differ by $O(a^2)$ effects. Fitting their difference as a function of a^2, as shown in Fig. 1, we observe that in the continuum limit the difference vanishes, as expected.

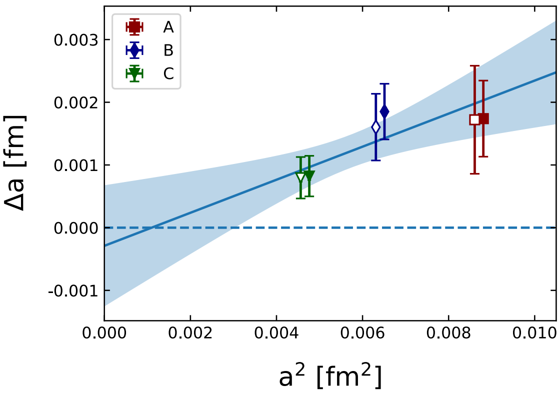
3. Determination of quark masses in the meson sector

To determine the light quark mass, we use SU(2) chiral perturbation theory (ChPT) for m_π and f_π given by

$$(m_\pi w_0)^2 = 2(Bw_0)(m_\ell w_0)\left[1 + \xi_\ell \log \xi_\ell + P_1\xi_\ell + P_2\, a^2/w_0^2\right] K_{M^2}^{FSE} \tag{3}$$

$$(f_\pi w_0) = (f w_0)\left[1 - 2\xi_\ell \log \xi_\ell + P_3\xi_\ell + P_4\, a^2/w_0^2 + a^2 m_\ell P_5\right] K_f^{FSE}, \tag{4}$$

where $\xi_\ell = \frac{2Bm_\ell}{(4\pi f)^2}$, $P_1 = -\bar{\ell}_3 - 2\log\left(m_\pi^{\text{isoQCD}}/(4\pi f)\right)$, $\quad P_3 = 2\bar{\ell}_4 + 4\log\left(m_\pi^{\text{isoQCD}}/(4\pi f)\right)$ and the quantities $K_{M^2}^{FSE}$ and K_f^{FSE} represent the finite size effects (FSE) on the squared pion mass and the pion decay constant, respectively.



Sector	a_A [fm]	a_B [fm]	a_C [fm]
Pion	0.09471(39)	0.08161(30)	0.06942(26)
Nucleon	0.09295(47)	0.07975(32)	0.06860(20)
Δa	0.00176(61)	0.00186(44)	0.00082(32)

Figure 1: Left: The difference Δa between the lattice spacings determined from the pion sector and the nucleon mass versus a^2. Full symbols are the lattice spacings determined using all the ensembles for which $m_\pi < 260$ MeV. Open symbols, shifted to the left for clarity, are obtained using ensembles for which the pion mass is below 190 MeV. The solid line shows the linear fit in a^2 to the results extracted by using ensembles with $m_\pi < 260$ MeV (full symbols), which is largely consistent with zero in the continuum limit. Right: The values of the lattice spacings.

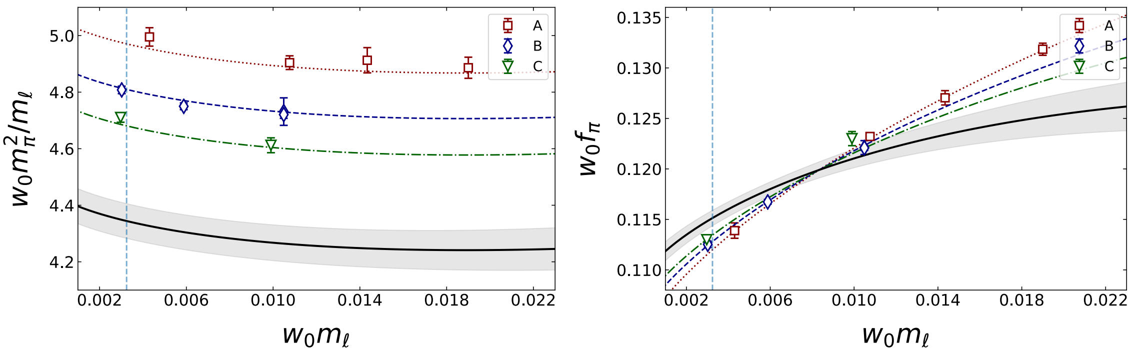


Figure 2: Chiral and continuum extrapolation of $w_0 m_\pi^2/m_\ell$ (left) and $w_0 f_\pi$ (right) as function of $w_0\, m_\ell$ using Eqs. (3) and (4) and Z_P for the M2b method. Different colored bands correspond to different lattice spacings (red for the A ensembles, blue for the B and green for the C). The grey band is the extrapolation to the continuum limit. Note that for $w_0 f_\pi$ discretization effects proportional both to a^2 and $a^2 m_\ell$ are visible.

We use the kaon mass $m_K^{\rm isoQCD} = 494.2(3)$ MeV as input for fixing the strange quark mass. To determine the charm quark mass, we use both the mass of the D- and D$_s$-mesons, $m_D^{\rm isoQCD} = 1867.0(4)$ MeV and $m_{D_s}^{\rm isoQCD} = 1969.0(4)$ MeV, respectively. We use three reference values for the strange and charm quark mass for all ensembles and interpolate linearly using $m_{K,D}^2 = a + b m_{s,c} s w_0$. In the case of the strange quark mass we use the NLO ChPT inspired Ansatz [7]

$$(m_K w_0)^2 = P_0(m_\ell w_0 + m_s w_0)\left[1 + P_1 m_\ell w_0 + P_2 m_\ell^2 w_0^2 + P_3\, a^2/w_0^2\right] . \tag{5}$$

In the case of the charm quark mass, given the weak dependence of D, D_s meson masses on m_ℓ, we use

$$m_{D,D_s} = P_0^{D,D_s} + P_1^{D,D_s} m_\ell w_0 + P_2^{D,D_s} a^2/w_0^2 . \tag{6}$$

The results of the chiral and continuum extrapolations are shown in Fig. 3.

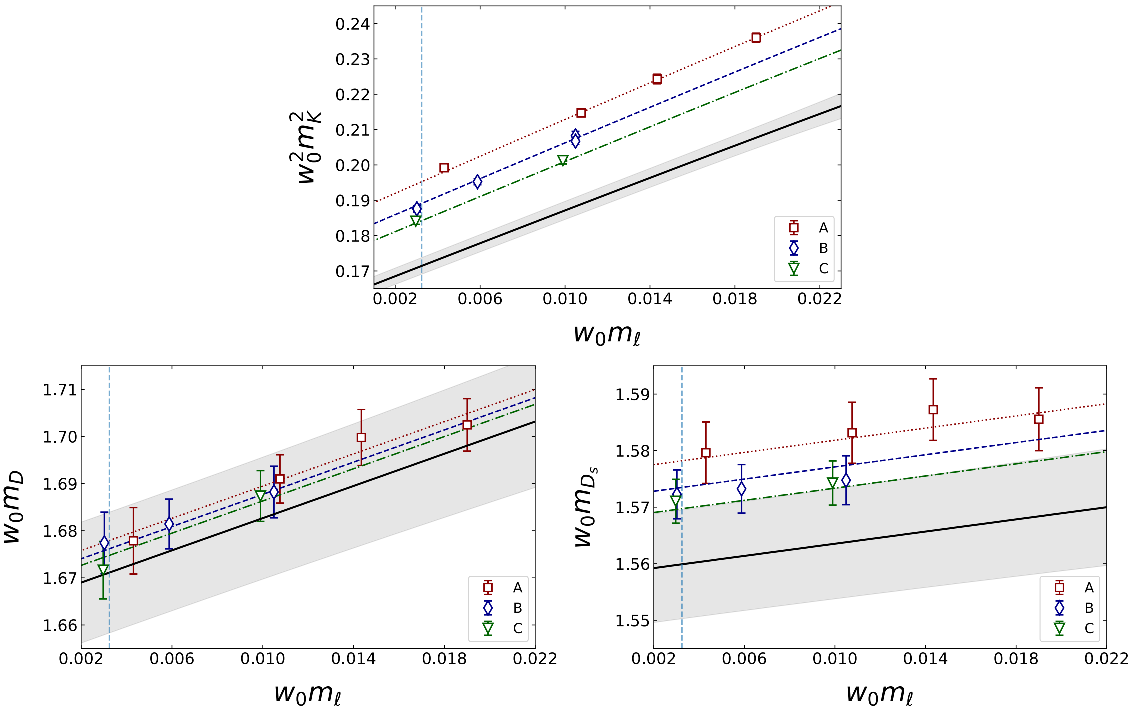


Figure 3: The red, blue and green solid lines show the resulting fits using Eq. (5) and Eq. (6) for the three ensembles A, B and C, respectively. The gray line shows the continuum extrapolation, for the determination of m_s (top) and m_c using the mass of the D-meson (left) and the mass of the D_s-meson (right).

4. Determination of quark masses in the baryon sector

For the determination of the light quark mass, we use the nucleon mass as an input and fit to the ChPT expression of Eq. (2) to extrapolate to the physical point. To one-loop order in ChPT (up to which the nucleon mass is expanded in Eq. (2)) we can parametrize the pion mass by $m_\pi^2 = 2Bm_{ud}(1 + c_2 a^2)$ obtaining the expansion

$$m_N(m_{ud}) = m_N^0 - 4c_1 \left(2Bm_{ud}(1 + c_2 a^2)\right) - \frac{3g_A^2}{16\pi f_\pi^2} \left(2Bm_{ud}(1 + c_2 a^2)\right)^{3/2}, \tag{7}$$

consistently with the order we are working and including $O(a^2)$ effects in the pion expansion with the coefficient c_2. We thus have two fit parameters, B and c_2, while the lattice spacings, m_N^0 and c_1 are determined from Eq. (2).

We determine the strange and charm quark masses using the experimental value of the $\Omega\,(sss)$ and $\Lambda_c\,(udc)$ masses and the lattice spacings obtained from the nucleon mass. Namely, we use $m_\Omega^{(phys.)} = 1672.5(3)$ and $m_{\Lambda_c}^{(phys)} = 2286.5(1)$ from the PDG [8]. We parametrize the Ω^- and Λ_c mass dependence on the strange and charm quark mass by expanding around $\tilde{m}_s$ and $\tilde{m}_c$, in the vicinity of the physical quark masses, using

$$m_\Omega = A_\Omega + B_\Omega\,(m_s - \tilde{m}_s), \tag{8}$$

$$m_{\Lambda_c} = A_{\Lambda_c} + B_{\Lambda_c}\,(m_c - \tilde{m}_c)\,. \tag{9}$$

We employ two methods to determine m_s and m_c: In method I we perform a chiral and continuum extrapolation of the A_{Ω,Λ_c} and B_{Ω,Λ_c} parameters separately using $A_{\Omega,\Lambda_c}(a, m_\pi^2) = c_1 + c_2 m_\pi^2 + c_3 a^2$ and an equivalent expression for B_{Ω,Λ_c}. In method II we adopt an iterative strategy: Namely, we start by fixing a value of the renormalized strange quark mass m_s in physical units for all the ensembles and then we extrapolate to the continuum limit and physical point using the ChPT result

$$m_{\Omega,\Lambda_c} = m^{(0)}_{\Omega,\Lambda_c} - 4c^{(1)}_{\Omega,\Lambda_c} m_\pi^2 + d^{(2)}_{\Omega,\Lambda_c} a^2 \,. \tag{10}$$

We iterate this procedure changing the value of $m_s(m_c)$ until the resulting value of m_{Ω,Λ_c} given in Eq. (10) at the physical point and continuum limit matches the physical value $m_\Omega^{(phys.)}$ $(m_{\Lambda_c}^{(phys.)})$.

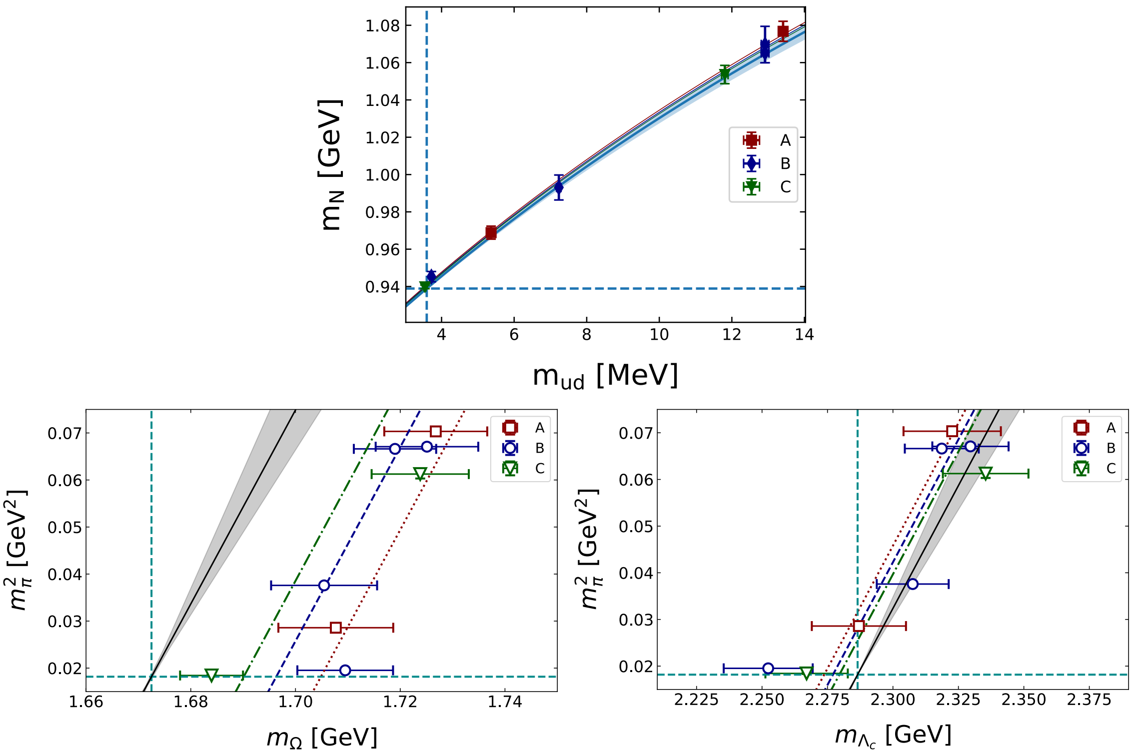


Figure 4: Top: The nucleon mass m_N for the A- (red), B- (blue) and C- (green) ensembles. The blue band shows the continuum extrapolation according to Eq. (7). Bottom: We show m_Ω (Λ_c) versus m_π^2, when m_s (m_c) takes the value that reproduces the physical mass of the Ω (Λ_c) at the continuum limit as described in method II. The dotted lines show the chiral extrapolation for the A- (red), B- (blue) and C- (green) ensembles. The solid black line shows the continuum extrapolation using Eq. (10) with the associated error (grey band). The horizontal and vertical dashed light blue lines represent, respectively, the physical pion and Ω (Λ_c) masses.

5. Results and Conclusions

In Table 2 we collect the values of the quark masses obtained in Sections 3 and 4 for the light and strange quark masses in the $\overline{\text{MS}}$ scheme at 2 GeV and for the charm quark mass at 3 GeV. The final results are given in the row labeled "Average" of Table 2 and are compared in Fig. 5 with those of the ETM analysis of Ref. [10] and the ones entering the $N_f = 2 + 1 + 1$ averages in the latest FLAG report [9]. Our results are larger by ~ 2.5 standard deviations in the case of m_{ud} and by ~ 2

	m_{ud} [MeV]	m_s [MeV]	m_c [MeV]	m_s/m_{ud}	m_c/m_s
Meson sector	3.689(80)(66)	101.0(1.9)(1.4)	1039(15)(8)	27.30(24)(14)	11.43(9)(10)
Baryon sector	3.608(58)$(^{+32}_{-19})$	94.9(2.4)$(^{+4.1}_{-1.0})$	1030(21)$(^{+22}_{-5})$	26.30(61)$(^{+1.17}_{-0.33})$	12.04(31)$(^{+58}_{-15})$
Average	3.636(66)$(^{+60}_{-57})$	98.7(2.4)$(^{+4.0}_{-3.2})$	1036(17)$(^{+15}_{-8})$	27.17(32)$(^{+56}_{-38})$	11.48(12)$(^{+25}_{-19})$
FLAG 2019	3.410(43)	93.44(68)	988(7)	27.23(10)	11.82(16)

Table 2: The renormalized quark masses determined in the meson sector (first row) and baryon sector (second row) in the $\overline{\text{MS}}$ scheme. In the third row we give the average over the values obtained in the meson and baryon sectors, while in the last row we give the latest FLAG averages [9] for $N_f = 2 + 1 + 1$.

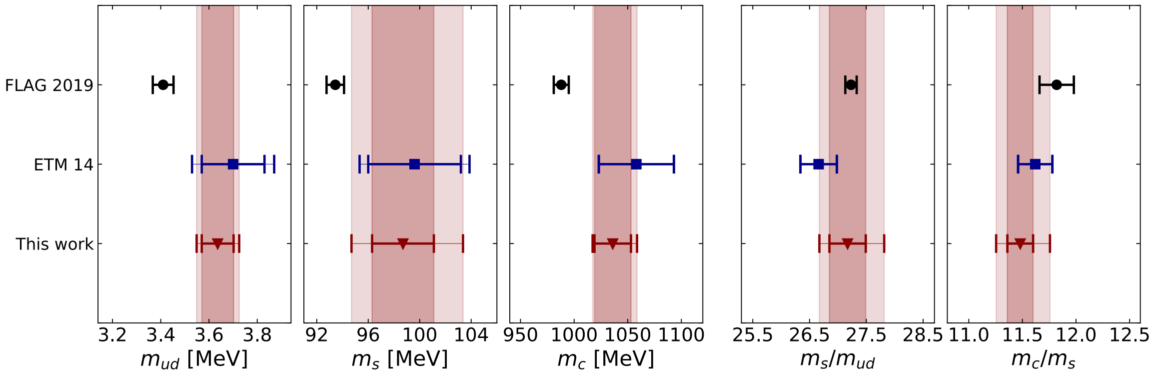


Figure 5: Comparison of the results average between the values determined in the meson and baryon sectors (red triangles) with the values obtained using twisted mass fermions in Ref. [10] (blue squares) and the $N_f = 2 + 1 + 1$ averages given in the last FLAG report [9] (black circles). The shorter error bars take into account the statistical error only, while the larger represent the total error, obtained by summing in quadrature the statistical and the systematic errors.

standard deviations in the case of m_c with respect to the corresponding FLAG values. The strange quark mass tends to also be larger, although, within the larger final error, deviates less from the FLAG result. A very good agreement is observed for the mass ratios m_s/m_{ud} and m_c/m_s and the ones reported by FLAG.

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