

# Adding Flavor to the SMEFT

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**ABSTRACT:** We study the flavor structure of the lepton and baryon number-conserving dimension-6 operators in the Standard Model effective field theory (SMEFT). Building on the work of [1], we define several well-motivated flavor symmetries and symmetry-breaking patterns that serve as competing hypotheses about the ultraviolet (UV) dynamics beyond the SM, not far above the TeV scale. In particular, we consider four different structures in the quark sector and seven in the charged lepton sector. The set of flavor-breaking spurions is (almost) always taken to be the minimal one needed to reproduce the observed charged fermion masses and mixings. For each case, we explicitly construct and count the operators to the first few orders in the spurion expansion, providing ready-for-use setups for phenomenological studies and global fits. We provide a `Mathematica` package `SMEFTflavor` (<https://github.com/aethomsen/SMEFTflavor>) to facilitate similar analyses for flavor symmetries not covered in this work.

**KEYWORDS:** Flavour Symmetries, SMEFT, Theories of Flavour

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**1 Introduction**

Our current understanding of particle physics is condensed into the Standard Model (SM). It is a quantum field theory with Poincaré spacetime symmetry and  $SU(3)_c \times SU(2)_L \times U(1)_Y$  gauge invariance. The field content includes five different gauge representations of Weyl fermions  $q_i, \ell_i, u_i, d_i, e_i$  each coming in three flavors (copies,  $i = 1, 2, 3$ ) and a single scalar field that condenses at the electroweak scale, breaking the gauge symmetry down to  $SU(3)_c \times U(1)_{EM}$ . The Lagrangian is constructed from all local operators consistent with the gauge symmetries up to the mass dimension four, known as the renormalizable operators. In the initial days of the SM, the renormalizability was considered to be an important consistency condition for valid theories. With better understanding, it became clear that this was only a low-energy property of an effective field theory [2–10]. With more precision

and/or higher energy, we will eventually discover the leading higher-dimensional operators in the infinite series organized in the inverse powers of the new physics (NP) cutoff  $\Lambda$  — an almost inevitable consequence of the known shortcomings of the SM in explaining the observed universe.

The SM effective field theory (SMEFT) is a framework, which has gained popularity in recent years [11–27]. The Large Hadron Collider (LHC) discovered the Higgs boson  $h$  [28, 29] completing the list of the light degrees of freedom in the SMEFT. The Higgs mechanism with  $h$  embedded in an  $SU(2)_L$  doublet (the linear realization) turned out to be a decent leading-order description of the electroweak symmetry breaking. At the same time, direct searches failed to discover any new particles despite tremendous efforts, suggesting a possible mass gap between the NP and SM states:  $\Lambda > G_F^{-1/2}$ . With this situation, the SMEFT analyses have become standard across different sectors of high- $p_T$  phenomenology, including top quark [30–39], electroweak vector bosons [38–47], Higgs [38–40, 48–51], jets [52–55], and high-mass Drell-Yan [56–82], aiming at finding smooth and correlated deviations from the SM predictions. To this end, a global SMEFT fit would optimally summarize the knowledge about the ultraviolet physics (UV) accessible to us at low energies.

The largest obstacle to such an analysis is the proliferation in the number of independent operators in the SMEFT. For instance, there are 2499 independent baryon and lepton number-conserving SMEFT operators that arise at the leading order (dimension 6) [14].<sup>1</sup> If the field content had included only a single generation instead of three, this number would have been 59 [13]. This is not surprising, as most of the parameters are flavorful already in the SM at dimension 4. While the gauge sector and the scalar potential have a total of 4 parameters (modulo the strong CP-violating term), the Yukawa sector introduces an additional 13 parameters to accommodate the charged fermion masses and the CKM mixing matrix.

In this paper, we address the flavor structures of the baryon and lepton number-conserving dimension-6 operators in the SMEFT suitable for global fits including high- $p_T$  data. For an anarchic flavor structure, with all real and imaginary coefficients of order one, charged lepton flavor violation, neutral meson oscillations, and electric dipole moments set a lower bound on the NP scale many decades above the TeV scale [86–91]. Only a few operators contributing to those rare transitions would be relevant for the phenomenology, leaving no hope of seeing new effects in the high- $p_T$  collider physics. This would also be in conflict with resolving the Higgs mass hierarchy problem at the TeV scale or, for instance, the  $B$ -anomalies [92–104] and  $(g-2)_\mu$  anomaly [105, 106] should those prove to originate in new physics.

There is, however, no reason to expect flavor anarchy in the dimension-6 operators given the peculiar form of the dimension-4 Yukawa interactions. Rather, we observe hierarchical masses for different generations of charged fermions with Yukawa couplings spanning six orders of magnitude and an almost diagonal CKM mixing matrix.<sup>2</sup> The elusive why of

<sup>1</sup>These many new operators also results in a large number of new CP-invariants [83–85].

<sup>2</sup>In this work, we consider neutrinos to be massless. They obtain a mass from a different set of operators from those considered here, such as the lepton number-violating, dimension-5 Weinberg operator. We only consider symmetry-breaking spurions that always preserve the total lepton number, thus giving no

these curious observations has long been sought after in the form of a theory of flavor. Almost regardless of what the solution is, an explanation will endow a flavor structure on the SMEFT above the EW scale.

The renormalizable SM without the Yukawa interactions has a large global  $U(3)^5$  flavor symmetry. The observed parameters break the symmetry in a particular way. The largest breaking is due to the top quark Yukawa down to  $U(2)^2 \times U(1) \times U(3)^3$  subgroup. This is a good approximate symmetry of the SM, which is further broken in steps by the smaller Yukawas. Eventually, the exact (classical) accidental symmetry of the dimension-4 Lagrangian,  $U(1)_B$  in the quark sector times  $U(1)^3$  for the leptons, is recovered. Higher-dimensional operators typically contribute to the breaking of the aforementioned (exact or approximate) accidental flavor symmetries.

Postulating a flavor symmetry and its breaking pattern (via a set of spurions) means making a hypothesis about the UV physics. A flavor spurion can be viewed as a non-dynamical (spurious) field that transforms under a nontrivial representation of the flavor group and whose background value breaks the flavor symmetry. In other words, we imagine that a UV theory will leave imprints on the flavor structure in the low-energy effective theory. The selection rules implied by the flavor symmetry have the advantage of reducing the number of important SMEFT operators (free parameters in the global fits) by truncating the flavor-spurion expansion to a given order. Global flavor symmetries and their breaking patterns, thus, provide a good organizing principle for the SMEFT, mapping the space of theories beyond the SM into universality classes. The induced model dependence should not be viewed as a drawback but rather as an opportunity to systematically learn about the UV from the data.

The prototypical example is the minimal flavor violation (MFV) hypothesis [107], which is a flavor structure based on the  $U(3)^5$  flavor symmetry broken by the SM Yukawa couplings  $Y_u$ ,  $Y_d$ , and  $Y_e$  promoted to spurions. Numerous analysis of the low-energy flavor physics data have shown that MFV allows for the NP cutoff as low as the TeV scale [108–117]. Such a flavor structure naturally arises as the low-energy limit of large classes of models, including supersymmetric models with anomaly or gauge mediation [118, 119]. The drawback of MFV is the ambiguity in the power counting given that  $y_t$  is a large parameter. A prominent competitor to MFV is the  $U(2)$  [120] (or general MFV [121]) flavor structures, which also provide a sufficient protection against dangerous flavor violation [122–126] and allow for a low NP scale.  $U(2)^5$  is an excellent approximate symmetry of the SM broken only at the level of  $\mathcal{O}(10^{-2})$ . The breaking spurions are already small parameters, thus, the power counting is more transparent. Unlike MFV, it is also a good starting point for the combined explanation of  $B$ -anomalies in charged and neutral currents, where the third family plays a special role [127–133]. Other flavor structures are also studied in the literature, e.g., refs. [134, 135].

Building on the work of ref. [1], we identify several symmetry hypotheses capable, in particular, of allowing new physics to be at the TeV scale. Apart from theoretical

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contributions to the Weinberg operator. Neutrino masses can easily be accounted for by introducing additional spurions fully contracting  $L^i L^j$  terms. Due to the smallness of the neutrino masses, the contribution from these spurions to dimension-6 operators is negligible.

SMEFT $\mathcal{O}(1)$ terms (dim-6, $\Delta B = 0$ )		Lepton sector															
		MFV <sub>L</sub>		U(3) <sub>V</sub>		U(2) <sup>2</sup> × U(1) <sup>2</sup>		U(2) <sup>2</sup>		U(2) <sub>V</sub>		U(1) <sup>6</sup>		U(1) <sup>3</sup>		No symm.	
Quark sector	MFV <sub>Q</sub>	41	6	45	9	59	6	62	9	67	13	81	6	93	18	207	132
	U(2) <sup>2</sup> × U(3) <sub>d</sub>	72	10	78	15	95	10	100	15	107	21	122	10	140	28	281	169
	U(2) <sup>3</sup> × U(1) <sub>d3</sub>	86	10	92	15	111	10	116	12	123	21	140	10	158	28	305	175
	U(2) <sup>3</sup>	93	17	100	23	118	17	124	23	132	30	147	17	168	38	321	191
	No symmetry	703	570	734	600	756	591	786	621	818	652	813	612	906	705	1350	1149

**Table 1.** Overview of the number of independent  $\mathcal{O}(1)$  terms for the dimension-6 SMEFT operators ( $\Delta B = 0$ ) for different assignments of quark and lepton symmetries considered in the paper. The left (right) entry in each column gives the number of CP even (odd) coefficients for each symmetry combination.

motivations such as the Higgs hierarchy problem, we are aiming at having an interesting interplay between low and high- $p_T$  physics, making the case for global fits. Our goal is to explore a broad spectrum of flavor structures beyond MFV and U(2)<sup>5</sup>. We extend the previous study [1] by including four structures in the quark sector and seven in the lepton sector, for a total of 28 different flavor structures. The set of flavor-breaking spurions is taken to be the minimal one needed to reproduce the observed charged fermion masses and mixings (in some cases other interesting spurions have been included for completeness). We benchmark the lepton sector densely compared to the quark sector. One motivation to do so is to pave the path for exploration of a broader class of models addressing only the neutral current  $B$ -anomalies and/or  $(g - 2)_\mu$  anomaly (see e.g. [136–139]). Unlike the CKM matrix of the quark sector, the PMNS matrix has only been probed by neutrino oscillation data and could come from physics at a very high scale completely decoupled from the low-energy flavor structure of the charged leptons. Without any firm clues as to the underlying flavor structure, we are open to many possibilities. For each of the flavor structures, we determine and provide the full basis of dimension-6 SMEFT operators beyond NLO in the spurion expansion.

A highlight of this work is table 1 summarizing the number of independent  $\Delta B = 0$  operators at dimension-6 in the SMEFT for different flavor symmetry assumptions in the absence of spurions (i.e.  $\mathcal{O}(1)$  terms). A general expectation is that low-energy flavor physics will not be very sensitive to these  $\mathcal{O}(1)$  operators thanks to the flavor symmetry protection leaving room for high- $p_T$  phenomenology [140] (see, however, refs. [141, 142]). The other operators, which come with the flavor-spurion suppression, will be constrained chiefly by low-energy flavor physics. For any of the symmetries in table 1, if an operator containing fermions turns out to be Hermitian, it is counted in the left column since it introduces a single real parameter, otherwise it appears both in the left and right columns for the real and imaginary parameters, respectively (some care has to be taken when connecting the imaginary parameters with CP violation [83]). Additionally, there are always 9 CP-even (added to the left column) and 6 CP-odd (added to the right column) parameters for the pure bosonic dimension-6 operators, which do not carry any flavor. Table 1 illustrates how the number of independent parameters gradually increases with smaller (less restrictive) symmetries. Different flavor symmetries are considered to finely chart the space of SMEFT

operators. For reference, these are compared with the no-symmetry case shown in the last row and column for quarks and leptons, respectively. All the examples have considerably less  $\mathcal{O}(1)$  parameters than the anarchic case, making global fits more feasible. During this project we developed a `Mathematica` package `SMEFTflavor` to automatically construct the SMEFT operators given a flavor group (for details see appendix B). Should the user have a different symmetry group or breaking spurions in mind, the package can be downloaded at the `github` page <https://github.com/aethomsen/SMEFTflavor>.

The paper is organized as follows: in section 2, we first discuss the decomposition and the counting of the pure quark operators, while in section 3, we consider pure lepton operators. Along the way, we always present an explicit parametrization of the spurions and Wilson coefficients ready to be employed in the phenomenological studies. The Warsaw basis used throughout this work is summarized in appendix A. The `Mathematica` package is documented in appendix B. Appendix C lists the counting of the mixed quark-lepton operators for all 28 flavor structures, while appendix D is reserved for useful group identities. We conclude in section 4.

## 2 Quark sector

The kinetic Lagrangian is invariant under flavor rotations of the matter, that is, unitary transformation between fields in the same gauge and Lorentz representations. In the SM quark sector, these transformations make up the group  $G_Q = \text{U}(3)_q \times \text{U}(3)_u \times \text{U}(3)_d$ . Assuming/imposing a flavor structure of NP, then comes down to proposing a finite set of spurions transforming in a subgroup of  $G \subset G_Q$ , such that the whole SMEFT Lagrangian is invariant under  $G$ . These assumptions can severely limit the number of operators, which can occur to a given order in the spurion expansion and will, generically, impose correlation between various operators.

In the SM, the  $G_Q$  quark flavor symmetry is broken classically by the two quark Yukawa coupling matrices  $Y_{u,d}$ :

$$\mathcal{L} \supset -\bar{q}_L Y_d d_R H - \bar{q}_L Y_u u_R \tilde{H} + \text{H.c.} \quad (2.1)$$

Formally, these couplings can be considered spurions transforming as  $Y_u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})$  and  $Y_d \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})$ , such as to leave the SM Lagrangian invariant under flavor rotations.<sup>3</sup> MFV [107] is a popular framework, which assumes that the SM contains the full flavor information of the underlying theory; no additional flavored spurions are introduced in the UV and all other couplings are singlet under  $G_Q$ . Accordingly, all flavor structure is contained in  $Y_u$  and  $Y_d$ . In this framework, flavor violation in operators, whether fundamental or effective, can only occur through some combination of the  $Y_u, Y_d$  spurions. This strongly constrains flavor transitions due to NP and presents a mechanism to avoid the strong flavor bounds from FCNC processes.

<sup>3</sup>Here we write only the SU(3) representation but assume that the global U(1) charges are also defined by eq. (2.1). There is no difference between U(3) and its SU(3) subgroup for dimension-6 baryon number conserving operators. The global U(1) charges will however play an important role for U(2) versus SU(2) symmetries.

If we wish to classify the SMEFT operators consistent with MFV, it is important to consider an organizing principle. One challenge is that e.g.  $(Y_u^\dagger Y_u)^{n \geq 0}$  all transform in a similar manner, thus operators can always be dressed with higher powers of  $Y_u^\dagger Y_u$ . However, not all of these are independent. In fact three of these are enough to span the space, and higher powers can be absorbed into the coefficients of the operators with lower powers: a finite set is sufficient to capture all physics. A proper organizing principle exists when the spurions are small (e.g., if  $Y_u$  always comes with a small parameter  $\varepsilon_u \ll 1$ ), and the MFV operators can be organized by powers of the spurions. This naive expansion in powers of  $Y_{u,d}$  is not necessarily possible, since  $y_t \sim 1$ , and in 2HDM type models even  $y_b$  can be order 1. The authors of ref. [121] were able to show that non-linearly realized MFV, where a power expansion is impossible, can be effectively captured as a special case of the later, much acclaimed  $U(2)^3$  flavor symmetry [120].

Here we consider a spectrum of viable flavor symmetries:

- i)*  $G = U(2)^3$  decouples the third generation quarks entirely, yet it gives a decent protection against FCNCs.
- ii)*  $G = U(2)^3 \times U(1)_b$  decouples only the third generation of down-quarks and keeps  $y_b$ , a spurion of  $U(1)_b$ , perturbatively small.
- iii)*  $G = U(2)^2 \times U(3)_d$  for when there is no suppression of  $y_t \simeq 1$  in the SMEFT operators. The enhanced symmetry allows for a spurion expansion of all but the top quark.
- iv)*  $G = U(3)^3$  linearly realized MFV, provides strong constraints on NP, and effectively protects against NP contributions to rare SM processes.

In this section, we explore these 4 different flavor structures for the quark sector. In each case, we will assume that a perturbative expansion in spurion insertions is possible. For each symmetry, we provide a parametrization of the spurions, list all flavor contractions that can occur up to dimension 6 in the SMEFT, and finally provide a counting of the quark operators at dimension 6.

## 2.1 $U(2)^3$ symmetry

We assume that the NP posses a symmetry  $G = U(2)_q \times U(2)_u \times U(2)_d \subset G_Q$ , under which the SM quarks decompose as

$$q = \begin{bmatrix} q^a \sim (\mathbf{2}, \mathbf{1}, \mathbf{1}) \\ q_3 \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}) \end{bmatrix}, \quad u = \begin{bmatrix} u^a \sim (\mathbf{1}, \mathbf{2}, \mathbf{1}) \\ u_3 \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}) \end{bmatrix}, \quad d = \begin{bmatrix} d^a \sim (\mathbf{1}, \mathbf{1}, \mathbf{2}) \\ d_3 \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}) \end{bmatrix}. \quad (2.2)$$

The minimal set of spurions needed to reproduce the SM masses and CKM matrix is

$$V_q \sim (\mathbf{2}, \mathbf{1}, \mathbf{1}), \quad \Delta_u \sim (\mathbf{2}, \bar{\mathbf{2}}, \mathbf{1}), \quad \Delta_d \sim (\mathbf{2}, \mathbf{1}, \bar{\mathbf{2}}). \quad (2.3)$$

These spurions generally allow for a slew of Yukawa operators, which contributes to the Yukawa coupling matrices as

$$Y_{u,d} = \begin{bmatrix} a_1^{u,d} \Delta_{u,d} + a_2^{u,d} \Delta_u \Delta_u^\dagger \Delta_{u,d} + \dots & b_1^{u,d} V_q + b_2^{u,d} \Delta_u \Delta_u^\dagger V_q + \dots \\ c_1^{u,d} V_q^\dagger \Delta_{u,d} + \dots & d_1^{u,d} + d_2^{u,d} V_q^\dagger V_q + \dots \end{bmatrix} \quad (2.4)$$

for  $\mathcal{O}(1)$  parameters  $a_n^{u,d}, \dots, d_n^{u,d}$ , parametrizing all covariant combinations of the spurions at each entry in the coupling matrix.

We now wish to point out a redundancy, which to our knowledge has been overlooked in previous literature on the  $U(2)^3$  flavor symmetry: despite the assumption of the  $G$  symmetry, the quark kinetic terms are, nevertheless, invariant under all  $G_Q$  transformations. In particular, rotations from the  $G_Q/G$  coset space have not been utilized because generic field transformations of this kind would ruin explicit  $G$  invariance of the model. The spurions, however, allow for the construction of covariant  $G_Q/G$  rotation matrices

$$\begin{aligned}
 U_q &= \exp \begin{bmatrix} 0 & \lambda_1^q V_q + \lambda_2^q \Delta_u \Delta_u^\dagger V_q + \dots \\ -(\lambda_1^q)^* V_q^\dagger - (\lambda_2^q)^* V_q^\dagger \Delta_u \Delta_u^\dagger - \dots & 0 \end{bmatrix}, \\
 U_{u,d} &= \exp \begin{bmatrix} 0 & \lambda_1^{u,d} \Delta_{u,d}^\dagger V_q + \dots \\ -(\lambda_1^{u,d})^* V_q^\dagger \Delta_{u,d} - \dots & 0 \end{bmatrix}.
 \end{aligned}
 \tag{2.5}$$

For a suitable choice of  $\lambda_n^f$ , a field transformation will bring the Yukawa couplings on the form<sup>4</sup>

$$\begin{aligned}
 Y'_u &= \begin{bmatrix} a_1^{u'} \Delta_u + a_2^{u'} \Delta_u \Delta_u^\dagger \Delta_u + \dots & b_1^{u'} V_q + b_2^{u'} \Delta_u \Delta_u^\dagger V_q + \dots \\ 0 & d_1^{u'} + d_2^{u'} V_q^\dagger V_q + \dots \end{bmatrix}, \\
 Y'_d &= \begin{bmatrix} a_1^{d'} \Delta_d + a_2^{d'} \Delta_u \Delta_u^\dagger \Delta_d + \dots & 0 \\ 0 & d_1^{d'} + d_2^{d'} V_q^\dagger V_q + \dots \end{bmatrix}.
 \end{aligned}
 \tag{2.6}$$

Finally, we can do a redefinition of the spurions (in the event that  $a_1^{u'}, a_1^{d'}, b_1^{u'}$  are not  $\mathcal{O}(1)$ , it can be necessary to maintain explicit real coefficients multiplying the new spurions to maintain the perturbative spurion expansion) to write

$$Y'_u = \begin{bmatrix} \Delta_u & V_q \\ 0 & y_t \end{bmatrix}, \quad Y'_d = \begin{bmatrix} \Delta_d & 0 \\ 0 & y_b \end{bmatrix}.
 \tag{2.7}$$

This spurion redefinition shuffles the coefficients of the higher-dimension SMEFT operators but otherwise does not change the spurions being generic matrices.  $G_Q/G$  contains a final phase transformation of the third generation quarks, which allows for setting  $y_t, y_b$  to be real parameters. The Yukawa matrices (2.7) are, thus, completely generic representations in the  $U(2)^3$  symmetry. This parametrization effectively breaks  $G_Q \rightarrow G \times U(1)_B$ , where the spurions transform under the  $G$  symmetry.

We next consider how the spurions break the  $G$  symmetry and how the broken symmetry allows us to choose a minimal parametrization of the spurions. For simplicity, we consider the realistic case where the singular values of each of the spurions are finite and non-degenerate in agreement with current data.<sup>5</sup> First, fixing the  $U(2)_q$  doublet gives<sup>6</sup>

$$V_q \longrightarrow \begin{bmatrix} 0 \\ \epsilon_q \end{bmatrix} : \quad U(2)_q \longrightarrow U(1)_{q_1}.
 \tag{2.8}$$

<sup>4</sup>This can be seen order by order in the perturbative expansion.

<sup>5</sup>Such degeneracy could cause an enhanced remnant symmetry (see e.g. ref. [83]).

<sup>6</sup>To see what parameters can be removed from the spurions, it is useful to write them in terms of their singular value decomposition such that the flavor rotations can directly remove phases and angles from the left and right rotation matrices.



Next  $\Delta_u$ , breaks

$$\Delta_u \longrightarrow \begin{bmatrix} c_u & -s_u \\ s_u & c_u \end{bmatrix} \begin{bmatrix} \delta_u & 0 \\ 0 & \delta'_u \end{bmatrix} : \quad \text{U}(1)_{q_1} \times \text{U}(2)_u \longrightarrow \emptyset, \quad (2.9)$$

where we adopt the notation  $s_u, c_u$  for sine and cosine of the same angle, and

$$\Delta_d \longrightarrow \begin{bmatrix} c_d & -s_d e^{i\alpha} \\ s_d e^{-i\alpha} & c_d \end{bmatrix} \begin{bmatrix} \delta_d & 0 \\ 0 & \delta'_d \end{bmatrix} : \quad \text{U}(2)_d \longrightarrow \emptyset. \quad (2.10)$$

The complete breaking of  $G \rightarrow \emptyset$  by the spurions makes it possible to remove 12 unphysical parameters from the spurions, reducing the naive 10 complex parameters down to a total of 5 real positive parameters, 2 mixing angles, and 1 phase. At dimension 4, together with  $y_b$  and  $y_t$ , these give the quark masses and the CKM mixing matrix. The breaking pattern and utilization of the full group of  $G_Q$  transformations at dimension 4 means that all coefficients of the baryon number-conserving SMEFT operators are physical.<sup>7</sup>

For convenience we compute numerical values for the relevant parameters reproducing the observed CKM mixing matrix [143] and quark masses (taken from ref. [144] at the renormalisation scale set to  $m_t$ ) from the Yukawa terms in eq. (2.7) at tree level:<sup>8</sup>

$$\begin{aligned} \delta_d &= 1.46 \times 10^{-5}, & \delta'_d &= 2.91 \times 10^{-4}, & y_b &= 0.0155, \\ \delta_u &= 6.72 \times 10^{-6}, & \delta'_u &= 3.38 \times 10^{-3}, & y_t &= 0.934, \\ \epsilon_q &= 0.0380, & \theta_d &= 0.210, & \theta_u &= 0.0888, & \alpha &= -1.57. \end{aligned} \quad (2.11)$$

Although there are corrections from higher order terms and radiative corrections, this provides a decent estimate. As already anticipated, the largest breaking of the symmetry occurs at  $\mathcal{O}(10^{-2})$  by  $\epsilon_q$ , while the symmetry-allowed parameter  $y_t$  is  $\mathcal{O}(1)$ . In other words, the approximate  $\text{U}(2)^3$  successfully explains these features. However, it fails to explain the smallness of  $y_b$  and the hierarchy in  $\Delta_{u,d}$ . This flavour structure also gives a suppression in FCNCs due to the smallness of spurions and allows for new physics scale not far above the TeV scale [122–126]. Since this is the least restrictive symmetry in the quark sector we consider, similar (stronger) conclusions will hold in the subsequent three sections as well.

We determine the dimension-6 SMEFT operators allowed by the flavor structure up to several insertions of the spurions. We base this on the Warsaw basis for the SMEFT (cf. appendix A), where we identify the unique fermion combinations that appear, the bilinear and quartic structures. We present the possible flavor contractions (including spurions) for these field combinations, and these, in turn, can be directly inserted back in to the appropriate Warsaw basis operators to recover the full set of SMEFT operators compatible

<sup>7</sup>Some of these coefficients are unphysical when using the redundant spurion parametrization of ref. [1]. With this in mind, the parametrization in ref. [1] can still be useful in model building.

<sup>8</sup>A suitable parameter point can be found by going to the down aligned basis, which allows for relating  $\delta_d^{(\prime)}$  and  $y_b$  directly to the down-type Higgs Yukawas. After this, the remaining parameters can be determined from  $V_{\text{CKM}}^\dagger \hat{Y}_u^2 V_{\text{CKM}} = \Lambda Y_u Y_u^\dagger \Lambda^\dagger$ . Here the left-hand side consists of the observed up-type Yukawa couplings (diagonal) and the CKM matrix while the right-hand side consists of our Yukawa matrix in the down-aligned basis and  $\Lambda = \text{diag}(e^{i\theta}, e^{i\phi}, e^{-i(\theta+\phi)})$  are additional unphysical phases. This provides a total of 9 constraints fixing all 9 parameters of the right-hand side.

$U(2)_q \times U(2)_u \times U(2)_d$		$\mathcal{O}(1)$		$\mathcal{O}(V)$		$\mathcal{O}(V^2)$		$\mathcal{O}(V^3)$		$\mathcal{O}(\Delta)$		$\mathcal{O}(\Delta V)$	
$\psi^2 H^3$	$Q_{uH}$	1	1	1	1					1	1	1	1
	$Q_{dH}$	1	1	1	1					1	1	1	1
$\psi^2 XH$	$Q_{u(G,W,B)}$	3	3	3	3					3	3	3	3
	$Q_{d(G,W,B)}$	3	3	3	3					3	3	3	3
$\psi^2 H^2 D$	$Q_{Hq}^{(1,3)}$	4		2	2	2							
	$Q_{Hu}, Q_{Hd}$	4										2	2
	$Q_{Hud}$	1	1									2	2
$(LL)(LL)$	$Q_{qq}^{(1,3)}$	10		6	6	10	2	2	2				
$(RR)(RR)$	$Q_{uu}, Q_{dd}$	10										6	6
	$Q_{ud}^{(1,8)}$	8										8	8
$(LL)(RR)$	$Q_{qu}^{(1,8)}, Q_{qd}^{(1,8)}$	16		8	8	8				4	4	12	12
$(LR)(LR)$	$Q_{quqd}^{(1,8)}$	2	2	4	4	2	2			8	8	12	12
Total		63	11	28	28	22	4	2	2	20	20	50	50

**Table 2.** Counting of the pure quark SMEFT operators (see appendix A) assuming  $U(2)_q \times U(2)_u \times U(2)_d$  symmetry in the quark sector. The counting is performed taking up to three insertions of the  $V_q$  spurion, one insertion of  $\Delta_{u,d}$  and one insertion of the  $\Delta_{u,d} V_q$  spurion product. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

with the flavor structure. We present the decompositions of the bilinear structures in eqs. (2.13)–(2.18) and decompositions of the unique quartic structures in eqs. (2.19)–(2.24). This also includes the expanded set of structures available in case the flavor symmetry assumption is reduced to  $SU(2)^3$ . The spurion counting of the pure quark SMEFT operators assuming  $U(2)^3$  ( $SU(2)^3$ ) symmetry in the quark sector is presented in table 2 3.

**Decomposition of bilinear structures.** In this section, we present the construction of bilinear structures invariant under the  $U(2)^3$  flavor symmetry, starting with the  $\mathcal{O}(1)$  structures. Since  $q, u$  and  $d$  all decompose as  $\mathbf{2}_{q,u,d} \oplus \mathbf{1}$ , respectively, under  $U(2)^3$  group, the  $\mathcal{O}(1)$  bilinears can be formed either by contracting two doublets or singlets of the same field or by contracting singlets of the different fields. By doing this, we end up with nine  $\mathcal{O}(1)$  bilinears:  $(\bar{q}q)$ ,  $(\bar{q}_3 q_3)$ ,  $(\bar{u}u)$ ,  $(\bar{u}_3 u_3)$ ,  $(\bar{d}d)$ ,  $(\bar{d}_3 d_3)$ ,  $(\bar{u}_3 d_3)$ ,  $(\bar{q}_3 u_3)$  and  $(\bar{q}_3 d_3)$ .

There are only three bilinears that can be formed with one insertion of the  $V_q$  spurion. All of these include the singlet contraction with the quark doublet and one additional singlet from the field decomposition:  $(\bar{q} V_q q_3)$ ,  $(\bar{q} V_q u_3)$  and  $(\bar{q} V_q d_3)$ . However, in the case of  $SU(2)^3$ , there are additional bilinears, such as  $V_q^a \varepsilon_{ab} (\bar{q}_3 q^b)$ . Following analogous reasoning for the case of  $\mathcal{O}(V^2)$  bilinears, we obtain  $(\bar{q} V_q V_q^\dagger q)$  structure for the  $U(2)^3$  case and  $\varepsilon_{bc} (\bar{q} V_q V_q^c q^b)$  for the  $SU(2)^3$ .

$SU(2)_q \times SU(2)_u \times SU(2)_d$		$\mathcal{O}(1)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(V^3)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta V)$
$\psi^2 H^3$	$Q_{uH}$	1 1	2 2			2 2	4 4
	$Q_{dH}$	1 1	2 2			2 2	4 4
$\psi^2 XH$	$Q_{u(G,W,B)}$	3 3	6 6			6 6	12 12
	$Q_{d(G,W,B)}$	3 3	6 6			6 6	12 12
$\psi^2 H^2 D$	$Q_{Hq}^{(1,3)}$	4	4 4	4 2			
	$Q_{Hu}, Q_{Hd}$	4					8 8
	$Q_{Hud}$	1 1					8 8
$(LL)(LL)$	$Q_{qq}^{(1,3)}$	10	14 14	20 12	8 8		
$(RR)(RR)$	$Q_{uu}, Q_{dd}$	10					28 28
	$Q_{ud}^{(1,8)}$	8					32 32
$(LL)(RR)$	$Q_{qu}^{(1,8)}, Q_{qd}^{(1,8)}$	16	16 16	16 8		16 16	56 56
$(LR)(LR)$	$Q_{quqd}^{(1,8)}$	4 4	8 8	8 8		16 16	60 60
Total		65 13	58 58	48 30	8 8	48 48	224 224

**Table 3.** Counting of the pure quark SMEFT operators (see appendix A) assuming  $SU(2)_q \times SU(2)_u \times SU(2)_d$  symmetry in the quark sector. The counting is performed taking up to three insertions of the  $V_q$  spurion, one insertion of  $\Delta_{u,d}$  and one insertion of the  $\Delta_{u,d}V_q$  spurion product. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting. Due to the presence of the additional  $SU(2)$  structures in the decompositions, the counting is different compared to table 2.

Before we proceed with our discussion about  $\mathcal{O}(\Delta)$  and  $\mathcal{O}(\Delta V)$  bilinears, let us introduce the shorthand notation

$$(\tilde{\Delta}_{u,d})^a{}_d = \varepsilon^{ab}(\Delta_{u,d}^*){}_b{}^c \varepsilon_{cd} \sim (\mathbf{2}, \bar{\mathbf{2}}, \mathbf{1}), (\mathbf{2}, \mathbf{1}, \bar{\mathbf{2}}), \quad (2.12)$$

which proves to be useful in constructing  $SU(2)^3$  invariant structures. The  $\mathcal{O}(\Delta)$  bilinears are formed with the  $(\bar{q}\Delta_{u,d})_a$  contractions, which can then be contracted to  $u^a$  or  $d^a$  yielding two bilinears  $(\bar{q}\Delta_u u)$  and  $(\bar{q}\Delta_d d)$ . In case of  $SU(2)^3$  we get two additional ones given by  $(\bar{q}\tilde{\Delta}_u u)$  and  $(\bar{q}\tilde{\Delta}_d d)$ .

For the remaining  $\mathcal{O}(\Delta V)$  bilinears, we observe that the contractions  $(V_q^\dagger \Delta_{u,d})_a$  transform in the anti-fundamental representation of  $U(2)_{u,d}$  meaning they can form singlets when contracted to  $u^a$  or  $d^a$ . We find six such structures:  $(\bar{u}\Delta_u^\dagger V_q u_3)$ ,  $(\bar{d}\Delta_d^\dagger V_q d_3)$ ,  $(\bar{u}\Delta_u^\dagger V_q d_3)$ ,  $(\bar{u}_3 V_q^\dagger \Delta_d d)$ ,  $(\bar{q}_3 V_q^\dagger \Delta_u u)$  and  $(\bar{q}_3 V_q^\dagger \Delta_d d)$ . The complete list of bilinears is presented below and the new structures that appear in case of  $SU(2)^3$  symmetry are denoted in blue:

$\boxed{(\bar{q}q)}$ 

$$\begin{aligned} \mathcal{O}(1): & (\bar{q}q), \quad (\bar{q}_3q_3), \quad \mathcal{O}(V): (\bar{q}V_qq_3), \quad V_q^a \varepsilon_{ab} (\bar{q}_3q^b), \quad \text{H.c.}, \\ \mathcal{O}(V^2): & (\bar{q}V_qV_q^\dagger q), \quad [\varepsilon_{bc} (\bar{q}V_qV_q^c q^b), \quad \text{H.c.}]. \end{aligned} \quad (2.13)$$

 $\boxed{(\bar{u}u)}$ 

$$\begin{aligned} \mathcal{O}(1): & (\bar{u}u), \quad (\bar{u}_3u_3), \\ \mathcal{O}(\Delta V): & (\bar{u}\Delta_u^\dagger V_q u_3), \quad (\bar{u}_a u_3) \varepsilon^{ab} (V_q^\dagger \Delta_u)_b, \quad \varepsilon^{ad} \varepsilon_{bc} [\bar{u}^a V_q^b (\Delta_u)^c d u_3], \quad \text{H.c.}, \\ & \varepsilon_{bc} [\bar{u}_3 V_q^b (\Delta_u)^c_a u^a], \quad \text{H.c.}. \end{aligned} \quad (2.14)$$

 $\boxed{(\bar{d}d)}$ 

$$\begin{aligned} \mathcal{O}(1): & (\bar{d}d), \quad (\bar{d}_3d_3), \\ \mathcal{O}(\Delta V): & (\bar{d}\Delta_d^\dagger V_q d_3), \quad (\bar{d}_a d_3) \varepsilon^{ab} (V_q^\dagger \Delta_d)_b, \quad \varepsilon^{ad} \varepsilon_{bc} [\bar{d}^a V_q^b (\Delta_d)^c d d_3], \quad \text{H.c.}, \\ & \varepsilon_{bc} [\bar{d}_3 V_q^b (\Delta_d)^c_a d^a], \quad \text{H.c.}. \end{aligned} \quad (2.15)$$

 $\boxed{(\bar{u}d)}$ 

$$\begin{aligned} \mathcal{O}(1): & (\bar{u}_3d_3), \\ \mathcal{O}(\Delta V): & (\bar{u}\Delta_u^\dagger V_q d_3), \quad (\bar{u}_3 V_q^\dagger \Delta_d d), \quad (\bar{u}_a d_3) \varepsilon^{ab} (V_q^\dagger \Delta_u)_b, \quad (\bar{u}_3 d^a) \varepsilon_{ab} (V_q \Delta_d^\dagger)^b, \\ & \varepsilon^{ad} \varepsilon_{bc} [\bar{u}_a V_q^b (\Delta_u)^c_d d_3], \quad \varepsilon^{bc} \varepsilon_{ad} [\bar{u}_3 (V_q^*)_b (\Delta_d^*)_c^d d^a], \\ & \varepsilon^{bc} [\bar{u}_a (V_q^*)_b (\Delta_u^*)_c^a d_3], \quad \varepsilon_{bc} [\bar{u}_3 V_q^b (\Delta_d)^c_a d^a]. \end{aligned} \quad (2.16)$$

 $\boxed{(\bar{q}u)}$ 

$$\begin{aligned} \mathcal{O}(1): & (\bar{q}_3u_3), \quad \mathcal{O}(V): (\bar{q}V_qu_3), \quad (V_q^*)_a \varepsilon^{ab} (\bar{q}_b u_3), \\ \mathcal{O}(\Delta): & (\bar{q}\Delta_u u), \quad (\bar{q}\tilde{\Delta}_u u), \\ \mathcal{O}(\Delta V): & (\bar{q}_3 V_q^\dagger \Delta_u u), \quad (\bar{q}_3 V_q^\dagger \tilde{\Delta}_u u), \quad \varepsilon_{bc} [\bar{q}_3 V_q^b (\Delta_u)^c_a u^a], \quad \varepsilon_{ac} [\bar{q}_3 V_q^b (\Delta_u^*)_b^c u^a]. \end{aligned} \quad (2.17)$$

 $\boxed{(\bar{q}d)}$ 

$$\begin{aligned} \mathcal{O}(1): & (\bar{q}_3d_3), \quad \mathcal{O}(V): (\bar{q}V_qd_3), \quad (V_q^*)_a \varepsilon^{ab} (\bar{q}_b d_3), \\ \mathcal{O}(\Delta): & (\bar{q}\Delta_d d), \quad (\bar{q}\tilde{\Delta}_d d), \\ \mathcal{O}(\Delta V): & (\bar{q}_3 V_q^\dagger \Delta_d d), \quad (\bar{q}_3 V_q^\dagger \tilde{\Delta}_d d), \quad \varepsilon_{bc} [\bar{q}_3 V_q^b (\Delta_d)^c_a d^a], \quad \varepsilon_{ac} [\bar{q}_3 V_q^b (\Delta_d^*)_b^c d^a]. \end{aligned} \quad (2.18)$$

**Decomposition of quartic structures.** Let us continue with the construction of the quartic structures. In what follows, we focus on the unique, non-factorizable<sup>9</sup> structures only. Starting with  $\mathcal{O}(1)$  structures, we follow the similar reasoning as in the case of bilinears, obtaining six structures:  $(\bar{q}_a q^b)(\bar{q}_b q^a)$ ,  $(\bar{q}_a q_3)(\bar{q}_3 q^a)$ ,  $(\bar{u}_a u^b)(\bar{u}_b u^a)$ ,  $(\bar{u}_a u_3)(\bar{u}_3 u^a)$ ,  $(\bar{d}_a d^b)(\bar{d}_b d^a)$  and  $(\bar{d}_a d_3)(\bar{d}_3 d^a)$ . In the case of  $SU(2)^3$  symmetry, only one additional  $\mathcal{O}(1)$  structure appears:  $(\bar{q}_a u_3)\varepsilon^{ab}(\bar{q}_b d_3)$ .

At  $\mathcal{O}(V)$ , the  $U(2)^3$  and  $SU(2)^3$  unique structures are  $(\bar{q}_a q_3)(\bar{q}V_q q^a)$ ,  $(\bar{q}_3 q^a)(\bar{q}_a \varepsilon_{bc} V_q^c q^b)$  and  $(\bar{q}_3 q^a)(\bar{q}V_q \varepsilon_{ac} q^c)$ , while at  $\mathcal{O}(V^2)$  there is only one structure of the form  $(\bar{q}_a V_q^\dagger q)(\bar{q}V_q q^a)$ . With one insertion of  $\Delta$  spurion, there are four  $U(2)^3$  unique ones:  $(\bar{q}_a V_q^\dagger q)(\Delta_u)^a_b(\bar{u}_3 u^b)$ ,  $(\bar{q}_a q_3)(\Delta_d)^a_b(\bar{d}_3 d^b)$ ,  $(\bar{q}_a u_3)(\Delta_d)^a_b(\bar{q}_3 d^b)$  and  $(\bar{q}_3 u^a)(\Delta_u)^b_a(\bar{q}_b d_3)$ .

With the insertion of both  $\Delta_{u,d}$  and  $V_q$  spurions, we obtain six  $\mathcal{O}(\Delta V)$   $U(2)^3$  structures given by  $(\bar{u}_a u_3)(\bar{u}\Delta_u^\dagger V_q u^a)$ ,  $(\bar{d}_a d_3)(\bar{d}\Delta_d^\dagger V_q d^a)$ ,  $(\bar{q}_a V_q^\dagger q)(\Delta_u)^a_b(\bar{u}_3 u^b)$ ,  $(\bar{q}_a V_q^\dagger q)(\Delta_d)^a_b(\bar{d}_3 d^b)$ ,  $(\bar{q}_a u_3)(\Delta_d)^a_b(\bar{q}V_q d^b)$  and  $(\bar{q}V_q u^a)(\Delta_u)^b_a(\bar{q}_b d_3)$ . There are, however, plenty of new  $SU(2)^3$  unique structures that emerge at both  $\mathcal{O}(\Delta)$  and  $\mathcal{O}(\Delta V)$ . The complete list is presented below and the  $SU(2)^3$  structures are denoted in blue:

$$\boxed{(\bar{q}q)(\bar{q}q)}$$

$$\begin{aligned}
 \mathcal{O}(1): & \quad (\bar{q}_a q^b)(\bar{q}_b q^a), (\bar{q}_a q_3)(\bar{q}_3 q^a), \\
 \mathcal{O}(V): & \quad (\bar{q}_a q_3)(\bar{q}V_q q^a), (\bar{q}_3 q^a)(\bar{q}_a \varepsilon_{bc} V_q^c q^b), (\bar{q}_3 q^a)(\bar{q}V_q \varepsilon_{ac} q^c), \text{ H.c.}, \\
 \mathcal{O}(V^2): & \quad (\bar{q}_a V_q^\dagger q)(\bar{q}V_q q^a).
 \end{aligned}
 \tag{2.19}$$

$$\boxed{(\bar{u}u)(\bar{u}u)}$$

$$\begin{aligned}
 \mathcal{O}(1): & \quad (\bar{u}_a u^b)(\bar{u}_b u^a), (\bar{u}_a u_3)(\bar{u}_3 u^a), \\
 \mathcal{O}(\Delta V): & \quad (\bar{u}_a u_3)(\bar{u}\Delta_u^\dagger V_q u^a), (\bar{u}_a u_3)\varepsilon^{ab}\varepsilon_{de}[\bar{u}_b V_q^d(\Delta_u)^e_c u^c], \varepsilon^{be}\varepsilon_{cd}(\bar{u}_a u_3)[\bar{u}_b V_q^c(\Delta_u)^d_e u^a], \text{ H.c.}, \\
 & \quad (\bar{u}_3 u^a)[\bar{u}_a V_q^c \varepsilon_{cd}(\Delta_u)^d_b u^b], (\bar{u}_3 u^a)[\bar{u}_a \varepsilon_{bd} V_q^c(\Delta_u^*)_c^d u^b], \varepsilon_{ac}(\bar{u}_3 u^a)[\bar{u}_b V_q^d(\Delta_u^*)_d^b u^c], \text{ H.c.}
 \end{aligned}
 \tag{2.20}$$

$$\boxed{(\bar{d}d)(\bar{d}d)}$$

$$\begin{aligned}
 \mathcal{O}(1): & \quad (\bar{d}_a d^b)(\bar{d}_b d^a), (\bar{d}_a d_3)(\bar{d}_3 d^a), \\
 \mathcal{O}(\Delta V): & \quad (\bar{d}_a d_3)(\bar{d}\Delta_d^\dagger V_q d^a), (\bar{d}_a d_3)\varepsilon^{ab}\varepsilon_{de}[\bar{d}_b V_q^d(\Delta_d)^e_c d^c], \varepsilon^{be}\varepsilon_{cd}(\bar{d}_a d_3)[\bar{d}_b V_q^c(\Delta_d)^d_e d^a], \text{ H.c.}, \\
 & \quad (\bar{d}_3 d^a)[\bar{d}_a V_q^c \varepsilon_{cd}(\Delta_d)^d_b d^b], (\bar{d}_3 d^a)[\bar{d}_a \varepsilon_{bd} V_q^c(\Delta_d^*)_c^d d^b], \varepsilon_{ac}(\bar{d}_3 d^a)[\bar{d}_b V_q^d(\Delta_d^*)_d^b d^c], \text{ H.c.}
 \end{aligned}
 \tag{2.21}$$

<sup>9</sup>Epithets ‘unique’ and ‘non-factorizable’ are used interchangeably when dealing with the quartic structures. This nomenclature refers simply to the quartic structures that cannot be formed as a product of two factorizing bilinears fully invariant under the discussed flavor group. Needless to say, the final spurion counting of the SMEFT operators is performed taking the full set of quartic structures.

$$\boxed{(\bar{q}q)(\bar{u}u)}$$

$$\begin{aligned} \mathcal{O}(\Delta): & \quad (\bar{q}_a q_3)(\Delta_u)^a{}_b (\bar{u}_3 u^b), \quad \varepsilon^{bc} (\bar{q}_a q_3)(\Delta_u)^a{}_c (\bar{u}_b u^3), \quad \varepsilon^{bd} \varepsilon_{ac} (\bar{q}_3 q^a)(\Delta_u)^c{}_d (\bar{u}_b u_3), \quad \text{H.c.}, \\ & \quad \varepsilon_{ac} (\bar{q}_3 q^a) [\bar{u}_3 (\Delta_u)^c{}_b u^b], \quad \text{H.c.}, \\ \mathcal{O}(\Delta V): & \quad (\bar{q}_a V_q^\dagger q)(\Delta_u)^a{}_b (\bar{u}_3 u^b), \quad \varepsilon^{ce} \varepsilon_{bd} (\bar{q} V_q q^b)(\Delta_u)^d{}_e (\bar{u}_c u_3), \quad \varepsilon^{ce} \varepsilon_{bd} (\bar{q}_a V_q^d q^b)(\Delta_u)^a{}_e (\bar{u}_c u_3), \quad \text{H.c.}, \\ & \quad \varepsilon_{bd} (\bar{q} V_q q^b)(\Delta_u)^d{}_c (\bar{u}_3 u^c), \quad \varepsilon_{bd} (\bar{q}_a V_q^d q^b)(\Delta_u)^a{}_c (\bar{u}_3 u^c), \quad \varepsilon_{cd} (\bar{q} V_q q^b)(\Delta_u^*){}_b{}^d (\bar{u}_3 u^c), \quad \text{H.c.} \end{aligned} \quad (2.22)$$

$$\boxed{(\bar{q}q)(\bar{d}d)}$$

$$\begin{aligned} \mathcal{O}(\Delta): & \quad (\bar{q}_a q_3)(\Delta_d)^a{}_b (\bar{d}_3 d^b), \quad \varepsilon^{bc} (\bar{q}_a q_3)(\Delta_d)^a{}_c (\bar{d}_b d^3), \quad \varepsilon^{bd} \varepsilon_{ac} (\bar{q}_3 q^a)(\Delta_d)^c{}_d (\bar{d}_b d_3), \quad \text{H.c.}, \\ & \quad \varepsilon_{ac} (\bar{q}_3 q^a) [\bar{d}_3 (\Delta_d)^c{}_b d^b], \quad \text{H.c.}, \\ \mathcal{O}(\Delta V): & \quad (\bar{q}_a V_q^\dagger q)(\Delta_d)^a{}_b (\bar{d}_3 d^b), \quad \varepsilon^{ce} \varepsilon_{bd} (\bar{q} V_q q^b)(\Delta_d)^d{}_e (\bar{d}_c d_3), \quad \varepsilon^{ce} \varepsilon_{bd} (\bar{q}_a V_q^d q^b)(\Delta_d)^a{}_e (\bar{d}_c d_3), \quad \text{H.c.}, \\ & \quad \varepsilon_{bd} (\bar{q} V_q q^b)(\Delta_d)^d{}_c (\bar{d}_3 d^c), \quad \varepsilon_{bd} (\bar{q}_a V_q^d q^b)(\Delta_d)^a{}_c (\bar{d}_3 d^c), \quad \varepsilon_{cd} (\bar{q} V_q q^b)(\Delta_d^*){}_b{}^d (\bar{d}_3 d^c), \quad \text{H.c.} \end{aligned} \quad (2.23)$$

$$\boxed{(\bar{q}u)(\bar{q}d)}$$

$$\begin{aligned} \mathcal{O}(1): & \quad (\bar{q}_a u_3) \varepsilon^{ab} (\bar{q}_b d_3), \\ \mathcal{O}(\Delta): & \quad (\bar{q}_a u_3)(\Delta_d)^a{}_b (\bar{q}_3 d^b), \quad (\bar{q}_3 u^a)(\Delta_u)^b{}_a (\bar{q}_b d_3), \quad (\bar{q}_a u_3) (\tilde{\Delta}_d)^a{}_b (\bar{q}_3 d^b), \quad (\bar{q}_3 u^a) (\tilde{\Delta}_u)^b{}_a (\bar{q}_b d_3), \\ \mathcal{O}(\Delta V): & \quad (\bar{q}_a u_3)(\Delta_d)^a{}_b (\bar{q} V_q d^b), \quad (\bar{q} V_q u^a)(\Delta_u)^b{}_a (\bar{q}_b d_3), \quad \varepsilon^{ac} \varepsilon_{be} (\bar{q}_a u^b) V_q^d (\Delta_u^*){}_d{}^e (\bar{q}_c d_3), \\ & \quad (\bar{q} V_q u^b) (\tilde{\Delta}_u)^c{}_b (\bar{q}_c d_3), \quad \varepsilon^{ac} [\bar{q}_a (V_q^\dagger \Delta_u)_b u^b] (\bar{q}_c d_3), \quad \varepsilon^{ad} [\bar{q}_a (V_q^*)_d u^b] (\Delta_u)^c{}_b (\bar{q}_c d_3), \\ & \quad \varepsilon^{ac} [\bar{q}_a (V_q^\dagger \tilde{\Delta}_u)_b u^b] (\bar{q}_c d^3), \quad \varepsilon^{ad} [\bar{q}_a (V_q^*)_d u^b] (\tilde{\Delta}_u)^c{}_b (\bar{q}_c d^3), \quad \varepsilon^{ab} \varepsilon_{ce} (\bar{q}_a u_3) V_q^d (\Delta_d^*){}_d{}^e (\bar{q}_b d^c), \\ & \quad (\bar{q}_a u_3) (\tilde{\Delta}_d)^a{}_c (\bar{q} V_q d^c), \quad \varepsilon^{ab} (\bar{q}_a u_3) [\bar{q}_b (V_q^\dagger \Delta_d)_c d^c], \quad \varepsilon^{bd} [\bar{q}_a (V_q^*)_d u_3] (\Delta_d)^a{}_c (\bar{q}_b d^c), \\ & \quad \varepsilon^{ab} (\bar{q}_a u_3) [\bar{q}_b (V_q^\dagger \tilde{\Delta}_d)_c d^c], \quad \varepsilon^{bd} [\bar{q}_a (V_q^*)_d u_3] (\tilde{\Delta}_d)^a{}_c (\bar{q}_b d^c). \end{aligned} \quad (2.24)$$

## 2.2 $U(2)^3 \times U(1)_{d_3}$ symmetry

If the NP possesses a  $G = U(2)_q \times U(2)_u \times U(2)_d \times U(1)_{d_3} \subset G_Q$ , it is possible to introduce the  $b$  Yukawa as a spurion for the perturbative counting. Under this symmetry, the quark fields decompose as<sup>10</sup>

$$q = \begin{bmatrix} q^a \sim (\mathbf{2}, \mathbf{1}, \mathbf{1}|0) \\ q_3 \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}|0) \end{bmatrix}, \quad u = \begin{bmatrix} u^a \sim (\mathbf{1}, \mathbf{2}, \mathbf{1}|0) \\ u_3 \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}|0) \end{bmatrix}, \quad d = \begin{bmatrix} d^a \sim (\mathbf{1}, \mathbf{1}, \mathbf{2}|0) \\ d_3 \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}|1) \end{bmatrix}, \quad (2.25)$$

and the minimal set of spurions required to write the Yukawa matrix is given as:

$$\Delta_u \sim (\mathbf{2}, \bar{\mathbf{2}}, \mathbf{1}|0), \quad \Delta_d \sim (\mathbf{2}, \mathbf{1}, \bar{\mathbf{2}}|0), \quad V_q \sim (\mathbf{2}, \mathbf{1}, \mathbf{1}|0), \quad X_b \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}|-1). \quad (2.26)$$

<sup>10</sup>The representations of the fields and spurions with respect to this particular group are labeled as  $(U(2)_q, U(2)_u, U(2)_d|U(1)_{d_3})$ .

$U(2)^3 \times U(1)_{d_3}$		$\mathcal{O}(1)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(V^3)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta V)$	$\mathcal{O}(X)$	$\mathcal{O}(VX)$	$\mathcal{O}(V^2X)$	$\mathcal{O}(\Delta X)$	$\mathcal{O}(\Delta VX)$
$\psi^2 H^3$	$Q_{uH}$	1 1	1 1			1 1	1 1					
	$Q_{dH}$					1 1	1 1	1 1	1 1			
$\psi^2 XH$	$Q_{u(G,W,B)}$	3 3	3 3			3 3	3 3					
	$Q_{d(G,W,B)}$					3 3	3 3	3 3	3 3			
$\psi^2 H^2 D$	$Q_{Hq}^{(1,3)}$	4	2 2	2								
	$Q_{Hu}$	2				1 1						
	$Q_{Hd}$	2									1 1	1 1
	$Q_{Hud}$					1 1	1 1				1 1	1 1
$(LL)(LL)$	$Q_{qq}^{(1,3)}$	10	6 6	10 2	2 2							
$(RR)(RR)$	$Q_{uu}$	5					3 3					
	$Q_{dd}$	5									3 3	3 3
	$Q_{ud}^{(1,8)}$	8					4 4				4 4	4 4
$(LL)(RR)$	$Q_{qu}^{(1,8)}$	8	4 4	4		2 2	6 6					
	$Q_{qd}^{(1,8)}$	8	4 4	4							2 2	6 6
$(LR)(LR)$	$Q_{quqd}^{(1,8)}$					4 4	6 6	2 2	4 4	2 2	4 4	6 6
Total		56 4	20 20	20 2	2 2	14 14	29 29	7 7	8 8	2 2	6 6	21 21

**Table 4.** Counting of the pure quark SMEFT operators (see appendix A) assuming  $U(2)^3 \times U(1)_{d_3}$  symmetry in the quark sector. The counting is done up to three insertions of the  $V_q$  spurion and one insertion of  $\Delta_{u,d}$  or  $X_b$ . For spurion products, the counting is presented for  $\Delta_{u,d}V_q$ ,  $V_qX_b$ , and  $\Delta_{u,d}X_b$  as well as for  $V_q^2X_b$  and  $\Delta_{u,d}V_qX_b$  insertions. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

Transforming the fields with  $G_Q/G$  rotations and redefining the spurions as in section 2.1, the Yukawa matrices can generically be written as:

$$Y_u = \begin{bmatrix} \Delta_u & V_q \\ 0 & y_t \end{bmatrix}, \quad Y_d = \begin{bmatrix} \Delta_d & 0 \\ 0 & X_b \end{bmatrix}, \quad (2.27)$$

for  $y_t > 0$ . The complete breaking of  $G$  by the spurions proceeds in the same way, leading to the parametrization of the  $\Delta_{u,d}$  and  $V_q$  spurions of eqs. (2.8)–(2.10). The only novelty compared to the  $U(2)^3$  case is the introduction of the  $X_b$  spurion, which breaks the  $U(1)_{d_3}$  symmetry, allowing us to remove an extra phase from  $X_b$  and make it real and positive. In total, we end up parametrizing spurions with 6 real and positive parameters, 2 mixing angles and 1 phase. The numerical values of the parameters are as in eq. (2.11) but with the replacement  $y_b = X_b$  explaining the smallness of the bottom Yukawa.

The spurion counting of the pure quark operators is presented in the table 4, while the bilinear and unique quartic structures are listed in eqs. (2.28)–(2.33) and eqs. (2.34)–(2.39) respectively.

**Decomposition of bilinear structures.** We follow the analysis of section 2.1 to obtain all SMEFT structures in the presence of the  $U(2)^3 \times U(1)_{d_3}$  symmetry. The only difference is that when the  $U(1)_{d_3}$  charge of a structure is nonzero, powers of the  $X_b$  spurion need to be included, giving further spurion suppression. The list of all invariant bilinears is presented below:

$\boxed{(\bar{q}q)}$ 

$$\mathcal{O}(1): (\bar{q}q), (\bar{q}_3q_3), \quad \mathcal{O}(V): (\bar{q}V_qq_3), \quad \text{H.c.}, \quad \mathcal{O}(V^2): (\bar{q}V_qV_q^\dagger q). \quad (2.28)$$

 $\boxed{(\bar{u}u)}$ 

$$\mathcal{O}(1): (\bar{u}u), (\bar{u}_3u_3), \quad \mathcal{O}(\Delta V): (\bar{u}\Delta_u^\dagger V_q u_3), \quad \text{H.c.} \quad (2.29)$$

 $\boxed{(\bar{d}d)}$ 

$$\mathcal{O}(1): (\bar{d}d), (\bar{d}_3d_3), \quad \mathcal{O}(\Delta V X): (\bar{d}\Delta_d^\dagger V_q X_b d_3), \quad \text{H.c.} \quad (2.30)$$

 $\boxed{(\bar{u}d)}$ 

$$\mathcal{O}(X): (\bar{u}_3 X_b d_3), \quad \mathcal{O}(\Delta V): (\bar{u}_3 V_q^\dagger \Delta_d d), \quad \mathcal{O}(\Delta V X): (\bar{u}\Delta_u^\dagger V_q X_b d_3). \quad (2.31)$$

 $\boxed{(\bar{q}u)}$ 

$$\mathcal{O}(1): (\bar{q}_3 u_3), \quad \mathcal{O}(V): (\bar{q}V_q u_3), \quad \mathcal{O}(\Delta): (\bar{q}\Delta_u u), \quad \mathcal{O}(\Delta V): (\bar{q}_3 V_q^\dagger \Delta_u u). \quad (2.32)$$

 $\boxed{(\bar{q}d)}$ 

$$\begin{aligned} \mathcal{O}(X): (\bar{q}_3 X_b d_3), \quad \mathcal{O}(V X): (\bar{q}V_q X_b d_3), \quad \mathcal{O}(\Delta): (\bar{q}\Delta_d d), \\ \mathcal{O}(\Delta V): (\bar{q}_3 V_q^\dagger \Delta_d d). \end{aligned} \quad (2.33)$$

**Decomposition of quartic structures.** Again following the construction of quartic SMEFT structures in section 2.1 and with suitable insertions of  $X_b$ , the complete list of  $U(2)^3 \times U(1)_{d_3}$  unique quartic structures is presented below:

 $\boxed{(\bar{q}q)(\bar{q}q)}$ 

$$\begin{aligned} \mathcal{O}(1): (\bar{q}_a q^b)(\bar{q}_b q^a), (\bar{q}_a q_3)(\bar{q}_3 q^a), \quad \mathcal{O}(V): (\bar{q}_a q_3)(\bar{q}V_q q^a), \quad \text{H.c.}, \\ \mathcal{O}(V^2): (\bar{q}_a V_q^\dagger q)(\bar{q}V_q q^a). \end{aligned} \quad (2.34)$$

 $\boxed{(\bar{u}u)(\bar{u}u)}$ 

$$\mathcal{O}(1): (\bar{u}_a u^b)(\bar{u}_b u^a), (\bar{u}_a u_3)(\bar{u}_3 u^a), \quad \mathcal{O}(\Delta V): (\bar{u}_a u_3)(\bar{u}\Delta_u^\dagger V_q u^a), \quad \text{H.c.} \quad (2.35)$$

 $\boxed{(\bar{d}d)(\bar{d}d)}$ 

$$\mathcal{O}(1): (\bar{d}_a d^b)(\bar{d}_b d^a), (\bar{d}_a d_3)(\bar{d}_3 d^a), \quad \mathcal{O}(\Delta V X): (\bar{d}_a X_b d_3)(\bar{d}\Delta_d^\dagger V_q d^a), \quad \text{H.c.} \quad (2.36)$$



$$\boxed{(\bar{q}q)(\bar{u}u)}$$

$$\mathcal{O}(\Delta): (\bar{q}_a q_3)(\Delta_u)^a{}_b (\bar{u}_3 u^b), \quad \text{H.c.}, \quad \mathcal{O}(\Delta V): (\bar{q}_a V_q^\dagger q)(\Delta_u)^a{}_b (\bar{u}_3 u^b), \quad \text{H.c.} \quad (2.37)$$

$$\boxed{(\bar{q}q)(\bar{d}d)}$$

$$\mathcal{O}(\Delta X): (\bar{q}_a q_3)(\Delta_d)^a{}_c (\bar{d}_3 X_b^* d^c), \quad \text{H.c.}, \quad \mathcal{O}(\Delta V X): (\bar{q}_a V_q^\dagger q)(\Delta_d)^a{}_c (\bar{d}_3 X_b^* d^c), \quad \text{H.c.} \quad (2.38)$$

$$\boxed{(\bar{q}u)(\bar{q}d)}$$

$$\begin{aligned} \mathcal{O}(\Delta): (\bar{q}_a u_3)(\Delta_d)^a{}_b (\bar{q}_3 d^b), \quad \mathcal{O}(\Delta X): (\bar{q}_3 u^a)(\Delta_u)^c{}_a (\bar{q}_c X_b d_3), \\ \mathcal{O}(\Delta V): (\bar{q}_a u_3)(\Delta_d)^a{}_b (\bar{q} V_q d^b), \quad \mathcal{O}(\Delta V X): (\bar{q} V_q u^a)(\Delta_u)^c{}_a (\bar{q}_c X_b d_3). \end{aligned} \quad (2.39)$$

### 2.3 $U(2)^2 \times U(3)_d$ symmetry

We consider the quark symmetry  $G = U(2)_q \times U(2)_u \times U(3)_d \subset G_Q$ , which is the flavor symmetry of the SM when only the top quark has a Yukawa coupling to the Higgs. This is also a good approximate symmetry of the realistic Yukawa sector. Here,  $q$  and  $u$  decompose as

$$q = \begin{bmatrix} q^a \sim (\mathbf{2}, \mathbf{1}, \mathbf{1}) \\ q_3 \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}) \end{bmatrix}, \quad u = \begin{bmatrix} u^a \sim (\mathbf{1}, \mathbf{2}, \mathbf{1}) \\ u_3 \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}) \end{bmatrix}, \quad (2.40)$$

under  $G$ , while  $d$  transforms as  $d^i \sim (\mathbf{1}, \mathbf{1}, \mathbf{3})$ . The minimal set of spurions needed to produce realistic Yukawa matrices for the quarks are

$$\Delta_u \sim (\mathbf{2}, \bar{\mathbf{2}}, \mathbf{1}), \quad \Sigma_d \sim (\mathbf{2}, \mathbf{1}, \bar{\mathbf{3}}), \quad \Lambda_d \sim (\mathbf{1}, \mathbf{1}, \mathbf{3}), \quad (2.41)$$

but we will also allow for  $V_q \sim (\mathbf{2}, \mathbf{1}, \mathbf{1})$  to allow for further mixing between top quarks and light generations. In the minimal analysis, one can simply set  $V_q = 0$ . In either case, a combination of rotations in  $G_Q/G$  and redefinitions of the spurions, as detailed in section 2.1, allows for the generic Yukawa matrices to be parametrized as

$$Y_u = \begin{bmatrix} \Delta_u & 0 \\ 0 & y_t \end{bmatrix}, \quad Y_d = \begin{bmatrix} \Sigma_d \\ \Lambda_d^\dagger \end{bmatrix}, \quad (2.42)$$

with a real coefficient  $y_t$ .

To see how the spurions break  $G$ , we begin with  $\Delta_u$ , which, after a suitable rotation, can be parametrized as

$$\Delta_u \longrightarrow \begin{bmatrix} \delta_u & 0 \\ 0 & \delta'_u \end{bmatrix}: \quad U(2)_q \times U(2)_u \longrightarrow U(1)_{q+u}^2. \quad (2.43)$$

Meanwhile, a  $U(3)_d$  rotation can be used to align  $\Lambda_b$  to the 3rd generation:

$$\Lambda_d \longrightarrow \begin{bmatrix} 0 & 0 & y_b \end{bmatrix}^\top: \quad U(3)_d \longrightarrow U(2)_d. \quad (2.44)$$

The last part of the symmetry is used to parametrize

$$\Sigma_d \longrightarrow \begin{bmatrix} c_d & -s_d e^{i\alpha} \\ s_d e^{-i\alpha} & c_d \end{bmatrix} \begin{bmatrix} \delta_d & 0 & 0 \\ 0 & \delta'_d & 0 \end{bmatrix} \begin{bmatrix} c_{13} & 0 & -s_{13} \\ -s_{13}s_{23} & c_{23} & -c_{13}s_{23} \\ s_{13}c_{23} & s_{23} & c_{13}c_{23} \end{bmatrix} : \quad \text{U}(1)_{q+u}^2 \times \text{U}(2)_d \longrightarrow \emptyset. \quad (2.45)$$

The singular values can always be taken to be  $\delta_u^{(\prime)}, \delta_d^{(\prime)}, y_b > 0$ . The complete breaking of  $G$  by the spurions makes it possible to remove 17 unphysical parameters from the spurions, reducing the naive 13 complex parameters down to a total of 5 real positive parameters, 3 mixing angles, and a phase. In case  $V_q$  is included, both of its complex parameters are physical.

Following a similar procedure to eq. (2.11), the observed CKM matrix and quark masses are reproduced for

$$\begin{aligned} \delta_d &= 5.70 \times 10^{-5}, & \delta'_d &= 6.91 \times 10^{-4}, & y_b &= 0.0155, \\ \delta_u &= 6.72 \times 10^{-6}, & \delta'_u &= 3.38 \times 10^{-3}, & y_t &= 0.934, \\ \theta_d &= 0.096, & \theta_{13} &= 0.412, & \theta_{23} &= 1.453, & \alpha &= -1.19. \end{aligned} \quad (2.46)$$

The absolute value of all symmetry-breaking parameters (including  $y_b$ ) is small compared with the symmetry-allowed  $y_t$ .

The spurion counting of the pure quark SMEFT operators assuming  $\text{U}(2)^2 \times \text{U}(3)_d$  symmetry is presented in table 5. The decompositions of the bilinear structures are listed in (2.47)–(2.52) and the unique quartic structures are given in (2.53)–(2.58).

**Decomposition of bilinear structures.** For the  $\mathcal{O}(1)$  bilinear structures, since  $d^i \sim (\mathbf{1}, \mathbf{1}, \mathbf{3})$ , there is only one  $\mathcal{O}(1)$  bilinear with two appearances of  $d$  given as  $(\bar{d}d)$  and four with two appearances of  $q$  and  $u$ :  $(\bar{q}q)$ ,  $(\bar{q}_3 q_3)$ ,  $(\bar{u}u)$  and  $(\bar{u}_3 u_3)$ . One additional  $\mathcal{O}(1)$  structure can be formed with two singlets:  $(\bar{q}_3 u_3)$ .

We proceed with the bilinear structures containing one insertion of the  $V_q$  or  $\Lambda_d$  spurion. Since  $V_q \sim (\mathbf{2}, \mathbf{1}, \mathbf{1})$ , the only possible singlet is formed contracting  $V_q$  and the quark doublet. There are two such structures of the form  $(\bar{q}V_q q_3)$  and  $(\bar{q}V_q u_3)$ . Similar reasoning holds for  $\Lambda_d$ , yielding two more bilinear structures:  $(\bar{q}_3 \Lambda_d^\dagger d)$  and  $(\bar{u}_3 \Lambda_d^\dagger d)$ .

For bilinear structures at  $\mathcal{O}(V^2)$  and  $\mathcal{O}(\Lambda_d^2)$ , we have two structures formed by contracting the insertion of the spurion with the appropriate field:  $(\bar{q}V_q V_q^\dagger q)$  and  $(\bar{d}\Lambda_d \Lambda_d^\dagger d)$ . At  $\mathcal{O}(\Delta)$  there is only one structure allowed:  $(\bar{q}\Delta_u u)$ , and similar conclusion holds for  $\mathcal{O}(\Sigma_d)$  with one structure:  $(\bar{q}\Sigma_d d)$ .

In order to construct structures with two insertions of different spurions, let us first note the transformation properties of the relevant spurion products. At  $\mathcal{O}(\Lambda_d \Sigma_d)$ , we have  $\Sigma_d \Lambda_d \sim (\mathbf{2}, \mathbf{1}, \mathbf{1})$  transformation, which, being the same as for the  $V_q$  spurion, gives two structures:  $(\bar{q}\Sigma_d \Lambda_d q_3)$  and  $(\bar{q}\Sigma_d \Lambda_d u_3)$ . The analogous product we have is  $V_q^\dagger \Sigma_d \sim (\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}})$ , which can be utilized to construct two more structures of the form  $(\bar{q}_3 V_q^\dagger \Sigma_d d)$  and  $(\bar{u}_3 V_q^\dagger \Sigma_d d)$ . There is also one structure at  $\mathcal{O}(V_q \Lambda_d)$  given as  $(\bar{q}V_q \Lambda_d^\dagger d)$ . Finally, we note the transformation  $V_q^\dagger \Delta_u \sim (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{1})$ , which can be used to construct the remaining  $\mathcal{O}(\Delta V_q)$  structures:  $(\bar{u}\Delta_u^\dagger V_q u_3)$  and  $(\bar{q}_3 V_q^\dagger \Delta_u u)$ . The complete list of bilinears is presented below:

$U(2)_q \times U(2)_u \times U(3)_d$		$\mathcal{O}(1)$	$\mathcal{O}(V_q)$	$\mathcal{O}(V_q^2)$	$\mathcal{O}(V_q^3)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta V_q)$	$\mathcal{O}(\Lambda_d)$	$\mathcal{O}(\Sigma_d)$	$\mathcal{O}(V_q \Lambda_d)$	$\mathcal{O}(\Lambda_d^2)$	$\mathcal{O}(V_q \Sigma_d)$	$\mathcal{O}(\Lambda_d \Sigma_d)$
$\psi^2 H^3$	$Q_{uH}$ $Q_{dH}$	1 1	1 1			1 1	1 1			1 1		1 1	1 1
$\psi^2 XH$	$Q_{u(G,W,B)}$ $Q_{d(G,W,B)}$	3 3	3 3			3 3	3 3			3 3		3 3	3 3
$\psi^2 H^2 D$	$Q_{Hq}^{(1,3)}$ $Q_{Hu}$ $Q_{Hd}$ $Q_{Hud}$	4 2 1	2 2	2			1 1				1		2 2
$(LL)(LL)$	$Q_{qq}^{(1,3)}$	10	6 6	10 2	2 2								6 6
$(RR)(RR)$	$Q_{uu}$ $Q_{dd}$ $Q_{ud}^{(1,8)}$	5 2 4					3 3				2 4		
$(LL)(RR)$	$Q_{qu}^{(1,8)}$ $Q_{qd}^{(1,8)}$	8 4	4 4	4		2 2	6 6				4		4 4 4 4
$(LR)(LR)$	$Q_{quqd}^{(1,8)}$							2 2	4 4	4 4		6 6	
Total		44 4	18 18	18 2	2 2	6 6	16 16	7 7	8 8	8 8	11	11 11	20 20

**Table 5.** Counting of the pure quark SMEFT operators (see appendix A) assuming  $U(2)_q \times U(2)_u \times U(3)_d$  symmetry in the quark sector. The counting is performed taking up to three insertions of  $V_q$ , one insertion of  $\Delta_u$  or  $\Sigma_d$ , and two insertions of  $\Lambda_d$  as well as one insertion of  $\Delta_u V_q$ ,  $V_q \Lambda_d$ ,  $V_q \Sigma_d$ , and  $\Lambda_d \Sigma_d$  products each. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

$(\bar{q}q)$

$$\begin{aligned}
 \mathcal{O}(1) : (\bar{q}q), \quad (\bar{q}_3 q_3), \quad \mathcal{O}(V_q) : (\bar{q} V_q q_3), \quad \text{H.c.}, \quad \mathcal{O}(V_q^2) : (\bar{q} V_q V_q^\dagger q), \\
 \mathcal{O}(\Lambda_d \Sigma_d) : (\bar{q} \Sigma_d \Lambda_d q_3), \quad \text{H.c.} .
 \end{aligned} \tag{2.47}$$

$(\bar{u}u)$

$$\mathcal{O}(1) : (\bar{u}u), \quad (\bar{u}_3 u_3), \quad \mathcal{O}(\Delta V_q) : (\bar{u} \Delta_u^\dagger V_q u_3), \quad \text{H.c.} . \tag{2.48}$$

$(\bar{d}d)$

$$\mathcal{O}(1) : (\bar{d}d), \quad \mathcal{O}(\Lambda_d^2) : (\bar{d} \Lambda_d \Lambda_d^\dagger d) . \tag{2.49}$$

$(\bar{u}d)$

$$\mathcal{O}(\Lambda_d) : (\bar{u}_3 \Lambda_d^\dagger d), \quad \mathcal{O}(V_q \Sigma_d) : (\bar{u}_3 V_q^\dagger \Sigma_d d) . \tag{2.50}$$

$(\bar{q}u)$

$$\begin{aligned}
 \mathcal{O}(1) : (\bar{q}_3 u_3), \quad \mathcal{O}(V_q) : (\bar{q} V_q u_3), \quad \mathcal{O}(\Delta) : (\bar{q} \Delta_u u), \quad \mathcal{O}(\Delta V_q) : (\bar{q}_3 V_q^\dagger \Delta_u u), \\
 \mathcal{O}(\Lambda_d \Sigma_d) : (\bar{q} \Sigma_d \Lambda_d u_3) .
 \end{aligned} \tag{2.51}$$

$\boxed{(\bar{q}d)}$ 

$$\begin{aligned} \mathcal{O}(\Lambda_d) &: (\bar{q}_3 \Lambda_d^\dagger d), & \mathcal{O}(\Sigma_d) &: (\bar{q} \Sigma_d d), & \mathcal{O}(V_q \Lambda_d) &: (\bar{q} V_q \Lambda_d^\dagger d), \\ \mathcal{O}(V_q \Sigma_d) &: (\bar{q}_3 V_q^\dagger \Sigma_d d). \end{aligned} \quad (2.52)$$

**Decomposition of quartic structures.** At  $\mathcal{O}(1)$  there is only one unique structure containing four appearances of  $d$ ,  $(\bar{d}_i d^j)(\bar{d}_j d^i)$ , two with four instances of  $q$ ,  $(\bar{q}_a q^b)(\bar{q}_b q^a)$  and  $(\bar{q}_a q_3)(\bar{q}_3 q^a)$ , and two similar ones with  $u$ . One insertion of  $V_q$  yields only one unique  $\mathcal{O}(V_q)$  quartic structure  $(\bar{q}_a q_3)(\bar{q} V_q q^a)$ , and there are no structures with one insertion of  $\Lambda_d$  only. There is one  $\mathcal{O}(\Delta)$  structure  $(\bar{q}_a q_3)(\Delta_u)^a{}_b(\bar{u}_3 u^b)$  and one at  $\mathcal{O}(\Sigma_d)$  given as  $(\bar{q}_a u_3)(\Sigma_d)^a{}_j(\bar{q}_3 d^j)$ .

With two insertions of  $V_q$  or  $\Lambda_d$ , there are only two unique structures:  $(\bar{q}_a V_q^\dagger q)(\bar{q} V_q q^a)$  and  $(\bar{d} \Lambda_d d^j)(\bar{d}_j \Lambda_d^\dagger d)$ . We also get two structures with one insertion of the  $\Lambda_d \Sigma_d$  product,  $(\bar{q} \Sigma_d \Lambda_d q^b)(\bar{q}_b q_3)$  and  $(\bar{q}_a q_3)(\Sigma_d)^a{}_j(\bar{d} \Lambda_d d^j)$ , and one at  $\mathcal{O}(V_q \Sigma_d)$  given as  $(\bar{q}_a u_3)(\Sigma_d)^a{}_j(\bar{q} V_q d^j)$ . At  $\mathcal{O}(\Delta V_q)$  we have two structures,  $(\bar{u}_a u_3)(\bar{u} \Delta_u^\dagger V_q u^a)$  and  $(\bar{q}_a V_q^\dagger q)(\Delta_u)^a{}_b(\bar{u}_3 u^b)$ , and there is one at  $\mathcal{O}(\Delta \Lambda_d)$ :  $(\bar{q}_3 u^a)(\Delta_u)^b{}_a(\bar{q}_b \Lambda_d^\dagger d)$ . We list all the quartic structures below:

 $\boxed{(\bar{q}q)(\bar{q}q)}$ 

$$\begin{aligned} \mathcal{O}(1) &: (\bar{q}_a q^b)(\bar{q}_b q^a), \quad (\bar{q}_a q_3)(\bar{q}_3 q^a), & \mathcal{O}(V_q) &: (\bar{q}_a q_3)(\bar{q} V_q q^a), \quad \text{H.c.}, \\ \mathcal{O}(V_q^2) &: (\bar{q}_a V_q^\dagger q)(\bar{q} V_q q^a), & \mathcal{O}(\Lambda_d \Sigma_d) &: (\bar{q} \Sigma_d \Lambda_d q^b)(\bar{q}_b q_3), \quad \text{H.c.} \end{aligned} \quad (2.53)$$

 $\boxed{(\bar{u}u)(\bar{u}u)}$ 

$$\mathcal{O}(1) : (\bar{u}_a u^b)(\bar{u}_b u^a), \quad (\bar{u}_a u_3)(\bar{u}_3 u^a), \quad \mathcal{O}(\Delta V_q) : (\bar{u}_a u_3)(\bar{u} \Delta_u^\dagger V_q u^a), \quad \text{H.c.} \quad (2.54)$$

 $\boxed{(\bar{d}d)(\bar{d}d)}$ 

$$\mathcal{O}(1) : (\bar{d}_i d^j)(\bar{d}_j d^i), \quad \mathcal{O}(\Lambda_d^2) : (\bar{d} \Lambda_d d^j)(\bar{d}_j \Lambda_d^\dagger d). \quad (2.55)$$

 $\boxed{(\bar{q}q)(\bar{u}u)}$ 

$$\mathcal{O}(\Delta) : (\bar{q}_a q_3)(\Delta_u)^a{}_b(\bar{u}_3 u^b), \quad \text{H.c.}, \quad \mathcal{O}(\Delta V_q) : (\bar{q}_a V_q^\dagger q)(\Delta_u)^a{}_b(\bar{u}_3 u^b), \quad \text{H.c.} \quad (2.56)$$

 $\boxed{(\bar{q}q)(\bar{d}d)}$ 

$$\mathcal{O}(\Lambda_d \Sigma_d) : (\bar{q}_a q_3)(\Sigma_d)^a{}_j(\bar{d} \Lambda_d d^j), \quad \text{H.c.} \quad (2.57)$$

 $\boxed{(\bar{q}u)(\bar{q}d)}$ 

$$\begin{aligned} \mathcal{O}(\Sigma_d) &: (\bar{q}_a u_3)(\Sigma_d)^a{}_j(\bar{q}_3 d^j), & \mathcal{O}(V_q \Sigma_d) &: (\bar{q}_a u_3)(\Sigma_d)^a{}_j(\bar{q} V_q d^j), \\ \mathcal{O}(\Delta \Lambda_d) &: (\bar{q}_3 u^a)(\Delta_u)^b{}_a(\bar{q}_b \Lambda_d^\dagger d). \end{aligned} \quad (2.58)$$

## 2.4 MFV<sub>Q</sub> symmetry

Minimal flavor violation assumes that the only spurions of the  $G_Q = U(3)_q \times U(3)_u \times U(3)_d$  symmetry in the quark sector are the SM Yukawa couplings. The quarks transform as

$$q \sim (\mathbf{3}, \mathbf{1}, \mathbf{1}), \quad u \sim (\mathbf{1}, \mathbf{3}, \mathbf{1}), \quad d \sim (\mathbf{1}, \mathbf{1}, \mathbf{3}) \quad (2.59)$$

under  $G_Q$ . As the Yukawa couplings are the sources of the symmetry breaking, they are promoted into spurions with the transformations assigned as

$$Y_u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}), \quad Y_d \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}). \quad (2.60)$$

Fixing the parameters of the SM, i.e., the values of the  $Y_{u,d,e}$  spurions, breaks  $G_Q$ . With no degenerate or vanishing eigenvalues nor any accidental alignment of  $Y_u$  and  $Y_d$ ,  $Y_u$  can be parametrized exclusively with the diagonal matrix of its singular values,  $\hat{Y}_u$ :

$$Y_u \longrightarrow \hat{Y}_u : \quad U(3)_q \times U(3)_u \longrightarrow U(1)_{q+u}^3. \quad (2.61)$$

The remaining quark sector symmetry can then be used to partially diagonalize  $Y_d$ , writing

$$Y_d \longrightarrow V\hat{Y}_d : \quad U(1)_{q+u}^3 \times U(3)_d \longrightarrow U(1)_B. \quad (2.62)$$

Here  $V$  is a special unitary matrix with 3 rotation angles but only 1 phase, as the others have been successfully factored out:  $V$  is nothing but the CKM matrix. Only the vectorial baryon number symmetry  $U(1)_B$  remains unbroken after the inclusion of the quark Yukawa couplings. Only 9 real parameters and 1 phase are physical; a total of 26 unphysical parameters have been removed. The remnant flavor symmetry of the quark sector is  $U(1)_B$ , which is consistent with 26 broken generators. No additional phases can be removed from the baryon number-conserving SMEFT operators with the remnant symmetry.

The spurion counting of the pure quark operators is presented in table 6, while the decompositions of the bilinear and quartic structures are listed in eqs. (2.63)–(2.68) and eqs. (2.69)–(2.74).

**Decomposition of bilinear structures.** We present the decompositions of the bilinear structures with up to three insertions of the spurions.  $\mathcal{O}(1)$  structures can be formed only by contracting a field with its conjugate. This gives the three distinct structures  $(\bar{q}q)$ ,  $(\bar{u}u)$ , and  $(\bar{d}d)$ .

At  $\mathcal{O}(Y_{u,d})$ , we get only two bilinears:  $(\bar{q}Y_u u)$  and  $(\bar{q}Y_d d)$ . Meanwhile, the  $\mathcal{O}(Y^2)$  structures can be obtained by contracting one index of the Yukawas with each other and the remaining two open indices with fields. There are five such structures:  $(\bar{q}Y_u Y_u^\dagger q)$ ,  $(\bar{q}Y_d Y_d^\dagger q)$ ,  $(\bar{u}Y_u^\dagger Y_u u)$ ,  $(\bar{d}Y_d^\dagger Y_d d)$ , and  $(\bar{u}Y_u^\dagger Y_d d)$ . Note that the singlets formed by tracing over the Yukawas, e.g.,  $(Y_u^\dagger Y_u)(\bar{q}q)$  drop from the counting, since contractions like this give bilinears, which are structurally the same as the  $\mathcal{O}(1)$  ones.

Lastly, there are four  $\mathcal{O}(Y^3)$  structures, which can be formed by inserting the products  $Y_{u,d} Y_{u,d}^\dagger$  and  $Y_{u,d}^\dagger Y_{u,d}$  in the  $\mathcal{O}(Y)$  bilinears. These four structures are  $(\bar{q}Y_d Y_d^\dagger Y_u u)$ ,  $(\bar{q}Y_u Y_u^\dagger Y_u u)$ ,  $(\bar{q}Y_u Y_u^\dagger Y_d d)$  and  $(\bar{q}Y_d Y_d^\dagger Y_d d)$ . The full list of bilinears follows below:

MFV <sub>Q</sub>		$\mathcal{O}(1)$	$\mathcal{O}(Y_u)$	$\mathcal{O}(Y_u^2)$	$\mathcal{O}(Y_d)$	$\mathcal{O}(Y_d^2)$	$\mathcal{O}(Y_u Y_d)$	$\mathcal{O}(Y_u^2 Y_d, Y_d^2 Y_u)$	$\mathcal{O}(Y_u^3, Y_d^3)$
$\psi^2 H^3$	$Q_{uH}$		1 1					1 1	1 1
	$Q_{dH}$				1 1			1 1	1 1
$\psi^2 XH$	$Q_{u(G,W,B)}$		3 3					3 3	3 3
	$Q_{d(G,W,B)}$				3 3			3 3	3 3
$\psi^2 H^2 D$	$Q_{Hq}^{(1,3)}$	2		2		2			
	$Q_{Hu}$	1		1					
	$Q_{Hd}$	1				1			
	$Q_{Hud}$						1 1		
$(LL)(LL)$	$Q_{qq}^{(1,3)}$	4		4		4			
$(RR)(RR)$	$Q_{uu}$	2		2					
	$Q_{dd}$	2				2			
	$Q_{ud}^{(1,8)}$	2		2		2			
$(LL)(RR)$	$Q_{qu}^{(1,8)}$	2		6		2			
	$Q_{qd}^{(1,8)}$	2		2		6			
$(LR)(LR)$	$Q_{quqd}^{(1,8)}$						4 4		
Total		18	4 4	19	4 4	19	5 5	8 8	8 8

**Table 6.** Counting of the pure quark SMEFT operators (see appendix A) assuming MFV symmetry in the quark sector and taking the combinations up to three insertions of  $Y_u$  and  $Y_d$  spurions in the decompositions. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

$(\bar{q}q)$

$$\mathcal{O}(1) : (\bar{q}q), \quad \mathcal{O}(Y_u^2) : (\bar{q}Y_u Y_u^\dagger q), \quad \mathcal{O}(Y_d^2) : (\bar{q}Y_d Y_d^\dagger q). \quad (2.63)$$

$(\bar{u}u)$

$$\mathcal{O}(1) : (\bar{u}u), \quad \mathcal{O}(Y_u^2) : (\bar{u}Y_u^\dagger Y_u u). \quad (2.64)$$

$(\bar{d}d)$

$$\mathcal{O}(1) : (\bar{d}d), \quad \mathcal{O}(Y_d^2) : (\bar{d}Y_d^\dagger Y_d d). \quad (2.65)$$

$(\bar{u}d)$

$$\mathcal{O}(Y_u Y_d) : (\bar{u}Y_u^\dagger Y_d d). \quad (2.66)$$

$(\bar{q}u)$

$$\mathcal{O}(Y_u) : (\bar{q}Y_u u), \quad \mathcal{O}(Y_d^2 Y_u) : (\bar{q}Y_d Y_d^\dagger Y_u u), \quad \mathcal{O}(Y_u^3) : (\bar{q}Y_u Y_u^\dagger Y_u u). \quad (2.67)$$

$(\bar{q}d)$

$$\mathcal{O}(Y_d) : (\bar{q}Y_d d), \quad \mathcal{O}(Y_u^2 Y_d) : (\bar{q}Y_u Y_u^\dagger Y_d d), \quad \mathcal{O}(Y_d^3) : (\bar{q}Y_d Y_d^\dagger Y_d d). \quad (2.68)$$

**Decomposition of quartic structures.** At  $\mathcal{O}(1)$  in the spurion counting, there are only three unique structures that can be formed:  $(\bar{q}_i q^j)(\bar{q}_j q^i)$ ,  $(\bar{u}_i u^j)(\bar{u}_j u^i)$  and  $(\bar{d}_i d^j)(\bar{d}_j d^i)$ . All the remaining unique quartic structures contain two insertions of  $Y_{u,d}$  spurions and can be formed accounting for the transformation properties of the bilinears, e.g.,  $\bar{q}_i (Y_u)_j^i q^k \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})$  and  $\bar{q}_k (Y_u^\dagger)^j_\ell q^\ell \sim (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1})$ , and contracting the two. Examining and counting these combinations, we conclude that there are in total seven unique  $\mathcal{O}(Y^2)$  quartic structures, which we list below:

$$\boxed{(\bar{q}q)(\bar{q}q)}$$

$$\begin{aligned} \mathcal{O}(1) &: (\bar{q}_i q^j)(\bar{q}_j q^i), \\ \mathcal{O}(Y_u^2) &: (\bar{q}_i q^j)(Y_u Y_u^\dagger)^i_k (\bar{q}_j q^k), \quad \mathcal{O}(Y_d^2) : (\bar{q}_i q^j)(Y_d Y_d^\dagger)^i_k (\bar{q}_j q^k). \end{aligned} \quad (2.69)$$

$$\boxed{(\bar{u}u)(\bar{u}u)}$$

$$\mathcal{O}(1) : (\bar{u}_i u^j)(\bar{u}_j u^i), \quad \mathcal{O}(Y_u^2) : (\bar{u}_i u^j)(Y_u^\dagger Y_u)^i_k (\bar{u}_j u^k). \quad (2.70)$$

$$\boxed{(\bar{d}d)(\bar{d}d)}$$

$$\mathcal{O}(1) : (\bar{d}_i d^j)(\bar{d}_j d^i), \quad \mathcal{O}(Y_d^2) : (\bar{d}_i d^j)(Y_d^\dagger Y_d)^i_k (\bar{d}_j d^k). \quad (2.71)$$

$$\boxed{(\bar{q}q)(\bar{u}u)}$$

$$\mathcal{O}(Y_u^2) : (Y_u)^i_\ell (Y_u^\dagger)^k_j (\bar{q}_i q^j)(\bar{u}_k u^\ell). \quad (2.72)$$

$$\boxed{(\bar{q}q)(\bar{d}d)}$$

$$\mathcal{O}(Y_d^2) : (Y_d)^i_k (Y_d^\dagger)^\ell_j (\bar{q}_i q^j)(\bar{d}_\ell d^k). \quad (2.73)$$

$$\boxed{(\bar{q}u)(\bar{q}d)}$$

$$\mathcal{O}(Y_u Y_d) : (Y_d)^i_j (Y_u)^\ell_k (\bar{q}_i u^k)(\bar{q}_\ell d^j). \quad (2.74)$$

### 3 Lepton sector

Similarly to quarks, leptons come in three flavors, allowing for flavor transformations, which leaves physics unchanged. The lepton kinetic terms are symmetric under flavor transformations from the group  $G_L = \text{U}(3)_\ell \times \text{U}(3)_e$ . In the SM, this symmetry is broken explicitly by the Yukawa term

$$\mathcal{L} \supset -\bar{\ell}_L Y_e e_R H + \text{H.c.}, \quad (3.1)$$

leaving an accidental  $\text{U}(1)^3$  symmetry conserving the individual lepton numbers. Clearly, the observation of neutrino oscillations indicates that BSM physics must necessarily violate this

accidental symmetry at some level. On the other hand, the non-observation of any charged lepton flavor-violating decays indicates that TeV-scale NP must suppress contribution to such processes.

In contrast to the quark Yukawa matrices, from which derives the CKM matrix, the charged lepton Yukawa matrix is not fixed by the PMNS mixing matrix, which could come from the neutrino sector. Neutrino masses in the SMEFT originate from lepton number-violating operators, such as the dimension-5 Weinberg operator. All flavor symmetries we consider here contain the usual lepton number symmetry as a subgroup, and this symmetry is preserved by the spurions populating the Yukawa matrix. The inclusion of neutrino masses, therefore, necessitates new lepton number-violating spurions. Due to the smallness of the neutrino masses these additional spurions must be vanishingly small (for TeV-scale NP) and can be neglected.<sup>11</sup> In extreme examples where the neutrino mass operators are highly suppressed by loop factors and/or operator dimension rather than the size of the spurions, the relevant spurions can simply be included to the symmetries.

In this work, we identify several viable scenarios for the lepton flavor structure of the SMEFT that can accommodate hierarchical Yukawa couplings while suppressing charged lepton flavor-violating contributions from the dimension-6 operators. Accordingly, we consider a variety of different options for a flavor symmetry  $G \subset G_L$ :

- i)*  $G = U(1)^3$  vectorial provides lepton flavor conservation but does not allow for any spurions providing a perturbative suppression of the electron mass;
- ii)*  $G = U(1)^6$  is also compatible with exact lepton number conservation but allows for a controlled expansion in lepton masses;
- iii)*  $G = U(2)$  vectorial symmetry gives additional correlation between the light leptons but no LFV;
- iv)*  $G = U(2)^2$  decouples the 3rd generation from the first two generations of leptons while completely forbidding LFV depending with the minimal set of spurions;
- v)*  $G = U(2)^2 \times U(1)^2$  decouples the 3rd generation from the first two generations of leptons and allows for a perturbative expansion in all lepton masses;
- vi)*  $G = U(3)$  vectorial symmetry is compatible with exact lepton flavor conservation;
- vii)*  $G = U(3)^2$ , linearly realized MFV symmetry gives lepton flavor conservation but only SM-like violation of lepton flavor universality.

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<sup>11</sup>Furthermore, for the lepton number-conserving operators we consider here, the neutrino mass spurions must combine in lepton number-conserving combinations requiring at least two such. This further suppresses the relevance of such insertions.



### 3.1 $U(1)^3$ vectorial symmetry

The first symmetry of the lepton sector we consider is the vectorial  $G = U(1)^3 = U(1)_e \times U(1)_\mu \times U(1)_\tau \subset G_L$  symmetry, under which the  $\ell$  and  $e$  fields decompose as

$$\ell = \begin{bmatrix} \ell_1 \sim (1, 0, 0) \\ \ell_2 \sim (0, 1, 0) \\ \ell_3 \sim (0, 0, 1) \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \sim (1, 0, 0) \\ e_2 \sim (0, 1, 0) \\ e_3 \sim (0, 0, 1) \end{bmatrix}, \quad (3.2)$$

with the  $U(1)$  charges indicated in the brackets. Since the  $U(1)^3$  vectorial symmetry is exact, there are no spurions present. The Yukawa matrix can be written as

$$Y_e = \begin{bmatrix} y_e & & \\ & y_\mu & \\ & & y_\tau \end{bmatrix}. \quad (3.3)$$

These coefficients break the axial  $U(1)_A^3 \subset G_L$  symmetry, which can be used to rotate away unphysical phases from  $y_{e,\mu,\tau}$ . Accordingly, they can be chosen to be real and positive. The numerical values of these parameters are

$$y_e = 2.793 \times 10^{-6}, \quad y_\mu = 5.884 \times 10^{-4}, \quad y_\tau = 9.994 \times 10^{-3}. \quad (3.4)$$

The smallness ( $y_\ell \ll 1$ ) and the hierarchy among different generations is an open question. In section 3.2, these parameters are actually spurions, which explains their smallness but not the hierarchy. The motivation to consider this flavor structure comes from stringent experimental constraints on charged lepton flavor-violating processes.

The neutrino masses can be minimally accounted for by the dimension-5 Weinberg operator and three additional spurions with the opposite charge to the leptons in eq. (3.2). The PMNS mixing matrix is accommodated by assuming no hierarchy in the three spurions. Note that, for the TeV-scale cutoff, these spurions take extremely small values in order to reproduce the observed neutrino masses. Therefore, their effect on charged lepton flavor-violating processes is negligible. For these reasons, we omit them from the counting of dimension-6 operators.

The counting of the pure lepton SMEFT operators is presented in the table 7 and the decompositions of the bilinear and unique quartic structures are listed in eqs. (3.5)–(3.9).

**Decomposition of bilinear and quartic structures.** Constructing bilinear and quartic structures is straightforward in the case of  $U(1)^3$  vectorial symmetry, since only  $\mathcal{O}(1)$  structures are present. These are given by  $(\bar{\ell}_i \ell_i)$ ,  $(\bar{e}_i e_i)$  and  $(\bar{\ell}_i e_i)$ . Similarly, the only possible non-factorizable quartic structures are given by  $(\bar{\ell}_i \ell_j)(\bar{\ell}_j \ell_i)$  and  $(\bar{\ell}_i \ell_j)(\bar{e}_j e_i)$ .

The structures of the form  $(\bar{e}_i e_j)(\bar{e}_j e_i)$  are identical to  $(\bar{e}_i e_i)(\bar{e}_j e_j)$  due to the Fierz identity for vector currents in the underlying operators. This is valid not only here but for all lepton symmetries we consider, that is, there are no unique  $(\bar{e}e)(\bar{e}e)$  structures. Thus, the quartic structures with four insertions of  $e$  are formed solely by multiplying  $(\bar{e}e)$  bilinears. We list the decompositions of the bilinear and unique quartic structures below:

$U(1)_e \times U(1)_\mu \times U(1)_\tau$		$\mathcal{O}(1)$	
$\psi^2 H^3$	$Q_{eH}$	3	3
$\psi^2 XH$	$Q_{e(W,B)}$	6	6
$\psi^2 H^2 D$	$Q_{H\ell}^{(1,3)}$	6	
	$Q_{He}$	3	
$(LL)(LL)$	$Q_{\ell\ell}$	9	
$(RR)(RR)$	$Q_{ee}$	6	
$(LL)(RR)$	$Q_{\ell e}$	12	3
Total		45	12

**Table 7.** Counting of the pure lepton SMEFT operators (see appendix A) assuming  $U(1)_V^3$  symmetry in the lepton sector. Since the  $U(1)_V^3$  symmetry is exact (no spurions), the counting is presented for the  $\mathcal{O}(1)$  operators only. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

$$\boxed{(\bar{\ell}e)}$$

$$\mathcal{O}(1) : (\bar{\ell}_1 e_1), (\bar{\ell}_2 e_2), (\bar{\ell}_3 e_3). \quad (3.5)$$

$$\boxed{(\bar{\ell}\ell)}$$

$$\mathcal{O}(1) : (\bar{\ell}_1 \ell_1), (\bar{\ell}_2 \ell_2), (\bar{\ell}_3 \ell_3). \quad (3.6)$$

$$\boxed{(\bar{e}e)}$$

$$\mathcal{O}(1) : (\bar{e}_1 e_1), (\bar{e}_2 e_2), (\bar{e}_3 e_3). \quad (3.7)$$

$$\boxed{(\bar{\ell}\ell)(\bar{\ell}\ell)}$$

$$\mathcal{O}(1) : (\bar{\ell}_1 \ell_2)(\bar{\ell}_2 \ell_1), (\bar{\ell}_1 \ell_3)(\bar{\ell}_3 \ell_1), (\bar{\ell}_2 \ell_3)(\bar{\ell}_3 \ell_2). \quad (3.8)$$

$$\boxed{(\bar{\ell}\ell)(\bar{e}e)}$$

$$\mathcal{O}(1) : (\bar{\ell}_1 \ell_2)(\bar{e}_2 e_1), (\bar{\ell}_2 \ell_3)(\bar{e}_3 e_2), (\bar{\ell}_3 \ell_1)(\bar{e}_1 e_3), \text{ H.c.} \quad (3.9)$$

$$\boxed{(\bar{e}e)(\bar{e}e)}$$

No unique structures present due to Fierz identities.

### 3.2 $U(1)^6$ symmetry

For a  $G = U(1)^6 = U(1)_{\ell_1} \times U(1)_{e_1} \times U(1)_{\ell_2} \times U(1)_{e_2} \times U(1)_{\ell_3} \times U(1)_{e_3} \subset G_L$  symmetry of the lepton sector, we have the field decompositions and  $U(1)^6$  charge assignments

$$\ell = \begin{bmatrix} \ell_1 \sim (1, 0, 0, 0, 0, 0) \\ \ell_2 \sim (0, 0, 1, 0, 0, 0) \\ \ell_3 \sim (0, 0, 0, 0, 1, 0) \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \sim (0, 1, 0, 0, 0, 0) \\ e_2 \sim (0, 0, 0, 1, 0, 0) \\ e_3 \sim (0, 0, 0, 0, 0, 1) \end{bmatrix}. \quad (3.10)$$

U(1) <sup>6</sup>		O(1)	O(y)
$\psi^2 H^3$	$Q_{eH}$		3 3
$\psi^2 XH$	$Q_{e(W,B)}$		6 6
$\psi^2 H^2 D$	$Q_{H\ell}^{(1,3)}$	6	
	$Q_{He}$	3	
$(LL)(LL)$	$Q_{\ell\ell}$	9	
$(RR)(RR)$	$Q_{ee}$	6	
$(LL)(RR)$	$Q_{\ell e}$	9	
Total		33	9 9

**Table 8.** Counting of the pure lepton SMEFT operators (see appendix A) assuming U(1)<sup>6</sup> symmetry in the lepton sector. The counting is performed taking up to one insertion of  $y_{e,\mu,\tau}$  spurion. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

The minimal set of spurions required to write the Yukawa matrix is

$$y_e \sim (1, -1, 0, 0, 0, 0), \quad y_\mu \sim (0, 0, 1, -1, 0, 0), \quad y_\tau \sim (0, 0, 0, 0, 1, -1), \quad (3.11)$$

which we use to write the Yukawa matrix

$$Y_e = \begin{bmatrix} y_e & & \\ & y_\mu & \\ & & y_\tau \end{bmatrix}, \quad (3.12)$$

The flavor symmetry breaking pattern is given by:

$$\begin{aligned} y_e &: \quad \text{U}(1)_{e_L} \times \text{U}(1)_{e_R} \longrightarrow \text{U}(1)_{L_e}, \\ y_\mu &: \quad \text{U}(1)_{\mu_L} \times \text{U}(1)_{\mu_R} \longrightarrow \text{U}(1)_{L_\mu}, \\ y_\tau &: \quad \text{U}(1)_{\tau_L} \times \text{U}(1)_{\tau_R} \longrightarrow \text{U}(1)_{L_\tau}, \end{aligned} \quad (3.13)$$

yielding a total of 3 broken generators. This allows for the elimination of redundant phases from  $y_{e,\mu,\tau}$ , which can all be taken to be positive, real numbers.

We present the spurion counting of the leptonic operators assuming U(1)<sup>6</sup> flavor symmetry in table 8. The decompositions are listed in eqs. (3.14)–(3.17).

**Decomposition of bilinear and quartic structures.** In this case, the O(1) bilinear structures can only be constructed with two appearances of the same field. These structures are  $(\bar{\ell}_i \ell_i)$  and  $(\bar{e}_i e_i)$ . Matching the charges, the bilinear structure with one insertion of a spurion  $(Y_e)_{ii}$  can only be of the form  $[\bar{\ell}_i (Y_e)_{ii} e_i]$ . The set of unique quartic structures is comprised of three O(1) structures of the form  $(\bar{\ell}_1 \ell_2)(\bar{\ell}_2 \ell_1)$ ,  $(\bar{\ell}_2 \ell_3)(\bar{\ell}_3 \ell_2)$  and  $(\bar{\ell}_3 \ell_1)(\bar{\ell}_1 \ell_3)$ , whereas there are no unique  $(\bar{\ell}\ell)(\bar{e}e)$  structures present. We list the decompositions below:

$$\boxed{(\bar{\ell}e)} \quad \mathcal{O}(y) : \quad (\bar{\ell}_1 y_e e_1), \quad (\bar{\ell}_2 y_\mu e_2), \quad (\bar{\ell}_3 y_\tau e_3). \quad (3.14)$$

$$\boxed{(\bar{\ell}\ell)}$$

$$\mathcal{O}(1) : (\bar{\ell}_1\ell_1), (\bar{\ell}_2\ell_2), (\bar{\ell}_3\ell_3). \quad (3.15)$$

$$\boxed{(\bar{e}e)}$$

$$\mathcal{O}(1) : (\bar{e}_1e_1), (\bar{e}_2e_2), (\bar{e}_3e_3). \quad (3.16)$$

$$\boxed{(\bar{\ell}\ell)(\bar{\ell}\ell)}$$

$$\mathcal{O}(1) : (\bar{\ell}_1\ell_2)(\bar{\ell}_2\ell_1), (\bar{\ell}_2\ell_3)(\bar{\ell}_3\ell_2), (\bar{\ell}_3\ell_1)(\bar{\ell}_1\ell_3). \quad (3.17)$$

$$\boxed{(\bar{\ell}\ell)(\bar{e}e)}$$

No unique structures present.

$$\boxed{(\bar{e}e)(\bar{e}e)}$$

No unique structures present due to Fierz identities.

### 3.3 U(2) vectorial symmetry

Next, we consider a  $U(2) \subset G_L$  vectorial flavor symmetry, under which the fields decompose as

$$\ell = \begin{bmatrix} \ell^a \sim \mathbf{2} \\ \ell_3 \sim \mathbf{1} \end{bmatrix}, \quad e = \begin{bmatrix} e^a \sim \mathbf{2} \\ e_3 \sim \mathbf{1} \end{bmatrix}. \quad (3.18)$$

The minimal choice of spurion necessary to produce a realistic Yukawa coupling is  $\Delta_\ell \sim \mathbf{3}$ , which we take to be real.<sup>12</sup> We use the simplifying notation

$$\Delta_\ell^I (T^I)^{a_b} = (\Delta_\ell)^{a_b}. \quad (3.19)$$

With this spurion, the Yukawa matrix generically takes the form

$$Y_e = \begin{bmatrix} \Delta_\ell + s_\ell \mathbf{1} & 0 \\ 0 & y_\tau \end{bmatrix}. \quad (3.20)$$

$\Delta_\ell$  breaks the  $U(2)_V$  symmetry as

$$\Delta_\ell \longrightarrow \begin{bmatrix} -\delta_\ell & 0 \\ 0 & \delta_\ell \end{bmatrix} : \quad U(2)_V \longrightarrow U(1)_{\ell+e}^2, \quad (3.21)$$

where we use the general properties of the special unitary matrices along with the flavor symmetry breaking pattern to parametrize the  $\Delta_\ell$  spurion with 1 real parameter. The Yukawa matrix (3.20) also preserves an accidental  $U(1)_{\ell_3+e_3}$  symmetry, while it breaks axial  $U(1)$ 's, which can be used to remove phases from the coefficients  $s_\ell$  and  $y_\tau$ . The numerical values of the relevant parameters are

$$s_\ell = 2.956 \times 10^{-4}, \quad \delta_\ell = 2.928 \times 10^{-4}, \quad y_\tau = 9.994 \times 10^{-3}. \quad (3.22)$$

This flavor structure fails to explain the hierarchy among generations. Furthermore, a tuning is needed between  $s_\ell$  and (the symmetry-breaking)  $\delta_\ell$  in order to accommodate for the observed  $e$  and  $\mu$  masses.

The spurion counting of the pure lepton operators assuming  $U(2)_V$  symmetry is given in table 9 and we list the decompositions in eqs. (3.24)–(3.28).

<sup>12</sup>A slightly less minimal choice would be to introduce a  $V_\ell \sim \mathbf{2}$  in place of  $\Delta_\ell$ .

U(2) <sub>V</sub>		O(1)		O(Δ <sub>ℓ</sub> )	
ψ <sup>2</sup> H <sup>3</sup>	Q <sub>eH</sub>	2	2	1	1
ψ <sup>2</sup> XH	Q <sub>e(W,B)</sub>	4	4	2	2
ψ <sup>2</sup> H <sup>2</sup> D	Q <sub>Hℓ</sub> <sup>(1,3)</sup>	4		2	
	Q <sub>He</sub>	2		1	
(LL)(LL)	Q <sub>ℓℓ</sub>	5		3	
(RR)(RR)	Q <sub>ee</sub>	3		2	
(LL)(RR)	Q <sub>ℓe</sub>	6	1	5	2
Total		26	7	16	5

**Table 9.** Counting of the pure lepton SMEFT operators (see appendix A) assuming U(2) vectorial symmetry in the lepton sector. The counting is performed taking up to one insertion of Δ<sub>ℓ</sub> spurion. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

**Decomposition of bilinear and quartic structures.** O(1) bilinear structures can only be constructed by directly contracting the doublets or the singlets of the ℓ and e fields. There are six such structures: (ℓe), (ℓ<sub>3</sub>e<sub>3</sub>), (ℓℓ), (ℓ<sub>3</sub>ℓ<sub>3</sub>), (ēe), and (ē<sub>3</sub>e<sub>3</sub>). To form the O(Δ<sub>ℓ</sub>) bilinears, the Δ<sub>ℓ</sub> has to be contracted to ℓ<sup>a</sup> and e<sup>a</sup>, giving three bilinears: (ℓ̄Δ<sub>ℓ</sub>e), (ℓ̄Δ<sub>ℓ</sub>ℓ), and (ē̄Δ<sub>ℓ</sub>e).

Analyzing the possible contractions involving four field appearances in an analogous way, we obtain four O(1) quartic structures: (ℓ<sub>a</sub>ℓ<sup>b</sup>)(ℓ̄<sub>b</sub>ℓ<sup>a</sup>), (ℓ<sub>a</sub>ℓ<sub>3</sub>)(ℓ̄<sub>3</sub>ℓ<sup>a</sup>), (ℓ<sub>a</sub>ℓ)(ē̄e<sup>a</sup>), and (ℓ<sub>a</sub>ℓ<sub>3</sub>)(ē̄<sub>3</sub>e<sup>a</sup>). Similarly, we find that there are three O(Δ<sub>ℓ</sub>) structures given by (ℓ<sub>a</sub>ℓ<sub>3</sub>)(Δ<sub>ℓ</sub>)<sup>a</sup><sub>b</sub>(ℓ̄<sub>3</sub>ℓ<sup>b</sup>), (ℓ<sub>a</sub>ℓ<sup>b</sup>)(Δ<sub>ℓ</sub>)<sup>a</sup><sub>c</sub>(ē̄<sub>b</sub>e<sup>c</sup>), and (ℓ<sub>a</sub>ℓ<sub>3</sub>)(Δ<sub>ℓ</sub>)<sup>a</sup><sub>b</sub>(ē̄<sub>3</sub>e<sup>b</sup>).

We remark that there is a overcounting if we include all the (ℓ̄ℓ)(ē̄e) factorizing quartic structures contain only the U(2)<sub>V</sub> doublets ℓ and e. Having the O(1) and O(Δ<sub>ℓ</sub>) decompositions of the bilinear structures, the factorizing O(Δ<sub>ℓ</sub>) quartic structures are formed simply by multiplying the corresponding bilinears. Applying this recipe trivially gives two O(Δ<sub>ℓ</sub>) structures of the form (ℓ̄ℓ)(ē̄Δ<sub>ℓ</sub>e), (ℓ̄Δ<sub>ℓ</sub>ℓ)(ē̄e). It turns out that due to the group identity (D.6), one of these structures can be expressed in terms of the other factorizing quartic structure and the unique contraction (ℓ<sub>a</sub>ℓ<sup>b</sup>)(Δ<sub>ℓ</sub>)<sup>a</sup><sub>c</sub>(ē̄<sub>b</sub>e<sup>c</sup>) and its Hermitian conjugate:

$$\begin{aligned}
 (\bar{\ell}\ell)(\bar{e}\Delta_{\ell}e) &= \Delta_{\ell}^I (t^I)^a{}_b \delta^c{}_d (\bar{\ell}_c \ell^d) (\bar{e}_a e^b) = \Delta_{\ell}^I [(t^I)^c{}_b \delta^a{}_d - (t^I)^c{}_d \delta^a{}_b + (t^I)^a{}_d \delta^c{}_b] (\bar{\ell}_c \ell^d) (\bar{e}_a e^b) \\
 &= \left[ (\bar{\ell}_a \ell^b) (\Delta_{\ell})^a{}_c (\bar{e}_b e^c) + \text{H.c.} \right] - (\bar{\ell}\Delta_{\ell}\ell)(\bar{e}e).
 \end{aligned}
 \tag{3.23}$$

The (ℓ̄ℓ)(ē̄Δ<sub>ℓ</sub>e) structure, therefore, drops from the counting. We list the complete decompositions below:

$$\boxed{(\bar{\ell}e)}$$

$$\mathcal{O}(1) : (\bar{\ell}e), (\bar{\ell}_3 e_3), \quad \mathcal{O}(\Delta_{\ell}) : (\bar{\ell}\Delta_{\ell}e). \tag{3.24}$$

$$\boxed{(\bar{\ell}\ell)}$$

$$\mathcal{O}(1): (\bar{\ell}\ell), (\bar{\ell}_3\ell_3), \quad \mathcal{O}(\Delta_\ell): (\bar{\ell}\Delta_\ell\ell). \quad (3.25)$$

$$\boxed{(\bar{e}e)}$$

$$\mathcal{O}(1): (\bar{e}e), (\bar{e}_3e_3), \quad \mathcal{O}(\Delta_\ell): (\bar{e}\Delta_\ell e). \quad (3.26)$$

$$\boxed{(\bar{\ell}\ell)(\bar{\ell}\ell)}$$

$$\mathcal{O}(1): (\bar{\ell}_a\ell^b)(\bar{\ell}_b\ell^a), (\bar{\ell}_a\ell_3)(\bar{\ell}_3\ell^a), \quad \mathcal{O}(\Delta_\ell): (\bar{\ell}_a\ell_3)(\Delta_\ell)_b^a(\bar{\ell}_3\ell^b). \quad (3.27)$$

$$\boxed{(\bar{\ell}\ell)(\bar{e}e)}$$

$$\begin{aligned} \mathcal{O}(1): & (\bar{\ell}_a\ell)(\bar{e}e^a), [(\bar{\ell}_a\ell_3)(\bar{e}_3e^a), \text{H.c.}], \\ \mathcal{O}(\Delta_\ell): & (\bar{\ell}_a\ell^b)(\Delta_\ell)_c^a(\bar{e}_be^c), (\bar{\ell}_a\ell_3)(\Delta_\ell)_b^a(\bar{e}_3e^b), \text{H.c.} \end{aligned} \quad (3.28)$$

$$\boxed{(\bar{e}e)(\bar{e}e)}$$

No unique structures present due to Fierz identities.

### 3.4 $U(2)^2$ symmetry

We consider the case where NP is invariant under  $G = U(2)_\ell \times U(2)_e \subset G_L$ . In this case, the fields decompose as

$$\ell = \begin{bmatrix} \ell^a \sim (\mathbf{2}, \mathbf{1}) \\ \ell_3 \sim (\mathbf{1}, \mathbf{1}) \end{bmatrix}, \quad e = \begin{bmatrix} e^a \sim (\mathbf{1}, \mathbf{2}) \\ e_3 \sim (\mathbf{1}, \mathbf{1}) \end{bmatrix}. \quad (3.29)$$

In order to write a realistic the lepton Yukawa matrix, a spurion

$$\Delta_e \sim (\mathbf{2}, \bar{\mathbf{2}}) \quad (3.30)$$

is required. It is then possible to write the Yukawa matrix

$$Y_e = \begin{bmatrix} \Delta_e & 0 \\ 0 & y_\tau \end{bmatrix}. \quad (3.31)$$

We also allow for the non-minimal inclusion of the spurion  $V_\ell \sim (\mathbf{2}, \mathbf{1})$  to allow for a mixing of third generation leptons with the light generations. Note that  $V_\ell$  will be included in the decompositions (SMEFT operators), but it is absent from the Yukawa matrix, since the inclusion of  $V_\ell$  leads to a non-minimal parametrization of Yukawa.

We determine the breaking pattern of the  $U(2)^2$  flavor symmetry by the spurions. Fixing the  $\Delta_e$  bi-doublet, we have

$$\Delta_e \longrightarrow \begin{bmatrix} \delta_\ell & 0 \\ 0 & \delta'_\ell \end{bmatrix}: \quad U(2)_\ell \times U(2)_e \longrightarrow U(1)_{\ell+e}^2. \quad (3.32)$$

$U(2)_\ell \times U(2)_e$		$\mathcal{O}(1)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(V^3)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta V)$
$\psi^2 H^3$	$Q_{eH}$	1 1	1 1			1 1	1 1
$\psi^2 XH$	$Q_{e(W,B)}$	2 2	2 2			2 2	2 2
$\psi^2 H^2 D$	$Q_{H\ell}^{(1,3)}$	4	2 2	2			
	$Q_{He}$	2					1 1
$(LL)(LL)$	$Q_{\ell\ell}$	5	3 3	5 1	1 1		
$(RR)(RR)$	$Q_{ee}$	3					2 2
$(LL)(RR)$	$Q_{\ell e}$	4	2 2	2		1 1	3 3
Total		21 3	10 10	9 1	1 1	4 4	9 9

**Table 10.** Counting of the pure lepton SMEFT operators (see appendix A) assuming  $U(2)_\ell \times U(2)_e$  symmetry in the lepton sector. Analogously to the counting performed in the quark sector assuming the same symmetry (see table 2), we once again take up to three insertions of  $V_\ell$  spurion, one insertion of  $\Delta_e$  and one insertion of the  $\Delta_e V_\ell$  spurion product. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

In the second step,  $V_\ell$  breaks

$$V_\ell \longrightarrow \begin{bmatrix} \epsilon_\ell \\ \epsilon'_\ell \end{bmatrix} : \quad U(1)_{\ell+e}^2 \longrightarrow \emptyset, \quad (3.33)$$

for  $\epsilon_\ell^{(i)} > 0$ . Thus, we conclude that these spurions completely break the flavor  $U(2)^2$  symmetry, making it possible to remove 8 unphysical parameters from the parametrization of the spurions.  $y_\tau$  breaks a third generation axial  $U(1)$  symmetry, and its unphysical phase can be removed. The numerical values of the relevant parameters for the Yukawa matrix are

$$\delta_\ell = 2.793 \times 10^{-6}, \quad \delta'_\ell = 5.884 \times 10^{-4}, \quad y_\tau = 9.994 \times 10^{-3}. \quad (3.34)$$

This flavor structure provides a rationale for why the tau is much heavier than the other two leptons. The light lepton masses can be accommodated without tuning in contrast to the previous section; however, the hierarchy among them is left unexplained.

The spurion counting of the pure lepton operators assuming  $U(2)^2$  symmetry of the lepton sector is presented in table 10. The decompositions of bilinear and quartic structures are listed in eqs. (3.35)–(3.39).

**Decomposition of bilinear and quartic structures.** Forming the structures invariant under  $U(2)^2$  symmetry follows the same approach as in the case of  $U(2)^3$  symmetry in the quark sector (see eqs. (2.13)–(2.24)). Therefore, in order to obtain the invariant structures it is sufficient to take the corresponding structures in the quark sector (either  $q$  and  $u$  or  $q$  and  $d$  structures) and do a relabeling  $q \rightarrow \ell$  and  $u/d \rightarrow e$ . The decompositions of the bilinear and quartic structures are listed below:

$$\boxed{(\bar{l}e)}$$

$$\mathcal{O}(1): (\bar{l}_3 e_3), \quad \mathcal{O}(V): (\bar{l} V_\ell e_3), \quad \mathcal{O}(\Delta): (\bar{l} \Delta_e e), \quad \mathcal{O}(\Delta V): [\bar{l}_3 (V_\ell^\dagger \Delta_e) e]. \quad (3.35)$$

$$\boxed{(\bar{l}\ell)}$$

$$\mathcal{O}(1): (\bar{l}\ell), \quad (\bar{l}_3 \ell_3), \quad \mathcal{O}(V): (\bar{l} V_\ell \ell_3), \quad \text{H.c.}, \quad \mathcal{O}(V^2): (\bar{l} V_\ell V_\ell^\dagger \ell). \quad (3.36)$$

$$\boxed{(\bar{e}e)}$$

$$\mathcal{O}(1): (\bar{e}e), \quad (\bar{e}_3 e_3), \quad \mathcal{O}(\Delta V): (\bar{e} \Delta_e^\dagger V_\ell e_3), \quad \text{H.c.} \quad (3.37)$$

$$\boxed{(\bar{l}\ell)(\bar{l}\ell)}$$

$$\begin{aligned} \mathcal{O}(1): & (\bar{l}_a \ell^b) (\bar{l}_b \ell^a), \quad (\bar{l}\ell_3) (\bar{l}_3 \ell), \quad \mathcal{O}(V): (\bar{l} V_\ell \ell^b) (\bar{l}_b \ell_3), \quad \text{H.c.}, \\ \mathcal{O}(V^2): & (\bar{l} V_\ell \ell^b) (\bar{l}_b V_\ell^\dagger \ell). \end{aligned} \quad (3.38)$$

$$\boxed{(\bar{l}\ell)(\bar{e}e)}$$

$$\mathcal{O}(\Delta): (\bar{l}_a \ell_3) (\Delta_e)^a_b (\bar{e}_3 e^b), \quad \text{H.c.}, \quad \mathcal{O}(\Delta V): (\bar{l} V_\ell \ell^b) (\Delta_e)^c_b (\bar{e}_c e_3), \quad \text{H.c.} \quad (3.39)$$

$$\boxed{(\bar{e}e)(\bar{e}e)}$$

No unique structures present due to Fierz identities.

### 3.5 $U(2)^2 \times U(1)^2$ symmetry

If we wish to include the  $\tau$  Yukawa as a spurion in our expansion, we can consider extending the lepton symmetry to  $G = U(2)^2 \times U(1)^2 \subset G_L$  (a similar construction is possible with one  $U(1)$  factor). Under this symmetry, the fields decompose as<sup>13</sup>

$$\ell = \begin{bmatrix} \ell^a \sim (\mathbf{2}, \mathbf{1}|0, 0) \\ \ell_3 \sim (\mathbf{1}, \mathbf{1}|1, 0) \end{bmatrix}, \quad e = \begin{bmatrix} e^a \sim (\mathbf{1}, \mathbf{2}|0, 0) \\ e_3 \sim (\mathbf{1}, \mathbf{1}|0, 1) \end{bmatrix}, \quad (3.40)$$

and the minimal set of spurions required to produce a realistic Yukawa matrix is given by

$$\Delta_e \sim (\mathbf{2}, \bar{\mathbf{2}}|0, 0), \quad X_\ell \sim (\mathbf{1}, \mathbf{1}|1, -1). \quad (3.41)$$

With these spurions, the Yukawa matrix can be written as

$$Y_e = \begin{bmatrix} \Delta_e & 0 \\ 0 & X_\ell \end{bmatrix}. \quad (3.42)$$

<sup>13</sup>The representations under the flavor group are indicated in the  $(U(2)_\ell, U(2)_e|U(1)_{\ell_3}, U(1)_{e_3})$  format.



We will also include  $V_\ell \sim (\mathbf{2}, \mathbf{1}|0, -1)$  and  $V_e \sim (\mathbf{1}, \mathbf{2}|-1, 0)$  spurions in the decompositions of the bilinear and quartic structures, but these spurions can be set to 0 in the minimal parametrization of the Yukawa matrix (see section 2.1 for details).

Let us take a look at how this set of spurions breaks the flavor symmetry. First,  $\Delta_e$  breaks

$$\Delta_e \longrightarrow \begin{bmatrix} \delta_\ell & 0 \\ 0 & \delta'_\ell \end{bmatrix} : \quad \text{U}(2)_\ell \times \text{U}(2)_e \longrightarrow \text{U}(1)_{\ell+e}^2. \quad (3.43)$$

In the next step,  $X_\ell$  breaks

$$X_\ell \longrightarrow \chi_\ell : \quad \text{U}(1)_{\ell_3} \times \text{U}(1)_{e_3} \longrightarrow \text{U}(1)_{\ell_3+e_3}. \quad (3.44)$$

Furthermore, inclusion of  $V_\ell$  breaks the flavor symmetry down to global lepton number

$$V_\ell \longrightarrow \begin{bmatrix} \epsilon_\ell \\ \epsilon'_\ell \end{bmatrix}, \quad \text{U}(1)_{\ell+e}^2 \times \text{U}(1)_{\ell_3+e_3} \longrightarrow \text{U}(1)_L, \quad (3.45)$$

with  $\epsilon_\ell^{(\prime)} > 0$ .  $V_e$  does not break the symmetry further and has 2 complex parameters. We are left with 5 real, positive parameters and 2 complex parameters in the spurion parametrization. The numerical values of the spurion parameters appearing in the Yukawa matrix are given by

$$\delta_\ell = 2.793 \times 10^{-6}, \quad \delta'_\ell = 5.884 \times 10^{-4}, \quad \chi_\ell = 9.994 \times 10^{-3}. \quad (3.46)$$

The added advantage with respect to the previous section is the explanation of the small  $y_\tau$ .

The flavor spurion counting of the leptonic operators assuming  $\text{U}(2)^2 \times \text{U}(1)^2$  symmetry is presented in table 11, and the flavor decompositions of the bilinear and unique quartic structures are listed in eqs. (3.47)–(3.51).

**Decomposition of bilinear and quartic structures.** Decomposing  $\mathcal{O}(1)$  bilinear structures proceeds in a similar way as in the case of quark  $\text{U}(2)^3$  symmetry. We obtain following  $\mathcal{O}(1)$  structures:  $(\bar{\ell}\ell)$ ,  $(\bar{e}e)$ ,  $(\bar{\ell}_3\ell_3)$  and  $(\bar{e}_3e_3)$ . At  $\mathcal{O}(\Delta)$  there is only one bilinear structure allowed:  $(\bar{\ell}\Delta_\ell e)$ . There are two  $\mathcal{O}(V)$  bilinears of the form  $(\bar{\ell}V_\ell e_3)$  and  $(\bar{\ell}_3V_e^\dagger e)$ . Two doublets of  $\ell$  or  $e$  can also be contracted to two insertions of  $V_\ell$  or  $V_e$  respectively, giving  $(\bar{\ell}V_\ell V_\ell^\dagger \ell)$  and  $(\bar{e}V_e V_e^\dagger e)$  structures. We also get one  $\mathcal{O}(X)$  structure of the form  $(\bar{\ell}_3X_\ell e_3)$ .

More interesting bilinears emerge for two or three spurion insertions. Two  $\mathcal{O}(XV)$  structures can be written based on  $X_\ell^*V_\ell \sim (\mathbf{2}, \mathbf{1}|-1, 0)$  and  $X_\ell V_e \sim (\mathbf{2}, \mathbf{1}|0, -1)$ :  $[\bar{\ell}(X_\ell^*V_\ell)\ell_3]$  and  $[\bar{e}(X_\ell V_e)e_3]$ . Similarly, at order  $\mathcal{O}(\Delta V)$ , we have  $\Delta_e V_e \sim (\mathbf{2}, \mathbf{1}|-1, 0)$  and  $\Delta_e^\dagger V_\ell \sim (\mathbf{1}, \mathbf{2}|0, -1)$ , giving two bilinears of the form  $[\bar{\ell}(\Delta_e V_e)\ell_3]$  and  $[\bar{\ell}(\Delta_e^\dagger V_\ell)\ell_3]$ . At order  $\Delta XV$ , we have the transformations  $X_\ell V_\ell^\dagger \Delta_e \sim (\mathbf{1}, \mathbf{2}|1, 0)$  and  $X_\ell \Delta_e V_e \sim (\mathbf{2}, \mathbf{1}|0, -1)$ , yielding two additional bilinears:  $[\bar{\ell}_3(X_\ell V_\ell^\dagger \Delta_e)e]$  and  $[\bar{\ell}(X_\ell \Delta_e V_e)e_3]$ .

The unique  $\mathcal{O}(1)$  quartic structures with four instances of  $\ell$  include  $(\bar{\ell}_a \ell^b)(\bar{\ell}_b \ell^a)$  and  $(\bar{\ell}\ell_3)(\bar{\ell}_3\ell)$ . Using the transformation properties of the  $X_\ell^*V_\ell$  product, we can construct one more structure:  $(\bar{\ell}_a \ell_3)[\bar{\ell}_b(X_\ell^*V_\ell)^b \ell^a]$ . Analogously, we deduce that there are three  $\mathcal{O}(\Delta V)$  structures,  $[\bar{\ell}_a(V_e \Delta_e)^a \ell^b](\bar{\ell}_b \ell_3)$ ,  $(\bar{\ell}V_\ell \ell^a)(\Delta_e^*)_a{}^b(\bar{e}_b e_3)$ , and  $(\bar{\ell}_a \ell^3)(\Delta_e)_a{}^b(\bar{e}V_e e^b)$ , and three  $\mathcal{O}(V^2)$  ones,  $(\bar{\ell}_a V_\ell^\dagger \ell)(\bar{\ell}V_\ell \ell^a)$ ,  $(\bar{\ell}_3 V_\ell^\dagger \ell)(\bar{\ell}V_\ell \ell_3)$ , and  $(\bar{\ell}V_\ell \ell_3)(\bar{e}V_e e_3)$ . We present the complete list below:

$U(2)^2 \times U(1)^2$		$\mathcal{O}(1)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(X)$	$\mathcal{O}(XV)$	$\mathcal{O}(\Delta V)$	$\mathcal{O}(XV\Delta)$
$\psi^2 H^3$	$Q_{eH}$		2 2		1 1	1 1			2 2
$\psi^2 XH$	$Q_{e(W,B)}$		4 4		2 2	2 2			4 4
$\psi^2 H^2 D$	$Q_{H\ell}^{(1,3)}$	4		2			2 2	2 2	
	$Q_{He}$	2		1			1 1	1 1	
$(LL)(LL)$	$Q_{\ell\ell}$	5		4			3 3	3 3	
$(RR)(RR)$	$Q_{ee}$	3		2			2 2	2 2	
$(LL)(RR)$	$Q_{\ell e}$	4		5 1			4 4	6 6	
Total		18	6 6	14 1	3 3	3 3	12 12	14 14	6 6

**Table 11.** Counting of the pure lepton SMEFT operators (see appendix A) assuming  $U(2)^2 \times U(1)^2$  symmetry in the lepton sector. The counting is performed up to two insertions of  $V$  and one insertion of  $\Delta_e$  and  $X_\ell$  spurion. Moreover, the counting is presented taking two  $(X_\ell V_{e,\ell}, \Delta_e V_{e,\ell})$  and three  $(X_\ell V_{e,\ell} \Delta_e)$  insertions in the spurion product. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

$(\bar{\ell}e)$

$$\begin{aligned}
 \mathcal{O}(\Delta): & (\bar{\ell}\Delta_e e), & \mathcal{O}(V): & (\bar{\ell}V_\ell e_3), \quad (\bar{\ell}_3 V_e^\dagger e), & \mathcal{O}(X): & (\bar{\ell}_3 X_\ell e_3), \\
 \mathcal{O}(\Delta XV): & [\bar{\ell}_3 (X_\ell V_\ell^\dagger \Delta_e) e], & & [\bar{\ell} (X_\ell \Delta_e V_e) e_3].
 \end{aligned} \tag{3.47}$$

$(\bar{\ell}\ell)$

$$\begin{aligned}
 \mathcal{O}(1): & (\bar{\ell}\ell), \quad (\bar{\ell}_3 \ell_3), & \mathcal{O}(XV): & [\bar{\ell} (X_\ell^* V_\ell) \ell_3], \quad \text{H.c.}, & \mathcal{O}(\Delta V): & [\bar{\ell} (\Delta_e V_e) \ell_3], \quad \text{H.c.}, \\
 \mathcal{O}(V^2): & (\bar{\ell} V_\ell V_\ell^\dagger \ell).
 \end{aligned} \tag{3.48}$$

$(\bar{e}e)$

$$\begin{aligned}
 \mathcal{O}(1): & (\bar{e}e), \quad (\bar{e}_3 e_3), & \mathcal{O}(XV): & [\bar{e} (X_\ell V_e) e_3], \quad \text{H.c.}, & \mathcal{O}(\Delta V): & [\bar{e} (\Delta_e^\dagger V_e) e_3], \quad \text{H.c.}, \\
 \mathcal{O}(V^2): & (\bar{e} V_e V_e^\dagger e).
 \end{aligned} \tag{3.49}$$

$(\bar{\ell}\ell)(\bar{\ell}\ell)$

$$\begin{aligned}
 \mathcal{O}(1): & (\bar{\ell}_a \ell^b)(\bar{\ell}_b \ell^a), \quad (\bar{\ell}\ell_3)(\bar{\ell}_3 \ell), & \mathcal{O}(V^2): & (\bar{\ell}_a V_\ell^\dagger \ell)(\bar{\ell} V_\ell \ell^a), \quad (\bar{\ell}_3 V_\ell^\dagger \ell)(\bar{\ell} V_\ell \ell_3), \\
 \mathcal{O}(\Delta V): & [\bar{\ell}_a (V_e \Delta_e)^a \ell^b](\bar{\ell}_b \ell_3), \quad \text{H.c.}, & \mathcal{O}(XV): & (\bar{\ell}_a \ell_3)[\bar{\ell}_b (X_\ell^* V_\ell)^b \ell^a], \quad \text{H.c.}
 \end{aligned} \tag{3.50}$$

$(\bar{\ell}\ell)(\bar{e}e)$

$$\mathcal{O}(V^2): (\bar{\ell} V_\ell \ell_3)(\bar{e} V_e e_3), \quad \text{H.c.}, \quad \mathcal{O}(\Delta V): (\bar{\ell} V_\ell \ell^b)(\Delta_e^*)^a_b (\bar{e}_a e_3), \quad (\bar{\ell}_a \ell^3)(\Delta_e)^a_b (\bar{e} V_e e^b), \quad \text{H.c.} \tag{3.51}$$

$(\bar{e}e)(\bar{e}e)$

No unique structures present due to Fierz identities.

### 3.6 U(3) vectorial symmetry

In this section, we take a look at the  $G = U(3) \subset G_L$  vectorial symmetry, under which the fields transform as

$$\ell \sim \mathbf{3}, \quad e \sim \mathbf{3}. \quad (3.52)$$

The minimal spurion required in this case, similar to the  $U(2)_V$  case, is  $\Delta_\ell \sim \mathbf{8}$ , which we take to be real. For simplicity, we once again use the implicit contraction with the generator:

$$\Delta_\ell^A (T^A)^i_j = (\Delta_\ell)^i_j. \quad (3.53)$$

With the  $\Delta_\ell$  spurion, the Yukawa matrix is parametrized by

$$Y_e = \Delta_\ell + x_\ell \mathbf{1}. \quad (3.54)$$

$\Delta_\ell$  breaks the  $U(3)_V$  flavor symmetry as

$$\Delta_\ell \longrightarrow \begin{bmatrix} -\delta_\ell & & \\ & -\delta'_\ell & \\ & & \delta_\ell + \delta'_\ell \end{bmatrix} : \quad U(3)_V \rightarrow U(1)_{\ell+e}^3. \quad (3.55)$$

The flavor symmetry, thus, removes 6 unphysical parameters from  $\Delta_\ell$ , leaving  $\delta_\ell^{(i)} > 0$ . Furthermore  $x_\ell$  breaks a  $U(1)$  axial symmetry, which allows us to remove an unphysical phase. We observe that the realistic lepton masses require a high degree of tuning between  $x_\ell$  and  $\delta_\ell^{(i)}$ , naturally of the order of the tau Yukawa coupling, to produce the small electron and muon Yukawa couplings. The numerical values of the relevant parameters are given by

$$\delta_\ell = 3.526 \times 10^{-3}, \quad \delta'_\ell = 2.940 \times 10^{-3}, \quad x_\ell = 3.529 \times 10^{-3}. \quad (3.56)$$

Note that one needs a tuning among parameters to accommodate for the smallness of  $y_e$  and  $y_\mu$  with respect to  $y_\tau$ .

We present the spurion counting in table 12 and list the flavor decompositions of the bilinear and unique quartic structures in eqs. (3.57)–(3.61).

**Decomposition of bilinear and quartic structures.** The  $\mathcal{O}(1)$  bilinears are given by  $(\bar{\ell}\ell)$ ,  $(\bar{e}e)$  or  $(\bar{\ell}e)$ . Bilinears containing one insertion of  $\Delta_\ell$  are formed similarly to the  $U(2)_V$  case and are given by  $(\bar{\ell}\Delta_\ell\ell)$ ,  $(\bar{e}\Delta_\ell e)$  and  $(\bar{\ell}\Delta_\ell e)$ . The  $\mathcal{O}(1)$  unique quartic structures we obtain are  $(\bar{\ell}_i\ell^j)(\bar{\ell}_j\ell^i)$  and  $(\bar{\ell}_i\ell^j)(\bar{e}_j e^i)$ . Considering quartic structures with one insertion of  $\Delta_\ell$ , there are two allowed structures:  $(\bar{\ell}_i\ell^j)(\Delta_\ell)^i_k(\bar{\ell}_j\ell^k)$  and  $(\bar{\ell}_i\ell^j)(\Delta_\ell)^i_k(\bar{e}_j e^k)$ . Allowed  $\mathcal{O}(1)$  and  $\mathcal{O}(\Delta_\ell)$  structures are presented below:

$$\boxed{(\bar{\ell}e)} \quad \mathcal{O}(1) : (\bar{\ell}e), \quad \mathcal{O}(\Delta_\ell) : (\bar{\ell}\Delta_\ell e). \quad (3.57)$$

$$\boxed{(\bar{\ell}\ell)} \quad \mathcal{O}(1) : (\bar{\ell}\ell), \quad \mathcal{O}(\Delta_\ell) : (\bar{\ell}\Delta_\ell\ell). \quad (3.58)$$

$U(3)_V$		$\mathcal{O}(1)$		$\mathcal{O}(\Delta_\ell)$	
$\psi^2 H^3$	$Q_{eH}$	1	1	1	1
$\psi^2 XH$	$Q_{e(W,B)}$	2	2	2	2
$\psi^2 H^2 D$	$Q_{H\ell}^{(1,3)}$	2		2	
	$Q_{He}$	1		1	
$(LL)(LL)$	$Q_{\ell\ell}$	2		2	
$(RR)(RR)$	$Q_{ee}$	1		1	
$(LL)(RR)$	$Q_{\ell e}$	2		3	1
Total		11	3	12	4

**Table 12.** Counting of the pure lepton SMEFT operators (see appendix A) assuming  $U(3)$  vectorial symmetry in the lepton sector. The counting is performed up to one insertion of  $\Delta_\ell$  spurion. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

$$\boxed{(\bar{e}e)} \quad \mathcal{O}(1) : (\bar{e}e), \quad \mathcal{O}(\Delta_\ell) : (\bar{e}\Delta_\ell e). \quad (3.59)$$

$$\boxed{(\bar{\ell}\ell)(\bar{\ell}\ell)} \quad \mathcal{O}(1) : (\bar{\ell}_i \ell^j)(\bar{\ell}_j \ell^i), \quad \mathcal{O}(\Delta_\ell) : (\bar{\ell}_i \ell^j)(\Delta_\ell)^i_k (\bar{\ell}_j \ell^k). \quad (3.60)$$

$$\boxed{(\bar{\ell}\ell)(\bar{e}e)} \quad \mathcal{O}(1) : (\bar{\ell}_i \ell^j)(\bar{e}_j e^i), \quad \mathcal{O}(\Delta_\ell) : (\bar{\ell}_i \ell^j)(\Delta_\ell)^i_k (\bar{e}_j e^k), \quad \text{H.c.} \quad (3.61)$$

$\boxed{(\bar{e}e)(\bar{e}e)}$   
No unique structures present due to Fierz identities.

### 3.7 MFV<sub>L</sub> symmetry

Lastly, let us take a look at minimal flavor violation in the lepton sector, with the full symmetry  $G = U(3)_\ell \times U(3)_e = G_L$ . The leptons are in the representations

$$\ell \sim (\mathbf{3}, \mathbf{1}), \quad e \sim (\mathbf{1}, \mathbf{3}), \quad (3.62)$$

and the leptonic Yukawa matrix, which serves as the sole spurion transforms as

$$Y_e \sim (\mathbf{3}, \bar{\mathbf{3}}). \quad (3.63)$$

As in the SM, the Yukawa matrix breaks the symmetry according to

$$Y_e \longrightarrow \hat{Y}_e : \quad U(3)_\ell \times U(3)_e \longrightarrow U(1)_{\ell+e}^3, \quad (3.64)$$

MFV <sub>L</sub>		$\mathcal{O}(1)$	$\mathcal{O}(Y_e)$
$\psi^2 H^3$	$Q_{eH}$		1 1
$\psi^2 XH$	$Q_{e(W,B)}$		2 2
$\psi^2 H^2 D$	$Q_{H\ell}^{(1,3)}$	2	
	$Q_{He}$	1	
$(LL)(LL)$	$Q_{\ell\ell}$	2	
$(RR)(RR)$	$Q_{ee}$	1	
$(LL)(RR)$	$Q_{\ell e}$	1	
Total		7	3 3

**Table 13.** Counting of the pure lepton SMEFT operators (see appendix A) assuming MFV<sub>L</sub> symmetry in the lepton sector. The counting is performed up to one insertion of  $Y_e$  spurion. The left (right) entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

where  $\hat{Y}_e$  is a real, positive, diagonal matrix. 15 unphysical parameters are removed from the spurion in this manner, and the remnant symmetry ensures conservation of individual lepton numbers.

The flavor spurion counting of the pure lepton operators is presented in table 13 and the decompositions are listed in eqs. (3.65)–(3.68).

**Decomposition of bilinear and quartic structures.** At  $\mathcal{O}(1)$ , we get two structures of the form  $(\bar{\ell}_i \ell^i)$  and  $(\bar{e}_i e^i)$ . Using one insertion of  $Y_e$  we obtain one structure given by  $(\bar{\ell} Y_e e)$ . Regarding the non-factorizing quartic structures, there is only one that can be formed with four appearances of  $\ell$  of the form  $(\bar{\ell}_i \ell^j)(\bar{\ell}_j \ell^i)$ . There are no unique structures of the form  $(\bar{\ell}\ell)(\bar{e}e)$ . Taking these remarks into account, the list of the  $\mathcal{O}(1)$  and  $\mathcal{O}(Y_e)$  structures is presented below:

$$\boxed{(\bar{\ell} e)} \qquad \mathcal{O}(Y_e) : (\bar{\ell} Y_e e) . \qquad (3.65)$$

$$\boxed{(\bar{\ell}\ell)} \qquad \mathcal{O}(1) : (\bar{\ell}\ell) . \qquad (3.66)$$

$$\boxed{(\bar{e}e)} \qquad \mathcal{O}(1) : (\bar{e}e) . \qquad (3.67)$$

$$\boxed{(\bar{\ell}\ell)(\bar{\ell}\ell)} \qquad \mathcal{O}(1) : (\bar{\ell}_i \ell^j)(\bar{\ell}_j \ell^i) . \qquad (3.68)$$

$$\boxed{(\bar{\ell}\ell)(\bar{e}e)}$$

No unique structures present.

$$\boxed{(\bar{e}e)(\bar{e}e)}$$

No unique structures present due to Fierz identities.

## 4 Conclusions

The hierarchical pattern of charged fermion masses and mixings observed in nature craves an explanation: the dimension-4 Yukawa interactions in the SM provide only a parametrization but not an understanding of flavor. To make progress in addressing this long-standing puzzle, we must uncover new, flavored interactions beyond the SM. The SMEFT is a powerful framework that can capture the low-energy physics of a high-energy model. There are 2499 leading dimension-6 baryon and lepton number-conserving operators, the great majority of which are flavorful. The hope is that experiments will observe some of these interactions and start clarifying their flavor patterns. This might provide a crucial clue towards solving the puzzle. Perhaps the ongoing flavor anomalies are the first step in this direction.

Patterns are closely related to symmetries. In this paper, we systematically explored the flavor structure of  $\Delta B = 0$  dimension-6 SMEFT operators using flavor symmetries as an organizing principle in an extension of ref. [1]. Our underlying assumption is that short-distance physics will leave a global flavor symmetry and a breaking pattern in the effective operators at low energies. From the IR perspective, postulating different flavor structures in the effective field theory means mapping the space of physics beyond the SM into the universality classes. These assumptions impose correlations among operators, which can be tested in experiments, providing a systematic way to learn about BSM.

Concretely, working in the Warsaw basis (appendix A), we imposed a variety of different global flavor symmetry assumptions in both the quark and the lepton sectors. The symmetries are carefully chosen to allow for new physics at (not far beyond) the TeV scale while respecting the experimental bounds from FCNC, LFV, and EDMs. In particular, they allow for potential new physics effects in the high- $p_T$  experiments and motivate global SMEFT fits in the top, Higgs, and EW sectors. The symmetry breaking spurions are non-dynamical objects formally transforming in a non-trivial representation of the imposed flavor group. For each flavor structure, we construct the basis of dimension-6 operators compatible with the flavor symmetry and breaking spurions.

As a supplement to this work, we also provide a `Mathematica` package `SMEFTflavor` for automatic generation of the operators should the user have a different symmetry group or breaking spurions in mind (appendix B).

As shown in table 1, the number of leading flavor-symmetric operators without spurion insertions, which are important for the high- $p_T$  fits, is significantly reduced from the initial 2499 when no symmetries are imposed. In section 2, we explicitly construct independent operators for  $U(2)^3$ ,  $U(2)^3 \times U(1)$ ,  $U(2)^2 \times U(3)$ , and  $U(3)^3$  quark flavor symmetries, focusing on the quark-only operators. We pay special attention to the  $SU(2)$  subgroup invariants.

We also derive minimal spurion parametrizations, which can be directly employed in the phenomenological studies. The novelty here is the use of the full  $U(3)^3$  field redefinitions consistent with the imposed symmetry to avoid overparametrizations often found in the literature. In section 3, we repeat the analysis for the lepton-only operators and lepton symmetries  $U(1)^6$ ,  $U(1)_V^3$ ,  $U(2)_V$ ,  $U(2)^2$ ,  $U(2)^2 \times U(1)^2$ ,  $U(3)_V$ , and  $U(3)^2$ . The counting of the mixed quark-lepton operators is worked out in appendix C for each combination of the four quark and the seven lepton symmetries.

Our methodology can be extended to the dimension-8 operators in the SMEFT, which will be presented in a separate publication. We hope the flavor structures proposed in this work will find its place in the future phenomenological studies of low and high- $p_T$  data.

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## A Warsaw basis

Here we list the  $\Delta B = 0$  dimension-6 fermionic SMEFT operators in the Warsaw basis [13] with division into classes as presented in [14].

5–7: Fermion Bilinears

non-hermitian ( $\bar{L}R$ )					
5: $\psi^2 H^3$		6: $\psi^2 XH$			
$Q_{eH}$	$(H^\dagger H)(\bar{\ell}_p e_r H)$	$Q_{eW}$	$(\bar{\ell}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$	$Q_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{H} G_{\mu\nu}^A$
$Q_{uH}$	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$	$Q_{eB}$	$(\bar{\ell}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$Q_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$
$Q_{dH}$	$(H^\dagger H)(\bar{q}_p d_r H)$			$Q_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G_{\mu\nu}^A$
				$Q_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H W_{\mu\nu}^I$
				$Q_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$

hermitian (+ $Q_{Hud}$ ) $\sim$ 7: $\psi^2 H^2 D$					
$(\bar{L}L)$		$(\bar{R}R)$		$(\bar{R}R')$	
$Q_{H\ell}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\ell}_p \gamma^\mu \ell_r)$	$Q_{He}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$	$Q_{Hud}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$
$Q_{H\ell}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{\ell}_p \tau^I \gamma^\mu \ell_r)$	$Q_{Hu}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$		
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$	$Q_{Hd}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$		
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$				

8: Fermion Quadrilinears

hermitian					
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$Q_{\ell\ell}$	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{\ell}_s\gamma^\mu\ell_t)$	$Q_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$Q_{\ell e}$	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$Q_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$Q_{\ell u}$	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$Q_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{\ell d}$	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{\ell q}^{(1)}$	$(\bar{\ell}_p\gamma_\mu\ell_r)(\bar{q}_s\gamma^\mu q_t)$	$Q_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$Q_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{\ell q}^{(3)}$	$(\bar{\ell}_p\gamma_\mu\tau^I\ell_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$Q_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

non-hermitian			
$(\bar{L}R)(\bar{R}L)$		$(\bar{L}R)(\bar{L}R)$	
$Q_{ledq}$	$(\bar{\ell}_p^j e_r)(\bar{d}_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar{\ell}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar{\ell}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

## B SMEFTflavor

In this section we briefly outline the details of the `Mathematica` package developed during this project, which has played an important, twofold role: first, it has been used to perform the cross-checks of all the decompositions and counting tables. Second, the code has been developed to allow for implementation of other flavor symmetries than those analyzed here to determine the SMEFT operators and corresponding spurion counting tables. The package can be downloaded from the `github` page <https://github.com/aethomsen/SMEFTflavor>. It can then be run from a notebook located in the base directory of the package (the one with the ‘Tutorial.nb’ notebook) by setting the directory to that of the notebook with `SetDirectory@ NotebookDirectory[]` and then running `<< SMEFTflavor``.

Let us now present the main functions of the program and describe their output. `SMEFTflavor` comes with implementations of the 4 quark and 7 lepton symmetries considered in this paper ready to use, and more symmetries can be added by the user. The details of the implemented symmetries can be found in the association `$flavorSymmetries`. The keys of this association, `Keys@ $flavorSymmetries`, are all the names of symmetries (strings) with the indicative labels, e.g., ‘3U2’ for  $U(2)^3$ , ‘6U1’ for  $U(1)^6$ , ‘2U2xU3’ for  $U(2)^2 \times U(3)$  etc. To access the details (groups, representations, spurions) of a particular combination of quark and lepton symmetry assignment, one simply picks out the element of the association, e.g., `$flavorSymmetries@ {"quark:MFV", "lep:2U2"}` for the MFV in quark and  $U(2)^2$



in the lepton sector. Similarly, details of the SMEFT operators (Warsaw basis) are found in the association `$smeftOperators` and the keys `Keys@ $smeftOperators` constitute a list of all the operators. As the pure quark, pure lepton and mixed quark-lepton operators are analyzed separately in this work, these subsets of the SMEFT operators are listed in the program using `leptonicOperators`, `quarkOperators` and `semiLeptonicOperators`

The main function of the package is `DetermineOperatorBasis[symmetry, options]` and it is used to construct the operator basis provided the symmetry (input as a string or a list of strings if mixed quark-lepton operators are considered) along with other options. The options this function can take are `SpurionCount` (taking an integer value — default 3 — and used to indicate the order in the counting of the spurions based on the corresponding input in the `FlavorSymmetries`) and `SMEFTOperators` (defining the set SMEFT operators to consider — default `All` — e.g., one of the three subsets of the SMEFT operators mentioned above). An illustrative example of this command is

```
DetermineOperatorBasis["lep:2U2",
  SpurionCount-> 1,
  SMEFTOperators->leptonicOperators]
```

The output of this function consists of an association of the pure lepton operator basis (organized by SMEFT operator and spurion insertions) up to order 1 in the specified spurion counting. In order to present operators in a more legible form with the contractions explicitly indicated, `//OpForm` can be added to the end of the previous line. `OpForm` generally formats all the operators to make them legible to humans.

The second important function we point out is the `CountingTable[symmetry, options]` function, which also takes `SpurionCount` and `SMEFTOperators` for options. This function returns the spurion counting table for particular symmetry based on the operator basis. Let us present three illustrative examples of this function. In order to return the spurion counting table for the pure quark MFV case (table 6) up to order 3 in the spurion counting one can call `CountingTable["quark:MFV", SpurionCount-> 3]`. Similarly, to get the spurion counting table for the pure lepton  $U(1)^3$  case (table 7) one can run the analogous command `CountingTable["lep:3U1", SpurionCount-> 1]` Lastly, to obtain the full spurion counting table for all operators assuming, MFV in the quark and  $U(1)^3$  in the lepton sector one can run `CountingTable[{"quark:MFV", "lep:3U1"}, SpurionCount->3]`

The last function we would like to point out is `AddSMEFTSymmetry`, which enables user to implement a new quark, lepton, or mixed quark-lepton symmetry. The syntax of this function is `AddSMEFTSymmetry["Type", "Name"-> GroupInfoAssociation]` The first argument can either be `"Lepton"`, `"Quark"`, or `"Mixed"`, designating which sector the newly added symmetry is associated to. `Name` refers to the string introduced as the name of the added symmetry and the `GroupInfoAssociation` associates all the symmetry properties (similar to those contained in `$flavorSymmetries`) to the new symmetry. To illustrate how this function is used, let us imagine that we would like to introduce a  $U(2)_{\text{diag}}$  symmetry group in the lepton sector with a real spurion transforming in the adjoint:

```

AddSMEFTSymmetry["Lepton", "U2diag"-> <|
  Groups-> <|"U21"-> SU@ 2|>,
  Spurions-> {"Δ1"},
  Charges-> <|"112"-> {1}, "13"-> 0, "e12"-> {1}, "e3"-> 0, "Δ1"-> 0,
    "V1"-> {1}|>,
  Representations-> <|"112"-> {"U21"@ fund}, "e12"-> {"U21"@ fund},
    "Δ1"-> {"U21"@ adj}|>,
  FieldSubstitutions-> <|"1"-> {"112", "13"}, "e"-> {"e12", "e3"}|>,
  SpurionCounting-> <|"Δ1"-> 2|>,
  SelfConjugate-> {"Δ1"}|>]
    
```

Note that only SU(2) and SU(3) factors are supported. This should cover the vast majority of cases. All the aforementioned functions and their outputs along with additional practical examples have been presented in the tutorial notebook provided in the package.

### C Mixed quark-lepton operators

The mixed quark-lepton four-fermion operators change for every combination of the 4 quark and 7 lepton flavor structures we have considered (for a total of 28 unique cases). In each case, the flavor structure factorizes straight-forwardly into a quark and a lepton bilinear, all of which we have presented in the main text. Here we report in tables below the counting for all 28 cases while the exhaustive results, including the explicit forms for the operators, can be generated using the `SMEFTflavor` package.

#### MFV<sub>Q</sub> × MFV<sub>L</sub>

MFV <sub>Q</sub> × MFV <sub>L</sub>		$\mathcal{O}(1)$	$\mathcal{O}(Y_u^2)$	$\mathcal{O}(Y_d^2)$	$\mathcal{O}(Y_e Y_u)$	$\mathcal{O}(Y_e Y_d)$
(LL)(LL)	$Q_{\ell q}^{(1,3)}$	2	2	2		
(RR)(RR)	$Q_{eu}$	1	1			
	$Q_{ed}$	1		1		
(LL)(RR)	$Q_{\ell u}$	1	1			
	$Q_{\ell d}$	1		1		
	$Q_{qe}$	1	1	1		
(LR)(RL)	$Q_{\ell edq}$					1 1
(LR)(LR)	$Q_{\ell equ}^{(1,3)}$				2 2	
Total		7	5	5	2 2	1 1

#### MFV<sub>Q</sub> × U(3)<sub>V,L</sub>

MFV <sub>Q</sub> × U(3) <sub>V,L</sub>		$\mathcal{O}(1)$	$\mathcal{O}(Y_u)$	$\mathcal{O}(Y_u^2)$	$\mathcal{O}(Y_d)$	$\mathcal{O}(Y_d^2)$	$\mathcal{O}(Y_u^2 Y_d)$	$\mathcal{O}(Y_u Y_d^2)$	$\mathcal{O}(\Delta_\ell)$	$\mathcal{O}(\Delta_\ell Y_u)$	$\mathcal{O}(\Delta_\ell Y_d)$
(LL)(LL)	$Q_{\ell q}^{(1,3)}$	2		2		2			2		
(RR)(RR)	$Q_{eu}$	1		1					1		
	$Q_{ed}$	1			1				1		
(LL)(RR)	$Q_{\ell u}$	1		1					1		
	$Q_{\ell d}$	1			1				1		
	$Q_{qe}$	1		1	1				1		
(LR)(RL)	$Q_{\ell edq}$				1 1		1 1				1 1
(LR)(LR)	$Q_{\ell equ}^{(1,3)}$		2 2					2 2		2 2	
Total		7	2 2	5	1 1	5	1 1	2 2	7	2 2	1 1

$\mathbf{MFV}_Q \times (\mathbf{U}(2)^2 \times \mathbf{U}(1)^2)_L$ 

$\mathbf{MFV}_Q \times (\mathbf{U}(2)^2 \times \mathbf{U}(1)^2)_L$		$\mathcal{O}(1)$	$\mathcal{O}(Y_u^2)$	$\mathcal{O}(Y_d^2)$	$\mathcal{O}(XV_{\ell,e})$	$\mathcal{O}(\Delta V_{\ell,e})$	$\mathcal{O}(V_{\ell,e}^2)$	$\mathcal{O}(\Delta Y_d, \Delta Y_u)$	$\mathcal{O}(V_{\ell,e} Y_d, V_{\ell,e} Y_u)$	$\mathcal{O}(XY_d, XY_u)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	4	4	4	2 2	2 2	2			
$(RR)(RR)$	$Q_{eu}$	2	2		1 1	1 1	1			
	$Q_{ed}$	2		2	1 1	1 1	1			
$(LL)(RR)$	$Q_{\ell u}$	2	2		1 1	1 1	1			
	$Q_{\ell d}$	2		2	1 1	1 1	1			
	$Q_{qe}$	2	2	2	1 1	1 1	1			
$(LR)(RL)$	$Q_{\ell dq}$							1 1	2 2	1 1
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$							2 2	4 4	2 2
Total		14	10	10	7 7	7 7	7	3 3	6 6	3 3

 $\mathbf{MFV}_Q \times \mathbf{U}(2)_L^2$ 

$\mathbf{MFV}_Q \times \mathbf{U}(2)_L^2$		$\mathcal{O}(1)$	$\mathcal{O}(Y_u)$	$\mathcal{O}(Y_d)$	$\mathcal{O}(Y_u^2)$	$\mathcal{O}(Y_d^2)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(\Delta V)$	$\mathcal{O}(VY_d, VY_u)$	$\mathcal{O}(\Delta Y_u, \Delta Y_d)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	4			4	4	2 2	2			
$(RR)(RR)$	$Q_{eu}$	2			2				1 1		
	$Q_{ed}$	2				2			1 1		
$(LL)(RR)$	$Q_{\ell u}$	2			2		1 1	1			
	$Q_{\ell d}$	2				2	1 1	1			
	$Q_{qe}$	2			2	2			1 1		
$(LR)(RL)$	$Q_{\ell dq}$			1 1						1 1	1 1
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$		2 2							2 2	2 2
Total		14	2 2	1 1	10	10	4 4	4	3 3	3 3	3 3

 $\mathbf{MFV}_Q \times \mathbf{U}(2)_{V,L}$ 

$\mathbf{MFV}_Q \times \mathbf{U}(2)_{V,L}$		$\mathcal{O}(1)$	$\mathcal{O}(Y_u)$	$\mathcal{O}(Y_u^2)$	$\mathcal{O}(Y_d)$	$\mathcal{O}(Y_d^2)$	$\mathcal{O}(\Delta_\ell)$	$\mathcal{O}(\Delta_\ell Y_u)$	$\mathcal{O}(\Delta_\ell Y_d)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	4		4		4	2		
$(RR)(RR)$	$Q_{eu}$	2		2			1		
	$Q_{ed}$	2				2	1		
$(LL)(RR)$	$Q_{\ell u}$	2		2			1		
	$Q_{\ell d}$	2				2	1		
	$Q_{qe}$	2		2		2	1		
$(LR)(RL)$	$Q_{\ell dq}$				2 2				1 1
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$		4 4					2 2	
Total		14	4 4	10	2 2	10	7	2 2	1 1

$\mathbf{MFV}_Q \times \mathbf{U}(1)_L^6$ 

$\mathbf{MFV}_Q \times \mathbf{U}(1)_L^6$		$\mathcal{O}(1)$	$\mathcal{O}(Y_u^2)$	$\mathcal{O}(Y_d^2)$	$\mathcal{O}(yY_u)$	$\mathcal{O}(yY_d)$
$(LL)(LL)$	$Q_{lq}^{(1,3)}$	6	6	6		
$(RR)(RR)$	$Q_{eu}$	3	3			
	$Q_{ed}$	3		3		
$(LL)(RR)$	$Q_{lu}$	3	3			
	$Q_{ld}$	3		3		
	$Q_{qe}$	3	3	3		
$(LR)(RL)$	$Q_{ledq}$					3 3
$(LR)(LR)$	$Q_{lequ}^{(1,3)}$				6 6	
Total		21	15	15	6 6	3 3

 $\mathbf{MFV}_Q \times \mathbf{U}(1)_L^3$ 

$\mathbf{MFV}_Q \times \mathbf{U}(1)_L^3$		$\mathcal{O}(1)$	$\mathcal{O}(Y_u)$	$\mathcal{O}(Y_d)$	$\mathcal{O}(Y_u^2)$	$\mathcal{O}(Y_d^2)$	$\mathcal{O}(Y_u^2 Y_d)$	$\mathcal{O}(Y_d^2 Y_u)$
$(LL)(LL)$	$Q_{lq}^{(1,3)}$	6			6	6		
$(RR)(RR)$	$Q_{eu}$	3			3			
	$Q_{ed}$	3				3		
$(LL)(RR)$	$Q_{lu}$	3			3			
	$Q_{ld}$	3				3		
	$Q_{qe}$	3			3	3		
$(LR)(RL)$	$Q_{ledq}$			3 3			3 3	
$(LR)(LR)$	$Q_{lequ}^{(1,3)}$		6 6					6 6
Total		21	6 6	3 3	15	15	3 3	6 6

 $(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{MFV}_L$ 

$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{MFV}_L$	$\mathcal{O}(1)$	$\mathcal{O}(V_q)$	$\mathcal{O}(Y_e)$	$\mathcal{O}(\Lambda_d^2)$	$\mathcal{O}(V_q^2)$	$\mathcal{O}(\Lambda_d \Sigma_d)$	$\mathcal{O}(\Lambda_d Y_e)$	$\mathcal{O}(V_q Y_e)$	$\mathcal{O}(V_q \Delta)$	$\mathcal{O}(\Sigma_d^2)$	$\mathcal{O}(\Sigma_d Y_e)$	$\mathcal{O}(Y_e^2)$	$\mathcal{O}(Y_e \Delta)$	
$(LL)(LL)$	$Q_{lq}^{(1,3)}$	4	2 2		2	2 2				2		4		
$(RR)(RR)$	$Q_{eu}$	2						1 1				2		
	$Q_{ed}$	1		1						1		1		
$(LL)(RR)$	$Q_{lu}$	2							1 1			2		
	$Q_{ld}$	1		1						1		1		
	$Q_{qe}$	2	1 1			1	1 1			1		2		
$(LR)(RL)$	$Q_{ledq}$						1 1				1 1			
$(LR)(LR)$	$Q_{lequ}^{(1,3)}$			2 2				2 2					2 2	
Total		12	3 3	2 2	2	3	3 3	1 1	2 2	2 2	5	1 1	12	2 2

$$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(3)_{V,L}$$

$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(3)_{V,L}$		$\mathcal{O}(1)$	$\mathcal{O}(\Lambda_d)$	$\mathcal{O}(V_q)$	$\mathcal{O}(\Sigma_d)$	$\mathcal{O}(\Delta_\ell)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Lambda_d^2, V_q^2, \Lambda_d V_q)$	$\mathcal{O}(\Lambda_d \Sigma_d, V_q \Sigma_d)$	$\mathcal{O}(\Lambda_d \Delta_\ell, V_q \Delta_\ell)$	$\mathcal{O}(V_q \Delta)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	4		2 2		4		2	2 2	2 2	
$(RR)(RR)$	$Q_{eu}$	2				2					1 1
	$Q_{ed}$	1				1		1			
$(LL)(RR)$	$Q_{\ell u}$	2				2					1 1
	$Q_{\ell d}$	1				1		1			
	$Q_{qe}$	2		1 1		2		1	1 1	1 1	
$(LR)(RL)$	$Q_{\ell dq}$		1 1		1 1			1 1	1 1	1 1	
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$	2 2		2 2		2 2	2 2		2 2	2 2	2 2
Total		14 2	1 1	5 5	1 1	14 2	2 2	6 1	6 6	6 6	4 4

$$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times (\mathbf{U}(2)^2 \times \mathbf{U}(1)^2)_L$$

$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times (\mathbf{U}(2)^2 \times \mathbf{U}(1)^2)_L$		$\mathcal{O}(1)$	$\mathcal{O}(V_{e,\ell,q})$	$\mathcal{O}(X)$	$\mathcal{O}(\Delta_e)$	$\mathcal{O}(V_{e,\ell,q}^2, \Lambda_d^2, \Lambda_d V_{e,\ell}, V_{e,\ell} V_q)$	$\mathcal{O}(X V_{e,\ell,q})$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	8	4 4			8	4 4
$(RR)(RR)$	$Q_{eu}$	4				2	2 2
	$Q_{ed}$	2				3	1 1
$(LL)(RR)$	$Q_{\ell u}$	4				2	2 2
	$Q_{\ell d}$	2				3	1 1
	$Q_{qe}$	4	2 2			4	2 2
$(LR)(RL)$	$Q_{\ell dq}$					2 2	
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$		4 4	2 2	2 2	4 4	2 2
Total		24	10 10	2 2	2 2	28 6	14 14

$$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(2)_L^2$$

$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(2)_L^2$		$\mathcal{O}(1)$	$\mathcal{O}(\Lambda_d, V_{\ell,q})$	$\mathcal{O}(\Sigma_d)$	$\mathcal{O}(\Delta_{e,u})$	$\mathcal{O}(\Lambda_d^2, V_{\ell,q}^2)$	$\mathcal{O}(\Lambda_d V_\ell, \Lambda_d V_q, V_\ell V_q)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	8	8 8			8	4 4
$(RR)(RR)$	$Q_{eu}$	4					
	$Q_{ed}$	2				2	
$(LL)(RR)$	$Q_{\ell u}$	4	2 2			2	
	$Q_{\ell d}$	2	1 1			3	
	$Q_{qe}$	4	2 2			2	
$(LR)(RL)$	$Q_{\ell dq}$		1 1	1 1			2 2
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$	2 2	4 4		4 4		2 2
Total		26 2	18 18	1 1	4 4	17	8 8

$$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(2)_{V,L}$$

$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(2)_{V,L}$		$\mathcal{O}(1)$	$\mathcal{O}(\Lambda_d)$	$\mathcal{O}(V_q)$	$\mathcal{O}(\Sigma_d)$	$\mathcal{O}(\Delta_\ell)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Lambda_d^2, V_q^2, \Lambda_d V_q)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	8		4 4		4		4
$(RR)(RR)$	$Q_{eu}$	4				2		
	$Q_{ed}$	2				1		2
$(LL)(RR)$	$Q_{\ell u}$	4				2		
	$Q_{\ell d}$	2				1		2
	$Q_{qe}$	4		2 2		2		2
$(LR)(RL)$	$Q_{\ell edq}$		2 2		2 2			2 2
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$	4 4		4 4		2 2	4 4	
Total		28 4	2 2	10 10	2 2	14 2	4 4	12 2

$$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(1)_L^6$$

$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(1)_L^6$		$\mathcal{O}(1)$	$\mathcal{O}(V_q)$	$\mathcal{O}(y)$	$\mathcal{O}(\Lambda_d^2, V_q^2)$	$\mathcal{O}(y\Lambda_d)$	$\mathcal{O}(yV_q)$	$\mathcal{O}(\Lambda_d \Sigma_d)$	$\mathcal{O}(V_q \Delta)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	12	6 6		6			6 6	
$(RR)(RR)$	$Q_{eu}$	6							3 3
	$Q_{ed}$	3			3				
$(LL)(RR)$	$Q_{\ell u}$	6							3 3
	$Q_{\ell d}$	3			3				
	$Q_{qe}$	6	3 3		3			3 3	
$(LR)(RL)$	$Q_{\ell edq}$					3 3			
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$			6 6			6 6		
Total		36	9 9	6 6	15	3 3	6 6	9 9	6 6

$$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(1)_L^3$$

$(\mathbf{U}(2)^2 \times \mathbf{U}(3))_Q \times \mathbf{U}(1)_L^3$		$\mathcal{O}(1)$	$\mathcal{O}(\Lambda_d)$	$\mathcal{O}(V_q)$	$\mathcal{O}(\Sigma_d)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Lambda_d^2, V_q^2, \Lambda_d V_q)$	$\mathcal{O}(\Lambda_d \Sigma_d, V_q \Sigma_d)$	$\mathcal{O}(V_q \Delta)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	12		6 6			6	6 6	
$(RR)(RR)$	$Q_{eu}$	6							3 3
	$Q_{ed}$	3				3			
$(LL)(RR)$	$Q_{\ell u}$	6							3 3
	$Q_{\ell d}$	3				3			
	$Q_{qe}$	6		3 3		3		3 3	
$(LR)(RL)$	$Q_{\ell edq}$		3 3		3 3		3 3	3 3	
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$	6 6		6 6		6 6		6 6	6 6
Total		42 6	3 3	15 15	3 3	6 6	18 3	18 18	12 12

$(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{MFV}_L$ 

$(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{MFV}_L$	$\mathcal{O}(1)$	$\mathcal{O}(V_q)$	$\mathcal{O}(Y_e)$	$\mathcal{O}(V_q^2)$	$\mathcal{O}(V_q Y_e)$	$\mathcal{O}(V_q \Delta)$	$\mathcal{O}(X_b Y_e)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	4	2 2	2			
$(RR)(RR)$	$Q_{eu},$	2				1 1	
	$Q_{ed}$	2					
$(LL)(RR)$	$Q_{\ell u}$	2				1 1	
	$Q_{\ell d}$	2					
	$Q_{qe}$	2	1 1		1		
$(LR)(RL)$	$Q_{\ell edq}$						1 1
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$			2 2		2 2	
Total		14	3 3	2 2	3	2 2	2 2

 $(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(3)_{v,L}$ 

$(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(3)_{v,L}$	$\mathcal{O}(1)$	$\mathcal{O}(V_q)$	$\mathcal{O}(X_b)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta_\ell)$	$\mathcal{O}(V_q^2)$	$\mathcal{O}(V_q X_b)$	$\mathcal{O}(V_q \Delta)$	$\mathcal{O}(V_q \Delta_\ell)$	$\mathcal{O}(X_b \Delta_\ell)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	4	2 2		4	2			2 2	
$(RR)(RR)$	$Q_{eu},$	2			2			1 1		
	$Q_{ed}$	2			2					
$(LL)(RR)$	$Q_{\ell u}$	2			2			1 1		
	$Q_{\ell d}$	2			2					
	$Q_{qe}$	2	1 1		2	1			1 1	
$(LR)(RL)$	$Q_{\ell edq}$		1 1	1 1			1 1	1 1		1 1
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$	2 2	2 2		2 2			2 2	2 2	
Total		16 2	5 5	1 1	3 3	16 2	3	1 1	5 5	5 5

 $(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times (\mathbf{U}(2)^2 \times \mathbf{U}(1)^2)_L$ 

$(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times (\mathbf{U}(2)^2 \times \mathbf{U}(1)^2)_L$	$\mathcal{O}(1)$	$\mathcal{O}(V_{e,\ell,q})$	$\mathcal{O}(X)$	$\mathcal{O}(\Delta_e)$	$\mathcal{O}(V_e^2, V_e V_q)$	$\mathcal{O}(V_e X, V_e X_b)$	$\mathcal{O}(V_\ell^2, V_\ell V_q)$	$\mathcal{O}(V_\ell X, V_\ell X_b)$	$\mathcal{O}(V_q^2)$	$\mathcal{O}(X X_b)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	8	4 4				4	4 4	4	
$(RR)(RR)$	$Q_{eu},$	4			2	2 2				
	$Q_{ed}$	4			2	2 2				
$(LL)(RR)$	$Q_{\ell u}$	4					2	2 2		
	$Q_{\ell d}$	4					2	2 2		
	$Q_{qe}$	4	2 2		2	2 2			2	
$(LR)(RL)$	$Q_{\ell edq}$					1 1		1 1		1 1
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$		4 4	2 2	2 2		2 2			
Total		28	10 10	2 2	2 2	8 2	7 7	10 2	9 9	6

 $(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(2)_L^2$ 

$(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(2)_L^2$	$\mathcal{O}(1)$	$\mathcal{O}(V_\ell)$	$\mathcal{O}(V_q)$	$\mathcal{O}(X_b)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta_e)$	$\mathcal{O}(V_\ell^2)$	$\mathcal{O}(V_q^2)$	$\mathcal{O}(V_\ell V_q)$	$\mathcal{O}(X_b V_{\ell,q})$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	8	4 4	4 4			4	4	4 4	
$(RR)(RR)$	$Q_{eu},$	4								
	$Q_{ed}$	4								
$(LL)(RR)$	$Q_{\ell u}$	4	2 2				2			
	$Q_{\ell d}$	4	2 2				2			
	$Q_{qe}$	4		2 2				2		
$(LR)(RL)$	$Q_{\ell edq}$				1 1	1 1				2 2
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$	2 2	2 2	2 2		2 2			2 2	
Total		30 2	10 10	8 8	1 1	3 3	2 2	8	6	6 6

$(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(2)_{V,L}$ 

$(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(2)_{V,L}$		$\mathcal{O}(1)$	$\mathcal{O}(V_q)$		$\mathcal{O}(X_b)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta_\ell)$		$\mathcal{O}(V_q^2)$	$\mathcal{O}(X_b V_q)$				
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	8	4	4			4	4						
$(RR)(RR)$	$Q_{eu}$	4					2							
	$Q_{ed}$	4					2							
$(LL)(RR)$	$Q_{lu}$	4					2							
	$Q_{ld}$	4					2							
	$Q_{qe}$	4	2	2			2	2						
$(LR)(RL)$	$Q_{ledq}$				2	2	2	2			2	2		
$(LR)(LR)$	$Q_{lequ}^{(1,3)}$	4	4	4	4		4	4	2	2				
Total		32	4	10	10	2	2	6	6	16	2	6	2	2

 $(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(1)_L^6$ 

$(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(1)_L^6$		$\mathcal{O}(1)$	$\mathcal{O}(V_q)$		$\mathcal{O}(V_q^2)$	$\mathcal{O}(y)$	$\mathcal{O}(V_q y)$		$\mathcal{O}(y X_b)$	$\mathcal{O}(V_q \Delta_u, V_q X_b \Delta_d)$			
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	12	6	6	6								
$(RR)(RR)$	$Q_{eu}$	6								3	3		
	$Q_{ed}$	6								3	3		
$(LL)(RR)$	$Q_{lu}$	6								3	3		
	$Q_{ld}$	6								3	3		
	$Q_{qe}$	6	3	3	3								
$(LR)(RL)$	$Q_{ledq}$							3	3				
$(LR)(LR)$	$Q_{lequ}^{(1,3)}$					6	6	6	6				
Total		42	9	9	9	6	6	6	6	3	3	12	12

 $(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(1)_L^3$ 

$(\mathbf{U}(2)^3 \times \mathbf{U}(1))_Q \times \mathbf{U}(1)_L^3$		$\mathcal{O}(1)$	$\mathcal{O}(V_q)$		$\mathcal{O}(X_b)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(V_q^2)$	$\mathcal{O}(X_b V_q)$		$\mathcal{O}(V_q \Delta)$				
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	12	6	6			6							
$(RR)(RR)$	$Q_{eu}$	6								3	3			
	$Q_{ed}$	6												
$(LL)(RR)$	$Q_{lu}$	6								3	3			
	$Q_{ld}$	6												
	$Q_{qe}$	6	3	3			3							
$(LR)(RL)$	$Q_{ledq}$				3	3	3	3	3	3	3			
$(LR)(LR)$	$Q_{lequ}^{(1,3)}$	6	6	6	6	6				6	6			
Total		48	6	15	15	3	3	9	9	9	3	3	15	15

 $\mathbf{U}(2)_Q^3 \times \mathbf{MFV}_L$ 

$\mathbf{U}(2)_Q^3 \times \mathbf{MFV}_L$		$\mathcal{O}(1)$	$\mathcal{O}(V)$		$\mathcal{O}(V^2)$	$\mathcal{O}(\Delta V)$	$\mathcal{O}(Y_e)$	$\mathcal{O}(Y_e V)$		$\mathcal{O}(Y_e \Delta)$		$\mathcal{O}(Y_e \Delta V)$	
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	4	2	2	2								
$(RR)(RR)$	$Q_{eu}, Q_{ed}$	4				2	2						
$(LL)(RR)$	$Q_{lu}, Q_{ld}$	4				2	2						
	$Q_{qe}$	2	1	1	1								
$(LR)(RL)$	$Q_{ledq}$						1	1	1	1	1	1	1
$(LR)(LR)$	$Q_{lequ}^{(1,3)}$						2	2	2	2	2	2	2
Total		14	3	3	3	4	4	3	3	3	3	3	3



$$\mathbf{U}(2)_Q^3 \times \mathbf{U}(3)_{V,L}$$

$\mathbf{U}(2)_Q^3 \times \mathbf{U}(3)_{V,L}$		$\mathcal{O}(1)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta V)$	$\mathcal{O}(\Delta_\ell)$	$\mathcal{O}(\Delta_\ell V)$	$\mathcal{O}(\Delta_\ell \Delta)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	4	2 2	2			4	2 2	
$(RR)(RR)$	$Q_{eu}, Q_{ed}$	4				2 2	4		
$(LL)(RR)$	$Q_{\ell u}, Q_{\ell d}$	4				2 2	4		
	$Q_{qe}$	2	1 1	1			2	1 1	
$(LR)(RL)$	$Q_{\ell edq}$	1 1	1 1		1 1	1 1	1 1	1 1	1 1
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$	2 2	2 2		2 2	2 2	2 2	2 2	2 2
Total		17 3	6 6	3	3 3	7 7	17 3	6 6	3 3

$$\mathbf{U}(2)_Q^3 \times (\mathbf{U}(2)^2 \times \mathbf{U}(1)^2)_L$$

$\mathbf{U}(2)_Q^3 \times (\mathbf{U}(2)^2 \times \mathbf{U}(1)^2)_L$		$\mathcal{O}(1)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(V^3)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(X)$	$\mathcal{O}(\Delta V)$	$\mathcal{O}(XV)$	$\mathcal{O}(X\Delta)$	$\mathcal{O}(XV\Delta)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	8	4 4	8	2 2			4 4	4 4		
$(RR)(RR)$	$Q_{eu}, Q_{ed}$	8		4				8 8	4 4		
$(LL)(RR)$	$Q_{\ell u}, Q_{\ell d}$	8		4				8 8	4 4		
	$Q_{qe}$	4	2 2	4	1 1			2 2	2 2		
$(LR)(RL)$	$Q_{\ell edq}$		2 2	2 2		1 1	1 1	3 3	1 1	1 1	3 3
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$		4 4	4 4		2 2	2 2	6 6	2 2	2 2	6 6
Total		28	12 12	26 6	3 3	3 3	3 3	31 31	17 17	3 3	9 9

$$\mathbf{U}(2)_Q^3 \times \mathbf{U}(2)_L^2$$

$\mathbf{U}(2)_Q^3 \times \mathbf{U}(2)_L^2$		$\mathcal{O}(1)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(V^3)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta V)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	8	8 8	12 4	4 4		
$(RR)(RR)$	$Q_{eu}, Q_{ed}$	8					8 8
$(LL)(RR)$	$Q_{\ell u}, Q_{\ell d}$	8	4 4	4			4 4
	$Q_{qe}$	4	2 2	2			2 2
$(LR)(RL)$	$Q_{\ell edq}$	1 1	2 2	1 1		2 2	4 4
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$	2 2	4 4	2 2		4 4	8 8
Total		31 3	20 20	21 7	4 4	6 6	26 26

$$\mathbf{U}(2)_Q^3 \times \mathbf{U}(2)_{V,L}$$

$\mathbf{U}(2)_Q^3 \times \mathbf{U}(2)_{V,L}$		$\mathcal{O}(1)$	$\mathcal{O}(V_q)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta_\ell)$	$\mathcal{O}(V_q^2)$	$\mathcal{O}(V_q \Delta)$	$\mathcal{O}(V_q \Delta_\ell)$
$(LL)(LL)$	$Q_{\ell q}^{(1,3)}$	8	4 4		4	4		2 2
$(RR)(RR)$	$Q_{eu}, Q_{ed}$	8			4		4 4	
$(LL)(RR)$	$Q_{\ell u}, Q_{\ell d}$	8			4		4 4	
	$Q_{qe}$	4	2 2		2	2		1 1
$(LR)(RL)$	$Q_{\ell edq}$	2 2	2 2	2 2	1 1		2 2	1 1
$(LR)(LR)$	$Q_{\ell equ}^{(1,3)}$	4 4	4 4	4 4	2 2		4 4	2 2
Total		34 6	12 12	6 6	17 3	6	14 14	6 6

$U(2)_Q^3 \times U(1)_L^6$ 

$U(2)_Q^3 \times U(1)_L^6$		$\mathcal{O}(1)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta V)$	$\mathcal{O}(y)$	$\mathcal{O}(yV)$	$\mathcal{O}(y\Delta)$	$\mathcal{O}(y\Delta V)$
$(LL)(LL)$	$Q_{lq}^{(1,3)}$	12	6 6	6						
$(RR)(RR)$	$Q_{eu}, Q_{ed}$	12				6 6				
$(LL)(RR)$	$Q_{lu}, Q_{ld}$	12				6 6				
	$Q_{qe}$	6	3 3	3						
$(LR)(RL)$	$Q_{ledq}$						3 3	3 3	3 3	3 3
$(LR)(LR)$	$Q_{lequ}^{(1,3)}$						6 6	6 6	6 6	6 6
Total		42	9 9	9		12 12	9 9	9 9	9 9	9 9

 $U(2)_Q^3 \times U(1)_L^3$ 

$U(2)_Q^3 \times U(1)_L^3$		$\mathcal{O}(1)$	$\mathcal{O}(V)$	$\mathcal{O}(V^2)$	$\mathcal{O}(\Delta)$	$\mathcal{O}(\Delta V)$
$(LL)(LL)$	$Q_{lq}^{(1,3)}$	12	6 6	6		
$(RR)(RR)$	$Q_{eu}, Q_{ed}$	12				6 6
$(LL)(RR)$	$Q_{lu}, Q_{ld}$	12				6 6
	$Q_{qe}$	6	3 3	3		
$(LR)(RL)$	$Q_{ledq}$	3 3	3 3		3 3	3 3
$(LR)(LR)$	$Q_{lequ}^{(1,3)}$	6 6	6 6		6 6	6 6
Total		51 9	18 18	9	9 9	21 21

## D Group identities

In  $SU(2)$  the following identities hold:

$$\varepsilon^{ij}\varepsilon_{k\ell} = \delta^i_\ell\delta^j_k - \delta^i_k\delta^j_\ell \quad (\text{D.1})$$

using the convention  $\varepsilon_{12} = -\varepsilon^{12}$ .

In  $SU(N)$  the following identities hold:

$$t^{ai}{}_j t^{ak}{}_\ell = \frac{1}{2}\delta^i_\ell\delta^k_j - \frac{1}{2N}\delta^i_j\delta^k_\ell, \quad (\text{D.2})$$

$$f^{abc}t^{bi}{}_j t^{ck}{}_\ell = \frac{i}{2}\left(t^{ai}{}_\ell\delta^k_j - t^{ak}{}_j\delta^i_\ell\right), \quad (\text{D.3})$$

$$d^{abc}t^{bi}{}_j t^{ck}{}_\ell = \frac{1}{2}\left(t^{ai}{}_\ell\delta^k_j + t^{ak}{}_j\delta^i_\ell\right) - \frac{1}{N}\left(t^{ai}{}_j\delta^k_\ell + t^{ak}{}_\ell\delta^i_j\right), \quad (\text{D.4})$$

where the defining identity for the symmetric tensor is

$$t^a t^b = \frac{1}{2}\left[\frac{1}{N}\delta^{ab}\mathbf{1} + \left(d^{abc} + if^{abc}\right)t^c\right]. \quad (\text{D.5})$$

In the case of  $SU(2)$  there is no 3-index symmetric tensor and eq. (D.4) implies the identity

$$t^{ai}{}_\ell\delta^k_j + t^{ak}{}_j\delta^i_\ell = t^{ai}{}_j\delta^k_\ell + t^{ak}{}_\ell\delta^i_j. \quad (\text{D.6})$$

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