Research paper

Foundations for future math achievement: Early numeracy, home learning environment, and the absence of math anxiety

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ABSTRACT

Background: Mathematics achievement is pivotal in shaping children's future prospects. Cognitive skills (numeracy), feelings (anxiety), and the social environment (home learning environment) influence early math development.
Method: A longitudinal study involved 85 children (mean age T1 = 6.4 years; T2 = 7.9) to explore these predictors holistically. Data were collected on early numeracy skills, home learning environment, math anxiety, and their impact on various aspects of math.
Results: The study found that early numeracy skills, home learning environment, and math anxiety significantly influenced math school achievement. However, they affected written computation, sequences, and comparisons differently. Early numeracy skills strongly predicted overall achievement and comparison subtest performance.
Conclusion: These findings underscore the substantial role of math anxiety and home learning environment in children's math achievement. The study emphasizes the need to consider the selective impacts of these factors in future research, shedding light on the multifaceted nature of mathematics achievement determinants.

1. Introduction

Math achievement is crucial for scholastic attainment and everyday and professional life prospects [1,2]. In addition, the continuous further development of technology requires good math skills [3]. Therefore, it is essential to better understand the determinants of math development. The development of math skills begins early in life. It is influenced by factors such as home learning environment, parenting style, the child's interests, feelings and attitudes, and innate and learned abilities (see [4], for a review). Numerous studies on the influence of these variables on math development have been published, specifically addressing predictors of math performance and development, including cognitive skills such as early numeracy skills [5,6], feelings such as math anxiety [7,8], or home learning environment [9,10]. Finally, cognitive abilities—such as working memory—are the most robust predictor for math learning [11–13].

All the aforementioned predictors are significantly related to math achievement and essential for math development. The present longitudinal study aims to consider the influences of early numeracy skills, home learning environment assessed in kindergarten, and math anxiety

on later second-grade math achievement while controlling for working memory. Certainly, these are not the only factors that influence mathematics achievements. They were selected due to their prominence in the existing literature, and because these factors provide multiple, diverse perspectives on mathematics learning and development. Namely, early numeracy skills and working memory tell us something about the role of children's innate capacity on math achievement. If we think of early numeracy skills and working memory as standing in for "nature" in nature-nurture debate, then investigating home learning environment, in contrast, provides a view into the role of "nurture." Finally, including mathematics anxiety can provide insight into the affective side of mathematics learning and development. In the following, we will overview previous research regarding early numeracy skills, home learning environment, math anxiety, and the relationship between those constructs and math achievement.

Children can demonstrate intuitive early numeracy skills at an early stage, such as processing small numerosities to count or establishing first numerical relationships even before they enter preschool [14]. In later development, early numeracy skills serve as building blocks for acquiring basic arithmetic and other more advanced math skills (see

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[15] for a developmental model of early numerical skills). Early numeracy skills are often investigated by tasks requiring children to compare magnitudes or quantities [16] or number line estimation (i.e., indicating the spatial position of a number on a given number line). These tasks can be carried out in symbolic (using Arabic digits; [17,18]) but also in non-symbolic format (using dot patterns or sets of other objects; [19]) or require the mapping of one format onto the other (e.g., conceptual subitizing; [20,21]). Importantly, such early numeracy skills are strong predictors of later mathematics achievement longitudinally [15,22–24], directly [25,26] and indirectly [27].

Of course, early numeracy skills develop in a context. Research suggests that children's home learning environment plays a decisive role in cognitive development in general [28,29] and numerical development in particular—especially in early childhood (e.g., [30,31]). Numerical henceforth HLE is defined as the mathematics-specific learning environment at home, comprising family activities including numbers, quantities, and measurements; more broadly, home learning environment includes attitudes towards mathematics and model behavior of parents [32,33]. The frequency of such numerical activities within a family appears positively related to children's math skills [9]. Similarly, children's prior numerical knowledge in kindergarten positively relates to the HLE [34]. Additionally, an intervention study indicated that enriching the numerical HLE improved early numeracy skills in pre-schoolers [35].

Within the field of research on HLE, a distinction is sometimes made between a formal home learning environment (i.e., Formal HLE) and an informal home learning environment (Informal HLE; [34,36]). Formal HLE consists of activities that specifically target early numeracy skills such as counting or magnitude understanding, which were observed to underlie later math development [36] by the frequency being predictive for children's growth of math skills [10,37] and even after controlling for children's numerical skills at baseline (see [38], for a review). Informal HLE, however, includes activities such as playing games requiring the processing of numbers (e.g., through using dice) or involving a numerical strategy, thus indirectly targeting numerical learning [36]. Informal HLE has been shown to predict non-symbolic numeracy skills (e.g., [10]), arithmetic fluency, and math achievement more generally in kindergarten, first, and second year of primary school [36]. However, recent meta-analyzes found that although the effects are positive, they remain small (Cohen's $d = 0.18$; [39]); and that the variation in the relationship between the HLE and children's math achievement cannot be attributed to a single feature [40].

In addition to more cognitive and environmental factors, we wished to account for the role of affect in math learning and development. Interest in math anxiety has increased in the last 30 years ([41,42]; for a summary, see [43]). Math anxiety is tension when a person is in a numerical context. It leads to avoidance of numerical/math activities, which in the long run further increases math anxiety to peak in high school years [44]. In severe cases, it has been shown to lead to failure or school phobia accompanied by psychosomatic complaints [45,46]. Likewise, math anxiety was observed to harm math achievement, not only because of avoidance but also because anxiety-related thoughts block attention and working memory capacities. Thus, cognitive resources for math processing are reduced [47,48]. This negative influence was found in different outcome measures such as processing of numerical magnitudes [49] calculation and math fluency skills [50,51], grades [52], but also performance in standardized math tests [53], as well as engagement with math [54]. Although it was believed that children remain unaffected by their academic beliefs and attitudes [55], in part because children are typically overly optimistic about their performance [56,57], research demonstrated that math anxiety may be present as early as first grade [58].

Early numeracy and home learning environment positively influence the development of math skills—in contrast, math anxiety negatively influences math skills development. Since these factors were considered in isolation in previous research, it is hard to evaluate potential selective influences of these factors on math achievement in general or on sub-dimensions (e.g., arithmetic) of math achievement. Because these factors interact in everyday life, the present longitudinal study considered influences of early numeracy skills, home learning environment, and math anxiety on children's development of math skills. To do so, 85 children were followed from kindergarten to 2nd grade. Moreover, as math achievement is a multidimensional concept and the chosen predictive factors were observed to correlate selectively with different subdimension [59], we evaluated influences of early numeracy skills, home learning environment, as well as math anxiety not only on math achievement in general but also on the three sub-dimensions of i) written calculation, ii) sequences, and iii) comparisons. In all analyzes, we controlled for working memory influences as an established domain-general predictor of mathematics achievement [60,61].

The present study was exploratory. The statistical approach we used was Bayesian multi-model inference to investigate which models, including which variables, are best predictors of math achievement and math achievement subdimensions. We expected that previously shown positive predictors (mostly symbolic skills and home learning environment; see Fig. 1) would be included in the model and that math anxiety would be a negative predictor for math achievement. As for the sub-dimensions, we tentatively hypothesized that math anxiety would most affect arithmetic skills.

2. Method

2.1. Participants

The sample consisted of 85 children recruited from different kindergartens in rural and urban areas near Bern, Switzerland. For 69.2 % of the participants, the spoken language at home was Swiss German, 10 % spoke other languages with both parents, and 12 % were bilinguals (information missing for 9 % of children). However, all children were fluent enough in Swiss German and Standard German understand task instructions. The sample's mean age at time point 1 (henceforth T1) was $M = 6.42$ years (SD = 0.31 years), including 46 boys and 42 girls. There was no significant difference in age between boys ($M = 6.46$ years) and girls ($M = 6.43$ years, t (156) = −0.47, p = .54). At the second measurement time point (henceforth T2), mean age was $M = 7.98$ years (SD = 0.31 years). Data on all predictor variables were collected at the first time point, whereas on the second time, data on math anxiety and a week after that the outcome variable of math achievement was collected (see Table 1 for descriptive data). Written informed consent was obtained from parents, whereas oral informed assent was obtained from children, the school directory, and teachers prior to the start of the study.

2.2. Measures

2.2.1. Early numeracy skills

Early numeracy skills in kindergarten were assessed using a *number line estimation*, a *magnitude comparison*, as well as a *conceptual subitizing* task. The computerized number line estimation task (the Number-to-Position version by [62], adapted by [63]) was administered in a symbolic and a non-symbolic version. In the symbolic version, children were asked to estimate the spatial location of 22 Arabic numbers on an otherwise empty number line ranging from 1 to 100. For the non-symbolic version, a racing car was introduced—as a cover story——and filled with 22 different numbers of drops of gasoline (reflecting 22 items as in the symbolic task version). Children were asked to point on the line (which began at one drop and ended at 100 drops) how far the car would go with the respective number of drops of gasoline. Each child's linear fit score (R2) indicated how closely their estimations matched a linear function on each task; the scores ranged from 0 (no fit at all) to 1 (precisely linear).

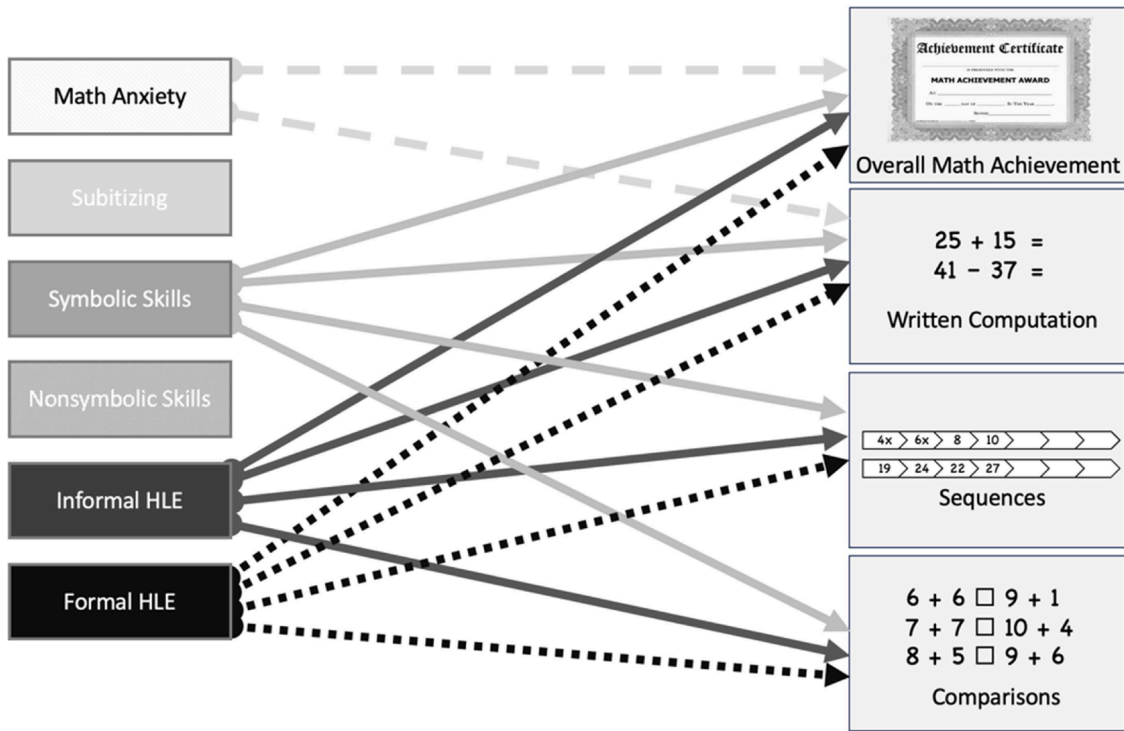In the magnitude comparison task (based on [62] and adapted by

Fig. 1. Hypothesized relations between outcome and predictor variables.

Table 1
Descriptive statistics.

	M	SD	Range
Age at T1	6.42	0.31	5.86–7.29
Working Memory (correct trials)	3.65	3.42	0.00–13.00
Early Numeracy Skills			
Number Line symbolic (linear fit)	0.46	0.28	0.00–0.97
Number Line non-symbolic (linear fit)	0.66	0.19	0.00–0.95
Magnitude Comparison symbolic (proportion correct)	0.73	0.16	0.37–1.00
Magnitude Comparison non-symbolic (proportion correct)	0.65	0.16	0.47–0.83
Conceptual Subitizing (reaction time ms)	67.99	31.62	13.81–180.00
Math Anxiety			
Concern	9.66	5.52	0.00–26.00
Sadness	10.07	4.68	0.00–24.00
Home Learning Environment (HLE)			
Informal HLE (%)	60.00	26.70	0.00–100.00
Formal HLE	23.55	10.13	1.00–47.00
Mathematics Achievement			
Written Computation (accuracy)	6.35	2.39	2.00–12.00
Sequences (correct trials)	10.77	2.74	4.00–15.00
Equations (correct trials)	17.46	5.47	4.00–35.50

Note. Conceptual Subitizing was reversed prior to z-standardization; HLE = Home Learning Environment.

[63]), children had to decide which one of two Arabic two-digit numbers presented on a laptop screen had the larger magnitude. Children had to press either a left or a right external response button to indicate that the right or left Arabic number was larger. The task started with six practice trials followed by 33 experimental trials, and the position of the higher number was randomized. Overall comparison accuracy (in percent correctly solved items) was used as the analysis's dependent variable.

Conceptual Subitizing was assessed using pictures of the faces of two different dice. Children had to tell the number of points on both dice without counting them (adding the points on both dice up). They were asked to answer verbally and as quickly as possible and to give only one answer. After their answer was given, the next picture was shown. The task started with one practice trial followed by nine experimental trials. The reaction time of the task was z-transformed and reversed (such that a higher number represents a higher performance) and used for the analysis.

2.2.2. Working memory

We adapted the subtest Matrix from the Working Memory Test Battery for Children (WMTB-C; [64]) for working memory. A four-by-four matrix was displayed on the screen to the children. During each trial, children were asked to recall which matrix squares became black and retrieve the sequence in reverse order on a similar matrix. As part of the experimental session, a span of two colored squares was used, and the level increased when at least four of the six trials on each span were correctly recalled. The number of accurately solved trials was taken as a score for working memory.

2.2.3. Math anxiety

Concern and sadness related to math events were assessed employing the German version of the Mathematics Attitude and Anxiety Questionnaire (FRA; [65]). The internal consistency of these tests varies between $r = 0.65$ and 0.85 depending on the different subscales for children in kindergarten, generally showing good reliability [65]. Children were asked to give answers on a pictorial 5-point scale (from 0 meaning not at all to 4 meaning very much) on four questions (covering the four scales of self-rating, liking, anxiety, and sadness) concerning seven subsequent math-related situations, this means math in general, written/mental calculations, easy/complex calculations, math homework, and listening and understanding during math lessons. The scores for each question ranged from 0 to 4. Thus, for each scale from 0 to 28. Since the scales "concern" and "sadness" are used to measure math anxiety, only the sum score of these two scales went into analysis.

2.2.4. Home learning environment

Home learning environment was assessed using a translation of the self-rating questionnaire from Skwarchuk et al. [10] with 58 items

answered in writing by children's parents at the first measurement. The answers to 13 questions identified formal home learning environment, which reflects the direct promotion of math skills in the family. Parents had to specify how often they accomplish several activities with numbers with their children. A sum score was computed across the 13 items and used as a dependent variable in the analyzes.

Reflecting informal home learning environment, indirect support in games with math content (e.g., using dice) was identified by a list of 25 games for 3- to 6-year-old children. The list included ten numerical games, ten non-numerical games, and five fictional games (again, according to [10]). Parents had to indicate which games they recognized from their own home or children without guessing and looking them up on the internet. The dependent variable is calculated by subtracting the number of chosen fictitious games from the number of chosen real numerical games, dividing by 10, and multiplying by 100, according to Skwarchuk and colleagues [10].

2.2.5. Mathematics achievement

Mathematics achievement was assessed using three subtests of a standardized and curriculum-based paper and pencil test (Heidelberger Rechentest [59]). The group testing lasted for approximately 15 min. The subtests used were (i) comparisons, (ii) sequences, and (iii) written calculation. We chose these specific subtests because of their content validity; they represent a typical selection from the spectrum of basic arithmetic operations and equation tasks used in primary schools in German-speaking countries. Moreover, there is a high and specific agreement with the school grade in mathematics ($r = 0.67$, see [59]). Thus, this battery and, specifically, these subtests are widely used to assess math achievement in German-speaking countries.

In the comparisons, subtest children had to indicate whether a number or brief arithmetic term on the left side of a page was $\langle$, =, or $\rangle$ than a number or term on the right side of the page by writing the respective sign in a prespecified field. There were 40 items, and children had to solve as many items as possible within two minutes. An accuracy score was computed according to the test manual by subtracting the number of erroneously solved trials from the number of correctly solved ones and then dividing the result by 2.

In the sequences subtest, children had to find the math rule underlying a sequence of four numbers to continue the respective sequence by three more numbers i.e., filling three blank spaces at the end of each trial or in other words identifying and extending the logical relationship between numbers (e.g., "3 4 5 6 _ _ _"). The dependent variable used in the analyzes was the sum of correctly solved trials per the test manual.

In the written calculation subtest, children had to solve as many simple arithmetic problems as possible, including additions and subtractions, taught in 2nd grade, within a time limit of two minutes and 30 s. Following the test manual, the dependent variable was the sum of correctly solved items with a maximum score of 12.

2.3. Procedure

This study was part of a larger research project approved by the Ethics Committee of the University of Bern, which follows APA guidelines and the principles of the Declaration of Helsinki. At T1, at the end of the kindergarten year, all tests were completed within two sessions lasting 45 min each. Computer-based tasks had to be completed individually on a laptop (Lenovo 3000 N200 with a 15.4 WXGA screen, aspect ratio of 16:10, run at a resolution of 1280×800 pixels) located in a separate room. Children also completed the math anxiety questionnaire individually with the experimenter in a separate room. After completing testing, children were rewarded with a small gift (e.g., a toy car, a pencil, and similar). T2 took place 18 months after T1 when children were at the beginning of second grade. Mathematics achievement was tested in classroom groups within a regular school hour. Again, participating children were rewarded with a small gift upon completion. As scaling was not comparable across tasks, all variables

were z-standardized prior to analyzes. A joint measure of symbolic and non-symbolic number line estimation was computed by adding the z-scores to a sum score. Similarly, the overall mathematics achievement score was computed by summing up the subtests' z-scores.

2.4. Data analysis

According to our research questions and addressing our hypotheses, we statistically predicted math achievement and the math achievement sub-scores (comparison, sequences, and written calculations) by early numeracy skills (symbolic, non-symbolic, subitizing), home learning environment (informal and formal), and math anxiety. We performed Bayesian correlations to explore the relations between the variables while expecting a relationship between the variables (alternative hypothesis). We used Bayesian multi-model inference in JASP [66] to perform regression analysis. We used this approach for the following reasons: (1) Bayesian analyzes do not depend on large samples, matching our sample size, (2) multi-model inference retains all models and calculates weights for each model, which reflect how much the data support that model, (3) and accomplishes variable selection and parameter estimation simultaneously instead of sequentially [67]. Although it is commonly used, inference from two-step methods leads to overestimating parameters [68], however, we also calculated hierarchical regression analysis — in the first step controlling for working memory — which is reported in the Supplementary Materials.

3. Results

The correlations between the included variables are shown in Table 2. Surprisingly, not many constructs were correlated with each other; There was moderate evidence that subitizing was positively related to symbolic skills and strong evidence for a positive relationship between math achievement and subitizing, symbolic, and nonsymbolic skills.

Since it is challenging to illustrate all models, model averaging was used. This way, the individual relevance of predictors was quantified. To read the following Tables 3–6, the first column denotes the predictor variable, followed by 'mean' and 'SD,' which represent the respective posterior mean and standard deviation of the parameter after model averaging. *(incl)* indicates the prior inclusion probability, and *P (incl|data)* represents the posterior inclusion probability. The Bayes factor *(BFincl)* stands for the change from prior to posterior inclusion odds, and the last two columns mark a 95 % credible interval (*CI*).

Table 3 confirms the impression given by the best models about subitizing, nonsymbolic skills, and informal HLE being significant predictors for math achievement by indicating moderate evidence (see *BFincl* in Table 3). In addition, strong evidence was found for symbolic skills to predict math achievement. Table 4 shows the results of the prediction for written computation. Moderate evidence was found for subitizing, math anxiety, symbolic, and nonsymbolic skills to predict future written computation. Interestingly there was no strong evidence for any of the predictors, suggesting that there might not be one dominant factor predicting written computation. For sequences, the results can be found in Table 5. Here moderate evidence was found for informal HLE and strong evidence for symbolic skills as predictors for future achievement in sequences tasks. Finally, Table 6 can be consulted for the subscore of comparisons. There is strong evidence that comparisons were best and uniquely predicted by symbolic skills, while no conclusive evidence could be found for any other factor. The best model also only included symbolic skills.

To summarize, the Bayesian model-averaged analysis showed that essential predictors for overall math achievement are symbolic skills, subitizing, nonsymbolic skills, and informal HLE; for the subscore written computation, significant predictors are subitizing, math anxiety, symbolic, and nonsymbolic skills; for sequences, it is symbolic skills and informal HLE; and for comparisons, it is only symbolic skills (see Fig. 2).

Table 2
Bayesian Pearson Correlations of Coefficients and Overall Math Achievement.

Variable		1	2	3	4	5	6	7
1. Subitizing	Pearson's r	—						
	BF_{10}	—						
2. Formal HLE	Pearson's r	0.064	—					
	BF_{10}	0.160	—					
3. Informal HLE	Pearson's r	0.160	0.107	—				
	BF_{10}	0.387	0.216	—				
4. Symbolic Skills	Pearson's r	0.327*	0.011	0.174	—			
	BF_{10}	13.234	0.136	0.470	—			
5. Nonsymbolic Skills	Pearson's r	0.201	−0.008	0.173	0.221	—		
	BF_{10}	0.724	0.136	0.467	1.039	—		
6. Math Anxiety	Pearson's r	−0.141	0.111	−0.018	−0.162	−0.153	—	
	BF_{10}	0.308	0.225	0.137	0.398	0.355	—	
7. Math Achievement (T2)	Pearson's r	0.409***	0.054	0.307	0.525***	0.391***	−0.259	—
	BF_{10}	228.638	0.152	7.464	63,941.385	114.151	2.290	—

Note. ** $BF_{10} > 30$,.
* $BF_{10} > 10$,.
*** $BF_{10} > 100$.

Table 3
Bayesian Linear Regression Coefficients predicting Overall Math Achievement.

Overall Math Achievement					95 % Credible Interval	
Coefficient	Mean (SD)	P (incl| data)	P (excl| data)	$BF_{inclusion}$	Lower	Upper
Working Memory	0.042 (0.03)	0.832	0.168	4.936*	0.000	0.096
Math Anxiety	−0.053 (0.08)	0.595	0.405	1.471	−0.238	0.037
Subitizing	0.180 (0.10)	0.895	0.105	8.497*	0.000	0.352
Symbolic Skills	0.192 (0.06)	0.994	0.006	170.939**	0.083	0.308
Nonsymbolic Skills	0.118 (0.06)	0.918	0.082	11.194*	0.000	0.217
Informal HLE	0.115 (0.10)	0.772	0.228	3.383*	−0.009	0.289
Formal HLE	0.012 (0.06)	0.494	0.506	0.974	−0.093	0.154

Note.
* Bayes Factor between 3 and 10 indicates moderate evidence,.
** Bayes Factor > 100 is considered strong evidence.

Table 5
Bayesian Linear Regression Coefficients predicting the score on the subtest Sequences.

Sequences					95 % Credible Interval	
Coefficient	Mean (SD)	P(incl| data)	P (excl| data)	$BF_{inclusion}$	Lower	Upper
Working Memory	0.049 (0.03)	0.767	0.233	3.298	−0.004	0.103
Math Anxiety	9.475e-4 (0.09)	0.448	0.552	0.813	−0.179	0.181
Subitizing	0.129 (0.09)	0.650	0.350	1.859	−0.052	0.310
Symbolic Skills	0.144 (0.06)	0.965	0.035	27.470**	0.028	0.259
Nonsymbolic Skills	0.098 (0.06)	0.773	0.227	3.411	−0.013	0.209
Informal HLE	0.187 (0.09)	0.864	0.136	6.335*	0.013	0.360
Formal HLE	−0.027 (0.09)	0.445	0.555	0.802	−0.196	0.143

Note.
* Bayes Factor between 3 and 10 indicates moderate evidence,.
** Bayes Factor > 100 is considered strong evidence.

Table 4
Bayesian Linear Regression Coefficients predicting the score on the subtest Written Computation.

Written Computation					95 % Credible Interval	
Coefficient	Mean (SD)	P(incl| data)	P (excl| data)	$BF_{inclusion}$	Lower	Upper
Working Memory	0.018 (0.03)	0.570	0.430	1.328	−0.035	0.071
Math Anxiety	−0.173 (0.09)	0.870	0.130	6.685*	−0.352	0.006
Subitizing	0.180 (0.09)	0.869	0.131	6.629*	6.500e-4	0.360
Symbolic Skills	0.092 (0.06)	0.806	0.194	4.159*	−0.023	0.207
Nonsymbolic Skills	0.135 (0.06)	0.950	0.050	18.805*	0.024	0.245
Informal HLE	0.155 (0.09)	0.795	0.205	3.882	−0.017	0.327
Formal HLE	−0.048 (0.09)	0.526	0.474	1.111	−0.217	0.121

Note. ** Bayes Factor > 100 is considered strong evidence.
* Bayes Factor between 3 and 10 indicates moderate evidence,.

Table 6
Bayesian Linear Regression Coefficients predicting the score on the subtest Comparisons.

Comparisons					95 % Credible Interval	
Coefficient	Mean (SD)	P (incl| data)	P (excl| data)	$BF_{inclusion}$	Lower	Upper
Working Memory	0.030 (0.03)	0.608	0.392	1.549	−5.905e-4	0.095
Math Anxiety	−0.014 (0.06)	0.341	0.659	0.518	−0.175	0.093
Subitizing	0.100 (0.11)	0.603	0.397	1.516	−0.012	0.312
Symbolic Skills	0.229 (0.07)	0.995	0.005	198.985**	0.103	0.367
Nonsymbolic Skills	0.029 (0.05)	0.441	0.559	0.788	−0.018	0.154
Informal HLE	0.006 (0.05)	0.324	0.676	0.480	−0.093	0.158
Formal HLE	0.068 (0.09)	0.517	0.483	1.070	−0.021	0.261

Note. *Bayes Factor between 3 and 10 indicates moderate evidence,.
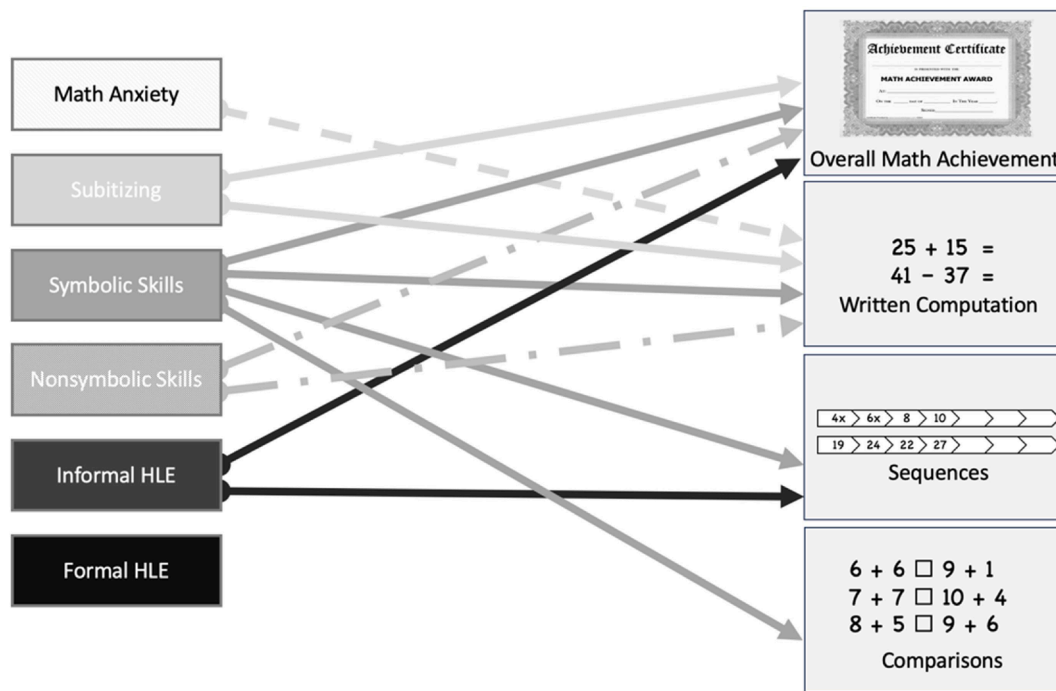** Bayes Factor > 100 is considered strong evidence.

Fig. 2. Summarized relations between outcome and predictor variables.

4. Discussion

This longitudinal study aimed to evaluate the potentially selective prediction of math achievement in 7- to 8-year-old children, considering influences of early numeracy skills, home learning environment, and math anxiety assessed when the same children were 6- to 7-years old.

Previous research indicated that early numeracy skills were essential and robust predictors of later math achievement even when controlling for influences of age, general intelligence, and working memory (e.g., $\beta = 0.287$ in [25]; $r^2 = .278$ in [26]). In this regard, early numeracy skills would again be identified as a robust predictor in the present study. In line with previous evidence and our expectations, our results substantiated that early numeracy skills assessed in 6-to-7-year old children (employing magnitude comparison, number line estimation, and subitizing) were not only strongly correlated but also highly predictive of overall math achievement at 18 months later when children were in second grade [15,22–24,69].

Furthermore, performance in the sub-dimensions number sequences and comparisons was predicted by symbolic skills. In contrast, the subdimension written calculation was predicted by symbolic and nonsymbolic skills and conceptual subitizing. These results provide support for both sides of the ongoing controversy about when and how symbolic and non-symbolic early numeracy skills contribute to later math achievement (e.g., [19,70,71]; for a review see [72]). In sum, these results indicate that certain factors are only predictors for certain subdimensions of math achievement (for similar findings, see [73]). Early numeracy skills are a stable predictor of later mathematics achievement. Nevertheless, at a closer look, it is also the case that specific early numeracy skills predict specific sub-dimensions of later math achievement.

The significant association with and prediction of the subdimension sequences by informal home learning environment substantiated previous findings suggesting that informal home learning environment predicted only certain subdimensions of mathematical achievement (e. g. [36,74–76]). Although previous research has shown an association of informal home learning environment with other subdimensions, we think that our findings are reasonable. The result suggests hat numerical board or card games that children play at home seem helpful for later

sequence knowledge or that children with better sequence knowledge also engage more often in numerical card or board games. A possible explanation for this specific association may be that numerical activities in games often involve processing sequences (e.g., children observe a pattern and use it to predict the subsequent steps/numbers/actions in a game) that, in turn, help children obtain the missing numbers in the sequence task. Identifying the rule with which the sequence continues was sometimes tricky and may have challenged children similarly to games and quizzes. A recent international conference yielded insights emphasizing the imperative for enhanced measurement and understanding of the home mathematics environment; including a comprehensive approach encompassing factors such as child, family, community, culture, caregiver traits, and children's cognitive and affective characteristics [77]. After all, the variation in the relationship between the home mathematics environment and children's mathematical achievement arises from the complex nature of children's learning environments, highlighting the significance of social interactions with caregivers and diverse environmental inputs, as guided by Vygotsky's socio-cultural theory [78] and Bronfenbrenner's ecological systems theory [79].

Nevertheless, home learning environment was positively related to early numeracy skills (confirming [34]). In earlier studies, home learning environment predicts early numeracy skills [35]. Our results suggest that family activities are important learning possibilities and should be promoted accordingly. Future research should investigate how children learn best in an incidental or playful manner and which kinds of activities are the most effectual and least complex, and costly. Furthermore, future research should address if children engage more in those games if they already have more developed numerical skills. Those research outcomes might inform parents, practitioners, and teachers how to support children's numeracy development more practically and cost-effectively.

Math anxiety also proved to be an important negative influence on subcomponents of math achievement (written computation), confirming previous findings (symbolic math; [61,65,80,81]). This finding is twofold. An optimistic interpretation of this finding is that although math anxiety does affect a subcomponent of math achievement, it does not (yet) have a detrimental effect on overall math achievement.

Previous studies believed that math anxiety is only experienced after 2nd grade [41,42]. On top of that, children between the ages of six and nine tend to give more positive responses, leading to fewer verbal reports of sadness and worry [7]. Possibly, math anxiety has been overlooked in the past as it only shows evidence in our study for one sub-dimension. Although the study of math anxiety in kindergarteners and early primary school children is still young, our study shows that it is essential to focus on this emotional construct and emphasize its importance concerning early mathematics performance. Even though kindergarteners and early primary school children report less negative and generally more positive feelings about their skills and achievement, our study showed that written computation is influenced negatively by math anxiety. This question is as follows: Why does math anxiety influence written computation? What kind of support can be offered to affected children—early on—so they do not get into the vicious cycle of disengaging and therefore missing opportunities to increase their knowledge and skills in mathematics [54]? Since this is one factor influencing career decisions [45], the potential beneficiary effects of STEM participation in this field of research should not be underestimated. For a more detailed discussion about research in math anxiety, see the work of Cipora et al. [43].

Surprisingly, although one might expect that a home learning environment would protect against math anxiety, the two factors did not correlate negatively (nor positively). These relations might not be seen in the present study because of the long (18 months) gap between the two measurement points. There might have been processes we could not capture because the children's behavior might have been changing drastically between the two measurements. Thus, more research is needed regarding math anxiety and the home learning environment, and it might address whether the home learning environment has a protective power on math anxiety longitudinally, using a more in-depth analysis of the relationship between them while it is changing through a micro genetic study design.

The present findings are of practical importance for typical and atypical development and for mathematics education. Since individual differences in early numeracy skills and math anxiety are relevant to a child's math achievement, children with poor early numeracy skills or a high rate of math anxiety could be at risk for developing math learning difficulties or math avoidance behavior. A home learning environment or at least an informal learning environment can be considered a possible intervention to promote essential skills and to allow early access to the numerical world, which is playful for children, cheerful, more accessible to learn, and more attractive. Furthermore, the relationship between math achievement and the numerical learning environment allows parents to support their children's interest in numbers and magnitudes at home with appropriate games. The increased numerical understanding can be gained early using the enriched numerical learning environment [34,35]. However, this also depends on the ability and willingness of the parents. Teachers and educators should consider using a variety of mathematical representations (symbolic, non-symbolic, or a combination of both) to teach mathematics knowledge to reduce math anxiety and increase math achievement. Using situational, playful, and verbal representations of math can assist young children in learning while maintaining low levels of math anxiety.

Since the investigated variables are only some of the ones involved in the development of math cognition, future research, including additional, diverse concepts, is needed. The self-concept decreases with more realistic assessments of one's abilities and the increase in social comparison [82,83]. Therefore, the influence of math anxiety increases during elementary school. This shows the need for longitudinal studies investigating math anxiety, early numeracy skills, home learning environment, school achievement, and self-concept throughout elementary school.

A limitation of the study is its sample size: it needed to be bigger to use more sophisticated models to explain the effects of the predictors or compare specific trajectories with each other. Another limitation is that the children were followed for 18 months and there were only two measurement points. Ideally, future studies should include more children, follow them for extended periods, and assess their skills at more time points.

In summary, there is moderate to strong evidence that math anxiety, home learning environment, and early numeracy skills in 6- to 7-year-olds predict their math achievement 18 months later. This study has made an additional contribution by investigating relevant factors for developing mathematical skills longitudinally, replicating previous findings, and showing that both nature and nurture are important in the development of mathematical cognition. Playful activities with indirect reference to numbers and magnitudes may be interventional support for improving preschool math skills and later math achievement.

Ethical statement

We assure that the following is fulfilled:

(1) This material is the authors' own original work, which has not been previously published elsewhere.

(2) The paper is not currently being considered for publication elsewhere.

(3) The paper reflects the authors' own research and analysis in a truthful and complete manner.

(4) The results are appropriately placed in the context of prior and existing research.

(5) All sources used are properly disclosed (correct citation).

(6) All authors have been personally and actively involved in substantial work leading to the paper, and will take public responsibility for its content.

(7) This study got ethical approval from the Ethics Committee of the University of Bern, which follows the Helsinki Guidelines.

(8) All participants gave oral consent, written informed consent was obtained by their legal guardians.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.tine.2023.100217.

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