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A Dispersive Treatment of $K_{\ell 4}$ Decays

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$K_{\ell 4}$ decays are interesting for several reasons: They allow an accurate measurement of a combination of S-wave $\pi\pi$ scattering lengths, one form factor of the decay is connected to the chiral anomaly and the decay is the best source for the determination of some low energy constants of ChPT. We present a dispersive approach to $K_{\ell 4}$ decays, which takes rescattering effects fully into account. Some fits to NA48/2 and E865 measurements and results of the matching to ChPT are shown.

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1. Motivation

$K_{\ell 4}$, the semileptonic decay of a kaon into two pions and a lepton-neutrino pair, plays a crucial role in the context of low energy hadron physics, because it provides almost unique information about some of the $\mathcal{O}(p^4)$ low energy constants (LECs) of Chiral Perturbation Theory (ChPT), the effective low energy theory of QCD. The physical region of $K_{\ell 4}$ starts already at the $\pi\pi$ threshold, thus it happens at lower energies than e.g. elastic $K\pi$ scattering, which would give access to the same LECs. Since ChPT is an expansion in the masses and momenta, it is expected to converge better at lower energies and to describe better $K_{\ell 4}$ than $K\pi$ scattering.

Besides, as the hadronic final state contains two pions, $K_{\ell 4}$ is also one of the best sources of information on the $\pi\pi$ scattering lengths a_0^0 and a_0^2 [1].

On the experimental side, we are confronted with impressive precision from high statistics measurements. During the last decade, the process has been measured in the E865 experiment at BNL [2] and in the NA48/2 experiment at CERN [1]. Recently, the NA48/2 collaboration has published the results on the branching ratio and form factors of $K_{\ell 4}$, based on more than a million events [3].

Here, we present preliminary results of a new dispersive treatment of $K_{\ell 4}$ decays. Dispersion relations are an interesting tool to treat low energy hadronic processes. They are based on the very general principles of analyticity and unitarity. In the present analysis, we do not solve the exact dispersion relation for this process, but an approximate form, which follows if the contribution of D- and higher waves to the discontinuities are neglected. This approximation is valid up to and including $\mathcal{O}(p^6)$ in the chiral counting. The effects due to $\pi\pi$ and $K\pi$ rescattering in S- and P-wave are however resummed to all orders. We expect this to capture the most important contributions beyond $\mathcal{O}(p^6)$. The dispersion relation is parametrised by five subtraction constants. As soon as these constants have been fixed, the energy dependence is fully determined by the dispersion relation.

2. Dispersion Relation for $K_{\ell 4}$ Decays

2.1 Matrix Element and Form Factors

We consider the charged decay mode of $K_{\ell 4}$:

$$K^+(p) \to \pi^+(p_1)\pi^-(p_2)\ell^+(p_\ell)\nu_\ell(p_\nu), \tag{2.1}$$

where $\ell \in \{e,\mu\}$ is either an electron or a muon.

After integrating out the W boson, we end up with a Fermi type current-current interaction and the matrix element splits up into a leptonic times a hadronic part. The leptonic matrix element can be treated in a standard way. The hadronic matrix element exhibits the usual $V-A$ structure of weak interaction. Its Lorentz structure allows us to write the two contributions as

$$\langle \pi^+(p_1)\pi^-(p_2)|V_\mu(0)|K^+(p)\rangle = -\frac{H}{M_K^3}\varepsilon_{\mu\nu\rho\sigma}L^\nu P^\rho Q^\sigma, \tag{2.2}$$

$$\langle \pi^+(p_1)\pi^-(p_2)|A_\mu(0)|K^+(p)\rangle = -i\frac{1}{M_K}\left(P_\mu F + Q_\mu G + L_\mu R\right). \tag{2.3}$$

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In the electron mode (up to now the only one where experimental data is available), both S- and P-wave of a specific linear combination of the form factors F and G are accessible:

$$F_1(s,t,u) = XF(s,t,u) + (u-t)\frac{PL}{2X}G(s,t,u), \tag{2.4}$$

where s, t and u are the usual Mandelstam variables, $X = \frac{1}{2}\lambda^{1/2}(M_K^2, s, s_\ell)$, $PL = \frac{1}{2}(M_K^2 - s - s_\ell)$ and $s_\ell = (p_\ell + p_\nu)^2$. $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$ is the Källén triangle function.

2.2 Decomposition of the Form Factor

The form factor F_1 has the following analytic properties:

- There is a right-hand branch cut in the complex s-plane, starting at the $\pi\pi$-threshold.
- Analogously, in the t- and u-channel, right-hand cuts start at the $K\pi$-threshold.

Due to crossing, the right-hand cuts in the t- and u-channel show up in the s-channel for negative values of s. The situation is analogous for the other channels.

Based on fixed-t and fixed-u dispersion relations for the form factor, we can derive its decomposition into functions of a single variable. Such a decomposition has first been worked out for the $\pi\pi$ scattering amplitude [4] and later for $K\pi$ scattering [5].

The basic idea is to define functions that only contain the right-hand cut of each partial wave and to split up in this way all the discontinuities of the form factor. For instance, if we define the s-channel partial wave expansion as

$$F_1(s,t,u) = f_0(s) + f_1(s)\cos\theta_\pi + f_{l\geq 2}(s,t,u), \tag{2.5}$$

where θ_π is the s-channel scattering angle, we define a function with only the right-hand cut of the S-wave as follows:

$$M_0(s) := P(s) + \frac{s^4}{\pi}\int_{4M_\pi^2}^{\Lambda^2} \frac{\mathrm{Im} f_0(s')}{(s'-s-i\varepsilon)s'^4} ds', \tag{2.6}$$

where $P(s)$ is a subtraction polynomial. After defining similar functions that take care of the right-hand cuts of f_1 and the S- and P-waves in the crossed channels, all the discontinuities are divided into functions of a single variable.

This procedure, also known as 'reconstruction theorem', neglects on the one hand the imaginary parts of D- and higher waves, on the other hand the high energy tails of the dispersion integrals from Λ^2 to ∞. Both effects are of $\mathcal{O}(p^8)$ in the chiral counting. In order to simplify substantially the dispersion relation, we also neglect at the present preliminary stage the dependence on s_ℓ, which is experimentally small. (In order to describe the dependence on s_ℓ, we will need to consider a coupled system for the form factors F and G.)

Respecting isospin properties, we obtain the following decomposition of the form factor:

$$F_1(s,t,u) = M_0(s) + \frac{2}{3}N_0(t) + \frac{1}{3}R_0(t) + R_0(u) + (u-t)M_1(s) - \frac{2}{3}\Big[t(u-s) - \Delta_{K\pi}\Delta_{\ell\pi}\Big]N_1(t), \tag{2.7}$$

where $\Delta_{K\pi} = M_K^2 - M_\pi^2$ and $\Delta_{\ell\pi} = s_\ell - M_\pi^2$.

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2.3 Integral Equations

One should not try to solve directly the dispersion relation (2.6) since it may not uniquely determine the solution of the problem [6]. Noting that each of the functions M_0, ... satisfies an Omnès equation, we apply the solution to the inhomogeneous Omnès problem:

$$M_0(s) = \Omega_0^0(s)\left\{\tilde{P}(s) + \frac{s^3}{\pi}\int_{4M_\pi^2}^{\Lambda^2} \frac{\hat{M}_0(s')\sin\delta_0^0(s')}{|\Omega_0^0(s')|(s'-s-i\varepsilon)s'^3}ds'\right\}, \tag{2.8}$$

with a new subtraction polynomial $\tilde{P}(s)$ and the Omnès function

$$\Omega_0^0(s) := \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\delta_0^0(s')}{s'(s'-s-i\varepsilon)}ds'\right\}. \tag{2.9}$$

Similar relations hold for the other functions. Due to Watson's final state theorem, we can identify the phases δ_l^I (l – angular momentum, I – isospin) below the inelastic thresholds with the phase shifts of the elastic scattering. As an input to our equations, we therefore need the following phase shifts:

- δ_0^0, δ_1^1: elastic $\pi\pi$ scattering [7]
- $\delta_0^{1/2}$, $\delta_1^{1/2}$, $\delta_0^{3/2}$: elastic $K\pi$ scattering [8, 9]

The inhomogeneities in the Omnès problem are given by the differences of the functions M_0, ... and the corresponding partial wave, e.g. $\hat{M}_0(s) = f_0(s) - M_0(s)$. These 'hat functions' contain the left-hand cut of the partial wave and we compute them by projecting out the partial wave of the decomposed form factor (2.7). E.g. $\hat{M}_0(s)$ is then given as angular averages of N_0, N_1, etc.

We can now solve the dispersion relation for the form factor. We have parametrised the problem by the constants appearing in the subtraction polynomials (in total, only five independent subtraction constants are needed due to an ambiguity in the decomposition) and we use the elastic scattering phase shifts as inputs. The energy dependence is then fully determined by the dispersion relation.

We face a set of coupled integral equations: The functions $M_0(s)$, $M_1(s)$, ... are given as dispersive integrals involving the hat functions $\hat{M}_0(s)$, $\hat{M}_1(s)$, ..., whereas the hat functions are themselves defined as angular integrals over $M_0(s)$, $M_1(s)$, etc. This system can be solved by iteration. The problem is linear in the subtraction constants that have to be determined by a fit to data.

3. Preliminary Results

3.1 Fit to Data

We perform a fit of the dispersion relation to both, the E865 [2] and NA48/2 data sets [3]. The S-wave dominantly determines the three subtraction constants in M_0, the P-wave the two in M_1. Figure 1 shows the result of the combined fit for the two partial waves. The χ^2/dof of this fit is 1.73. This rather large value is a consequence of the small uncertainties in the S-wave of the relative form factor in the NA48/2 data.

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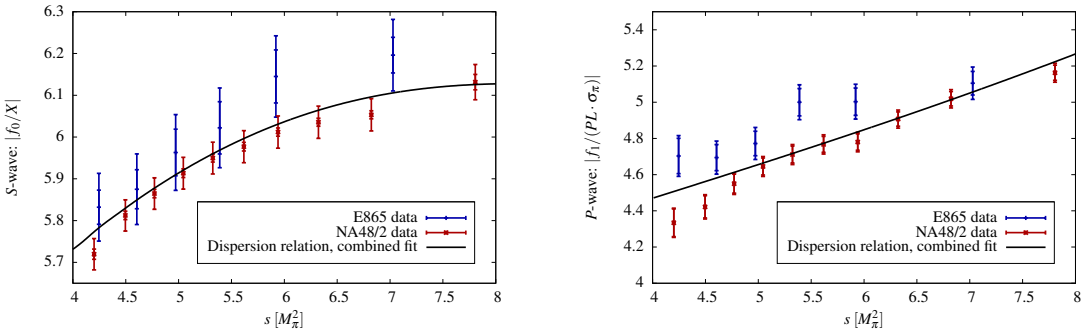

Figure 1: Fit of the S- and P-wave to both E865 [2] and NA48/2 [3] data. The error bars indicate separately the uncorrelated part of the uncertainties and the total errors (containing also the uncertainty from the normalisation).

3.2 Matching to ChPT

After having fixed the subtraction constants by fitting the data, we can match our dispersive representation to the ChPT result. In this matching procedure, we determine the LECs L_1^r, L_2^r and L_3^r.

The dispersion relation allows us to choose a convenient matching point that may lie outside the physical region. We choose the point $s = t - u = 0$ below the threshold, where ChPT should even converge better. In table 1, we show preliminary values of the low energy constants, resulting from a matching to the $\mathcal{O}(p^4)$ ChPT result. For comparison, we also quote the values of the global fit by Bijnens and Jemos [10] that takes as input the $K_{\ell 4}$ data from NA48/2 [1].

Table 1: Preliminary results for the LECs ($\mu = 770$ MeV).

	$10^3 L_1^r$	$10^3 L_2^r$	$10^3 L_3^r$
Dispersive treatment, fit to E865 [2]	0.42 ± 0.41	0.41 ± 0.34	-2.19 ± 1.41
Dispersive treatment, fit to NA48/2 [3]	0.60 ± 0.29	0.63 ± 0.28	-3.16 ± 1.19
Bijnens, Jemos, 'fit All' [10]	0.88 ± 0.09	0.61 ± 0.20	-3.04 ± 0.43

We find that our fit to NA48/2 agrees with this global fit. A large part of our uncertainties stems from the $\pi\pi$ scattering phase shifts and contains at present a conservative estimate of the systematics.

Our preliminary values are subject to change as we are going to perform a matching to $\mathcal{O}(p^6)$ ChPT and include the neglected s_ℓ dependence as well as isospin breaking corrections [11].

3.3 Summary

We have presented a dispersive representation of $K_{\ell 4}$ decays that provides a model independent parametrisation valid up to and including $\mathcal{O}(p^6)$. It includes a full summation of final state rescattering effects. It is parametrised by subtraction constants that we fix by fitting experimental data. The matching to ChPT can be performed below the physical threshold, where ChPT should converge better. Hence, we expect to find more reliable values for the LECs than with a pure ChPT treatment.

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