

Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery

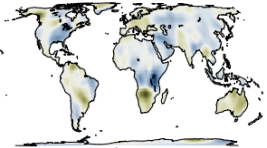
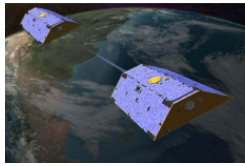
Martin Lasser, Ulrich Meyer, Daniel Arnold, Adrian Jäggi

Astronomical Institute, University of Bern, Switzerland

Gravity, Geoid, and Height Systems 2022 Symposium
12 September 2022
Austin, TX, USA



Operational processing



Basic parametrisation

- initial conditions 2x[6]
- accelerometer bias 2x[3]
- accelerometer scaling 2x[3]

parameters per arc 24

Additional parameters

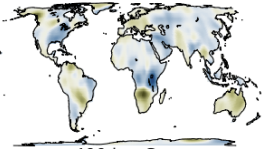
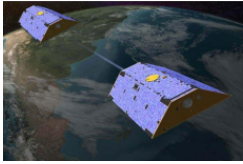
- 15 min PCA per satellite in
 - radial 2x[96]
 - along-track 2x[96]
 - cross-track 2x[96]

parameters per arc 576

in daily arcs (30 days):

- 18000 parameters,
- 17280 for the noise model
- + gravity field

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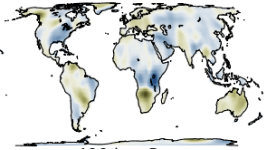
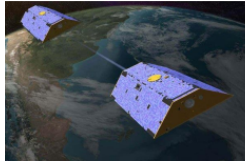
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Force models

Gravity field	Internal AIUB static GRACE field
Astromomic bodies	JPL DE421 (all planets + Pluto)
Mean pole	Linear
Solid Earth tides	IERS2010
Solid Earth pole tides	IERS2010
Ocean tides	FES2014b (+ admittances from TUG)
Ocean pole tides	Desai
Atmospheric tides	AOD RL06
Atmospheric & oeanic dealiasing	AOD RL06
Relativistic effects	IERS2010

Non-conservative forces:
ACT from TUG

VCE – constraints



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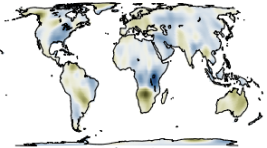
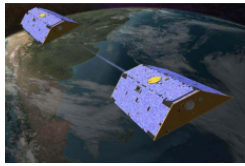
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Errors in background models will (mostly) sum up in 1/rev
- frequently used in the Celestial Mechanics Approach [Beutler et al., 2010]

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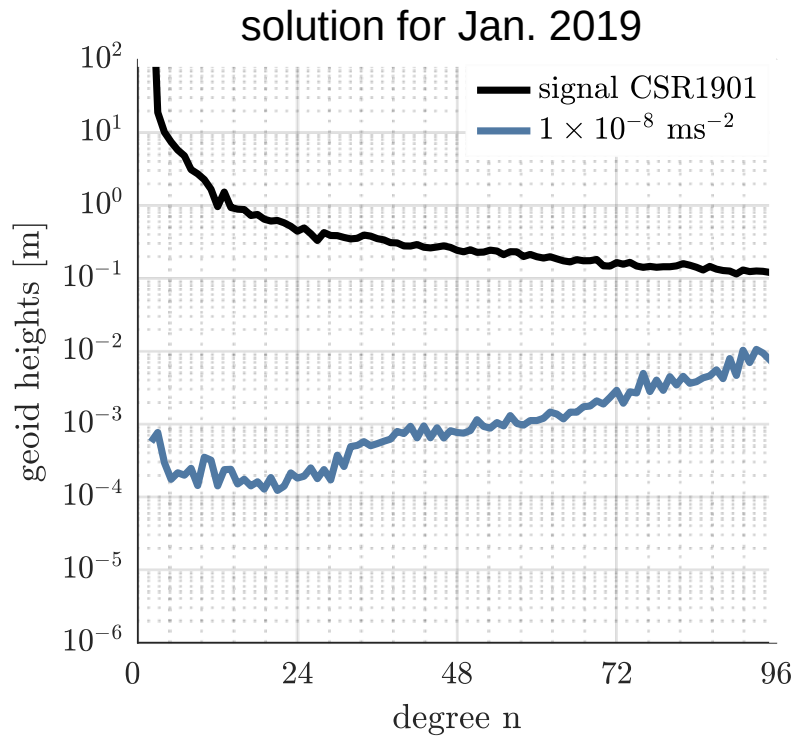
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How to constrain their impact to the correct magnitude?

Impact of different constraints

loose
PCAs
may
become
large



$$1 \times 10^{-8} \text{ ms}^{-2}$$

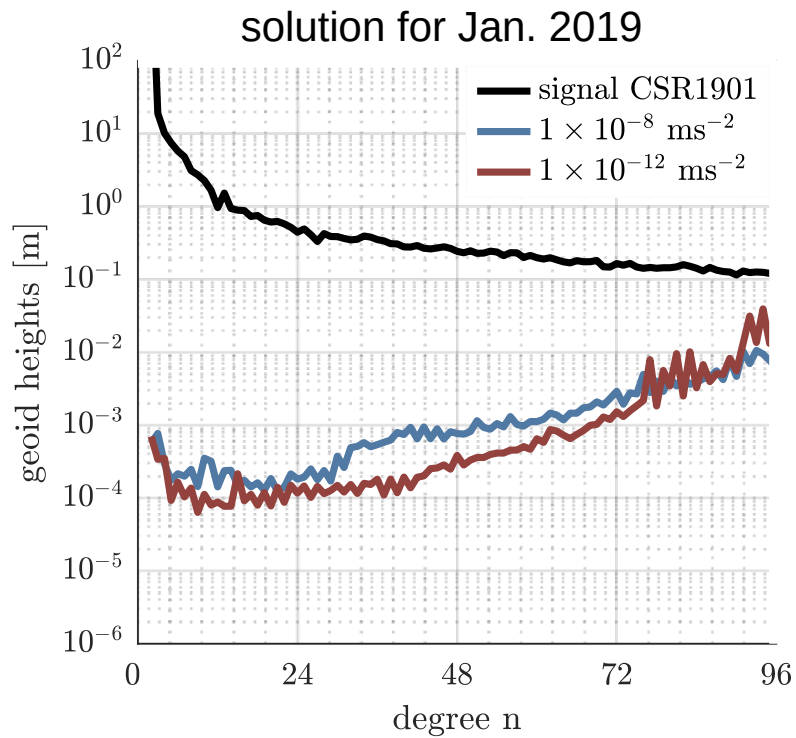
«loose» constraint

(gravity field signal absorbed in PCAs)

Impact of different constraints

loose
PCAs
may
become
large

tight
PCAs
strongly
confined



$$1 \times 10^{-12} \text{ ms}^{-2}$$

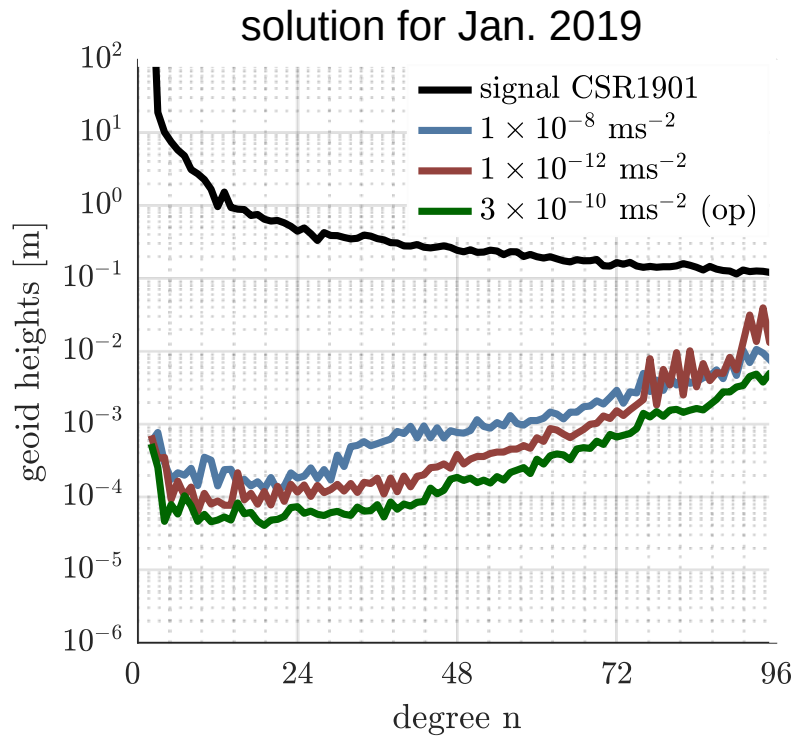
«tight» constraint

(not enough to absorb mis-modellings)

Impact of different constraints

loose
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$$3 \times 10^{-10} \text{ ms}^{-2}$$

reasonable balance

(applied in the operational solutions)

Constraining

A design matrix

P weight matrix

l observations

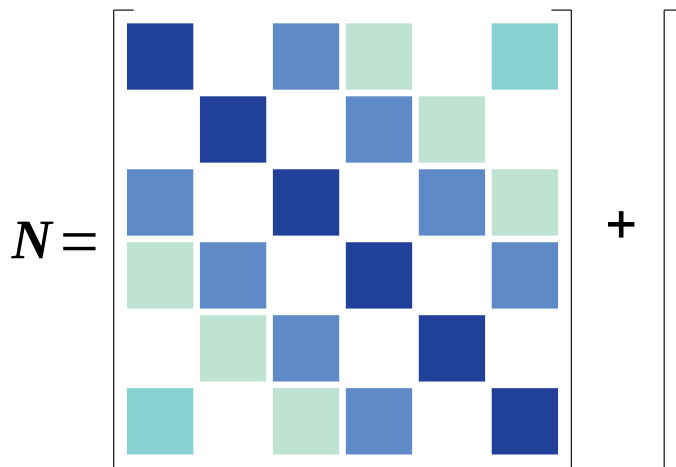
$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A}) \quad \text{and} \quad \mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l} \quad \longrightarrow \quad \hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{b}$$
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Constraining

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$$\frac{\sigma_0^2}{\sigma_{PCA}^2},$$

$$\sigma_{PCA}^2 = \text{e.g., } 3 \times 10^{-10} \text{ ms}^{-2}$$

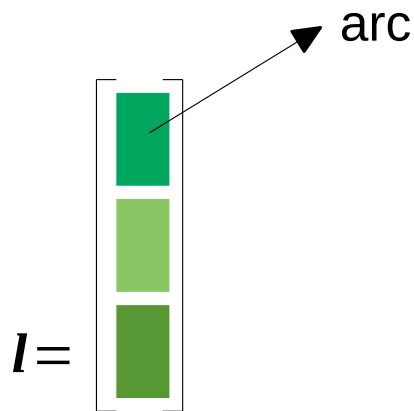
VCE and constraints

A design matrix $\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A})$ and $\mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l}$ \longrightarrow $\hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{b}$

P weight matrix $\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A} + \mathbf{W})$

l observations

- The observations of each arc are used to set up the normal equations (NEQs)



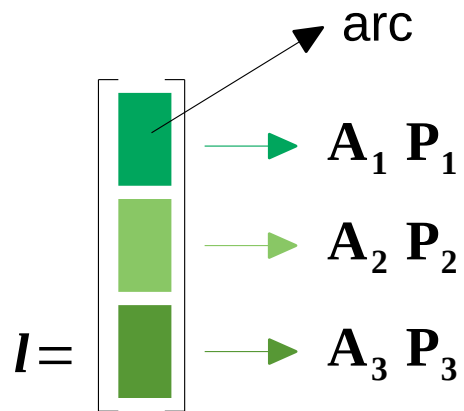
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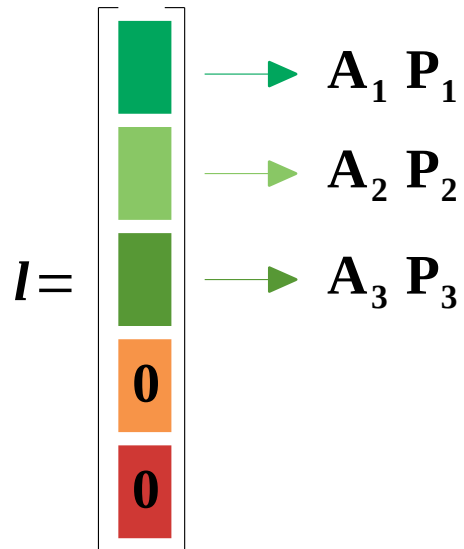
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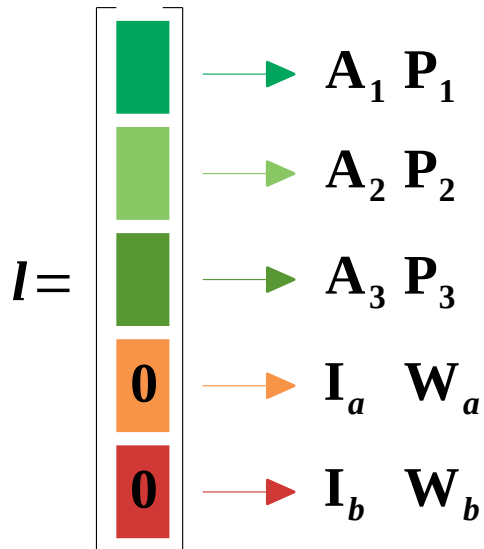
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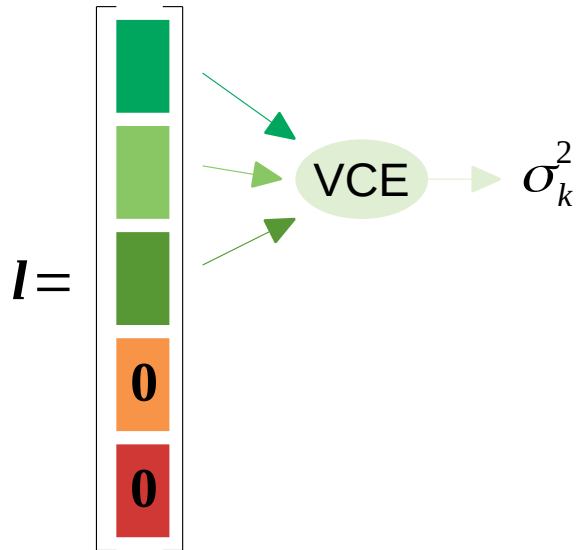


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VCE: Each group of observations gets a weight based on its contribution to the final solution

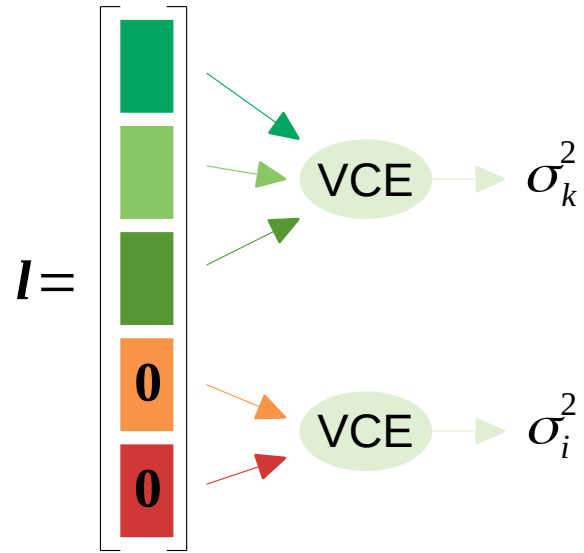


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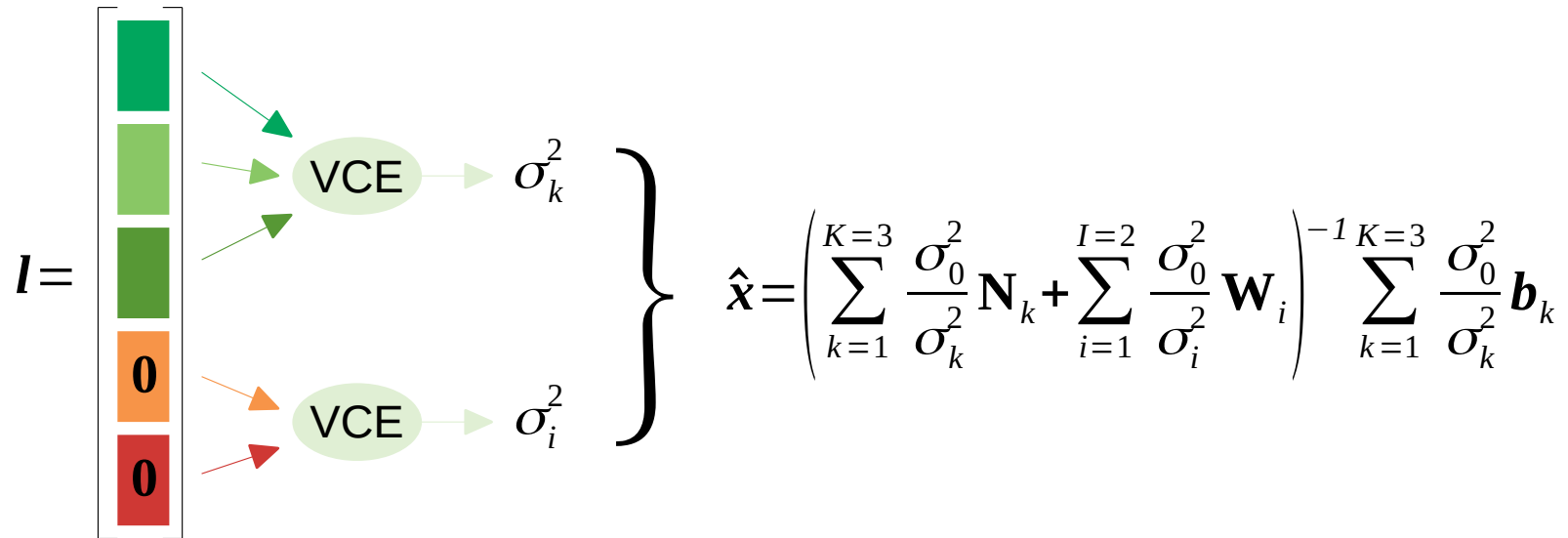


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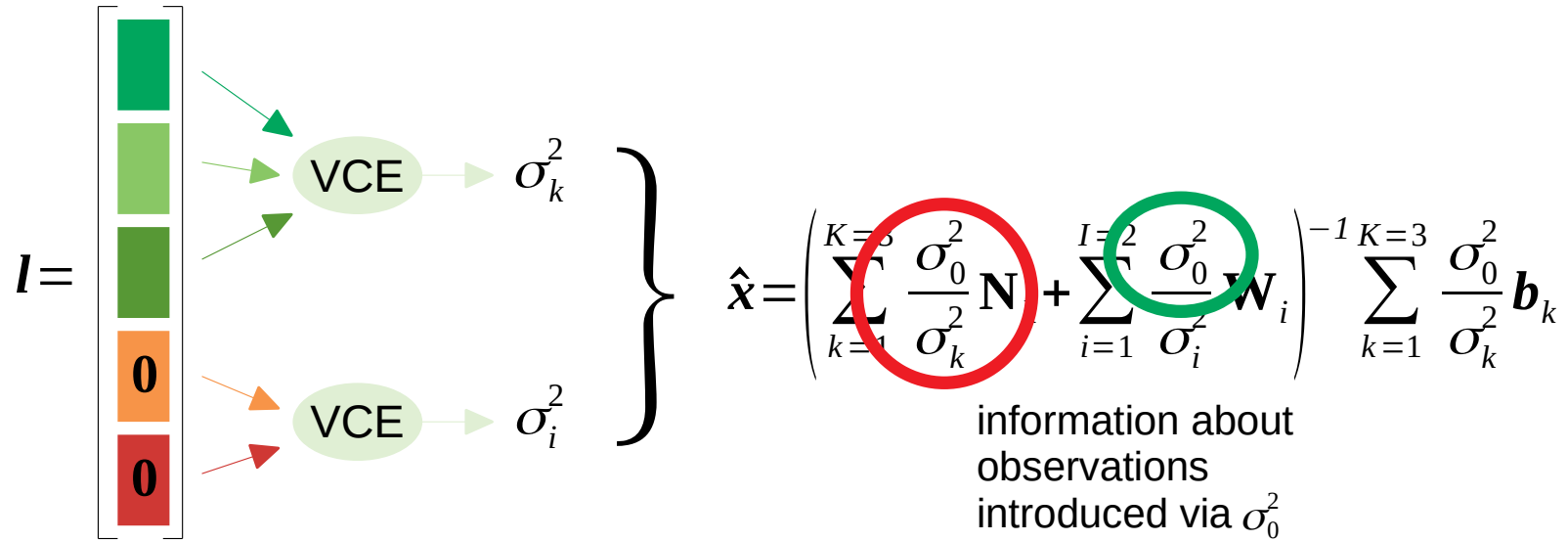


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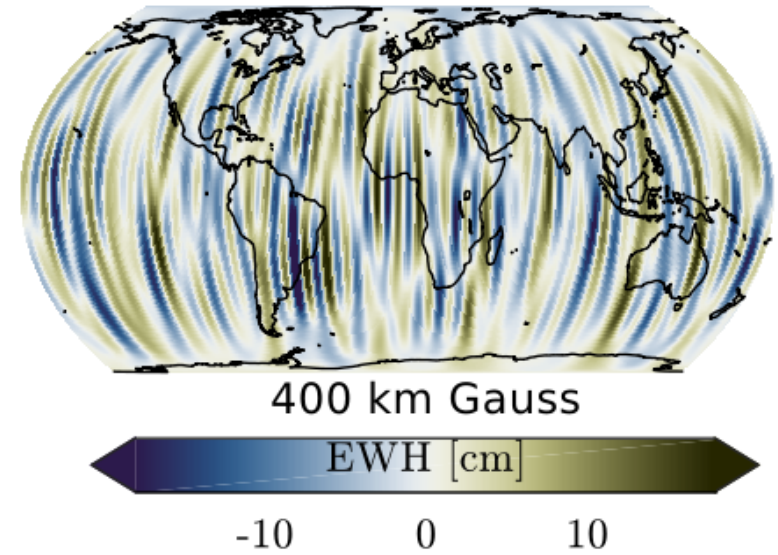
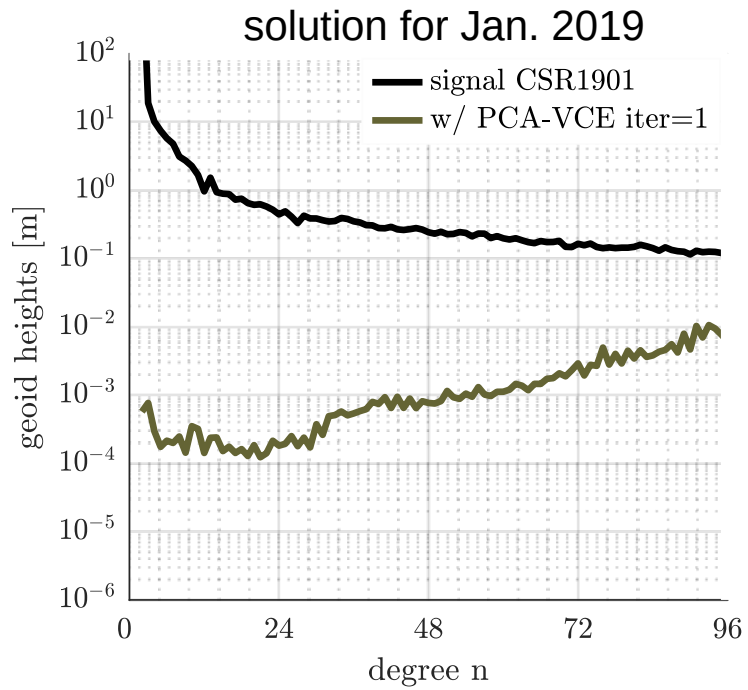
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Results for VCE on constraints

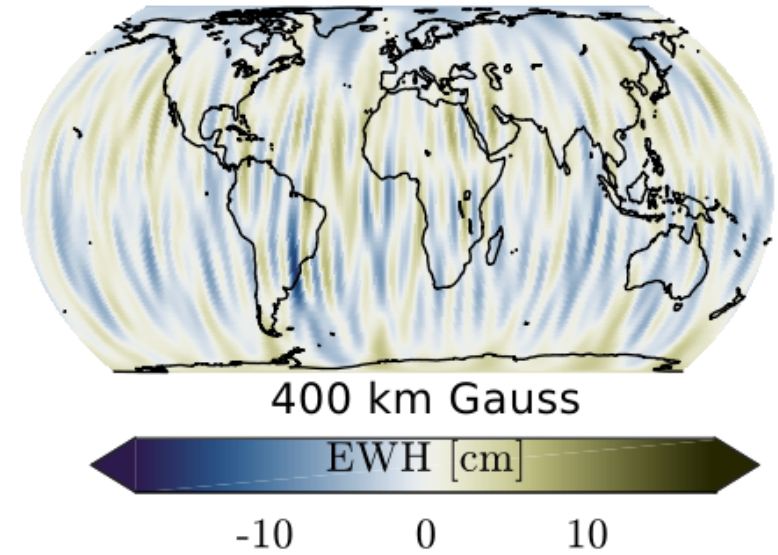
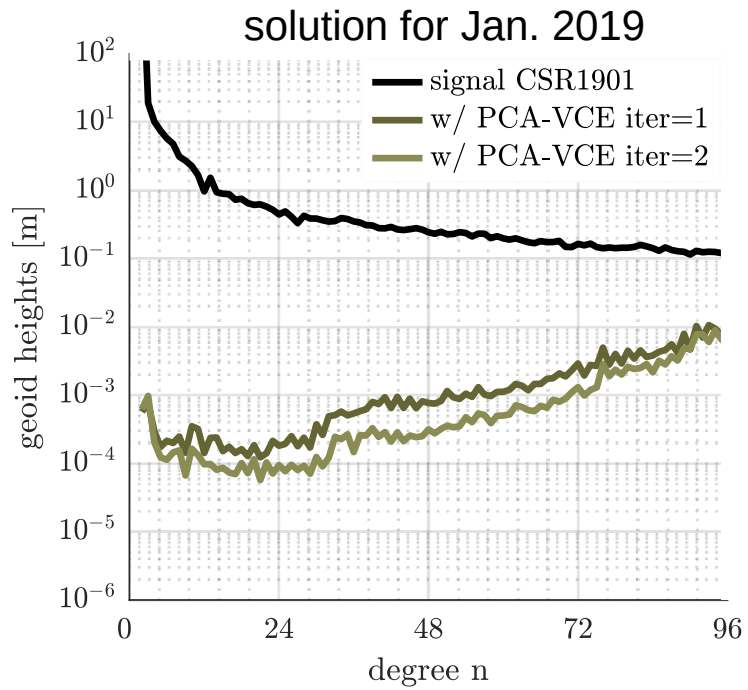


radial $1 \times 10^{-8} \text{ ms}^{-2}$

along-track $1 \times 10^{-8} \text{ ms}^{-2}$

cross-track $1 \times 10^{-8} \text{ ms}^{-2}$

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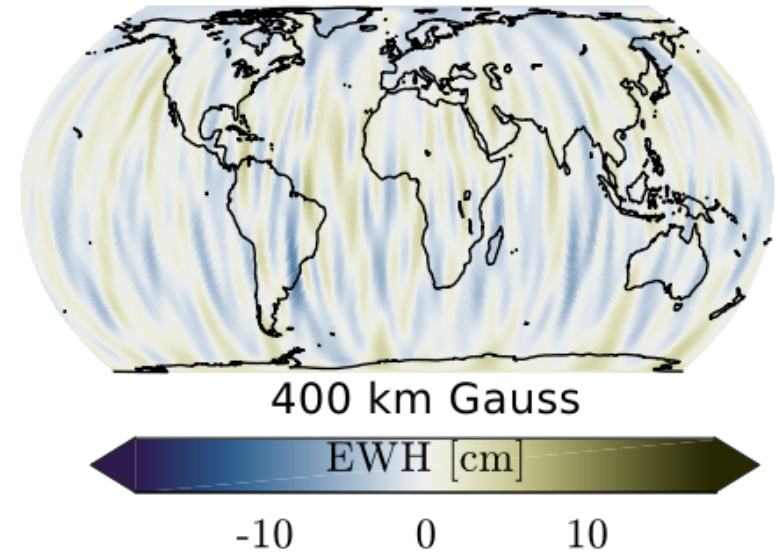
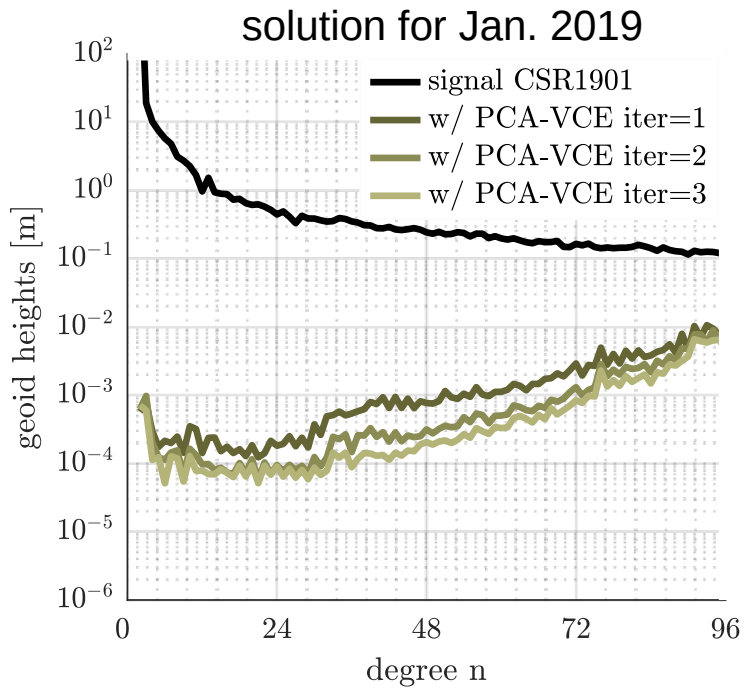


radial $1.9 \times 10^{-8} \text{ ms}^{-2}$

along-track $9.8 \times 10^{-9} \text{ ms}^{-2}$

cross-track $8.6 \times 10^{-9} \text{ ms}^{-2}$

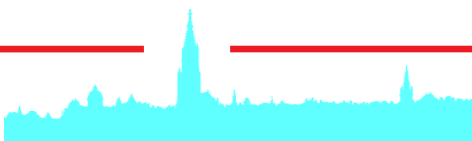
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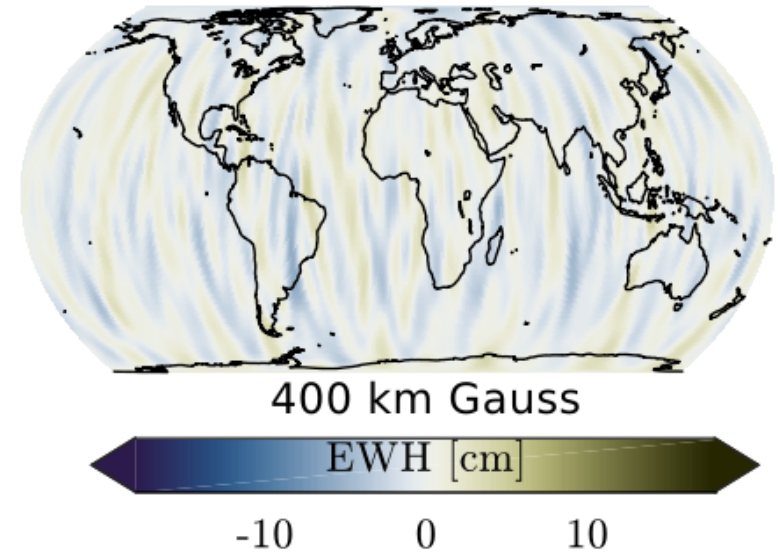
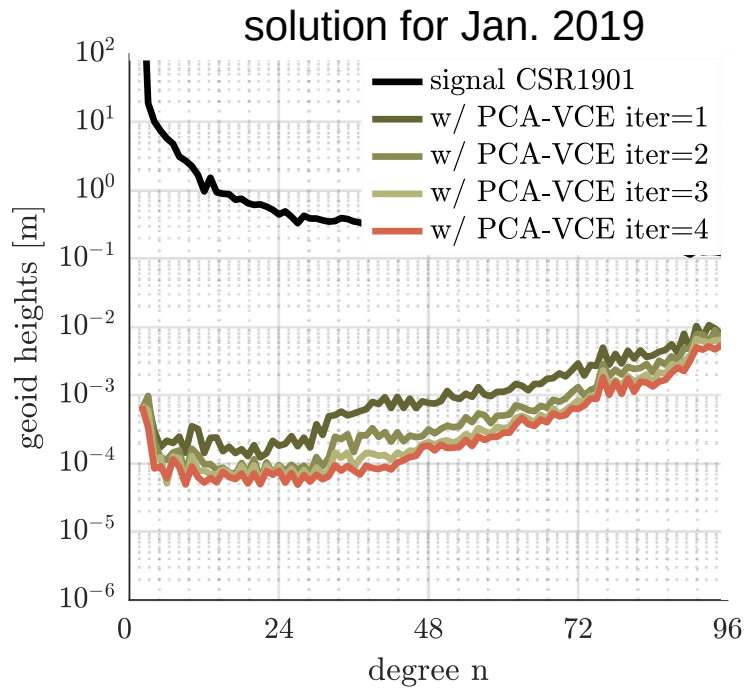
radial $2.9 \times 10^{-9} \text{ ms}^{-2}$

along-track $1.5 \times 10^{-9} \text{ ms}^{-2}$

cross-track $1.3 \times 10^{-9} \text{ ms}^{-2}$



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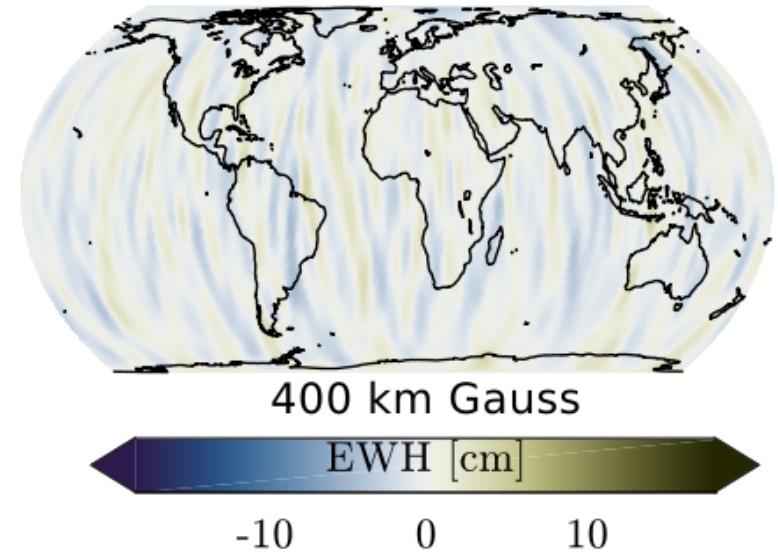
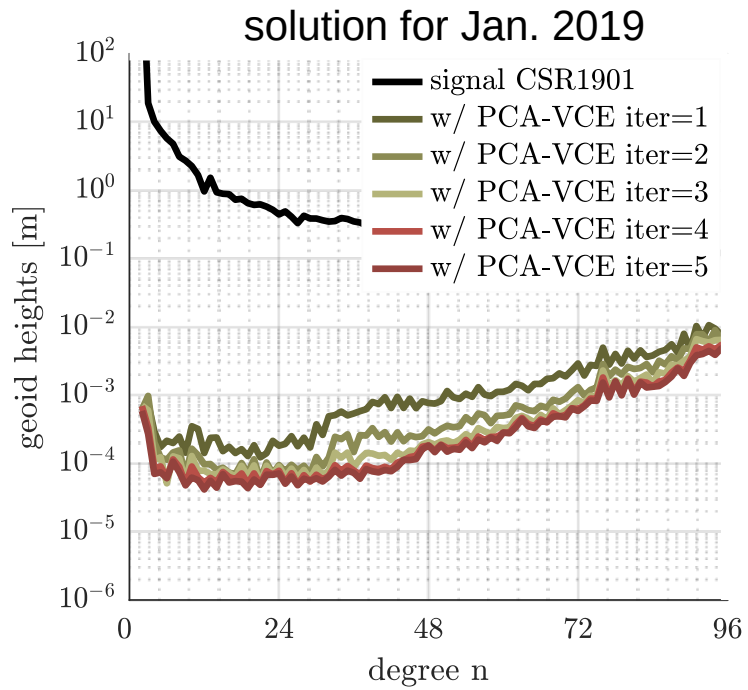


radial $1.2 \times 10^{-9} \text{ ms}^{-2}$

along-track $6.2 \times 10^{-10} \text{ ms}^{-2}$

cross-track $6.9 \times 10^{-10} \text{ ms}^{-2}$

Results for VCE on constraints

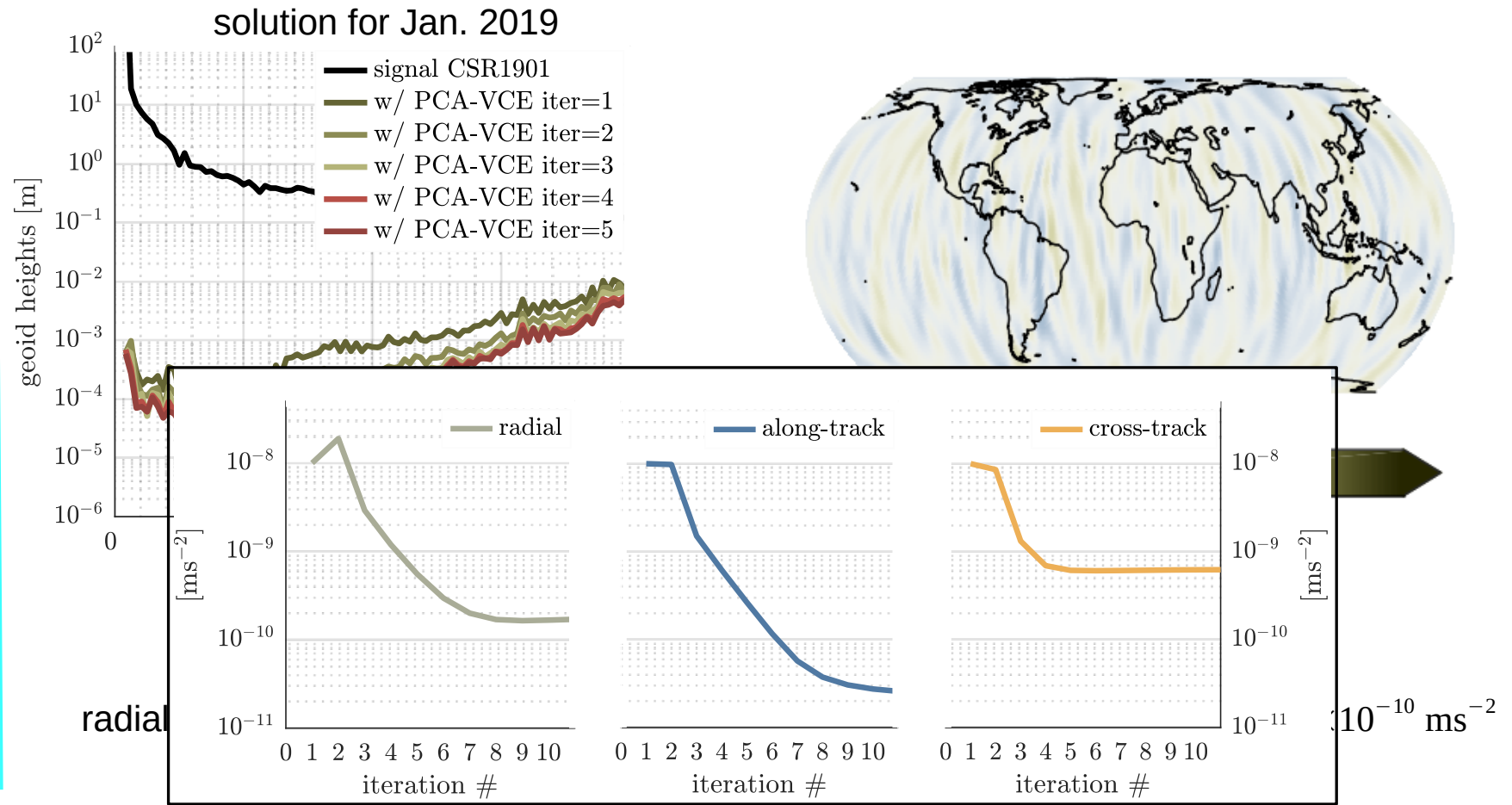


radial $5.6 \times 10^{-10} \text{ ms}^{-2}$

along-track $2.6 \times 10^{-10} \text{ ms}^{-2}$

cross-track $6.1 \times 10^{-10} \text{ ms}^{-2}$

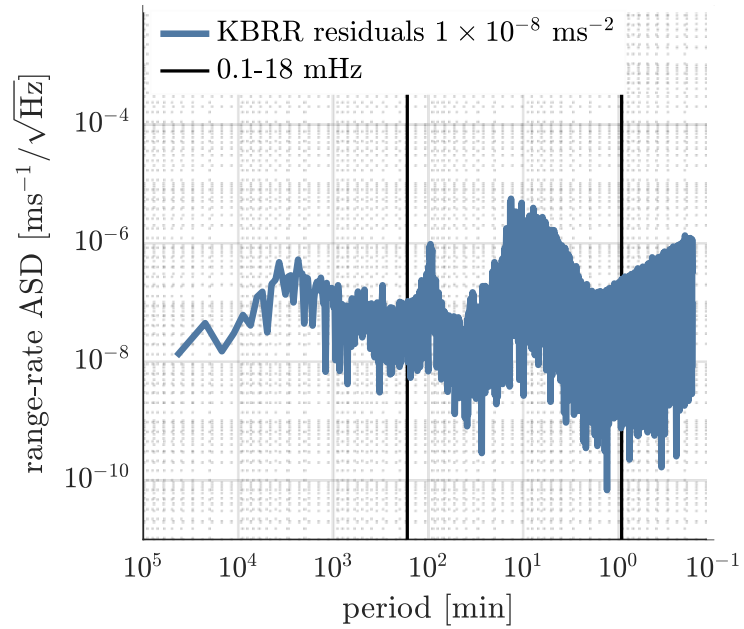
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M. Lasser et al.: Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery, Gravity, Geoid, and Height Systems 2022 Symposium, 12 September 2022

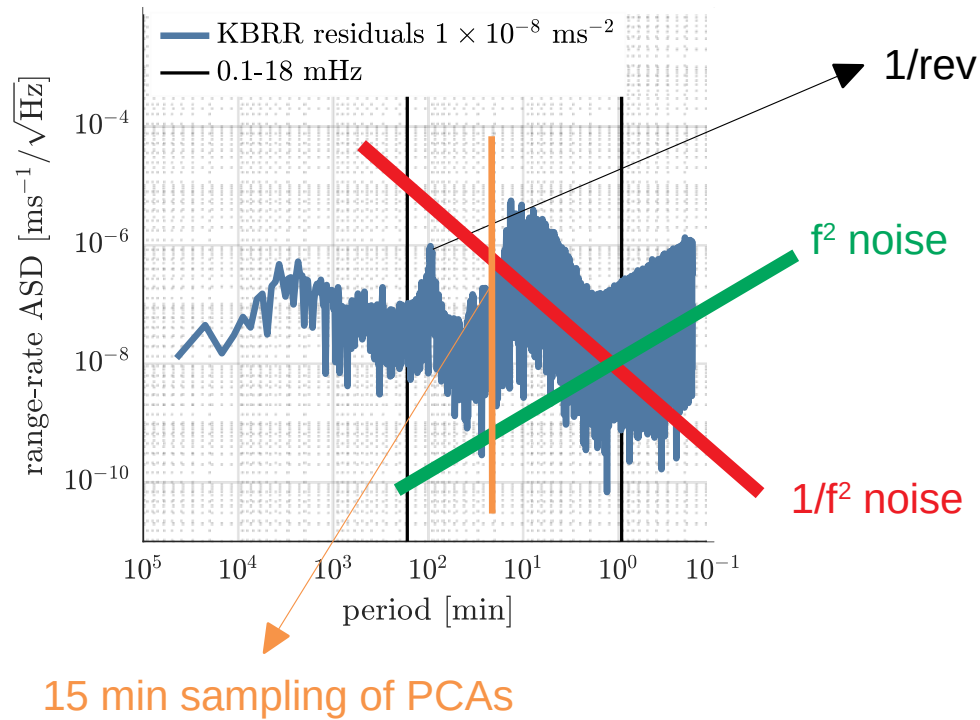
Results for VCE on constraints – post-fit residuals

$$\hat{e} = l - A \hat{x}$$



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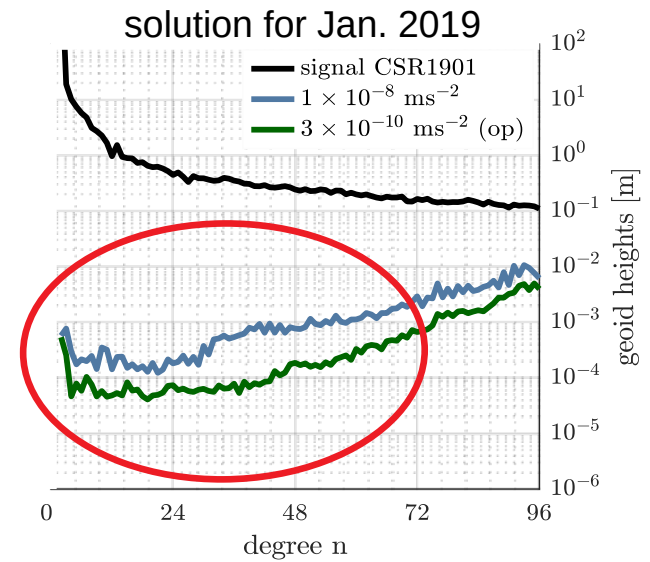
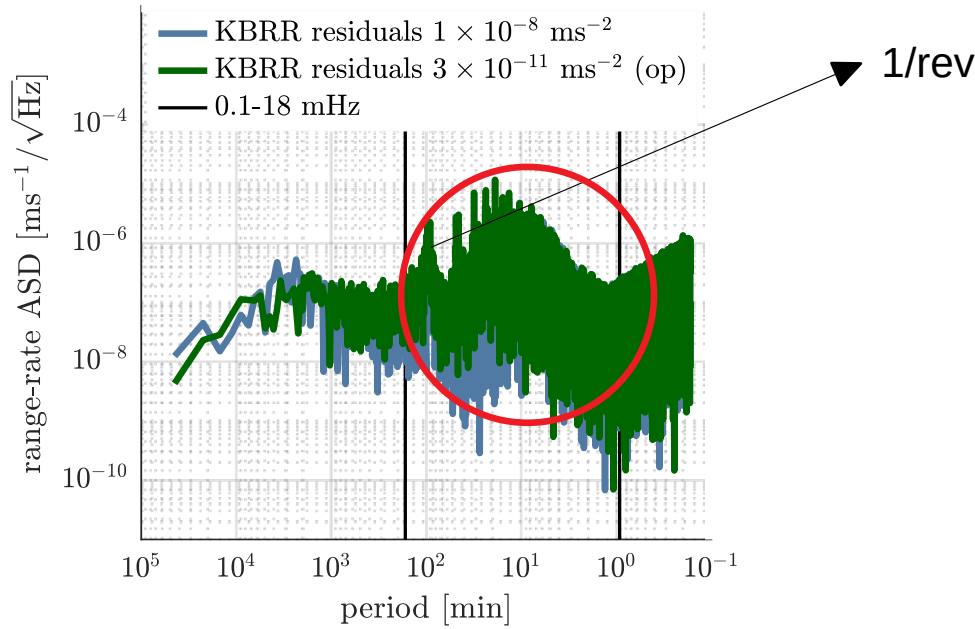
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15 min sampling of PCAs

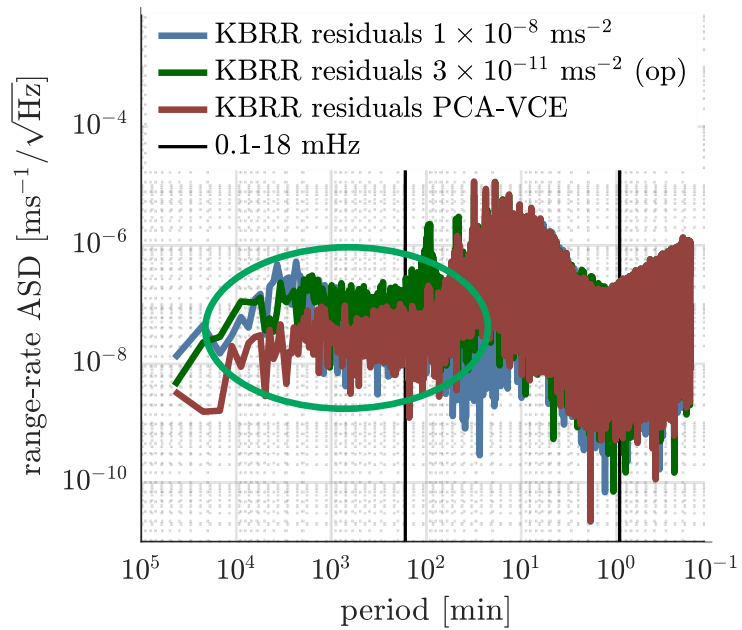
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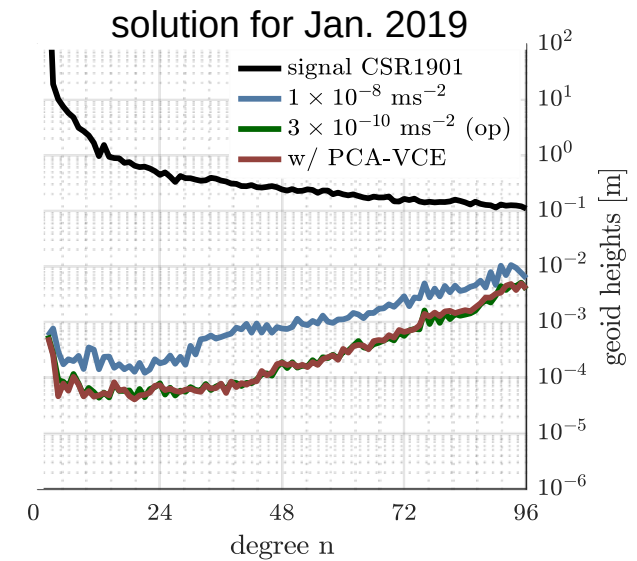


Results for VCE on constraints – post-fit residuals

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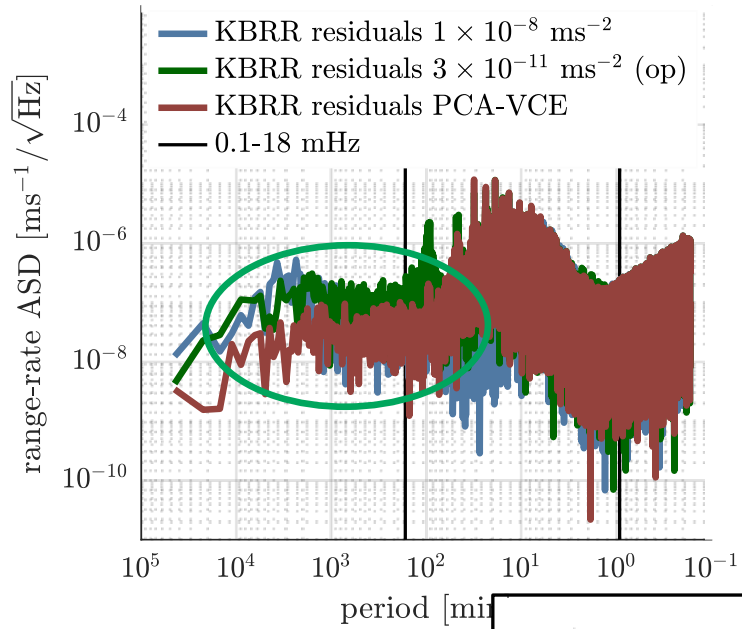


- almost flat spectrum for long periods
- indication for a “good” parametrisation of the respective signal



Results for VCE on constraints – summary

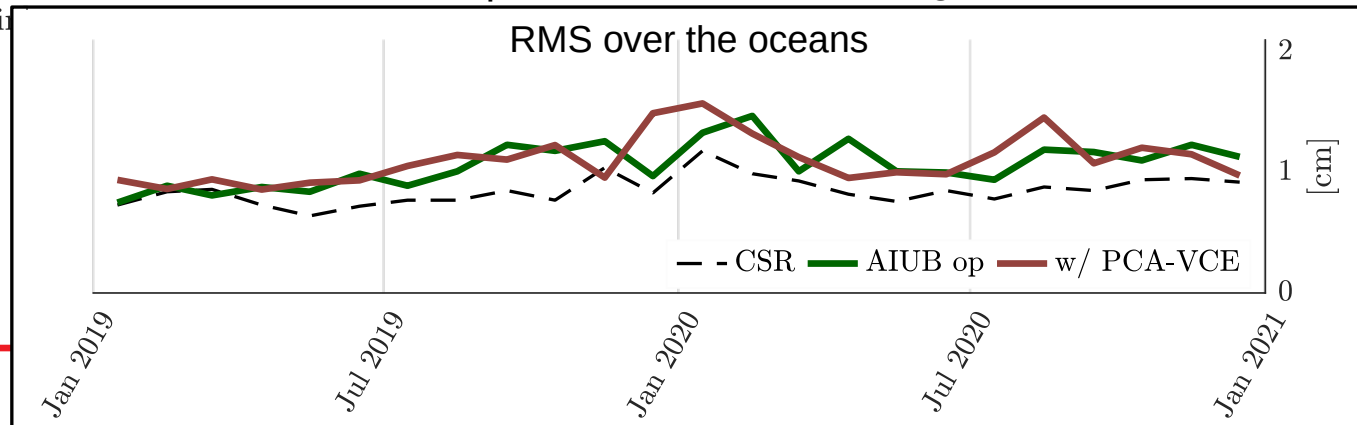
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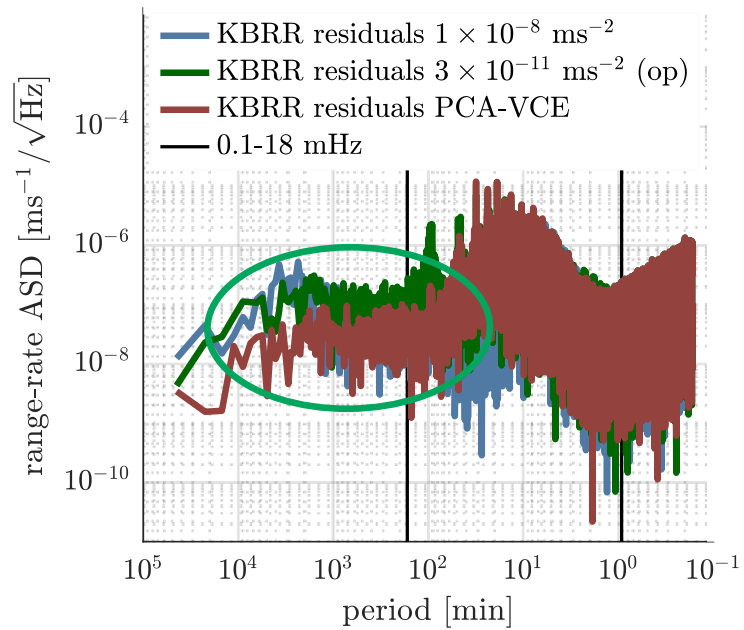
■ 12 months improved

■ 12 months degraded



Results for VCE on constraints – summary

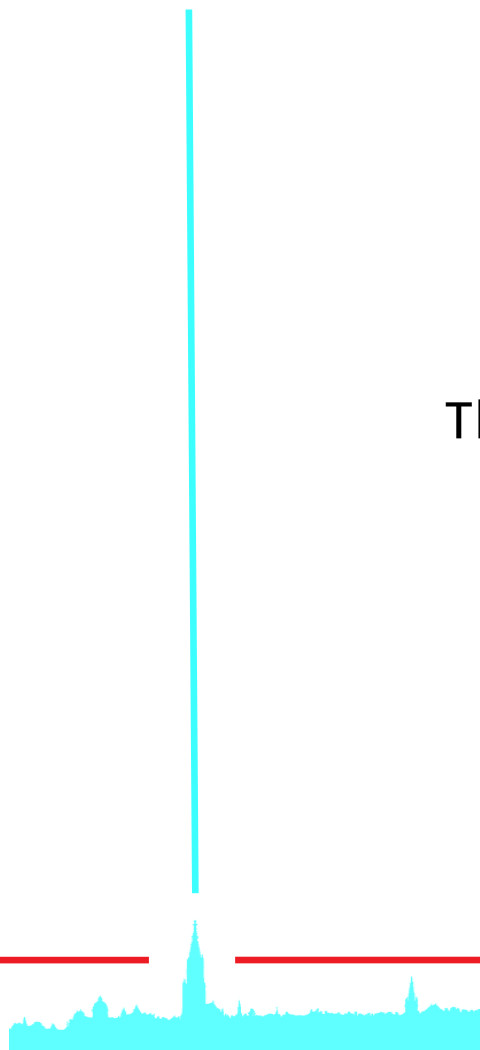
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- observation-based approach
- computed together with the solution
- provides a good solution (if PCAs sample correctly)

- computational efficiency...
- observation-based – outliers

Thank you for your attention



References

Beutler, G., Jäggi, A., Mervart, L. and Meyer, U. [2010]: The celestial mechanics approach: theoretical foundations. Journal of Geodesy, vol. 84(10), pp. 605-624. <https://doi.org/10.1007/s00190-010-0401-7>

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