Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery

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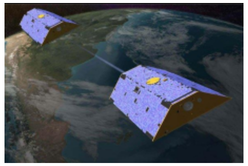
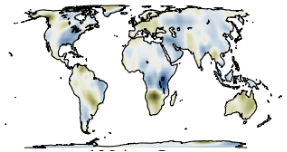
Austin, TX, USA

Operational processing





Basic parametrisation

- initial conditions 2x[6]
- accelerometer bias 2x[3]
- accelerometer scaling 2x[3]

parameters per arc 24

Additional parameters

- 15 min PCA per satellite in
  - radial 2x[96]
  - along-track 2x[96]
  - cross-track 2x[96]

parameters per arc 576

in daily arcs (30 days):

- 18000 parameters,
- 17280 for the noise model
- + gravity field

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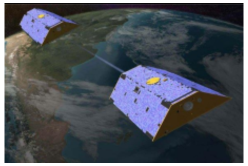

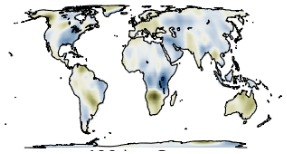

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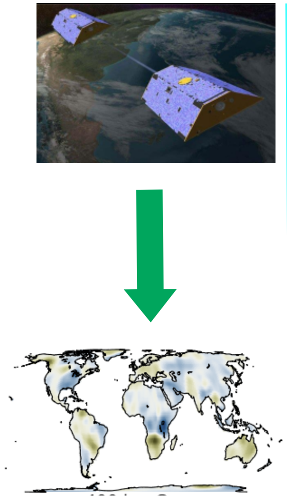
Force models	
Gravity field	Internal AIUB static GRACE field
Astromomic bodies	JPL DE421 (all planets + Pluto)
Mean pole	Linear
Solid Earth tides	IERS2010
Solid Earth pole tides	IERS2010
Ocean tides	FES2014b (+ admittances from TUG)
Ocean pole tides	Desai
Atmospheric tides	AOD RL06
Atmospheric & oeanic dealiasing	AOD RL06
Relativistic effects	IERS2010

Non-conservative forces:
ACT from TUG

VCE – constraints



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  Errors in background models will (mostly) sum up in 1/rev

→ frequently used in the Celestial Mechanics Approach [Beutler et al., 2010]

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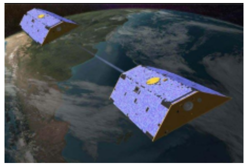

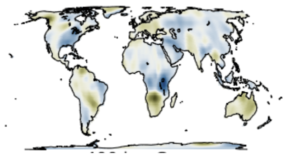

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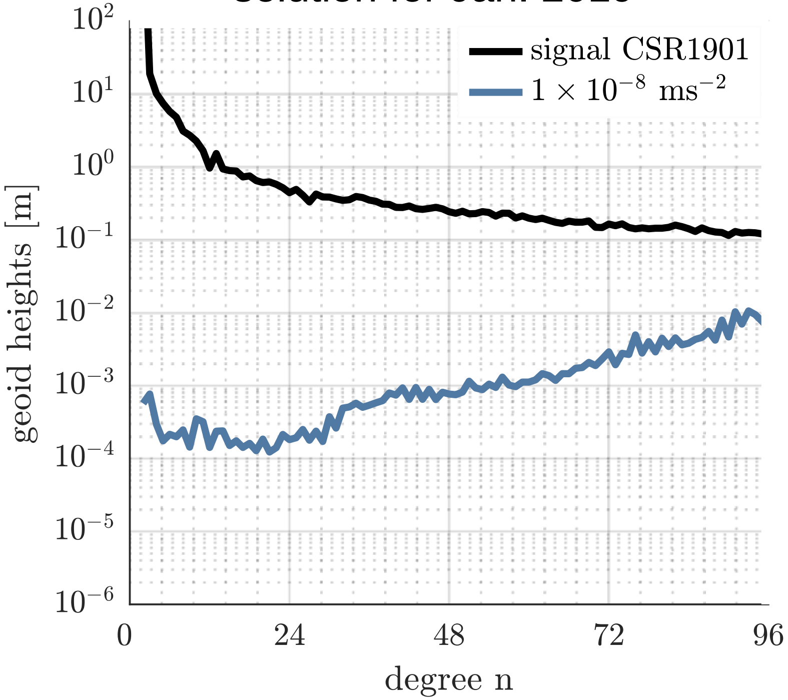
How to constrain their impact to the correct magnitude?

AIUB

Impact of different constraints

loose
PCAs
may
become
large

solution for Jan. 2019



1×10^{-8} ms^{-2}

«loose» constraint

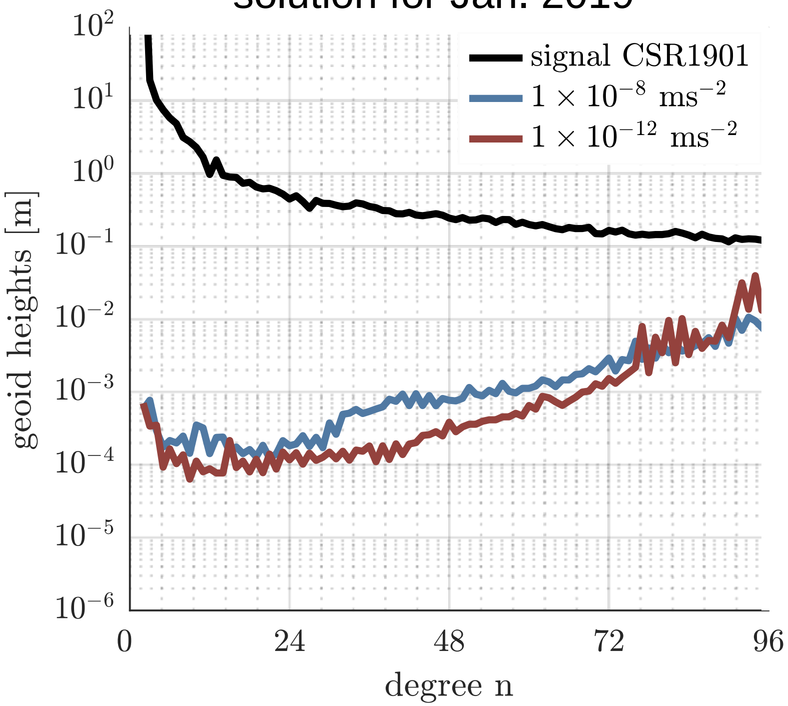
(gravity field signal absorbed in PCAs)

Impact of different constraints

loose PCAs may become large

tight PCAs strongly confined

solution for Jan. 2019



$1 \times 10^{-12}\ \mathrm{ms}^{-2}$

«tight» constraint

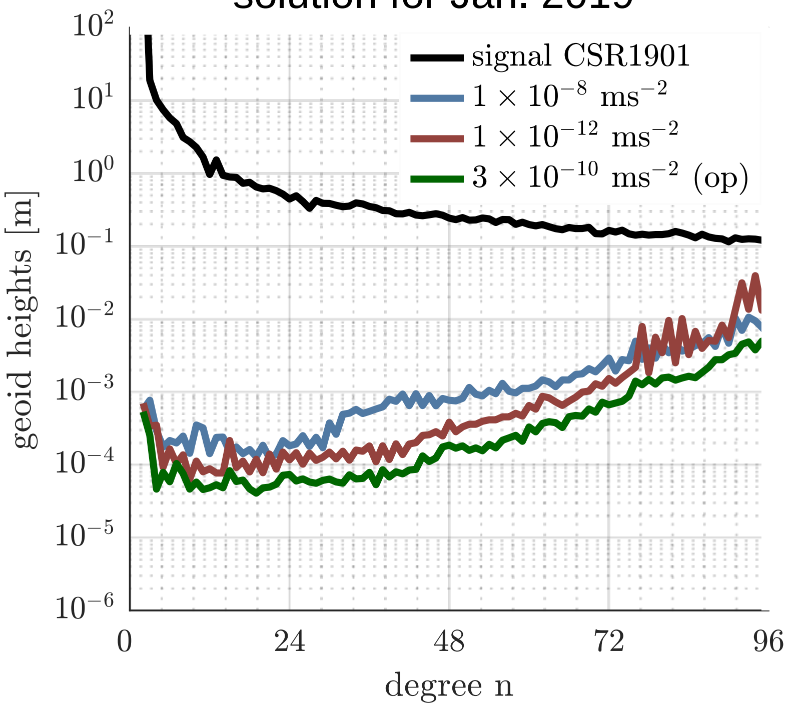
(not enough to absorb mis-modellings)

Impact of different constraints

loose PCAs may become large

tight PCAs strongly confined

solution for Jan. 2019



3×10^{-10} ms^{-2}

reasonable balance

(applied in the operational solutions)

Constraining

$\mathbf{A}$ design matrix

$\mathbf{P}$ weight matrix

$\boldsymbol{l}$ obser-vations

$$\mathbf{N}=(\mathbf{A}^T\mathbf{P}\mathbf{A})$$ and $$\boldsymbol{b}=\mathbf{A}^T\mathbf{P}\boldsymbol{l}$$ → $$\hat{\boldsymbol{x}}=\mathbf{N}^{-1}\boldsymbol{b}$$

$$\mathbf{N}=(\mathbf{A}^T\mathbf{P}\mathbf{A}+\mathbf{W})$$

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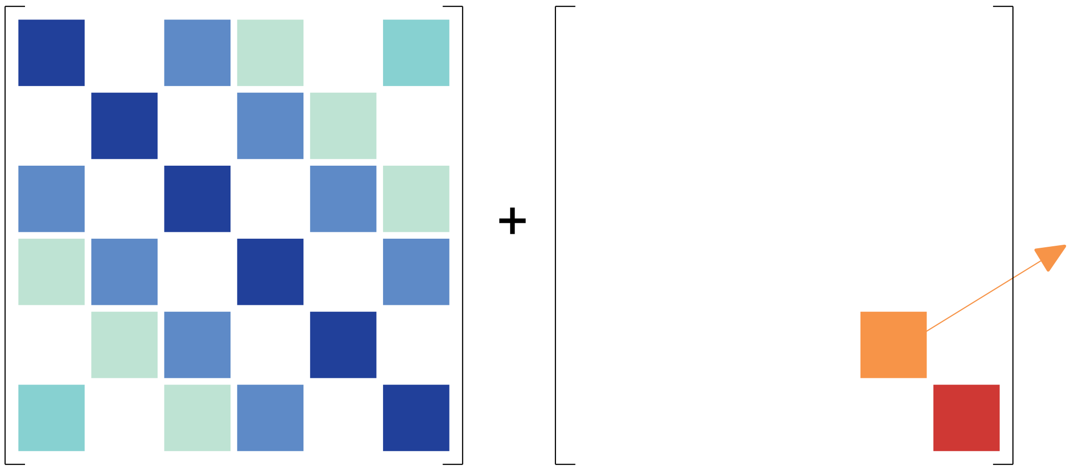
$N =$



$$\frac{\sigma_0^2}{\sigma_{PCA}^2},$$

$\sigma_{PCA}^2 =$ e.g., $3 \times 10^{-10}\ \mathrm{ms}^{-2}$

VCE and constraints

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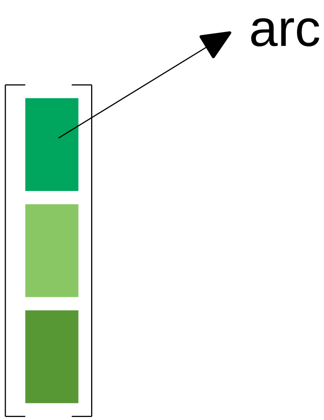
- The observations of each arc are used to set up the normal equations (NEQs)

$\boldsymbol{l}=$ 

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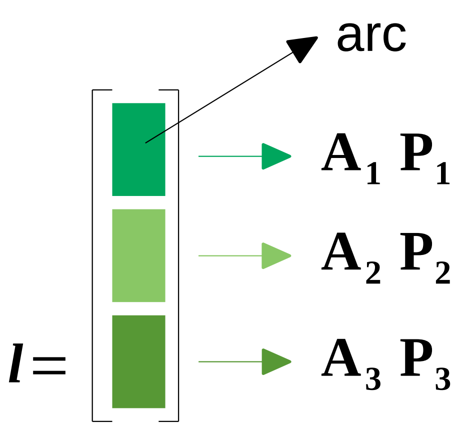
- The observations of each arc are used to set up the normal equations (NEQs)
- Each arc is treated as being independent



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$$\boldsymbol{l} = \begin{bmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad \begin{matrix} \rightarrow \mathbf{A}_1\,\mathbf{P}_1 \\ \rightarrow \mathbf{A}_2\,\mathbf{P}_2 \\ \rightarrow \mathbf{A}_3\,\mathbf{P}_3 \\ \\ \\ \end{matrix}$$

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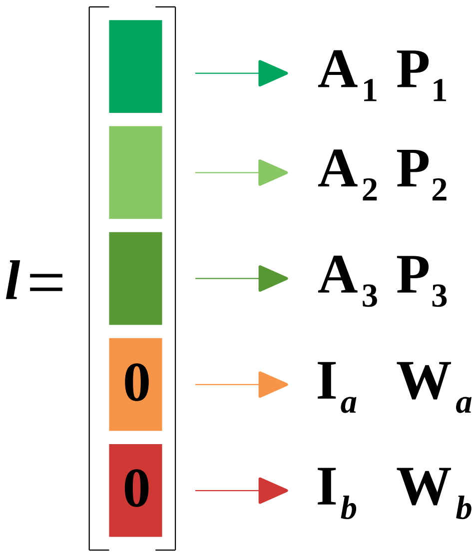

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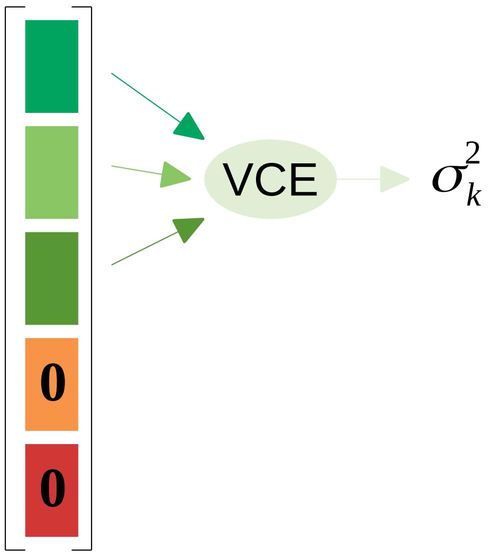
VCE: Each group of observations gets a weight
based on its contribution to the final solution

$\boldsymbol{l} =$



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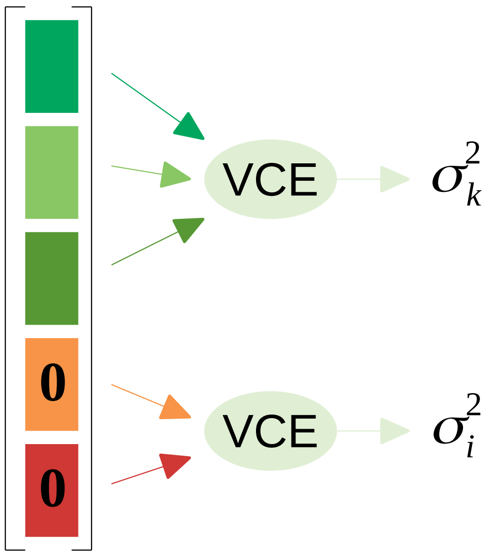

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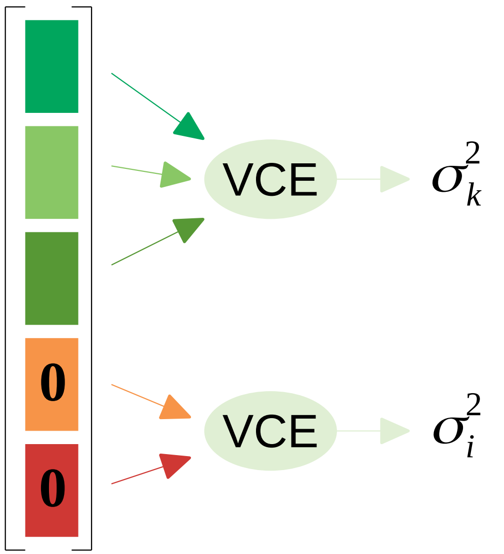

$$\hat{\boldsymbol{x}} = \left(\sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} \mathbf{N}_k + \sum_{i=1}^{I=2} \frac{\sigma_0^2}{\sigma_i^2} \mathbf{W}_i \right)^{-1} \sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} \boldsymbol{b}_k$$

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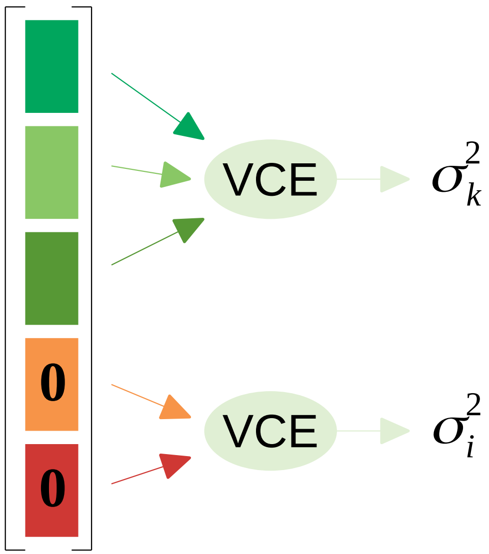
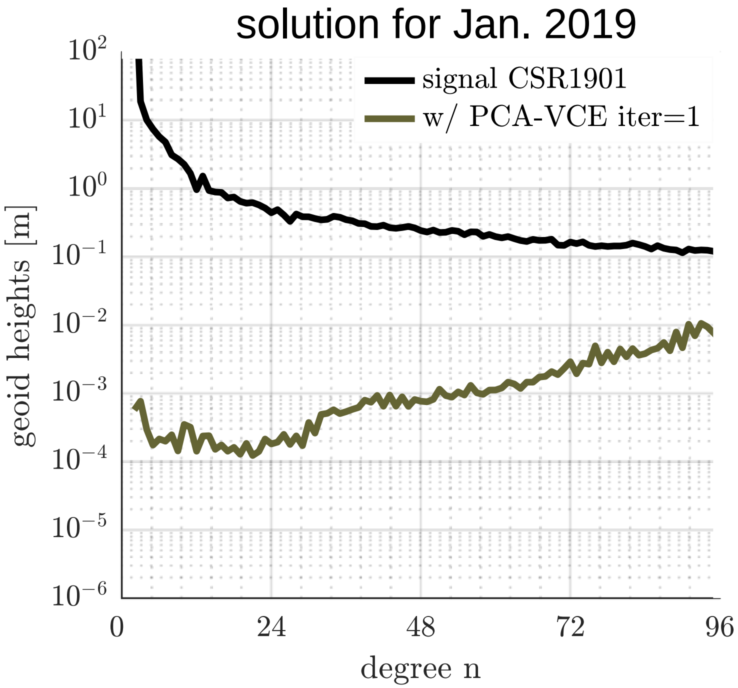
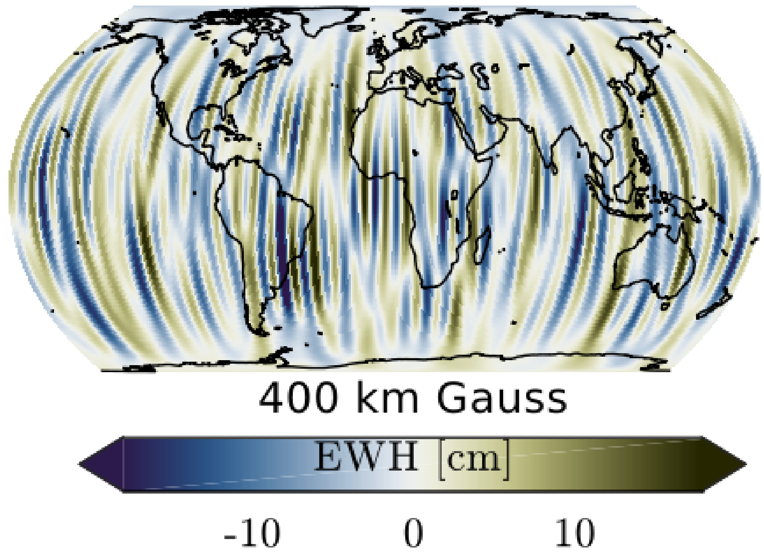


$$\hat{\boldsymbol{x}}=\left(\sum_{k=1}^{K=3}\frac{\sigma_0^2}{\sigma_k^2}\mathbf{N}+\sum_{i=1}^{I=2}\frac{\sigma_0^2}{\sigma_i^2}\mathbf{W}_i\right)^{-1}\sum_{k=1}^{K=3}\frac{\sigma_0^2}{\sigma_k^2}\boldsymbol{b}_k$$

information about observations introduced via σ_0^2

Results for VCE on constraints





radial 1×10^{-8} ms^{-2} along-track 1×10^{-8} ms^{-2} cross-track 1×10^{-8} ms^{-2}

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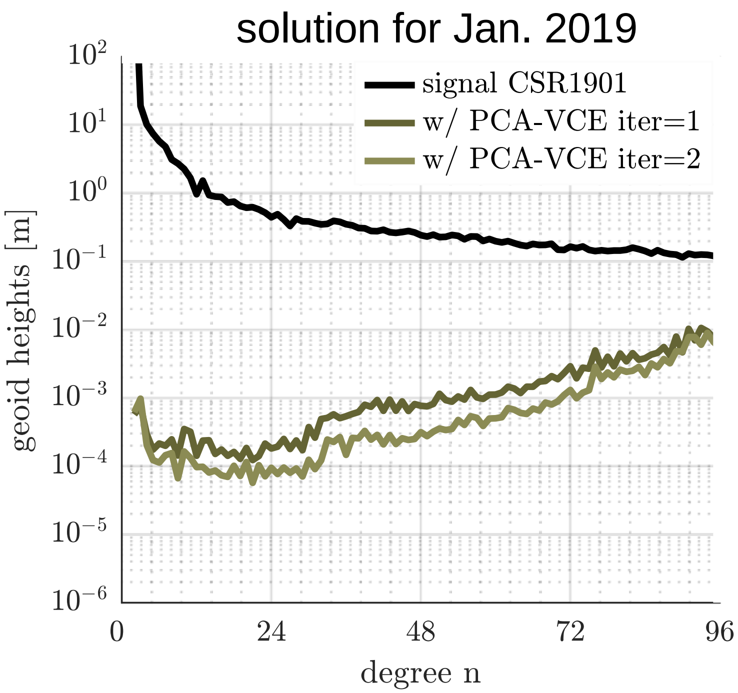
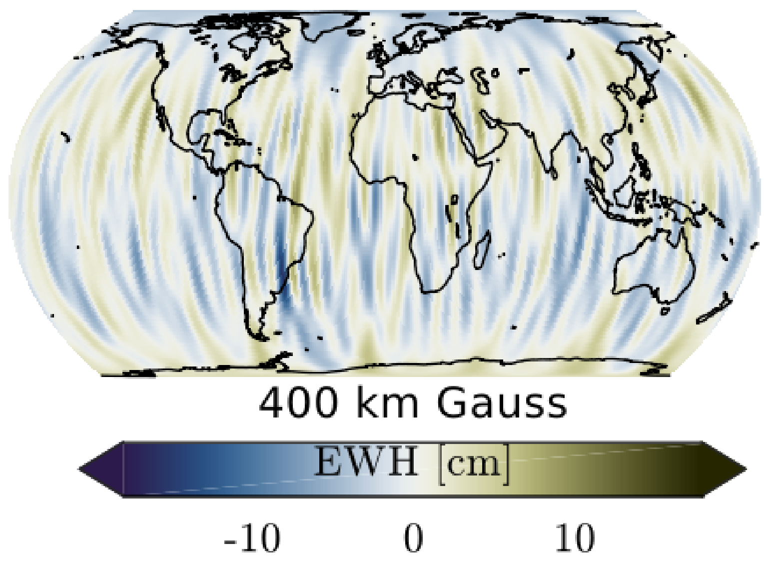
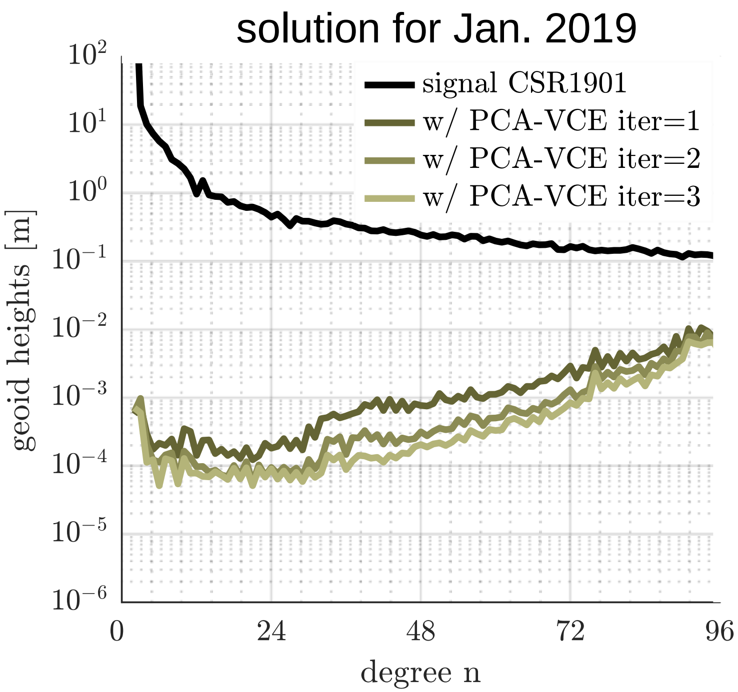


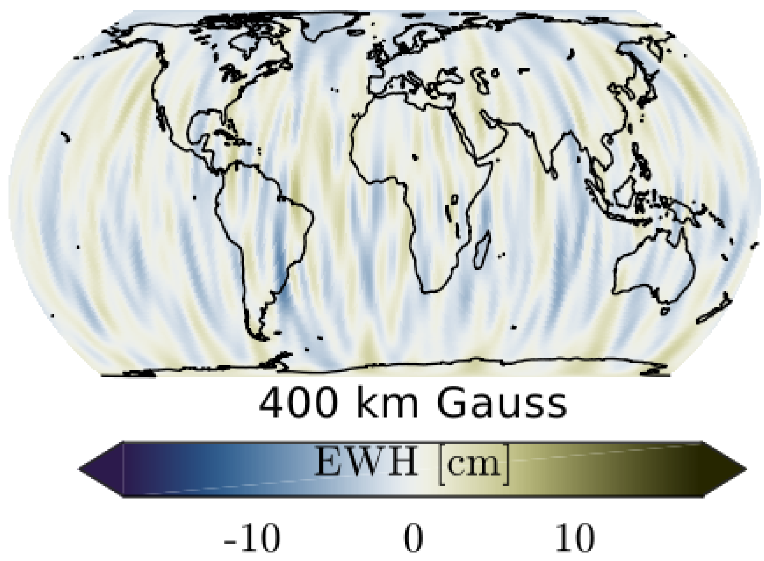


radial 1.9×10^{-8} ms^{-2} along-track 9.8×10^{-9} ms^{-2} cross-track 8.6×10^{-9} ms^{-2}

AIUB

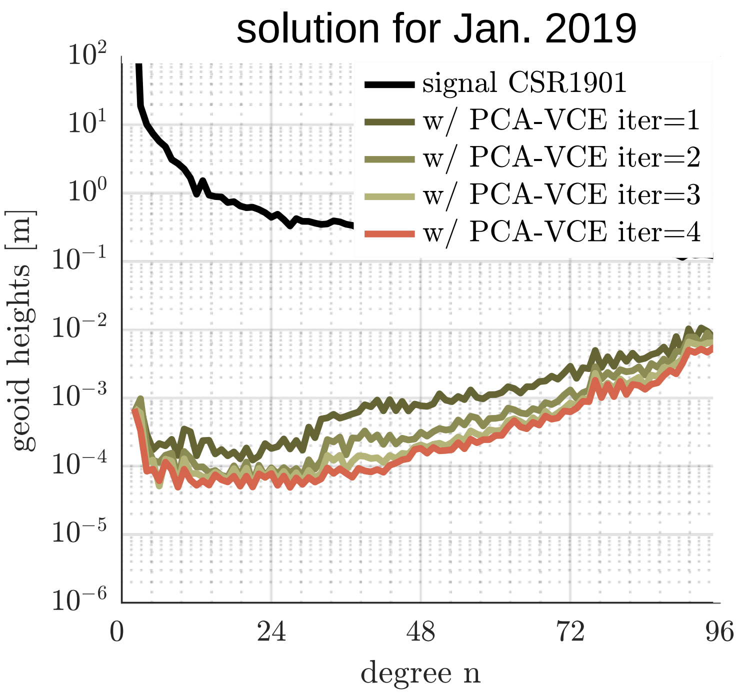
Results for VCE on constraints

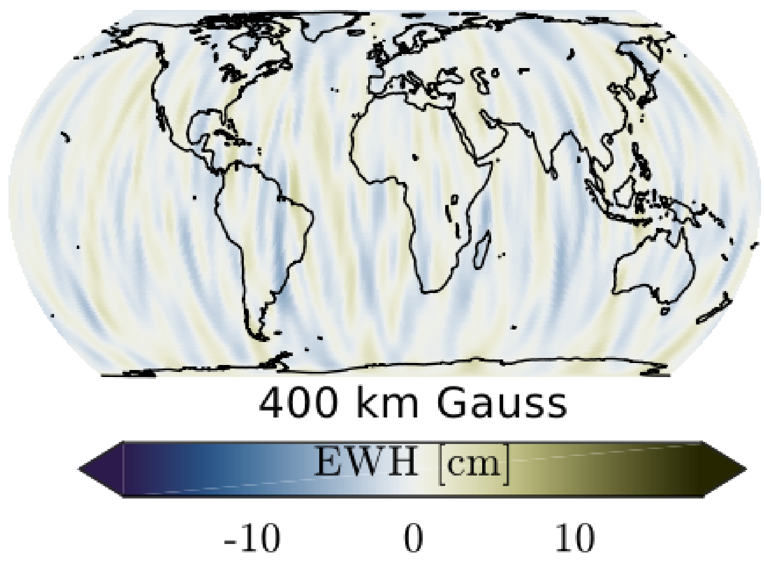




radial 2.9×10^{-9} ms^{-2} along-track 1.5×10^{-9} ms^{-2} cross-track 1.3×10^{-9} ms^{-2}

Results for VCE on constraints





radial 1.2×10^{-9} ms^{-2} along-track 6.2×10^{-10} ms^{-2} cross-track 6.9×10^{-10} ms^{-2}

AIUB

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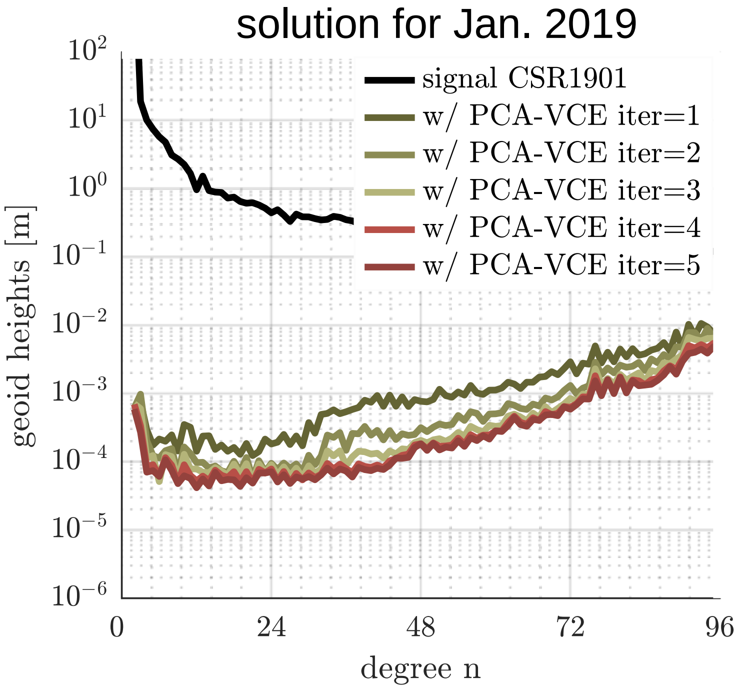
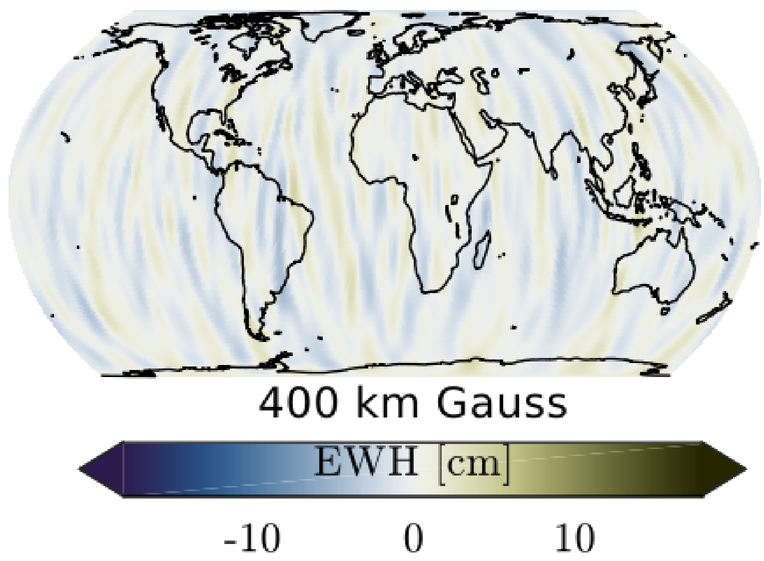
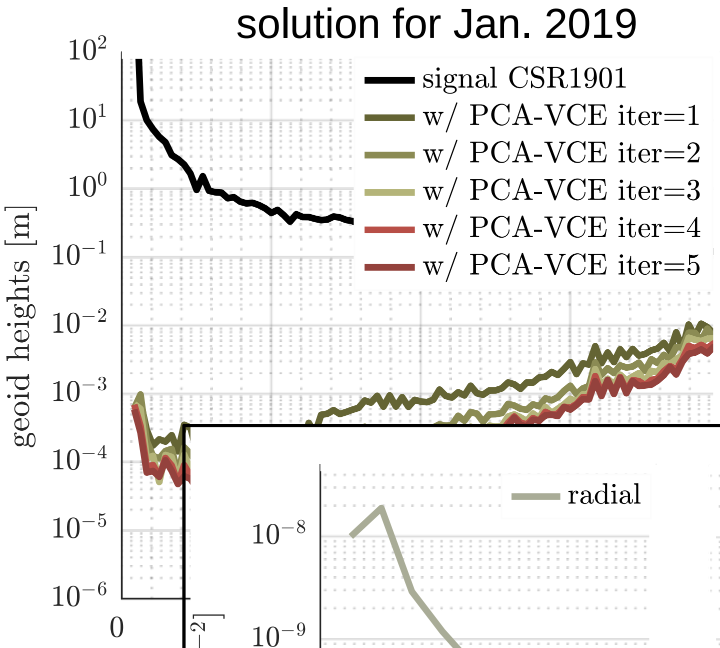


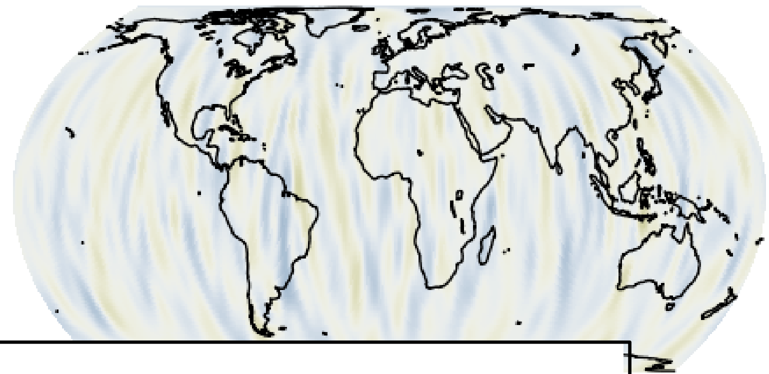


radial $5.6\times10^{-10}\,\mathrm{ms}^{-2}$ along-track $2.6\times10^{-10}\,\mathrm{ms}^{-2}$ cross-track $6.1\times10^{-10}\,\mathrm{ms}^{-2}$

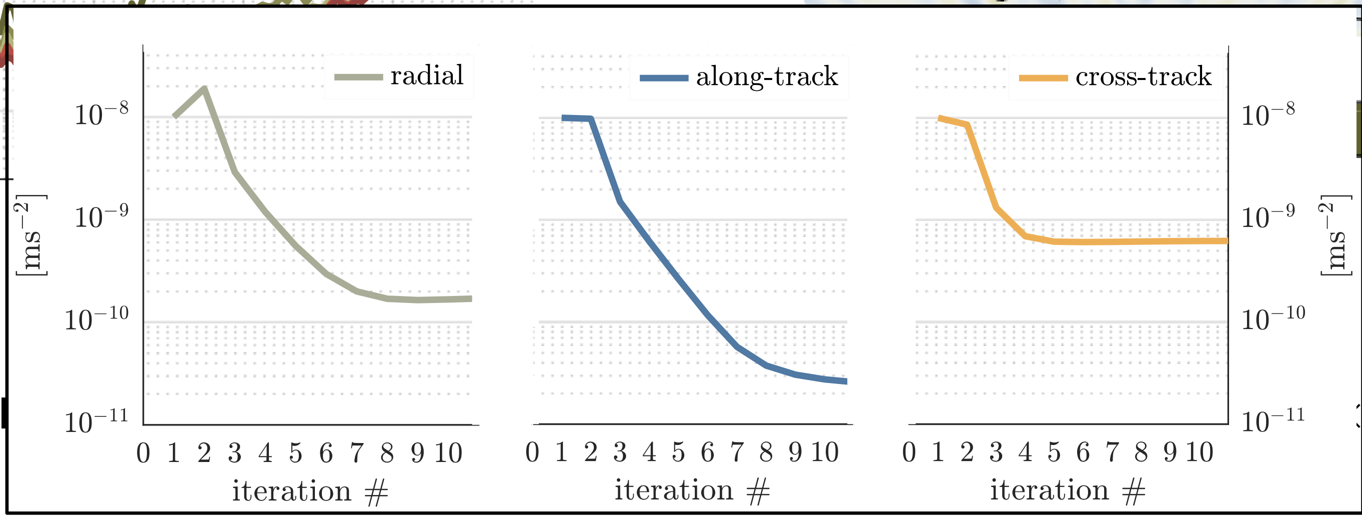
AIUB

Results for VCE on constraints





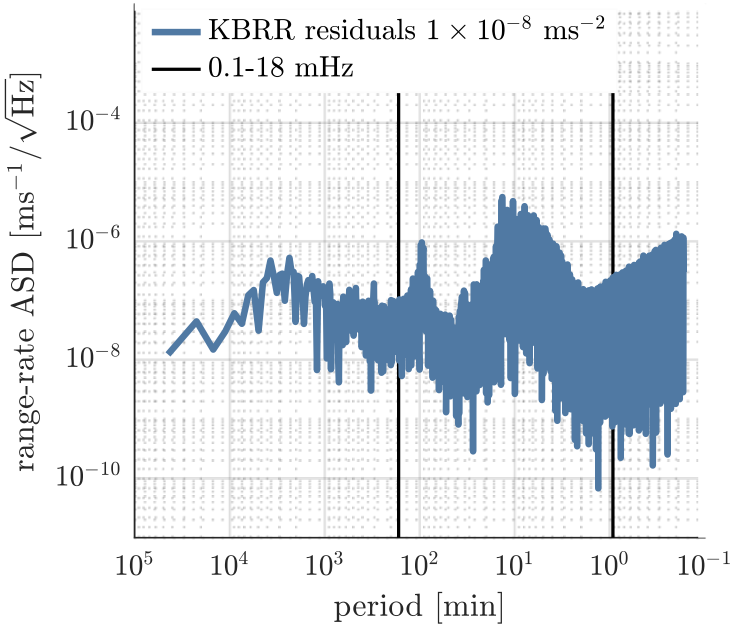
radial



10^{-10} ms^{-2}

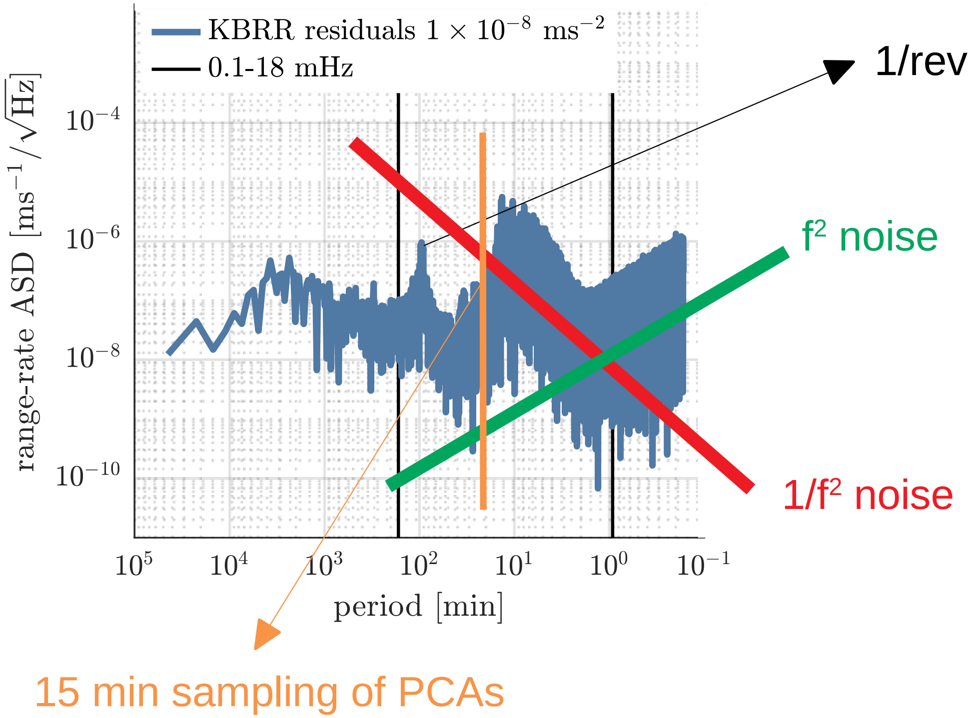
Results for VCE on constraints – post-fit residuals

$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$



Results for VCE on constraints – post-fit residuals

$\hat{e} = l - A\hat{x}$



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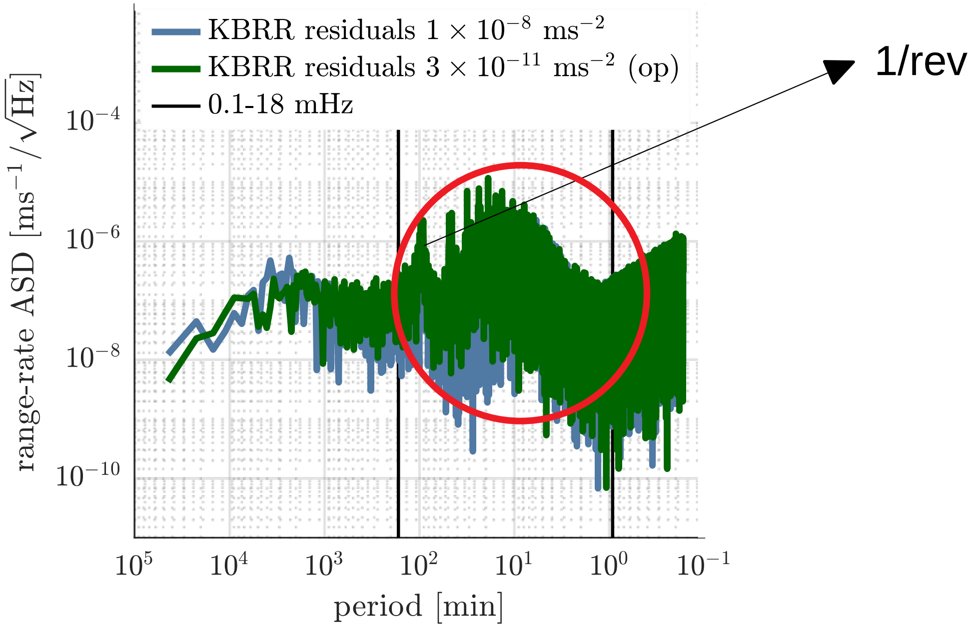

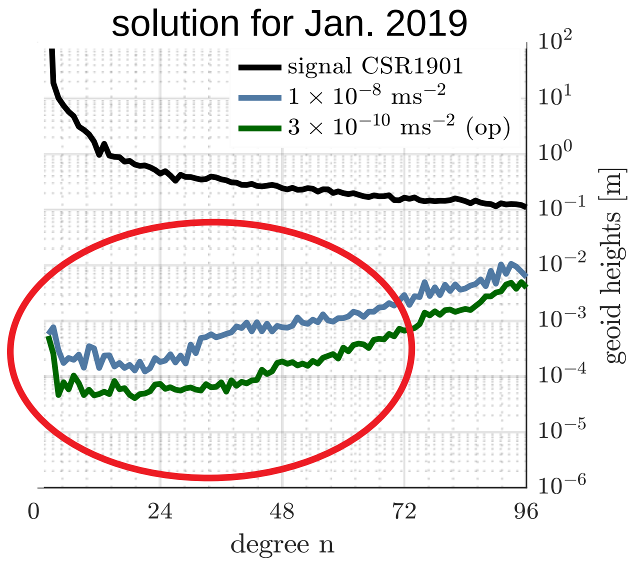

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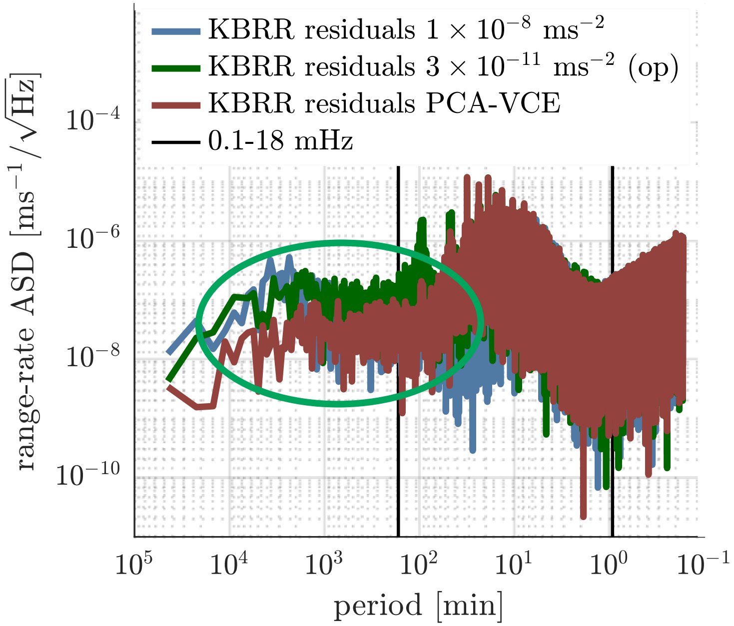
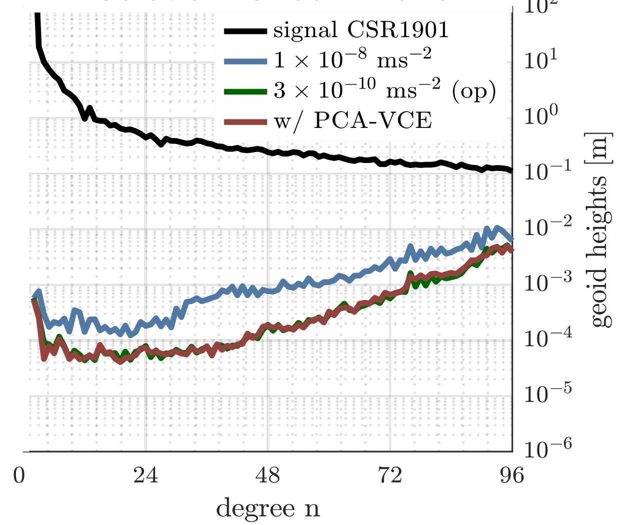
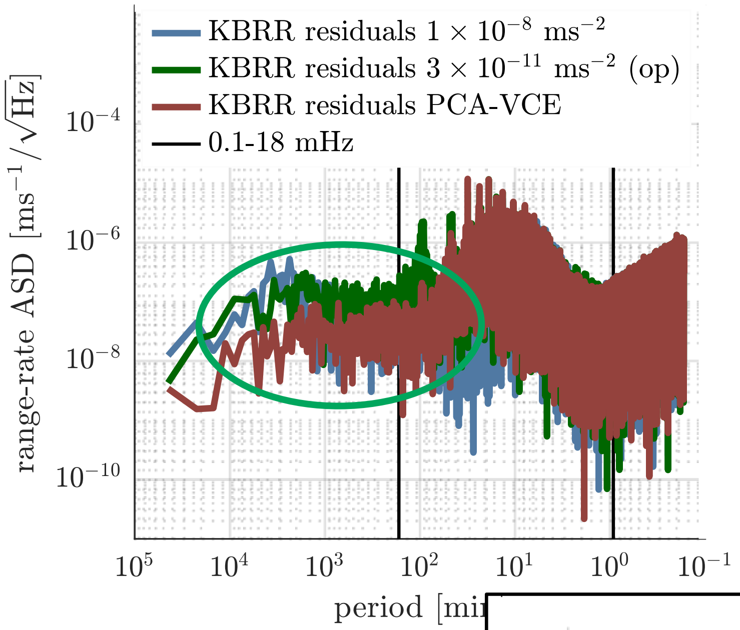


- almost flat spectrum for long periods
- indication for a "good" parametrisation of the respective signal



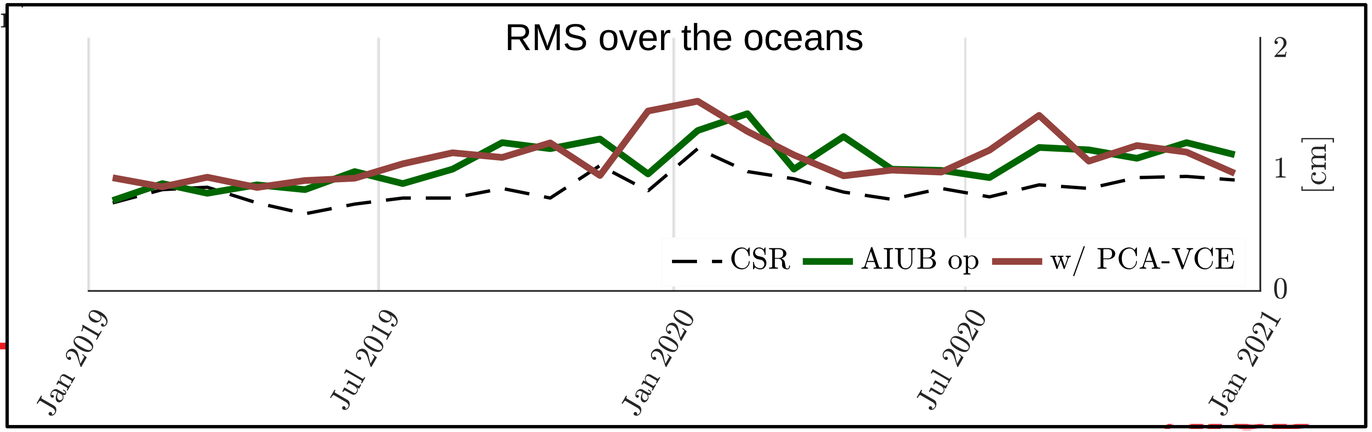
Results for VCE on constraints – summary

$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$



- almost flat spectrum for long periods
- indication for a “good” parametrisation of the respective signal

- 12 months improved
- 12 months degraded



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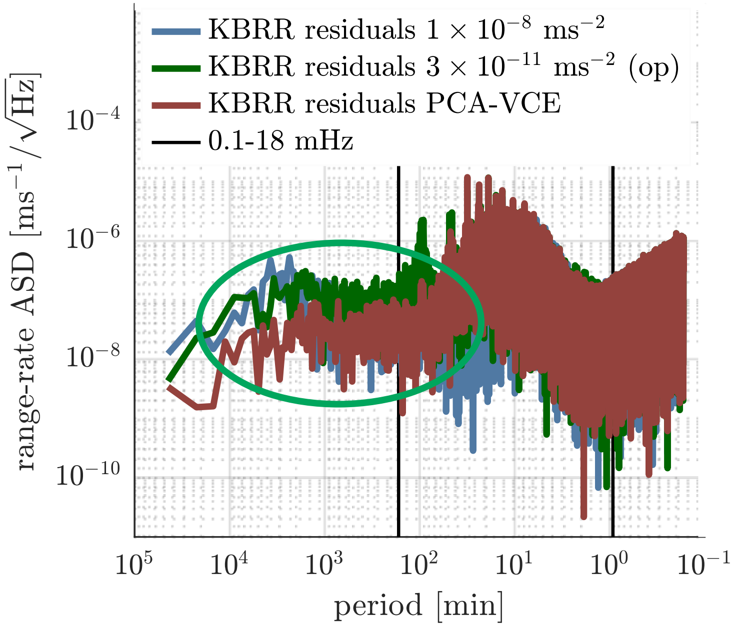

- observation-based approach
- computed together with the solution
- provides a good solution (if PCAs sample correctly)

- computational efficiency...
- observation-based – outliers

Thank you for your attention

References

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