EGU General Assembly 2022

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Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery

Martin Lasser, Ulrich Meyer, Daniel Arnold, Adrian Jäggi

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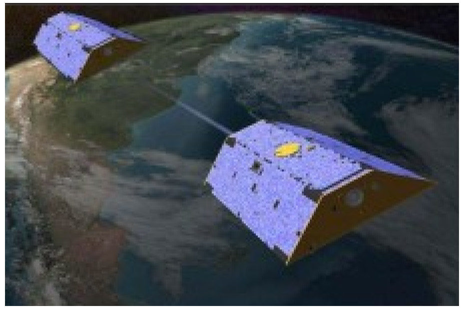
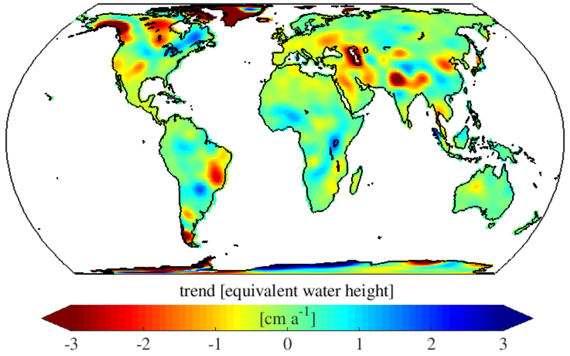
EGU General Assembly 2022
May 23-27, 2022
Vienna, Austria

Introduction



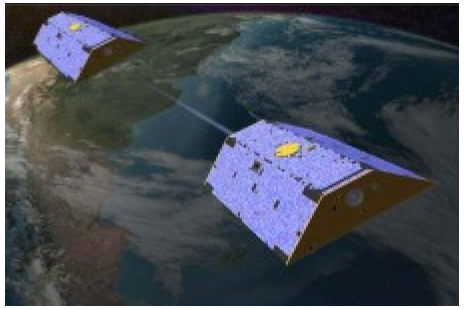


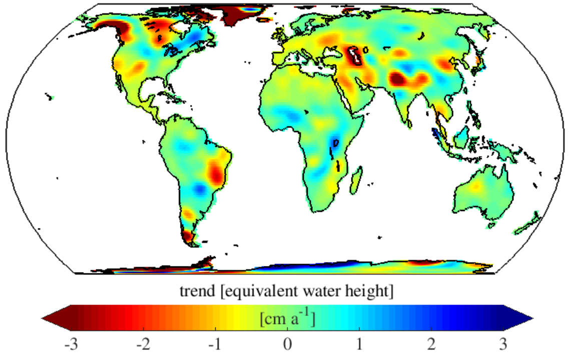
basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelermeter scaling 2x(3)

parameters per arc 24

Introduction





Perturbation theory [Kim, 2000]:
Errors in background models will (mostly) sum up in 1/rev

→ frequently used in the Celestial Mechanics Approach [Beutler et al., 2010]

basic parametrisation:

- initial conditions 2x(6)
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additional parameters:

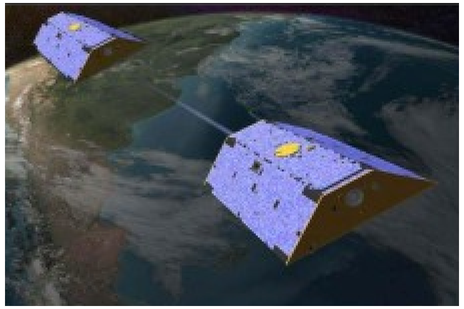
- 15 min PCA per satellite in
  - ➔ radial 2x(96)
  - ➔ along-track 2x(96)
  - ➔ cross-track 2x(96)

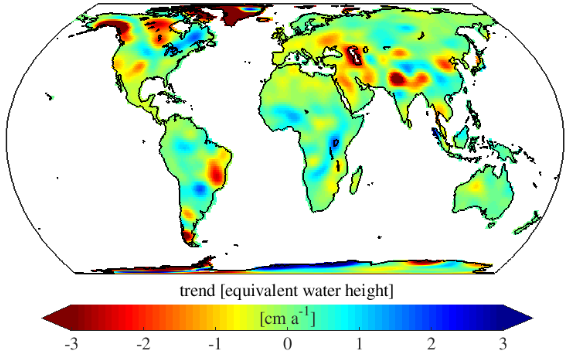
parameters per arc 576

in daily arcs (30 days):

- 18000 parameters,
- 17280 for the noise model
- + gravity field

Introduction





Perturbation theory [Kim, 2000]:
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[Beutler et al., 2010]

How to constrain their impact to the correct magnitude?

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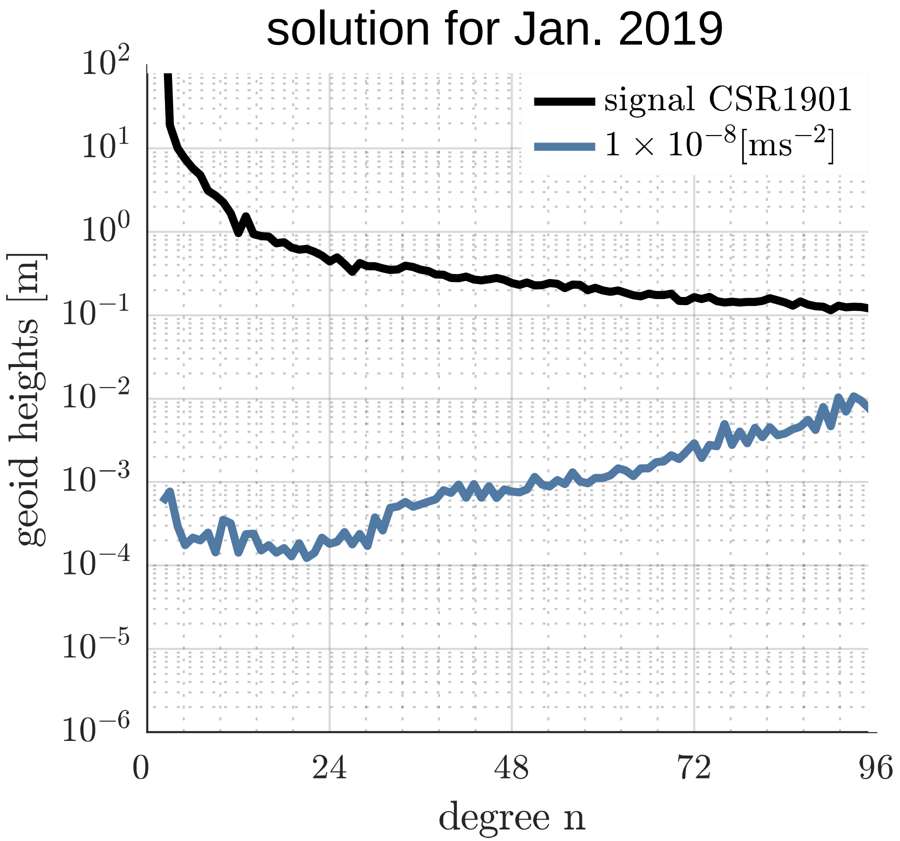
Impact of different constraints

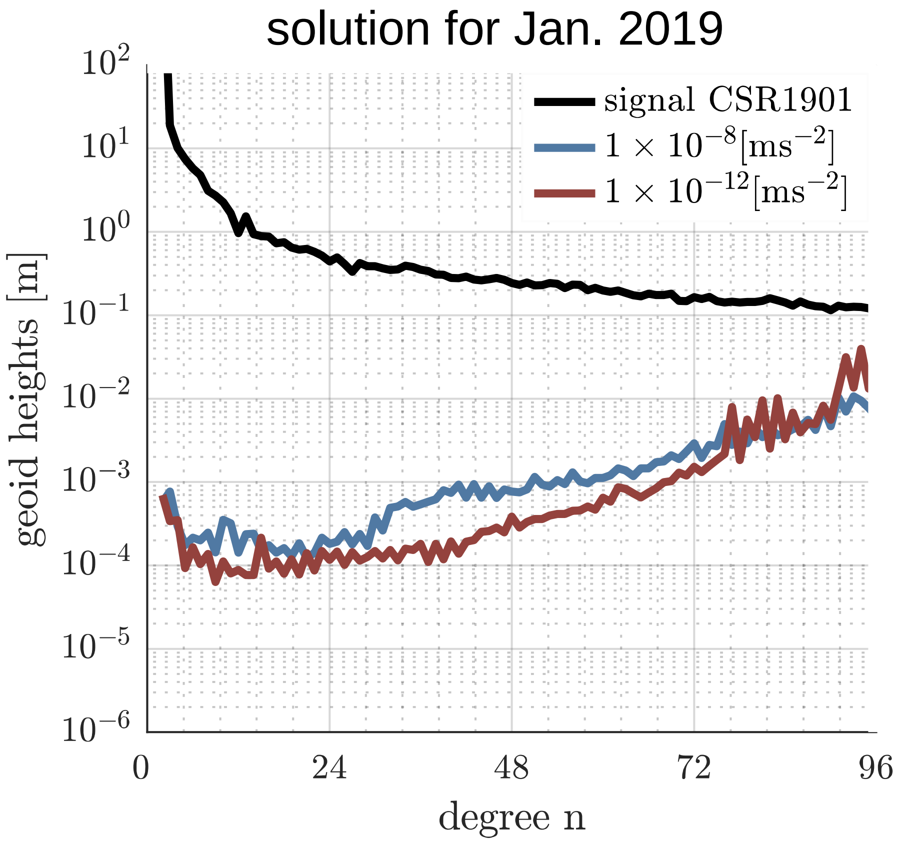


1×10^{-8} ms^{-2}

«loose» constraint

(gravity field signal absorbed in PCAs)

Impact of different constraints



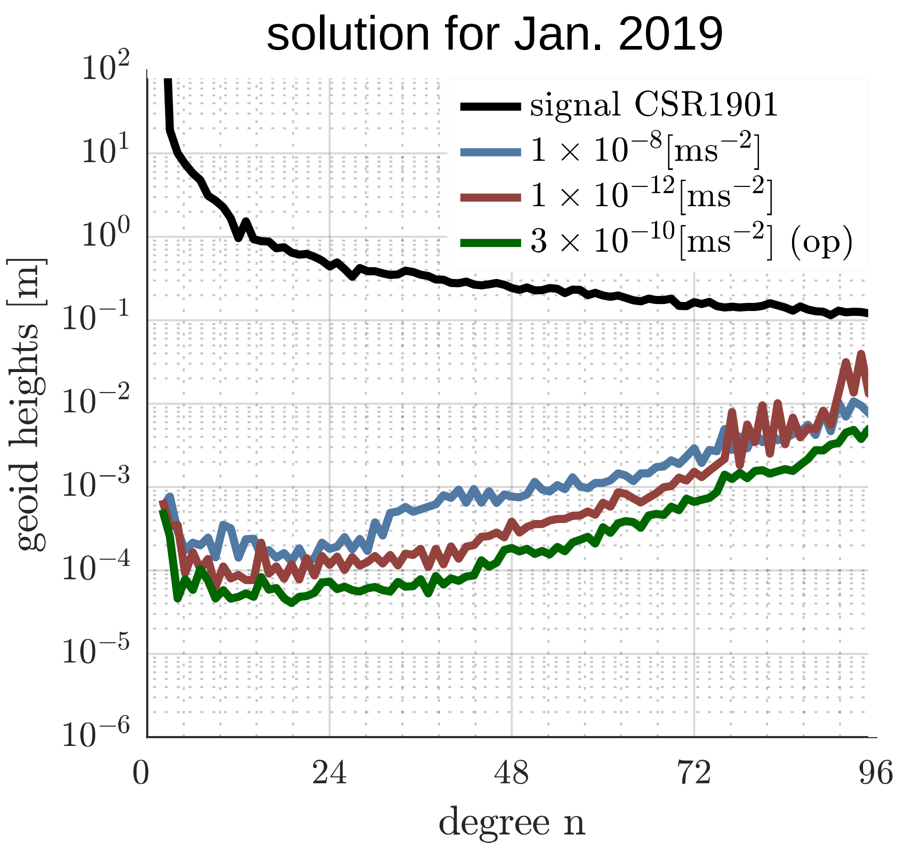
$1\times10^{-12}\ ms^{-2}$

«tight» constraint

(not enough to absorb mis-modellings)

M. Lasser, U. Meyer, D. Arnold, A. Jäggi: Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery, EGU General Assembly 2022, 26 May, 2022

Impact of different constraints



$3\times10^{-10}\,\mathrm{ms}^{-2}$

«reasonable» balance

(applied in the operational solutions)

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Constraining

$\mathbf{N}=(\mathbf{A}^T \mathbf{P} \mathbf{A})$ and $\boldsymbol{b}=\mathbf{A}^T \mathbf{P} \boldsymbol{l}$ → $\hat{\boldsymbol{x}}=\mathbf{N}^{-1} \boldsymbol{b}$

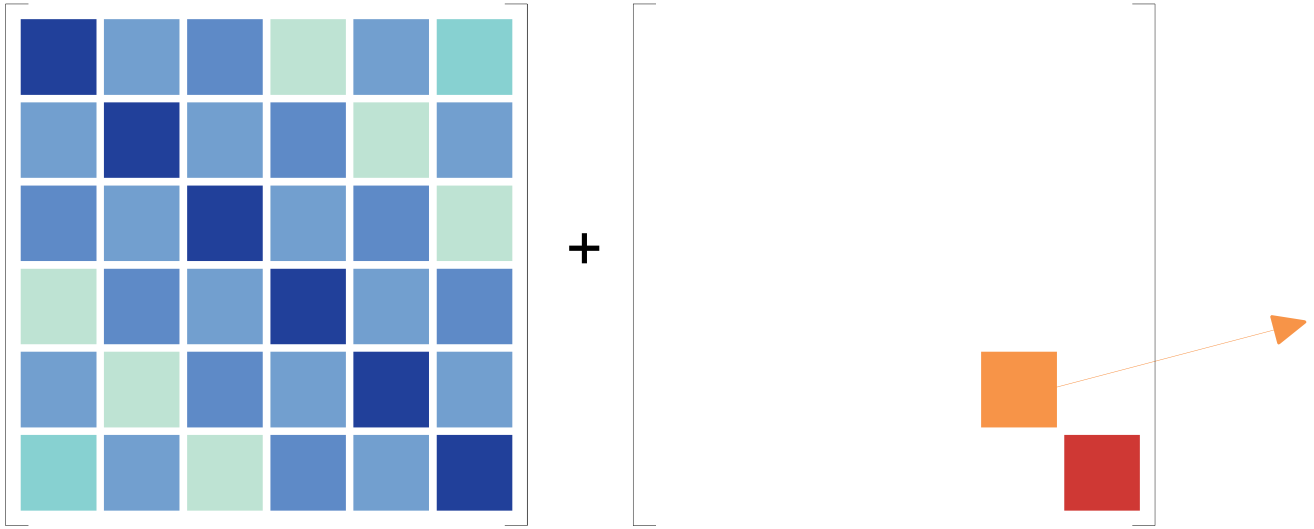
- $\mathbf{A}$: design matrix
- $\mathbf{P}$: weight matrix
- $\boldsymbol{l}$: vector of observations

Constraining

$$\mathbf{N}=\left(\mathbf{A}^T\mathbf{P}\,\mathbf{A}\right) \quad \text{and} \quad \boldsymbol{b}=\mathbf{A}^T\mathbf{P}\,\boldsymbol{l} \longrightarrow \hat{\mathbf{x}}=\mathbf{N}^{-1}\boldsymbol{b}$$

$$\mathbf{N}=\left(\mathbf{A}^T\mathbf{P}\,\mathbf{A}+\mathbf{W}\right)$$

$N=$



$$\frac{\sigma_0^2}{\sigma_{PCA}^2},$$

$\sigma_{PCA}^2=$ e.g., $3\times10^{-10}\ \mathrm{ms}^{-2}$

Variance Component Estimation

$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A})$ and $\boldsymbol{b} = \mathbf{A}^T \mathbf{P} \boldsymbol{l}$ → $\hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$

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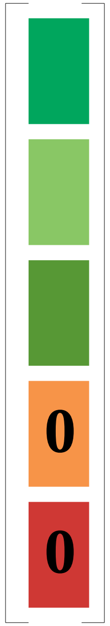
$\boldsymbol{l} =$



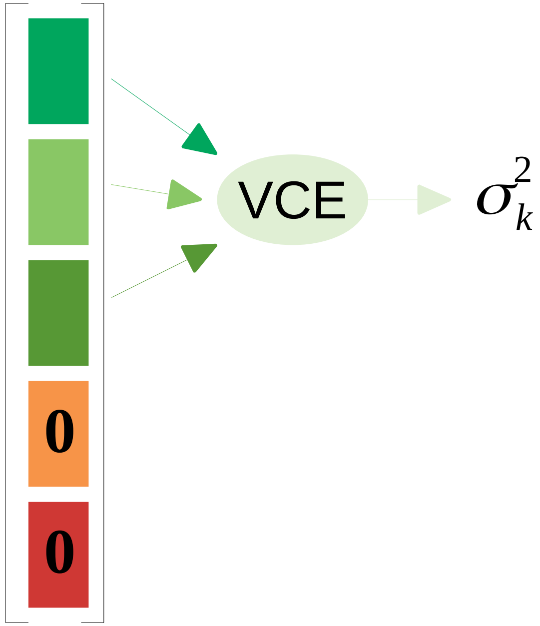
Variance Component Estimation

$$\mathbf{N}=\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)$$ and $$\boldsymbol{b}=\mathbf{A}^T\mathbf{P}\boldsymbol{l} \longrightarrow \hat{\boldsymbol{x}}=\mathbf{N}^{-1}\boldsymbol{b}$$

$$\mathbf{N}=\left(\mathbf{A}^T\mathbf{P}\mathbf{A}+\mathbf{W}\right)$$

VCE: Each group of observations gets a weight based on its contribution to the final solution

$$\boldsymbol{l}=$$



M. Lasser, U. Meyer, D. Arnold, A. Jäggi: Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery, EGU General Assembly 2022, 26 May, 2022

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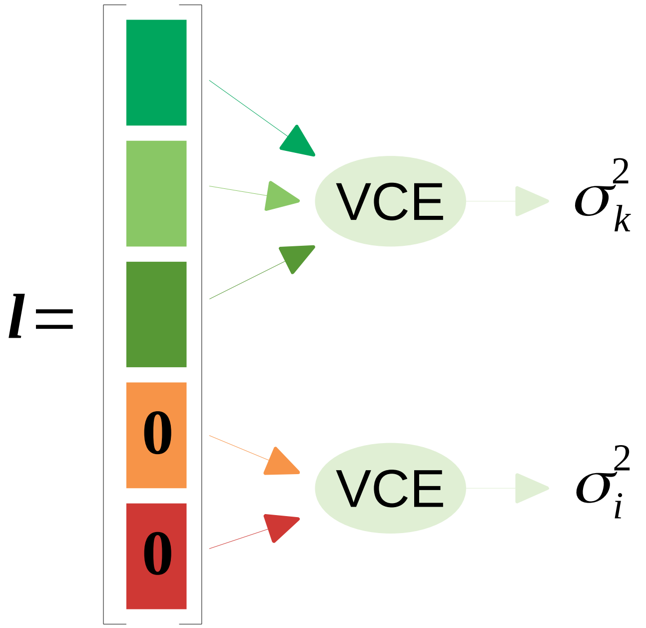

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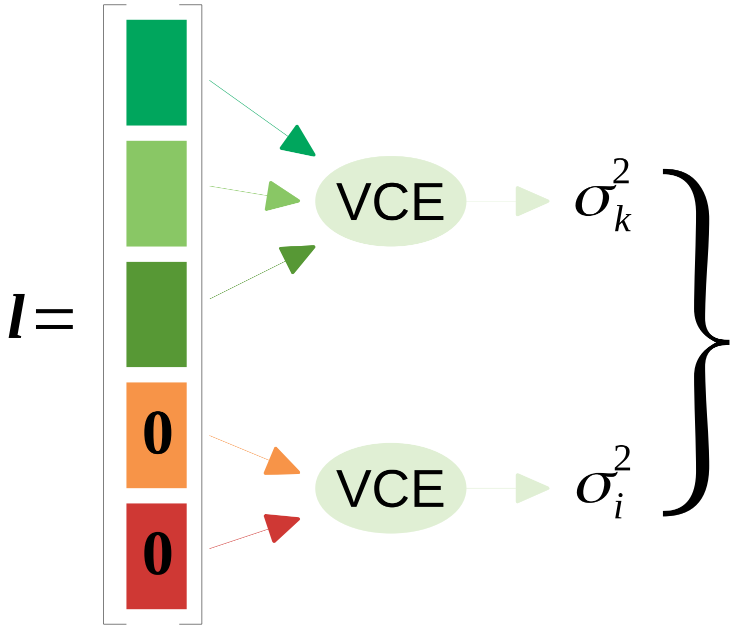

$$\hat{\boldsymbol{x}}=\left(\sum_{k=1}^{K=3}\frac{\sigma_0^2}{\sigma_k^2}\mathbf{N}_k+\sum_{i=1}^{I=2}\frac{\sigma_0^2}{\sigma_i^2}\mathbf{W}_i\right)^{-1}\sum_{k=1}^{K=3}\frac{\sigma_0^2}{\sigma_k^2}\boldsymbol{b}_k$$

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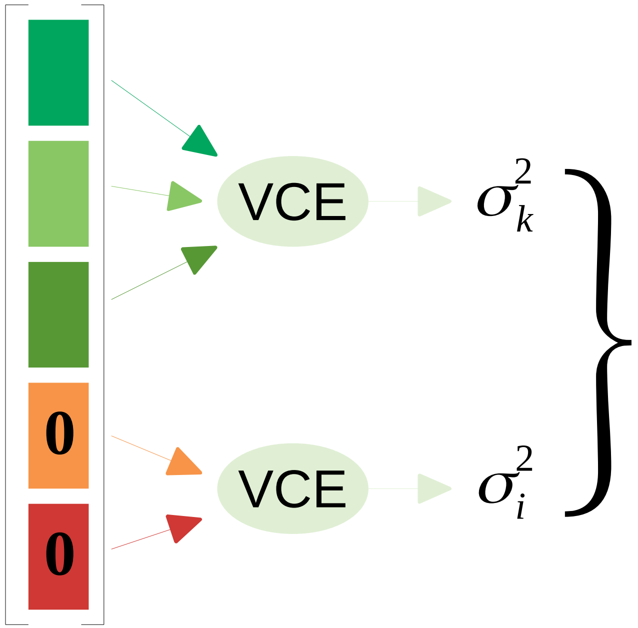
$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A} + \mathbf{W})$$

VCE: Each group of observations gets a weight based on its contribution to the final solution

$\boldsymbol{l} =$



$$\hat{\boldsymbol{x}} = \left(\sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} \mathbf{N} + \sum_{i=1}^{I=2} \frac{\sigma_0^2}{\sigma_i^2} \mathbf{W}_i \right)^{-1} \sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} \boldsymbol{b}_k$$

information about observations introduced via σ_0^2

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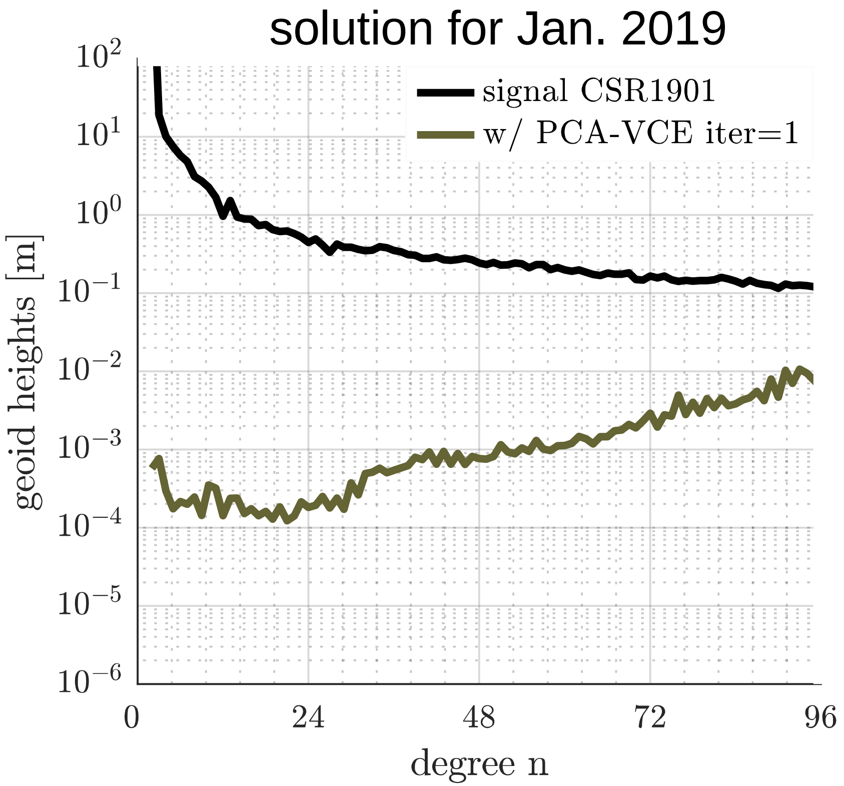
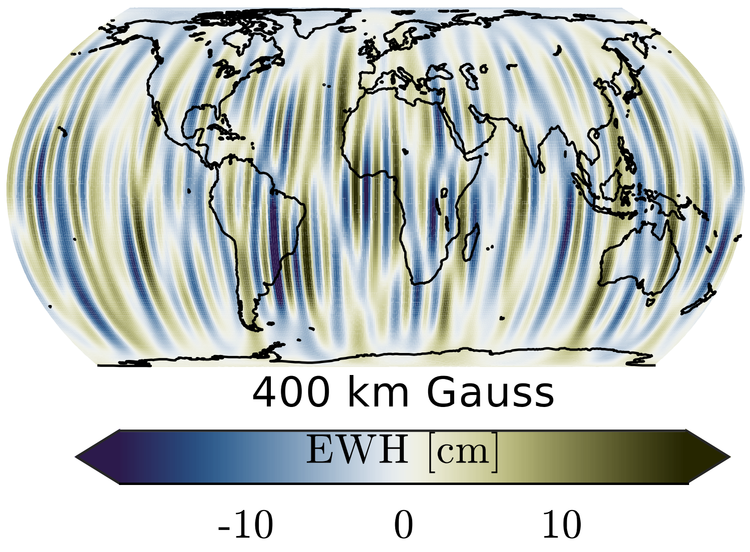
Results



	ms^{-2}
radial	1×10^{-8}
along-track	1×10^{-8}
cross-track	1×10^{-8}



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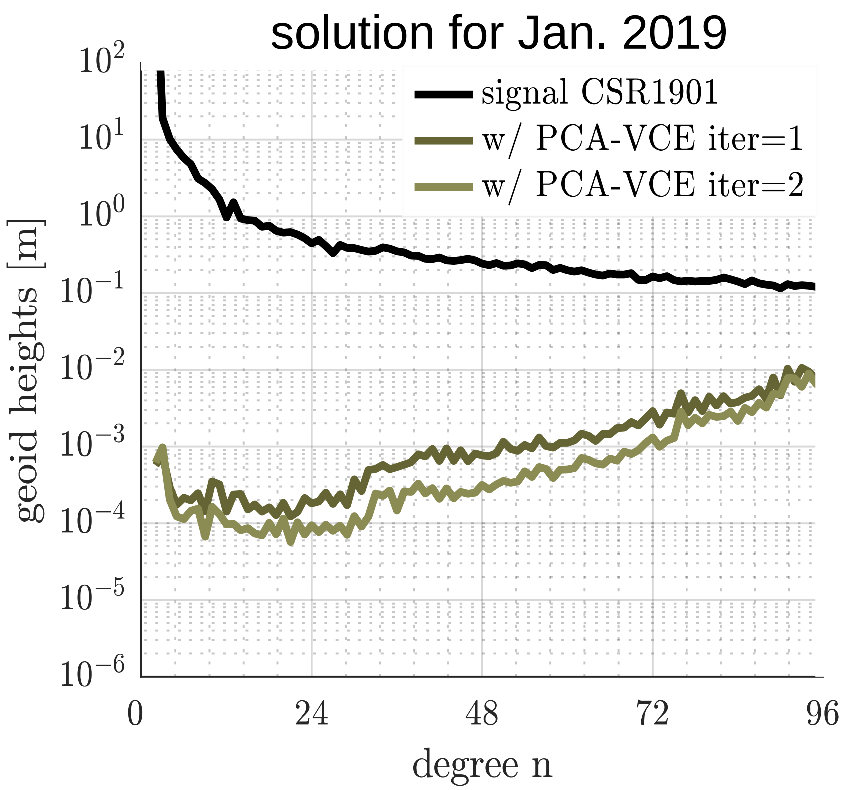
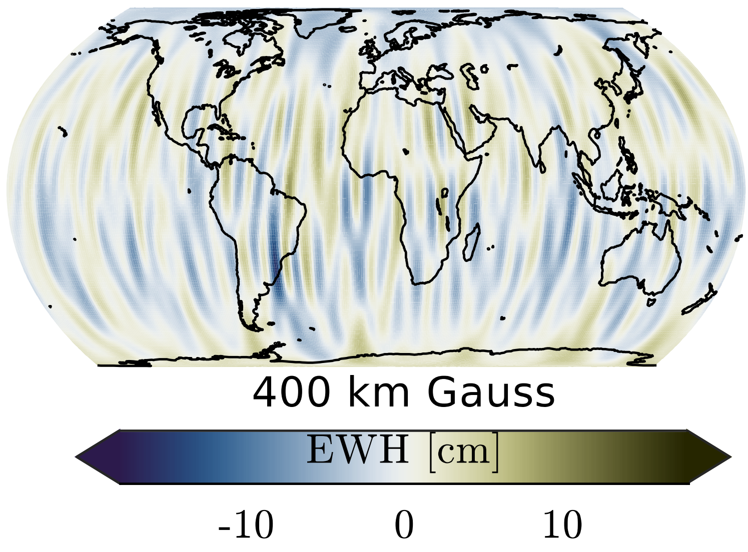


	ms^{-2}
radial	1.9×10^{-8}
along-track	9.8×10^{-9}
cross-track	8.6×10^{-9}



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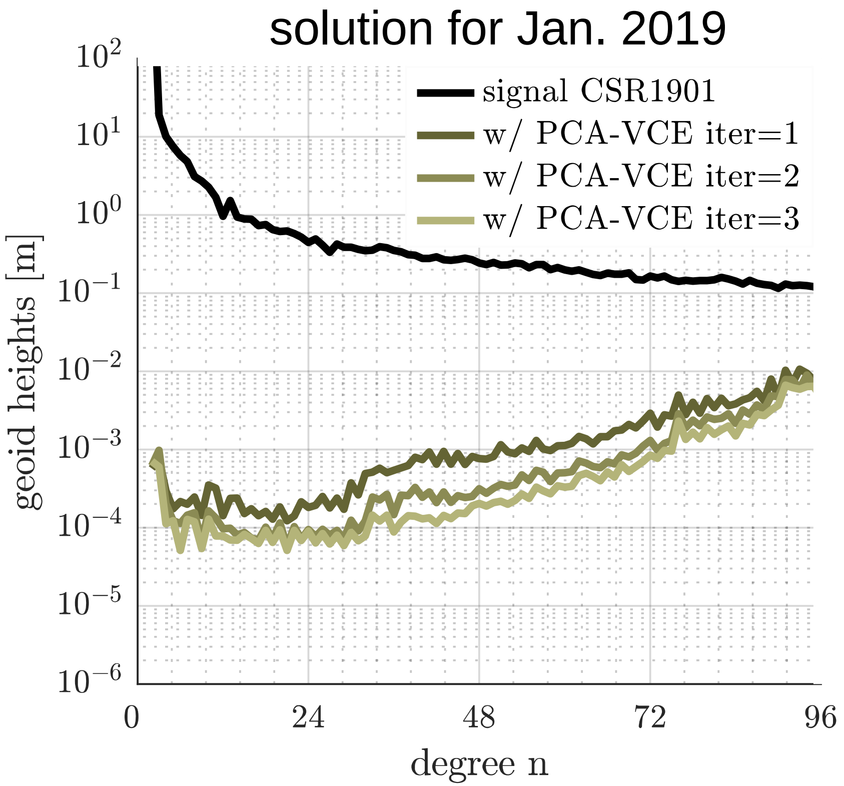
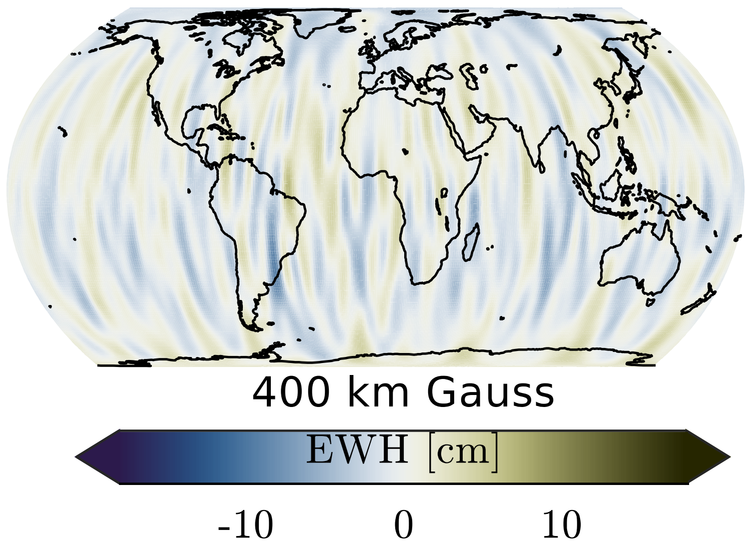


	ms^{-2}
radial	2.9×10^{-9}
along-track	1.5×10^{-9}
cross-track	1.3×10^{-9}



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Results

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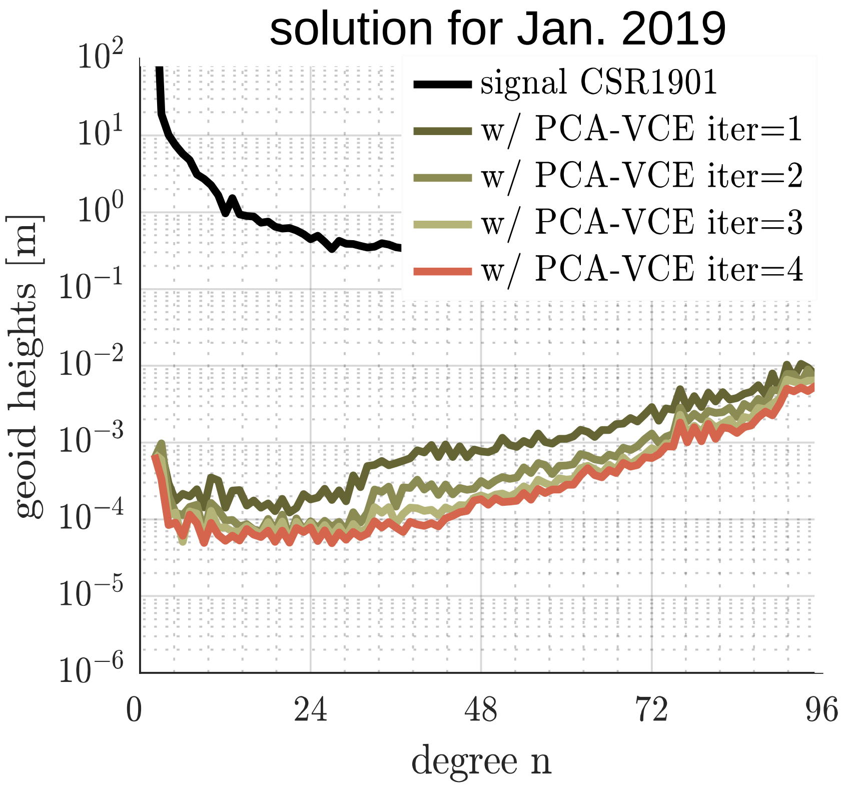
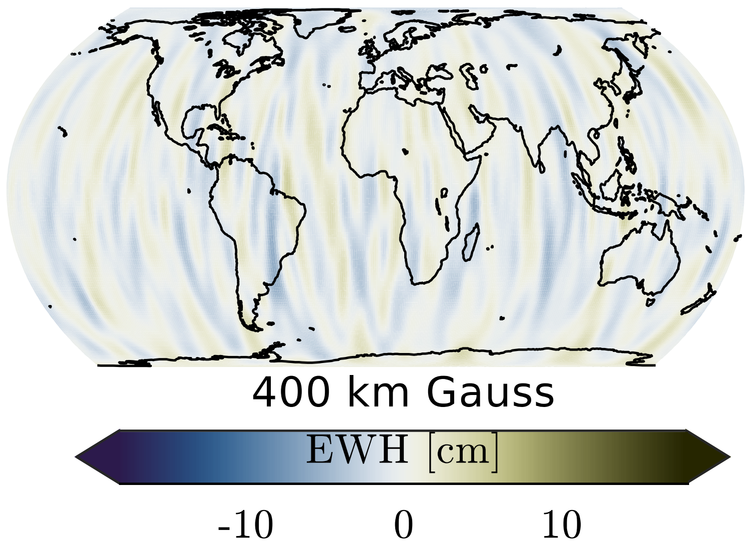
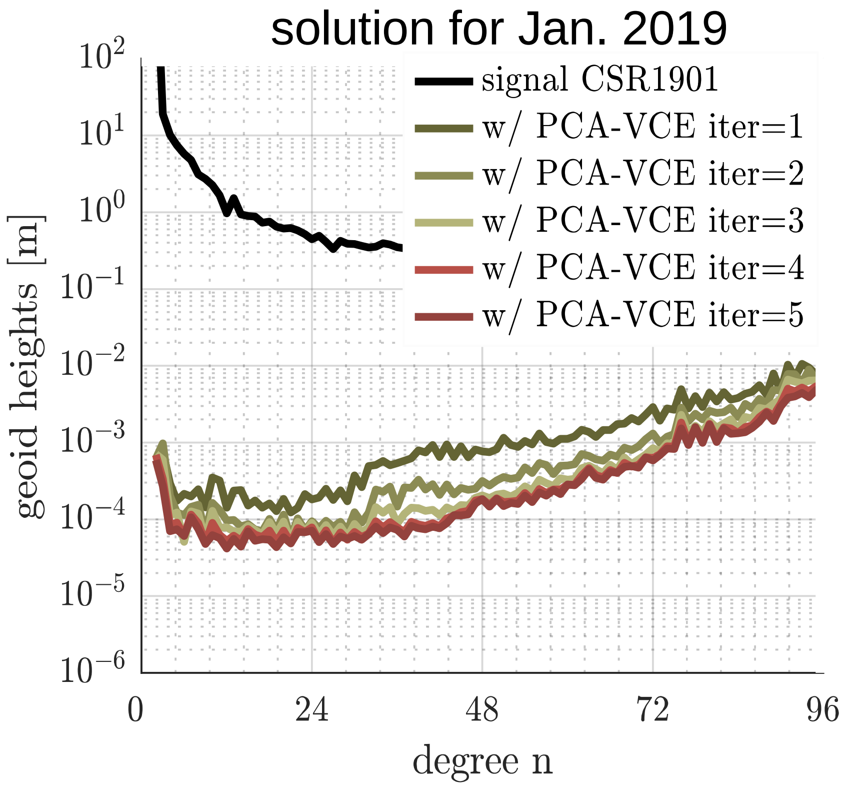


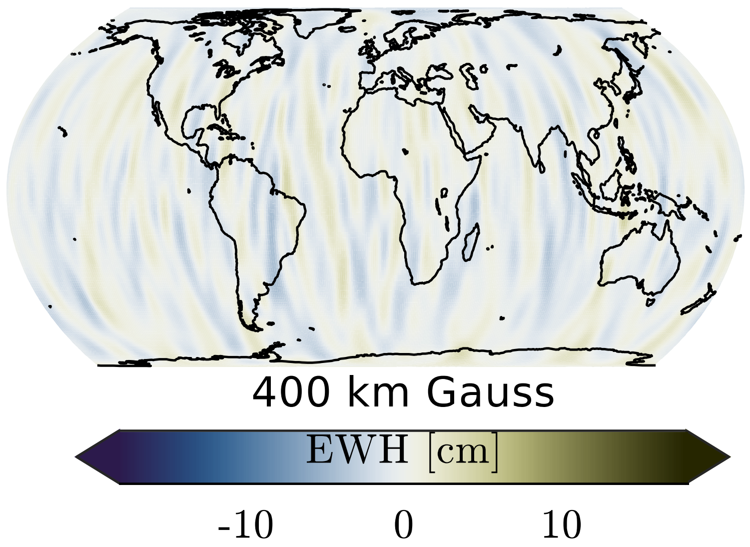
	ms^{-2}
radial	1.2×10^{-9}
along-track	6.2×10^{-10}
cross-track	6.9×10^{-10}



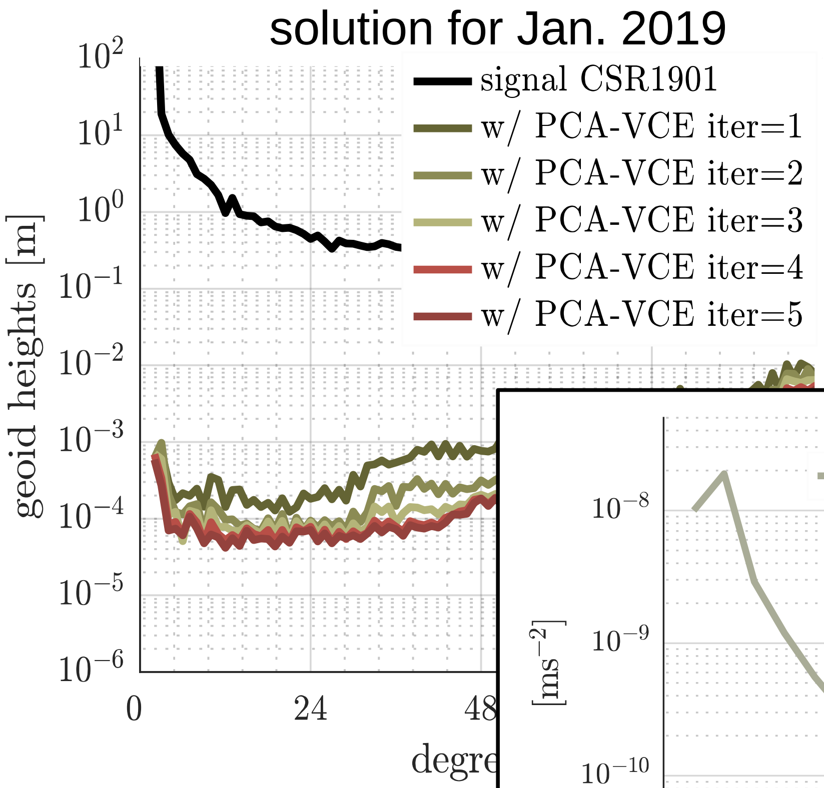
Results



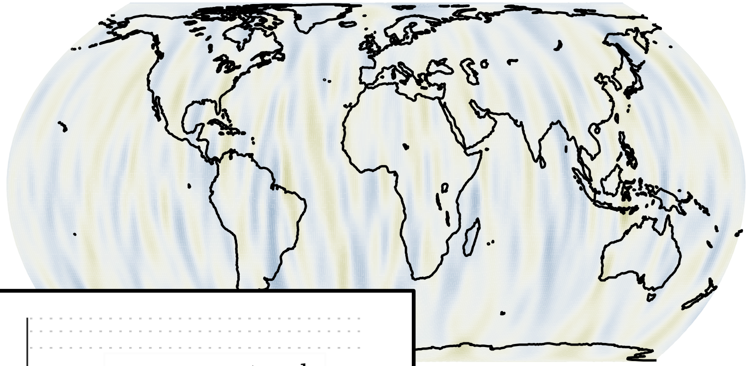
	ms^{-2}
radial	5.6×10^{-10}
along-track	2.6×10^{-10}
cross-track	6.1×10^{-10}



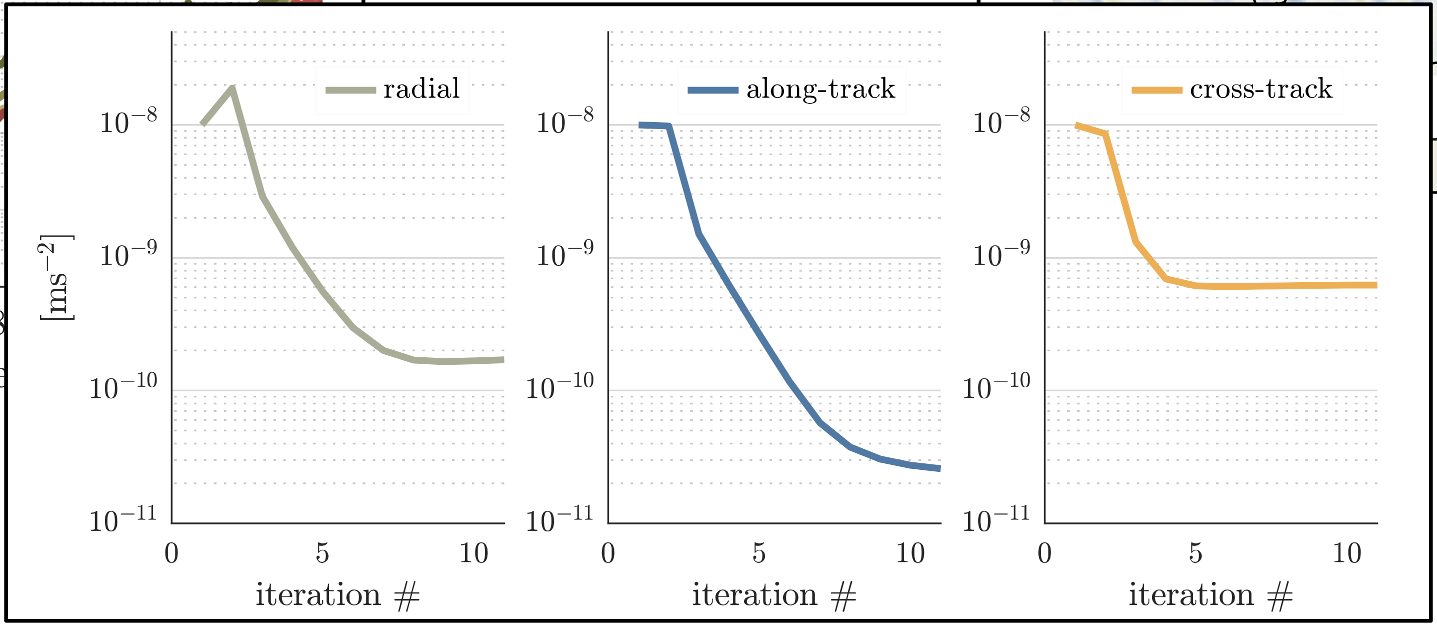
Results



	ms^{-2}
radial	5.6×10^{-10}
along-track	2.6×10^{-10}



Gauss



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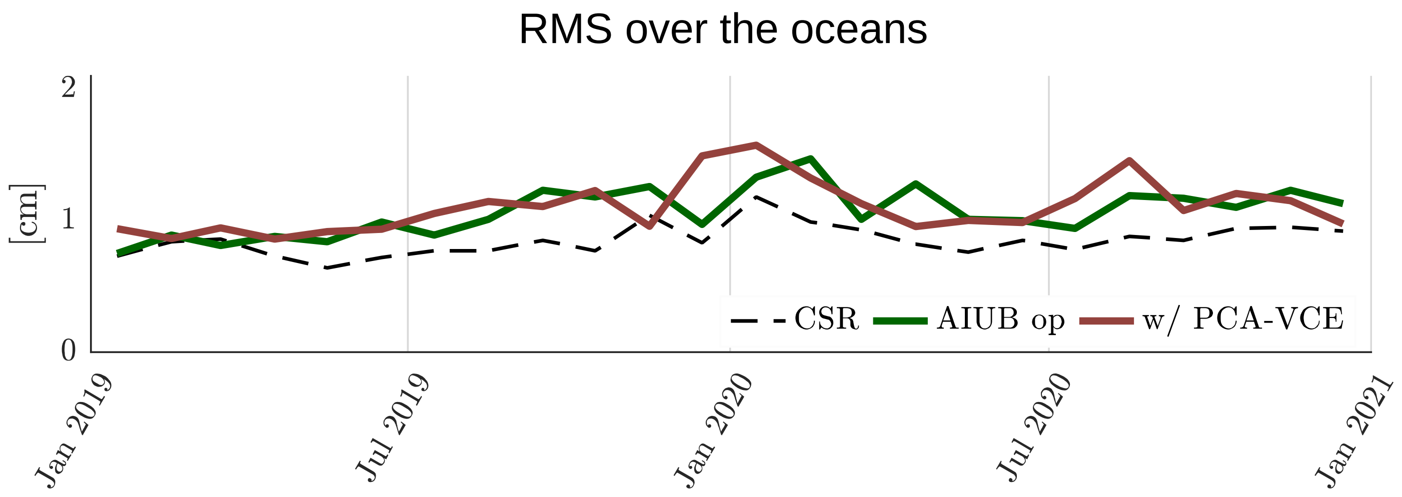
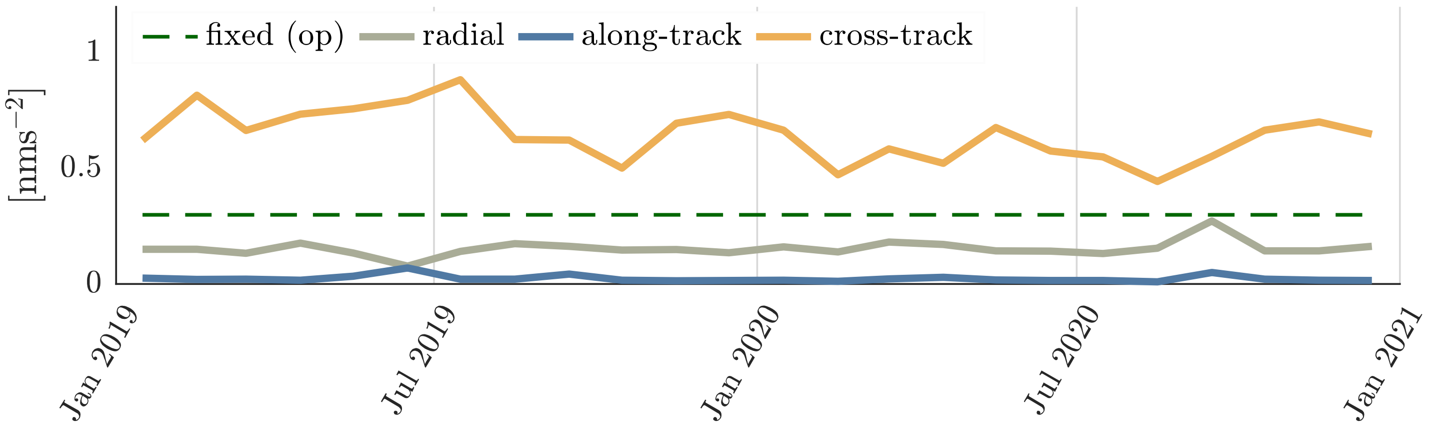
Conclusion

- observation-based approach
- computed together with the solution
- provides a good solution (if PCAs sample correctly)
- improvement...



- computational efficiency?
- observation-based – outliers
- improvement...



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