Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery

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Introduction

basic parametrisation:
- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24
Introduction

Perturbation theory [Kim, 2000]: Errors in background models will (mostly) sum up in 1/rev → frequently used in the Celestial Mechanics Approach [Beutler et al., 2010]

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<td>parameters per arc      24</td>
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<tr>
<td>• 15 min PCA per satellite in</td>
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<td>• radial    2x(96)</td>
</tr>
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<td>• along-track 2x(96)</td>
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<td>• cross-track 2x(96)</td>
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<td>parameters per arc      576</td>
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in daily arcs (30 days):
• 18000 parameters,
• 17280 for the noise model
• + gravity field
Introduction

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How to constrain their impact to the correct magnitude?

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in daily arcs (30 days):
• 18000 parameters,
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• + gravity field
Impact of different constraints

solution for Jan. 2019

1 × 10⁻⁸ m⁻²
«loose» constraint

(gravity field signal absorbed in PCAs)
Impact of different constraints

solution for Jan. 2019

1 $\times 10^{-12}$ ms$^{-2}$

«tight» constraint

(not enough to absorb mis-modellings)
Impact of different constraints

solution for Jan. 2019

3×10^{-10} \text{ ms}^{-2}

«reasonable» balance

(applied in the operational solutions)
Constraining

\[ \mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A}) \quad \text{and} \quad \mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l} \]

\[ \hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{b} \]

- design matrix
- weight matrix
- vector of observations
Constraining

\[ N = (A^T P A) \quad \text{and} \quad b = A^T P l \quad \rightarrow \quad \hat{x} = N^{-1} b \]

\[ N = (A^T P A + W) \]

\[ N = \begin{bmatrix} \text{Matplotlib} \\
\text{grid} \\
\text{plot} \end{bmatrix} + \begin{bmatrix} \text{PCA noise} \end{bmatrix} \]

\[ \sigma_{PCA}^2 = \text{e.g., } 3 \times 10^{-10} \text{ ms}^{-2} \]
Variance Component Estimation

\[ N = (A^T PA) \quad \text{and} \quad b = A^T Pl \quad \Rightarrow \quad \hat{x} = N^{-1}b \]

\[ N = (A^T PA + W) \]
Variance Component Estimation

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VCE: Each group of observations gets a weight based on its contribution to the final solution.
Variance Component Estimation

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VCE: Each group of observations gets a weight based on its contribution to the final solution.

\[ l = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

\[ \sigma_k^2 \]

\[ \sigma_i^2 \]
Variance Component Estimation

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VCE: Each group of observations gets a weight based on its contribution to the final solution.

The equation for \( \hat{x} \) is:

\[ \hat{x} = \left( \sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} N + \sum_{i=1}^{I=2} \frac{\sigma_0^2}{\sigma_i^2} W_i \right)^{-1} \sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} b_k \]

This represents the information about the observations introduced via \( \sigma_0^2 \).
Results

solution for Jan. 2019

radial $1 \times 10^{-8}$

along-track $1 \times 10^{-8}$

cross-track $1 \times 10^{-8}$
Results

solution for Jan. 2019

<table>
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<th>Value</th>
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<tr>
<td>radial</td>
<td>$1.9 \times 10^{-8}$ ms$^{-2}$</td>
</tr>
<tr>
<td>along-track</td>
<td>$9.8 \times 10^{-9}$ ms$^{-2}$</td>
</tr>
<tr>
<td>cross-track</td>
<td>$8.6 \times 10^{-9}$ ms$^{-2}$</td>
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Results

solution for Jan. 2019

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<tr>
<td>radial</td>
<td>$2.9 \times 10^{-9}$ ms$^{-2}$</td>
</tr>
<tr>
<td>along-track</td>
<td>$1.5 \times 10^{-9}$</td>
</tr>
<tr>
<td>cross-track</td>
<td>$1.3 \times 10^{-9}$</td>
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Results

solution for Jan. 2019

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<tr>
<td>radial</td>
<td>$1.2 \times 10^{-9}$ ms$^{-2}$</td>
</tr>
<tr>
<td>along-track</td>
<td>$6.2 \times 10^{-10}$</td>
</tr>
<tr>
<td>cross-track</td>
<td>$6.9 \times 10^{-10}$</td>
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Results

solution for Jan. 2019

\[
\begin{array}{ll}
\text{radial} & 5.6 \times 10^{-10} \\
\text{along-track} & 2.6 \times 10^{-10} \\
\text{cross-track} & 6.1 \times 10^{-10}
\end{array}
\]
Results

solution for Jan. 2019

- radial: $5.6 \times 10^{-10}$ ms$^{-2}$
- along-track: $2.6 \times 10^{-10}$ ms$^{-2}$

Graph showing geoid heights in meters as a function of degree and iteration.

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Conclusion

- observation-based approach
- computed together with the solution
- provides a good solution (if PCAs sample correctly)
- improvement...

- computational efficiency?
- observation-based – outliers
- improvement...
References


