Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling

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## Operational processing

<table>
<thead>
<tr>
<th>Basic parametrisation</th>
<th>Additional parameters</th>
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<td>15 min PCA per satellite in radial 2x[96]</td>
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<td>cross-track 2x[96]</td>
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- parameters per arc 24
- parameters per arc 576

- frequently used in the Celestial Mechanics Approach [Beutler et al., 2010]

- in daily arcs (30 days):
  - 18000 parameters,
  - 17280 for the noise model
  - + gravity field
Operational processing

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Forces models

- Gravity field: Internal AIUB static GRACE field
- Astromomic bodies: JPL DE421 (all planets + Pluto)
- Mean pole: Linear
- Solid Earth tides: IERS2010
- Solid Earth pole tides: IERS2010
- Ocean tides: FES2014b (+ admittances from TUG)
- Ocean pole tides: Desai
- Atmospheric tides: AOD RL06
- Atmospheric & oceanic dealiasing: AOD RL06
- Relativistic effects: IERS2010

Non-conservative forces:
- ACT from TUG

in daily arcs (30 days):
- 18000 parameters,
- 17280 for the noise model
- + gravity field
Processing – operational solution

\[ \mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A}) \]
\[ \mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l} \]
\[ \hat{x} = \mathbf{N}^{-1} \mathbf{b} \]
Processing – operational solution

\[ N = (A^T P A) \]
\[ b = A^T P l \]
\[ \hat{x} = N^{-1} b \]

Features
- updated background models
- data screening with variance component estimation
- use of alternative GF2 Level 1B ACT product from TUG

Facts
- operational since September 2020
- available at ICGEM as AIUB-GRACE-FO_operational
- continuation of AIUB-RL02
- current status: 49 months from June 2018 until August 2022
Empirical Noise Modelling
Empirical noise modelling based on post-fit residuals

\[ \hat{\mathbf{e}} = \mathbf{l} - \mathbf{A} \hat{\mathbf{x}} \]

\[ \mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l} \]

\[ \hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{b} \]

- information in the residuals (noise?)
- information in the parameters (signal?)

Least-squares:
\[ \mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A}) \]
\[ \mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l} \]
\[ \hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{b} \]
What do we expect from residuals?

\[ \hat{e} = l - A \hat{x} \]

The estimator is BLUE (best – linear – unbiased) if

- \( E(\hat{x}) = x \)
- \( E(e|\epsilon) = E(\hat{e}) = 0 \)
- \( D(e|\epsilon, \sigma_0^2) = D(l|\sigma_0^2) = \sigma_0^2 P^{-1} \)
  \[ = \sigma_0^2 Q_{ee} = \sigma_0^2 Q_{ll} \]
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\[
P = \frac{1}{\sigma_0^2} I\]
What do we expect from residuals?

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- \( \mathbb{D}(e|x, \sigma_0^2) = \mathbb{D}(l|\sigma_0^2) = \sigma_0^2 \mathbf{P}^{-1} = \sigma_0^2 \mathbf{Q}_{ee} = \sigma_0^2 \mathbf{Q}_{ll} \)

\[ \mathbf{P} = \frac{1}{\sigma_0^2} \mathbf{I} \]
What do we expect from residuals?

\[ \hat{e} = I - A \hat{x} \]

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\[ \hat{x} = N^{-1} b \quad \text{unbiased} \]

\[ C_{\hat{x}\hat{x}} = \sigma_0^2 \left( A^T P A \right)^{-1} \quad \text{biased} \]

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What do we expect from residuals?

\[ \hat{e} = l - A \hat{x} \]

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Auto-covariance function

\[ \hat{e} = I - A \hat{x} \quad \text{(post-fit residuals)} \]

\[ \text{cov}(\Delta t_k) = \frac{1}{N} \sum_{i=0}^{N} \hat{e}(t_i) \hat{e}(t_i + \Delta t_k) \]
Auto-covariance function

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- stationarity assumed
- biased estimation of auto-covariance
  $\rightarrow$ covariance matrix nondegenerate
\[ \hat{e} = l - A \hat{x} \quad \text{(post-fit residuals)} \]

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- stationarity assumed
- biased estimation of auto-covariance
  \rightarrow \text{covariance matrix nondegenerate}
Auto-covariance function and weight matrix

\[ \hat{e} = I - A \hat{x} \quad \text{(post-fit residuals)} \]

\[ \text{cov} (\Delta t_k) = \frac{1}{N} \sum_{i=0}^{N} \hat{e}(t_i) \hat{e}(t_i+\Delta t_k) \]

- stationarity assumed
- biased estimation of auto-covariance \( \rightarrow \) covariance matrix nondegenerate
Auto-covariance function – examples

\[ \hat{e} = I - A \hat{x} \]
Auto-covariance function – examples

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\[ \text{cov}(\Delta t_k) = \frac{1}{N} \sum_{i=0}^{N} \hat{e}(t_i)\hat{e}(t_i + \Delta t_k) \]
Auto-covariance function – examples

\[ \hat{e} = I - A \hat{x} \]

\[ \text{cov}(\Delta t_k) = \frac{1}{N}. \]

\[ \sum_{i=0}^{N} \hat{e}(t_i)\hat{e}(t_i + \Delta t_k) \]

15 min sampling of PCAs

1/rev

f^2 noise

1/f^2 noise

range-rate ASD [ms^-1/\sqrt{Hz}]

KBRR residuals w/ PCAs (op)

emperical model

0.1-18 mHz
Auto-covariance function – examples

\[ \hat{e} = I - A \hat{x} \]

\[ \text{cov}(\Delta t_k) = \frac{1}{N} \sum_{i=0}^{N} \hat{e}(t_i)\hat{e}(t_i + \Delta t_k) \]

truncation at d/o=60 (artefact → bias) is reflected
Results of empirical modelling – gravity field

Solution for Jan. 2019

- formal errors too optimistic compared to the assessed noise

[Graph showing geoid heights vs. degree n with two lines: signal CSR1901 and AIUB op]
Results of empirical modelling (K-band) – gravity field

- formal errors reflect assessed noise very well
- including features of resonance orders
Results of empirical modelling (KIN) – gravity field

- Formal errors reflect assessed noise very well
- Basically the formal error curve is shifted
Results of empirical modelling (KIN & K-band) – gravity field

- formal errors reflect assessed noise very well
- including features of resonance orders
Results of empirical modelling – formal errors
Results of empirical modelling - formal errors

Solution for Jan. 2019

- Signal CSR1901
- AIUB op
- Emp KIN & K-band

Degree n vs. geoid heights [m]

Empirical modelling results for Jan. 2019
Results of empirical modelling – RMS over the ocean
Results of empirical modelling – RMS over the ocean
Results of empirical modelling – RMS over the ocean
Results of empirical modelling – summary

- possible on any (stationary) residuals time series
- additional parameters can be reduced as stationary behaviour can be absorbed
- formal errors much more realistic and show resonance orders (if correlation length > 3 h)
- no constraints needed
- no/few a priori knowledge needed

- iterations required (might be time consuming)
- memory consumption and inversion time dependent on length of auto-covariance function
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- Combination not at the level of the best individual solutions
Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability

- (Combination not at the level of the best individual solutions)
- Combination significantly improved by the new processing scheme of one analysis centre
Thank you for your attention
References


