Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery

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Astronomical Institute, University of Bern, Switzerland

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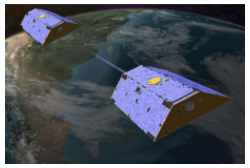
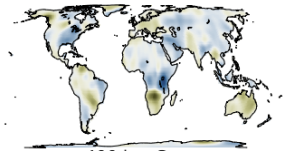
12 September 2022
Austin, TX, USA

Operational processing





Basic parametrisation

- initial conditions 2x[6]
- accelerometer bias 2x[3]
- accelerometer scaling 2x[3]

parameters per arc 24

Additional parameters

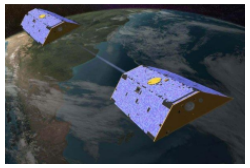
- 15 min PCA per satellite in
  - radial 2x[96]
  - along-track 2x[96]
  - cross-track 2x[96]

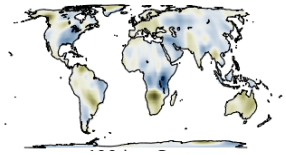
parameters per arc 576

in daily arcs (30 days):

- 18000 parameters,
- 17280 for the noise model
- + gravity field

Operational processing





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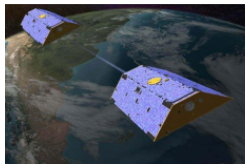
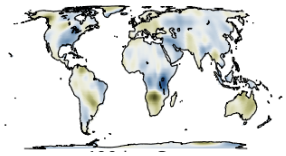
Force models	
Gravity field	Internal AIUB static GRACE field
Astromomic bodies	JPL DE421 (all planets + Pluto)
Mean pole	Linear
Solid Earth tides	IERS2010
Solid Earth pole tides	IERS2010
Ocean tides	FES2014b (+ admittances from TUG)
Ocean pole tides	Desai
Atmospheric tides	AOD RL06
Atmospheric & oeanic dealiasing	AOD RL06
Relativistic effects	IERS2010

Non-conservative forces:
ACT from TUG

VCE – constraints





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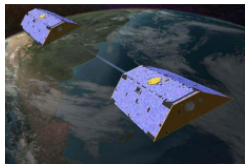
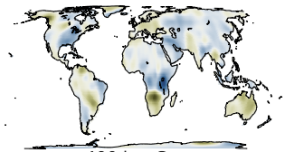
  Errors in background models will (mostly) sum up in 1/rev

→ frequently used in the Celestial Mechanics Approach [Beutler et al., 2010]

VCE – constraints





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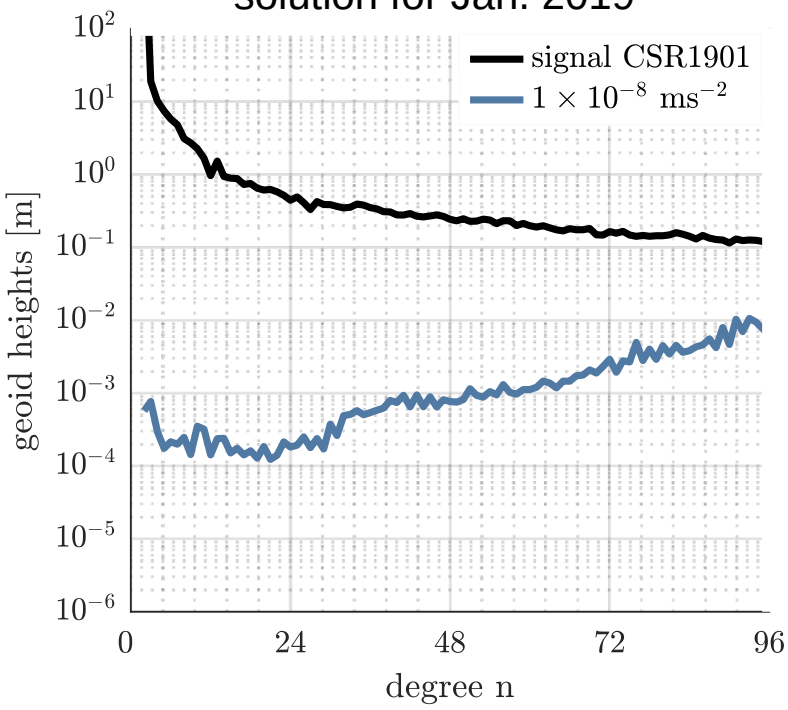
How to constrain their impact to the correct magnitude?

Impact of different constraints

loose
PCAs
may
become
large

solution for Jan. 2019



1×10^{-8} ms^{-2}

«loose» constraint

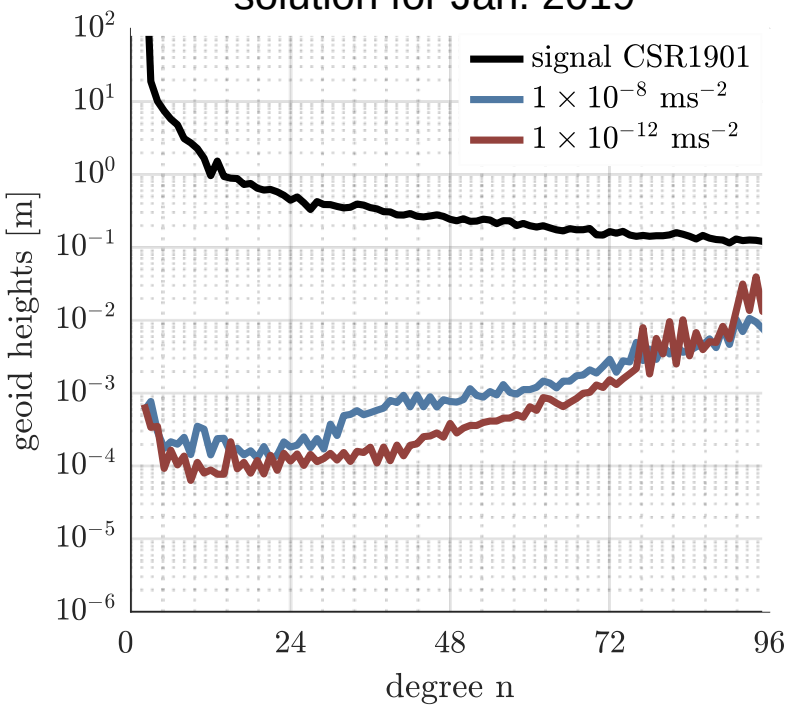
(gravity field signal absorbed in PCAs)

Impact of different constraints

loose
PCAs
may
become
large

tight
PCAs
strongly
confined

solution for Jan. 2019



1×10^{-12} ms^{-2}

«tight» constraint

(not enough to absorb mis-modellings)

Impact of different constraints

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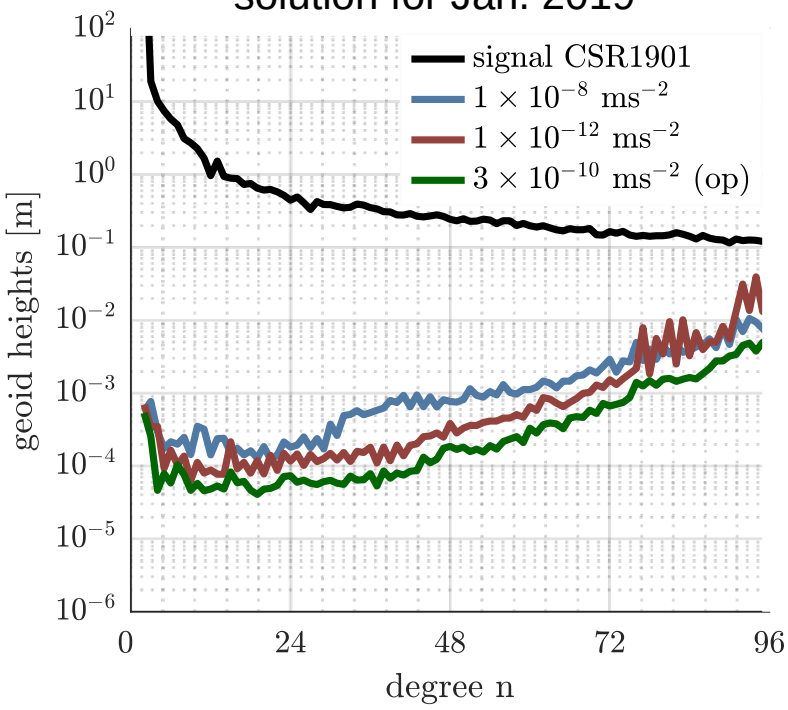
solution for Jan. 2019



3×10^{-10} ms^{-2}

reasonable balance

(applied in the operational solutions)

Constraining

$\mathbf{A}$ design matrix

$\mathbf{P}$ weight matrix

$\boldsymbol{l}$ obser-vations

$$\mathbf{N} = \left(\mathbf{A}^T \mathbf{P} \mathbf{A}\right) \quad \text{and} \quad \boldsymbol{b} = \mathbf{A}^T \mathbf{P} \boldsymbol{l} \longrightarrow \hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$$

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$$\boldsymbol{N} = \begin{bmatrix} \blacksquare & & \blacksquare & \blacksquare & & \blacksquare \\ & \blacksquare & & \blacksquare & \blacksquare & \\ \blacksquare & & \blacksquare & & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & & \blacksquare & & \blacksquare \\ & \blacksquare & \blacksquare & & \blacksquare & \\ \blacksquare & & \blacksquare & \blacksquare & & \blacksquare \end{bmatrix} + \begin{bmatrix} & & \\ & & \\ & & \\ & & \\ & & \blacksquare \\ & \blacksquare & \end{bmatrix}$$

$$\frac{\sigma_0^2}{\sigma_{PCA}^2},$$

$$\sigma_{PCA}^2 = \text{e.g., } 3 \times 10^{-10}\ \text{ms}^{-2}$$

VCE and constraints

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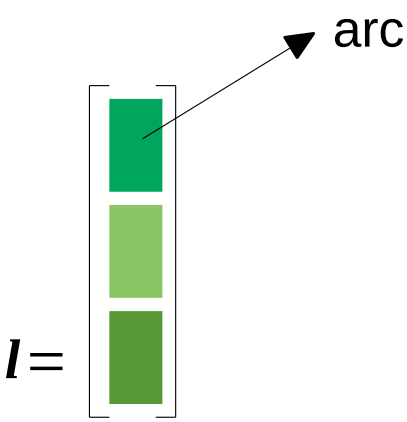
- The observations of each arc are used to set up the normal equations (NEQs)



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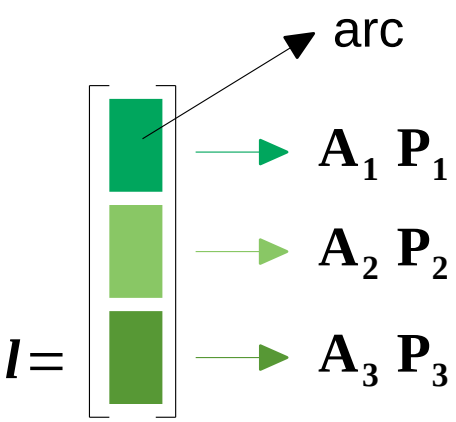

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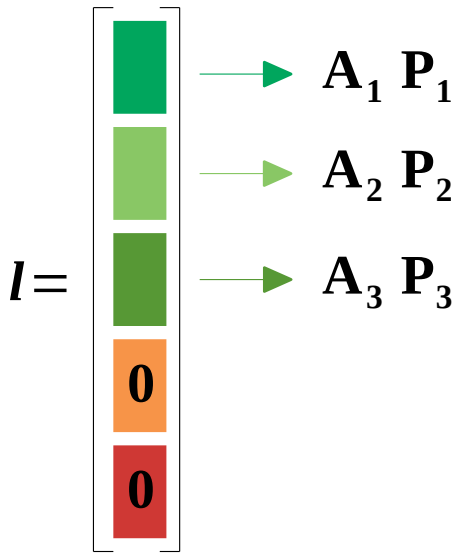

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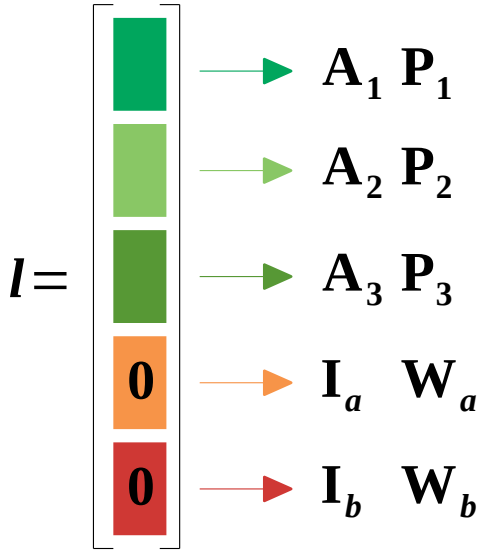

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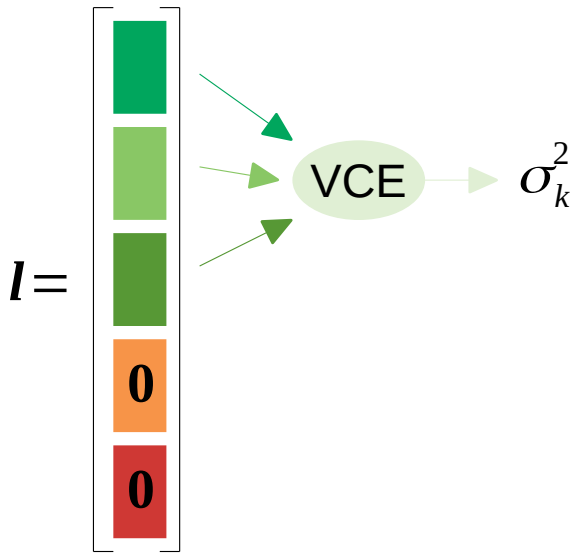
VCE: Each group of observations gets a weight based on its contribution to the final solution



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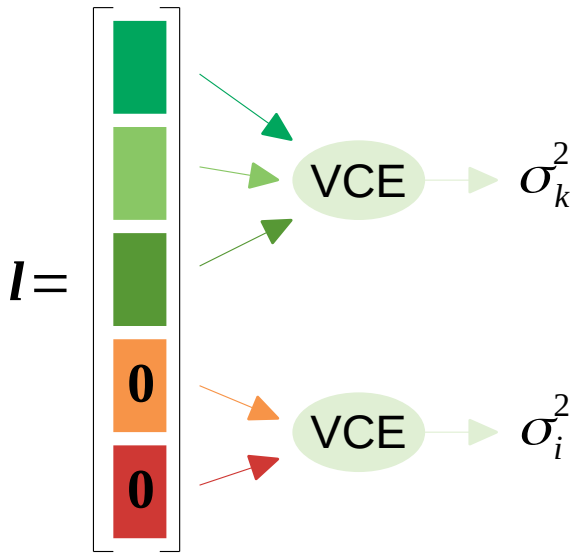

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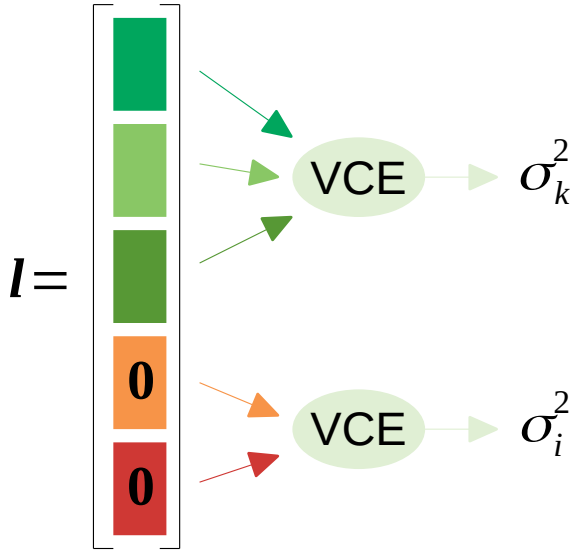

$$\hat{\boldsymbol{x}} = \left(\sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} \mathbf{N}_k + \sum_{i=1}^{I=2} \frac{\sigma_0^2}{\sigma_i^2} \mathbf{W}_i \right)^{-1} \sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} \boldsymbol{b}_k$$

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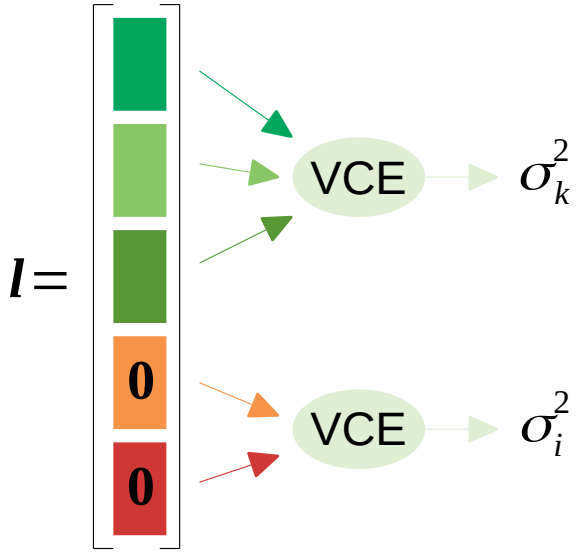

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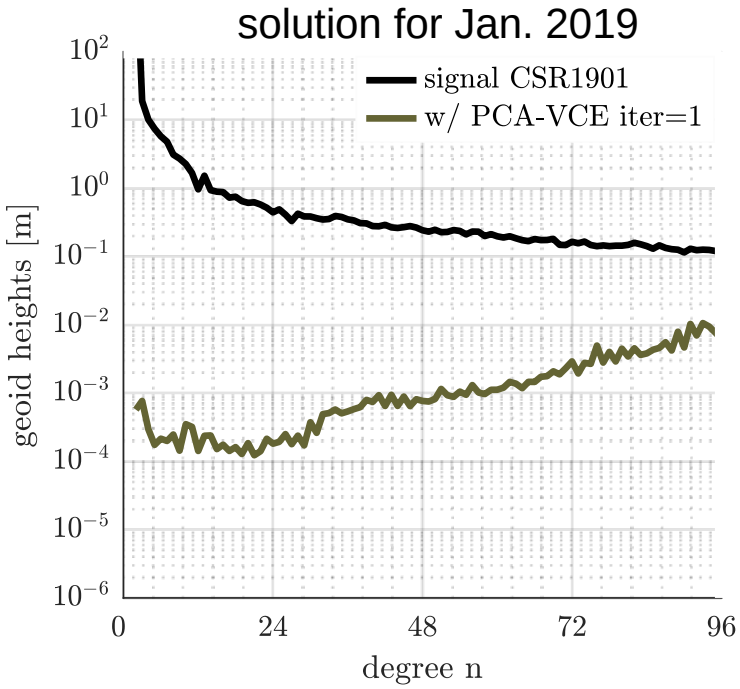
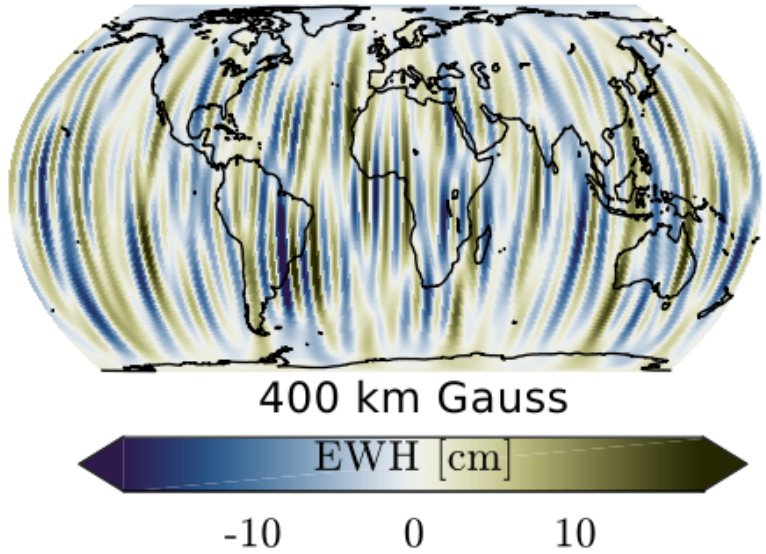
information about observations introduced via σ_0^2

Results for VCE on constraints





radial 1×10^{-8} ms^{-2} along-track 1×10^{-8} ms^{-2} cross-track 1×10^{-8} ms^{-2}

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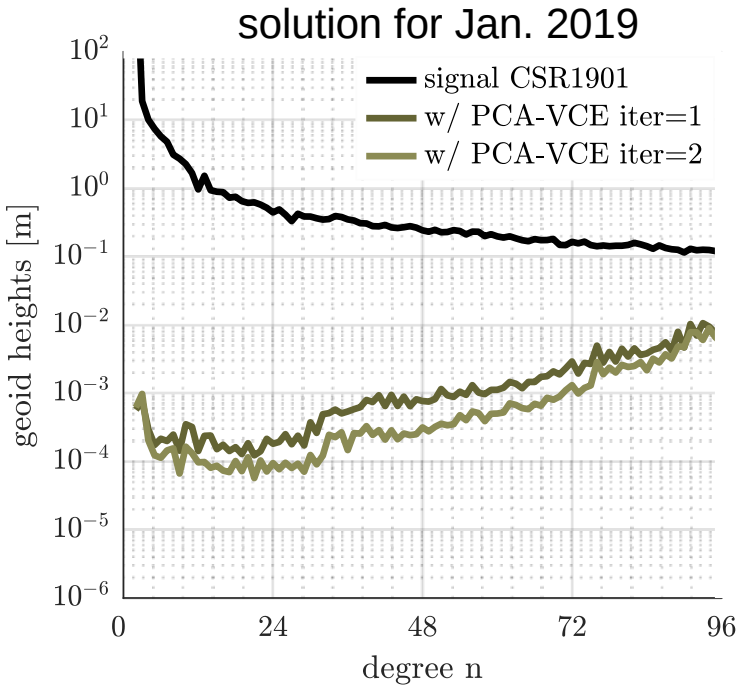

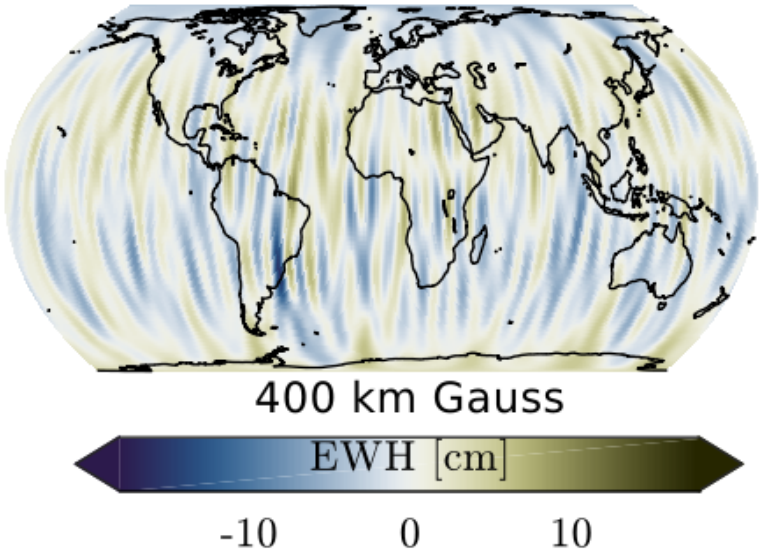

radial 1.9×10^{-8} ms^{-2} along-track 9.8×10^{-9} ms^{-2} cross-track 8.6×10^{-9} ms^{-2}

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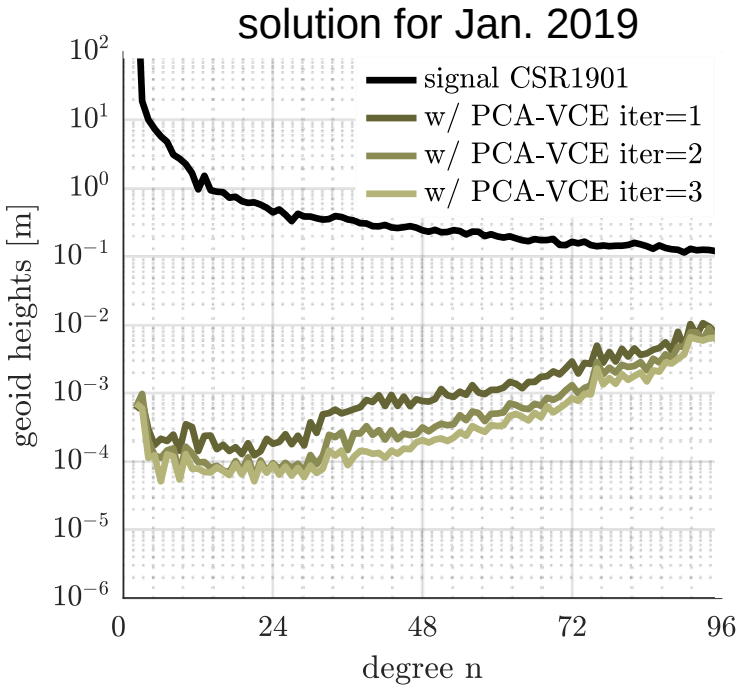

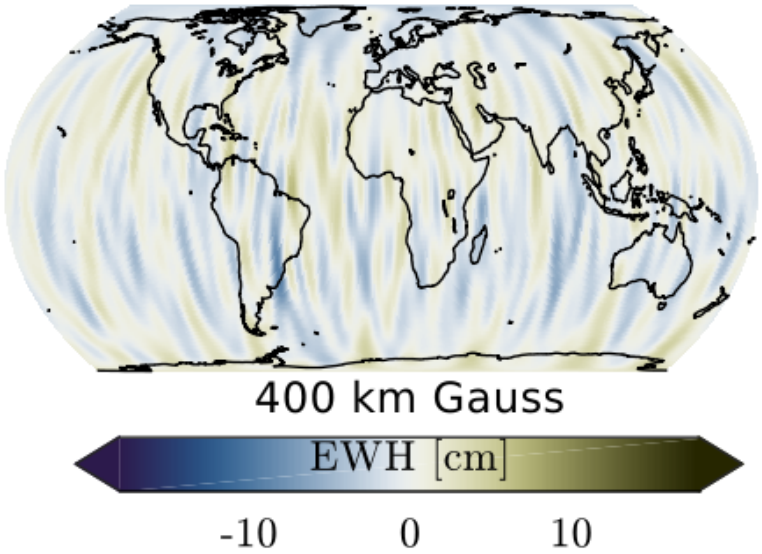

radial 2.9×10^{-9} ms^{-2} along-track 1.5×10^{-9} ms^{-2} cross-track 1.3×10^{-9} ms^{-2}

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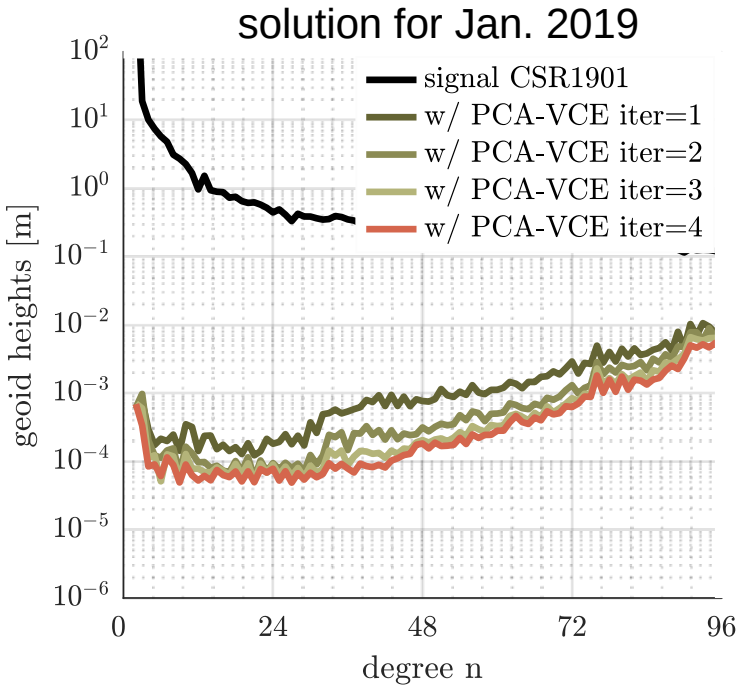

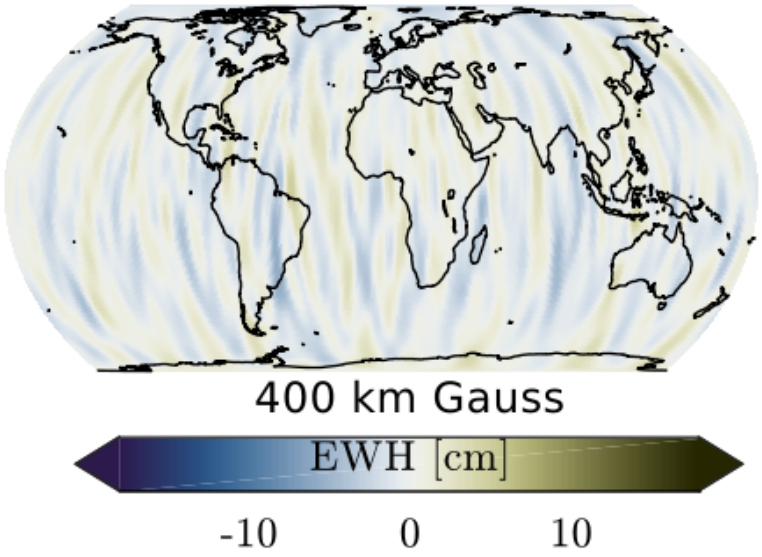

radial $1.2\times10^{-9}\ \mathrm{ms}^{-2}$ along-track $6.2\times10^{-10}\ \mathrm{ms}^{-2}$ cross-track $6.9\times10^{-10}\ \mathrm{ms}^{-2}$

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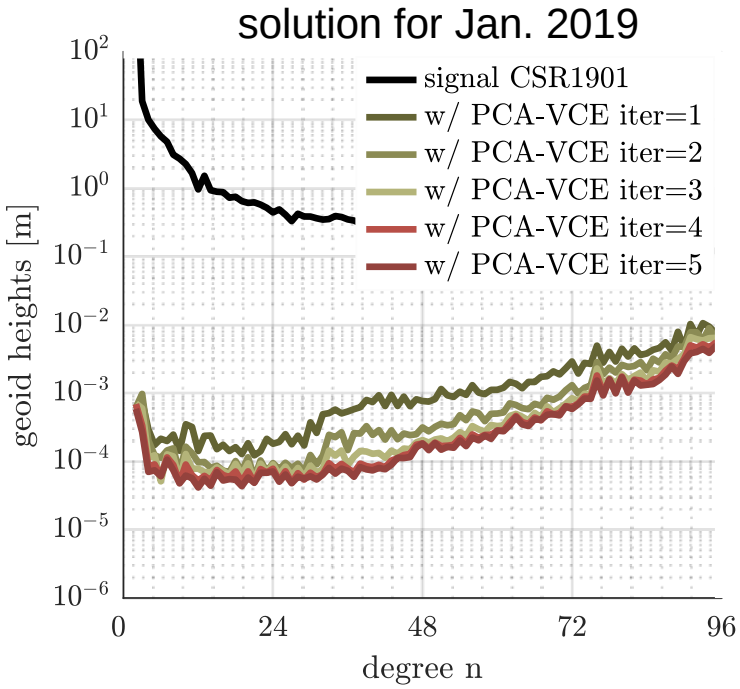
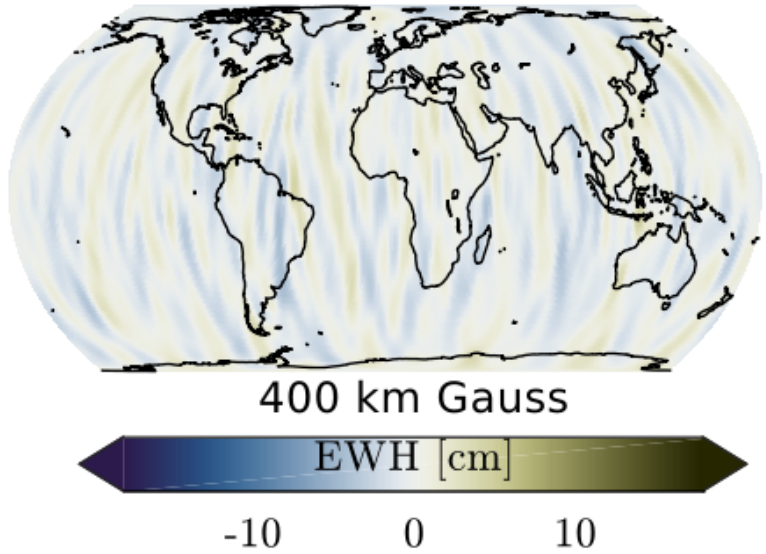
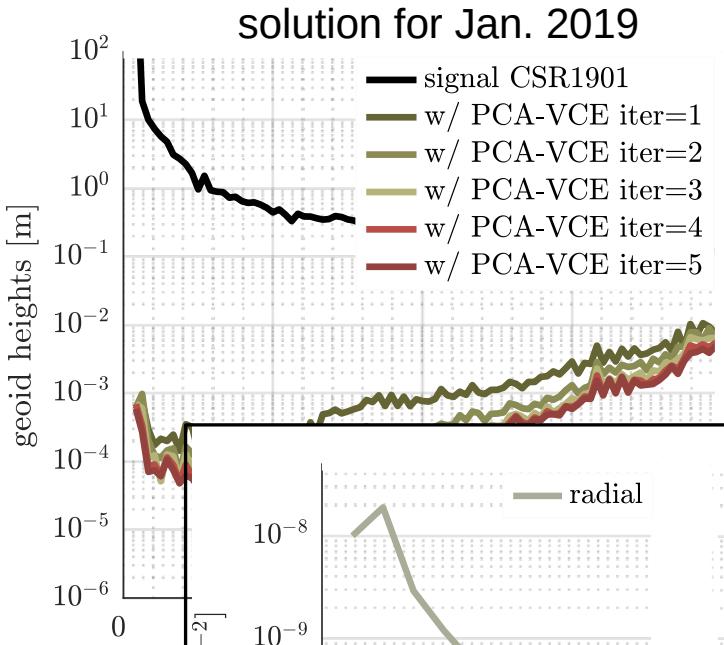
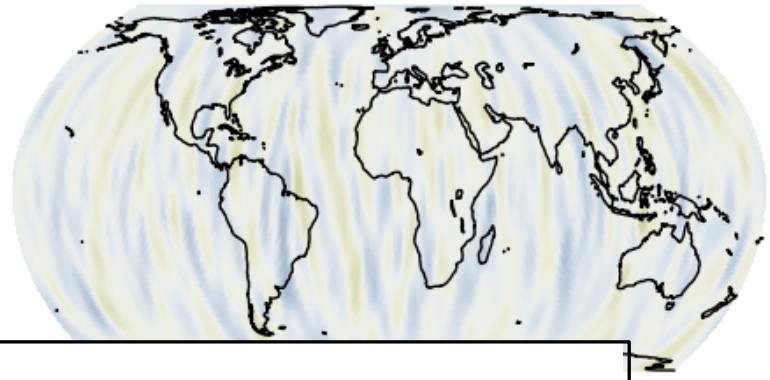


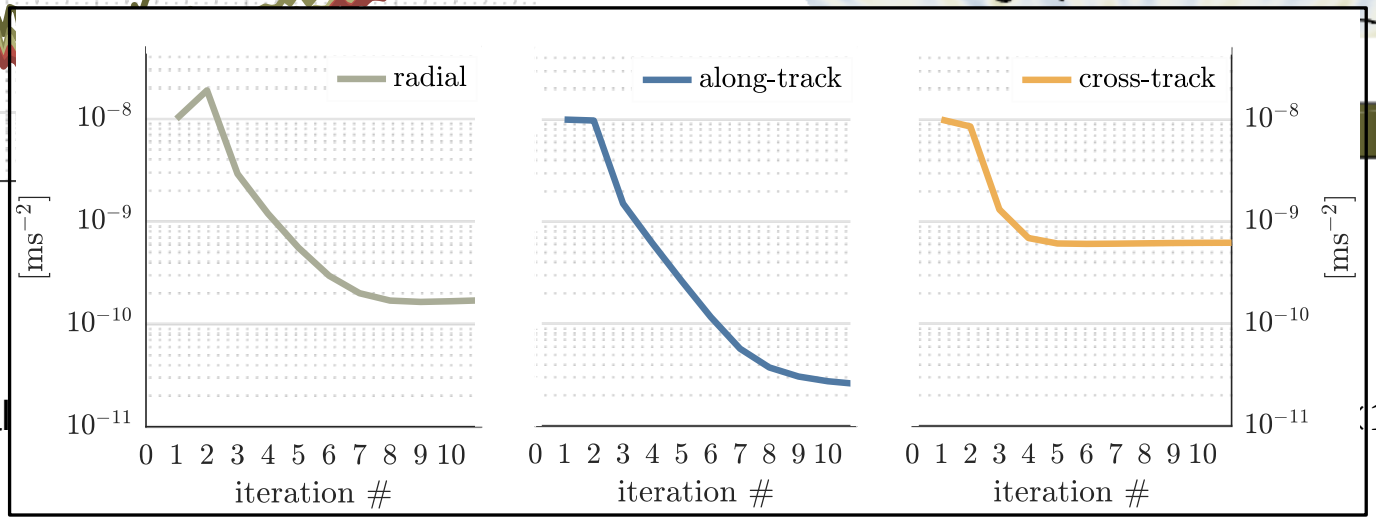


radial $5.6\times10^{-10}\,\mathrm{ms}^{-2}$ along-track $2.6\times10^{-10}\,\mathrm{ms}^{-2}$ cross-track $6.1\times10^{-10}\,\mathrm{ms}^{-2}$

Results for VCE on constraints





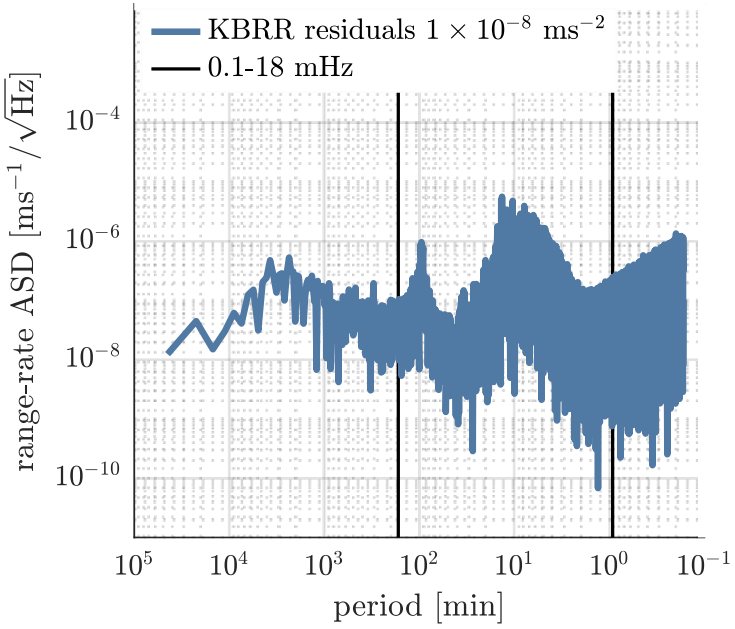


radial

10^{-10} ms^{-2}

Results for VCE on constraints – post-fit residuals

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



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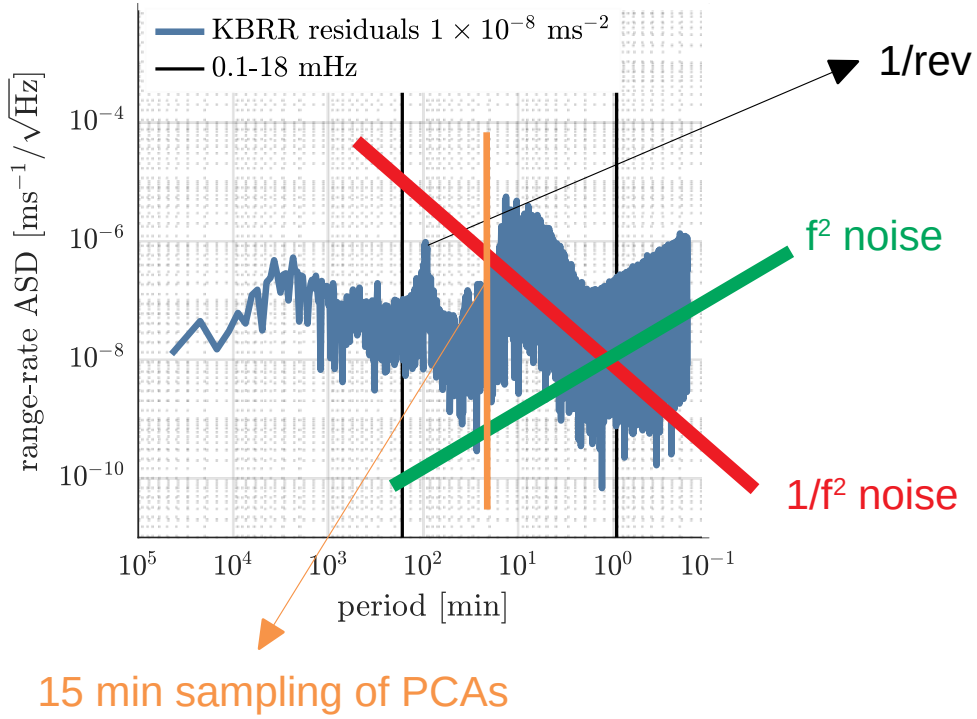

15 min sampling of PCAs

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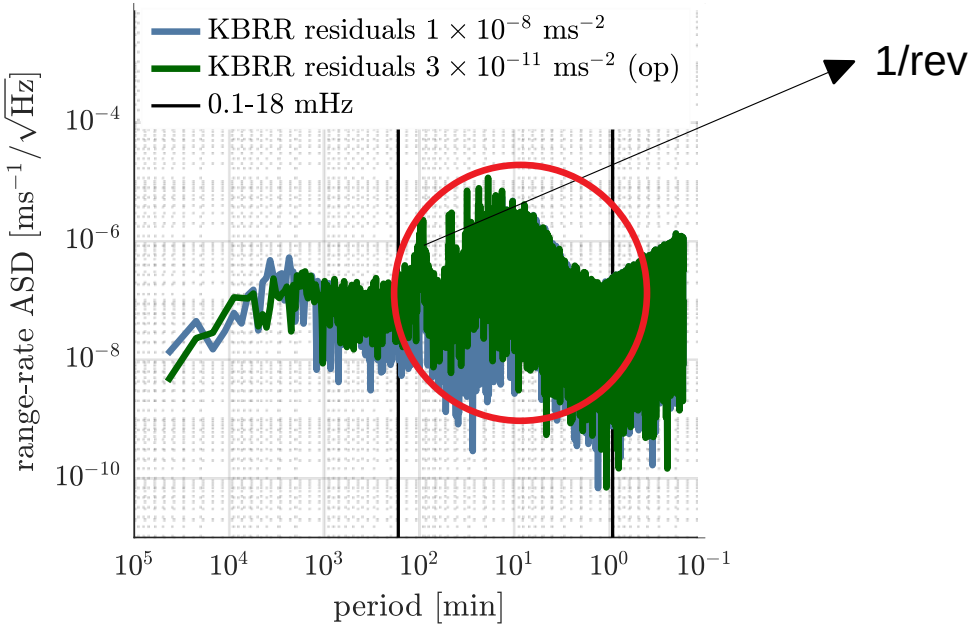
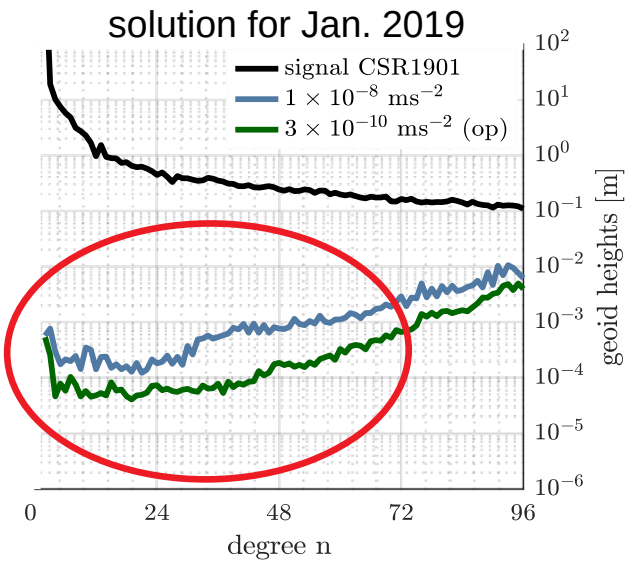
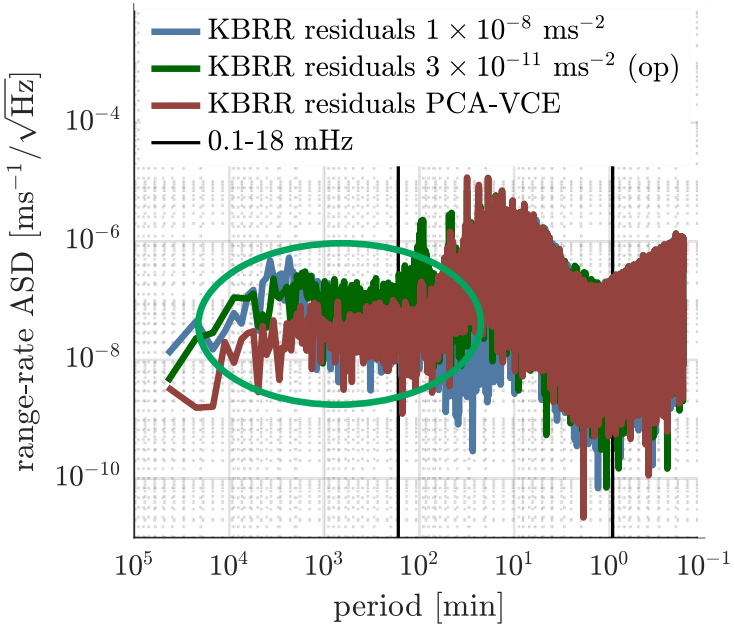
$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$





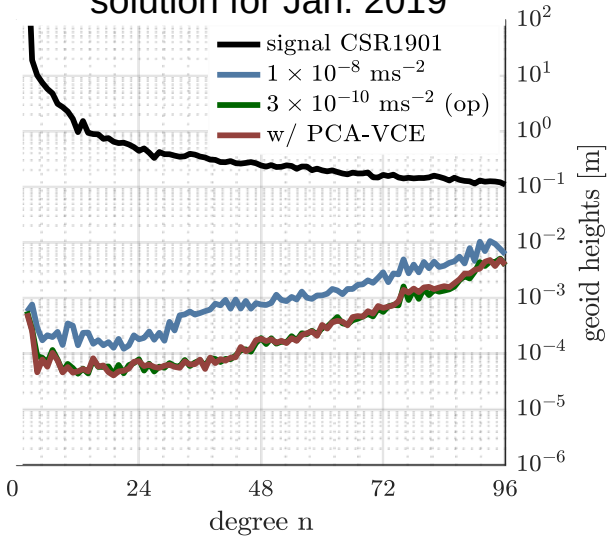
Results for VCE on constraints – post-fit residuals

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



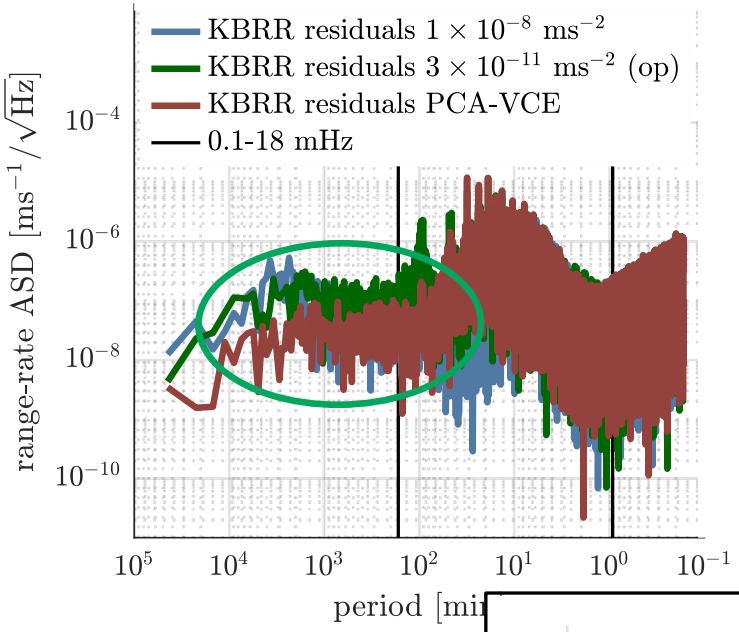
- almost flat spectrum for long periods
- indication for a “good” parametrisation of the respective signal

solution for Jan. 2019



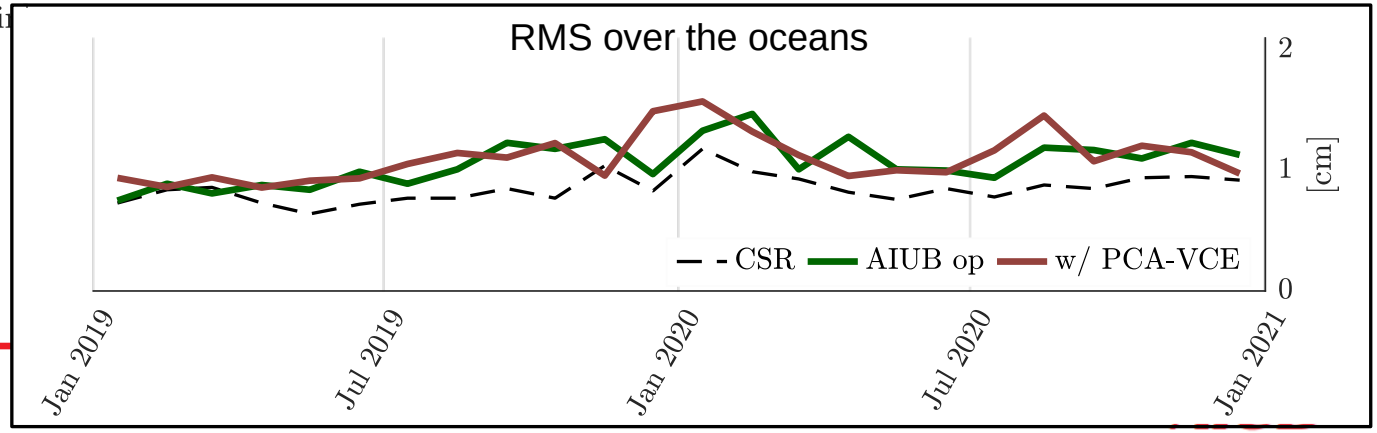
Results for VCE on constraints – summary

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



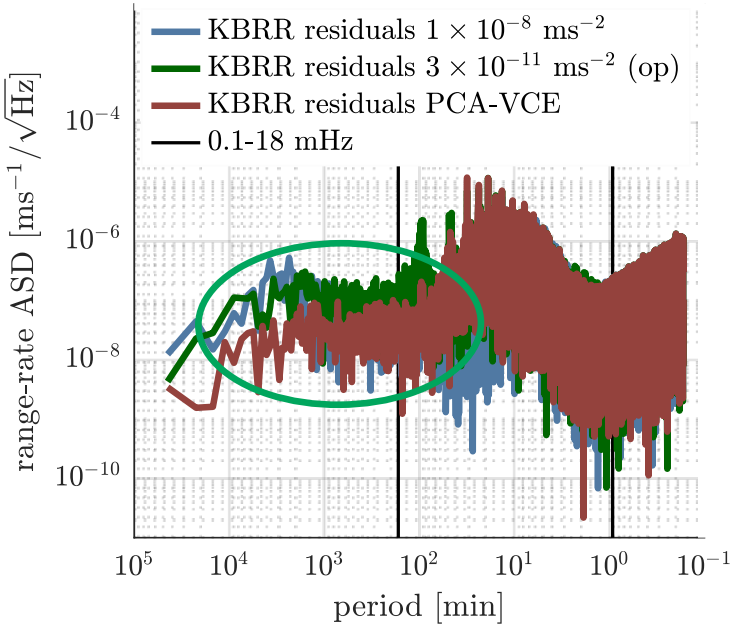
- almost flat spectrum for long periods
- indication for a “good” parametrisation of the respective signal

- 12 months improved
- 12 months degraded



Results for VCE on constraints – summary

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



- observation-based approach
- computed together with the solution
- provides a good solution (if PCAs sample correctly)

- computational efficiency...
- observation-based – outliers

Thank you for your attention

AIUB

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