Decomposition Methods in the Social Sciences

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5. Difference-in-difference decompositions

Contents

Difference-in-difference decompositions

- Up to now we were concerned with a single outcome differential (e.g. a gender wage gap) at a specific point in time and in a specific region or population.
- Often, however, comparisons over time or between countries or regions are of interest.
  - How did the gender wage gap change over time and how much of this change is due to changes with respect to covariates?
  - How would the gender wage gap in country A look like if it had the wage structure of country B?
- One way of analyzing changes over time or between populations is to compare separate decomposition results. Some questions, however, require a "double" or "difference-in-difference" decomposition.
- A very famous application of such methodology, for example, is the "Swimming upstream" paper by Blau and Kahn (1997).

Smith-Welch decomposition

(Smith and Welch 1987, also see e.g. Heckman et al. 2000, Kröger and Hartmann 2021)

- Given is a linear model

$$Y^{gt} = X^{gt}\beta^{gt} + \epsilon^{gt}, \quad \mathrm{E}(e^{gt}|X^{gt}) = 0$$

  for two groups, $g = m, f$ (males and females), at two time points, $t = 0, 1$.

- Using the male coefficients, β^{mt}, as reference, the decomposition of the group difference in average Y at time t can be written as

$$\Delta^{\mu t} = (\bar{X}^{mt} - \bar{X}^{ft})\beta^{mt} + \bar{X}^{ft}(\beta^{mt} - \beta^{ft}) = \Delta_X^{\mu t} + \Delta_S^{\mu t}$$

- We are now interested in decomposing the change in the wage gap over time.

Smith-Welch decomposition

- Let $\Delta\bar{X}^t = \bar{X}^{mt} - \bar{X}^{ft}$ and $\Delta\beta^t = \beta^{mt} - \beta^{ft}$. Using the male coefficients from the first time point, β^{m0}, as reference, this double decomposition can be written as

$$\begin{aligned} d\Delta^\mu = \Delta^{\mu 1} - \Delta^{\mu 0} &= \left\{(\Delta\bar{X}^1 - \Delta\bar{X}^0)\beta^{m0} + \Delta\bar{X}^1(\beta^{m1} - \beta^{m0})\right\} \\ &\quad + \left\{\bar{X}^{f1}(\Delta\beta^1 - \Delta\beta^0) + (\bar{X}^{f1} - \bar{X}^{f0})\Delta\beta^0\right\} \\ &= d\Delta^\mu_X + d\Delta^\mu_S \end{aligned}$$

- Interpretation:

$(\Delta\bar{X}^1 - \Delta\bar{X}^0)\beta^{m0}$	main endowments effect: shows how the wage gap changed because men and women became more similar or dissimilar in X (negative, if they became more similar; positive, if they became more dissimilar)
$\Delta\bar{X}^1(\beta^{m1} - \beta^{m0})$	secondary endowments effect due to change in reference wage structure over time

Smith-Welch decomposition

$\bar{X}^{f1}(\Delta\beta^1 - \Delta\beta^0)$	primary coefficients effect: effect of change in wage structure difference between men and women (negative, if coefficients became more similar; positive, if coefficients became more dissimilar)
$(\bar{X}^{f1} - \bar{X}^{f0})\Delta\beta^0$	secondary coefficients effect due to change in reference endowments over time

- Of course, various other types of decompositions are possible depending on the choice of the reference group and the reference year. The index problem of the standard OB decomposition is now a double index problem, which can make it hard to keep an overview.
- See `help smithwelch` for a systematic discussion. It starts with the threefold decomposition and then shows how the formulas change if a reference group and/or a reference year is introduced. Of course, reference groups/years can also be results from pooled or averaged models. See Kröger and Hartmann (2021) for further variants of difference-in-differences decompositions.

Example analysis (using smithwelch by Jann 2005b)

```
. use gsoep-extract, clear
(Example data based on the German Socio-Economic Panel)

. keep if inlist(wave,1995,2015)
(23,792 observations deleted)

. keep if inrange(age, 25, 55)
(8,147 observations deleted)

. generate lnwage = ln(wage)
(2,734 missing values generated)

. generate expft2 = expft^2
(56 missing values generated)

. generate byte t = wave==2015   // 0 = 1995, 1 = 2015

. generate byte female = sex==2  // 0 = male, 1 = female

. summarize lnwage yeduc expft expft2 t female
```

Variable	Obs	Mean	Std. dev.	Min	Max
lnwage	8,277	2.71872	.483595	1.108563	4.86638
yeduc	10,735	12.04332	2.700811	7	18
expft	10,955	12.21353	9.640926	0	40.5
expft2	10,955	242.1094	305.9972	0	1640.25
t	11,011	.6637908	.4724329	0	1
female	11,011	.5466352	.497843	0	1

```
. drop if missing(lnwage,yeduc,expft) // remove unused observation
(2,940 observations deleted)

. svyset psu [pw=weight], strata(strata)

Sampling weights: weight
             VCE: linearized
     Single unit: missing
        Strata 1: strata
 Sampling unit 1: psu
           FPC 1: <zero>
```

Example analysis

- Outcome by time point and gender

```
. svy: mean lnwage if !missing(yeduc, expft), over(t female)
(running mean on estimation sample)

Survey: Mean estimation

Number of strata =    15              Number of obs   =      8,071
Number of PSUs   = 2,459              Population size = 23,859,587
                                      Design df       =      2,444

--------------------------------------------------------------------
                  |             Linearized
                  |       Mean   std. err.     [95% conf. interval]
------------------+-------------------------------------------------
c.lnwage@t#female |
             0 0  |    2.84812   .0132636      2.822111    2.874129
             0 1  |   2.627384   .0186491      2.590815    2.663954
             1 0  |   2.862735   .0164148      2.830547    2.894923
             1 1  |   2.657428   .0151483      2.627723    2.687132
--------------------------------------------------------------------
```

Example analysis

- Characteristics by time point and gender

```
. svy: mean yeduc expft if !missing(lnwage), over(t female)
(running mean on estimation sample)

Survey: Mean estimation

Number of strata =    15            Number of obs   =      8,071
Number of PSUs   = 2,459            Population size = 23,859,587
                                    Design df       =      2,444
```

	Mean	Linearized std. err.	[95% conf.	interval]
c.yeduc@t#female				
0 0	12.04901	.0924429	11.86774	12.23029
0 1	11.79327	.091392	11.61406	11.97249
1 0	12.7098	.0980728	12.51749	12.90212
1 1	12.94121	.0904373	12.76387	13.11855
c.expft@t#female				
0 0	17.49546	.3094411	16.88867	18.10226
0 1	12.39256	.3700076	11.667	13.11812
1 0	17.17692	.3427645	16.50478	17.84906
1 1	10.79882	.2760263	10.25755	11.34009

Example analysis

- OB decomposition in 1995

```
. oaxaca lnwage yeduc (experience: expft expft2) ///
>     if t==0, by(female) weight(1) nodetail svy

Blinder-Oaxaca decomposition

Number of strata =   4                     Number of obs     =      2,609
Number of PSUs   = 794                     Population size   = 11,707,370
                                           Design df         =        790
                                           Model             =     linear
Group 1: female = 0                        N of obs 1        =      1,486
Group 2: female = 1                        N of obs 2        =      1,123

    explained: (X1 - X2) * b1
  unexplained: X2 * (b1 - b2)

------------------------------------------------------------------------------
             |             Linearized
      lnwage | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
overall      |
     group_1 |    2.84812    .013042   218.38   0.000     2.822519    2.873721
     group_2 |   2.627384   .0178997   146.78   0.000     2.592248    2.662521
  difference |   .2207358   .0206432    10.69   0.000     .1802138    .2612579
   explained |   .0758792   .0115696     6.56   0.000     .0531684    .0985899
 unexplained |   .1448567   .0216315     6.70   0.000     .1023946    .1873188
------------------------------------------------------------------------------
```

Example analysis

- OB decomposition in 2015

```
. oaxaca lnwage yeduc (experience: expft expft2) ///
>     if t==1, by(female) weight(1) nodetail svy

Blinder-Oaxaca decomposition

Number of strata =     15                    Number of obs      =      5,462
Number of PSUs   = 2,037                     Population size    = 12,152,217
                                             Design df          =      2,022
                                             Model              =     linear
Group 1: female = 0                          N of obs 1         =      2,642
Group 2: female = 1                          N of obs 2         =      2,820

    explained: (X1 - X2) * b1
  unexplained: X2 * (b1 - b2)
```

lnwage	Coefficient	Linearized std. err.	t	P>|t|	[95% conf.	interval]
overall						
group_1	2.862735	.0162749	175.90	0.000	2.830818	2.894652
group_2	2.657428	.0149503	177.75	0.000	2.628108	2.686747
difference	.2053074	.0204701	10.03	0.000	.1651628	.245452
explained	.0904872	.0151554	5.97	0.000	.0607654	.120209
unexplained	.1148202	.0211416	5.43	0.000	.0733585	.1562819

Example analysis

- Estimate outcome models by time point and gender; these model will be used as input to the Smith-Welch decomposition.

```
. svy: regress lnwage yeduc expft expft2 if female==0 & t==0
  (output omitted)
. estimates store male_t0
. svy: regress lnwage yeduc expft expft2 if female==1 & t==0
  (output omitted)
. estimates store female_t0
. svy: regress lnwage yeduc expft expft2 if female==0 & t==1
  (output omitted)
. estimates store male_t1
. svy: regress lnwage yeduc expft expft2 if female==1 & t==1
  (output omitted)
. estimates store female_t1
```

Example analysis

- Overview of models.

```
. esttab male_t0 female_t0 male_t1 female_t1, nogap mti nonum r2
```

	male_t0	female_t0	male_t1	female_t1
yeduc	0.0567***	0.0586***	0.0829***	0.0789***
	(9.51)	(9.63)	(14.88)	(14.91)
expft	0.0318***	0.0206**	0.0357***	0.0313***
	(5.62)	(3.11)	(6.04)	(6.36)
expft2	-0.000614***	-0.000373	-0.000593***	-0.000541***
	(-4.09)	(-1.93)	(-3.79)	(-3.67)
_cons	1.855***	1.769***	1.430***	1.404***
	(20.65)	(20.61)	(14.96)	(18.93)
N	1486	1123	2642	2820
R-sq	0.145	0.123	0.280	0.277

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Example analysis

- Smith-Welch decomposition

```
. smithwelch male_t0 female_t0 male_t1 female_t1, reference(1) benchmark(1)
Decompositions of individual differentials:
```

	D	E	C
Sample 1	.2207358	.0758792	.1448567
Sample 2	.2053074	.0904872	.1148202

```
Difference in (components of) differentials:
```

	dD	dE	dC
	-.0154284	.014608	-.0300365

```
Decomposition of difference in differentials:
```

	D	E	C
dE	.014608	-.0087573	.0233653
dC	-.0300365	-.0105462	-.0194903

```
D  = differential / difference in component of differential
E  = part of D due to differences in endowments
C  = part of D due to differences in coefficients
```

- Detailed Smith-Welch decomposition

```
. smithwelch male_t0 female_t0 male_t1 female_t1, reference(1) benchmark(1) ///
>     detail(schooling=yeduc, experience=expft*)
```

Decompositions of individual differentials:

Sample 1	D	E	C
schooling	-.0084441	.0144885	-.0229326
experience	.1426075	.0613907	.0812168
_cons	.0865725	0	.0865725
Total	.2207358	.0758792	.1448567

Sample 2	D	E	C
schooling	.0328791	-.0191954	.0520745
experience	.1466832	.1096826	.0370006
_cons	.0257451	0	.0257451
Total	.2053074	.0904872	.1148202

Difference in (components of) differentials:

	dD	dE	dC
schooling	.0413232	-.0336838	.0750071
experience	.0040757	.0482919	-.0442162
_cons	-.0608274	0	-.0608274

Total	-.0154284	.014608	-.0300365

```
Decomposition of difference in differentials:
```

dE	D	E	C
schooling	-.0336838	-.0275985	-.0060853
experience	.0482919	.0188412	.0294507
_cons	0	0	0
Total	.014608	-.0087573	.0233653
dC	**D**	**E**	**C**
schooling	.0750071	-.0022322	.0772393
experience	-.0442162	-.0083139	-.0359022
_cons	-.0608274	0	-.0608274
Total	-.0300365	-.0105462	-.0194903

```
D  = differential / difference in component of differential
E  = part of D due to differences in endowments
C  = part of D due to differences in coefficients
```

Juhn-Murphy-Pierce 1991 decomposition

(Juhn et al. 1991)

- Juhn et al. (1991) use an alternative setup by considering changes in the residual variance.
- The argument is that the gender wage gap will be large if the residual variance, that is, the variance of wages once controlling for observables such as education or work experience, is large.
- Conceptually, the residual variance can be viewed as the price of unobservables. The idea is, that there may be differences between men and women in such unobservables. If the prices increase, the gender wage gap will increase as well.
- For example, Blau and Kahn (1997) use the method in an analysis in which they argue that the gender wage gap would have declined more than it actually did, hadn't there been a strong increase in general wage inequality that had nothing to do with gender.

Juhn-Murphy-Pierce 1991 decomposition

- Consider a model
$$Y^{gt} = X^{gt}\beta^t + \epsilon^{gt}$$
for two groups, $g = m, f$ (males and females), at two time points, $t = 0, 1$, where β^t are some reference regression parameters (non-discriminatory prices of observables).
- The model can also be expressed as
$$Y^{gt} = X^{gt}\beta^t + r^{gt}\sigma^t$$
where σ is a reference residual standard deviation (non-discriminatory prices of unobservables) and $r = (Y - X\beta^t)/\sigma^t$ is a standardized residual.
- Thus, the equation now has a two-component error term. The residuals are expressed as a function of the general residual inequality at time t and the positions of the residuals in the residual distribution.

Juhn-Murphy-Pierce 1991 decomposition

- The mean outcome differential between men an women at time t can then be decomposed as follows:

$$\Delta^{\mu,t} = \bar{Y}^{mt} - \bar{Y}^{ft} = (\bar{X}^{mt} - \bar{X}^{ft})\beta^t + (\bar{r}^{mt} - \bar{r}^{ft})\sigma^t$$
$$= \Delta_X^\mu + \Delta_S^\mu$$

- The first term is the "predicted gap" and the second term is the "residual gap". They are equal to the "explained" part and the "unexplained" part in a standard OB decomposition using β^t as reference coefficients.
- Let $\Delta\bar{X}^t = (\bar{X}^{mt} - \bar{X}^{ft})$ and $\Delta\bar{r}^t = (\bar{r}^{mt} - \bar{r}^{ft})$. Given two time points $t = 0$ and $t = 1$ the change in the outcome differential can then be written as

$$\Delta^{\mu,1} - \Delta^{\mu,0} = (\Delta\bar{X}^1\beta^1 - \Delta\bar{X}^0\beta^0) + (\Delta\bar{r}^1\sigma^1 - \Delta\bar{r}^0\sigma^0)$$
$$= d\Delta_X^\mu + d\Delta_S^\mu$$

Juhn-Murphy-Pierce 1991 decomposition

- The change in the "predicted gap" can be further decomposed as

$$d\Delta_X^{\mu} = (\Delta\bar{X}^1 - \Delta\bar{X}^0)\beta^0 + \Delta\bar{X}^0(\beta^1 - \beta^0) + (\Delta\bar{X}^1 - \Delta\bar{X}^0)(\beta^1 - \beta^0)$$

  where the first term is the main "observed quantities" effect due to a change in gender differences in X, the second term is a secondary effect due a change in "prices" for observed quantities, and the third term is an interaction term.

- Likewise, the change in the "residual gap" can be decomposed as

$$d\Delta_S^{\mu} = (\Delta\bar{r}^1 - \Delta\bar{r}^0)\sigma^0 + \Delta\bar{r}^0(\sigma^1 - \sigma^0) + (\Delta\bar{r}^1 - \Delta\bar{r}^0)(\sigma^1 - \sigma^0)$$

  where the first term is the so-called "gap effect" due to changes in the group differences in residual positions (i.e. changes in the group differences in "unobserved quantities" and changes in discrimination), the second term is the part due to changes in residual inequality (i.e. changes in "prices" for unobserved quantities), and the third term is again an interaction term.

Juhn-Murphy-Pierce 1991 decomposition

- Similar to other decompositions, it is common practice to use the "prices" of one of the years as the reference prices (or use some average or pooled results). For example, if we use $t = 0$ as the reference, the decomposition simplifies to:

$$d\Delta_X^\mu = (\Delta\bar{X}^1 - \Delta\bar{X}^0)\beta^0 + \Delta\bar{X}^1(\beta^1 - \beta^0)$$

$$d\Delta_S^\mu = (\Delta\bar{r}^1 - \Delta\bar{r}^0)\sigma^0 + \Delta\bar{r}^1(\sigma^1 - \sigma^0)$$

- Furthermore, a detailed decomposition can be obtained for the components of $d\Delta_X^\mu$ in the usual way (but obviously not for $d\Delta_S^\mu$).
- Note that results for $d\Delta_X^\mu$ are the same as for the Smith-Welch decomposition (if using the same setup). For $d\Delta_S^\mu$ only the total is the same; that is, Smith-Welch and Juhn-Murphy-Pierce lead to a different breakup of $d\Delta_S^\mu$.

Juhn-Murphy-Pierce 1991 decomposition: estimation

- Estimation of the components of $d\Delta_X^\mu$ is straightforward.
- Estimation of the components of $d\Delta_S^\mu$ is more involved and requires some discussion. Two approaches are used in the literature, a **parametric** approach and a **nonparametric** approach.
- Parametric
  - Since, by definition, $\epsilon^t = r^t\sigma^t$, expression $\Delta\bar{r}^t\sigma^t$ can simply be estimated as the mean difference in residuals ϵ between men an women at time t.
  - But what about expressions such as $\Delta\bar{r}^1\sigma^0$?
  - An obvious solution is to estimate the residual standard deviation at time $t = 0$ and then multiply it by the mean difference in standardized residuals of $t = 1$.
- Nonparametric
  - The parametric approach is simple, but neglects changes in the distributional shape (apart from the variance).
  - The following nonparametric procedure has therefore been proposed.

Juhn-Murphy-Pierce 1991 decomposition: estimation

- Let $F^t()$ be the distribution function of the residuals at time t. Furthermore, let p^t represent the relative positions of the residuals in the residual distribution at time t, that is

$$p^{gt} = F^t(\epsilon^{gt}) \quad \text{and thus} \quad \epsilon^{gt} = Q^t(p^{gt})$$

  where $Q() = F^{-1}()$ is the quantile function (inverse of $F()$).
- The solution now is to apply the quantile function of one time point to the residual ranks of the other time point.
- For example, $\Delta\bar{r}^1\sigma^0$ is estimated by assigning each individual at $t = 1$ a percentile number corresponding to its position in the residual distribution of $t = 1$ (i.e., compute p^1), then using these relative ranks to derive hypothetical residuals given the $t = 0$ residual distribution (i.e. compute $Q^0(p^1)$), and finally taking the mean difference in these hypothetical residuals between men and women.

- The JMP 1991 procedure relies on some strong assumptions and is not free of critique (e.g. Yun 2009).

Example analysis (using jmpierce2 by Jann 2005b)

- Estimate the outcome models to be used in the JMP 1991 decomposition.
  - jmpierce2 does not fully support models estimated with the svy prefix; this is why we just apply the weights rather than svy.

```
. regress lnwage yeduc expft expft2 [pw=weight] if female==0 & t==0
  (output omitted)
. estimates store male_t0
. regress lnwage yeduc expft expft2 [pw=weight] if female==1 & t==0
  (output omitted)
. estimates store female_t0
. regress lnwage yeduc expft expft2 [pw=weight] if female==0 & t==1
  (output omitted)
. estimates store male_t1
. regress lnwage yeduc expft expft2 [pw=weight] if female==1 & t==1
  (output omitted)
. estimates store female_t1
```

Nonparametric JMP 1991 decomposition

```
. jmpierce2 male_t0 female_t0 male_t1 female_t1, reference(1) benchmark(1)
```

Decomposition of individual differentials:

	raw differential	quantity effect	residual gap
Sample 1	.2207358	.0758792	.1448567
Sample 2	.2053074	.0904872	.1148202

Difference in (components of) differentials:

	D	E	U
Total	-.0154284	.014608	-.0300365

Decomposition of difference in predicted gap:

	E	Q	P
Total	.014608	-.0087573	.0233653

Decomposition of diffence in residual gap:

	U	Q	P
Total	-.0300365	-.0394444	.0094079

```
D  = difference in differential
E  = difference in predicted gap
U  = difference in residual gap
Q  = quantity effect
P  = price effect
```

Parametric JMP 1991 decomposition

```
. jmpierce2 male_t0 female_t0 male_t1 female_t1, reference(1) benchmark(1) parametric
```

Decomposition of individual differentials:

	raw differential	quantity effect	residual gap
Sample 1	.2207358	.0758792	.1448567
Sample 2	.2053074	.0904872	.1148202

Difference in (components of) differentials:

	D	E	U
Total	-.0154284	.014608	-.0300365

Decomposition of difference in predicted gap:

	E	Q	P
Total	.014608	-.0087573	.0233653

Decomposition of diffence in residual gap:

	U	Q	P
Total	-.0300365	-.0385607	.0085242

```
D  = difference in differential
E  = difference in predicted gap
U  = difference in residual gap
Q  = quantity effect
P  = price effect
```

Detailed JMP 1991 decomposition

```
. jmpierce2 male_t0 female_t0 male_t1 female_t1, reference(1) benchmark(1) ///
>     detail(schooling=yeduc, experience=expft*)
```

Decomposition of individual differentials:

	raw differential	quantity effect	residual gap
Sample 1	.2207358	.0758792	.1448567
Sample 2	.2053074	.0904872	.1148202

Difference in (components of) differentials:

	D	E	U
Total	-.0154284	.014608	-.0300365

Decomposition of difference in predicted gap:

	E	Q	P
Total	.014608	-.0087573	.0233653
schooling	-.0336838	-.0275985	-.0060853
experience	.0482919	.0188412	.0294507

Decomposition of diffence in residual gap:

	U	Q	P
Total	-.0300365	-.0394444	.0094079

D = difference in differential
E = difference in predicted gap
U = difference in residual gap
Q = quantity effect
P = price effect

Interventionist approach (Kröger/Hartmann)

- Kröger and Hartmann (2021) give an overview of several difference-in-differences decompositions and propagate an easy to interpret interventionist approach.
- The approach is somewhat different from Smith-Welch and Juhn-Murphy-Pierce in that it does not try to provide separate breakups of the change/difference in the explained part and the change/difference in the unexplained part.
- Again, let there be two groups two groups, $g \in \{m, f\}$ (males and females), and two time points, $t \in \{0, 1\}$.
- The decomposition then asks:
  1. How does the group difference change if endowments are adjusted to the level of time 1, but coefficients are kept at their values of time 0 (endowments effect).
  2. How does the group difference change if coefficients are adjusted to the values of time 1, but endowments are kept at their level of time 0 (coefficients effect)

Interventionist approach (Kröger/Hartmann)

- That is, the decomposition is as follows:

$$\Delta E = (\bar{X}^{m1} - \bar{X}^{m0})\beta^{m0} - (\bar{X}^{f1} - \bar{X}^{f0})\beta^{f0}$$
$$\Delta C = \bar{X}^{m0}(\beta^{m1} - \beta^{m0}) - \bar{X}^{f0}(\beta^{f1} - \beta^{f0})$$
$$\Delta I = (\bar{X}^{m1} - \bar{X}^{m0})(\beta^{m1} - \beta^{m0}) - (\bar{X}^{f1} - \bar{X}^{f0})(\beta^{f1} - \beta^{f0})$$

- The sum of ΔE, ΔC, and ΔI is equal to the overall change in the group difference between time 0 and time 1.
- Primary interest lies in ΔE (endowments effect) and ΔC (coefficients effect). As usual, the interaction term ΔI is less straightforward to interpret.

Exercise 5

References

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