Improved Limits on Lepton-Flavor-Violating Decays of Light Pseudoscalars via Spin-Dependent $\mu \to e$ Conversion in Nuclei

Martin Hoferichter,[1] Javier Menéndez,[2] and Frederic Noël[1]

[1]*Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland*

[2]*Department of Quantum Physics and Astrophysics and Institute of Cosmos Sciences, University of Barcelona, 08028 Barcelona, Spain*

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Lepton-flavor-violating decays of light pseudoscalars, $P = \pi^0, \eta, \eta' \to \mu e$, are stringently suppressed in the standard model up to tiny contributions from neutrino oscillations, so that their observation would be a clear indication for physics beyond the standard model. However, in effective field theory such decays proceed via axial-vector, pseudoscalar, or gluonic operators, which are, at the same time, probed in spin-dependent $\mu \to e$ conversion in nuclei. We derive master formulas that connect both processes in a model-independent way in terms of Wilson coefficients and study the implications of current $\mu \to e$ limits in titanium for the $P \to \mu e$ decays. We find that these indirect limits surpass direct ones by many orders of magnitude.

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Introduction.—In the standard model (SM) of particle physics, the flavor of charged leptons is conserved apart from tiny corrections due to nonvanishing neutrino masses. Nonetheless, neutrino oscillations contribute to charged lepton-flavor-violating (LFV) decays suppressed by the ratio of the neutrino to the W-boson masses $(m_\nu/M_W)^4$, with resulting branching ratios of order 10^{-50}. Thus any observation of LFV in the charged sector would constitute a discovery of physics beyond the SM (BSM) [1–5]; see, e.g., Refs. [6–8] for reviews.

The leading limits on such LFV decays are obtained from $\mu \to e$ transitions, with $\mathrm{Br}[\mu \to e\gamma] < 4.2 \times 10^{-13}$ [9], $\mathrm{Br}[\mu \to 3e] < 1.0 \times 10^{-12}$ [10] for purely leptonic processes (all limits given at 90% confidence level), and [11,12]

$$\begin{aligned} \mathrm{Br}[\mu \to e, \mathrm{Ti}] &< 6.1 \times 10^{-13}, \\ \mathrm{Br}[\mu \to e, \mathrm{Au}] &< 7 \times 10^{-13}, \end{aligned} \tag{1}$$

for $\mu \to e$ conversion in the field of an atomic nucleus, with branching fractions normalized to the respective rate for nuclear capture [13]. (Reference [11] represents the final result by the SINDRUM-II experiment for $\mu \to e$ conversion in Ti, superseding the earlier limit $\mathrm{Br}[\mu \to e, \mathrm{Ti}] < 4.3 \times 10^{-12}$ [14].) While leptonic limits will improve at the MEG II [15] and Mu3e [16] experiments (and potentially beyond [17]), especially significant improvements up to 4 orders of magnitude beyond the present limits (1) are projected for $\mu \to e$ conversion at Mu2e [18] and COMET [19].

Independent constraints on $\mu \to e$ transitions can be obtained from LFV decays of light pseudoscalars, $P = \pi^0$, $\eta, \eta' \to \mu e$, for which the current limits read [20] (see Refs. [21–25])

$$\begin{aligned} \mathrm{Br}[\pi^0 \to \mu^+ e^-] &< 3.8 \times 10^{-10}, \\ \mathrm{Br}[\pi^0 \to \mu^- e^+] &< 3.2 \times 10^{-10}, \\ \mathrm{Br}[\pi^0 \to \mu^+ e^- + \mu^- e^+] &< 3.6 \times 10^{-10}, \\ \mathrm{Br}[\eta \to \mu^+ e^- + \mu^- e^+] &< 6 \times 10^{-6}, \\ \mathrm{Br}[\eta' \to \mu^+ e^- + \mu^- e^+] &< 4.7 \times 10^{-4}. \end{aligned} \tag{2}$$

In particular, Ref. [22] improves the previous limit [26] on the $\pi^0 \to \mu^- e^+$ channel by an order of magnitude, leading to three independent constraints on $\pi^0 \to \mu e$ all at the level of 10^{-10} [21–23]. Limits on the analogous η, η' decays are much weaker, but could be improved substantially at the planned JEF [27] and REDTOP [28] experiments.

In this Letter, we study the relation between LFV pseudoscalar decays (2) and $\mu \to e$ conversion limits (1). In an effective-field-theory (EFT) approach to LFV [29–36], only axial-vector, pseudoscalar, or gluonic operators can contribute to the pseudoscalar decays, with scalar and vector operators forbidden by parity. Accordingly, the responses for the relevant operators only give rise to so-called spin-dependent (SD) $\mu \to e$ conversion [33,37,38] which is not enhanced by the coherent sum over the entire nucleus—these operators probe the spins of the nucleons, which combine in spin-zero pairs due to the nuclear pairing interaction. In addition to this lack of coherence, weaker limits are expected compared to vector or scalar operators because SD responses vanish for nuclei with even number of protons and neutrons, which are spinless. Thus only nuclei with odd number of nucleons contribute at all. Moreover, controlling the nuclear structure for a nucleus as heavy as ^{197}Au is challenging, leaving in practice ^{47}Ti and ^{49}Ti, with low natural abundances of 7.44% and 5.41%, respectively, that further dilute the interpretation of the experimental limit (1). For these reasons, one might expect that limits derived from pseudoscalar decays could be competitive for these operators.

To address this question systematically, we derive master formulas that express the $P \to \mu e$ branching ratio and the $\mu \to e$ conversion rate in terms of the same effective Wilson coefficients, and provide all hadronic matrix elements and nuclear structure factors required for a model-independent comparison. Since $\mu \to e$ conversion and pseudoscalar decays probe different linear combinations of Wilson coefficients, we study which $P \to \mu e$ regions in parameter space are least subject to independent limits, and comment on the role of renormalization group (RG) corrections in closing the resulting flat directions.

Formalism.—The relevant operators up to dimension seven that can generate SD responses in $\mu \to e$ conversion are

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{SD}} = \frac{1}{\Lambda^2} \sum_{\substack{Y=L,R\\ q=u,d,s}} [&C_Y^{P,q} (\overline{e_Y}\mu)(\bar{q}\gamma_5 q) \\ &+ C_Y^{A,q}(\overline{e_Y}\gamma^\mu\mu)(\bar{q}\gamma_\mu\gamma_5 q) + C_Y^{T,q}(\overline{e_Y}\sigma^{\mu\nu}\mu)(\bar{q}\sigma_{\mu\nu}q)] \\ &+ \frac{i\alpha_s}{\Lambda^3}\sum_{Y=L,R}(\overline{e_Y}\mu)C_Y^{G\tilde{G}}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a + \text{H.c.},\end{aligned} \tag{3}$$

while the leading spin-independent (SI) contributions arise from the analogous scalar, vector, and gluon operators (with Wilson coefficients denoted by $C_Y^{S,q}$, $C_Y^{V,q}$, and C_Y^{GG} in the following). (In the SI case there is also a contribution from the dipole operator, which we do not need for the present analysis and thus omit for simplicity.) The projectors are introduced as $\overline{e_Y} = \bar{e}P_{\bar{Y}}$, with $Y \in \{L, R\}$ and $P_{L/R} = (\mathbb{1} \mp \gamma_5)/2$, to make explicit that the left- and right-handed components e_L and e_R decouple in the limit $m_e \to 0$, which we assume throughout this Letter. The BSM scale Λ is introduced to make the Wilson coefficients dimensionless.

In these conventions, the decay rate becomes

$$\text{Br}[P \to \mu^\mp e^\pm] = \frac{(M_P^2 - m_\mu^2)^2}{16\pi\Gamma_P M_P^3}\sum_{Y=L,R}|C_Y^P|^2, \tag{4}$$

where the Wilson coefficients and hadronic matrix elements are combined in

$$C_Y^P = \sum_q \frac{b_q}{\Lambda^2}\left(\pm C_Y^{A,q} f_P^q m_\mu - C_Y^{P,q}\frac{h_P^q}{2m_q}\right) + \frac{4\pi}{\Lambda^3}C_Y^{G\tilde{G}}a_P, \tag{5}$$

and the upper or lower sign applies to $\mu^\mp e^\pm$. The matrix elements f_P^q, h_P^q, a_P are defined by [39]

$$\begin{aligned}\langle 0|\bar{q}\gamma^\mu\gamma_5 q|P(k)\rangle &= ib_q f_P^q k^\mu, \\ \langle 0|m_q\bar{q}i\gamma_5 q|P(k)\rangle &= \frac{b_q h_P^q}{2}, \\ \langle 0|\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a|P(k)\rangle &= a_P,\end{aligned} \tag{6}$$

with dual field strength tensor $\tilde{G}^{\mu\nu}_a = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G^a_{\alpha\beta}$, $\epsilon^{0123} = +1$, and satisfy the Ward identity,

$$b_q f_P^q M_P^2 = b_q h_P^q - a_P, \tag{7}$$

while the matrix element of the tensor current vanishes. The numerical coefficients are $b_u = b_d = 1/\sqrt{2}$, $b_s = 1$, and M_P, m_μ, and m_q denote the pseudoscalar, muon, and quark masses, respectively. Phenomenologically, the $2 \times 3 \times 3 + 3 = 21$ parameters can be further reduced using isospin symmetry and neglecting strangeness and gluonic contributions to the pion matrix elements. This leaves as free parameters the pion decay constant F_π, the singlet and octet decay constants F^0, F^8, the corresponding mixing angles θ_0, θ_8, as well as gluon parameters a_0, θ_y. The explicit parametrization reads

$$\begin{aligned}&f_\pi^u = -f_\pi^d = \sqrt{2}F_\pi, \qquad f_\pi^s = 0, \qquad a_\pi = 0, \\ &f_\eta^u = f_\eta^d = \sqrt{\frac{2}{3}}F^8\cos\theta_8 - \frac{2}{\sqrt{3}}F^0\sin\theta_0, \\ &f_\eta^s = -\frac{2}{\sqrt{3}}F^8\cos\theta_8 - \sqrt{\frac{2}{3}}F^0\sin\theta_0, \\ &f_{\eta'}^u = f_{\eta'}^d = \sqrt{\frac{2}{3}}F^8\sin\theta_8 + \frac{2}{\sqrt{3}}F^0\cos\theta_0, \\ &f_{\eta'}^s = -\frac{2}{\sqrt{3}}F^8\sin\theta_8 + \sqrt{\frac{2}{3}}F^0\cos\theta_0, \\ &a_\eta = -a_0\sin\theta_y, \qquad a_{\eta'} = a_0\cos\theta_y,\end{aligned} \tag{8}$$

which determines the pseudoscalar matrix elements h_P^q via Eq. (7). Table I collects selected numerical values for these parameters.

A decomposition analogous to Eq. (4) applies to the rate for $\mu \to e$ conversion in nuclei; see Ref. [43] for the general form. In addition to nucleon matrix elements, these processes involving atomic nuclei depend on nuclear structure factors, which encode the structure of the

TABLE I. Numerical values for the axial-vector and gluonic matrix elements contributing to the $P \to \mu e$ decays, from a phenomenological extraction via η, η' transition form factors [40] and the recent lattice-QCD calculation [41] ($\overline{\rm MS}$ scale $\mu = 2$ GeV). The last line indicates the value of a_P extracted from f_P^u in the Feldmann-Kroll-Stech (FKS) scheme [42]. We use $F_\pi = 92.28$ MeV [20].

	π	η		η'	
		Ref. [40]	Ref. [41]	Ref. [40]	Ref. [41]
$b_u f_P^u/F_\pi$	1	0.80	0.77	0.66	0.56
$b_d f_P^d/F_\pi$	−1	0.80	0.77	0.66	0.56
$b_s f_P^s/F_\pi$	0	−1.26	−1.17	1.45	1.50
a_P (GeV^3)	0	⋯	−0.017	⋯	−0.038
$a_P^{\rm FKS}$ (GeV^3)	0	−0.022	−0.021	−0.056	−0.048

many-body nuclear state. These are often included in terms of a multipole decomposition [44–48], and two-body corrections can be addressed in chiral EFT; see Refs. [49–55]. As a final step, the $\mu \to e$ conversion rate involves atomic wave functions, describing the bound-state physics of the initial muon in the $1S$ state of the atom as well as the overlap with the final-state electron. For the SI process, these effects have traditionally been parametrized in terms of overlap integrals [56], where effectively only the leading M multipole is kept, convolved with the solution of the Dirac equation for the electromagnetic potential of the nuclear charge distribution [57]. Keeping only scalar and vector operators, the SI branching fraction becomes

$$\mathrm{Br}_{\rm SI}[\mu \to e] = \frac{4m_\mu^5}{\Gamma_{\rm cap}} \sum_{Y=L,R} \left| \sum_{\substack{N=p,n\\ \mathcal{O}=S,V}} \bar{C}_Y^{\mathcal{O},N} \mathcal{O}^{(N)} \right|^2, \qquad (9)$$

where $\Gamma_{\rm cap}$ is the capture rate, $\mathcal{O}^{(N)}$ are the (dimensionless) overlap integrals [56], and

$$\bar{C}_Y^{S,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{S,q} \frac{m_N}{m_q} f_q^N + \frac{4\pi}{\Lambda^3} C_Y^{GG} a_N,$$
$$\bar{C}_Y^{V,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{V,q} f_{V_q}^N \qquad (10)$$

subsume Wilson coefficients and nucleon matrix elements. At leading order in the momentum expansion only the scalar/vector couplings $f_q^N/f_{V_q}^N$ enter [58–63], while the gluon operator can be expressed via the trace anomaly of the energy-momentum tensor [64]. As the SI contribution only affects the pseudoscalar decays indirectly, via RG and relativistic corrections, it suffices to consider the leading contributions (10) in this Letter; see Refs. [43,65–67] for two-body and momentum-dependent corrections as well as other nuclear multipoles.

TABLE II. Overlap integrals for ^{48}Ti compared to Ref. [56]. We find $Z_{\rm eff} = 17.65$, using the charge distribution from Ref. [57] in the solution of the Dirac equation (for ^{27}Al we have $Z_{\rm eff} = 11.64$). Methods 1 and 3 differ mainly in the estimate of the neutron distribution.

	$S^{(p)}$	$V^{(p)}$	$S^{(n)}$	$V^{(n)}$
Ref. [56], method 1	0.0368	0.0396	0.0435	0.0468
Ref. [56], method 3	0.0371	0.0399	0.0462	0.0495
This Letter	0.039		0.044	

Approximating the electron and muon wave function by a plane wave and its average value in the nucleus, respectively, the overlap integrals become

$$S^{(N)} = V^{(N)} = \frac{(\alpha Z)^{3/2}}{4\pi} \left(\frac{Z_{\rm eff}}{Z}\right)^2 \mathcal{F}_N^M(m_\mu^2), \qquad (11)$$

where $\mathcal{F}_N^M(m_\mu^2)$ denote the structure factors for the M multipole, evaluated at momentum transfer $\mathbf{q}^2 = m_\mu^2$ and normalized to the number of protons (Z) or neutrons (N), $\mathcal{F}_p^M(0) = Z$, $\mathcal{F}_n^M(0) = N$, and $Z_{\rm eff}$ parametrizes the wave-function average [56]. For the numerical analysis we use nuclear structure factors obtained using the nuclear shell model [68,69] with the code ANTOINE [68,70]. Our calculations for Ti isotopes use the KB3G interaction [71] in a configuration space consisting of the $0f_{7/2}$, $1p_{3/2}$, $1p_{1/2}$, and $0f_{5/2}$ proton and neutron orbitals, with a ^{40}Ca core. For ^{27}Al we use the USDB interaction [72] and the $0d_{5/2}$, $0d_{3/2}$, $1s_{1/2}$ configuration space with an ^{16}O core [73]. In particular, for ^{48}Ti Table II compares the approximation (11) to Ref. [56], showing reasonable agreement. Note that differences at this level are even expected, as we rely on the neutron distribution predicted by the nuclear shell model, not the assumptions from Ref. [56]. For this Letter the approximation (11) thus proves sufficient; see Ref. [43] for the full analysis.

Under the same assumptions, the decay rate for SD $\mu \to e$ conversion can be written as

$$\mathrm{Br}_{\rm SD}[\mu \to e] = \frac{4m_\mu^5 \alpha^3 Z^3}{\pi \Gamma_{\rm cap}(2J+1)} \left(\frac{Z_{\rm eff}}{Z}\right)^4 \times \sum_{\substack{Y=L,R\\ \tau=\mathcal{L},\mathcal{T}}} [C_Y^{\tau,00} S_{00}^\tau + C_Y^{\tau,11} S_{11}^\tau + C_Y^{\tau,01} S_{01}^\tau], \qquad (12)$$

where J is the spin of the nucleus, S_{ij}^τ are the transverse ($\mathcal{T}$) and longitudinal ($\mathcal{L}$) structure factors [55] (corresponding to the multipoles Σ' and Σ'', respectively), and the coefficients receive contributions from all operators in Eq. (3). Defining

$$\bar{C}_Y^{P,N} = \frac{1}{\Lambda^2}\sum_q C_Y^{P,q}\frac{m_N}{m_q}g_5^{q,N} - \frac{4\pi}{\Lambda^3}C_Y^{G\tilde{G}}\tilde{a}_N,$$
$$\bar{C}_Y^{A,N} = \frac{1}{\Lambda^2}\sum_q C_Y^{A,q}g_A^{q,N},$$
$$\bar{C}_Y^{T,N} = \frac{1}{\Lambda^2}\sum_q C_Y^{T,q}f_{1,T}^{q,N}, \tag{13}$$

with nucleon matrix elements at vanishing momentum transfer in the conventions of Ref. [55],

$$\langle N|\bar{q}\gamma^\mu\gamma_5 q|N\rangle = g_A^{q,N}\langle N|\bar{N}\gamma^\mu\gamma_5 N|N\rangle,$$
$$m_q\langle N|\bar{q}i\gamma_5 q|N\rangle = m_N g_5^{q,N}\langle N|\bar{N}i\gamma_5 N|N\rangle,$$
$$\langle N|\bar{q}\sigma^{\mu\nu}q|N\rangle = f_{1,T}^{q,N}\langle N|\bar{N}\sigma^{\mu\nu}N|N\rangle,$$
$$\langle N|\frac{\alpha_s}{4\pi}G_{\mu\nu}^a\tilde{G}_a^{\mu\nu}|N\rangle = \tilde{a}_N\langle N|\bar{N}i\gamma_5 N|N\rangle, \tag{14}$$

we have

$$C_Y^{\mathcal{T},ij} = [\bar{C}_Y^{A,i}(1+\delta')^i \pm 2\bar{C}_Y^{T,i}] \times (i \leftrightarrow j),$$
$$C_Y^{\mathcal{L},ij} = \left[\bar{C}_Y^{A,i}(1+\delta'')^i - \frac{m_\mu}{2m_N}\bar{C}_Y^{P,i} \pm 2\bar{C}_Y^{T,i}\right] \times (i \leftrightarrow j), \tag{15}$$

where the upper or lower sign refers to $Y = L$ or R. For all coefficients $\bar{C}^N$ the isoscalar and isovector components are defined as

$$\bar{C}^0 = \frac{\bar{C}^p + \bar{C}^n}{2}, \qquad \bar{C}^1 = \frac{\bar{C}^p - \bar{C}^n}{2}, \tag{16}$$

and δ', δ'' encode the corrections from the induced pseudoscalar form factor, the axial radius, and two-body currents [55]—note that they are not included in the S_{ij}^τ structure factors. At $\mathbf{q}^2 = m_\mu^2$ they take the values $\delta' = -0.28(5)$, $\delta'' = -0.44(4)$. Especially the two-body corrections lead to a sizable reduction of the $\mu \to e$ matrix elements, as also well established for nuclear β decays [80], and thus need to be included. The uncertainties are derived from the corresponding low-energy constants [81,82] and the convergence properties of the chiral expansion, as detailed in Ref. [55], and also cover nuclear shell-model uncertainties [73].

The nucleon matrix elements are related by the Ward identity,

$$g_A^{q,N} = g_5^{q,N} - \frac{\tilde{a}_N}{2m_N}, \tag{17}$$

in close analogy to Eq. (7). For the isovector combination we also keep the momentum-dependent correction from the induced pseudoscalar form factor, which amounts to shifting $g_A^{u,p}$ and $g_A^{d,p}$ by $\mp g_A/2 \times m_\mu^2/(M_\pi^2 + m_\mu^2)$, respectively, when applying Eq. (17) (the neutron couplings are obtained assuming isospin symmetry). Once the value of $\tilde{a}_N$ is determined, all $g_5^{q,N}$ thus follow from the $g_A^{q,N}$, for which we use the values from Refs. [20,55,83] (in reasonable agreement with recent lattice-QCD calculations [84–86]). Contrary to a_P, for $\tilde{a}_N$ only estimates based on large-N_c arguments are available so far [87,88], while lattice-QCD techniques employed for the QCD θ term could allow for an *ab initio* determination [89,90]. We use the estimate

$$\tilde{a}_N = -2m_N g_A^{u,0} = -0.39(12)\ \text{GeV}, \tag{18}$$

with $g_A^{u,0} = (g_A^{u,p} + g_A^{u,n})/2$, as can be derived in analogy to a_P^{FKS} in Table I [73], and assign a 30% uncertainty motivated by $1/N_c$ corrections. The tensor coefficients [91,92] are not needed as the tensor operator does not contribute to the pseudoscalar decays.

Finally, the operators of interest for $P \to \mu e$ could also contribute to $\text{Br}_{\text{SI}}[\mu \to e]$ via relativistic corrections, in analogy to the SI contribution that arises from the tensor operator at $\mathcal{O}(1/m_N)$ [33]. However, given that the matrix element of the tensor operator in $P \to \mu e$ vanishes, such corrections are suppressed further than could be overcome by the coherent enhancement of the SI response.

Limits on $P \to \mu e$.—In general, pseudoscalar decays (4) and SD $\mu \to e$ conversion (13) are not sensitive to the same linear combination of Wilson coefficients. Therefore, the translation of limits depends on the underlying BSM scenario as parametrized by the Wilson coefficients $C_Y^{A,q}$, $C_Y^{P,q}$, $C_Y^{G\tilde{G}}$. In the special case where only a single linear combination of Wilson coefficients contributes, the transition is immediate. Table III shows the results if the triplet, octet, or singlet components of $C_Y^{A,q}$ or $C_Y^{P,q}$ are dominant, together with the case in which only $C_Y^{G\tilde{G}}$ is nonvanishing. The octet, singlet, and gluonic operators do not contribute to $\pi^0 \to \mu e$, nor do the triplet operators to $\eta, \eta' \to \mu e$, so that considering all these flavor combinations should provide a realistic assessment of the sensitivities:

$$\text{Br}[\pi^0 \to \mu e] \lesssim 4 \times 10^{-17},$$
$$\text{Br}[\eta \to \mu e] \lesssim 4 \times 10^{-12},$$
$$\text{Br}[\eta' \to \mu e] \lesssim 5 \times 10^{-13}. \tag{19}$$

To derive rigorous limits requires a scan over Wilson coefficients to minimize the effect in $\mu \to e$ conversion while retaining a sizable $P \to \mu e$ rate. (We set $C_L = C_R$ throughout, as left- and right-handed components do not interfere in either rate.) Moreover, theory uncertainties due to the hadronic and nuclear matrix elements need to be taken into account. To obtain robust limits, we take the meson matrix elements either from the phenomenological

TABLE III. Limits for $\mathrm{Br}[P \to \mu e] \equiv \mathrm{Br}[P \to \mu^+ e^- + \mu^- e^+]$ that follow from $\mathrm{Br}[\mu \to e, \mathrm{Ti}] < 6.1 \times 10^{-13}$ assuming the dominance of a single Wilson coefficient. $C_Y^{A,i}$, $C_Y^{P,i}$, $i = 3$, 8, 0, refer to triplet, octet, and singlet components, respectively. In all cases we take $C_L = C_R$. We show the worst limits obtained when scanning over the two sets of matrix elements from Table I, the couplings $g_A^{q,N}$ from Refs. [83–85], and $\tilde{a}_N$ including a 30% error. The η and η' limits for $C_Y^{P,i}$ are sensitive to the uncertainty of $\tilde{a}_N$; we show the weakest limit obtained within its assigned error, but note that for the central value the limits are stronger by an order of magnitude.

	π^0	η	η'
$C_Y^{A,3}$	1.3×10^{-17}	...	...
$C_Y^{A,8}$	...	1.5×10^{-17}	4.0×10^{-20}
$C_Y^{A,0}$	...	2.9×10^{-19}	2.1×10^{-19}
$C_Y^{P,3}$	4.1×10^{-17}	...	...
$C_Y^{P,8}$	...	1.6×10^{-12}	2.1×10^{-14}
$C_Y^{P,0}$	...	4.1×10^{-12}	5.4×10^{-13}
$C_Y^{G\tilde{G}}$	...	5.8×10^{-15}	4.7×10^{-16}

or the lattice-QCD determinations quoted in Table I, similarly for the couplings $g_A^{q,N}$ from Refs. [83–85], and for $\tilde{a}_N$ as well δ', δ'' we include the uncertainties as given above. All quoted limits then refer to the worst limit obtained under this variation of the hadronic and nuclear input.

Equation (13) shows that each multipole in the transverse and longitudinal responses is squared separately, which in ^{47}Ti ($L = 1, 3, 5$) and ^{49}Ti ($L = 1, 3, 5, 7$) leads to a total of $2 \times 3 + 2 \times 4 = 14$ positive definite quantities. Accordingly, the only way to tune the rate to zero is to consider the couplings directly to protons and neutrons. Such a cancellation occurs at

$$C_Y^{A,u} = C_Y^{A,d}, \qquad C_Y^{A,s} = -\frac{2C_Y^{A,u} g_A^{u,0}}{g_A^{s,N}},$$
$$\frac{C_Y^{P,u}}{m_u} = \frac{C_Y^{P,d}}{m_d}, \qquad \frac{C_Y^{P,s}}{m_s} = \frac{4\pi}{\Lambda} C_Y^{G\tilde{G}} \frac{2g_A^{u,0}}{g_A^{u,0} - g_A^{s,N}}. \tag{20}$$

Since the conditions not involving strangeness remove any isovector contribution, this implies that for this choice of Wilson coefficients $\mathrm{Br}[\pi^0 \to \mu e]$ vanishes as well. In this case, the limit is thus protected against accidental cancellation, and a scan over the parameter space establishes

$$\mathrm{Br}[\pi^0 \to \mu e] < 1.2 \times 10^{-13} \tag{21}$$

as a rigorous limit. For η, η' a nonvanishing contribution remains, but such fine-tuned solutions are not viable due to RG corrections. As an example, we consider the dimension-six contribution from $C_Y^{A,u} = C_Y^{A,d}$. If generated at a

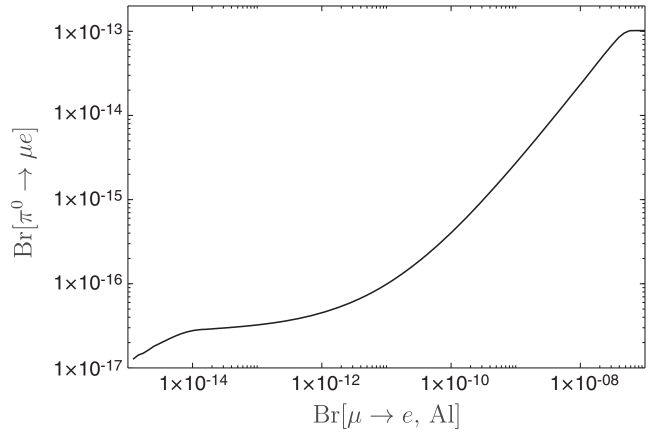

FIG. 1. Limits on $\mathrm{Br}[\pi^0 \to \mu e]$ derived from $\mathrm{Br}[\mu \to e, \mathrm{Ti}]$ [11] in combination with a future limit on $\mathrm{Br}[\mu \to e, \mathrm{Al}]$. Already a moderate precision for the latter suffices to substantially reduce cancellations that otherwise dilute the limit from Eq. (19) to Eq. (21).

high scale Λ above the electroweak scale M_W, already the one-loop QED corrections below M_W produce a vector operator [32,33],

$$C_Y^{V,q} \simeq -3Q_q \frac{\alpha}{\pi} \log\frac{M_W}{m_N} C_Y^{A,q}, \tag{22}$$

with quark charges $Q_u = 2/3$, $Q_d = -1/3$, and thus a contribution to the SI rate (10). This indirect constraint gives

$$\mathrm{Br}[\eta \to \mu e] < 3.8 \times 10^{-18},$$
$$\mathrm{Br}[\eta' \to \mu e] < 9.1 \times 10^{-20}, \tag{23}$$

and thus excludes the solution via $C_Y^{A,q}$. Therefore, a fine-tuning of Wilson coefficients can relax the limits (19), but, realistically, only by a few orders of magnitude. Moreover, the cancellations that arise from the interference of isoscalar and isovector contributions can be substantially reduced by considering other $\mu \to e$ targets. Figure 1 illustrates this for $\mathrm{Br}[\pi^0 \to \mu e]$ as a function of a future limit for $\mu \to e$ conversion in Al in combination with the current Ti constraint.

Conclusions.—In this Letter, we studied the connection between LFV decays of the light pseudoscalars $P = \pi^0, \eta, \eta'$ and $\mu \to e$ conversion in nuclei. The EFT approach shows that up to dimension seven only a few operators—axial-vector, pseudoscalar, and gluonic ones—contribute to the pseudoscalar decays, which at the same time can mediate the $\mu \to e$ conversion process albeit only by coupling to the nuclear spin. We derived master formulas for both processes to quantify their interplay, including all required hadronic matrix elements and the nuclear responses for Ti. Despite the lack of coherent enhancement for the spin-dependent response, we found that, in general, the indirect limits for $P \to \mu e$ as derived from the current $\mu \to e$ conversion limit in Ti surpass the direct ones by many orders of magnitude. Fine-tuning Wilson coefficients can relax these limits to some extent, especially for η, η', but RG corrections curtail the amount of cancellations. The indirect limits presented here will further advance in the future with forthcoming measurements of $\mu \to e$ conversion in Al at the Mu2e and COMET experiments.

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